7. PROBLEMS ON NUMBERS

In this section, questions involving a set of numbers are put in the form of a puzzle. You have to analyse the given conditions, assume the unknown numbers and form equations accordingly, which on solving yield the unknown numbers.

SOLVED EXAMPLES

Ex. 1. ***A number is as much greater than 36 as is less than 86. Find the number.***

Sol. Let the number be x. Then, $x - 36 = 86 - x \Leftrightarrow 2x = 86 + 36 = 122 \Leftrightarrow x = 61$.

Hence, the required number is 61.

Ex. 2. ***Find a number such that when 15 is subtracted from 7 times the number, the result is 10 more than twice the number.*** **(Hotel Management, 2002)**

Sol. Let the number be x. Then, $7x - 15 = 2x + 10 \Leftrightarrow 5x = 25 \Leftrightarrow x = 5$.

Hence, the required number is 5.

Ex. 3. ***The sum of a rational number and its reciprocal is $\frac{13}{6}$. Find the number.*** **(S.S.C. 2000)**

Sol. Let the number be x.

Then, $x + \frac{1}{x} = \frac{13}{6} \Leftrightarrow \frac{x^2+1}{x} = \frac{13}{6} \Leftrightarrow 6x^2 - 13x + 6 = 0$

$\Leftrightarrow 6x^2 - 9x - 4x + 6 = 0 \Leftrightarrow (3x - 2)(2x - 3) = 0$

$\Leftrightarrow x = \frac{2}{3}$ or $x = \frac{3}{2}$.

Hence, the required number is $\frac{2}{3}$ or $\frac{3}{2}$.

Ex. 4. ***The sum of two numbers is 184. If one-third of the one exceeds one-seventh of the other by 8, find the smaller number.***

Sol. Let the numbers be x and $(184 - x)$. Then,

$$\frac{x}{3} - \frac{(184 - x)}{7} = 8 \Leftrightarrow 7x - 3(184 - x) = 168 \Leftrightarrow 10x = 720 \Leftrightarrow x = 72.$$

So, the numbers are 72 and 112. Hence, smaller number = 72.

Ex. 5. ***The difference of two numbers is 11 and one-fifth of their sum is 9. Find the numbers.***

Sol. Let the numbers be x and y. Then,

$x - y = 11$ ...(*i*) and $\frac{1}{5}(x + y) = 9 \Rightarrow x + y = 45$ ...(*ii*)

Adding (*i*) and (*ii*), we get : $2x = 56$ or $x = 28$. Putting $x = 28$ in (*i*), we get : $y = 17$.

Hence, the numbers are 28 and 17.

Ex. 6. ***If the sum of two numbers is 42 and their product is 437, then find the absolute difference between the numbers.*** **(S.S.C. 2003)**

Sol. Let the numbers be x and y. Then, $x + y = 42$ and $xy = 437$.

$$x - y = \sqrt{(x+y)^2 - 4xy} = \sqrt{(42)^2 - 4 \times 437} = \sqrt{1764 - 1748} = \sqrt{16} = 4.$$

$\therefore$ Required difference = 4.

Ex. 7. ***The sum of two numbers is 15 and the sum of their squares is 113. Find the numbers.***

Sol. Let the numbers be x and $(15 - x)$.

Then, $x^2 + (15 - x)^2 = 113 \Leftrightarrow x^2 + 225 + x^2 - 30x = 113$

$\Leftrightarrow 2x^2 - 30x + 112 = 0 \Leftrightarrow x^2 - 15x + 56 = 0$

$\Leftrightarrow (x - 7)(x - 8) = 0 \Leftrightarrow x = 7$ or $x = 8$.

So, the numbers are 7 and 8.

Ex. 8. ***The average of four consecutive even numbers is 27. Find the largest of these numbers.***

Sol. Let the four consecutive even numbers be x, $x + 2$, $x + 4$ and $x + 6$.

Then, sum of these numbers $= (27 \times 4) = 108$.

So, $x + (x + 2) + (x + 4) + (x + 6) = 108$ or $4x = 96$ or $x = 24$.

$\therefore$ Largest number $= (x + 6) = 30$.

Ex. 9. ***The sum of the squares of three consecutive odd numbers is 2531. Find the numbers.***

Sol. Let the numbers be x, $x + 2$ and $x + 4$.

Then, $x^2 + (x + 2)^2 + (x + 4)^2 = 2531 \Leftrightarrow 3x^2 + 12x - 2511 = 0$

$\Leftrightarrow x^2 + 4x - 837 = 0 \Leftrightarrow (x - 27)(x + 31) = 0 \Leftrightarrow x = 27$.

Hence, the required numbers are 27, 29 and 31.

Ex. 10. ***Of two numbers, 4 times the smaller one is less than 3 times the larger one by 5. If the sum of the numbers is larger than 6 times their difference by 6, find the two numbers.***

Sol. Let the numbers be x and y, such that $x > y$.

Then, $3x - 4y = 5$ ...(*i*) and $(x + y) - 6(x - y) = 6 \Rightarrow -5x + 7y = 6$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x = 59$ and $y = 43$.

Hence, the required numbers are 59 and 43.

Ex. 11. ***The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit in the unit's place is 3 more than the digit in the ten's place, what is the number ?***

Sol. Let the ten's digit be x. Then, unit's digit $= (x + 3)$.

Sum of the digits $= x + (x + 3) = 2x + 3$. Number $= 10x + (x + 3) = 11x + 3$.

$\therefore \dfrac{11x + 3}{2x + 3} = \dfrac{4}{1} \Leftrightarrow 11x + 3 = 4(2x + 3) \Leftrightarrow 3x = 9 \Leftrightarrow x = 3$.

Hence, required number $= 11x + 3 = 36$.

Ex. 12. ***A number consists of two digits. The sum of the digits is 9. If 63 is subtracted from the number, its digits are interchanged. Find the number.***

Sol. Let the ten's digit be x. Then, unit's digit $= (9 - x)$.

Number $= 10x + (9 - x) = 9x + 9$.

Number obtained by reversing the digits $= 10(9 - x) + x = 90 - 9x$.

$\therefore (9x + 9) - 63 = 90 - 9x \Leftrightarrow 18x = 144 \Leftrightarrow x = 8$.

So, ten's digit $= 8$ and unit's digit $= 1$.

Hence, the required number is 81.

Ex. 13. ***A fraction becomes $\frac{2}{3}$ when 1 is added to both, its numerator and denominator. And, it becomes $\frac{1}{2}$ when 1 is subtracted from both the numerator and denominator. Find the fraction.***

Sol. Let the required fraction be $\frac{x}{y}$. Then,

$$\frac{x+1}{y+1} = \frac{2}{3} \Rightarrow 3x - 2y = -1 \quad ...(i) \text{ and } \frac{x-1}{y-1} = \frac{1}{2} \Rightarrow 2x - y = 1 \quad ...(ii)$$

Solving (*i*) and (*ii*), we get : $x = 3, y = 5$.

$\therefore$ Required fraction $= \frac{3}{5}$.

Ex. 14. ***50 is divided into two parts such that the sum of their reciprocals is*** $\frac{1}{12}$. ***Find the two parts.***

Sol. Let the two parts be x and $(50 - x)$.

Then, $\frac{1}{x} + \frac{1}{50-x} = \frac{1}{12} \Leftrightarrow \frac{50-x+x}{x(50-x)} = \frac{1}{12} \Rightarrow x^2 - 50x + 600 = 0$

$\Rightarrow (x - 30)(x - 20) = 0 \Rightarrow x = 30$ or $x = 20$.

So, the parts are 30 and 20.

Ex. 15. ***If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers.*** **(S.S.C. 2000)**

Sol. Let the numbers be x, y and z. Then,

$x + y = 10 \quad ...(i) \quad y + z = 19 \quad ...(ii) \quad x + z = 21 \quad ...(iii)$

Adding (*i*), (*ii*) and (*iii*), we get : $2(x + y + z) = 50$ or $(x + y + z) = 25$.

Thus, $x = (25 - 19) = 6$; $y = (25 - 21) = 4$; $z = (25 - 10) = 15$.

Hence, the required numbers are 6, 4 and 15.

