9 Applied Statistics

Walter Shewhart
(March 18, 1891-
March 11, 1967)

Introduction

The term Applied Statistics refers to the use of statistical theory to conduct operational activities in a variety of fields in real life situations. Today, the applications of statistics are an indispensable part of any and every activity. There are many fields in which statistical concepts can be applied, some of them are business decision making, finance, marketing, economics, social sciences, industry, agriculture etc… An important aspect of applied statistics is to study about the present and future behaviour of the activities performed in an industry.

Walter Andrew Shewhart (pronounced like "shoe-heart",) was an American physicist, engineer and statistician, sometimes known as the father of statistical quality control and also related to the Shewhart cycle.

Learning Objectives

After studying this chapter the students are able to understand

- Time series data
- Components of Time Series
- Moving Averages
- Seasonal Variation
- Index Numbers
- Weighted Index Number
- Tests for an Ideal Index Number
- Statistical Quality Control
- Causes for Variation
- Process Control and Product Control

In this chapter we would be studying about the theoretical and application of the statistical methods of Time Series, Index Number and Statistical Quality Control. Each one of them has its importance in its field of application. Statistical analysis has been widely used for scientific research, surveys, experiments etc. The reliability of the interpretation of the statistical analysis depends upon the informations collected and represented.

9.1 Time Series Analysis

Time Series analysis is one of the statistical methods used to determine the patterns in data collected for a period of time. Generally, each of us should know about the past data to observe and understand the changes that have taken place in the past and current time. One can also identify the regular or irregular occurrence of any specific feature over a time period in a time series data. Most of the time series data relates to fields like Economics, Business, Commerce, etc… For example Production of a product, Cost of a product, Sales of a product, National income, Salary of an individual, etc.. By close observation of time series data, one can predict and plan for future operations in industries and other fields.

Definition 9.1

A Time Series consists of data arranged chronologically – *Croxton & Cowden.*

When quantitative data are arranged in the order of their occurrence, the resulting series is called the Time Series – *Wessel & Wallet.*

9.1.1 Meaning, Uses and Basic Components

Meaning:

A time series consists of a set of observations arranged in chronological order (either ascending or descending). Time Series has an important objective to identify the variations and try to eliminate the variations and also helps us to estimate or predict the future values.

Why should we learn Time Series?

It helps in the analysis of the past behavior.

It helps in forecasting and for future plans.

It helps in the evaluation of current achievements.

It helps in making comparative studies between one time period and others.

Therefore time series helps us to study and analyze the time related data which involves in business fields, economics, industries, etc…

Components of Time Series

There are four types of components in a time series. They are as follows;

(i) Secular Trend

(ii) Seasonal variations

(iii) Cyclic variations

(iv) Irregular variations

(i) Secular Trend

It is a general tendency of time series to increase or decrease or stagnates during a long period of time. An upward tendency is usually observed in population of a country, production, sales, prices in industries, income of individuals etc., A downward tendency is observed in deaths, epidemics, prices of electronic gadgets, water sources, mortality rate etc.… It is not necessarily that the increase or decrease should be in the same direction throughout the given period of time.

(ii) Seasonal Variations

As the name suggests, tendency movements are due to nature which repeat themselves periodically in every seasons. These variations repeat themselves in less than one year time. It is measured in an interval of time. Seasonal variations may be influenced by natural force, social customs and traditions. These variations are the results of such factors which uniformly and regularly rise and fall in the magnitude. For example, selling of umbrellas' and raincoat in the rainy season, sales of cool drinks in summer season, crackers in Deepawali season, purchase of dresses in a festival season, sugarcane in Pongal season.

(iii) Cyclic Variations

These variations are not necessarily uniformly periodic in nature. That is, they may or may not follow exactly similar patterns after equal intervals of time. Generally one cyclic period ranges from 7 to 9 years and there is no hard and fast rule in the fixation of years for a cyclic period. For example, every business cycle has a Start- Boom- Depression- Recover, maintenance during booms and depressions, changes in government monetary policies, changes in interest rates.

(iv) Irregular Variations

These variations do not have particular pattern and there is no regular period of time of their occurrences. These are accidently changes which are purely random or unpredictable. Normally they are short-term variations, but its occurrence sometimes has its effect so intense that they may give rise to new cyclic or other movements of variations. For example floods, wars, earthquakes, Tsunami, strikes, lockouts etc…

Mathematical Model for a Time Series

There are two common models used for decomposition of a time series into its components, namely additive and multiplicative model.

(i) Additive Model:

This model assumes that the observed value is the sum of all the four components of time series. (i.e) $Y= T+S+C+I$

where Y = Original value, T = Trend Value, S = Seasonal component, C = Cyclic component, I = Irregular component

The additive model assumes that all the four components operate independently. It also assumes that the behavior of components is of an additive character.

(ii) Multiplicative Model:

This model assumes that the observed value is obtained by multiplying the trend (T) by the rates of other three components. $Y = T \times S \times C \times I$

where Y = Original value , T = Trend Value, S = Seasonal component, C = Cyclic component, I = Irregular component

This model assumes that the components due to different causes are not necessarily independent and they can affect one another. It also assumes that the behavior of components is of a multiplicative character.

9.1.2 Measurements of Trends

Following are the methods by which we can measure the trend.

(i) Freehand or Graphic Method.

(ii) Method of Semi-Averages.

(iii) Method of Moving Averages.

(iv) Method of Least Squares.

(i) Freehand or Graphic Method.

It is the simplest and most flexible method for estimating a trend. We will see the working procedure of this method.

Procedure:

(a) Plot the time series data on a graph.

(b) Draw a freehand smooth curve joining the plotted points.

(c) Examine the direction of the trend based on the plotted points.

(d) Draw a straight line which will pass through the maximum number of plotted points.

Example 9.1

Fit a trend line by the method of freehand method for the given data.

