PHYSICS

Heat & Thermodynamics

Heat & Thermodynamics

IIT-JEE SYLLABUS

Thermal expansion of solids, liquids and gases; Calorimetry, latent heat; Heat conduction in one dimension; Elementary concepts of convection and radiational; Newton's law of cooling Ideal gas laws; Specific heats (C_v and C_p for monatomic and diatomic gases); Isothermal and adiabatic processes, bulk modulus of gases; Equivalence of heat and work; First law of thermodynamics and its applications (only for ideal gases). Blackbody radiation : absorptive and emissive powers; Kirchhoff's law, Wien's displacement law, Stefan's law.

Heat is a form of energy. This form of energy can be used in different walks of our life. Vehicles moving on the road use I.C. engines which utilise heat energy and this heat energy is converted into mechanical energy. All these engines follow some cycles. These cycles are called Thermodynamic cycles. Each Thermodynamic cycle again consists of number of processes. Efficiency of a Thermodynamic cycle depends on these processes. So it is important to study all these processes, which are elaborately dealt with in THERMODYNAMICS part of this module.

Heat is transitory in nature. It can move from one object to another or from one part of a body to another. This flow of heat can take place in any one of the three ways – conduction, convection and radiation. In HEAT TRANSFER we will study this aspect of heat.

We will also study about the expansion of solid on heating, which is the result of increase in kinetic energy of the molecules or atoms, the solid is consisting of.

::: Contents :::

Heat & Thermodynamics

Definition of Heat :

Heat is energy in transit which is transferred from one body to the other, due to difference in temperature, without any mechanical work involved.

1. Thermal Expansion:

Expansion due to increase in temperature.

Cause of thermal expansion : Molecules are held together by elastic forces and they vibrate with some constant mean distance between them.

As temperature increases, vibration energy of the constituent particles increases which results in increase in separation between the particles and hence there is thermal expansion.

Types of Thermal expansion:

Coefficient of expansion For temperature change Δt

(1) Linear

$\alpha = \lim_{\Delta t \to 0} \frac{1}{l_0} \frac{\Delta l}{\Delta t}$ change in length $\Delta \ell = \ell - \ell_0 = \ell_0 \alpha \Delta t$

(l_0 is initial length of the rod, where α is co-efficient of linear expansion)

(2) Superficial

$\beta = \lim_{\Delta t \to 0} \frac{1}{A_0} \frac{\Delta A}{\Delta t}$ change in Area $\Delta A = A - A_0 = A_0 \beta \Delta t$

(A_0 is initial area of the rod, where β is co-efficient of superficial expansion)

(3) Volume $\gamma = \lim_{\Delta t \to 0} \frac{1}{V_0} \frac{\Delta V}{\Delta t}$ change in volume $\Delta V = V - V_0 = V_0 \gamma \Delta t$

(V_0 is initial volume of the rod, where γ is co-efficient of volume expansion)

(4) For isotropic solid,

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha \text{ (say)}$$

So $\beta = 2\alpha$ and $\gamma = 3\alpha$

(5) For anisotropic solids,

$\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$

Here α_1, α_2 and α_3 are coefficients of linear expansion in X, Y and Z directions respectively.

Illustration1:

A copper and a tungsten plate having a thickness $\delta = 2\ mm$ each are riveted together so that at 0ºC they form a flat bimetallic plate. Find the average radius of curvature of this plate at $t = 200°C$. The coefficients of linear expansion for copper and tungsten are $\alpha_c = 1.7 \times 10^{-5}\ K^{-1}$ and $\alpha_t = 0.4 \times 10^{-5}\ K^{-1}$.

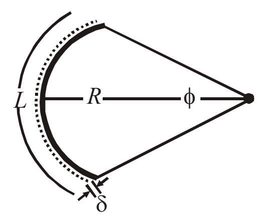
Solution : From figure,

$L = R\phi$ take logarithm and differentiate

$In\ L = In\,R + In\,\phi$

$$\frac{dL}{L} = \frac{dR}{R} \qquad \text{(as } \Delta L \text{ is very small)}$$

$$\frac{\Delta L}{L} = \frac{\Delta R}{R}$$



$$\Rightarrow \qquad \frac{L(\alpha_1 - \alpha_2)\Delta t}{L} = \frac{\delta}{R}$$

$$R = \frac{\delta}{(\alpha_1 - \alpha_2)\Delta t} = 0.769\,m.$$

Variation in Density:

With increase in temperature, volume increases, so density decreases and vice-versa.

$$d = \frac{d_0}{(1 + \gamma\Delta t)}$$

For solids, values of γ are generally small so we can write

$d = d_0(1 - \gamma\Delta t)$ (using binomial expansion)

Note: (i) γ for liquids are in order of 10^{-3}.

(ii) For water, density increases from 0 to 4ºC so γ is –ve (0 to 4ºC) and for 4ºC to higher temperature γ is +ve. At 4ºC density is maximum.

Illustration 2:

A sphere of diameter 7cm and mass 266.5 gm floats in a bath of liquid. As the temperature is raised, the sphere just begins to sink at a temperature of 35ºC. If the density of the liquid at 0ºC is $1.527\ gm/cm^3$, find the co-efficient of cubical expansion of the liquid. Neglect the expansion of the sphere.

Solution :

The sphere will sink in the liquid at 35ºC, when its density becomes equal to the density of liquid at 35ºC.

The density of sphere, $\rho_{35} = \dfrac{266.5}{\frac{4}{3} \times \left(\frac{22}{7}\right) \times \left(\frac{7}{2}\right)^3}$

$\rho_{35} = 1.483\,gm/cm^3$

Now, $\rho_0 = \rho_{35}[1 + \gamma\Delta T]$

$1.527 = 1.483[1 + \gamma \times 35]$

$1.029 = 1 + \gamma \times 35 \quad \Rightarrow \quad \gamma = \dfrac{1.029 - 1}{35} = 0.00083/°C.$

Apparent Expansion of Liquid in a container:

Initially container was completely filled with liquid. Temperature is increased by Δt

Change in volume of liquid $V_L = V_0(1+\gamma_L\,\Delta t)$

Change in volume of container $V_C = V_0(1+\gamma_C\,\Delta t)$

So, overflow volume of liquid relative to container $\Delta V = V_L - V_C$

$\Delta V = V_0(\gamma_L - \gamma_C)\Delta t$

So coefficient of apparent expansion of liquid w.r.t. container

$\gamma_{\text{apparent}} = \gamma_L - \gamma_C$

Expansion In Enclosed Volume:

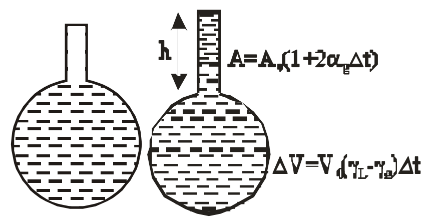
Increase in height of liquid level in tube when bulb was initially completely filled :

$$h = \frac{\text{apparent change in volume of liquid}}{\text{area of tube}}$$

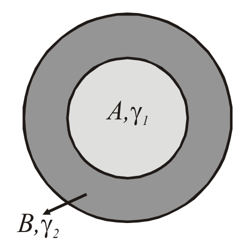
$$h = \frac{V_0(\gamma_L - \gamma_g)\Delta t}{A_0(1+2\alpha_g\Delta t)};$$

$\gamma_{\text{apparent}} = \gamma_L - \gamma_g$

Initially   Finally



A sphere of metal $A(\gamma_1)$ just inside a spherical shell of metal $B(\gamma_2)$ at any temperature *t*. If the temperature of metals is increased and $\gamma_1 > \gamma_2$ then volume stress will be developed (compressive in metal A and tensile in metal B). But if $(\gamma_1 < \gamma_2)$ the free space will be created between metals.



With same amount of change in temp. the expansion of radius will be same $R = R_0(1+\alpha\Delta t)$

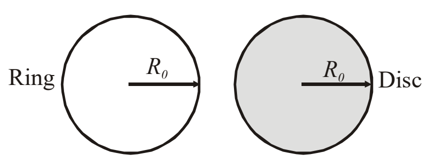


Illustration 3:

A one litre glass flask contains some mercury. It is found that at different temperature the volume of air inside the flask remains the same. What is the volume of mercury in this flask if coefficient of linear expansion of glass is $9\times10^{-6}/°C$ *while volume expansion of mercury is* $1.8\times10^{-4}/C°$ *?*

Solution:

If V is the volume of flask, V_L of mercury and V_A of air in it,

$$V = V_L + V_A$$

Now as with change in temperature volume of air remains constant, the expansion of mercury will be equal to that of the whole flask i.e.,

$$\Delta V = \Delta V_L$$

or $\quad V\gamma_G\Delta\theta = V_L\gamma_L\Delta\theta \qquad$ [as $\Delta V = V\gamma\Delta\theta$]

Here $\quad V = 1\text{ litre} = 1000\text{ cc}$ and $\gamma_G = 3\alpha_G = 27\times10^{-6}/°C$

So $\quad V_L = (1000\times27\times10^{-6}/1.8\times10^{-4}) = 150\text{ cc}.$

Thermal Stress:

A rod of length l_0 clamped between two fixed walls

For Δt change in temperature

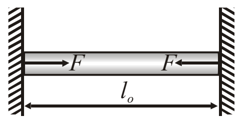
stress $=\dfrac{F}{A}$ (area assumed to be constant)

strain $=\dfrac{\Delta l}{l_0}$

$\Delta l = l_0 \alpha \Delta t$

so, $\quad Y=\dfrac{F/A}{\Delta l/l_0}=\dfrac{Fl_0}{A\Delta l}=\dfrac{F}{A\alpha\Delta t}$

or, $\quad F = YA\alpha\ |\Delta t|$



or, $\quad F = YA\dfrac{\Delta l}{l_0}=\left(\dfrac{YA}{l_0}\right)\Delta l$

Energy stored in rod $E=\dfrac{1}{2}\times$ stress $\times$ strain $\times$ volume

For increase in temperature stress will be compressive and for decrease in temperatue stress will be tensile

Illustration 4:

A light steel wire of length ℓ and area of cross-section A is hanging vertically downward with a ceiling. It will cool to the room temperature (30ºC) from the initial temperature 100ºC. Calculate the weight which should be attached at its lower end such that its length remains same. Young's Modulus of steel is Y and coefficient of linear expansion is α.

Solution :

Thermal Stress due to temperature change $=\alpha Y(100-30)$

Stress due to weight $=W/A$

Since no change in length take place

$\therefore \quad \dfrac{W}{A}=\alpha Y.(100-30)$

$W = 70\ A\alpha Y$.

Variation of Time Period of Pendulum Clocks:

$$T_0 = 2\pi\sqrt{\frac{l_0}{g}}$$

If temperature is increased by Δt, $\quad T=2\pi\sqrt{\dfrac{l_0(1+\alpha\Delta t)}{g}}$

$T=2\pi\sqrt{\dfrac{l_0}{g}}\ (1+\dfrac{\alpha}{2}\Delta t)$ (by using Binomial expansion)

$T=T_0(1+\dfrac{\alpha}{2}\Delta t) \qquad \Rightarrow \qquad T-T_0=T_0\dfrac{\alpha}{2}\Delta t$

$\dfrac{\Delta T}{T_0}=\dfrac{1}{2}(\alpha\Delta t)$ sec/sec $\quad \Delta T=$ increase in time period

Illustration 5.

A pendulum clock with a pendulum made of Invar $(\alpha = 0.7 \times 10^{-6} / C^{o})$ *has a period of 0.5 s and is accurate at 25º C. If the clock is used in a country where the temperature averages 35º C, what correction is necessary at the end of a month (30 days) to the time given by the clock?*

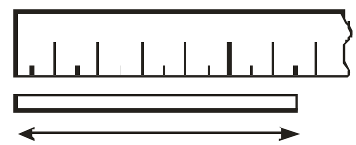
Solution:

In time interval t, the clock will become slow (or will lose time) by

$$\Delta t = \frac{1}{2}\alpha t \Delta\theta$$

So, $$\Delta t = \frac{1}{2} \times (7 \times 10^{-7}) \times (30 \times 86400) \times (35 - 25) = 9.1 s.$$

Measurement of Length by Metallic Scale:



$$l_{\text{actual}} = l_{\text{measure}} \times l_{\text{one division}}$$

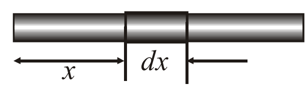
and $$l_{\text{one division}} = (1\text{cm})\ (1 + \alpha\Delta t)$$

For variable :

(1) variation of α with distance

Let $\alpha = ax + b$

Total expansion $= \int (\text{expansion of length dx}) = \int_0^l (ax + b) dx \Delta t$



(2) variation of α with temperature

Let $\alpha = f(T)$

$$\Delta l = \int_{T_1}^{T_2} l_0 f(T) dT$$

Caution : If α is in ºC then put T_1 and T_2 in ºC

Similarly if α is in ºK, put T_1 and T_2 in K.

Practice Problems # 01

1. A surveyor's 30m stell tape is correct at a temperature of 20ºC. The distance between two points, as measured by this tape on a day when the temperature is 35ºC, is 26m. What is the true distance between the points ? ($\alpha_{\text{steel}} = 1.2 \times 10^{-5}$ /ºC)
2. A clock with a brass pendulum shaft keeps correct time set a certain tempeature.
   (a) How closely must the temperature be controlled if the clock is not to gain or lose more than 1 sec a day ? Does the answer depend on the period of the pendulum?
   (b) Will an increase of temperature cause the clock to gain or lose? ($\alpha_{\text{brass}} = 2 \times 10^{-5}$/ºC)
3. A steel ring of 3.00 inches inside diameter at 20ºC is to be heated and slipped over a brass shift measuring 3.003 inches in diameter at 20ºC. To what temperature should ring be heated?
4. A pendulum clock loses 12sec a day if the temperature is 40ºC and goes fast by 4 sec. a day if the temperature is 20ºC. Find the temperature at which the clock will show correct time and the coefficient of linear expansion of the metal of the pendulum shaft.
5. A copper and a tungsten plate having a thickness δ = 2mm each are riveted together so that at 0ºC they form a flat bimetallic plate. Find the average radius of the curvature of this plate at t = 200ºC. The coefficient of linear expansion for copper and tungsten are $\alpha_{Cu} = 1.7 \times 10^{-5}$ /K and $\alpha_W = 0.4 \times 10^{-5}$ /K.

6. A glass flask whose volume is exactly $1000cm^3$ at 0ºC is filled level full of mercury at this temperature. When the flask and mercury are heated to 100ºC, $15.2cm^3$ of mercury overflow. The coefficient of cubical expansion for Hg is 1.82×10^{-2}/ºC. Compute the coefficient of linear expansion of glass.