EXERCISE 7A

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer*** :

1. The difference between a number and its three-fifth is 50. What is the number ?
   (*a*) 75 (*b*) 100 (*c*) 125 (*d*) None of these
   (Bank P.O. 2003)
2. If a number is decreased by 4 and divided by 6, the result is 8. What would be the result if 2 is subtracted from the number and then it is divided by 5 ?
   (*a*) $9\frac{2}{3}$ (*b*) 10 (*c*) $10\frac{1}{5}$ (*d*) $11\frac{1}{5}$ (*e*) None of these
   (Bank P.O. 2000)
3. If one-third of one-fourth of a number is 15, then three-tenth of that number is :
   (*a*) 35 (*b*) 36 (*c*) 45 (*d*) 54
   (N.I.F.T. 2003)
4. A number is doubled and 9 is added. If the resultant is trebled, it becomes 75. What is that number ? **(S.S.C. 1999)**
   (*a*) 3.5 (*b*) 6 (*c*) 8 (*d*) None of these
5. Three-fourth of a number is 60 more than its one-third. The number is :
   (*a*) 84 (*b*) 108 (*c*) 144 (*d*) None of these
6. When 24 is subtracted from a number, it reduces to its four-seventh. What is the sum of the digits of that number ?
   (*a*) 1 (*b*) 9 (*c*) 11
   (*d*) Data inadequate (*e*) None of these

7. Find the number which when multiplied by 15 is increased by 196. (L.I.C. 2003)
(a) 14 (b) 20 (c) 26 (d) 28

8. If a number, when divided by 4, is reduced by 21, the number is :
(a) 18 (b) 20 (c) 28 (d) 38

9. A number whose fifth part increased by 4 is equal to its fourth part diminished by 10, is :
(a) 240 (b) 260 (c) 270 (d) 280

10. The difference of two numbers is 20% of the larger number. If the smaller number is 12, the larger one is :
(a) 15 (b) 16 (c) 18 (d) 20

11. If one-seventh of a number exceeds its eleventh part by 100, then the number is :
(a) 770 (b) 1100 (c) 1825 (d) 1925

12. If the sum of one-half and one-fifth of a number exceeds one-third of that number by $7\frac{1}{3}$, the number is : (C.B.I. 1998)
(a) 15 (b) 18 (c) 20 (d) 30

13. If doubling a number and adding 20 to the result gives the same answer as multiplying the number by 8 and taking away 4 from the product, the number is :
(a) 2 (b) 3 (c) 4 (d) 6
(S.S.C. 2000)

14. If 50 is subtracted from two-third of a number, the result is equal to sum of 40 and one-fourth of that number. What is the number ? (R.R.B. 2002)
(a) 174 (b) 216 (c) 246 (d) 336

15. If the sum of a number and its square is 182, what is the number ?
(a) 15 (b) 26 (c) 28 (d) 91 (e) None of these
(Bank P.O. 1999)

16. Twenty times a positive integer is less than its square by 96. What is the integer ?
(a) 20 (b) 24 (c) 30
(d) Cannot be determined (e) None of these (Bank P.O. 2003)

17. Thrice the square of a natural number decreased by 4 times the number is equal to 50 more than the number. The number is : (S.S.C. 2003)
(a) 4 (b) 5 (c) 6 (d) 10

18. The sum of a number and its reciprocal is one-eighth of 34. What is the product of the number and its square root ? (Hotel Management, 2001)
(a) 8 (b) 27 (c) 32 (d) None of these

19. Two-third of a positive number and $\frac{25}{216}$ of its reciprocal are equal. The number is :
(a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{25}{144}$ (d) $\frac{144}{25}$
(S.S.C. 1999)

20. Find a positive number which when increased by 17 is equal to 60 times the reciprocal of the number. (I.M.T. 2002)
(a) 3 (b) 10 (c) 17 (d) 20

21. A positive number when decreased by 4 is equal to 21 times the reciprocal of the number. The number is :
(a) 3 (b) 5 (c) 7 (d) 9

22. The sum of a positive number and its reciprocal is thrice the difference of the number and its reciprocal. The number is :

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

23. The product of two natural numbers is 17. Then, the sum of the reciprocals of their squares is : (S.S.C. 1999)

(a) $\frac{1}{289}$ (b) $\frac{289}{290}$ (c) $\frac{290}{289}$ (d) 289

24. If $2\frac{1}{2}$ is added to a number and the sum multiplied by $4\frac{1}{2}$ and 3 is added to the product and then dividing the sum by $1\frac{1}{5}$, the quotient becomes 25. What is the number ? (R.R.B. 2002)

(a) $2\frac{1}{2}$ (b) $3\frac{1}{2}$ (c) $4\frac{1}{2}$ (d) $5\frac{1}{2}$

25. Three numbers are in the ratio 4 : 5 : 6 and their average is 25. The largest number is :

(a) 30 (b) 32 (c) 36 (d) 42

26. Three numbers are in the ratio of 3 : 4 : 6 and their product is 1944. The largest of these numbers is :

(a) 6 (b) 12 (c) 18 (d) None of these

27. Two numbers are such that the square of one is 224 less than 8 times the square of the other. If the numbers be in the ratio of 3 : 4, the numbers are :

(a) 6, 8 (b) 9, 12 (c) 12, 16 (d) None of these

28. Two numbers are such that the ratio between them is 4 : 7. If each is increased by 4, the ratio becomes 3 : 5. The larger number is :

(a) 36 (b) 48 (c) 56 (d) 64

29. The sum of three numbers is 264. If the first number be twice the second and third number be one-third of the first, then the second number is : (R.R.B. 2004)

(a) 48 (b) 54 (c) 72 (d) 84

30. The sum of two numbers is 22. Five times one number is equal to 6 times the other. The bigger of the two numbers is : (C.B.I. 1998)

(a) 10 (b) 12 (c) 15 (d) 16

31. One-fifth of a number is equal to $\frac{5}{8}$ of another number. If 35 is added to the first number, it becomes four times of the second number. The second number is :

(a) 25 (b) 40 (c) 70 (d) 125

(Bank P.O. 1999)

32. The sum of two numbers is 25 and their difference is 13. Find their product.

(a) 104 (b) 114 (c) 315 (d) 325

(L.I.C. 2003)

33. If the sum of two numbers is 33 and their difference is 15, the smaller number is :

(a) 9 (b) 12 (c) 15 (d) 18

(C.B.I. 1997)

34. The sum of two numbers is 40 and their difference is 4. The ratio of the numbers is :

(a) 11 : 9 (b) 11 : 18 (c) 21 : 19 (d) 22 : 9

(S.S.C. 2000)

35. The product of two numbers is 192 and the sum of these two numbers is 28. What is the smaller of these two numbers ? **(Bank P.O. 1999)**
(a) 12 (b) 14 (c) 16 (d) 18 (e) None of these

36. The difference between two integers is 5. Their product is 500. Find the numbers.
(a) 15, 20 (b) 20, 25 (c) 30, 25 (d) 21, 26
(Hotel Management, 2003)

37. Two numbers differ by 5. If their product is 336, then the sum of the two numbers is :
(a) 21 (b) 28 (c) 37 (d) 51
(S.S.C. 1999)

38. Two different natural numbers are such that their product is less than their sum. One of the numbers must be :
(a) 1 (b) 2 (c) 3 (d) None of these

39. The product of two numbers is 9375 and the quotient, when the larger one is divided by the smaller, is 15. The sum of the numbers is : **(S.S.C. 2004)**
(a) 380 (b) 395 (c) 400 (d) 425

40. The difference between two numbers is 1365. When the larger number is divided by the smaller one, the quotient is 6 and the remainder is 15. The smaller number is :
(a) 240 (b) 270 (c) 295 (d) 360

41. The sum of two numbers is 40 and their product is 375. What will be the sum of their reciprocals ? **(S.S.C. 1999)**
(a) $\frac{1}{40}$ (b) $\frac{8}{75}$ (c) $\frac{75}{4}$ (d) $\frac{75}{8}$

42. The sum of two positive integers multiplied by the bigger number is 204, and their difference multiplied by the smaller number is 35. The numbers are :
(a) 12, 5 (b) 13, 4 (c) 14, 3 (d) 24, 10

43. If the sum and difference of two numbers are 20 and 8 respectively, then the difference of their squares is : **(S.S.C. 2000)**
(a) 12 (b) 28 (c) 160 (d) 180

44. The product of two numbers is 120 and the sum of their squares is 289. The sum of the numbers is : **(R.R.B. 2004)**
(a) 20 (b) 23 (c) 169 (d) None of these

45. The product of two numbers is 45 and the sum of their squares is 106. The numbers are : **(R.R.B. 2002)**
(a) 3 and 5 (b) 5 and 9 (c) 5 and 19 (d) 45 and 1

