# Mathematics
# Class XII
# Sample Paper – 7

**Time: 3 hours** **Total Marks: 100**

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

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**SECTION – A**

**1.** If A =[$a_{ij}$], such that $a_{ij} = \frac{i+2j}{2}$, find the value of element at 3rd column and 2nd row.

**2.** Find $\frac{dy}{dx}$, if y + sin y = cos x

**3.** Determine the order and degree of the following differential equation:

$$\left(\frac{dy}{dx}\right)^2 + \frac{1}{\frac{dy}{dx}} = 2$$

**4.** Find the vector equation of line through point (5, 2, -4) and which is parallel to the vector $3\hat{i}+2\hat{j}-8\hat{k}$.

**OR**

Find the vector equation of a line passing through the points (-1, 0, 2) and (3, 4, 6).

**SECTION – B**

**5.** Prove that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b): |a – b| is even}, is an equivalence relation.

**6.** Find matrix A if

$$A + \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}.$$

**7.** $\int \sin x \sin 2x \sin 3x \, dx$

**8.** Evaluate: $\int \frac{2}{1-x \quad 1+x^2}\,dx$

**OR**

Evaluate:

$$\int \frac{\sin\ x-a}{\sin\ x+a}\,dx$$

**9.** Form a differential equation corresponding to $y^2 = a(b - x)(b + x)$, by eliminating parameters a and b

**10.** If $|\vec{a}| = 5, |\vec{b}| = 13, \vec{a}\cdot\vec{b} = 60$, find $|\vec{a}\times\vec{b}|$

**OR**

Find the value of $\lambda$ which makes the vectors $\vec{a}, \vec{b}$, and $\vec{c}$ coplanar, where $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - \lambda\hat{j} + 5\hat{k}$.

**11.** Find the probability distribution of the number of Kings drawn when two cards are drawn one by one, without replacement, from a pack of 52 playing cards

**12.** Two bags I and II contain 4 red, 3 black balls and 2 red and 4 black balls respectively. One bag is selected at random and from the bag selected, a ball is drawn. Find the probability that the ball is red.

**OR**

A company has two factories to manufacture machinery. Factory I manufactures 70% of the machinery and factory II manufactures 30% of the machinery. At factory I, 80% of the machinery are rated to be of a standard quality and at factory II 90% of the machinery are rated to be of a standard quality. A machine is chosen at random and is found to be of a standard quality. What is the probability that it came from factory II?

## SECTION – C

**13.** Let A = R × R and * be a binary operation on A defined by
(a, b) * (c, d) = (a + c, b + d) Show that * is commutative and associative.
Find the identity element for * on A. Also find the inverse of every element $(a, b) \in A$.

**OR**

Consider $f : R - \left\{-\frac{4}{3}\right\} \rightarrow R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$ Show that f is bijective.

Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x)=2$

**14.** Prove that $\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$

**15.** If a, b, and c are in A.P., find the value of the determinant

$$\Delta = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

**16.** If $y=\tan^{-1}\left\{\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right\}, -1<x<1, x \neq 0$, find $\frac{dy}{dx}$.

**OR**

If $\log\left(x^2+y^2\right)=2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx}=\frac{x+y}{x-y}$

**17.** If $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\tan^{-1}a$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

**18.** A balloon, which always remains spherical on inflation, is being by inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

**19.** Evaluate:

$$\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$$

**20.** Prove that, $\int_0^{\pi} \frac{x}{1+\sin x}\, dx=\pi$

**21.** Solve the differential equation:

$$\sin^{-1}\left(\frac{dy}{dx}\right)=x+y$$

**OR**

Solve the differential equation:

$$\frac{dy}{dx}=(4x+y+1)^2$$

**22.** The scalar product of the vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of $\lambda$ and hence find the unit vector along $\vec{b}+\vec{c}$.

**23.** Find the shortest distance between the lines given by $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$

**SECTION – D**

**24.** If $A=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$, then prove that $A^n=\begin{bmatrix}3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\end{bmatrix}$ for every positive integer n.

**OR**

Let $A=\begin{bmatrix}0&-\tan\left(\frac{x}{2}\right)\\\tan\left(\frac{x}{2}\right)&0\end{bmatrix}$ and I be the identity matrix of order 2.

Show that: $I + A = (I - A)\begin{bmatrix}\cos x&-\sin x\\\sin x&\cos x\end{bmatrix}$

**25.** A square piece of tin with side 18 cm is to be made into a box without a top by cutting a square piece from each corner and folding up the flaps. What should be the side of the square to be cut off, so that the volume of the box is the largest? Also find the maximum volume of the box.

**26.** Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bonded by x = 0, x = 4, y = 4, and y = 0 into three equal parts.

**OR**

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the straight line $\frac{x}{3}+\frac{y}{2}=1$.

**27.** Find the angle between the following pair of lines:

$$\frac{-x+2}{-2}=\frac{y-1}{7}=\frac{z+3}{-3} \text{ and } \frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$$

And check whether the lines are parallel or perpendicular.

**OR**

Write the vector equations of the following lines and hence determine the distance between them:

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6};\frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}$$

**28.** A brick manufacturer has two depots, A and B, with stocks of 30,000 and 20,000 bricks respectively. He receives orders from three builders P, Q and R for 15,000, 20,000 and 15,000 bricks respectively. The cost in rupees for transporting 1000 bricks to the builders from the depots are given below:

| From \ To | P | Q | R |
|---|---|---|---|
| A | 40 | 20 | 30 |
| B | 20 | 60 | 40 |

How should the manufacturer fulfil the orders so as to keep the cost of transportation minimum?

**29.** A factory manufactures screws. Machines X, Y and Z manufacture respectively 1000, 2000, and 3000 screws, of which 1%, 1.5% and 2 % of their outputs are respectively defective. A screw is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by machine X?