7. A $250cm^3$ glass bottle is completely filled with water at 50ºC. The bottle and water are heated to 60ºC. How much water runs over if :

   (a) the expansion of the bottle is neglected :

   (b) the expansion of the bottle is included ? Given the coefficient of areal expansion of glass $\beta = 1.2 \times 10^{-5}$ /ºC and $\gamma_{water} = 60 \times 10^{-5}$ /ºC

8. A sinker of weight W_0 has an apparent weight W_1 when placed in a liquid at a temperature T_1 and W_2 when weighed in the same liquid at a temperature T_2. The coefficient of cubical expansion of the material of the sinker is β. What is the coefficient of volume expansion of the liquid?

9. A vessel is filled completely with 500gm of water and 1000gm of mercury. When 21200 cal of heat is given to it, water of mass 3.52gm overflows. Calculate the coefficient of volume expansion of mercury. The expansion of the vessel may be neglected. Coefficient of volume expansion of water $= 1.5 \times 10^{-4}$ /ºC, density of mercury = 13.6 g/cc, density of water = 1g/cc and specific heat of mercury = 0.03 cal/g/ºC .

10. A U tube contains mercury. The left limb of tube is maintained at a temperature of T_1ºC and the right limb at a temperature of T_2ºC. The height of mercury columns in the left and right limbs are h_1 and h_2 respectively. Find the coefficient of volume expansion of Hg. Neglect the expansion of tube.

2. Calorimetry:

Calorie:

The amount of heat needed to increase the temperature of 1 gm of water from 14.5ºC to 15.5ºC at STP is known as 1 calorie

Specific Heat:

It is heat required to raise temperature by 1º C or 1º K for unit mass of the body.

$$dQ = mc\,dT$$

$$Q = m\int_{T_1}^{T_2} c\,dT$$ (be careful about unit of temperature, use units according to the given units of c)

Latent Heat:

The amount of heat required to change one phase of 1 gm of a substance to another phase.

$Q = mL$ — L = latent heat of substance in cal/gm or in Kcal/kg.

$L_{ice} = 80\,cal/gm$ – Specific latent heat of fusion of ice

$L_{steam} = 540$ cal/gm – Specific latent heat of vaporization of water

Molar Heat Capacity:

If instead of unit mass we consider one mole of a substance, the heat required to change the temperature of one mole of a substance through 1 ºC (or K) is called molar heat capacity or molar specific heat and is represented by C. If the molecular weight of a substance is M:

$$C = Mc = \frac{Q}{\mu \Delta T} \qquad \left[\text{as } c = \frac{Q}{m\Delta T} \text{ and } \mu = \frac{m}{M}\right]$$

Its SI units are (J/mol K) while dimensions $[ML^2T^{-2}\theta^{-1}\mu^{-1}]$

Thermal-capacity :

If instead of unit mass we consider the whole body, (of mass m), the heat required to raise the temperature of a given body by 1 ºC is called its thermal capacity , i.e.,

Thermal capacity $= mc = \mu C = (Q/\Delta T)$

Thermal capacity of a body depends on the mass and nature of body. It has units (J/K) or cal/ºC and dimensions $[ML^2T^{-2}\theta^{-1}]$.

Water-Equivalent :

If thermal capacity of a body is expressed in terms of mass of water it is called water-equivalent of the body, i.e., water-equivalent of a body is the mass of water which when given same amount of heat as to the body, changes the temperature of water through same range as that of the body, i.e.,

$W = (m \times c) g$

The unit of water equivalent W is g while its dimension [M].

Principle of Calorimetry :

When two bodies (one being solid and other liquid or both being liquid) at different temperature are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that :

Heat lost = Heat gained,

i.e. principle of calorimetry represents the law of conservation of heat energy.

Illuatration 6.

The temperature of equal masses of three different liquids A, B and C are 12ºC, 19º C and 28º C respectively. The temperature when A and B are mixed is 16º C and when B and C are mixed is 23º C. What would be the temperature when A and C are mixed?

Solution: In accordance with principle of calorimetry :

When A and B are mixed

$$mc_A(16-12) = mc_B(19-16)$$

$$\Rightarrow \quad c_A = (3/4)c_B$$

and when B and C are mixed

$$mc_B(23-19) = mc_C(28-23)$$

$$\Rightarrow \quad c_C = (4/5)c_B$$

Now when A and C are mixed if T is the common temperature of mixture:

$$mc_A(T-12) = mc_C(28-T)$$

Substituting c_A and c_C from above,

$$(3/4)(T-12) = (4/5)(28-T)$$

which on solving gives,

$$T = 20.25^\circ C \cdot$$

Illustration 7.

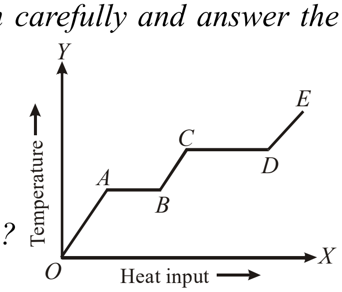
A solid material is supplied with heat at a constant rate. The temperature of the material is changing with the heat input as shown in figure. Study the graph carefully and answer the following questions :

(i) What do the horizontal regions AB and CD represent?

(ii) If $CD = 2\ BA$, what do you infer?

(iii) What does slope DE represent?

(iv) The slope of $OA >$ the slope of BC. What does this indicate?



Solution :

(i) The horizontal portions AB and CD of the graph represent the change of phase.

The portion AB represents the change of phase from solid to liquid at constant temperature and the portion CD represents the change of phase from liquid to vapour at constant temperature or the portion CD represents the latent heat of vaporization.

(ii) $CD = 2AB$,

i.e., latent heat of vaporization is twice the latent heat of fusion.

(iii) The slope DE is equal to dT/dQ for vapour, i.e., this gives the rate of increase of temperature of vapour with heat input.

$$\therefore \text{ Slope of } DE \propto \frac{1}{\text{Specific heat of the vapour}}$$

$$\text{or, Specific heat of vapour} \propto \frac{1}{\text{Slope of } DE}$$

(iv) Slope $OA >$ slope BC

The slope OA represents that

$$\text{Specific heat of solid} \propto \frac{1}{\text{Slope of } OA}$$

Now slope $OA >$ slope BC, represents that specific heat of the liquid is more than that of the solid.

Practice Problems # 02

1. The heat required to increase the temperature of 10 kg water by 10º C is :

(A) 20 kcal (B) 100 kcal (C) 10 kcal (D) 1 kcal

2. When 400 J of heat are added to a 0.1 kg sample of metal, its temperature increases by 20ºC. The specific heat of the metal is : (in J/kgºC)

(A) 100 (B) 200 (C) 300 (D) 50

3. 420 J of energy is supplied to 20 gm of water. The rise of temperature is :

(A) 10ºC (B) 7ºC (C) 100ºC (D) 5ºC

4. The specific heat of metal at low temperature varies as $S = aT^3$ where a is a constant and T is the absolute temperature. The heat energy needed to raise unit mass of the metal from T = 1 K and T = 2K is :

(A) 3a (B) 15a/4 (C) 2a/3 (D) 12a/5

5. The ratio of densities of two bodies is 3 : 4 and the ratio of specific heat is 4 : 3. The ratio of their thermal capacities for unit volume is :

(A) 3 : 4 (B) 4 : 3 (C) 1 : 1 (D) 9 : 16

6. The density of a material A is 1500 kg/m^3 and that of another material is 2000 kg/m^3. It is found that the heat capacity of 8 volumes of A is equal to heat capacity of 12 volume of B. The ratio of specific heats of A and B will be :

(A) 1 : 2 (B) 3 : 1 (C) 3 : 2 (D) 2 : 1

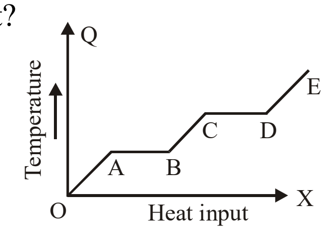
7. 200 gm water is filled in a calorimeter of negligible heat capacity. It is heated till its temperature is increased by 20ºC. The heat supplied to water is :

(A) 1000 cal (B) 4000 cal (C) 2000 cal (D) 3000 cal

8. Heat released by 1 kg steam at 150º C, if it is converted into 1 kg water at 50ºC is :

(A) 315 kcal (B) 115 kcal (C) 150 kcal (D) 615 kcal

9. A solid material is supplied with heat at a constant rate. The temperature of material is changing with heat input as shown in the figure. What does slope DE represent?



(A) Latent heat of liquid
(B) Latent heat of vapour
(C) Heat capacity of vapour
(D) Inverse of heat capacity of vapour.

10. How much heat is required to convert 8.0 gm of ice at –15ºC to steam at 100ºC ? (Given S_{ice} = 0.53 cal/gºC, L_f = 80 cal/g and L_v = 539 cal/g and S_{water} = 1cal/gºC)

Practice Problems # 03

1. A block of mass 2.5kg is heated to temperature of 500ºC and placed on a large ice block. What is the maximum amount of ice that can melt (approx). Specific heat for the body = 0.1Cal/gmºC.

(A) 1kg (B) 1.5kg (C) 2kg (D) 2.5 kg

2. 10gm of ice at 0ºC is kept in a calorimeter of water equivalent 10gm. How much heat should be supplied to the apparatus to evaporate the water thus formed? (Neglect loss of heat)

(A) 6200 cal (B) 7200 cal (C) 13600 cal (D) 8200 cal

3. Heat is being supplied at constant rate to a sphere of ice which is melting at the rate of 0.1gm/sec. It melts completely in 100sec. The rate of rise of temperature thereafter will be (assume no loss of heat)

(A) 0.8 ºC/sec (B) 5.4 ºC/sec (C) 3.6 ºC/sec (D) will change with time

4. 1kg of ice at –10ºC is mixed with 4.4kg of water at 30ºC. The final temperature of mixture is (specific heat of ice is 2100 J/kg/K)

(A) 2.3ºC (B) 4.4C (C) 5.3ºC (D) 8.7ºC

5. Steam at 100ºC is added slowly to 1400gm of water at 16ºC until the temperature of water is raised to 80ºC. The mass of steam required to do this (L_v = 540 cal/gm)

(A) 160 gm (B) 125mg (C) 250gm (D) 320gm

6. A 2100 W continuous flow geyser (instant geyser) has water inlet temperature = 10ºC while the water flows out at the rate of 20g/sec. The outlet temperature of water must be about

(A) 20ºC (B) 30ºC (C) 35ºC (D) 40ºC

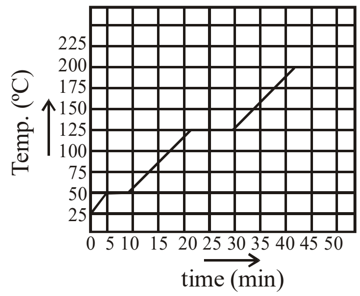
7. The temperature of equal masses of three different liquids A, B, C and 12ºC, 19ºC and 28ºC respectively. The temperature when A and B are mixed 16ºC while when B and C are mixed, it is 23ºC. The temperature when A and C are mixted is .

(A) 16.5ºC (B) 20.26ºC (C) 10.36ºC (D) none of these

8. Two identical conducting rods are first connected independently to two vessels, one containing water at 100ºC and the other containing ice at 0ºC. In the second case, the rods are joined end to end and connected to the same vessels. Let q_1 and q_2 g/s be the rate of melting of ice in the two cases respectively. The ratio q_1/q_2 is

(A) $\frac{1}{2}$ (B) $\frac{2}{1}$ (C) $\frac{4}{1}$ (D) $\frac{1}{4}$

9. The graph shown in figure represent change in the temperature of 5kg of a substance at it absorbs heat at a costant rate of 42kJ min^{-1}. The latent heat of vaporization of the substance is



(A) 630 kJ kg^{-1} (B) 126 kJ kg^{-1} (C) 84 kJ kg^{-1} (D) 12.6 kJ kg^{-1}

10. Calorie is defined as the amount of heat required to raise temperature of 1g of water by 1ºC and it is defined under which of the following conditions?

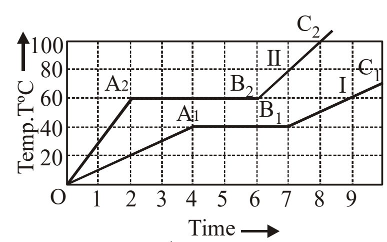
(A) from 14.5ºC to 15.5ºC at 760 mm of Hg (B) from 98.5ºC to 99.5ºC at 760 mm of Hg
(C) from 13.5ºC to 14.5ºC at 76 mm of Hg (D) from 3.5ºC to 4.5ºC at 76 mm of Hg

Practice Problems # 04

1. An ice cube of mass 0.1 kg at 0ºC is placed in an isolated container which is at 227ºC. The specific heat S of the container varies with temperature T according to the empirical relation S = A + BT, where A = 100 cal/kg-K and B = 2×10^{-2} cal/kg-K^2. If the final temperature of the container is 27ºC, determine the mass of the container. (Latent heat of fusion of water = 8×10^4 cal kg,sp. heat of water = 10^3 cal/kg-K).

2. The specific heat of a substance varies with temperature T as $C = AT^2 + BT$ cal/gºC, with temperature in degree kelvin. Calculate the amount of heat required to heat 50g of substance from 27ºC to 57ºC. Also find the time taken if the heat is supplied by a heater of resistance 200 ohm operating on 220V. (Given A = 2.5×10^{-3} cal/g(ºC)^3 ; B = 12×10^{-2}cal/g(ºC)^2.

3. Two bodies of equal mass m are heated at a uniform rate under identical conditions. Their change in temperatures are shown graphically in figure.



(a) What are their melting points ?
(b) What is the ratio of their latent heats ?
(c) What is the ratio of their specific heats ?

4. A mixture of 250 g of water and 200g of ice at 0ºC is kept in a calorimeter which has water equivalent of 50g. If 200 g of steam at 100ºC is passed through the mixture, calculate the final temperature and weight of content of the calorimeter. Given that specific heat of ice and water are 0.5cal/gºC and 1cal/gºC while latent heat 80cal/g and 540 cal/g respectively.

5. A lump of 0.10 kg of ice at –10ºC put in 0.15kg of water at 20ºC. How much water and ice will be found in the mixture when it has reached in thermal equilibrium? (specific heat of ice = 0.50 kcal/kg while its latent heat 80 kcal/kg)

6. 5g of water at 30ºC and 5g of ice at –20ºC are mixed together in a calorimeter. Find the final temperature of the mixture. Assume water equivalent of calorimeter to be negligible,sp. heat of ice and water are 0.5cal/gºC and 1cal/gºC, and latent heat of ice is 80 cal/g.

7. How many grams of ice at –14ºC is needed to cool 200g of water from 25ºC to 10ºC? [c_{ice} = 0.5cal/gºC and L_F for ice = 80 cal/g]

8. Determine the final result when 200gm of water and 20gm of ice at 0ºC are in a calorimeter having a water equivalent of 30gm and 50gm of steam is passed into it at 100ºC.

9. What will be the final temperature, when 150gm of ice at 0ºC is mixed with 300gm of water at 50ºC. Specific heat of water = 1 cal/gm/ºC. Latent heat of fusion of ice = 80 cal/gm.