46. The sum of the squares of two numbers is 3341 and the difference of their squares is 891. The numbers are :
(a) 25, 36 (b) 25, 46 (c) 35, 46 (d) None of these

47. The difference between two positive integers is 3. If the sum of their squares is 369, then the sum of the numbers is : **(S.S.C. 2003)**
(a) 25 (b) 27 (c) 33 (d) 81

48. If the sum of two numbers is 22 and the sum of their squares is 404, then the product of the numbers is : **(S.S.C. 2000)**
(a) 40 (b) 44 (c) 80 (d) 88

49. The difference between the squares of two numbers is 256000 and the sum of the numbers is 1000. The numbers are :
(a) 600, 400 (b) 628, 372 (c) 640, 360 (d) None of these

50. If the difference of two numbers is 3 and the difference of their squares is 39, then the larger number is :
(a) 8 (b) 9 (c) 12 (d) 13

51. The sum of three consecutive numbers is 87. The greatest among these three numbers is : **(Hotel Management, 2003)**

(*a*) 26 (*b*) 28 (*c*) 29 (*d*) 30

52. Three times the first of three consecutive odd integers is 3 more than twice the third. The third integer is : **(M.B.A. 1998)**

(*a*) 9 (*b*) 11 (*c*) 13 (*d*) 15

53. The sum of four consecutive even integers is 1284. The greatest of them is :

(*a*) 320 (*b*) 322 (*c*) 324 (*d*) 326 **(S.S.C. 2002)**

54. The sum of three consecutive odd numbers is 20 more than the first of these numbers. What is the middle number ? **(S.B.I.P.O. 1997)**

(*a*) 7 (*b*) 9 (*c*) 11

(*d*) Data inadequate (*e*) None of these

55. The product of three consecutive even numbers when divided by 8 is 720. The product of their square roots is : **(Hotel Management, 2001)**

(*a*) $12\sqrt{10}$ (*b*) $24\sqrt{10}$ (*c*) 120 (*d*) None of these

56. The sum of three consecutive multiples of 3 is 72. What is the largest number ?

(*a*) 21 (*b*) 24 (*c*) 27 (*d*) 36 **(S.S.C. 1999)**

57. What is the sum of two consecutive even numbers, the difference of whose squares is 84 ? **(S.S.C. 2003)**

(*a*) 34 (*b*) 38 (*c*) 42 (*d*) 46

58. The sum of the squares of three consecutive natural numbers is 2030. What is the middle number ? **(S.S.C. 2000)**

(*a*) 25 (*b*) 26 (*c*) 27 (*d*) 28

59. There are two numbers such that the sum of twice the first and thrice the second is 39, while the sum of thrice the first and twice the second is 36. The larger of the two is :

(*a*) 6 (*b*) 8 (*c*) 9 (*d*) 12

60. In a two-digit number, the digit in the unit's place is four times the digit in ten's place and sum of the digits is equal to 10. What is the number ? **(Bank P.O. 1999)**

(*a*) 14 (*b*) 41 (*c*) 82

(*d*) Data inadequate (*e*) None of these

61. A number of two digits has 3 for its unit's digit, and the sum of digits is $\frac{1}{7}$ of the number itself. The number is : **(L.I.C. 2003)**

(*a*) 43 (*b*) 53 (*c*) 63 (*d*) 73.

62. A two-digit number exceeds the sum of the digits of that number by 18. If the digit at the unit's place is double the digit in the ten's place, what is the number ?

(*a*) 24 (*b*) 42 (*c*) 48 (*d*) Data inadequate

63. The sum of the digits of a two-digit number is 15 and the difference between the digits is 3. What is the two-digit number ? **(B.S.R.B. 2003)**

(*a*) 69 (*b*) 78 (*c*) 96

(*d*) Cannot be determined (*e*) None of these

64. In a two-digit number, if it is known that its unit's digit exceeds its ten's digit by 2 and that the product of the given number and the sum of its digits is equal to 144, then the number is : **(C.B.I. 2003)**

(*a*) 24 (*b*) 26 (*c*) 42 (*d*) 46

65. A number consists of two digits. If the digits interchange places and the new number is added to the original number, then the resulting number will be divisible by :
(a) 3 (b) 5 (c) 9 (d) 11
(S.S.C. 2003)

66. The sum of the digits of a two-digit number is 9 less than the number. Which of the following digits is at unit's place of the number ?
(a) 1 (b) 2 (c) 4 (d) Data inadequate

67. The difference between a two-digit number and the number obtained by interchanging the positions of its digits is 36. What is the difference between the two digits of that number ? **(Bank P.O. 2003)**
(a) 3 (b) 4 (c) 9
(d) Cannot be determined (e) None of these

68. The difference between a two-digit number and the number obtained by interchanging the two digits is 63. Which is the smaller of the two numbers ? **(Bank P.O. 2003)**
(a) 29 (b) 70 (c) 92
(d) Cannot be determined (e) None of these.

69. The sum of the digits of a two-digit number is $\frac{1}{5}$ of the difference between the number and the number obtained by interchanging the positions of the digits. What is definitely the difference between the digits of that number ? **(Bank P.O. 2000)**
(a) 5 (b) 7 (c) 9
(d) Data inadequate (e) None of these

70. If the digit in the unit's place of a two-digit number is halved and the digit in the ten's place is doubled, the number thus obtained is equal to the number obtained by interchanging the digits. Which of the following is definitely true ? **(Bank P.O. 2003)**
(a) Sum of the digits is a two-digit number.
(b) Digit in the unit's place is twice the digit in the ten's place.
(c) Digits in the unit's place and the ten's place are equal.
(d) Digit in the unit's place is half of the digit in the ten's place.
(e) None of these

71. If the number obtained on interchanging the digits of a two-digit number is 18 more than the original number and the sum of the digits is 8, then what is the original number ? **(S.B.I.P.O. 2002)**
(a) 26 (b) 35 (c) 53
(d) Cannot be determined (e) None of these

72. The difference between a two-digit number and the number obtained by interchanging the digits is 36. What is the difference between the sum and the difference of the digits of the number if the ratio between the digits of the number is 1 : 2 ? **(M.A.T. 1999)**
(a) 4 (b) 8 (c) 16 (d) None of these

73. A number consists of 3 digits whose sum is 10. The middle digit is equal to the sum of the other two and the number will be increased by 99 if its digits are reversed. The number is : **(Hotel Management, 2003)**
(a) 145 (b) 253 (c) 370 (d) 352

74. A two-digit number becomes five-sixth of itself when its digits are reversed. The two digits differ by one. The number is :
(a) 45 (b) 54 (c) 56 (d) 65

75. A number consists of two digits such that the digit in the ten's place is less by 2 than the digit in the unit's place. Three times the number added to $\frac{6}{7}$ times the number obtained by reversing the digits equals 108. The sum of the digits in the number is :
(*a*) 6 (*b*) 7 (*c*) 8 (*d*) 9
(S.S.C. 2003)

76. The digit in the unit's place of a number is equal to the digit in the ten's place of half of that number and the digit in the ten's place of that number is less than the digit in unit's place of half of the number by 1. If the sum of the digits of the number is 7, then what is the number ? **(S.B.I.P.O. 2001)**
(*a*) 34 (*b*) 52 (*c*) 162
(*d*) Data inadequate (*e*) None of these

77. In a two-digit number, the digit in the unit's place is more than twice the digit in ten's place by 1. If the digits in the unit's place and the ten's place are interchanged, difference between the newly formed number and the original number is less than the original number by 1. What is the original number ? **(Bank P.O. 1999)**
(*a*) 25 (*b*) 37 (*c*) 49 (*d*) 52 (*e*) 73

78. A certain number of two digits is three times the sum of its digits and if 45 be added to it, the digits are reversed. The number is : **(L.I.C.A.A.O. 2003)**
(*a*) 23 (*b*) 27 (*c*) 32 (*d*) 72

79. A two-digit number is such that the product of the digits is 8. When 18 is added to the number, then the digits are reversed. The number is : **(M.B.A. 2003)**
(*a*) 18 (*b*) 24 (*c*) 42 (*d*) 81

80. The product of two fractions is $\frac{14}{15}$ and their quotient is $\frac{35}{24}$. The greater fraction is :
(*a*) $\frac{4}{5}$ (*b*) $\frac{7}{6}$ (*c*) $\frac{7}{4}$ (*d*) $\frac{7}{3}$
(S.S.C. 2002)