**Mathematics**
**Class XII**
**Sample Paper – 7 Solution**

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## SECTION – A

**1.** Element at 3rd column and 2nd row

So i =2 and j =3

Substituting in $a_{ij}=\frac{i+2j}{2}$ we get

$$a_{23}=\frac{2+2\times3}{2}=\frac{8}{2}=4$$

**2.** y + sin y = cos x

differentiating w.r.t. x, we get,

$$\frac{d}{dx}\left(y+\sin y\right)=\frac{d}{dx}\cos x$$
$$\frac{dy}{dx}+\cos y\frac{dy}{dx}=-\sin x$$
$$\frac{dy}{dx}=\frac{-\sin x}{1+\cos y}$$

**3.**

$$\left(\frac{dy}{dx}\right)^2+\frac{1}{\frac{dy}{dx}}=2$$

rearranging

$$\left(\frac{dy}{dx}\right)^3+1=2\frac{dy}{dx}$$

Order: 1

Degree:3

**4.** The vector equation of the line passing through the point (5, 2,-4) and parallel to $3\hat{i}+2\hat{j}-8\hat{k}$.

$$\vec{r}=\left(5\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}-8\hat{k}\right)$$

**OR**

The vector equation of the line passing through the points (-1, 0, 2) and (3, 4, 6) is

$\vec{r} = \left(-\hat{i} + 2\hat{k}\right) + \lambda\left(4\hat{i} + 4\hat{j} + 4\hat{k}\right)$

## SECTION – B

**5.**

A = {1, 2, 3, 4, 5}

R = {(a, b): $|a-b|$ is even}

For R to be an equivalence relation it must be

(i) Reflexive, $|a-a| = 0$

$\therefore (a,a) \in R$ for $\forall a \in A$

So R is reflexive.

(ii) Symmetric,

if $(a,b) \in R \Rightarrow |a-b|$ is even

$\Rightarrow |b-a|$ is also even

So R is symmetric.

(iii) Transitive

If (a, b) $\in$R (b, c) $\in$ R then (a, c) $\in$R

(a, b) $\in R \Rightarrow |a-b|$ is even

(b, c) $\in$ R $\Rightarrow |b-c|$ is even

Sum of two even numbers is even

So, $|a-b| + |b-c|$

$= |a-b+b-c| = |a-c|$ is even since, $|a-b|$ and $|b-c|$ are even

So (a, c) $\in$ R

Hence, R is transitive.

Therefore, R is an equivalence relation.

**6.**

Let $B=\begin{bmatrix}2 & 3\\ -1 & 4\end{bmatrix}$ and $C=\begin{bmatrix}3 & -6\\ -3 & 8\end{bmatrix}$.Then, the given matrix equation is A + B = C

now,

A + B – B = C – B

A = C – B

$$A=\begin{bmatrix}3 & -6\\ -3 & 8\end{bmatrix}-\begin{bmatrix}2 & 3\\ -1 & 4\end{bmatrix}$$

$$A=\begin{bmatrix}3-2 & -6-3\\ -3+1 & 8-4\end{bmatrix}$$

$$A=\begin{bmatrix}1 & -9\\ -2 & 4\end{bmatrix}$$

**7.**

It is known that, $\sin A\sin B=\frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$

$$\therefore \int \sin x\sin 2x\sin 3x\,dx=\int\left[\sin x\times\frac{1}{2}\left\{\cos(2x-3x)-\cos(2x+3x)\right\}\right]$$

$$=\frac{1}{2}\int\left(\sin x\cos(-x)-\sin x\cos 5x\right)dx$$

$$=\frac{1}{2}\int\left(\sin x\cos x-\sin x\cos 5x\right)dx$$

$$=\frac{1}{2}\int\frac{\sin 2x}{2}dx-\frac{1}{2}\int\sin x\cos 5x$$

$$=\frac{1}{4}\left[\frac{-\cos 2x}{2}\right]-\frac{1}{2}\int\frac{1}{2}\left\{\sin(x+5x)+\sin(x-5x)\right\}dx$$

$$=\frac{-\cos 2x}{8}-\frac{1}{4}\int\left(\sin 6x+\sin(-4x)\right)dx$$

$$=\frac{-\cos 2x}{8}-\frac{1}{4}\left[\frac{-\cos 6x}{6}+\frac{\cos 4x}{4}\right]+C$$

$$=\frac{-\cos 2x}{8}-\frac{1}{8}\left[\frac{-\cos 6x}{3}+\frac{\cos 4x}{2}\right]+C$$

$$=\frac{-6\cos 2x}{48}-\frac{1}{8}\left[\frac{-2\cos 6x+3\cos 4x}{6}\right]+C$$

$$=\frac{1}{48}\left[2\cos 6x-3\cos 4x-6\cos 2x\right]+C$$

**8.**

Let $\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$

$2 = A(1+x^2) + (Bx+C)(1-x)$

$2 = A + Ax^2 + Bx - Bx^2 + C - Cx$

Equating the coefficient of $x^2$, x, and constant term, we obtain
A − B = 0
B − C = 0
A + C = 2
On solving these equations, we obtain
A = 1, B = 1, and C = 1

$\therefore \frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$

$\Rightarrow \int \frac{2}{(1-x)(1+x^2)}dx = \int \frac{1}{1-x}dx + \int \frac{x}{1+x^2}dx + \int \frac{1}{1+x^2}dx$

$= -\int \frac{1}{x-1}dx + \frac{1}{2}\int \frac{2x}{1+x^2}dx + \int \frac{1}{1+x^2}dx$

$= -\log|x-1| + \frac{1}{2}\log|1+x^2| + \tan^{-1}x + C$

**OR**

$I = \int \frac{\sin(x-a)}{\sin(x+a)}dx$

Let (x + a) = t $\Rightarrow$ dx = dt

$\therefore I = \int \frac{\sin(t-2a)}{\sin t}dt$

$= \int \frac{\sin t \cos 2a - \cos t \sin 2a}{\sin t}dt$

$= \int (\cos 2a - \cot t \sin 2a)\,dt$

$= (\cos 2a)t - \sin 2a \log|\sin t| + C$

$= \cos 2a(x+a) - \sin 2a \log|\sin(x+a)| + C$

**9.** $y^2 = a(b - x)(b + x)$

$y^2 = a(b^2 - x^2)$

There are two arbitrary constants so we have to differentiate it two times

Differentiating w.r.t. x

$$2y\frac{dy}{dx} = -2ax$$

$$\frac{y}{x}\frac{dy}{dx} = -a.......(i)$$

$$y\frac{dy}{dx} = -a$$

Differentiating again

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -a$$

putting value of -a

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx} \qquad \text{from}(i)$$

**10.**

Let the angle between $\vec{a}$ and $\vec{b}$ be $\theta$ .