10. In a calorimeter (water equivalent = 40gm) are 200gm of water and 50gm of ice all at 0ºC. Into this is poured 30gm of water at 90ºC. What will be the final condition of the system.

3. Laws of Thermodynamics :

Zeroth Law of Thermodynamics:

If two bodies A and B are in thermal equilibrium and A and C are also in thermal equilibrium, then B and C are also in thermal equilibrium.

Prevost Theory of Exchanges:

According to this theory, every body is continuously emitting radiant energy in all directions at a rate depending only on the nature of its surface and its temperature and it is absorbing radiant energy from all surrounding bodies at a rate depending on its surface and the temperature of the surrounding bodies.

Mechanical Equivalent of Heat:

Whenever mechanical work is transformed into heat or heat into mechanical work, there is a constant ratio between the work and the amount of heat. This ratio is called "mechanical equivalent of heat" and is denoted by J . Thus, if W be the amount of work done and Q the amount of heat produced, we have

$$\frac{W}{Q} = J, \qquad W = JQ$$

If $Q = 1$ unit then $J = W$. Therefore, J is numerically equal to the mechanical work required to produce one unit of heat.

First law of Thermodynamics :

It is the consequence of conservation of energy for gaseous system.

Heat supplied to the gas = Increase in internal energy + work done by the gas.

$Q = \Delta U + W$ — $Q = $+ve $\Rightarrow$ heat is supplied to the gas

in differential form $dQ = dU + dW$ — $Q = $–ve $\Rightarrow$ heat is taken out from the gas

and $dQ = nCdT$ — C = molar specific heat

$C = C_P$ (constant pressure); — $C = C_V$ =(constant volume)

and $dU = d[(f/2)nRT]$ — f = degree of freedom

$dU = (f/2)nRdT$ — dU = +ve for increase in temperature

dU = –ve for decrease in temperature

$$dW = \int_{V_1}^{V_2} PdV$$ (P = pressure of the gas of which work is to be calculated)

W = +ve for work done by gas (in expansion of gas)

W = –ve for work done by gas (in contraction of gas)

Process	**C**	**Monoatomic**	**Diatomic**	**Polyatomic**
V=constant	$C_V = (f/2)R$	(3/2) R	(5/2) R	3R
P=constant	$C_P = \frac{f+2}{2}R$	(5/2) R	(7/2) R	4R

Mayor's Relation $C_P = C_V + R$

Note: C of a gas depends on the process of that gas, (which is infinite in Isothermal Process).

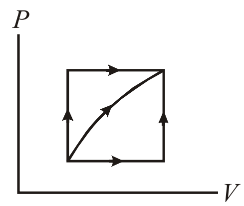
Ratio of specific heat of gases : $\gamma = \dfrac{C_P}{C_V} = \dfrac{f+2}{f}$

Monoatomic	**5/3=1.67**
Diatomic	**7/5=1.4**
Polyatomic	**4/3=1.33**

And $f = \dfrac{2}{\gamma - 1}$; $C_V = \dfrac{R}{\gamma - 1}$; $C_P = \dfrac{\gamma R}{\gamma - 1}$.

Indicator Diagram:

This is a graph between pressure and volume of a system under going operation,



(1) Every point of Indicator Diagram represents a unique state (P, V, T) of gases.

(2) Every curve on Indicator Diagram represents a unique process.

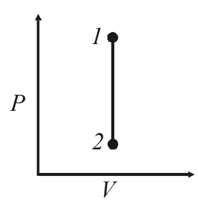
Isochoric Process (V = constant)

$dV = 0 \Rightarrow dW = 0$

By First Law of Thermodynamic

$dQ = dU = nC_V dT$

$$Q = \int_{T_1}^{T_2} nC_V dT = nC_V (T_2 - T_1)$$



Isochoric

Isobaric Process (P = constant)

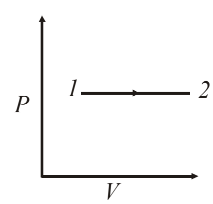
$dP = 0$

By First Law of Thermodynamics

$dQ = dU + dW$

$$nC_p(T_2 - T_1) = \left(\frac{f}{2}\right) nR(T_2 - T_1) + nR(T_2 - T_1)$$

$W = nR(T_2 - T_1)$



Isobaric

* Be careful if $\Delta V = 0$ then not necessarily an Isochoric Process.
* If $\Delta P = 0$ then not necessarily an Isobaric Process.

Isothermal Process (T = constant):

$dU = 0$ $(\because\ dT = 0)$

$PV = K$

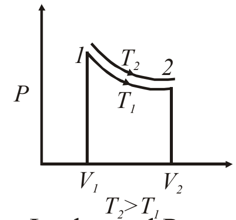
By First Law of Thermodynamics

$$\int dQ = \int dW$$

$$\Rightarrow \quad \int dQ = \int PdV$$

$$Q = W = (nRT)\int_{V_1}^{V_2} dV/V$$

$$W = nRT\ln\frac{V_2}{V_1} = nRT\ \ln\frac{P_1}{P_2}.$$



Isothermal Process

Adiabatic Process :

dQ = 0 but if $\Delta Q = 0$, it is not necessarily adiabatic.

$dW = -dU$ By First Law of Thermodynamics

$$W = -\int_{T_1}^{T_2}\frac{nRdT}{\gamma-1} = \frac{nR(T_1-T_2)}{\gamma-1} = \frac{P_1V_1-P_2V_2}{\gamma-1}$$

How to get the process Equation for adiabatic

(i) First Law of Thermodynamics with process condition

$$dU = -dW = \frac{nRdT}{\gamma-1} \qquad \ldots\text{(i)}$$

(ii) Differential form of gas law

$$d(PV) = d(nRT)$$

$$PdV + VdP = nRdT$$

But $dW = PdV = \dfrac{-nRdT}{\gamma-1}$

So $PdV + VdP = -(\gamma-1)PdV \qquad \ldots\text{(ii)}$

$$VdP = -(\gamma PdV)$$

$$\frac{dP}{P} = -\gamma\frac{dV}{V}$$

$$\ln P = -\gamma\ln V + \ln C$$

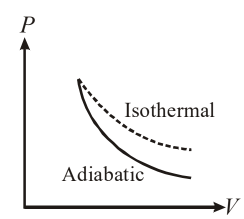
$$PV^{\gamma} = \text{Const.}$$

$$TV^{\gamma-1} = \text{Const.}$$

$$T^{\gamma}P^{1-\gamma} = \text{Const.}$$

For Adiabatic Process $PV^{\gamma} =$ constant

$$\left|\frac{dP}{dV}\right|_{\text{adiabatic}} = \gamma\left|\frac{dP}{dV}\right|_{\text{isothermal}}$$



Slope of adiabatic curve is more in magnitude in comparison to the slope of the isothermal curve.

Bulk Modulus of Gases:

$$\beta = -\frac{\Delta P}{(\Delta V)/V} = -V\frac{\Delta P}{\Delta V}$$

Isothermal bulk modulus of Elasticity

$$E_T = -\frac{dP}{dV/V} = -V\left(\frac{\partial P}{\partial V}\right)_T$$

Adiabatic bulk modulus of Elasticity

$$E_{adia} = -\frac{dP}{dV/V} = -V\left(\frac{\partial P}{\partial V}\right)_{adia}$$

$$\frac{E_{adia}}{E_T} = \gamma.$$

Molar Specific Heat For Polytropic Process

$$PV^n = K$$

molar heat capacity of polytropic process $C = \frac{R}{\gamma-1} + \frac{R}{1-n} = \frac{R}{\gamma-1} - \frac{R}{n-1}$

So C is constant for polytropic process

Illustration 8:

Two moles of diatomic ideal gas is taken through the process $PT =$ const. Its temperature is increased from $T_0 K$ to $2T_0 K$. Find work done by the system ?

Solution : $W = \int PdV$

Here $PT = P_1T_1 = P_2T_2 = c$ (Constant)

$\therefore$ $PT = c$

$$P.\frac{PV}{nR} = c$$

$\therefore$ $P^2V = ncR$

$$P = \sqrt{\frac{ncR}{V}}$$

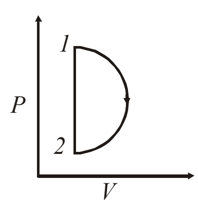
$\therefore$ $\int PdV = \sqrt{ncR}\int_{V_1}^{V_2}\frac{1}{\sqrt{V}}dV$

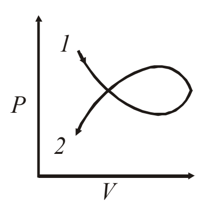
$$= \sqrt{ncR}\left[2\left(\sqrt{V_2} - \sqrt{V_1}\right)\right]$$

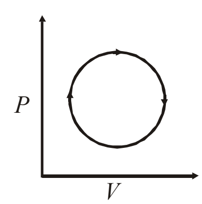
$$= 2[\sqrt{nR.P_2T_2V_2} - \sqrt{nRP_1T_1V_1}\ ] = 2[\sqrt{nRT_2(nRT_2)} - \sqrt{nRT_1(nRT_1)}$$

$$= 2nR(T_2 - T_1) = 4RT_0.$$

Work done on gas in some process :







$\Delta V = 0$ but $V \neq 0$
work done $= +$ ve

ΔW can be zero
but $dW \neq 0$ so work is done

For clockwise $\Delta W = +$ ve
For anticlockwise $\Delta W = -$ ve

Work done is least for monoatomic gas in expansion :

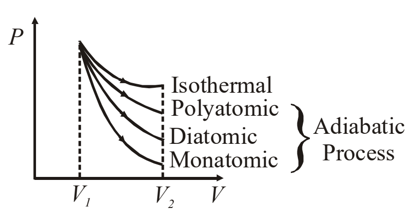


Illustration 9:

In a given gas during a process one third of heat supplied is used to raise internal energy of gas. Find molar specific heat of the gas and their process.

Solution: Heat supplied = $nCdT$ C = molar specific heat

$$\frac{nCdT}{3} = \frac{f}{2}nRdT \Rightarrow C = \frac{3fR}{2} = \frac{3R}{\gamma - 1} \left(\because \frac{dQ}{3} = dU\right).$$

Illustration 10:

An ideal gas is taken through a process in which the pressure and the volume are changed according to the equation P = KV. Show that the molar heat capacity of the gas for the process is given by $C = C_V + R/2$.

Solution : PV = nRT ..... (i)

P = KV ..... (ii)

From (i) and (ii),

$KV^2 = nRT$

Differentiating

2 KVdV = nRdT PdV = nRdT/2

dQ = dU + PdV nCdT = $nC_V dT$ + nRdT/2

$C = C_V + R/2$

Illustration 11:

An ideal gas, whose adiabatic exponent is equal to γ, is expanded so that the amount of heat transferred to the gas is equal to the decrease of internal energy. Find

(a) The molar heat capacity of the gas in this process,

(b) The equation of the process in the variables T, V;

(c) The work performed by one mole of the gas when its volume increases η times if the initial temperature of the gas is T_0.

Solution : ***(a)*** $\Delta Q = U_1 - U_2 = -(U_2 - U_1) = -\Delta U$

(as given in the problem)

so $dQ = -dU = -nC_V\,dT = -n\dfrac{R}{\gamma - 1}dT$

Hence, molar heat capacity $C = \dfrac{dQ}{ndT} = -\dfrac{R}{\gamma - 1}$

(b) $dQ = dU + dW$

For this problem

$\Rightarrow$ $-dU = dU + dW$

$\Rightarrow$ $-2dU = dW$

$\Rightarrow$ $-2nC_V dT = pdV$

$\Rightarrow$ $-2n\dfrac{R}{\gamma - 1}dT = \dfrac{nRT}{V}dV$

$$\Rightarrow \quad \frac{dT}{T} + \frac{(\gamma-1)}{2}\frac{dV}{V} = 0$$

$$\Rightarrow \quad TV^{(\gamma-1)/2} = \text{constant}$$

(c) $dW = -2dU$

$$\Rightarrow \quad W = -2\int dU = -2\Delta U$$

$$= -2C_V(T - T_0) = -2C_V T_0\left(\frac{T}{T_0} - 1\right) = 2C_V T_0\left(1 - \frac{T}{T_0}\right)$$

Since $TV^{(\gamma-1)/2} = \text{constant} = T_0 V_0^{(\gamma-1)/2}$

$$\Rightarrow \quad \frac{T}{T_0} = \left(\frac{V_0}{V}\right)^{(\gamma-1)/2} = \left(\frac{1}{\eta}\right)^{(\gamma-1)/2}$$

so, $W = 2C_V T_0\left(1 - \frac{T}{T_0}\right) = 2C_V T_0\left(1 - \frac{1}{\eta^{(\gamma-1)/2}}\right)$

$$= \frac{2RT_0[1 - 1/\eta^{(\gamma-1)/2}]}{(\gamma-1)}.$$

Efficiency of a Cyclic Process

So $\Delta U = 0 \Rightarrow$ no rise in internal energy

$\Delta Q = \Delta W$

Efficiency $\eta = \dfrac{\text{work done by gas}}{\text{heat input}}$

$$\eta = \frac{W}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

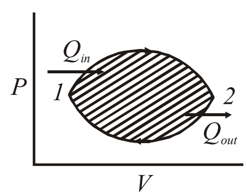


Illustration 12:

An ideal gas is taken through a cyclic thermodynamical process through four steps. The amount of heat involved in these steps are $Q_1 = 5960\ J$; $Q_2 = -5585\ J$ $Q_3 = -2980\ J$; *and* $Q_4 = 3645\ J$ *respectively. The corresponding works involved are* $W_1 = 2200\,J$; $W_2 = -825\ J$; $W_3 = -1100\ J$ *and* W_4 *respectively.*

(a) *Find the value of* W_4

(b) *What is the efficiency of the cycle?*

Solution : ***(a)*** According to the given problem

$$\Delta Q = Q_1 + Q_2 + Q_3 + Q_4 = 5960 - 5585 - 2980 + 3645$$

$$\Delta Q = 9605 - 8565 = 1040\ \text{J}$$

$$\Delta W = W_1 + W_2 + W_3 + W_4 = 2200 - 825 - 1100 + W_4 = 275 + W_4$$

and as for cyclic process $U_F = U_I$, $\qquad \Delta U = U_F - U_I = 0$

So from first law of thermodynamics, i.e., $\Delta Q = \Delta U + \Delta W$, we have

$1040 = (275 + W_4) + 0$, i.e. $W_4 = 765$ J

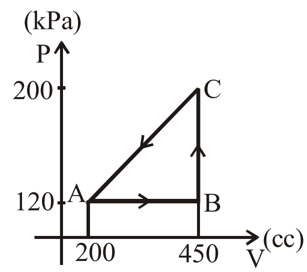
(b) As efficiency of a cycle is defined as

$$\eta = \frac{\text{Network}}{\text{Input heat}} = \frac{\Delta W}{(Q_1 + Q_4)} = \frac{\Delta Q}{(Q_1 + Q_4)}$$

$$\eta = \frac{1040}{9605} = 0.1082 = 10.82\% .$$

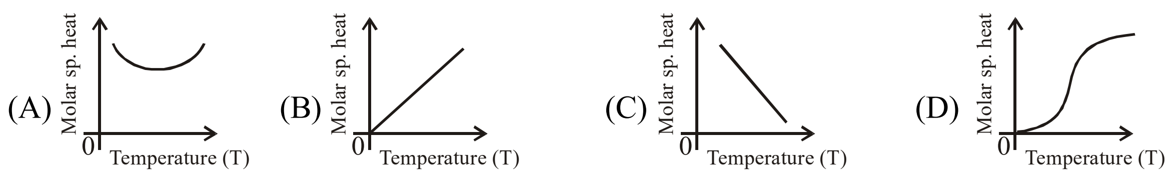
Practice Problems # 05

1. Calculate the work done by the gas in the state diagram shown.



(A) 30J (B) 20J (C) –20J (D) –10J

2. Variation of molar specific heat of a metal with temperature is best depicted by



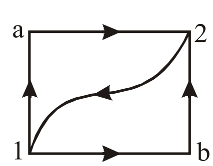
3. 1g of H_2O changes from liquid to vapour phase at constant pressure at 1atm. The voulme increases from 1 cc to 1671 cc. The heat of vaporization at this pressure is 540 cal/g. The increase in internal energy of water is

(A) 2099 J (B) 3000 J (C) 992 J (D) 2122 J

4. A sound wave passing through air at NTP produces a pressure of 0.001 dyne/cm^2 during a compression. The corresponding change in temperature (given $\gamma = 1.5$ and assume gas to be ideal) is

(A) 8.97×10^{-4} K (B) 8.97×10^{-6} K (C) 8.97×10^{-8} K (D) none of these

5. When a system is taken from state 1 to 2 along the path 1a2 it absorbs 50 cal of heat and work done is 20 cal. Along the path 1b2, Q = 36 cal. What is the work done along 1b2?