Year	2000	2001	2002	2003	2004	2005	2006	2007
Sales	30	46	25	59	40	60	38	65

Solution:

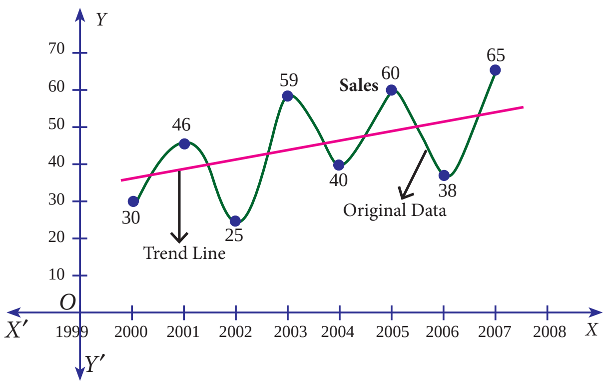

Fig: 9.1

Note

The trend drawn by the freehand method can be extended to predict the future values of the given data. However, this method is subjective in nature, predictions obtained by this method depends on the personal bias and judgement of the investigator handling the data.

(ii) Method of Semi-Averages

In this method, the semi-averages are calculated to find out the trend values. Now, we will see the working procedure of this method.

Procedure:

(i) The data is divided into two equal parts. In case of odd number of data, two equal parts can be made simply by omitting the middle year.

(ii) The average of each part is calculated, thus we get two points.

(iii) Each point is plotted at the mid-point (year) of each half.

(iv) Join the two points by a straight line.

(v) The straight line can be extended on either side.

(vi) This line is the trend line by the methods of semi-averages.

Example 9.2

Fit a trend line by the method of semi-averages for the given data.

Year	2000	2001	2002	2003	2004	2005	2006
Production	105	115	120	100	110	125	135

Solution:

Since the number of years is odd(seven), we will leave the middle year's production value and obtain the averages of first three years and last three years.

Year	Production	Average
2000	105	
2001	115	$\frac{105+115+120}{3}=113.33$
2002	120	
2003	100 (left out)	
2004	110	
2005	125	$\frac{110+125+135}{3}=123.33$
2006	135	

Table 9.1

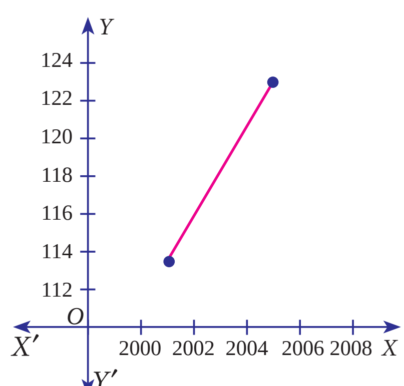

Fig: 9.2

Example 9.3

Fit a trend line by the method of semi-averages for the given data.

Year	1990	1991	1992	1993	1994	1995	1996	1997
Sales	15	11	20	10	15	25	35	30

Solution:

Since the number of years is even(eight), we can equally divide the given data it two equal parts and obtain the averages of first four years and last four years.

Year	Production	Average
1990	15	$\frac{15+11+20+10}{4}=14$
1991	11	
1992	20	
1993	10	
1994	15	$\frac{15+25+35+30}{4}=26.25$
1995	25	
1996	35	
1997	30	

Table 9.2

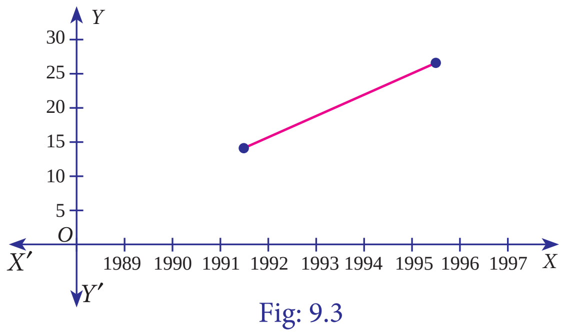

Fig: 9.3

Note

(i) The future values can be predicted.

(ii) The trend values obtained by this method and the predicted values are not precise.

9.1.3 Method of Moving Averages

Moving Averages Method gives a trend with a fair degree of accuracy. In this method, we take arithmetic mean of the values for a certain time span. The time span can be three- years, four -years, five- years and so on depending on the data set and our interest. We will see the working procedure of this method.

Procedure:

(i) Decide the period of moving averages (three- years, four -years).

(ii) In case of odd years, averages can be obtained by calculating,

$$\frac{a+b+c}{3}, \frac{b+c+d}{3}, \frac{c+d+e}{3}, \frac{d+e+f}{3}, \ldots$$

(iii) If the moving average is an odd number, there is no problem of centering it, the average value will be centered besides the second year for every three years.

(iv) In case of even years, averages can be obtained by calculating,

$$\frac{a+b+c+d}{4}, \frac{b+c+d+e}{4},$$

$$\frac{c+d+e+f}{4}, \frac{d+e+f+g}{4}, \ldots$$

(v) If the moving average is an even number, the average of first four values will be placed between 2^{nd} and 3^{rd} year, similarly the average of the second four values will be placed between 3^{rd} and 4^{th} year. These two averages will be again averaged and placed in the 3^{rd} year. This continues for rest of the values in the problem. This process is called as centering of the averages.

Example 9.4

Calculate three-yearly moving averages of number of students studying in a higher secondary school in a particular village from the following data.

Year	Number of students
1995	332
1996	317
1997	357
1998	392
1999	402
2000	405
2001	410
2002	427
2003	435
2004	438

Solution:

Computation of three- yearly moving averages.

Year	Number of students	3- yearly moving Total	3- yearly moving Averages
1995	332	---	---
1996	317	1006	335.33
1997	357	1066	355.33
1998	392	1151	383.67
1999	402	1199	399.67
2000	405	1217	405.67
2001	410	1242	414.00
2002	427	1272	424.00
2003	435	1300	433.33
2004	438	---	---

Table 9.3

Example 9.5

Calculate four-yearly moving averages of number of students studying in a higher secondary school in a particular city from the following data.

Year	Number of students
2001	124
2002	120
2003	135
2004	140
2005	145
2006	158
2007	162
2008	170
2009	175

Solution:

Computation of four- yearly moving averages.

Year	Sales	4-yearly centered moving total	4-yearly moving Average	4-yearly centered moving Average
2001	124	--	--	--
2002	120	--	--	--
		519	129.75	
2003	135	--		132.37
		540	135.00	
2004	140	--		139.75
		578	144.50	
2005	145	--		147.87
		605	151.25	
2006	158	--		155.00
		635	158.75	
2007	162	--		162.50
		665	166.25	
2008	170	--	--	-
2009	175	--	--	-

Table 9.4

Note

The calculated 4-yearly centered moving average belongs to the particular year present in that row. In example 9.5, the value 132.37 belongs to the year 2003.