81. In a pair of fractions, fraction A is twice the fraction B and the product of two fractions is $\frac{2}{25}$. What is the value of fraction A ? **(Bank P.O. 1999)**
(*a*) $\frac{1}{5}$ (*b*) $\frac{1}{25}$ (*c*) $\frac{2}{5}$ (*d*) Data inadequate

82. The sum of the numerator and denominator of a fraction is 11. If 1 is added to the numerator and 2 is subtracted from the denominator, it becomes $\frac{2}{3}$. The fraction is :
(*a*) $\frac{5}{6}$ (*b*) $\frac{6}{5}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{8}{3}$

83. The denominator of a fraction is 3 more than the numerator. If the numerator as well as the denominator is increased by 4, the fraction becomes $\frac{4}{5}$. What was the original fraction ? **(S.B.I.P.O. 1999)**
(*a*) $\frac{8}{11}$ (*b*) $\frac{5}{8}$ (*c*) $\frac{10}{13}$ (*d*) $\frac{7}{10}$

84. The difference between the numerator and the denominator of a fraction is 5. If 5 is added to its denominator, the fraction is decreased by $1\frac{1}{4}$. Find the value of the fraction.
(*a*) $\frac{1}{6}$ (*b*) $2\frac{1}{4}$ (*c*) $3\frac{1}{4}$ (*d*) 6
(M.B.A. 1997)

85. The numerator and denominator of a fraction are in the ratio of 2 : 3. If 6 is subtracted from the numerator, the result is a fraction that has a value $\frac{2}{3}$ of the original fraction. The numerator of the original fraction is : (S.S.C. 1999)
(a) 6 (b) 18 (c) 27 (d) 36

86. If 1 is added to the denominator of a fraction, the fraction becomes $\frac{1}{2}$. If 1 is added to the numerator of the fraction, the fraction becomes 1. The fraction is : (C.B.I. 1997)
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{3}{2}$

87. If the numerator of a fraction is increased by 2 and the denominator is increased by 3, the fraction becomes $\frac{7}{9}$ and if both the numerator as well as the denominator are decreased by 1, the fraction becomes $\frac{4}{5}$. What is the original fraction ?
(a) $\frac{5}{6}$ (b) $\frac{9}{11}$ (c) $\frac{13}{16}$ (d) $\frac{17}{21}$
(S.B.I.P.O. 1999)

88. When the numerator of a fraction increases by 4, the fraction increases by $\frac{2}{3}$. The denominator of the fraction is :
(a) 2 (b) 3 (c) 4 (d) 6

89. 54 is to be divided into two parts such that the sum of 10 times the first and 22 times the second is 780. The bigger part is :
(a) 24 (b) 34 (c) 30 (d) 32

90. 243 has been divided into three parts such that half of the first part, one-third of the second part and one-fourth of the third part are equal. The largest part is :
(a) 74 (b) 86 (c) 92 (d) 108

91. The sum of four numbers is 64. If you add 3 to the first number, 3 is subtracted from the second number, the third is multiplied by 3 and the fourth is divided by 3, then all the results are equal. What is the difference between the largest and the smallest of the original numbers ? (S.B.I.P.O. 2000)
(a) 21 (b) 27 (c) 32
(d) Cannot be determined (e) None of these

92. The sum of the squares of three numbers is 138, while the sum of their products taken two at a time is 131. Their sum is : (Hotel Management, 1999)
(a) 20 (b) 30 (c) 40 (d) None of these

93. The sum of three numbers is 136. If the ratio between first and second be 2 : 3 and that between second and third is 5 : 3, then the second number is :
(a) 40 (b) 48 (c) 60 (d) 72

94. Of the three numbers, the sum of the first two is 45; the sum of the second and the third is 55 and the sum of the third and thrice the first is 90. The third number is :
(a) 20 (b) 25 (c) 30 (d) 3

ANSWERS

1. (c)	2. (b)	3. (d)	4. (c)	5. (c)	6. (c)	7. (a)	8. (c)	9. (d)
10. (a)	11. (d)	12. (c)	13. (c)	14. (b)	15. (e)	16. (b)	17. (b)	18. (a)
19. (a)	20. (a)	21. (c)	22. (a)	23. (c)	24. (b)	25. (a)	26. (c)	27. (a)
28. (c)	29. (c)	30. (b)	31. (b)	32. (b)	33. (a)	34. (a)	35. (a)	36. (b)

37. (c)	38. (a)	39. (c)	40. (b)	41. (b)	42. (a)	43. (c)	44. (b)	45. (b)
46. (c)	47. (b)	48. (a)	49. (b)	50. (a)	51. (d)	52. (d)	53. (c)	54. (b)
55. (b)	56. (c)	57. (c)	58. (b)	59. (c)	60. (e)	61. (c)	62. (a)	63. (d)
64. (a)	65. (d)	66. (d)	67. (b)	68. (d)	69. (d)	70. (b)	71. (b)	72. (b)
73. (b)	74. (b)	75. (a)	76. (b)	77. (b)	78. (b)	79. (b)	80. (b)	81. (c)
82. (c)	83. (a)	84. (b)	85. (b)	86. (b)	87. (a)	88. (d)	89. (b)	90. (d)
91. (c)	92. (a)	93. (c)	94. (c)					

SOLUTIONS

1. Let the number be x. Then, $x - \frac{3}{5}x = 50 \Leftrightarrow \frac{2}{5}x = 50 \Leftrightarrow x = \left(\frac{50 \times 5}{2}\right) = 125$.

2. Let the number be x. Then, $\frac{x-4}{6} = 8 \Leftrightarrow x - 4 = 48 \Leftrightarrow x = 52$.

   $\therefore \quad \frac{x-2}{5} = \frac{52-2}{5} = \frac{50}{5} = 10$.

3. Let the number be x. Then, $\frac{1}{3}$ of $\frac{1}{4}$ of $x = 15 \Leftrightarrow x = 15 \times 12 = 180$.

   So, required number $= \left(\frac{3}{10} \times 180\right) = 54$.

4. Let the number be x. Then, $3(2x + 9) = 75 \Leftrightarrow 2x + 9 = 25 \Leftrightarrow 2x = 16 \Leftrightarrow x = 8$.

5. Let the number be x. Then, $\frac{3}{4}x - \frac{1}{3}x = 60 \Leftrightarrow \frac{5x}{12} = 60 \Leftrightarrow x = \left(\frac{60 \times 12}{5}\right) = 144$.

6. Let the number be x. Then,

   $x - 24 = \frac{4}{7}x \Leftrightarrow x - \frac{4}{7}x = 24 \Leftrightarrow \frac{3}{7}x = 24 \Leftrightarrow x = \left(\frac{24 \times 7}{3}\right) = 56$.

   $\therefore$ Sum of the digits $= (5 + 6) = 11$.

7. Let the number be x. Then, $15x - x = 196 \Leftrightarrow 14x = 196 \Leftrightarrow x = 14$.

8. Let the number be x. Then, $\frac{x}{4} = x - 21 \Leftrightarrow x = 4x - 84 \Leftrightarrow 3x = 84 \Leftrightarrow x = 28$.

9. Let the number be x. Then, $\left(\frac{1}{5}x + 4\right) = \left(\frac{1}{4}x - 10\right) \Leftrightarrow \frac{x}{20} = 14 \Leftrightarrow x = 14 \times 20 = 280$.

10. Let the larger number be x.

    Then, $x - 12 = 20\%$ of $x \Leftrightarrow x - \frac{x}{5} = 12 \Leftrightarrow \frac{4x}{5} = 12 \Leftrightarrow x = \left(\frac{12 \times 5}{4}\right) = 15$.

11. Let the number be x. Then, $\frac{1}{7}x - \frac{1}{11}x = 100 \Leftrightarrow \frac{4x}{77} = 100 \Leftrightarrow x = \frac{7700}{4} = 1925$.

12. Let the number be x.

    Then, $\left(\frac{1}{2}x + \frac{1}{5}x\right) - \frac{1}{3}x = \frac{22}{3} \Leftrightarrow \frac{11x}{30} = \frac{22}{3} \Leftrightarrow x = \left(\frac{22 \times 30}{3 \times 11}\right) = 20$.

13. Let the number be x. Then, $2x + 20 = 8x - 4 \Leftrightarrow 6x = 24 \Leftrightarrow x = 4$.

14. Let the number be x.

    Then, $\frac{2}{3}x - 50 = \frac{1}{4}x + 40 \Leftrightarrow \frac{2}{3}x - \frac{1}{4}x = 90 \Leftrightarrow \frac{5x}{12} = 90 \Leftrightarrow x = \left(\frac{90 \times 12}{5}\right) = 216$.