(A) 56 cal (B) 66 cal (C) 16 cal (D) 6 cal

6. 1g mole of an ideal gas at STP is subjected to a reversible adiabatic expansion to double its volume. Find the change in internal energy ($\gamma = 1.4$)

(A) 1169.5 J (B) 769.5 J (C) 1369.5 J (D) 969.5 J

7. A gram mole of a gas at 127ºC expands isothermally until its volume is doubled. Find the amount of work done.

(A) 238 cal (B) 548 cal (C) 548 J (D) 238 J

8. Find the work required to compress adiabatically 1g of air initially at NTP to half its volume. Density of air at NTP = 0.001129 gcm^{-3} and $\frac{C_P}{C_V} = 1.4$.

(A) 62.64 J (B) 32.64 J (C) –32.64 J (D) –62.64 J

9. An ideal gas expands according to the law $PV^{3/2}$ = constant. We conclude

(A) The adiabatic exponent at the gas K = 1.5

(B) The molar heat capacity $C = C_V - 2R$

(C) Temperature increases during the process

(D) Such a gas is not feasible

10. The ratio of work done by an ideal diatomic gas to the heat supplied by the gas in an isobaric process is

(A) $\frac{5}{7}$ (B) $\frac{3}{5}$ (C) $\frac{2}{7}$ (D) $\frac{5}{3}$

Practice Problems # 06

1. A monatomic gas is supplied heat Q very slowly keeping the pressure constant. Find the work done by the gas.

(A) $\frac{2}{5}Q$ (B) $\frac{3}{5}Q$ (C) $\frac{Q}{5}$ (D) $\frac{2}{3}Q$

2. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

(A) 4RT (B) 15RT (C) 9RT (D) 11 RT

3. For a thermodynamic process $\delta Q = -50$ calorie and $W = -20$ calorie. If the initial internal energy is –30 calorie then, final internal energy will be :

(A) –100 calorie (B) –60 calorie (C) 100 calorie (D) 91.20 calorie

4. The isothermal bulk modulus of elasticity of a gas is $1.5 \times 10^5\ Nm^{-2}$. Its adiabatic bulk modulus of elasticity will be (if $\gamma = 1.4$)

(A) $3 \times 10^5\ Nm^{-2}$ (B) $2.1 \times 10^5\ Nm^{-2}$ (C) $1.5 \times 10^5\ Nm^{-2}$ (D) ∞

5. When the temperature of a gas in a vessel is increased by 1ºC then its pressure is increased by 0.5%. The initial temperature is

(A) 100 K (B) 200 K (C) 273 K (D) 300 K

6. The internal energy of air in a room of volume $50m^3$ at atmospheric pressure will be

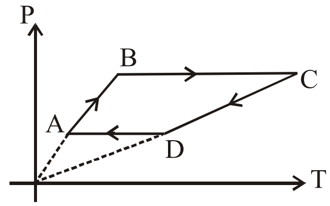
(A) 2.5×10^7 erg (B) 2.5×10^7 joule (C) 5.25×10^7 joule (D) 1.25×10^7 joule

7. The volume of a gas is reduced to 1/4 of its initial volume adiabatically at 27ºC. The final temperature of gas (if $\gamma = 1.4$) will be

(A) $27 \times (4)^{0.4}$ K (B) $300 \times (1.4)^{0.4}$ K (C) $100 \times (4)^{0.4}$ K (D) $300 \times (4)^{0.4}$ K

8. One mole of an ideal gas is contained under a weightless piston of a vertical cylinder at a temperature T. The space over the piston opens into the atmosphere. What work has to be performed in order to increase isothermally the gas volume under the piston η times by slowly rising the piston? Neglect friction.

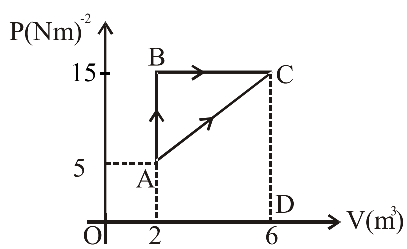
9. 3 moles of an ideal monatomic gas perform a cycle shown in figure. The gas temperature $T_A = 400K$, $T_B = 800K$, $T_C = 2400$ K, $T_D = 1200$ K. Find the work done by the gas.



10. One mole of Argon is heated using $PV^{3/2}$ = constant. Find the amount of heat obtained by the process when the temperature changes by $\Delta T = -26K$.

Practice Problems # 07

1. In the given diagram an ideal gas changes its state from A to C via two points AC and AB.

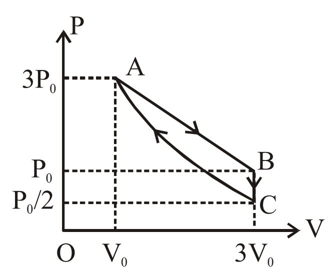


(a) Find work done along each path

(b) The internal energy of the gas at A is 10J and the amount of heat supplied to the gas to change its state to C through the path AC is 200J. Calculate internal energy at C .

(c) The internal energy at state B is 20J. Find the amount of heat supplied to the gas in process AB and process BC.

2. One mole of a monatomic gas is taken through the cycle ABCA as shown in the figure. In the process $C \rightarrow A$ the gas obeys the relation



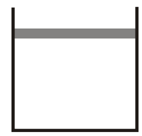
$$Q_{C \rightarrow A} + W_{C \rightarrow A} = 0$$

(where $Q_{C \rightarrow A}$ + is the heat supplied in the process $C \rightarrow A$ and $W_{C \rightarrow A}$ is the work done by the gas in that process) AB is an isothermal process.

(a) Find the work done and change in internal energy in each process and also find the heat exchanged in each process.

(b) Find the average molar specific heat for processes AB and CA.

3. The shown container has all insulating surface except bottom through which heat can flow. The bottom has area of cross-section A, thickness a and thermal conductivity k. The movable piston is insulating and massless and no leakage is possible through it. Initially (at time $t = 0$) the gas (monatomic) inside the container is at temperature T_0 and volume V_0. The surrounding temperature is T_s ($> T_0$) and pressure P_0.

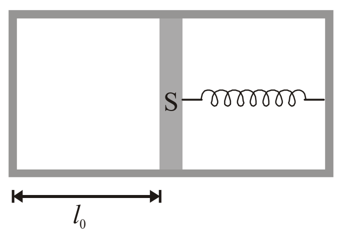


Find

(a) the temperature of the container as a function of time.

(b) the height of piston from the bottom as a function of time if initial height is h_0.

4. S is piston of mass M which can slide inside a cylinder without friction. The walls of the cylinder is adiabatic and the piston is diathermic and area of cross section of cylinder is A, length of the cylinder is l_0 initial pressure of each chamber is p_0. Volume of each chambers are equal. Initially spring is unstretched and has spring constant k. The spring is fixed with piston and wall of the cylinder as shown in the figure.



Find out its time period when piston is given small displacement.

5. Three moles of an ideal gas $\left(C_P = \frac{7}{2}R\right)$ at pressure P_0 and temperature T_0 is isothermally expanded to twice its initial volume, it is then compressed at a constant pressure to its original volume.

(a) Sketch P-V and P-T diagram for complete process.

(b) Calculate net work done by the gas.

(c) Calculate net heat supplied to the gas during complete process.

(Write your answer in terms of gas constant = R)

6. On a P-V diagram starting from an initial state (P_0, V_0) plot an adiabatic expansion to $2V_0$, an isothermal expansion to $2V_0$ and an isobaric expansion to $2V_0$.

(a) Use this graph to determine in which process the least work in done by the system.

(b) Plot the processes in part (a) on a P-T diagram starting from (P_0, T_0).

7. There are two vessels, each of them containing one mole of an ideal monatomic gas. Initial volume of each gas in each vessel is 8.3×10^{-3} m^3 at 27ºC. Equal amount of heat is supplied to each vessel. In one of the vessels, the volume of gas is doubled without change in its internal energy, whereas the volume of gas is held constant in the other vessel. The vessels are now connected to allow free mixing of the gas. Find the final temperature and pressure of the combined gas system.

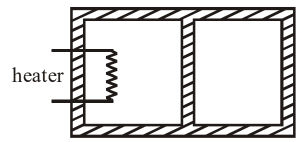
8. Two moles of a gas ($\gamma = 5/3$) are initially at temperature 27ºC and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled. Then it is subjected to an adiabatic change until the temperature returns to its initial value.

(a) Sketch the process on a P-V diagram.

(b) What are final volume and pressure of the gas.

(c) What is the work done by the gas.

9. The rectangular box shown in figure has a partition which can slide without friction along the length of the box. Initially each of two chambers of the box has one mole of a monatomic ideal gas ($\gamma = 5/3$) at a pressure of P_0, volume V_0 and temperature T_0. The chamber on the partition are thermally insulated. Heat loss through lead wires of heater is negligible. The gas in the left chamber expands by pushing the partition until the final pressure in both chambers becomes (243/32) P_0. Determine



(a) the final temperature of the gas in each chamber.
(b) the work done by the gas in the right chamber.

10. An ideal mono-atomic gas is confined in a cylinder by a spring loaded piston of cross-section 8×10^{-3} m^2. Initially the gas is at 300K and occupies a volume of 2.4×10^{-3} m^3 and the spring is in its relaxed (unstretched, uncompressed) state. The gas is heated by a small electric heater until and piston moves out slowly by 0.1m. Calculate the final temperature of the gas and the heat supplied in joules by the heater. The force constant of the spring is 8000 N/m, atmospheric pressure is 1×10^5 N/m^2. The cylinder and the piston are thermally insulated. The piston is massless and there is no friction between the piston and the cylinder. Neglect heat loss through the lead wires of the heater. The heat capacity of the heater coil is negligible. Assume the spring to be massless.

4. Heat–Transfer:

(A) Conduction:

Heat energy is transferred (usually through solids) from one part of the material medium to other without transferring the material particles.

(i) **Steady State :** In this state heat absorption stops and temperature gradient throughout the rod becomes constant i.e. $\frac{dT}{dx} = \text{constant}$

(ii) **Before steady State :** Temperature of rod at any point changes

Note:
If specific heat of any substance is zero, it can be considered always in steady state.

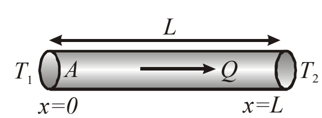
Ohm's Law For Thermal Conduction in Steady State:

In steady state heat passing through a bar of length L and cross-section A in time t when its ends are at temperatures T_1 and $T_2 (< T_1)$, it is given by:

$$Q = KA\frac{(T_1 - T_2)}{L}t \qquad \ldots \text{(i)}$$

So rate of flow of heat will be

$$\frac{dQ}{dt} = -KA\frac{dT}{dx} \qquad \ldots \text{(ii)}$$

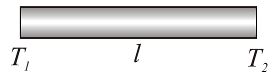


The quantity (dT/dx) is called **temperature gradient** (minus sign indicates that with increase in x, temperature θ decreases) and the constant K depends on the nature of metal and is called coefficient of thermal conductivity or simply thermal conductivity and is a measure of the ability of a substance to conduct heat through it. The dimensions of coefficient of thermal conductivity are $[MLT^{-3}T^{-1}]$ while its SI units are W/mK.

Let the two ends of rod of length ℓ is maintained at temperature T_1 and T_2

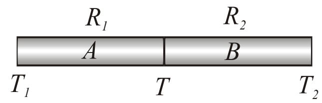
Thermal current $\dfrac{dQ}{dt} = \dfrac{T_1 - T_2}{R_{Th}}$

Where thermal resistance $R_{Th} = \dfrac{\ell}{KA}$



(a) Two rods joined

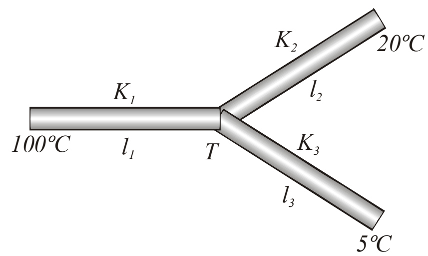
$$\frac{dQ}{dt} = \frac{T_1 - T_2}{R_1 + R_2}$$



$(T_1 > T_2)$

(b) Three rods joined to a common point

$$\frac{T-100}{\frac{\ell_1}{K_1 A}} + \frac{T-20}{\frac{\ell_2}{K_2 A}} + \frac{T-5}{\frac{\ell_3}{K_3 A}} = 0$$



Series and Parallel Connection of Rods in Steady State:

Series Connection

K_1 = thermal conductivity of B

K_2 = thermal conductivity of C

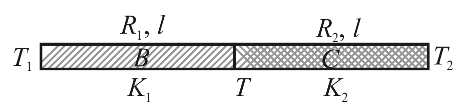
$T_1 > T_2$

length and cross section area of both rods are same

length = ℓ and cross section area = A

$R_1 = \dfrac{\ell}{K_1 A}$ $\qquad R_2 = \dfrac{\ell}{K_2 A}$

Thermal current $i = \dfrac{\Delta Q}{\Delta t} = \dfrac{T_1 - T}{R_1} = \dfrac{T - T_2}{R_2} = \dfrac{T_1 - T_2}{R_1 + R_2}$



$\Rightarrow \quad (T - T_2)R_1 = (T_1 - T)R_2$

$TR_1 - T_2 R_1 = T_1 R_2 - TR_2$

$\Rightarrow \quad T(R_1 + R_2) = T_1 R_2 + T_2 R_1$

$T = \dfrac{T_1 R_2 + T_2 R_1}{R_1 + R_2}$

$\Rightarrow \quad \dfrac{T_1 - T_2}{R} = \dfrac{T_1 - T_2}{R_1 + R_2}$

$\Rightarrow \quad R = R_1 + R_2$

Two rods together is equivalent to a single rod of thermal resistance $R_1 + R_2$

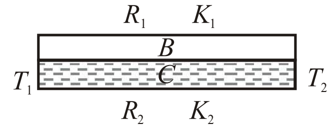
Parallel Connection

$i_1 = \dfrac{\Delta Q_1}{\Delta t} = \dfrac{T_1 - T_2}{R_1}$

$i_2 = \dfrac{\Delta Q_2}{\Delta t} = \dfrac{T_1 - T_2}{R_2}$

$i = i_1 + i_2$

$$\frac{T_1 - T_2}{R} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

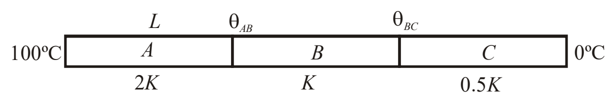


The system of the two rods is equivalent to a single rod of thermal resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Illustration 13.