9.1.4 Method of Least Squares

The line of best fit is a line from which the sum of the deviations of various points is zero. This is the best method for obtaining the trend values. It gives a convenient basis for calculating the line of best fit for the time series. It is a mathematical method for measuring trend. Further the sum of the squares of these deviations would be least when compared with other fitting methods. So, this method is known as the Method of Least Squares and satisfies the following conditions:

(i) The sum of the deviations of the actual values of Y and $\hat{Y}$ (estimated value of Y) is Zero. that is $\Sigma(Y-\hat{Y}) = 0$.

(ii) The sum of squares of the deviations of the actual values of Y and $\hat{Y}$ (estimated value of Y) is least. that is $\Sigma(Y-\hat{Y})^2$ is least.

Procedure:

(i) The straight line trend is represented by the equation $Y = a + bX$ ...(1)

where Y is the actual value, X is time, a, b are constants

(ii) The constants 'a' and 'b' are estimated by solving the following two normal Equations

$$\Sigma Y = n\,a + b\,\Sigma X \qquad ...(2)$$

$$\Sigma XY = a\,\Sigma X + b\,\Sigma X^2 \qquad ...(3)$$

Where 'n' = number of years given in the data.

(iii) By taking the mid-point of the time as the origin, we get $\Sigma X = 0$

(iv) When $\Sigma X = 0$, the two normal equations reduces to

$$\Sigma Y = n\,a + b\,(0);\ a = \frac{\sum Y}{n} = \overline{Y}$$

$$\Sigma XY = a(0) + b\,\Sigma X^2\ ;\ b = \frac{\sum XY}{\sum X^2}$$

The constant 'a' gives the mean of Y and 'b' gives the rate of change (slope).

(v) By substituting the values of 'a' and 'b' in the trend equation (1), we get the Line of Best Fit.

DO YOU KNOW?

The constant 'a' takes only positive values, but constant 'b' takes both positive and negative values

Example 9.6

Given below are the data relating to the production of sugarcane in a district.

Fit a straight line trend by the method of least squares and tabulate the trend values.

Year	2000	2001	2002	2003	2004	2005	2006
Prod. of Sugarcane	40	45	46	42	47	50	46

Solution:

Computation of trend values by the method of least squares (ODD Years).

Year (x)	Production of Sugarcane (Y)	$X = (x - 2003)$	X^2	XY	Trend values (Yt)
2000	40	–3	9	–120	42.04
2001	45	–2	4	–90	43.07
2002	46	–1	1	–46	44.11
2003	42	0	0	0	45.14
2004	47	1	1	47	46.18
2005	50	2	4	100	47.22
2006	46	3	9	138	48.25
$N = 7$	$\Sigma Y = 316$	$\Sigma X = 0$	$\Sigma X^2 = 28$	$\Sigma XY = 29$	$\Sigma Yt = 316$

Table 9.5

$$a = \frac{\sum Y}{n} = \frac{316}{7} = 45.143$$

$$b = \frac{\sum XY}{\sum X^2} = \frac{29}{28} = 1.036$$

Therefore, the required equation of the straight line trend is given by

$$Y = a + bX$$

$$Y = 45.143 + 1.036\,(x - 2003)$$

The trend values can be obtained by

When $X = 2000$,

$Yt = 45.143 + 1.036\,(2000–2003) = 42.035$

When $X = 2001$,

$Yt = 45.143 + 1.036\,(2001–2003) = 43.071$,

similarly other values can be obtained.

Example 9.7

Given below are the data relating to the sales of a product in a district.

Fit a straight line trend by the method of least squares and tabulate the trend values.

Year	1995	1996	1997	1998	1999	2000	2001	2002
Sales	6.7	5.3	4.3	6.1	5.6	7.9	5.8	6.1

Solution:

Computation of trend values by the method of least squares.

In case of EVEN number of years, let us consider

$$X = \frac{(x - \text{Arithimetic mean of two middle years})}{0.5}$$

Year (x)	Sales (Y)	$X = \dfrac{(x - 1998.5)}{0.5}$	XY	X^2	Trend values (Y_t)
1995	6.7	–7	–46.9	49	5.6167
1996	5.3	–5	–26.5	25	5.7190
1997	4.3	–3	–12.9	9	5.8214
1998	6.1	–1	–6.1	1	5.9238
1999	5.6	1	5.6	1	6.0262
2000	7.9	3	23.7	9	6.1286
2001	5.8	5	29.0	25	6.2310
2002	6.1	7	42.7	49	6.3333
N=8	47.8	$\Sigma X = 0$	8.6	168	

Table 9.6

$$a = \frac{\sum Y}{n} = \frac{47.8}{8} = 5.975$$

$$b = \frac{\sum XY}{\sum X^2} = \frac{8.6}{168} = 0.05119$$

Therefore, the required equation of the straight line trend is given by

$Y = a + bX$; $Y = 5.975 + 0.05119\,X$.

When $X = 1995$,

$$X = 1995,\ Y_t = 5.975 + 0.05119\left(\frac{1995 - 1998.5}{0.5}\right) = 5.6167$$

When $X = 1996$,

$$X = 1996,\ Y_t = 5.975 + 0.05119\left(\frac{1996 - 1998.5}{0.5}\right) = 5.7190$$

similarly other values can be obtained.

Note

(i) Future forecasts made by this method are based only on trend values.

(ii) The predicted values are more reliable in this method than the other methods.

9.1.5 Methods of measuring Seasonal Variations By Simple Averages :

Seasonal Variations can be measured by the method of simple average. The data should be available in season wise likely weeks, months, quarters.

Method of Simple Averages:

This is the simplest and easiest method for studying Seasonal Variations. The procedure of simple average method is outlined below.

Procedure:

(i) Arrange the data by months, quarters or years according to the data given.

(ii) Find the sum of the each months, quarters or year.

(iii) Find the average of each months, quarters or year.

(iv) Find the average of averages, and it is called Grand Average (G)

(v) Compute Seasonal Index for every season (i.e) months, quarters or year is given by

$$\text{Seasonal Index (S.I)} = \frac{\text{Seasonal Average}}{\text{Grand average}} \times 100$$

(vi) If the data is given in months

$$\text{S.I for Jan} = \frac{\text{Monthly Average (for Jan)}}{\text{Grand average}} \times 100$$

$$\text{S.I for Feb} = \frac{\text{Monthly Average (for Feb)}}{\text{Grand average}} \times 100$$

Similarly we can calculate SI for all other months.