15. Let the number be x.
Then, $x + x^2 = 182 \Leftrightarrow x^2 + x - 182 = 0 \Leftrightarrow (x + 14)(x - 13) = 0 \Leftrightarrow x = 13$.

16. Let the integer be x.
Then, $x^2 - 20x = 96 \Leftrightarrow x^2 - 20x - 96 = 0 \Leftrightarrow (x + 4)(x - 24) = 0 \Leftrightarrow x = 24$.

17. Let the number be x.
Then, $3x^2 - 4x = x + 50 \Leftrightarrow 3x^2 - 5x - 50 = 0 \Leftrightarrow (3x + 10)(x - 5) = 0 \Leftrightarrow x = 5$.

18. Let the number be x. Then, $x + \frac{1}{x} = \frac{34}{8} \Leftrightarrow \frac{x^2+1}{x} = \frac{34}{8} \Leftrightarrow 8x^2 - 34x + 8 = 0$

$\Leftrightarrow 4x^2 - 17x + 4 = 0 \Leftrightarrow (4x - 1)(x - 4) = 0 \Leftrightarrow x = 4$.

$\left[\text{neglecting } x = \frac{1}{4}, \text{ as } x \text{ is a natural no.}\right]$

$\therefore$ Required number $= 4 \times \sqrt{4} = 4 \times 2 = 8$.

19. Let the number be x.

Then, $\frac{2}{3}x = \frac{25}{216} \times \frac{1}{x} \Leftrightarrow x^2 = \frac{25}{216} \times \frac{3}{2} = \frac{25}{144} \Leftrightarrow x = \sqrt{\frac{25}{144}} = \frac{5}{12}$.

20. Let the number be x.

Then, $x + 17 = \frac{60}{x} \Leftrightarrow x^2 + 17x - 60 = 0 \Leftrightarrow (x + 20)(x - 3) = 0 \Leftrightarrow x = 3$.

21. Let the number be x.

Then, $x - 4 = \frac{21}{x} \Leftrightarrow x^2 - 4x - 21 = 0 \Leftrightarrow (x - 7)(x + 3) = 0 \Leftrightarrow x = 7$.

22. Let the number be x. Then, $x + \frac{1}{x} = 3\left(x - \frac{1}{x}\right) \Leftrightarrow \frac{x^2+1}{x} = 3\left(\frac{x^2-1}{x}\right)$

$\Leftrightarrow x^2 + 1 = 3x^2 - 3 \Leftrightarrow 2x^2 = 4 \Leftrightarrow x^2 = 2 \Leftrightarrow x = \sqrt{2}$.

23. Let the numbers be a and b. Then, $ab = 17 \Rightarrow a = 1$ and $b = 17$.

So, $\frac{1}{a^2} + \frac{1}{b^2} = \frac{a^2+b^2}{a^2b^2} = \frac{1^2+(17)^2}{(1 \times 17)^2} = \frac{290}{289}$.

24. Let the number be x. Then,

$$\frac{4\frac{1}{2}\left(x + 2\frac{1}{2}\right) + 3}{1\frac{1}{5}} = 25 \Leftrightarrow \frac{\frac{9}{2}\left(x + \frac{5}{2}\right) + 3}{\frac{6}{5}} = 25$$

$$\Leftrightarrow \frac{9x}{2} + \frac{45}{4} + 3 = 25 \times \frac{6}{5} = 30 \Leftrightarrow \frac{9x}{2} = 30 - \frac{57}{4} \Leftrightarrow \frac{9x}{2} = \frac{63}{4}$$

$$\Leftrightarrow x = \left(\frac{63}{4} \times \frac{2}{9}\right) = \frac{7}{2} = 3\frac{1}{2}.$$

25. Let the numbers be $4x$, $5x$ and $6x$. Then, $\frac{4x + 5x + 6x}{3} = 25 \Leftrightarrow 5x = 25 \Leftrightarrow x = 5$.

$\therefore$ Largest number $= 6x = 30$.

26. Let the numbers be $3x$, $4x$ and $6x$.
Then, $3x \times 4x \times 6x = 1944 \Leftrightarrow 72x^3 = 1944 \Leftrightarrow x^3 = 27 \Leftrightarrow x = 3$.
$\therefore$ Largest number $= 6x = 18$.

27. Let the numbers be $3x$ and $4x$. Then,
$(4x)^2 = 8 \times (3x)^2 - 224 \Leftrightarrow 16x^2 = 72x^2 - 224 \Leftrightarrow 56x^2 = 224 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2$.
So, the numbers are 6 and 8.

28. Let the numbers be $4x$ and $7x$. Then, $\frac{4x+4}{7x+4} = \frac{3}{5} \Leftrightarrow 5(4x+4) = 3(7x+4) \Leftrightarrow x = 8$.

$\therefore$ Larger number $= 7x = 56$.

29. Let the second number be x. Then, first number $= 2x$ and third number $= \frac{2x}{3}$.

$\therefore \quad 2x + x + \frac{2x}{3} = 264 \Leftrightarrow \frac{11x}{3} = 264 \Leftrightarrow x = \left(\frac{264 \times 3}{11}\right) = 72.$

30. Let the numbers be x and $(22 - x)$. Then, $5x = 6(22 - x) \Leftrightarrow 11x = 132 \Leftrightarrow x = 12$.
So, the numbers are 12 and 10.

31. Let the numbers be x and y. Then, $\frac{1}{5}x = \frac{5}{8}y \Leftrightarrow y = \frac{8}{25}x$.

Now, $x + 35 = 4y \Leftrightarrow x + 35 = \frac{32}{25}x \Leftrightarrow \frac{7}{25}x = 35 \Leftrightarrow x = \left(\frac{35 \times 25}{7}\right) = 125.$

$\therefore$ Second number $= y = \frac{8}{25}x = \left(\frac{8}{25} \times 125\right) = 40.$

32. Let the numbers be x and y. Then, $x + y = 25$ and $x - y = 13$.
$4xy = (x + y)^2 - (x - y)^2 = (25)^2 - (13)^2 = 625 - 169 = 456 \Rightarrow xy = 114.$

33. Let the numbers be x and y.
Then, $x + y = 33$ ...(*i*) and $x - y = 15$ ...(*ii*)
Solving (*i*) and (*ii*), we get : $x = 24$, $y = 9$.
$\therefore$ Smaller number $= 9$.

34. Let the numbers be x and y. Then,

$\frac{x+y}{x-y} = \frac{40}{4} = 10 \Leftrightarrow (x + y) = 10(x - y) \Leftrightarrow 9x = 11y \Leftrightarrow \frac{x}{y} = \frac{11}{9}$.

35. Let the numbers be x and $(28 - x)$. Then,
$x(28 - x) = 192 \Leftrightarrow x^2 - 28x + 192 = 0 \Leftrightarrow (x - 16)(x - 12) = 0$
$\Leftrightarrow x = 16$ or $x = 12$.
So, the numbers are 16 and 12.

36. Let the integers be x and $(x + 5)$. Then,
$x(x + 5) = 500 \Leftrightarrow x^2 + 5x - 500 = 0 \Leftrightarrow (x + 25)(x - 20) = 0 \Leftrightarrow x = 20.$
So, the numbers are 20 and 25.

37. Let the numbers be x and y. Then, $x - y = 5$ and $xy = 336$.
$(x + y)^2 = (x - y)^2 + 4xy = 25 + 4 \times 336 = 1369 \Rightarrow x + y = \sqrt{1369} = 37.$

38. Since $1.x < 1 + x$, so one of the numbers is 1.

39. Let the numbers be x and y. Then, $xy = 9375$ and $\frac{x}{y} = 15$.

$\frac{xy}{(x/y)} = \frac{9375}{15} \Leftrightarrow y^2 = 625 \Leftrightarrow y = 25 \Rightarrow x = 15y = (15 \times 25) = 375.$

$\therefore$ Sum of the numbers $= 375 + 25 = 400$.

40. Let the numbers be x and $(x + 1365)$.
Then, $x + 1365 = 6x + 15 \Leftrightarrow 5x = 1350 \Leftrightarrow x = 270.$

41. Let the numbers be x and y. Then, $x + y = 40$ and $xy = 375$.

$\therefore \quad \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{40}{375} = \frac{8}{75}.$

42. Let the numbers be x and y such that $x > y$. Then,
$x(x + y) = 204 \Rightarrow x^2 + xy = 204$ ...(*i*) and $y(x - y) = 35 \Rightarrow xy - y^2 = 35$ ...(*ii*)
Subtracting (*ii*) from (*i*), we get : $x^2 + y^2 = 169$.
The only triplet satisfying this condition is (12, 5, 13). Thus, $x = 12$, $y = 5$.