Three cylindrical rods A, B and C of equal lengths and equal diameters are joined in series as shown in figure. Their thermal conductivities are $2K$, K and $0.5K$ respectively. In steady state, if the free ends of rods A and C are at $100^o C$ and $0^o C$ respectively, calculate the temperature at the two junction points. Assume negligible loss through the curved surface. What will be the equivalent thermal conductivity?



Solution: As the rods are in series, $R_{eq} = R_A + R_B + R_C$ with $R = (L/KA)$

i.e., $$R_{eq} = \frac{L}{2KA} + \frac{L}{KA} + \frac{L}{0.5KA} = \frac{7L}{2KA} \qquad \ldots (i)$$

And hence, $$H = \frac{dQ}{dt} = \frac{\Delta\theta}{R} = \frac{(100-0)}{(7L/2KA)} = \frac{200KA}{7L}$$

Now in series, rate of flow of heat remains same, i.e., $H = H_A = H_B = H_C$.

So for rod A,

$$\left[\frac{dQ}{dt}\right]_A = \left[\frac{dQ}{dt}\right]$$

i.e., $$\frac{(100-\theta_{AB})2KA}{L} = \frac{200KA}{7L}$$

or, $\theta_{AB} = 100 - (100/7) = (600/7) = 85.7^o C$

And for rod C,

$$\left[\frac{dQ}{dt}\right]_C = \left[\frac{dQ}{dt}\right]$$

i.e., $$\frac{(\theta_{BC}-0)\times 0.5KA}{L} = \frac{200KA}{7L}$$

or, $\theta_{BC} = (400/7) = 57.1^o C$

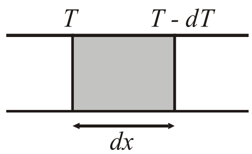
Furthermore if K_{eq} is equivalent thermal conducitivity,

$$R_{eq} = \frac{L+L+L}{K_{eq}A} = \frac{7L}{2KA} \qquad \text{[from equation (i)]}$$

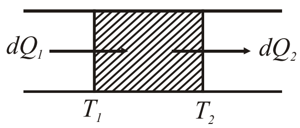
i.e., $K_{eq} = (6/7)K$.

Conduction Before Steady State:
Differential form of Ohm's Law

$$\frac{dQ}{dt} = KA\frac{dT}{dx} \qquad \frac{dT}{dx} = \text{temperature gradient}$$



Conduction in a Section of Medium Before Steady State:



$(T_1 > T_2)$

$dQ = dQ_1 - dQ_2$ (In steady state $dQ = 0$, i.e. $dQ_1 = dQ_2$)

$$\therefore\ mS\ dT = KA\frac{dT_1}{dx}dt - KA\frac{dT_2}{dx}dt$$

$$\frac{dT}{dt} = \frac{KA}{mS}\left(\frac{dT_1}{dx} - \frac{dT_2}{dx}\right)$$

$dT =$ increase in temperature of the section in time *dt*.

Illsturation 14:

Find temperature as a function of radius r in case of spherical shell. Inner and outer surfaces temperature are fixed at θ_1 and θ_2 respectively. Inner and outer radius of shell are r_1 and r_2 respectively.

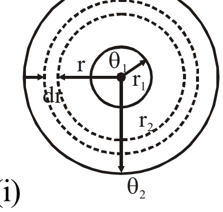
Solution :

Considering a spherical shell of inner radius r_1 and outer radius r_2, maintained at temperature θ_1 and θ_2 respectively $(\theta_2 < \theta_1)$.

Considering an elementary spherical shell of thickness *dr* at a temperature difference $d\theta$. Rate of radial flow of heat in steady state

$$H = \frac{dQ}{dt} = -K(4\pi r^2)\frac{d\theta}{dr}$$

or $$\int_{r_1}^{r_2}\frac{dr}{r^2} = -\frac{K4\pi}{H}\int_{\theta_1}^{\theta_2}d\theta$$



$$\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{K4\pi}{H}[\theta_1 - \theta_2] \qquad \ldots(i)$$

Rate of flow

$$H = \frac{dQ}{dt} = \frac{K4\pi(\theta_1 - \theta_2)}{\left[\frac{1}{r_1} - \frac{1}{r_2}\right]} = \frac{4\pi K r_1 r_2(\theta_1 - \theta_2)}{(r_2 - r_1)}$$

Considering the temperature of the layer as θ at a distance r from the centre.

$$\int_{r_1}^{r}\frac{dr}{r^2} = -\frac{K4\pi}{H}\int_{\theta_1}^{\theta}d\theta$$

$$\left[\frac{1}{r_1}-\frac{1}{r}\right]=\frac{K.4\pi}{H}[\theta_1-\theta] \qquad \ldots\text{(ii)}$$

Using (i) and (ii), we get

$$\frac{\frac{1}{r_1}-\frac{1}{r}}{\frac{1}{r_1}-\frac{1}{r_2}}=\frac{\theta_1-\theta}{\theta_1-\theta_2}, \qquad \theta=\theta_1-\frac{\frac{1}{r_1}-\frac{1}{r}}{\frac{1}{r_1}-\frac{1}{r_2}}(\theta_1-\theta_2).$$

Growth Of Ice:

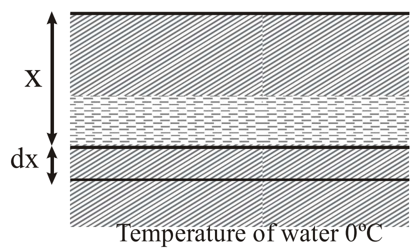
Considering a layer of ice of thickness X. The air temperature is $\theta^o C$ and water temperature below the ice is 0^o C.

Considering unit cross-section area of ice, if a layer of thickness dx grows in time dt,

Then heat given by this layer

= mass $\times$ latent heat = A.dx . ρ . L

ρ = density of ice $L =$ latent heat of fusion of ice. Temperature of air $-\theta^o C$



If this quantity of heat is conducted upwards through the ice layer in time dt.

$$\therefore \qquad Adx\cdot\rho\cdot L=K\frac{\{0-(-\theta)\}}{x}dt\,A$$

time taken $t=\dfrac{\rho L}{K\theta}\displaystyle\int_{x_1}^{x_2}x\cdot dx=\dfrac{\rho L}{2K\cdot\theta}(x_2^2-x_1^2)$

Rate of increase of thickness of the ice layer

$$\left(\frac{dx}{dt}=\frac{K\cdot\theta}{\rho Lx}\right).$$

Convection:

Heat energy is transferred (usually through liquids and gases) by mass movement of molecules from one point to another. (Due to gravity & buoyant force).

Radiation:

Heat energy is transferred by electromagnetic waves even in absence of medium.

Absorptive power a

Absorptive power of a body is defined as the fraction of the incident radiation that is absorbed by the body.

Absorptive power $a=\dfrac{\text{Energy absorbed}}{\text{Energy incident}}$

Emissive power E

The emissive power denotes the energy radiated per unit area per unit time per unit solid angle along the normal to the area.

Emissivity ε

Emissivity of a surface is the ratio of the emissive power of the surface to the emissive power of black body at the same temperature.

Emissivity $\varepsilon =$ (Emissive power of the surface) $\div$ (Emissive power of black body at the same temperature)

Black body

A perfectly black body is one which absorbs completely all the radiation, of whatever wave-length, incident on it.

Kirchhoff's Law

It states that the ratio of the emissive power to the absorptive power for radiation of a given wave length is the same for all bodies at the same temperature, and is equal to the emissive power of a perfectly black body at that temperature.

Stefan's law of radiation

The total radiant energy emitted E per unit time by a black body of surface area A is proportional to the fourth power of its absolute temperature.

$E \propto T^4$

or, $\quad E = \sigma A T^4 \qquad \sigma =$ Stefan's constant

For a body which is not a black body

$E = \varepsilon\sigma A T^4 \qquad \varepsilon =$ emissivity of the body

Using Kirchoff's law

$$\frac{E_{(body)}}{E_{(black\,body)}} = a$$

or, $\quad \dfrac{\varepsilon\sigma A T^4}{\sigma A T^4} = a \qquad$ or, $\quad \varepsilon = a$

Emissivity and absorptive power have the same value.

Net Loss Of Thermal Energy:

If a body of surface area A is kept at absolute temperature T in a surrounding of temperature T_0 $(T_0 < T)$. Then energy emitted by the body per unit time

$E = \varepsilon\sigma A T^4$

And energy absorbed per unit time by the body

$E_0 = \varepsilon\sigma A T_0^4$

Net, loss of thermal energy per unit time.

$\Delta E = E - E_0 = \varepsilon\sigma A(T^4 - T_0^4)$.

This is known as Stefan Boltzmann's Law

Newton's Law Of Cooling:

For a small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference.

If a body of surface area A is kept at absolute temperature T in a surrounding of temperature $T_0 (T_0 < T)$. Then net loss of thermal energy per unit time.

$$\frac{dQ}{dt} = \varepsilon\sigma A(T^4 - T_0^4)$$

If the temperature difference is small

$\therefore \quad T = T_0 + \Delta T$

$$= \varepsilon\sigma A\{(T_0 + \Delta T)^4 - T_0^4\} \quad = \varepsilon\sigma A\left\{T_0^4\left(1+\frac{\Delta T}{T_0}\right)^4 - T_0^4\right\}$$

$$= \varepsilon\sigma A T_0^4\left\{1 + 4\frac{\Delta T}{T_0} + \text{higher powers of }\frac{\Delta T}{T_0} - 1\right\}$$

$$= 4\varepsilon\sigma A T_0^3 \Delta T \qquad \ldots \text{(i)}$$

Now, rate of loss of heat at temperature T

$$\frac{dQ}{dt} = -ms\frac{dT}{dt} \qquad \ldots \text{(ii)}$$

$$\therefore \quad ms\frac{dT}{dt} = -4\varepsilon\sigma A T_0^3 (T - T_0)$$

$$\frac{dT}{dt} = \frac{-4\varepsilon\sigma A T_0^3}{ms}(T - T_0)$$

$$\frac{dT}{dt} = -k(T - T_0) \text{ where } k = 4\varepsilon\sigma A T_0^3$$

$$\frac{dT}{dt} \propto (T - T_0)\ .$$

Average form of Newton's law of cooling :

If a body cools from T_1 to T_2 in time δt

$$\frac{T_1 - T_2}{\delta t} = \frac{K}{ms}\left(\frac{T_1 + T_2}{2} - T_0\right)$$

$$\frac{dT}{dt} = \frac{K}{ms}(T - T_0)$$

By solving and integrating T (from T_1 to T) and time (0 to t). We get

$$T = T_0 + (T_1 - T_0)e^{-Kt/ms}$$

Illustration 15:

In a room where the temperature is 30ºC, a body cools from 61ºC to 59ºC in 4 minutes. Find the time taken by the body to cool from 51ºC to 49ºC .

Solution :

Rate of cooling α difference in temperature

$$\frac{dT}{dt}\alpha\,\Delta\theta \qquad \Rightarrow \qquad \frac{dT}{dt} = K\Delta\theta$$

In first case

$dT = 61 - 59 = 2$

$\Delta\theta = 60 - 30 = 30$

$dt = 4$ minutes

$$\therefore \quad K = \frac{dT}{\Delta\theta dt} = \frac{2}{30\times 4} = \frac{1}{60}$$

For second case

$dT = 2$

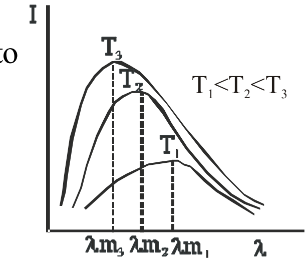
$\Delta\theta = 50 - 30 = 20$

$$\therefore \quad dt = \frac{dT}{K\Delta\theta} = \frac{2}{1/60 \times 20} = 6 \text{ min.}$$

Wien's black body radiation :

At every temperature (>0K) a body radiates energy radiations of all wavelengths.

According to Wien's displacement law if the wavelength corresponding to maximum energy is λ_m



Then $\lambda_m T = b$ where b is a constant (Wien's constant)

T = temperature of body

Intensity at a specific temp. T, $I_m \alpha T^5$

This is wien's fifth power law.

Illustration 16:

Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K calculate:

(a) The temperature of B and (b) wavelength λ_B.

Solution : (a) According to Stefan's law the power radiated by a body is given by – P = $e\sigma AT^4$

According to the given problem $P_A = P_B$ with $A_A = A_B$

So that $e_A T_A^4 = e_B T_B^4$,. i.e. $0.01 \times (5802)^2 = 0.81(T_B)^4$

or $T_B = (1/3)(5802) = 1934K$

(b) According to Wien's displacement law

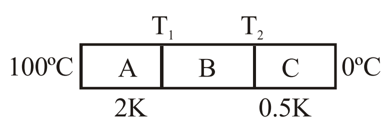
$\lambda_A T_A = \lambda_B T_B$, i.e. $\lambda_B = (5802/1934)\lambda_A$

i.e. $\lambda_B = 3\lambda_A$ and also $\lambda_B - \lambda_A = 1\mu m$ (given)

so $\lambda_B - (1/3)\lambda_B = 1\mu m$, i.e. $\lambda_B = 1.5\ \mu m$.

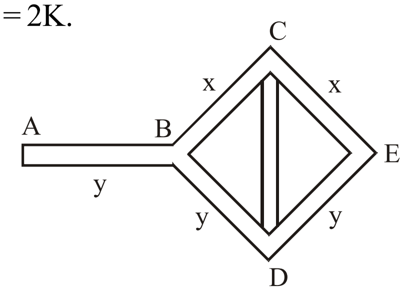
Practice Problems # 08

1. Three identical rods A, B and C of equal lengths and equal diameters are joined in series as shown in figure. Their thermal conductivities are 2K, K and K/2 respectively. Calculate the temperature at two junction points.