We know that $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Given $\vec{a}\cdot\vec{b} = 60$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = 60$

$\Rightarrow 13\times5\times\cos\theta = 60$

$\Rightarrow \cos\theta = \dfrac{60}{13\times5} = \dfrac{12}{13}$

$\Rightarrow \sin\theta = \sqrt{1 - \dfrac{144}{169}} = \dfrac{5}{13}$

Also we know that, $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

$|\vec{a}\times\vec{b}| = 5\times13\times\dfrac{5}{13} = 25$

**OR**

Vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar if their scalar product is zero

$$\vec{a}.\left(\vec{b}\times\vec{c}\right)=0$$

$$\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-3\hat{k} \text{ and } \vec{c}=3\hat{i}-\lambda\hat{j}+5\hat{k}$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & -\lambda & 5 \end{vmatrix}=0$$

$$2(10-3\lambda)+1(5+9)+1(-\lambda-6)=0$$
$$\Rightarrow 20-6\lambda+14-\lambda-6=0$$
$$\Rightarrow -7\lambda+28=0$$
$$\Rightarrow -7\lambda=-28$$
$$\Rightarrow \lambda=4$$

**11.** Let X represent the number of Kings drawn and the event here is to successfully draw a King. So X = 0, 1, 2

$$P(0)=\frac{^{48}C_2}{^{52}C_2}=\frac{48\times47}{52\times51}=\frac{188}{221}$$
$$P(1)=\frac{^{48}C_1\times{}^{4}C_1}{^{52}C_2}=\frac{4\times48\times2}{52\times51}=\frac{32}{221}$$
$$P(2)=\frac{^{48}C_0\times{}^{4}C_2}{^{52}C_2}=\frac{4\times3}{52\times51}=\frac{1}{221}$$

**12.** Let $E_1$: First Bag is selected; $E_2$: Second Bag is selected

and let E: A red ball is drawn

We note that $E_1 \cap E_2 = \phi$ and $E_1 \cup E_2 = S$

$\Rightarrow E_1$ and $E_2$ are mutually exclusive and exhaustive events .

Now $P(E_1)=\frac{1}{2}; P(E_2)=\frac{1}{2}$

$$P(E)=P(E|E_1)P(E_1)+P(E|E_2)P(E_2)$$

$$P(E)=P(E|E_1)\frac{1}{2}+P(E|E_2)\frac{1}{2}=\frac{4}{4+3}\times\frac{1}{2}+\frac{2}{2+4}\times\frac{1}{2}=\frac{2}{7}+\frac{1}{6}=\frac{19}{42}$$

**OR**

Let $E_1$: First factory manufactured the machinery;

$E_2$ : Second factory manufactured the machinery

and let E: selected machinery is of standard quality

We note that $E_1 \cap E_2 = \phi$ and $E_1 \cup E_2 = S$

$\Rightarrow E_1$ and $E_2$ are mutually exclusive and exhaustive events .

Now $P(E_1) = \frac{7}{10}; P(E_2) = \frac{30}{100} = \frac{3}{10}$

$P(E|E_1) = \frac{80}{100} = \frac{8}{10}; P(E|E_2) = \frac{90}{100} = \frac{9}{10}$

$$P(E_2|E) = \frac{P(E|E_2)P(E_2)}{P(E|E_2)P(E_2) + P(E|E_1)P(E_1)} = \frac{\frac{3}{10} \times \frac{9}{10}}{\frac{3}{10} \times \frac{9}{10} + \frac{7}{10} \times \frac{8}{10}} = \frac{27}{56+27} = \frac{27}{83}$$

**SECTION – C**

**13.** (a, b) * (c, d) = (a + c, b + d)

(i) Commutative

(a, b) * (c, d) = (a + c, b + d)

(c, d) * (a, b) = (c + a, d + b)

for all, a, b, c, d $\in$ R

* is commutative on A

(ii) Associative : ______

(a, b), (c, d), (e, f) $\in$A

{ (a, b) * (c, d) } * (e, f)

= (a + c, b + d) * (e, f)

= ((a + c) + e, (b + d) + f)

= (a + (c + e), b + (d + f))

= (a*b) * ( c + d, d + f)

= (a*b) {(c, d) * (e, f)}

is associative on A

Let (x, y) be the identity element in A.

then,
(a, b) * (x, y) = (a, b) for all (a, b) $\in$A

(a + x, b + y) = (a, b) for all (a, b) $\in$A

a + x = a, b + y = b for all (a, b) $\in$A

x = 0, y = 0

(0, 0) $\in$A

(0, 0) is the identity element in A.

Let (a, b) be an invertible element of A.

(a, b) * (c, d) = (0, 0) = (c, d) * (a, b)

(a + c, b + d) = (0, 0) = (c + a, d + b)

a + c = 0,  b + d = 0

a = - c      b = - d

c = - a      d = - b

(a, b) is an invertible element of A, in such a case the inverse of (a, b) is (-a, -b)

**OR**

Given that $f(x)=\frac{4x+3}{3x+4}$

For one – one function,

$f(x_1)=f(x_2)$

$\frac{4x_1+3}{3x_1+4}=\frac{4x_2+3}{3x_2+4}$

$(4x_1+3)(3x_2+4)=(4x_2+3)(3x_1+4)$

$12x_1x_2+16x_1+9x_2+12=12x_1x_2+16x_2+9x_1+12$

$7x_1=7x_2$

$x_1=x_2$

Hence, f(x) is one – one.

For onto function,

$y=\frac{4x+3}{3x+4}$

$y(3x+4)=4x+3$

$3xy+4y=4x+3$

$4y-3=4x-3xy$

$4y-3=x(4-3y)$

$x=\frac{4y-3}{4-3y}$

$$f\left(\frac{4y-3}{4-3y}\right)=\frac{4\left(\frac{4y-3}{4-3y}\right)+3}{3\left(\frac{4y-3}{4-3y}\right)+4}$$

$$=\frac{16y-12+12-9y}{12y-9+16-12y}$$

$$=y$$

So, the function is onto.

Hence, function is bijective.

$f^{-1}(x)=\frac{4x-3}{4-3x}$

$f^{-1}(0)=\frac{-3}{4}$

To find x such that $f^{-1}(x)=2$,

$\frac{4x-3}{4-3x}=2$

$4x-3=2(4-3x)$

$4x-3=8-6x$

$10x=11$

$x=\frac{11}{10}$

**14.**

$$\text{To prove}: \sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$$

$$\text{Let } \sin^{-1}\left(\frac{4}{5}\right)=x$$

$$\Rightarrow \sin x=\frac{4}{5}$$

$$\Rightarrow \cos x=\sqrt{1-\sin^2 x}=\frac{3}{5}$$

$$\sin^{-1}\left(\frac{5}{13}\right)=y$$

$$\Rightarrow \sin y=\frac{5}{13}$$

$$\Rightarrow \cos y=\sqrt{1-\sin^2 y}=\frac{12}{13}$$

$$\sin^{-1}\left(\frac{16}{65}\right)=z$$

$$\Rightarrow \sin z=\frac{16}{65}$$

$$\Rightarrow \cos z=\sqrt{1-\sin^2 z}=\frac{63}{65}$$

$$\tan x=\frac{4}{3}, \tan y=\frac{5}{12}, \tan z=\frac{16}{63}$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x.\tan y}=\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{20}{36}}=\frac{63}{16}=\cot z$$

$$\tan(x+y)=\tan\left(\frac{\pi}{2}-z\right)\Rightarrow x+y+z=\frac{\pi}{2}$$

$$\therefore \sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$$

**15.**

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

a, b, and c are in A.P.