(vii) If the data is given in quarter

$$\text{S.I for I Quarter} = \frac{\text{Average of I quarter}}{\text{Grand average}} \times 100$$

$$\text{S.I for II Quarter} = \frac{\text{Average of II quarter}}{\text{Grand average}} \times 100$$

$$\text{S.I for III Quarter} = \frac{\text{Average of III quarter}}{\text{Grand average}} \times 100$$

$$\text{S.I for IV Quarter} = \frac{\text{Average of IV quarter}}{\text{Grand average}} \times 100$$

Example 9.8

Calculate the seasonal index for the monthly sales of a product using the method of simple averages.

Months	Year		
	2001	2002	2003
Jan	15	20	18
Feb	41	21	16
Mar	25	27	20
Apr	31	19	28
May	29	17	24
June	47	25	25
July	41	29	30
Aug	19	31	34
Sep	35	35	30
Oct	38	39	38
Nov	40	30	37
Dec	30	44	39

Solution: Computation of seasonal Indices by method of simple averages.

Months	Year			Monthly Total	Monthly Averages
	2001	2002	2003		
Jan	15	20	18	53	17.67
Feb	41	21	16	78	26
Mar	25	27	20	72	24
Apr	31	19	28	78	26
May	29	17	24	70	23.33
June	47	25	25	97	32.33
July	41	29	30	100	33.33
Aug	19	31	34	84	28
Sep	35	35	30	100	33.33
Oct	38	39	38	115	38.33
Nov	40	30	37	107	35.67
Dec	30	44	39	113	37.67

Table 9.7

$$\text{S.I for Jan} = \frac{Monthly\ Average\,(for\ Jan)}{Grand\ average} \times 100$$

$$\text{Grand Average} = \frac{355.66}{12} = 29.64$$

$$\text{S.I for Jan} = \frac{17.67}{29.64} X100 = 59.62$$

$$\text{S.I for Feb} = \frac{26}{29.64} X100 = 87.72$$

Similarly other seasonal index values can be obtained.

Months	Seasonal Index
Jan	59.62
Feb	87.72
Mar	80.97
Apr	87.72
May	78.71
June	109.08
July	112.45
Aug	94.47
Sep	112.45
Oct	129.32
Nov	120.34
Dec	127.09

Example 9.9

Calculate the seasonal index for the quarterly production of a product using the method of simple averages.

Year	I Quarter	II Quarter	III Quarter	IV Quarter
2005	255	351	425	400
2006	269	310	396	410
2007	291	332	358	395
2008	198	289	310	357
2009	200	290	331	359
2010	250	300	350	400

Solution :

Computation of Seasonal Index by the method of simple averages.

Year	I Quarter	II Quarter	III Quarter	IV Quarter
2005	255	351	425	400
2006	269	310	396	410
2007	291	332	358	395
2008	198	289	310	357
2009	200	290	331	359
2010	250	300	350	400
Quarterly Total	1463	1872	2170	2321
Quarterly Averages	243.83	312	361.67	386.83

Table 9.8

$$\text{S.I for I Quarter} = \frac{Average\ of\ I\ quarter}{Grand\ average} \times 100$$

$$\text{Grand Average} = \frac{1304.333}{4} = 326.0833$$

$$\text{S.I for I Q} = \frac{243.8333}{326.0833} \times 100 = 74.77$$

$$\text{S.I for II Q} = \frac{312}{326.0833} \times 100 = 95.68$$

$$\text{S.I for III Q} = \frac{361.6667}{326.0833} \times 100 = 110.91$$

$$\text{S.I for IV Q} = \frac{386.833}{326.0833} \times 100 = 118.63$$

Exercise 9.1

1. Define Time series.
2. What is the need for studying time series?
3. State the uses of time series.
4. Mention the components of the time series.
5. Define secular trend.
6. Write a brief note on seasonal variations
7. Explain cyclic variations
8. Discuss about irregular variation
9. Define seasonal index.
10. Explain the method of fitting a straight line.
11. State the two normal equations used in fitting a straight line.
12. State the different methods of measuring trend.
13. Compute the average seasonal movement for the following series

year	Quarterly Production			
	I	II	III	IV
2002	3.5	3.8	3.7	3.5
2003	3.6	4.2	3.4	4.1
2004	3.4	3.9	3.7	4.2
2005	4.2	4.5	3.8	4.4
2006	3.9	4.4	4.2	4.6

14. The following figures relates to the profits of a commercial concern for 8 years

Year	Profit (₹)
1986	15,420
1987	15,470
1988	15,520
1989	21,020
1990	26,500
1991	31,950
1992	35,600
1993	34,900

Find the trend of profits by the method of three yearly moving averages.

15. Find the trend of production by the method of a five-yearly period of moving average for the following data:

Year	Production ('000)
1979	126
1980	123
1981	117
1982	128
1983	125
1984	124
1985	130

1986	114
1987	122
1988	129
1989	118
1990	123

16. The following table gives the number of small-scale units registered with the Directorate of Industries between 1985 and 1991. Show the growth on a trend line by the free hand method.

Year	No. of units (in'000)
1985	10
1986	22
1987	36
1988	62
1989	55
1990	40
1991	34
1992	50

17. The annual production of a commodity is given as follows :

Year	Production (in tones)
1995	155
1996	162
1997	171
1998	182
1999	158
2000	180
2001	178

Fit a straight line trend by the method of least squares.

18. Determine the equation of a straight line which best fits the following data

Year	2000	2001	2002	2003	2004
Sales (₹ '000)	35	36	79	80	40

Compute the trend values for all years from 2000 to 2004

19. The sales of a commodity in tones varied from January 2010 to December 2010 as follows:

in year 2010	Sales (in tones)
Jan	280
Feb	240
Mar	270
Apr	300
May	280
Jun	290
Jul	210
Aug	200
Sep	230
Oct	200
Nov	230
Dec	210

Fit a trend line by the method of semi-average.

20. Use the method of monthly averages to find the monthly indices for the following data of production of a commodity for the years 2002, 2003 and 2004.