(A) 85.7, 57.1ºC (B) 80.85, 50.3ºC (C) 77.3, 48.3ºC (D) 75.8, 49.3ºC

2. Three rods of material x and three rods of material y are connected as shown in figure. All the rods are of identical length and cross-section. If the end A is maintained at 60º and the junction E at 10ºC, find effective thermal resistance. Given length of each rod = l, area of cross-section = A, conductivity of x = K and conductivity of y = 2K.



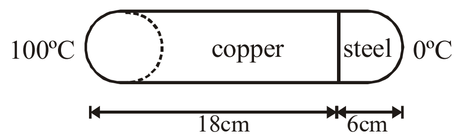
(A) $\frac{4l}{3KA}$ (B) $\frac{7l}{3KA}$ (C) $\frac{4KA}{3l}$ (D) $\frac{7KA}{3l}$

3. If wavelength of maximum intensity of radiation emitted by sun and moon are 0.5×10^{-6} m and 10^{-4}m respectively, the ratio of their temperature is
(A) 1/10 (B) 1/50 (C) 100 (D) 200

4. One end of a metal rod of length 1.0m and area of cross-section $100cm^2$ is maintained at 100ºC. If the other end of the rod is maintained at 0ºC, the quantity of heat transmitted through the rod per minute is (coefficient of thermal conductivity of material of rod = 100 $Wkg^{-1} K^{-1}$)
(A) 3×10^3 J (B) 6×10^3 J (C) 9×10^3 J (D) 12×10^3 J

5. In a steady state, the temperature at the end A and B of 20cm long rod AB are 100ºC and 0ºC. The temperature of a point 9 cm from A is
(A) 45ºC (B) 60ºC (C) 55ºC (D) 65ºC

6. The coefficient of thermal conductivity of copper is nine times that of steel. In the composite cylindrical bar shown in the figure, what will be the temperature at the junction of copper and steel?



(A) 75ºC (B) 33ºC (C) 67ºC (D) 25ºC

7. Two rods of length d_1 and d_2 and coefficient of thermal conductivities K_1 and K_2 are kept touching each other. Both have the same area of cross-section. The equivalent of thermal conductivity is

(A) $K_1 + K_2$ (B) $K_1 d_1 + K_2 d_2$ (C) $\frac{d_1K_2 + d_2K_2}{d_1 + d_2}$ (D) $\frac{d_1 + d_2}{(d_1K_2) + (d_2K_2)}$

8. The total area of the walls of a room is $137m^3$. An electric heater is used to maintain the temperature inside the room at +20ºC, while the outside temperature is –10ºC. Walls are made of three layers of different materials. The innermost layer is made of wood 2.5cm thick middle layer is made of cement 1cm thick and outermost layer is made of bricks 25cm thick. What will be the power of electric heater? Assume that there is no loss of heat from the roof and the floor. The coefficient of thermal conductivity of wood, cement and brick are 0.125, 1.5 and 1 watt/mºC respectively.
(A) 9000 watt (B) 8000 watt (C) 7000 watt (D) 5000 watt

9. A cylindrical rod has temperature T_1 and T_2 at its ends. The rate of flow of heat is Q_1 $cals^{-1}$. If all the linear dimensions are doulbed keeping temperature constant, then the rate of flow of heat is $cals^{-1}$ Q_2 will be
(A) $4Q_1$ (B) $2Q_1$ (C) $Q_1/4$ (D) $Q_1/2$

10. Two rods of same length and material transfer a given amount of heat in 12 seconds, when they are joined end to end. But when they are joined lengthwise, then they will transfer same heat in same conditions in
(A) 24s (B) 3s (C) 1.5s (D) 48 s

Practice Problems # 09

1. A man has a total surface area of $1.m^2$. Find the total rate of radiation of energy from the body.
(A) 566 J (B) 682 J (C) 732 J (D) 782 J

2. Two spheres of same material have radii 1m and 4m and temperature 4000K and 2000K respectively. The ratio of energy radiated per second is
(A) 1 (B) 2 (C) 4 (D) none of these

3. The emissivity of a body of surface area $5cm^2$ and at temperature 727ºC radiating 300J of energy per minute is
(A) 0.48 (B) 0.38 (C) 0.28 (D) 0.18

4. A body cools is 7 minutes from 60ºC to 40ºC. The temperature after next 7 minutes will be .... Given temperature of surrounding is 10ºC.

(A) 32ºC (B) 38ºC (C) 22ºC (D) none of these

5. Bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface area of the two bodies are same. The two bodies emit total radiant power at the same rate. Find the temperature of B if the temperature of A is 5802 K.

(A) 1634 K (B) 1734 K (C) 1934 K (D) none of these

6. The temperature of a body falls from 40ºC to 36ºC in 5 minutes. The temperature of the body will becomes 32ºC in

(A) less than 10 minutes (B) 10 minutes

(C) more than 10 minutes (D) none of these

7. A solid at temperature T_1 is kept in an evacuated chamber at temperature $T_2 > T_1$. The rate of growth of temperature is proportional to

(A) $T_2 - T_1$ (B) $T_2^2 - T_1^2$ (C) $T_2^3 - T_1^3$ (D) $T_2^4 - T_1^4$

8. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$ where $\Delta\theta$ is the difference of temperature of the body and the surrounding and n is equal to

(A) 3 (B) 4 (C) 1 (D) 2

9. A block body at a temperature 77ºC radiates heat at a rate of 10 calcm^{-2} -s. The rate at which this body would radiate heat is units of calcm^{-2}–s at 427ºC is closest to

(A) 40 (B) 160 (C) 200 (D) 400

10. Find the heat radiated per second by a body of surface area 12cm^2 kept in thermal equilibrium in a room at temperature 20ºC. The emissivity of the surface is 0.8 and $\sigma = 6 \times 10^{-8}$ Wm^{-2} K^{-4}.

(A) 4.2 J (B) 0.42 J (C) 0.042 J (D) 42 J

Practice Problems # 10

1. Water is being boiled in flat bottom kettle placed on a stove. The area of the bottom is 3000cm^2 and the thickness is 2mm. If the amount of steam produced is 1g/min, calculate the difference of temperature between the inner and outer surface of the bottom. K for the material of kettle is 0.5 cal/ºC/s/cm, and the latent heat of steam is 540 cal/gm.

2. A uniform copper bar 100cm long is insulated on sides, and has its ends exposed to the ice and steam respectively. If there is a layer of water 0.1 mm thick at each end, calculate the temperature gradient in the bar $K_{Cu} = 1.04$ and $K_{water} = 0.0014$ in C.G.S. units.

3. The rods of copper, brass and steel are welded together to form a Y-shaped structure. The cross-sectional area of each rod is 4cm^2. The end of copper rod is maintained at 100ºC and the ends of the brass and steel rods at 0ºC. Assume that there is no loss of heat from the surface of the rods. The length of rods are :

(A) What is the temperature of the junction point?

(B) What is the heat current in the copper rod ?

K(Cu) =0.92, K (steel) = 0.12 and K(brass) = 0.26 C.G.S. units.

4. One end of a copper rod of uniform cross-section and of length 1.5 m is in contact with ice and the other end with water. At what point along its length should a temperature of 200ºC be maintained so that in steady state, the mass of ice melting is equal to that of steam produced in the same interval of time? Assume that whole system is insulated from surroundings.

5. A steam pipe 1m in length with an outside diameter of 5.00cm has a uniform temperature of 100ºC. The pipe is insulated with a 6.00 cm layer of asbestos fibre. If the room temperature is 20ºC, what is the heat loss to room per hour per meter of pipe length? What is the temperature in the middle of the layer of insulation? K (asbestos) = 0.19×10^{-4} cal/s/ºC/m.

6. The emissivity of tungsten is approximately 0.35. A tungsten sphere 1cm in radius is suspended within a large evacuated enclosure whose walls are at 300K. What power input is required to maintain the sphere at a temperature of 3000K if heat conduction along the supports is neglected? $\sigma = 5.67 \times 10^{-8}$ S.I. units.

7. The rate at which the radiant energy reaches the surface of earth from the sun is about 1.4 kW/m^2. The distance from earth to the sun is about 1.5×10^{11} m, and the radius of sun is about 0.7×10^{9} m.

   (a) What is the rate of radiation of energy, per unit area, from the sun's surface?

   (b) If the sun radiates as an ideal blackbody, what is the temperature of its surface?

8. Two solid copper spheres of radii r_1 = 15 cm and r_2 = 20cm are both at a temperature of 60ºC. If the temperature of surroundings is 50ºC, then find :

   (a) the ratio of the heat loss per second from their surfaces initially

   (b) the ratio of rates of cooling initially.

9. Two identical sphere A and B are suspended in an air chamber which is maintained at a temperature of 50ºC. Find the ratio of the heat lost per sec. from the surface of the sphere if :

   (a) A and B are at temperature 60ºC and 55ºC respectively.

   (b) A and B are at temperature 250ºC and 200ºC respectively.

10. A body cools down from 60ºC to 55ºC in 30 seconds. Using Newton's law of coiling, calculate the time taken by same body to cool down from 55ºC to 50ºC. Assume that the temperature of surrounding is 45ºC.

Solved Example Objective

Problem 1: *During adiabatic process pressure (P) versus density (ρ) equation is*

(a) $P.\rho^{\gamma} = \text{constant}$ (b) $P.\rho^{-\gamma} = \text{constant}$

(c) $P^{\gamma}.\rho^{1+\gamma} = \text{constant}$ (d) $P^{1/\gamma}.\rho^{\gamma} = = \text{constant}$

Solution : In adiabatic process

PV^{γ} = constant … (1)

Density $\rho = \dfrac{m}{V}$ or $\rho \propto V^{-1}$

$\therefore$ equation (1) can be written as $P.\rho^{-\gamma} = \text{constant}$

Problem 2: *Two cylinders fitted with pistons contains equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of gas in B is*

(a) 30 K *(b) 18 K*

(c) 50 K *(d) 42 K*

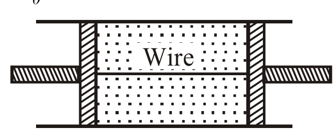
Solution : In cylinder A heat is supplied at constant pressure while in cylinder B heat is supplied at constant volume.

$\therefore$ $(\Delta Q)_A = nC_P\,(\Delta T)_A$ and $(\Delta Q)_B = nC_V\,(\Delta T)_B$

Given that $(\Delta Q)_A = (\Delta T)_B$ $\therefore$ $(\Delta T)_B = \dfrac{C_P}{C_V}(\Delta T)_A$

$$= (1.4)(30)\left(\frac{C_P}{C_V} = 1.4\right) = 42\text{ K}$$

Problem 3: *A cylindrical tube of uniform cross-sectional area A is fitted with two air tight friction less pistons. The pistons are connected to each other by a metallic wire. Initially the pres sure of the gas is P_0 and temperature is T_0. Atmospheric pressure is also P_0. Now the temperature of the gas is increased to $2T_0$, the tension in the wire will be*



(a) $2P_0A$ *(b) P_0A*

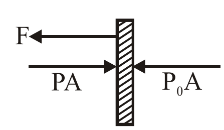
(c) $\dfrac{P_0A}{2}$ *(d) $4P_0A$*

Solution : Volume of the gas is constant V = constant

$\therefore P \propto T$

i.e. pressure will be doubled if temperature is doubled

$\therefore P = 2P_0$



Now let F be the tension in the wire. Then equilibrium of any one piston gives

$F = (P - P_0)A = (2P_0 - P_0)A = P_0A$

Problem 4:. *Internal energy of n_1 moles of hydrogen at temperature T is equal to the internal energy of n_2 moles of helium at temperature 2T. Then the ratio n_1/n_2 is*

(a) $\frac{3}{5}$ *(b)* $\frac{2}{3}$

(c) $\frac{6}{5}$ *(d)* $\frac{3}{7}$

Solution : Internal energy of n moles of an ideal gas at temperature T is given by:

$$U = \frac{f}{2}nRT \quad (f = \text{degrees of freedom})$$

$$U_1 = U_2$$

$$\therefore\ f_1n_1T_1 = f_2n_2T_2$$

$$\therefore\ \frac{n_1}{n_2} = \frac{f_2T_2}{f_1T_1} = \frac{(3)(2)}{(5)(1)} = \frac{6}{5}$$

Here f_2 = degrees of freedom of He = 3 and f_1 = degrees of freedom of H_2 = 5

Problem 5: *If 2 moles of an ideal monoatomic gas at temperature T_0 is mixed with 4 moles of another ideal monoatomic gas at temperature $2T_0$, then the temperature of the mixture is*

(a) $\frac{5}{3}T_0$ *(b)* $\frac{3}{2}T_0$

(c) $\frac{4}{3}T_0$ *(d)* $\frac{5}{4}T_0$

Solution : Let T be the temperature of the mixture.

Then $U = U_1 + U_2$

or $\frac{f}{2}(n_1 + n_2)RT = \frac{f}{2}(n_1)RT_0 + \frac{f}{2}(n_2)(R)(2T_0)$

or $(2 + 4)\ T = 2T_0 + 8T_0\ (n_1 = 2,\ n_2 = 4)$

or $T = \frac{5}{3}T_0$

Problem 6: *In the process PV = constant, pressure (P) versus density (ρ) graph of an ideal gas is*
(a) a straight line parallel to P-axis
(b) a straight line parallel to ρ-axis
(c) a straight line passing through origin
(d) a parabola

Solution : PV = constant

$\therefore$ T = constant

Now $\rho = \frac{PM}{RT}$

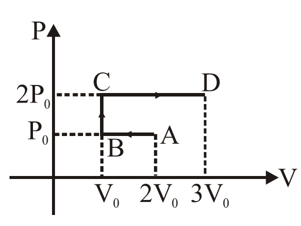
or $\rho \propto P$ for T = constant

Hence, P-ρ graph is a straight line passing through origin.