43. Let the numbers be x and y. Then, $x + y = 20$ and $x - y = 8$.
$\therefore\ x^2 - y^2 = (x + y)(x - y) = 20 \times 8 = 160$.

44. Let the numbers be x and y. Then, $xy = 120$ and $x^2 + y^2 = 289$.
$\therefore\ (x + y)^2 = x^2 + y^2 + 2xy = 289 + 240 = 529$.
$\therefore\ x + y = \sqrt{529} = 23$.

45. Let the numbers be x and y. Then, $xy = 45$ and $x^2 + y^2 = 106$.
$(x + y) = \sqrt{(x^2 + y^2) + 2xy} = \sqrt{106 + 90} = \sqrt{196} \Rightarrow x + y = 14$ ...(*i*)
$(x - y) = \sqrt{(x^2 + y^2) - 2xy} = \sqrt{106 - 90} = \sqrt{16} \Rightarrow x - y = 4$ ...(*ii*)
Solving (*i*) and (*ii*), we get : $x = 9$ and $y = 5$.

46. Let the numbers be x and y. Then,
$x^2 + y^2 = 3341$ ...(*i*) and $x^2 - y^2 = 891$ ...(*ii*)
Adding (*i*) and (*ii*), we get : $2x^2 = 4232$ or $x^2 = 2116$ or $x = 46$.
Subtracting (*ii*) from (*i*), we get : $2y^2 = 2450$ or $y^2 = 1225$ or $y = 35$.
So, the numbers are 35 and 46.

47. Let the numbers be x and $(x + 3)$. Then,
$x^2 + (x + 3)^2 = 369 \Leftrightarrow x^2 + x^2 + 9 + 6x = 369$
$\Leftrightarrow 2x^2 + 6x - 360 = 0 \Leftrightarrow x^2 + 3x - 180 = 0 \Leftrightarrow (x + 15)(x - 12) = 0 \Leftrightarrow x = 12$.
So, the numbers are 12 and 15.
$\therefore$ Required sum = (12 + 15) = 27.

48. Let the numbers be x and y. Then, $(x + y) = 22$ and $x^2 + y^2 = 404$.
Now, $2xy = (x + y)^2 - (x^2 + y^2) = (22)^2 - 404 = 484 - 404 = 80 \Rightarrow xy = 40$.

49. Let the numbers be x and y. Then, $x^2 - y^2 = 256000$ and $x + y = 1000$.
On dividing, we get : $x - y = 256$.
Solving $x + y = 1000$ and $x - y = 256$, we get : $x = 628$ and $y = 372$.

50. Let the numbers be x and y. Then, $x^2 - y^2 = 39$ and $x - y = 3$.
On dividing, we get : $x + y = 13$.
Solving $x + y = 13$ and $x - y = 3$, we get : $x = 8$ and $y = 5$.
$\therefore$ Larger number = 8.

51. Let the numbers be x, $x + 1$ and $x + 2$.
Then, $x + (x + 1) + (x + 2) = 87 \Leftrightarrow 3x = 84 \Leftrightarrow x = 28$.
$\therefore$ Greatest number = $(x + 2) = 30$.

52. Let the three integers be x, $x + 2$ and $x + 4$. Then, $3x = 2(x + 4) + 3 \Leftrightarrow x = 11$.
$\therefore$ Third integer = $x + 4 = 15$.

53. Let the four integers be x, $x + 2$, $x + 4$ and $x + 6$.
Then, $x + (x + 2) + (x + 4) + (x + 6) = 1284 \Leftrightarrow 4x = 1272 \Leftrightarrow x = 318$.
$\therefore$ Greatest integer = $x + 6 = 324$.

54. Let the numbers be x, $x + 2$ and $x + 4$.
Then, $x + (x + 2) + (x + 4) = x + 20 \Leftrightarrow 2x = 14 \Leftrightarrow x = 7$.
$\therefore$ Middle number = $x + 2 = 9$.

55. Let the numbers be x, $x + 2$ and $x + 4$.

Then, $\frac{x(x+2)(x+4)}{8} = 720 \Rightarrow x(x+2)(x+4) = 5760$.

$\therefore \sqrt{x} \times \sqrt{(x+2)} \times \sqrt{(x+4)} = \sqrt{x(x+2)(x+4)} = \sqrt{5760} = 24\sqrt{10}$.

56. Let the numbers be $3x$, $3x + 3$ and $3x + 6$.

Then, $3x + (3x + 3) + (3x + 6) = 72 \Leftrightarrow 9x = 63 \Leftrightarrow x = 7$.

$\therefore$ Largest number $= 3x + 6 = 27$.

57. Let the numbers be x and $x + 2$.

Then, $(x + 2)^2 - x^2 = 84 \Leftrightarrow 4x + 4 = 84 \Leftrightarrow 4x = 80 \Leftrightarrow x = 20$.

$\therefore$ Required sum $= x + (x + 2) = 2x + 2 = 42$.

58. Let the numbers be x, $x + 1$ and $x + 2$.

Then, $x^2 + (x + 1)^2 + (x + 2)^2 = 2030 \Leftrightarrow 3x^2 + 6x - 2025 = 0$

$\Leftrightarrow \quad x^2 + 2x - 675 = 0 \Leftrightarrow (x + 27)(x - 25) = 0 \Leftrightarrow x = 25$.

$\therefore$ Middle number $= (x + 1) = 26$.

59. Let the numbers be x and y. Then, $2x + 3y = 39$ ...(*i*) and $3x + 2y = 36$ ...(*ii*)

On solving (*i*) and (*ii*), we get : $x = 6$ and $y = 9$.

$\therefore$ Larger number $= 9$.

60. Let the ten's digits be x. Then, unit's digit $= 4x$.

$\therefore \quad x + 4x = 10 \Leftrightarrow 5x = 10 \Leftrightarrow x = 2$.

So, ten's digit $= 2$, unit's digit $= 8$.

Hence, the required number is 28.

61. Let the ten's digit be x. Then, number $= 10x + 3$ and sum of digits $= (x + 3)$.

So, $(x + 3) = \frac{1}{7}(10x + 3) \Leftrightarrow 7x + 21 = 10x + 3 \Leftrightarrow 3x = 18 \Leftrightarrow x = 6$.

Hence, the number is 63.

62. Let the ten's digit be x. Then, unit's digit $= 2x$.

Number $= 10x + 2x = 12x$; Sum of digits $= x + 2x = 3x$.

$\therefore \quad 12x - 3x = 18 \Leftrightarrow 9x = 18 \Leftrightarrow x = 2$.

Hence, required number $= 12x = 24$.

63. Let the ten's digit be x and unit's digit be y.

Then, $x + y = 15$ and $x - y = 3$ or $y - x = 3$.

Solving $x + y = 15$ and $x - y = 3$, we get : $x = 9$, $y = 6$.

Solving $x + y = 15$ and $y - x = 3$, we get : $x = 6$, $y = 9$.

So, the number is either 96 or 69. Hence, the number cannot be determined.

64. Let the ten's digit be x. Then, unit's digit $= x + 2$.

Number $= 10x + (x + 2) = 11x + 2$; Sum of digits $= x + (x + 2) = 2x + 2$.

$\therefore \quad (11x + 2)(2x + 2) = 144 \Leftrightarrow 22x^2 + 26x - 140 = 0 \Leftrightarrow 11x^2 + 13x - 70 = 0$

$\Leftrightarrow (x - 2)(11x + 35) = 0 \Leftrightarrow x = 2$.

Hence, required number $= 11x + 2 = 24$.

65. Let the ten's digit be x and unit's digit be y. Then, number $= 10x + y$.

Number obtained by interchanging the digits $= 10y + x$.

$\therefore \quad (10x + y) + (10y + x) = 11(x + y)$, which is divisible by 11.

66. Let the ten's digit be x and unit's digit be y. Then, $(10x + y) - (x + y) = 9$ or $x = 1$.

From this data, we cannot find y, the unit's digit. So, the data is inadequate.

67. Let the ten's digit be x and unit's digit be y.
Then, $(10x + y) - (10y + x) = 36 \Leftrightarrow 9(x - y) = 36 \Leftrightarrow x - y = 4$.