2002	2003	2004
15	20	18
18	18	25
17	16	21
19	13	11
16	12	14
20	15	16
21	22	19
18	16	20
17	18	17
15	20	16
14	17	18
18	15	20

21. Calculate the seasonal indices from the following data using the average from the following data using the average method:

	I Quarter	II Quarter	III Quarter	IV Quarter
2008	72	68	62	76
2009	78	74	78	72
2010	74	70	72	76
2011	76	74	74	72
2012	72	72	76	68

22. The following table shows the number of salesmen working for a certain concern:

Year	1992	1993	1994	1995	1996
No. of salesmen	46	48	42	56	52

Use the method of least squares to fit a straight line and estimate the number of salesmen in 1997.

9.2 Index Number

Introduction:

Index Numbers are the indicators which reflect the changes over a specified period of time in price of different commodities, production, sales, cost of living etc... Index Numbers are statistical methods used to measure the relative change in the level of a variable or group of variables with respect to time, geographical location or other characteristics such as income, profession etc. The variables may be

(i) The price of a particular commodity. For example gold, silver, iron (or) a group of commodities. For example consumer goods, household food items etc..

(ii) The volume of export and import, agricultural and industrial production.

(iii) National income of a country, cost of living of persons belonging to a particular income group.

9.2.1 Meaning, Classifications and Uses

Suppose we want to measure the general changes in the price level of consumer goods, the price changes are not directly measurable, as the prices of various commodities are in different units For example rice, wheat and sugar are in kilograms, milk, petrol, oil are in litres, clothes are in metres etc… Further, the price and quantity of some commodities may increase or decrease during the two time periods. Therefore, index number gives a single representative value which gives the general level of the prices of the commodities in a given group over a specified time period.

Definition 9.2

"An Index Number is a device which shows by its variations the Changes in a magnitude which is not capable of accurate measurements in itself or of direct valuation in practice". - *Wheldon*

"An Index number is a statistical measure of fluctuations in a variable arranged in the form of a series and using a base period for making comparisons" – *Lawrence J Kalpan*

Classification of Index Numbers:

Index number can be classified as follows,

(i) Price Index Number

It measures the general changes in the retail or wholesale price level of a particular or group of commodities.

(ii) Quantity Index Number

These are indices to measure the changes in the quantity of goods manufactured in a factory.

(iii) Cost of living Index Number

These are intended to study the effect of change in the price level on the cost of living of different classes of people.

Uses of Index number

i. It is an important tool for the formulating decision and management policies.

ii. It helps in studying the trends and tendencies.

iii. It determines the inflation and deflation in an economy.

Construction of Index Number

There are two types in construction of index number.

(i) Unweighted Index Number

(ii) Weighted Index Number

We confine ourselves to Weighted Index Number.

9.2.2 Weighted Index Number

In general, all the commodities cannot be given equal importance, so we can assign weights to each commodity according to their importance and the index number computed from these weights are called as weighted index number. The weights can be production, consumption values. If 'w' is the weight attached to a commodity, then the price index is given by,

$$\text{Price Index } (P_{01}) = \frac{\sum p_1 w}{\sum p_0 w} \times 100,$$

Let us consider the following notations,

p_1 - current year price

p_0 - base year price

q_1 - current year quantity

q_0 - base year quantity

where suffix '0' represents base year and '1' represents current year.

Laspeyre's price index number

$$P_{01}^{L} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

Paasche's price index number

$$P_{01}^{P} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

Fisher's price index number

$$P_{01}^{F} = \sqrt{P_{01}^{L} \times P_{01}^{P}}$$

$$P_{01}^{F} = \sqrt{\frac{\sum p_1 q_0 \times \sum p_1 q_1}{\sum p_0 q_0 \times \sum p_0 q_1}} \times 100$$

Note

To get exact Fisher's price index number, one should use formula method rather than using $P_{01}^{F} = \sqrt{P_{01}^{L} \times P_{01}^{P}}$.

In Laspeyre's price index number, the quantity of the base year is used as weight.

In Paasche's price index number, the quantity of the current year is used as weight.

DO YOU KNOW?

Fisher's price index number is the geometric mean between Laspeyre's and Paasche's price index number

Example 9.10

Calculate the Laspeyre's, Paasche's and Fisher's price index number for the following data. Interpret on the data.

Commodities	Price		Quantity	
	2000	2010	2000	2010
Rice	38	35	6	7
Wheat	12	18	7	10
Rent	10	15	10	15
Fuel	25	30	12	16
Miscellaneous	30	33	8	10

Solution

Commodities	Price		Quantity		p_0q_0	p_0q_1	p_1q_0	p_1q_1
	2000 (p_0)	2010 (p_1)	2000 (q_0)	2010 (q_1)				
Rice	38	35	6	7	228	266	210	245
Wheat	12	18	7	10	84	120	126	180
Rent	10	15	10	15	100	150	150	225
Fuel	25	30	12	16	300	400	360	480
Miscellaneous	30	33	8	10	240	300	264	330
				Total	**952**	**1236**	**1110**	**1460**

Table 9.9

Laspeyre's price index number

$$P_{01}^{L}=\frac{\sum p_1 q_0}{\sum p_0 q_0}\times 100=\frac{1110}{952}\times 100=116.60$$

On an average, there is an increase of 16.60 % in the price of the commodities when the year 2000 compared with the year 2010.

Paasche's price index number

$$P_{01}^{P}=\frac{\sum p_1 q_1}{\sum p_0 q_1}\times 100=\frac{1460}{1236}\times 100=118.12$$

On an average, there is an increase of 18.12 % in the price of the commodities when the year 2000 compared with the year 2010.

Fisher's price index number

$$P_{01}^{F}=\sqrt{\frac{\sum p_1q_0\times\sum p_1q_1}{\sum p_0q_0\times\sum p_0q_1}}\times 100$$

$$=\sqrt{\frac{1110\times 1460}{952\times 1236}}\times 100=117.36$$

On an average, there is an increase of 17.36 % in the price of the commodities when the year 2000 compared with the year 2010.

Example 9.11

Construct the Laspeyre's , Paasche's and Fisher's price index number for the following data. Comment on the result.