Problem 7: *P-V diagram of an ideal gas is shown in figure. Work done by the gas in the process ABCD is*

(a) $4P_0V_0$ *(b)* $2P_0V_0$

(c) $3P_0V_0$ *(d)* P_0V_0



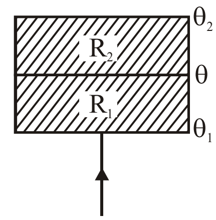
Solution : $W_{AB} = -\ P_0V_0$

$W_{BC} = 0$

and $W_{CD} = 4P_0V_0$

$\therefore W_{ABCD} = -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$

Problem 8: *Consider the two insulating sheets with thermal resistance R_1 and R_2 as shown in figure. The temperature θ is*



(a) $\dfrac{\theta_1\theta_2R_1R_2}{(\theta_1+\theta_2)(R_1R_2)}$ *(b)* $\dfrac{\theta_1R_1+\theta_2R_2}{R_1+R_2}$

(c) $\dfrac{(\theta_1+\theta_2)R_1R_2}{R_1^2+R_2^2}$ *(d)* $\dfrac{\theta_1R_2+\theta_2R_1}{R_1+R_2}$

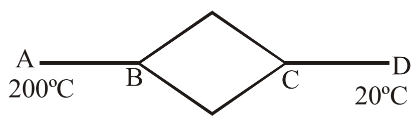
Solution : For the two sheets

$H_1 = H_2$ (H = rate of heat transfer)

or $\dfrac{\theta_1-\theta}{R_1}=\dfrac{\theta-\theta_2}{R_2}$

Solving this we get $\theta=\dfrac{\theta_1R_2+\theta_2R_1}{R_1+R_2}$

Problem 9 *Six identical conducting rods are joined as shown in figure. Points A and D are main tained at temperatures 200°C and 20°C respectively. The temperature of junction B will be*



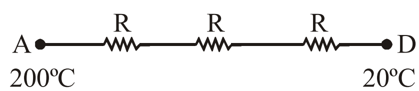
(a) 120°C *(b) 100°C*

(c) 140°C *(d) 80°C*

Solution : Equivalent electrical circuit will be as shown in figure.

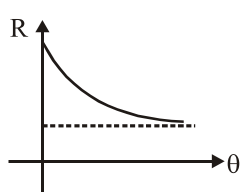
Temperature difference between A and D is 180°C which is equally distributed in all the rods.

Therefore, temperature difference between A and B will be 60°C or temperature of B should be 140°C.

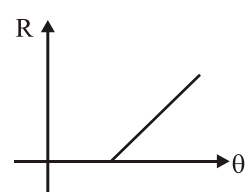


Problem 10: *Temperature of a body θ is slightly more than the temperature of the surrounding θ_0. Its rate of cooling (R) versus temperature of body (θ) is plotted, its shape would be*

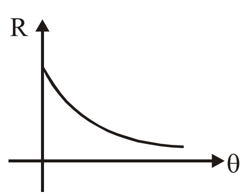
(a)



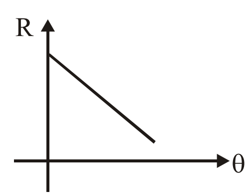
(b)



(c)



(d)



Solution : According to Newton's law of cooling rate of cooling R $\propto$ temperature difference

Hence, $R = K(\theta - \theta_0)$

This is an equation of a straight line.

Problem 11: *One end of a conducting rod is maintained at temperature 50°C and at the other end ice is melting at 0°C. The rate of melting of ice is doubled if*

(a) the temperature is made 200°C and the area of cross-section of the rod is doubled
(b) the temperature is made 100°C and length of the rod is made four times
(c) area of cross section of rod is halved and length is doubled
(d) the temperature is made 100°C and area of cross-section of rod and length both are doubled

Solution : Rate of melting of ice $\propto$ rate of heat transfer (dQ/dt).

Further $$\frac{dQ}{dt} = \frac{\text{temperature difference}}{\left(\frac{l}{KA}\right)}$$

or $$\frac{dQ}{dt} \propto \frac{(\text{temperature difference})}{l} A$$

If temperature difference, A and l are all doubled then $\frac{dQ}{dt}$ and hence, rate of melting of ice will be doubled.

Problem 12: *The ratio of specific heat of a gas at constant pressure to that at constant volume is γ. The change in internal energy of a mass of gas when the volume changes from V to 2V at constant pressure P is*

(a) $\frac{R}{\gamma - 1}$ *(b)* PV

(c) $\frac{PV}{\gamma - 1}$ *(d)* $\frac{\gamma PV}{\gamma - 1}$

Solution : $\Delta U = nC_V\Delta T$

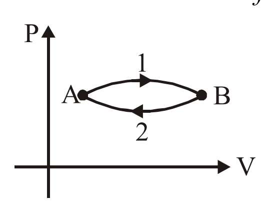
$$= n\left(\frac{R}{\gamma - 1}\right)(T_f - T_i)$$

$$= \frac{nRT_f - nRT_i}{\gamma - 1}$$

$$= \frac{P(2V) - P(V)}{\gamma - 1} = \frac{PV}{\gamma - 1}$$

Problem 13: *The figure shows two paths for the change of state of a gas from A to B. The ratio of molar heat capacities in path 1 and path 2 is*

(a) >1 *(b) < 1*
(c) 1 *(d) data insufficient*



Solution : Molar heat capacity $C = \frac{\Delta Q}{\Delta T}$

or $$C = \frac{\Delta U + \Delta W}{dT}$$

ΔU is same in both the paths but
$\Delta W_2 > \Delta W_1$
$\therefore\ C_2 > C_1$
or $C_1/C_2 < 1$

Problem 14: *The molar heat capacity in a process of a diatomic gas if it does a work of* $\frac{Q}{4}$ *when a heat of Q is supplied to it is*

(a) $\frac{2}{5}R$ (b) $\frac{5}{2}R$

(c) $\frac{10}{3}R$ (d) $\frac{6}{7}R$

Solution : $dU = C_V dT = \left(\frac{5}{2}R\right)dT$

or $dT = \frac{2(dU)}{5R}$ ...(1)

From first law of thermodynamics

$$dU = dQ - dW$$

$$= Q - \frac{Q}{4}$$

$$= \frac{3Q}{4}$$

Now molar heat capacity

$$C = \frac{dQ}{dT} = \frac{Q}{\frac{2(dU)}{5R}} = \frac{5RQ}{2\left(\frac{3Q}{4}\right)} = \frac{10}{3}R$$

Problem 15: *Two meal rods of the same length and area of cross-section are fixed end to end between rigid supports. The materials of the rods have Young modulii Y_1 and Y_2, and coefficients of linear expansion α_1 and α_2. The junction between the rods does not shift if the rods are cooled.*

(a) $Y_1\alpha_1 = Y_2\alpha_2$ (b) $Y_1\alpha_2 = Y_2\alpha_1$

(c) $Y_1\alpha_1^2 = Y_2\alpha_2^2$ (d) $Y_1^2\alpha_1 = Y_2^2\alpha_2$

Solution : Tension must be the same in both the rod for their junction to be in equilibrium.

$$Y_1A\alpha_1t = Y_2A\alpha_2t$$

Problem 16: *When the temperature of a body increases from t to t + Δt, its moment of inertia increases from I to I + ΔI. The coefficient of linear expansion of the body is α. The ratio* $\frac{\Delta I}{I}$ *is equal to*

(a) $\frac{\Delta t}{t}$ (b) $\frac{2\Delta t}{t}$

(c) $\alpha\Delta t$ (d) $2\alpha\,\Delta t$

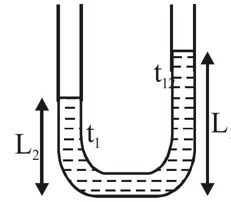
Solution : $I = \Sigma mr^2$

$I + \Delta I = \Sigma m[r(1 + \alpha\,\Delta t)]^2$

or $I + \Delta I = \Sigma[mr^2(1 + 2\alpha\,\Delta t)] = I(1 + 2\alpha\,\Delta t)$

or $\Delta I/I = 2\alpha\,\Delta t$.

Problem 17: *In a vertical U-tube containing a liquid, the two arms are maintained at different temperatures, t_1 and t_2. The liquid columns in the two arms have heights l_1 and l_2 respectively. The coefficient of volume expansion of the liquid is equal to*



(a) $\dfrac{l_1 - l_2}{l_2 t_1 - l_1 t_2}$ (b) $\dfrac{l_1 - l_2}{l_1 t_2 - l_2 t_2}$

(c) $\dfrac{l_1 + l_2}{l_2 t_1 + l_1 t_2}$ (d) $\dfrac{l_1 + l_2}{l_1 t_1 + l_2 t_2}$

Solution : Let ρ_0, ρ_1 and ρ_2 be the densities of the liquid at temperatures 0, t_1 and t_2 respectively
To balance pressure, $\rho_1 l_1 g = \rho_2 l_2 g$

$$\text{or } \left(\frac{\rho_0}{1+\gamma t_1}\right) l_1 = \left(\frac{\rho_0}{1+\gamma t_2}\right) l_2$$

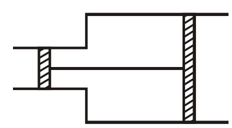
Problem 18: *Two containers of equal volume contain the same gas at pressure p_1 and p_2 and absolute temperatures T_1 and T_2 respectively. On joining the vessels, the gas reaches a common pressure p and a common temperature T. The ratio p/T is equal to*

(a) $\dfrac{p_1}{T_1} + \dfrac{p_2}{T_2}$ (b) $\dfrac{1}{2}\left[\dfrac{p_1}{T_1} + \dfrac{p_2}{T_2}\right]$

(c) $\dfrac{p_1 T_2 + p_2 T_1}{T_1 + T_2}$ (d) $\dfrac{p_1 T_2 - p_2 T_1}{T_1 - T_2}$

Solution : For a closed system, the total mass of gas or the number of moles remains constant.
$p_1 V = n_1 R T_1$, $p_2 V = n_2 R T_2$, $p(2V) = (n_1 + n_2)RT$

Problem 19: *A horizontal cylinder has two sections of unequal cross-sections, in which two pistons can move freely. The pistons are joined by a string. Some gas is trapped between the pistons. If this gas is heated, the pistons will*



(a) move to the left
(b) move to the right
(c) remain stationary
(d) either (a) or (b) depending on the initial pressure of the gas

Solution : The pressure of the gas remains constant, and is equal to the atmospheric pressure (for equilibrium of the gas pus pistons system). If the temperature of the gas is increased, its volume must increases. For this, the pistons must move to the right.

Problem 20: *A gas expands from 1 litre to 3 litres at atmospheric pressure. The work done by the gas is about*

(a) 2J *(b) 200 J*
(c) 300 J *(d) 2×10^5J*

Solution : AT constant pressure, work = pressure × change in volume
$= 1 \times 10^5 \times 2 \times 10^{-3}$ m^3
= 200 J

Solved Example Subjective

Problem 1: *One mole of an ideal gas whose pressure changes with volume as $P=\alpha V$, where α is a constant, is expanded so that its volume increase η times. Find the change in internal energy and heat capacity of the gas.*

Solution : Let V be the initial volume of the gas. It is expanded to a volume ηV. The work done in this process is given by

$$W=\int_V^{\eta V} PdV=\int_V^{\eta V}\alpha V dV=\alpha\left[\frac{V^2}{2}\right]_V^{\eta V}$$

$$=\frac{\alpha V^2}{2}\left[\eta^2-1\right]$$

The pressure of the gas varies with volume as $P=\alpha V$. So, the initial and final pressure will be α V and $\eta\alpha$ V. The change in internal energy is given by

$$dU=nC_V dT=\frac{R\left(T_f-T_i\right)}{\gamma-1}=\frac{P_fV_f-P_iV_i}{\gamma-1}=\frac{\eta^2\alpha V^2-\alpha V^2}{\gamma-1}=\frac{\alpha V^2}{\gamma-1}\left(\eta^2-1\right)$$

The heat exchange in this process is given by

$Q=U+W$

$$=\frac{\alpha V^2}{\gamma-1}\left[\eta^2-1\right]+\frac{\alpha V^2}{2}\left[\eta^2-1\right]=\frac{\alpha V^2}{2}\left[\eta^2-1\right]\left[\frac{\gamma+1}{\gamma-1}\right]$$

Here $T_i=\frac{P_iV_i}{nR}=\frac{\alpha V^2}{nR}$ and $T_f=\frac{P_fV_f}{nR}=\frac{\eta^2\alpha V^2}{nR}$

Now heat capacity $C=\frac{Q}{T_f-T_1}$

$$C=\frac{1}{T_f-T_i}\left[\frac{\alpha V^2}{2}\left(\eta^2-1\right)\left\{\frac{\gamma+1}{\gamma-1}\right\}\right]$$

$$=\frac{nR}{\alpha V^2\left(\eta^2-1\right)}\left[\frac{\alpha V^2}{2}\left(\eta^2-1\right)\left\{\frac{\gamma+1}{\gamma-1}\right\}\right]$$

$$=\frac{nR}{2}\left[\frac{\gamma+1}{\gamma-1}\right]$$

Here n = 1

$$\therefore \qquad C=\frac{R}{2}\left(\frac{\gamma+1}{\gamma-1}\right).$$

Problem 2: *One mole of monoatomic ideal gas is taken through the cycle shown in figure.*

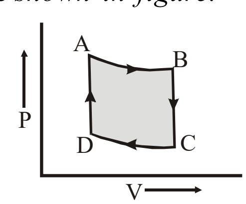
$A\rightarrow B$ Adiabatic expansion

$B\rightarrow C$ Cooling at constant volume

$C\rightarrow D$ Adiabatic compression

$D\rightarrow A$ Heating at constant volume

The pressure and temperature at A, B etc., are denoted by $P_A, T_A; P_B, T_B$ etc/ respectively.



Given $T_A = 1000K$, $P_B = (2/3)P_A$ *and* $P_C = (1/3)P_A$. *Calculate*

(a) The work done by the gas in the process $A \to B$

(b) The heat lost by the gas in the process $B \to C$ *and*

(c) Temperature T_D *given* $(2/3)^{2/5} = 0.85$ *and* $R = 8.31$ *J/mol K.*

Solution :

(a) As for adiabatic change $PV^\gamma = \text{constant}$

i.e. $P\left(\frac{\mu RT}{P}\right)^\gamma = \text{constant}$ [as $PV = \mu RT$]

i.e. $\frac{T^\gamma}{P^{\gamma-1}} = \text{constant}$ so $\left(\frac{T_B}{T_A}\right)^\gamma = \left(\frac{P_B}{P_A}\right)^{\gamma-1}$ where $\gamma = \frac{5}{3}$

i.e. $T_B = T_A\left(\frac{2}{3}\right)^{1-\frac{1}{\gamma}} = 1000\left(\frac{2}{3}\right)^{2/5} = 850K$

so $W_{AB} = \frac{\mu R[T_i - T_f]}{[\gamma - 1]} = \frac{1 \times 8.31[1000 - 850]}{[(5/3) - 1]}$

i.e. $W_{AB} = (3/2) \times 8.31 \times 150 = 1869.75$ J

(b) For $B \to C$, $V = \text{constant}$ so $\Delta W = 0$

so from first law of thermodynamics

$\Delta Q = \Delta U + \Delta W = \mu C_V \Delta T + 0$

or $\Delta Q = 1 \times \left(\frac{3}{2}R\right)(T_C - 850)$ as $C_v = \frac{3}{2}R$

Now along path BC, V = constant; $P \propto T$

i.e. $\frac{P_C}{P_B} = \frac{T_C}{T_B}$, $T_C = \frac{(1/3)P_A}{(2/3)P_A} \times T_B = \frac{T_B}{2} = \frac{850}{2} = 425$ K …(ii)

So $\Delta Q = 1 \times \frac{3}{2} \times 8.31(425 - 850) = -5297.625$ J

[Negative heat means, heat is lost by the system]

(c) D $\to$ A process is isochoric

$\frac{P_D}{P_A} = \frac{T_D}{T_A}$, i.e. $P_D = P_A \frac{T_D}{T_A}$

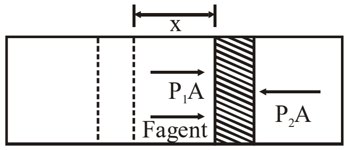
But C and D are on the same adiabatic

$$\left(\frac{T_D}{T_C}\right)^\gamma = \left(\frac{P_D}{P_C}\right)^{\gamma-1} = \left(\frac{P_A T_D}{P_C T_A}\right)^{\gamma-1}$$

or $(T_D)^{1/\gamma} = T_C\left[\frac{P_A}{P_C T_A}\right]^{1-\frac{1}{\gamma}}$, i.e. $T_C^{3/5} = \left(\frac{T_B}{2}\right)\left[\frac{P_A}{(1/3)P_A 1000}\right]^{2/5}$

i.e. $T_D^{3/5} = \left[\frac{1}{2}\left(\frac{2}{3}\right)^{2/3} \times 1000\right]\left[\frac{3}{1000}\right]^{2/5}$ i.e. $T_D = 500$ K

Problem 3: *A piston can freely move inside a horizontal cylinder closed from both ends. Initially, the piston separates the inside space of the cylinder into two equal parts each of volume V_0, in which an ideal gas is contained under the same pressure P_0 and at the same temperature. What work has to be performed in order to increase isothermally the volume of one part of gas η times to that of the other by slowly moving the piston ?*



Solution : Let the agent move as shown.