$b - a = c - b$

$\Rightarrow 2b = a + c \Rightarrow (a+c) - 2b = 0 \qquad \text{(i)}$

Performing the operation : $R_1 \to R_1 + R_3 - 2R_2$

$$\Delta = \begin{vmatrix} 0 & 0 & a+c-2b \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

But, $(a+c) - 2b = 0$ using this in above determinant

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

Since a row of the determinant is zero so $\Delta = 0$

**16.**

Putting $x^2 = \cos 2\theta$, we get

$$y = \tan^{-1}\left\{\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}\right\}$$

$$= \tan^{-1}\left\{\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right\}$$

$$= \tan^{-1}\left\{\frac{1+\tan\theta}{1-\tan\theta}\right\}$$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4}+\theta\right)\right\}$$

$$y = \frac{\pi}{4}+\theta$$

$$y = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$$

$$\frac{dy}{dx} = 0+\frac{-1}{2}\times\frac{1}{\sqrt{1-x^4}}\times 2x$$

$$= \frac{-x}{\sqrt{1-x^4}}$$

**OR**

Differentiating both sides of the given relation w.r.t. x, we get

$$\Rightarrow \frac{d}{dx}\left\{\log\left(x^2+y^2\right)\right\} = 2\frac{d}{dx}\left\{\tan^{-1}\left(\frac{y}{x}\right)\right\}$$

$$\Rightarrow \frac{1}{x^2+y^2}\times\frac{d}{dx}\left(x^2+y^2\right) = 2\times\frac{1}{1+\left(\frac{y}{x}\right)^2}\times\frac{d}{dx}\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{1}{x^2+y^2}\times\left\{2x+2y\frac{dy}{dx}\right\} = \frac{2}{x^2+y^2}\left\{x\frac{dy}{dx}-y\right\}$$

$$\Rightarrow x+y\frac{dy}{dx} = x\frac{dy}{dx}-y$$

$$\Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y}$$

**17.**

we have,

$$\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\tan^{-1}a$$

$$\Rightarrow \frac{x^2-y^2}{x^2+y^2}=\cos\left(\tan^{-1}a\right)=k.....\text{say a constant}$$

by componendo − dividendo

$$\Rightarrow \frac{2x^2}{-2y^2}=\frac{k+1}{k-1}$$

$$\Rightarrow \frac{x^2}{y^2}=\frac{1+k}{1-k}$$

differentiating, w.r.t. x

$$\frac{d}{dx}\left(\frac{x^2}{y^2}\right)=\frac{d}{dx}\left(\frac{1+k}{1-k}\right)$$

$$\frac{y^2\frac{d}{dx}x^2-x^2\frac{d}{dx}y^2}{y^4}=0$$

$$y^2\frac{d}{dx}x^2-x^2\frac{d}{dx}y^2=0$$

$$2xy^2-2x^2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{y}{x}$$

**18.**

The volume of a sphere(V) with radius (r) is given by,

$$V = \frac{4}{3}\pi r^3$$

$\therefore$ Rate of change of volume (V) w.r.t. (t) is given by,

$$\frac{dV}{dt} = \frac{dV}{dr}.\frac{dr}{dt}$$
$$= \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right).\frac{dr}{dt}$$
$$= 4\pi r^2.\frac{dr}{dt}$$

It is given that $\frac{dV}{dt} = 900\ cm^3 / s$

$$\therefore 900 = \frac{dr}{dt}$$
$$\Rightarrow \frac{dr}{dt} = \frac{900}{4\pi r^2} = \frac{225}{\pi r^2}$$

Therefore, when radius = 15 cm

$$\frac{dr}{dt} = \frac{225}{\pi(15)^2} = \frac{1}{\pi}$$

Hence, the rate at which the radius of the balloon increases when the radius is 15 cm is $\frac{1}{\pi} cm / s.$

**19.**

We need to evaluate the integral

$$\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$$

Let $I=\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$

Consider the integrand as follows:

$$\frac{x+2}{\sqrt{x^2+5x+6}}=\frac{A\frac{d}{dx}\left(x^2+5x+6\right)+B}{\sqrt{x^2+5x+6}}$$

$$\Rightarrow x+2=A\left(2x+5\right)+B$$

$$\Rightarrow x+2=\left(2A\right)x+5A+B$$

Comparing the coefficients, we have

$2A=1; 5A+B=2$

Solving the above equations, we have

$A=\frac{1}{2}$and $B=-\frac{1}{2}$

Thus,

$$I=\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$$

$$=\int \frac{\frac{2x+5}{2}-\frac{1}{2}}{\sqrt{x^2+5x+6}}dx$$

$$=\frac{1}{2}\int \frac{2x+5}{\sqrt{x^2+5x+6}}dx-\frac{1}{2}\int \frac{1}{\sqrt{x^2+5x+6}}dx$$

$$I=\frac{1}{2}I_1-\frac{1}{2}I_2,$$

where $I_1 = \int \frac{2x+5}{\sqrt{x^2+5x+6}} dx$

and $I_2 = \int \frac{1}{\sqrt{x^2+5x+6}} dx$

Now consider $I_1$ :

$$I_1 = \int \frac{2x+5}{\sqrt{x^2+5x+6}} dx$$

Substitute

$x^2+5x+6 = t; (2x+5)dx = dt$

$$I_1 = \int \frac{dt}{\sqrt{t}}$$

$$= 2\sqrt{t}$$

$$= 2\sqrt{x^2+5x+6}$$

Now consider $I_2$ :

$$I_2 = \int \frac{1}{\sqrt{x^2+5x+6}} dx$$

$$= \int \frac{1}{\sqrt{x^2+5x+\left(\frac{5}{2}\right)^2+6-\left(\frac{5}{2}\right)^2}} dx$$

$$= \int \frac{1}{\sqrt{\left(x+\frac{5}{2}\right)^2+6-\frac{25}{4}}} dx$$

$$= \int \frac{1}{\sqrt{\left(x+\frac{5}{2}\right)^2-\frac{1}{4}}} dx$$

$$= \int \frac{1}{\sqrt{\left(x+\frac{5}{2}\right)^2-\left(\frac{1}{2}\right)^2}} dx$$

$$I_2 = \log\left|x+\frac{5}{2}-\sqrt{x^2+5x+6}\right| + C$$

Thus, $I = \frac{1}{2}I_1 - \frac{1}{2}I_2$

$$I = \sqrt{x^2+5x+6} - \frac{1}{2}\log\left|x+\frac{5}{2}-\sqrt{x^2+5x+6}\right| + C$$