68. Let the ten's digit be x and unit's digit be y.
Then, $(10x + y) - (10y + x) = 63 \Leftrightarrow 9(x - y) = 63 \Leftrightarrow x - y = 7$.
Thus, none of the numbers can be determined.

69. Let the ten's digit be x and unit's digit be y.

Then, $x + y = \frac{1}{5}[(10x + y) - (10y + x)] \Leftrightarrow 5x + 5y = 9x - 9y \Leftrightarrow 4x = 14y$.

Thus, the value of $(x - y)$ cannot be determined from the given data.

70. Let the ten's digit be x and unit's digit be y.

Then, $10 \times 2x + \frac{1}{2}y = 10y + x \Leftrightarrow 20x - x = 10y - \frac{y}{2} \Leftrightarrow 19x = \frac{19}{2}y \Leftrightarrow y = 2x$.

Thus, the unit's digit is twice the ten's digit.

71. Let ten's digit = x. Then, unit's digit = $(8 - x)$.
$\therefore [10(8 - x) + x] - [10x + (8 - x)] = 18 \Leftrightarrow 18x = 54 \Leftrightarrow x = 3$.
So, ten's digit = 3 and unit's digit = 5. Hence, original number = 35.

72. Since the number is greater than the number obtained on reversing the digits, so the ten's digit is greater than the unit's digit.
Let the ten's and unit's digits be $2x$ and x respectively.
Then, $(10 \times 2x + x) - (10x + 2x) = 36 \Leftrightarrow 9x = 36 \Leftrightarrow x = 4$.
$\therefore$ Required difference = $(2x + x) - (2x - x) = 2x = 8$.

73. Let the middle digit be x. Then, $2x - 10$ or $x = 5$. So, the number is either 253 or 352.
Since the number increases on reversing the digits, so the hundred's digit is smaller than the unit's digit. Hence, required number = 253.

74. Since the number reduces on reversing the digits, so ten's digit is greater than the unit's digit.
Let the unit's digit be x. Then, ten's digit = $(x + 1)$.

$\therefore \quad 10x + (x + 1) = \frac{5}{6}[10(x + 1) + x] \Leftrightarrow 66x + 6 = 55x + 50 \Leftrightarrow 11x = 44 \Leftrightarrow x = 4$.

Hence, required number = 54.

75. Let the unit's digit be x. Then, ten's digit = $(x - 2)$.

$\therefore \quad 3[10(x - 2) + x] + \frac{6}{7}[10x + (x - 2)] = 108$

$\Leftrightarrow \quad 231x - 420 + 66x - 12 = 756 \Leftrightarrow 297x = 1188 \Leftrightarrow x = 4$.
Hence, sum of the digits = $x + (x - 2) = 2x - 2 = 6$.

76. Let the ten's digit be x and unit's digit be y. Then, $\frac{10x + y}{2} = 10y + (x + 1)$

$\Leftrightarrow \quad 10x + y = 20y + 2x + 2 \Leftrightarrow 8x - 19y = 2 \quad ...(i)$ and $x + y = 7 \quad ...(ii)$
Solving, (i) and (ii), we get : $x = 5$, $y = 2$. Hence, required number = 52.

77. Let the ten's digit be x. Then, unit's digit = $2x + 1$.
$[10x + (2x + 1)] - [(10(2x + 1) + x] - (10x + (2x + 1))] = 1$
$\Leftrightarrow \quad (12x + 1) - (9x + 9) = 1 \Leftrightarrow 3x = 9 \Leftrightarrow x = 3$.
So, ten's digit = 3 and unit's digit = 7. Hence, original number = 37.

78. Let the ten's digit be x and unit's digit be y.
Then, $10x + y = 3(x + y) \Rightarrow 7x - 2y = 0 \quad ...(i)$
$10x + y + 45 = 10y + x \Rightarrow y - x = 5 \quad ...(ii)$
Solving (i) and (ii), we get : $x = 2$ and $y = 7$.
$\therefore$ Required number = 27.

79. Let the ten's and unit's digit be x and $\frac{8}{x}$ respectively.

Then, $\left(10x+\frac{8}{x}\right)+18=10\times\frac{8}{x}+x \Leftrightarrow 10x^2+8+18x=80+x^2$

$\Leftrightarrow \quad 9x^2+18x-72=0 \Leftrightarrow x^2+2x-8=0 \Leftrightarrow (x+4)(x-2)=0 \Leftrightarrow x=2.$

So, ten's digit = 2 and unit's digit = 4. Hence, required number = 24.

80. Let the two fractions be a and b. Then, $ab=\frac{14}{15}$ and $\frac{a}{b}=\frac{35}{24}$.

$\frac{ab}{(a/b)}=\left(\frac{14}{15}\times\frac{24}{35}\right) \Leftrightarrow b^2=\frac{16}{25} \Leftrightarrow b=\frac{4}{5}.\ ab=\frac{14}{15} \Rightarrow a=\left(\frac{14}{15}\times\frac{5}{4}\right)=\frac{7}{6}.$

Since $a > b$, so greater fraction is $\frac{7}{6}$.

81. $A = 2B \Rightarrow B=\frac{1}{2}A$. So, $AB=\frac{2}{25} \Rightarrow \frac{1}{2}A^2=\frac{2}{25} \Rightarrow A^2=\frac{4}{25} \Rightarrow A=\frac{2}{5}$.

82. Let the fraction be $\frac{x}{y}$. Then, $x+y=11$. ...(*i*)

$\frac{x+1}{y-2}=\frac{2}{3} \Rightarrow 3(x+1)=2(y-2) \Rightarrow 3x-2y=-7$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x = 3$ and $y = 8$. So, the fraction is $\frac{3}{8}$.

83. Let the numerator be x. Then, denominator = $x + 3$.

Now, $\frac{x+4}{(x+3)+4}=\frac{4}{5} \Leftrightarrow 5(x+4)=4(x+7) \Leftrightarrow x=8.$

$\therefore$ The fraction is $\frac{8}{11}$.

84. Let the denominator be x. Then, numerator = $x + 5$.

Now, $\frac{x+5}{x}-\frac{x+5}{x+5}=\frac{5}{4} \Leftrightarrow \frac{x+5}{x}=\frac{5}{4}+1=\frac{9}{4}=2\frac{1}{4}.$

So, the fraction is $2\frac{1}{4}$.

85. Let the fraction be $\frac{2x}{3x}$. Then, $\frac{2x-6}{3x}=\frac{2}{3}\times\frac{2x}{3x} \Leftrightarrow \frac{2x-6}{3x}=\frac{4x}{9x} \Leftrightarrow 18x^2-54x=12x^2$

$\Leftrightarrow 6x^2=54x \Leftrightarrow x=9.$

Hence, numerator of the original fraction = $2x = 18$.

86. Let the fraction be $\frac{x}{y}$. Then,

$\frac{x}{y+1}=\frac{1}{2} \Leftrightarrow 2x-y=1$ ...(*i*) and $\frac{x+1}{y}=1 \Leftrightarrow x-y=-1$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x = 2$, $y = 3$. Hence, the required fraction is $\frac{2}{3}$.

87. Let the fraction be $\frac{x}{y}$. Then,

$\frac{x+2}{y+3}=\frac{7}{9} \Leftrightarrow 9x-7y=3$ ...(*i*) and $\frac{x-1}{y-1}=\frac{4}{5} \Leftrightarrow 5x-4y=1$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x = 5$, $y = 6$. Hence, the original fraction is $\frac{5}{6}$.

88. Let the fraction be $\frac{x}{y}$. Then, $\frac{x+4}{y} - \frac{x}{y} = \frac{2}{3} \Leftrightarrow \frac{4}{y} = \frac{2}{3} \Leftrightarrow y = \left(\frac{4 \times 3}{2}\right) = 6.$

$\therefore$ Denominator = 6.

89. Let the two parts be $(54 - x)$ and x.

Then, $10(54 - x) + 22x = 760 \Leftrightarrow 12x = 240 \Leftrightarrow x = 20.$

$\therefore$ Bigger part = $(54 - x) = 34$.

90. Let the three parts be A, B and C.

Let $\frac{A}{2} = \frac{B}{3} = \frac{C}{4} = x$. Then, $A = 2x$, $B = 3x$ and $C = 4x$. So, $A : B : C = 2 : 3 : 4$.