Commodities	Base Year		Current Year	
	Price	Quantity	Price	Quantity
Rice	15	5	16	8
Wheat	10	6	18	9
Rent	8	7	15	8
Fuel	9	5	12	6
Transport	11	4	11	7
Miscellaneous	16	6	15	10

Solution:

Commodities	Base Year		Current Year		p_0q_0	p_0q_1	p_1q_0	p_1q_1
	Price (p_0)	Quantity (p_0)	Price	Quantity				
Rice	15	5	16	8	75	120	80	128
Wheat	10	6	18	9	60	90	108	162
Rent	8	7	15	8	56	64	105	120
Fuel	9	5	12	6	45	54	60	72
Transport	11	4	11	7	44	77	44	77
Miscellaneous	16	6	15	10	96	160	90	150
				Total	**376**	**565**	**487**	**709**

Table 9.10

Laspeyre's price index number

$$P_{01}^{L}=\frac{\sum p_1 q_0}{\sum p_0 q_0}\times 100=\frac{487}{376}\times 100=129.5212$$

Paasche's price index number

$$P_{01}^{P}=\frac{\sum p_1 q_1}{\sum p_0 q_1}\times 100=\frac{709}{565}\times 100=125.4867$$

Fisher's price index number

$$P_{01}^{F}=\left(\sqrt{\frac{\sum p_1q_0\times\sum p_1q_1}{\sum p_0q_0\times\sum p_0q_1}}\right)\times 100$$

$$=\left(\sqrt{\frac{487\times 709}{376\times 565}}\right)\times 100=127.4879$$

On an average, there is an increase of 29.52 % , 25.48% and 27.48% in the price of the commodities by Laspeyre's, Paasche's, Fisher's price index number respectively, when the base year compared with the current year.

9.2.3 Test of adequacy for an Index Number

Index numbers are studied to know the relative changes in price and quantity for any two years compared. There are two tests which are used to test the adequacy for an index number. The two tests are as follows,

(i) Time Reversal Test

(ii) Factor Reversal Test

The criterion for a good index number is to satisfy the above two tests.

Time Reversal Test

It is an important test for testing the consistency of a good index number. This test maintains time consistency by working both forward and backward with respect to time (here time refers to base year and current year). Symbolically the following relationship should be satisfied, $P_{01} \times P_{10} = 1$

Fisher's index number formula satisfies the above relationship

$$P_{01}^{F} = \sqrt{\frac{\sum p_1 q_0 \times \sum p_1 q_1}{\sum p_0 q_0 \times \sum p_0 q_1}}$$

when the base year and current year are interchanged, we get

$$P_{10}^{F} = \sqrt{\frac{\sum p_0 q_1 \times \sum p_0 q_0}{\sum p_1 q_1 \times \sum p_1 q_0}}$$

$$P_{01}^{F} \times P_{10}^{F} = 1$$

Note

Ignore the factor 100 in each Index number

Factor Reversal Test

This is another test for testing the consistency of a good index number. The product of price index number and quantity index number from the base year to the current year should be equal to the true value ratio. That is, the ratio between the total value of current period and total

value of the base period is known as true value ratio. Factor Reversal Test is given by,

$$P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

where $P_{01} = \sqrt{\dfrac{\sum p_1 q_0 \times \sum p_1 q_1}{\sum p_0 q_0 \times \sum p_0 q_1}}$

Now interchanging P by Q, we get

$$Q_{01} = \sqrt{\frac{\sum q_1 p_0 \times \sum q_1 p_1}{\sum q_0 p_0 \times \sum q_0 p_1}}$$

where P_{01} is the relative change in price.

Q_{01} is the relative change in quantity.

Example 9.12

Calculate Fisher's price index number and show that it satisfies both Time Reversal Test and Factor Reversal Test for data given below.

Do You Know?

Since Fisher's Price Index Number satisfies both TRT & FRT, it is termed as an Ideal Index Number

Commodities	Price		Quantity	
	2003	2009	2003	2009
Rice	10	13	4	6
Wheat	15	18	7	8
Rent	25	29	5	9
Fuel	11	14	8	10
Miscellaneous	14	17	6	7

Solution:

Commodities	Price		Quantity		p_0q_0	p_0q_1	p_1q_0	p_1q_1
	2003 (p_0)	2009 (p_1)	2003 (q_0)	2009 (q_1)				
Rice	10	13	4	6	40	60	52	78
Wheat	15	18	7	8	105	120	126	144
Rent	25	29	5	9	125	225	145	261
Fuel	11	14	8	10	88	110	112	140
Miscellaneous	14	17	6	7	84	98	102	119
				Total	442	613	537	742

Table 9.11

Fisher's price index number

$$P_{01}^{F} = \left(\sqrt{\frac{\sum p_1 q_0 \times \sum p_1 q_1}{\sum p_0 q_0 \times \sum p_0 q_1}}\right) \times 100$$

$$= \left(\sqrt{\frac{537 \times 742}{442 \times 613}} \right) \times 100 = 121.2684$$

Time Reversal Test: $P_{01} \times P_{10} = 1$

$$P_{01} \times P_{10} = \sqrt{\left(\frac{\sum p_1q_0 \times \sum p_1q_1 \times \sum p_0q_1 \times \sum p_0q_0}{\sum p_0q_0 \times \sum p_0q_1 \times \sum p_1q_1 \times \sum p_1q_0} \right)}$$

$$P_{01} \times P_{10} = \sqrt{\left(\frac{537 \times 742 \times 613 \times 442}{442 \times 613 \times 742 \times 537} \right)}$$

$$P_{01} \times P_{10} = 1$$

Factor Reversal Test : $P_{01} \times Q_{01} = \dfrac{\sum p_1q_1}{\sum p_0q_0}$

$$P_{01} \times Q_{01} = \sqrt{\left(\frac{\sum p_1q_0 \times \sum p_1q_1 \times \sum q_1p_0 \times \sum q_1p_1}{\sum p_0q_0 \times \sum p_0q_1 \times \sum q_0p_0 \times \sum q_0p_1} \right)}$$

$$P_{01} \times Q_{01} = \sqrt{\left(\frac{537 \times 742 \times 613 \times 742}{442 \times 613 \times 442 \times 537} \right)}$$

$$P_{01} \times Q_{01} = \sqrt{\left(\frac{742 \times 742}{442 \times 442} \right)} = \frac{742}{442}$$

$$\Rightarrow \quad P_{01} \times Q_{01} = \frac{\sum p_1q_1}{\sum p_0q_0}$$

Example 9.13

Calculate Fisher's price index number and show that it satisfies both Time Reversal Test and Factor Reversal Test for data given below.