**20.**

Let $I = \int_0^{\pi} \frac{x}{1+\sin x}\,dx$ ……………..(i)

[By property of definite integrals]

$$I = \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)}\,dx \qquad \text{using} \quad \int_0^{a} f(x)dx = \int_0^{a} f(a-x)dx$$

$$\Rightarrow \quad I = \int_0^{\pi} \frac{\pi - x}{1+\sin x}\,dx$$

$$= \pi \int_0^{\pi} \frac{1}{1+\sin x}\,dx - \int_0^{\pi} \frac{x}{1+\sin x}\,dx$$

$$= \pi \int_0^{\pi} \frac{1 - \sin x}{1 - \sin^2 x}\,dx - I; \qquad \text{(using (i))}$$

$$\Rightarrow \quad 2I = \pi \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x}\,dx$$

$$= \pi \int_0^{\pi} \left(\sec^2 x - \tan x.\sec x\right)dx$$

$$= \pi\left[\tan x - \sec x\right]_0^{\pi}$$

$2I = \pi\ [(\tan\pi - \sec\pi) - (\tan 0 - \sec 0)]$

$2I = \pi\ [0 - (-1) - (0 - 1)] = 2\pi$

$\therefore\ I = \pi$

**21.**

$$\sin^{-1}\left(\frac{dy}{dx}\right)=x+y$$

$$\Rightarrow \frac{dy}{dx}=\sin(x+y)$$

put $x+y=v$

$$1+\frac{dy}{dx}=\frac{dv}{dx}$$

$$\Rightarrow \frac{dv}{dx}-1=\sin(v)$$

$$\Rightarrow \frac{dv}{dx}=1+\sin(v)$$

$$\Rightarrow \frac{1}{1+\sin(v)}dv=dx$$

$$\Rightarrow \int\frac{1}{1+\sin(v)}dv=\int dx$$

$$\Rightarrow \int\frac{1-\sin(v)}{1-\sin^2(v)}dv=\int dx$$

$$\Rightarrow \int\frac{1-\sin(v)}{\cos^2(v)}dv=\int dx$$

$$\Rightarrow \int dx=\int\left(\sec^2 v-\tan v.\sec v\right)dv$$

$$\Rightarrow x=\tan v-\sec v+c$$

$\Rightarrow x=\tan(x+y)-\sec(x+y)+c$......which is required solution

**OR**

$$\frac{dy}{dx}=\left(4x+y+1\right)^2$$

$$\Rightarrow \text{let } 4x+y+1=v$$

$$\Rightarrow 4+\frac{dy}{dx}=\frac{dv}{dx}$$

$$\Rightarrow \frac{dv}{dx}-4=v^2$$

$$\Rightarrow \frac{dv}{dx}=v^2+4$$

$$\Rightarrow \int \frac{1}{v^2+4}dv=\int dx$$

$$\Rightarrow \frac{1}{2}\tan^{-1} v=x+c$$

$$\Rightarrow \frac{1}{2}\tan^{-1}\left(4x+y+1\right)=x+c$$

as required

**22.** Given that

$\vec{b} = 2\hat{i} + 4\hat{j} - 5k$

$\vec{c} = \lambda\hat{i} + 2\hat{j} + 3k$

Now consider the sum of the vectors $\vec{b} + \vec{c}$:

$\vec{b} + \vec{c} = \left(2\hat{i} + 4\hat{j} - 5k\right) + \left(\lambda\hat{i} + 2\hat{j} + 3k\right)$

$\Rightarrow \vec{b} + \vec{c} = (2 + \lambda)\hat{i} + 6\hat{j} - 2k$

Let $\hat{n}$ be the unit vector along the sum of vectors $\vec{b} + \vec{c}$ :

$$\hat{n} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2k}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}}$$

The scalar product of $\vec{a}$ and n is 1. Thus,

$$\vec{a} \cdot \hat{n} = \left(\hat{i} + \hat{j} + k\right) \cdot \left( \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2k}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}} \right)$$

$$\Rightarrow 1 = \frac{1(2+\lambda) + 1 \cdot 6 - 1 \cdot 2}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}}$$

$$\Rightarrow \sqrt{(2+\lambda)^2 + 6^2 + 2^2} = 2 + \lambda + 6 - 2$$

$$\Rightarrow \sqrt{(2+\lambda)^2 + 6^2 + 2^2} = \lambda + 6$$

$$\Rightarrow (2+\lambda)^2 + 40 = (\lambda + 6)^2$$

$$\Rightarrow \lambda^2 + 4\lambda + 4 + 40 = \lambda^2 + 12\lambda + 36$$

$$\Rightarrow 4\lambda + 44 = 12\lambda + 36$$

$$\Rightarrow 8\lambda = 8$$

$$\Rightarrow \lambda = 1$$

Thus, n is:

$$n = \frac{(2+1)\hat{i} + 6\hat{j} - 2k}{\sqrt{(2+1)^2 + 6^2 + 2^2}}$$

$$\Rightarrow n = \frac{3\hat{i} + 6\hat{j} - 2k}{\sqrt{3^2 + 6^2 + 2^2}}$$

$$\Rightarrow \hat{n} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{49}}$$

$$\Rightarrow \hat{n} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7} \Rightarrow \hat{n} = \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$$

**23.**

$$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$$

The vector form of this equation is:

$$\vec{r}=\left(3\hat{i}+5\hat{j}+7\hat{k}\right)+\lambda\left(\hat{i}-2\hat{j}+\hat{k}\right)$$

$$\vec{r}=\overrightarrow{a_1}+\lambda\vec{b}_1 \qquad \ldots (1)$$

$$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$$

The vector form of this equation is:

$$\vec{r}=\left(-\hat{i}-\hat{j}-\hat{k}\right)+\lambda\left(7\hat{i}-6\hat{j}+\hat{k}\right)$$

$$\vec{r}=\vec{a}_2+\lambda\vec{b}_2$$

Therefore, $\vec{a}_1=3\hat{i}+5\hat{j}+7\hat{k}, \vec{b}_1=\hat{i}-2\hat{j}+\hat{k}, \vec{a}_2=-\hat{i}-\hat{j}-\hat{k}$ and $\vec{b}_2=7\hat{i}-6\hat{j}+\hat{k}$