In equilibrium position,

$$P_1A + F_{\text{agent}} = P_2A$$

$$F_{\text{agent}} = (P_2 - P_1)A$$

Elementary work done by the agent

$$F_{\text{agent}}\,dx = (P_2 - P_1)A \times dx = (P_2 - P_1)dV \qquad \ldots \text{(i)}$$

Applying PV = constant for two parts, we have

$P_1(V_0 + Ax) = P_0V_0$ and $P_2(V_0 - Ax) = P_0V_0$

$$P_1 = \frac{P_0V_0}{(V_0 + Ax)} \quad \text{and} \quad P_2 = \frac{P_0V_0}{(V_0 - Ax)}$$

$$\therefore \qquad P_2 - P_1 = \frac{P_0V_0(2Ax)}{V_0^2 - A^2x^2} = \frac{2P_0V_0V}{V_0^2 - V^2}$$

When the volume of the left end is η times the volume of right end, we have

$$(V_0 + V) = \eta(V_0 - V)$$

$$V = \left(\frac{\eta - 1}{\eta + 1}\right)V_0 \qquad \ldots \text{(ii)}$$

The work done by the agent is given by

$$W = \int_0^V (P_2 - P_1)dV = \int_0^V \frac{2P_0V_0V}{(V_0^2 - V^2)}dV$$

$$= -P_0V_0[\ln\left(V_0^2 - V^2\right)]_0^V = -P_0V_0[\ln\left(V_0^2 - V^2\right) - \ln V_0^2]$$

$$= -P_0V_0\left[\ln\left\{V_0^2 - \left(\frac{\eta - 1}{\eta + 1}\right)^2 V_0^2\right\} - \ln V_0^2\right]$$

$$= -P_0V_0\left[\ln\{4\eta/(\eta + 1)^2\}\right] = P_0V_0\ln\left[\frac{(\eta + 1)^2}{4\eta}\right].$$

Problem 4: *Three moles of an ideal gas being initially at a temperature $T_0 = 273\ K$ were isothermally expanded $\eta = 5.0$ time its initial volume and then isochorically heated so that the pressure in the final state became equal to that in the initial state. The total amount of heat transferred to the gas during the process equals $Q = 80$ KJ. Find the ratio $\gamma = C_P / C_V$ for this gas.*

Solution : In isothermal process, the heat transferred to the gas is given by

$$Q_1 = nRT_0 \ln(V_2/V_1) = nRT_0 \ln\eta \qquad \ldots(i)$$

$[\because \quad \eta = (V_2/V_1) = (P_1/P_2)]$

In isochroric process, $Q_2 = \Delta U \qquad (W = 0)$

$$\therefore \qquad Q_2 = nC_V\Delta T = n\{R/(\gamma-1)\}\Delta T \qquad \ldots(ii)$$

Now $\dfrac{P_2}{P_1} = \dfrac{T_0}{T}$ or $T = T_0\left(\dfrac{P_1}{P_2}\right) = \eta T_0$

$$\therefore \qquad \Delta T = \eta T_0 - T_0 = (\eta - 1)T_0 \qquad \ldots(iii)$$

substituting the value of ΔT from equation (iii) in equation (ii), we get

$$Q_2 = n\left(\frac{R}{\gamma-1}\right)(\eta-1)T_0$$

$$\therefore \qquad Q = nRT_0\ln\eta + n\left(\frac{R}{\gamma-1}\right)(\eta-1)T_0$$

or $$\frac{Q}{nRT_0} - \ln\eta = \left(\frac{\eta-1}{\gamma-1}\right)$$

or $$\gamma - 1 = \frac{\eta-1}{\dfrac{Q}{nRT_0} - \ln\eta}$$

$$\therefore \qquad \gamma = 1 + \frac{\eta-1}{\dfrac{Q}{nRT_0} - \ln\eta}$$

Substituting given values, we get

$$\gamma = 1 + \frac{(5-1)}{\dfrac{80\times10^3}{3\times8.3\times273} - \ln5}$$

Solving we get $\gamma = 1.4$

Problem 5: *One end of a rod of length 20 cm is inserted in a furnace at 800 K. the sides of the rod are covered with an insulating material and the other end emits radiation like a blackbody. The temperature of this end is 750 K in the steady state. The temperature of the surrounding air is 300 K. Assuming radiation to be the only important mode of energy transfer between the surrounding and the open end of the rod, find the thermal conductivity of the rod. Stefan constant* $\sigma = 6.0\times10^{-8} W/m^2 - K^4$

Solution : Quantity of heat flowing through the rod in steady state

$$\frac{dQ}{dt} = \frac{K.A.d\theta}{x} \qquad \ldots(i)$$

Quantity of heat radiated from the end of the rod in steady state

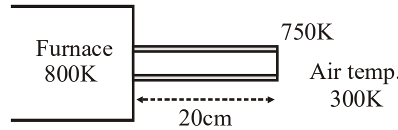
$$\frac{dQ}{dt} = A\sigma\left(T^4 - T_0^4\right) \qquad \ldots(ii)$$

From (i) and (ii)

$$\frac{K.d\theta}{x} = \sigma\left(T^4 - T_0^4\right)$$

$$\frac{K \times 50}{0.2} = 6.0 \times 10^{-8}[(7.5)^4 - (3)^4] \times 10^8$$

or $\quad$ K = 74 W/m – K.



Problem 6. *The intensity of solar radiation, just outside the earth's atmosphere, is measured to be 1.4* $\mathrm{kW/m^2}$. *If the radius of the sun* $7 \times 10^8\mathrm{m}$, *while the earth-sun distance is* $150 \times 10^6\,\mathrm{km}$, *then find*

(i) *the intensity of solar radiation at the surface of the sun,*

(ii) *the temperature at the surface of the sun assuming it to be a black body,*

(iii) *the most probable wavelength in solar radiation,*

Solution: Assuming the sun to be a "blackbody" at a temperature T_0, we can write,

W = intensity of solar radiation on the sun's surface $= \sigma T_0^{\,4}$,

Where σ is the Stefan-Boltzmann constant

(i) The radiation emitted from the solar surface per unit time is spread over the surface of a sphere having a radius equal to earth-sun distance where it is received on the earth (just outside the atmosphere)

$$\therefore \qquad W \times 4\pi R_S^2 = I_0 \times 4\pi D_{Se}^2$$

where D_{Se} is the distance between the sun and the earth, and I_0 is the intensity outside the earth's atmosphere.

$$I_0 = W \times \left(\frac{R_S}{D_{Se}}\right)^2$$

Now, $\quad R_S = 7 \times 10^8\,\mathrm{m}, D_{Se} = 150 \times 10^9\,\mathrm{m}$

and $\quad I_0 = 1.4 \times 10^3\ \mathrm{W/m^2}$.

$$\therefore \qquad 1.4 \times 10^3 = W \times \left(\frac{7 \times 10^8}{150 \times 10^9}\right)^2 = W \times \frac{49}{225} \times 10^{-4}$$

or $\quad W = 6.4 \times 10^7\,\mathrm{W/m^2}$

(ii) $\quad$ Assuming the sun to be a blackbody,

$$6.4 \times 10^7 = \sigma T_0^4 = \left(5.67 \times 10^{-8}\right) T_0^4$$

$$\therefore \qquad T_0^4 = \frac{6.4}{5.67} \times 10^{15}$$

or $\quad T_0 \approx 0.58 \times 10^4\,\mathrm{K} = 5800\mathrm{K}$

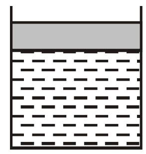
(iii) $\quad$ Using Wien's displacement law,

$\lambda_{mp} T_0 = 0.29\text{cm-K} = 2.9 \times 10^{-3}\text{m-K}$

or $$\lambda_{mp} = \frac{2.0 \times 10^{-3}}{5800} = 5 \times 10^{-7}\text{m} = 5000 A^\circ$$

[Note : λ_{mp} is also referred to as λ_{max}]

Problem 7. *Consider a lake that is getting frozen at an atmospheric temperature of -10 ° C. Assuming that most of the heat that is lost comes from the latent heat of fusion released when the water freezes. Find the rate at which the thickness of ice increases as a function of time. Take the conductivity of ice as K and the density of ice $\approx$ density of water $= \rho$*



Solution: The water just beneath the ice is almost at $0°C$. Assume that the thickness of ice at time t is $x(t)$, that the area of the lake is A_0 and that the density of ice is ρ.

If the latent heat of ice is *L*, then

$$\frac{dQ}{dt} = \frac{LA_0 dx\rho}{dt} = \frac{KA_0}{x}\left[0-(-10)\right] = \frac{10KA_0}{x}$$

or, $$\frac{dx}{dt} = \frac{10K}{xL\rho}$$

or, $$\int xdx = \frac{10K}{L\rho}\int dt$$

or $$\frac{x^2}{2} = \frac{10K}{L\rho}t + \text{constant}$$

At $t = 0$, we assume that $x = 0$: i.e. initially the lake is not frozen.

Therefore, $$x^2 = \frac{20K}{L\rho}t$$

or $$x(t) = \sqrt{\frac{20K}{L\rho}}\sqrt{t} = C\sqrt{t},$$

where $C = \sqrt{\frac{20K}{L\rho}}$ is a constant.

Problem 8. *A solid copper sphere of density ρ, specific heat c and radius r is at temperature T_1. It is suspended inside a chamber whose walls are at temperature 0 K. What is the time required for the temperature of sphere to drop to T_2? Take the emmissivity of the sphere to be equal to e.*

Solution: The rate of loss of energy due to radiation,

$P = eA\sigma T^4$

This rate must be equal to $mc\dfrac{dT}{dt}$.

Hence, $-mc\dfrac{dT}{dt} = eA\sigma T^4$

Negative sign is used as temperature decreases with time. In this equation,

$m = \left(\dfrac{4}{3}\pi r^3\right)\rho$ and $A = 4\pi r^2$

$$\therefore \qquad -\frac{dT}{dt} = \frac{3e\sigma}{\rho c r}T^4$$

$$\text{or,} \qquad -\int_0^t dt = \frac{r\rho c}{3e\sigma}\int_{T_1}^{T_2}\frac{dT}{T^4}$$

Solving this, we get $t = \dfrac{r\rho c}{9e\sigma}\left(\dfrac{1}{T_2^3} - \dfrac{1}{T_1^3}\right)$.

Problem 9. *As insulated container is divided into two equal portions. One portion contains an ideal gas at pressure P and temperature T, while the other portion is a perfect vacuum. If a hole is opened between the two portions, find the change in internal energy and temperature of the gas.*

Solution : As the system is thermally insulated,

$\Delta Q = 0$

Further as here the gas is expanding against vacuum (surroundings), the process is called free expansion and for it,

$\Delta W = \int PdV = 0$ [as for vacuum P = 0]

So in accordance with first law of thermodynamics, i.e. $\Delta Q = \Delta U + \Delta W$, we have

$0 = \Delta U + 0$, i.e. $\Delta U = 0$ or $U =$ constant

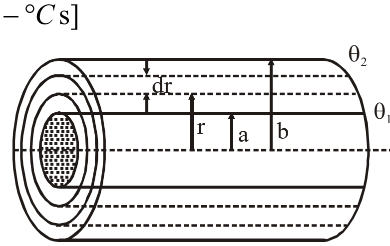
So in this problem internal energy of the gas remains constant, i.e. $\Delta U = 0$. Now as for an ideal gas

$U = \dfrac{3}{2}\mu RT$, i.e. $U \propto T$

So temperature of the gas will also remain constant, i.e. $\Delta T = 0$.

Problem 10: *A 2m long wire of resistance 4 ohm and diameter 0.64 mm is coated with plastic insulation of thickness 0.06 mm. When a current of 5 ampere flows through the wire, find the temperature difference across the insulation in steady state if*

$[K = 0.16 \times 10^{-2}\, cal/cm - °C\, s]$



Solution : Considering a concentric cylindrical shell of radius *r* and thickness *dr* as shown in figure. The radial rate of flow of heat through this shell in steady state will be

$H = \frac{dQ}{dt} = -KA\frac{d\theta}{dr}$ Negative sign is used as with increase in r, θ decreases

Now as for cylindrical shell A = $2\pi r$ L

$$H = -2\pi rLK\frac{d\theta}{dr}$$

or $$\int_a^b \frac{dr}{r} = -\frac{-2\pi LK}{H}\int_{\theta_1}^{\theta_2} d\theta$$

which on integration and simplification gives

$$H = \frac{dQ}{dt} = -\frac{2\pi LK(\theta_1 - \theta_2)}{\ln\left(b/a\right)} \quad \ldots\text{(i)}$$

Here, $H = \frac{I^2R}{4.2} = \frac{(5)^2 \times 4}{4.2} = 24\frac{\text{cal}}{\text{s}}$

$L = 2m = 200\,\text{cm}$

$r_1 = (0.64/2)$ mm= 0.032 cm and $R_2 = r_1 + d = 0.032 + 0.006 = 0.038$ cm

So $$(\theta_1 - \theta_2) = \frac{24 \times \ln\left(38/32\right)}{2 \times 2.3026[\log_{10} 38 - \log_{10} 32]}$$

$$= \frac{24 \times 2.3026[\log_{10} 38 - \log_{10} 32]}{3.14 \times 0.64}$$

or $$(\theta_1 - \theta_2) = \frac{55 \times [1.57 - 1.50]}{2} = 2\,^{\circ}\text{C}.$$