Commodities	Base Year		Current Year	
	Price	Quantity	Price	Quantity
Rice	10	5	11	6
Wheat	12	6	13	4
Rent	14	8	15	7
Fuel	16	9	17	8
Transport	18	7	19	5
Miscellaneous	20	4	21	3

Solution:

Commodities	Base Year		Current Year		p_0q_0	p_0q_1	p_1q_0	p_1q_1
	Price (p_0)	Quantity (q_0)	Price (p_1)	Quantity (q_1)				
Rice	10	5	11	6	50	60	55	66
Wheat	12	6	13	4	72	48	78	52
Rent	14	8	15	7	112	98	120	105
Fuel	16	9	17	8	144	128	153	136
Transport	18	7	19	5	126	90	133	95
Miscellaneous	20	4	21	3	80	60	84	63
				Total	584	484	623	517

Table 9.12

Fisher's price index number

$$P_{01}^{F} = \left(\sqrt{\frac{\sum p_1q_0 \times \sum p_1q_1}{\sum p_0q_0 \times \sum p_0q_1}} \right) \times 100$$

$$= \left(\sqrt{\frac{623 \times 517}{584 \times 484}} \right) \times 100 = 106.74$$

Time Reversal Test: $P_{01} \times P_{10} = 1$

$$P_{01} \times P_{10} = \sqrt{\left(\frac{\sum p_1q_0 \times \sum p_1q_1 \times \sum p_0q_1 \times \sum p_0q_0}{\sum p_0q_0 \times \sum p_0q_1 \times \sum p_1q_1 \times \sum p_1q_0} \right)}$$

$$P_{01} \times P_{10} = \sqrt{\left(\frac{623 \times 517 \times 484 \times 584}{584 \times 484 \times 517 \times 623} \right)}$$

$$P_{01} \times P_{10} = 1$$

Factor Reversal Test : $P_{01} \times Q_{01} = \dfrac{\sum p_1q_1}{\sum p_0q_0}$

$$P_{01} \times Q_{01} = \sqrt{\left(\frac{\sum p_1q_0 \times \sum p_1q_1 \times \sum q_1p_0 \times \sum q_1p_1}{\sum p_0q_0 \times \sum p_0q_1 \times \sum q_0p_0 \times \sum q_0p_1} \right)}$$

$$P_{01} \times Q_{01} = \sqrt{\left(\frac{623 \times 517 \times 484 \times 517}{584 \times 484 \times 584 \times 623} \right)}$$

$$P_{01} \times Q_{01} = \sqrt{\left(\frac{517 \times 517}{585 \times 584} \right)} = \frac{517}{584}$$

$$P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

Example 9.14

Construct Fisher's price index number and prove that it satisfies both Time Reversal Test and Factor Reversal Test for data following data.

Commodities	Base Year		Current Year	
	Price	Quantity	Price	Quantity
Rice	40	5	48	4
Wheat	45	2	42	3
Rent	90	4	95	6
Fuel	85	3	80	2
Transport	50	5	65	8
Miscellaneous	65	1	72	3

Solution:

Commodities	Base Year		Current Year		p_0q_0	p_0q_1	p_1q_0	p_1q_1
	Price (p_0)	Quantity (q_0)	Price (p_1)	Quantity (q_1)				
Rice	40	5	48	4	200	160	240	192
Wheat	45	2	42	3	90	135	84	126
Rent	90	4	95	6	360	540	380	570
Fuel	85	3	80	2	255	170	240	160
Transport	50	5	65	8	250	400	325	520
Miscellaneous	65	1	72	3	65	195	72	216
				Total	1220	1600	1341	1784

Table 9.13

Fisher's price index number

$$P_{01}^{F} = \left(\sqrt{\frac{\sum p_1 q_0 \times \sum p_1 q_1}{\sum p_0 q_0 \times \sum p_0 q_1}}\right) \times 100$$

$$= \left(\sqrt{\frac{1341 \times 1784}{1220 \times 1600}}\right) \times 100 = 110.706$$

Time Reversal Test: $P_{01} \times P_{10} = 1$

$$P_{01} \times P_{10} = \sqrt{\left(\frac{\sum p_1 q_0 \times \sum p_1 q_1 \times \sum p_0 q_1 \times \sum p_0 q_0}{\sum p_0 q_0 \times \sum p_0 q_1 \times \sum p_1 q_1 \times \sum p_1 q_0}\right)}$$

$$P_{01} \times P_{10} = \sqrt{\left(\frac{1341 \times 1784 \times 1600 \times 1220}{1220 \times 1600 \times 1784 \times 1341}\right)}$$

$$P_{01} \times P_{10} = 1$$

Factor Reversal Test

$$P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

$$P_{01} \times Q_{01} = \sqrt{\left(\frac{\sum p_1 q_0 \times \sum p_1 q_1 \times \sum q_1 p_0 \times \sum q_1 p_1}{\sum p_0 q_0 \times \sum p_0 q_1 \times \sum q_0 p_0 \times \sum q_0 p_1}\right)}$$

$$P_{01} \times Q_{01} = \sqrt{\left(\frac{1341 \times 1784 \times 1600 \times 1784}{1220 \times 1600 \times 1220 \times 1341}\right)}$$

$$P_{01} \times Q_{01} = \sqrt{\left(\frac{1784 \times 1784}{1220 \times 1220}\right)} = \frac{1784}{1220}$$

$$\Rightarrow \quad P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

9.2.4 Construction of Cost of Living Index Number

Cost of Living Index Number is constructed to study the effect of changes in the price of goods and services of consumers for a current period as compared with base period. The change in the cost of living index number between any two periods means the change in income which will be necessary to maintain the same standard of living in both the periods. Therefore the cost of living index number measures the average increase in the cost to maintain the same standard of life. Further, the consumption habits of people differ widely from class to class (rich, poor, middle class) and even with the region. The changes in the price level affect the different classes of people, consequently the general price index numbers fail to reflect the effect of changes in their cost of living of different classes of people. Therefore, cost of living index number measures the general price movement of the commodities consumed by different classes of people.