Now, the shortest distance between these two lines is given by:

$$d=\left|\frac{\left(\vec{b}_1\times\vec{b}_2\right).\left(\vec{a}_2-\vec{a}_1\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 1\\ 7 & -6 & 1\end{vmatrix}$$

$$=\hat{i}\left(-2+6\right)-\hat{j}\left(1-7\right)+\hat{k}\left(-6+14\right)$$

$$=4\hat{i}+6\hat{j}+8\hat{k}$$

$$\Rightarrow\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{4^2+6^2+8^2}=\sqrt{116}$$

$$\vec{a}_2-\vec{a}_1=\left(-\hat{i}-\hat{j}-\hat{k}\right)-\left(3\hat{i}+5\hat{j}+7\hat{k}\right)$$

$$=-4\hat{i}-6\hat{j}-8\hat{k}$$

$$\therefore d=\left|\frac{\left(4\hat{i}+6\hat{j}+8\hat{k}\right).\left(-4\hat{i}-6\hat{j}-8\hat{k}\right)}{\sqrt{116}}\right|=\left|\frac{-16-36-64}{\sqrt{116}}\right|=\left|\frac{-116}{\sqrt{116}}\right|=\sqrt{116}$$

## SECTION – D

**24.**

We shall prove this result by using mathematical induction on n

Step 1:

When n = 1, we have

$$A^1 = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

So, result is true for n = 1

Step 2:

Let the result be true for n = k

$$A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

Step 3:

Now we shall prove that the result is true for n = k + 1

$$A^{k+1} = \begin{bmatrix} 3^{k+1-1} & 3^{k+1-1} & 3^{k+1-1} \\ 3^{k+1-1} & 3^{k+1-1} & 3^{k+1-1} \\ 3^{k+1-1} & 3^{k+1-1} & 3^{k+1-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

So it holds true for n = k + 1

Thus in general we can say that

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$$

**OR**

We have,

$$I+A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & -\tan\left(\frac{x}{2}\right)\\ \tan\left(\frac{x}{2}\right) & 0\end{bmatrix}=\begin{bmatrix}1 & -\tan\left(\frac{x}{2}\right)\\ \tan\left(\frac{x}{2}\right) & 1\end{bmatrix}$$

$$I-A=\begin{bmatrix}1 & \tan\left(\frac{x}{2}\right)\\ -\tan\left(\frac{x}{2}\right) & 1\end{bmatrix}$$

$$(I-A)\begin{bmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{bmatrix}=\begin{bmatrix}1 & \tan\left(\frac{x}{2}\right)\\ -\tan\left(\frac{x}{2}\right) & 1\end{bmatrix}\begin{bmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{bmatrix}$$

$$=\begin{bmatrix}1 & \tan\left(\frac{x}{2}\right)\\ -\tan\left(\frac{x}{2}\right) & 1\end{bmatrix}\begin{bmatrix}\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)} & -\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\\ \frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)} & \frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\end{bmatrix}$$

for simplicity take $\tan\left(\frac{x}{2}\right)=t$

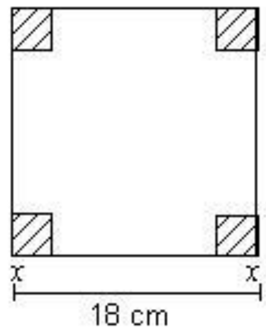
$$=\begin{bmatrix}1 & t\\ -t & 1\end{bmatrix}\begin{bmatrix}\frac{1-t}{1+t^2} & -\frac{2t}{1+t^2}\\ \frac{2t}{1+t^2} & \frac{1-t}{1+t^2}\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1+t^2}{1+t^2} & -\frac{t(1+t^2)}{1+t^2}\\ \frac{t(1+t^2)}{1+t^2} & \frac{1+t^2}{1+t^2}\end{bmatrix}$$

$$=\begin{bmatrix}1 & -t\\ t & 1\end{bmatrix}=\begin{bmatrix}1 & -\tan\left(\frac{x}{2}\right)\\ \tan\left(\frac{x}{2}\right) & 1\end{bmatrix}=I+A\text{.......hence proved}$$

**25.**

Let the side of the square piece cut from each corner of the given square plate (side = 18 cm) be x cm. Then the open box has dimensions 18 - 2x, 18 - 2x, x (in cm)




V = Volume of the open box

$= (18 - 2x)^2 . x$

$= 324x - 72x^2 + 4x^3$

$\therefore \frac{dV}{dx} = 324 - 144x + 12x^2$

and $\frac{d^2V}{dx^2} = -144 + 24x.$

For maxima or minima,

$\frac{dV}{dx} = 0$

$\Rightarrow 324 - 144x + 12x^2 = 0$

$\therefore x^2 - 12x + 27 = 0$

So x = 3, 9

Clearly x = 3

$$\left(\frac{d^2V}{dx^2}\right)_{x=3} = -144 + 24 \times 3 = -72 < 0.$$

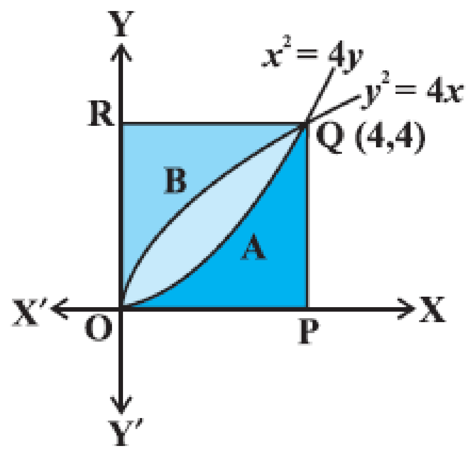
So x = 3 is a point of maxima

∴Volume is maximum, when side of the square cut off is 3 cm.

Maximum volume of box $= \left(18 - 2 \times 3\right)^2 \times 3 = 432 \text{ cm}^3$

**26.**

The point of intersection of the parabolas $y^2 = 4x$ and $x^2 = 4y$ are (0, 0) and (4, 4)




Now, the area of the region OAQBO bounded by curves $y^2 = 4x$ and $x^2 = 4y$

$$\int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \left[2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{12}\right]_0^4 = \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{ sq. units} \qquad \text{(i)}$$

Again, the area of the region OPQAO bounded by the curves $x^2 = 4y$, $x = 0$, $x = 4$ and the x-axis.

$$\int_0^4 \frac{x^2}{4} dx = \left[\frac{x^3}{12}\right]_0^4 = \left(\frac{64}{12}\right) = \frac{16}{3} \text{ sq units} \qquad \text{(ii)}$$

Similarly, the area of the region OBQRO bounded by the curve $y^2 = 4x$ and the y-axis,

$y = 0$ and $y = 4$

$$\int_0^4 \frac{y^2}{4} dy = \left[\frac{y^3}{12}\right]_0^4 = \frac{16}{3} \text{ sq units} \qquad \text{(iii)}$$

From (i) (ii), and (iii) it is concluded that

the area of the region OAQBO = area of the region OPQAO = area of the region OBQRO, i.e., area bounded by parabolas $y^2 = 4x$ and $x^2 = 4y$ divides the area of the square in three equal parts.