$\therefore$ Largest part = $\left(243 \times \frac{4}{9}\right) = 108.$

91. Let the four numbers be A, B, C and D. Let $A + 3 = B - 3 = 3C = \frac{D}{3} = x$.

Then, $A = x - 3$, $B = x + 3$, $C = \frac{x}{3}$ and $D = 3x$.

$$A + B + C + D = 64 \Rightarrow (x - 3) + (x + 3) + \frac{x}{3} + 3x = 64$$

$$\Leftrightarrow 5x + \frac{x}{3} = 64 \Leftrightarrow 16x = 192 \Rightarrow x = 12.$$

Thus, the numbers are 9, 15, 4 and 36.

$\therefore$ Required difference = $(36 - 4) = 32$.

92. Let the numbers be a, b and c. Then, $a^2 + b^2 + c^2 = 138$ and $(ab + bc + ca) = 131$.

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = 138 + 2 \times 131 = 400$

$\Rightarrow (a + b + c) = \sqrt{400} = 20.$

93. $A : B = 2 : 3$ and $B : C = 5 : 3 = \frac{3}{5} \times 5 : \frac{3}{5} \times 3 = 3 : \frac{9}{5}$.

So, $A : B : C = 2 : 3 : \frac{9}{5} = 10 : 15 : 9$.

$\therefore$ Second number = $\left(136 \times \frac{15}{34}\right) = 60.$

94. Let the numbers be x, y and z. Then, $x + y = 45$, $y + z = 55$ and $3x + z = 90$

$\Rightarrow y = 45 - x$, $z = 55 - y = 55 - (45 - x) = 10 + x$.

$\therefore$ $3x + 10 + x = 90$ or $x = 20$.

$y = (45 - 20) = 25$ and $z = (10 + 20) = 30$.

$\therefore$ Third number = 30.

EXERCISE 7B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 6*) : ***Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the question. Read both the statements and***

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. What is the two-digit number ? **(S.B.I.P.O. 2003)**
   I. The difference between the two digits is 9.
   II. The sum of the digits is equal to the difference between the two digits.
2. What is the difference between the digits of a two-digit number ? **(Bank P.O. 1999)**
   I. The sum of the digits of that number is 8.
   II. One-fifth of that number is 15 less than half of 44.
3. What is the ratio between the two numbers ?
   I. The sum of two numbers is twice their difference.
   II. The smaller number is 6.
4. What is the two-digit number whose first digit is a and the second digit is b ? The number is greater than 9. **(M.A.T. 2000)**
   I. The number is a multiple of 51.
   II. The sum of the digits a and b is 6.
5. The difference between the digits of a two-digit number is 4. What is the digit in the unit's place ?
   I. The difference between the number and the number obtained by interchanging the positions of the digits is 36.
   II. The sum of the digits of that number is 12.
6. What is the number ? **(Bank P.O. 2000)**
   I. The sum of the two digits is 8. The ratio of the two digits is 1 : 3.
   II. The product of two digits of a number is 12. The quotient of two digits is 3.

Directions (***Questions 7 to 10***) : ***Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the given question.***

7. What is the two-digit number ? **(M.B.A. 2002)**
   I. Sum of the digits is 7.
   II. Difference between the number and the number obtained by interchanging the digits is 9.
   III. Digit in the ten's place is bigger than the digit in the unit's place by 1.
   (*a*) I and II only (*b*) II and III only (*c*) I and III only
   (*d*) All I, II and III (*e*) None of these
8. What is the sum of the digits of the two-digit number ?
   I. The ratio between the ten's digit and unit's digit of the number is 3 : 2.
   II. The number obtained on reversing the order of its digits is 18 less than the original number.
   III. The product of the digits is 24.
   (*a*) Any two of the three (*b*) I only or II and III only (*c*) All I, II and III
   (*d*) I and II only (*e*) None of these

9. What will be the sum of two numbers ? (S.B.I.P.O. 2000)

I. Among the two numbers, the bigger number is greater than the smaller number by 6.

II. 40% of the smaller number is equal to 30% of the bigger number.

III. The ratio between half of the bigger number and one-third of the smaller number is 2 : 1.

(*a*) I and II only (*b*) II and III only (*c*) All I, II and III

(*d*) Any two of the three (*e*) None of these

10. What is the two-digit number ? (R.B.I. 2002)

I. The difference between the two-digit number and the number formed by interchanging the digits is 27.

II. The difference between the two digits is 3.

III. The digit at unit's place is less than that at ten's place by 3.

(*a*) I and II only (*b*) I and III only (*c*) All I, II and III

(*d*) I, and either II or III (*e*) Even with all I, II and III, answer cannot be given.

ANSWERS

1. (*e*) 2. (*e*) 3. (*a*) 4. (*e*) 5. (*b*) 6. (*b*) 7. (*e*) 8. (*a*)
9. (*a*) 10. (*c*)

SOLUTIONS

1. Let the tens and unit digits be x and y respectively. Then,

I. $x - y = 9$.

II. $x + y = x - y$.

From I and II, we get $x - y = 9$ and $x + y = 9$.

On solving, we get $x = 9$ and $y = 0$.

∴ Required number is 90.

Thus, both I and II are needed to get the answer.

∴ Correct answer is (*e*).

2. Let the tens and unit digits be x and y respectively. Then,

I. $x + y = 8$ ...(*i*)

II. $\left(\frac{1}{2} \times 44\right) - \frac{1}{5}(10x + y) = 15 \Rightarrow 10x + y = 35$ ...(*ii*)

On solving (*i*) and (*ii*), we get $x = 3$ and $y = 5$.

Thus, I and II together give the answer.

∴ Correct answer is (*e*).

3. Let the two numbers be x and y.

I gives, $x + y = 2(x - y) \Leftrightarrow x = 3y \Leftrightarrow \frac{x}{y} = \frac{3}{1} \Leftrightarrow x : y = 3 : 1$.

Thus, I only gives the answer.

II does not give the answer.

∴ Correct answer is (*a*).

4. Number = $10b + a$.

I. $10b + a = 51 \times c$, where $c = 1, 2, 3$ etc. ...(*i*)

II. $a + b = 6$ ...(*ii*)

Taking $c = 1$, we get $10b + (6 - b) = 51 \Leftrightarrow 9b = 45 \Leftrightarrow b = 5$.

$\therefore$ $a = 1$, $b = 5$. So, number = 51.

Thus, I and II together give the answer.

$\therefore$ Correct answer is (e).

5. Let the ten's digit be x and unit's digit be y.

Then, $x - y = \pm 4$ ...(i)

I. $(10x + y) - (10y + x) = 36 \Leftrightarrow x - y = 4$ ...(ii)

II. $x + y = 12$ ...(iii)

Thus, (i) and (iii) together give the answer.

$\therefore$ II alone gives the answer and I alone does not give the answer.

$\therefore$ Correct answer is (b).

6. Let the tens and units digit be x and y respectively. Then,

I. $x + y = 8$ and $\frac{x}{y} = \frac{1}{3}$.

II. $xy = 12$ and $\frac{x}{y} = \frac{3}{1}$.

$\therefore$ II gives, $x^2 = 36 \Leftrightarrow x = 6$. So, $3y = 6 \Leftrightarrow y = 2$.

Thus, II alone gives the number. Clearly, I alone does not give the answer.

$\therefore$ Correct answer is (b).

7. Let the tens and units digit be x and y respectively.

I. $x + y = 7$.

II. $(10x + y) - (10y + x) = 9 \Rightarrow x - y = 1$.

III. $x - y = 1$.

Thus, I and II as well as I and III give the answer.

$\therefore$ Correct answer is (e).

8. I. Let the tens and units digit be $3x$ and $2x$ respectively.

II. $(30x + 2x) - (20x + 3x) = 18 \Leftrightarrow x = 2$.

III. $3x \times 2x = 24 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2$.

Thus, any two of the three will give the answer.

$\therefore$ Correct answer is (a).

9. Let the required numbers be x and y, where $x > y$.

I. $x - y = 6$ ...(i)

II. $\frac{30}{100}x = \frac{40}{100}y \Leftrightarrow 3x - 4y = 0$ ...(ii)

III. $\frac{\frac{1}{2}x}{\frac{1}{3}y} = \frac{2}{1} \Leftrightarrow \frac{3x}{2y} = \frac{2}{1} \Leftrightarrow \frac{x}{y} = \frac{4}{3}$.

Thus, I and II only give the answer.

$\therefore$ Correct answer is (a).

10. Let the tens and units digit be x and y respectively.

I. $(10x + y) - (10y + x) = 27 \Leftrightarrow x - y = 3$.

II. $x - y = 3$.

III. $x - y = 3$.

Thus, even all the given three statements together do not give the answer.

$\therefore$ Correct answer is (e).