**OR**

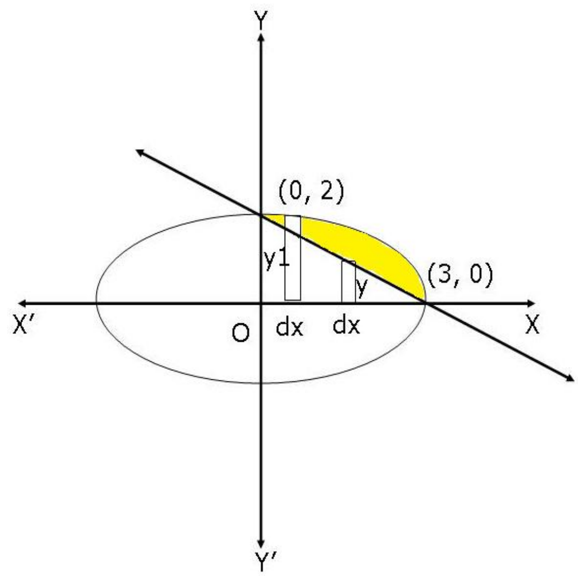
Given ellipse

$$\Rightarrow \frac{x^2}{9}+\frac{y^2}{4}=1$$

$$\Rightarrow y=\frac{2}{3}\sqrt{9-x^2}$$

Given line $\frac{x}{3}+\frac{y}{2}=1$

$$\Rightarrow y=\left(2-\frac{2x}{3}\right)$$




Required Area = $\int_0^3 (y_1 - y_2)\, dx$

$$=\int_0^3\left[\frac{2}{3}\sqrt{9-x^2}-\left(2-\frac{2x}{3}\right)\right]dx$$

$$=\left[\frac{2}{3}\left(\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right)-2x+\frac{x^2}{3}\right]_0^3$$

$$=\left[\frac{2}{3}\left(\frac{9}{2}\sin^{-1}1\right)-6+3\right]-0$$

$$=3\times\frac{\pi}{2}-3=\frac{3}{2}(\pi-2)\text{ square units}$$

**27.** Let $\vec{b}_1$ and $\vec{b}_2$ be the vector parallel to the pair to lines,

$\frac{-x+2}{-2}=\frac{y-1}{7}=\frac{z+3}{-3}$ and $\frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$, respectively.

Now, $\frac{-x+2}{-2}=\frac{y-1}{7}=\frac{z+3}{-3}$

$\Rightarrow \frac{x-2}{2}=\frac{y-1}{7}=\frac{z+3}{-3}$

$\frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$

$\Rightarrow \frac{x+2}{-1}=\frac{y-4}{2}=\frac{z-5}{4}$

$\therefore \vec{b}_1 = 2\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{b}_2 = -\hat{i} + 2\hat{j} + 4\hat{k}$

$|\vec{b}_1| = \sqrt{(2)^2+(7)^2+(-3)^2} = \sqrt{62}$

$|\vec{b}_2| = \sqrt{(-1)^2+(2)^2+(4)^2} = \sqrt{21}$

$\vec{b}_1.\vec{b}_2 = (2\hat{i} + 7\hat{j} - 3\hat{k}).(-\hat{i} + 2\hat{j} + 4\hat{k})$

$= 2(-1) + 7\times 2 + (-3).4$

$= -2 + 14 - 12$

$= 0$

The angle θ between the given pair of lines is given by the relation,

$\cos\theta = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

$\Rightarrow \cos\theta = \frac{0}{\sqrt{62}\times\sqrt{21}} = 0$

$\Rightarrow \theta = \cos^{-1}(0) = \frac{\pi}{2}$

Thus, the given lines are perpendicular to each other and the angle between them is 90°.

**OR**

Given equation of line is $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$

This can also be written in the standard form as $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-(-4)}{6}$

The vector form of the above equation is,

$$\vec{r}=(\hat{i}+2\hat{j}-4k)+\lambda(2\hat{i}+3\hat{j}+6k)$$
$$\Rightarrow \vec{r}=\overrightarrow{a_1}+\lambda\vec{b} \quad \ldots(1)$$
where, $\overrightarrow{a_1}=\hat{i}+2\hat{j}-4k$ and $\vec{b}=2\hat{i}+3\hat{j}+6k$

The second equation of line is $\frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}$

The above equation can also be written as $\frac{x-3}{4}=\frac{y-3}{6}=\frac{z-(-5)}{12}$

The vector form of this equation is

$$\vec{r}=(3\hat{i}+3\hat{j}-5k)+\mu(4\hat{i}+6\hat{j}+12k)$$
$$\Rightarrow \vec{r}=(3\hat{i}+3\hat{j}-5k)+2\mu(2\hat{i}+3\hat{j}+6k)$$
$$\Rightarrow \vec{r}=\overrightarrow{a_2}+2\mu\vec{b}\ldots(2)$$
where $\overrightarrow{a_2}=3\hat{i}+3\hat{j}-5k$ and $\vec{b}=2\hat{i}+3\hat{j}+6k$

Since $\vec{b}$ is same in equations (1) and (2), the two lines are parallel.

Distance d, between the two parallel lines is given by the formula,

$$d=\left|\frac{\vec{b}\times(\overrightarrow{a_2}-\overrightarrow{a_1})}{|\vec{b}|}\right|$$

Here, $\vec{b}=(2\hat{i}+3\hat{j}+6k)$ $\quad \overrightarrow{a_2}=(3\hat{i}+3\hat{j}-5k)$ and $\overrightarrow{a_1}=(\hat{i}+2\hat{j}-4k)$

On substitution, we get

$$d = \left| \frac{(2\hat{i} + 3\hat{j} + 6k) \times ((3\hat{i} + 3\hat{j} - 5k - (\hat{i} + 2\hat{j} - 4k))}{\sqrt{4 + 9 + 36}} \right|$$

$$= \frac{1}{\sqrt{49}} \left| (2\hat{i} + 3\hat{j} + 6k) \times (2\hat{i} + \hat{j} - k) \right|$$

$$= \frac{1}{7} \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \frac{1}{7} \left| \hat{i}(-3 - 6) - \hat{j}(-2 - 12) + k(2 - 6) \right|$$

$$= \frac{1}{7} \left| -9\hat{i} + 14\hat{j} - 4k \right|$$

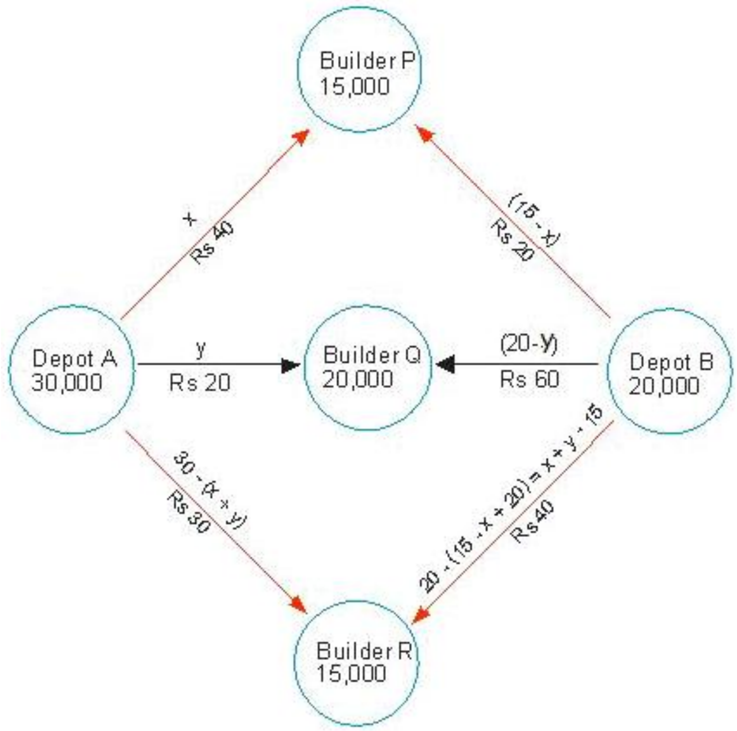
$$= \frac{1}{7} \left| \sqrt{81 + 196 + 16} \right|$$

$$= \frac{\sqrt{293}}{7}$$

Thus, the distance between the two given lines is $\frac{\sqrt{293}}{7}$

**28.**

Let the depot A transport x thousand bricks to builder P and y thousand bricks to builder Q and 30 - (x + y) thousand bricks to builder R. Let the depot B transport (15 – x) thousand bricks to builder P and (20 – y) thousand bricks to builder Q and (x + y) - 15 thousand bricks to builder R




Then, the LPP can be stated mathematically as follows:

Minimise Z = 30x -30y+ 1800

Subject to

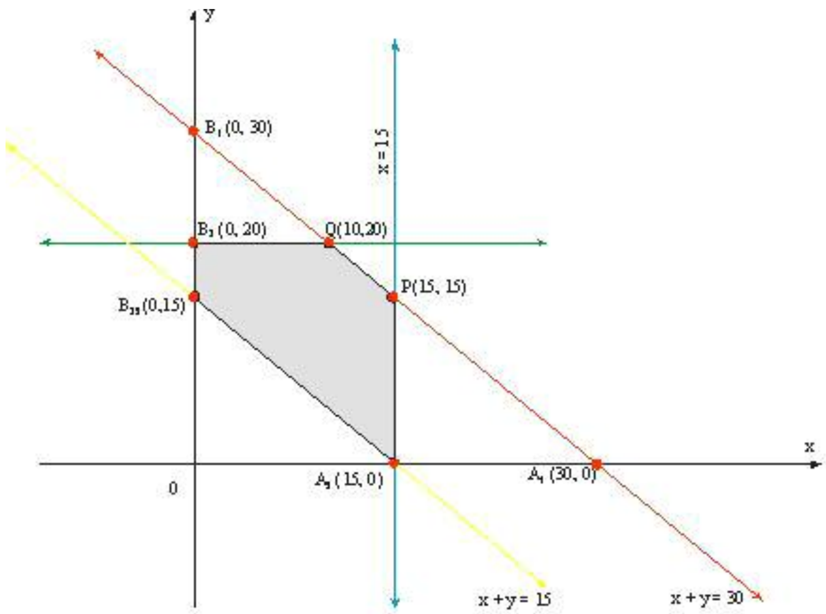
$x + y \leq 30$

$x + y \leq 15$

$x \leq 20$

$y \leq 15$ and $x \geq 0, y \geq 0$

To solve this LPP graphically, we first convert inequations into equations and then draw the corresponding lines. The feasible region of the LPP is shaded in the figure given below. The co-ordinates of the corner points of the feasible region $A_2$ PQ $B_3$ $B_2$ are $A_2$ (15, 0), P (15, 15), Q (10, 20), $B_3$ (0, 20) and $B_2$ (0, 15). These points have been obtained by solving the corresponding intersecting lines simultaneously.




The value of the objective function at the corner points of the feasible region are given in the following table

| Point (x, y) | Value of the objective function X = 30x – 30y + 1800 |
|---|---|
| $A_2$ (15, 0) | Z = 30 × 15 – 30 × 0 + 1800 = 2250 |
| P (15, 15) | Z = 30 × 15 – 30 × 15 + 1800 = 1800 |
| Q (10, 20) | Z = 30 × 10 – 30 × 20 + 1800 = 1500 |
| $B_3$ (0, 20) | Z = 30 × 0 – 30 × 20 + 1800 = 1200 |
| $B_2$ (0, 15) | Z = 30 × 0 – 30 × 15 + 1800 = 1350 |

Clearly, Z is minimum at x = 0, y = 20 and the minimum value Z is 1200.

Thus, the manufacturer should supply 0, 20 and 10 thousand bricks to builders P, Q and R from depot A and 15, 0 and 5 thousand bricks to builders P, Q and R from dept B respectively. In this case the minimum transportation cost will be Rs. 1200.

**29.**

$S_1$: the screw is manufactured by machine X

$S_2$: the screw is manufactured by machine Y

$S_3$: the screw is manufactured by machine Z

E: the screw manufactured is defective

Required probability: $P(S_1/E)$

$P(S_1) = 1/6$

$P(S_2) = 1/3$

$P(S_3) = ½$

$$P(E|S_1) = \frac{1}{100}$$

$$P(E|S_2) = \frac{3}{200}$$

$$P(E|S_3) = \frac{2}{100}$$

$$P(S_1|E) = \frac{P(S_1)(P(E|S_1)}{P(S_1)(P(E|S_1)) + P(S_2)(P(E|S_2)) + P(S_3)(P(E|S_3))}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{200} + \frac{1}{2} \times \frac{2}{100}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{2} + 1}$$

$$= \frac{1}{1+3+6} = \frac{1}{10}$$