Uses of Cost of Living Index Number

(i) It indicates whether the real wages of workers are rising or falling for a given time.

(ii) It is used by the administrators for regulating dearness allowance or grant of bonus to the workers.

Methods of constructing Cost of Living Index Number

The cost of living index number can be constructed by the following methods,

(i) Aggregate Expenditure Method (or) Weighted Aggregative Method.

(ii) Family Budget Method.

Aggregate Expenditure Method

This is the most common method used to calculate cost of living index number. In this method, weights are assigned to various commodities consumed by a group in the base year. In this method the quantity of the base year is used as weight.

The formula is given by,

$$\text{Cost of Living Index Number} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

Note

The formula for Aggregate Expenditure Method to calculate Cost of Living Index Number is same as formula of Laspeyre's Method.

Family Budget Method

In this method, the weights are calculated by multiplying prices and quantity of the base year. (i.e.) $V = \sum p_0 q_0$. The formula is given by,

$$\text{Cost of Living Index Number} = \frac{\sum PV}{\sum V}$$

where $P = \frac{p_1}{p_0} \times 100$ is the price relative.

$V = \sum p_0 q_0$ is the value relative.

Note

This method is same as the weighted average of price relative method.

When to Use ?

When the Price and Quantity are given, Aggregate Expenditure Method is used.

When the Price and Weight are given, Family Budget Method is used.

Example 9.15

Calculate the cost of living index number for the following data.

Commodities	Quantity 2005	Price 2005	Price 2010
A	10	7	9
B	12	6	8
C	17	10	15
D	19	14	16
E	15	12	17

Solution:

Commodities	Quantity 2005 (Q_0)	Price 2005 (P_0)	Price 2010 (P_1)	P_1Q_0	P_0Q_0
A	10	7	9	90	70
B	12	6	8	96	72
C	17	10	15	255	170
D	19	14	16	304	266
E	15	12	17	255	180
			Total	1000	758

Table 9.14

Cost of Living Index Number

$$= \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 = \frac{1000}{758} \times 100 = 131.926$$

Example 9.16

Calculate the cost of living index number for the year 2015 with respect to base year 2010 of the following data.

Commodities	Number of Units (2010)	Price (2010)	Price (2015)
Rice	5	1500	1750
Sugar	3.5	1100	1200
Pulses	3	800	950
Cloth	2	1200	1550
Ghee	0.75	550	700
Rent	12	2500	3000
Fuel	8	750	600
Misc	10	3200	3500

Solution:

Here the base year quantities are given, therefore we can apply Aggregate Expenditure Method.

Commo-dities	Number of Units (2010) q_0	Price (2010) p_0	Price (2015) p_1	p_0q_0	p_1q_0
Rice	5	1500	1750	7500	8750
Sugar	3.5	1100	1200	3850	4200
Pulses	3	800	950	2400	2850
Cloth	2	1200	1550	2400	3100
Ghee	0.75	550	700	412.5	525
Rent	12	2500	3000	30000	36000
Fuel	8	750	600	6000	4800
Misc	10	3200	3500	32000	35000
			Total	84562.5	95225

Table 9.15

Cost of Living Index Number

$$= \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 = \frac{95225}{84562.5} \times 100 = 112.609$$

Hence, the Cost of Living Index Number for a particular class of people for the year 2015 is increased by 12.61 % as compared to the year 2010.

Example 9.17

Calculate the cost of living index number by consumer price index number for the year 2016 with respect to base year 2011 of the following data.

Commo-dities	Price		Quantity
	Base year	Current year	
Rice	32	48	25
Sugar	25	42	10
Oil	54	85	6
Coffee	250	460	1
Tea	175	275	2

Solution:

Here the base year quantities are given, therefore we can apply Aggregate Expenditure Method.

Commo-dities	Price		Quantity (q_0)	p_0q_0	p_1q_0
	Base year (p_0)	Current year (p_1)			
Rice	32	48	25	800	1200
Sugar	25	42	10	250	420
Oil	54	85	6	324	510
Coffee	250	460	1	250	460
Tea	175	275	2	350	550
			Total	1974	3140

Table 9.16

Cost of Living Index Number

$$= \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 = \frac{3140}{1974} \times 100 = 159.0679$$

Hence, the Cost of Living Index Number for a particular class of people for the year 2016 is increased by 59.0679 % as compared to the year 2011.

Example 9.18

Construct the cost of living index number for 2011 on the basis of 2007 from the given data using family budget method.

Commodities	Price		Weights
	2007	2011	
A	350	400	40
B	175	250	35
C	100	115	15
D	75	105	20
E	60	80	25

Solution:

Commodities	Price		Weights (V)	$P=\frac{P_1}{P_0}\times 100$	PV
	2007 (P_0)	2011 (P_1)			
A	350	400	40	114.286	4571.44
B	175	250	35	142.857	4999.995
C	100	115	15	115	1725
D	75	105	20	140	2800
E	60	80	25	133.333	3333.325
		Total	135		17429.76

Table 9.17

Cost of Living Index Number

$$= \frac{\sum PV}{\sum V} = \frac{17429.76}{135} = 129.1093$$

Hence, the Cost of Living Index Number for a particular class of people for the year 2011 is increased by 29.1093% as compared to the year 2007.

Exercise 9.2

1. Define Index Number.
2. State the uses of Index Number.
3. Mention the classification of Index Number.
4. Define Laspeyre's price index number.
5. Explain Paasche's price index number.
6. Write note on Fisher's price index number.
7. State the test of adequacy of index number.
8. Define Time Reversal Test.
9. Explain Factor Reversal Test.
10. Define true value ratio.
11. Discuss about Cost of Living Index Number.
12. Define Family Budget Method.
13. State the uses of Cost of Living Index Number.
14. Calculate by a suitable method, the index number of price from the following data:

Commodity	2002		2012	
	Price	Quantity	Price	Quantity
A	10	20	16	10
B	12	34	18	42
C	15	30	20	26

15. Calculate price index number for 2005 by (a) Laspeyre's (b) Paasche's method

Commodity	1995		2005	
	Price	Quantity	Price	Quantity
A	5	60	15	70
B	4	20	8	35
C	3	15	6	20

16. Compute (i) Laspeyre's (ii) Paasche's (iii) Fisher's Index numbers for the 2010 from the following data.

Commodity	Price		Quantity	
	2000	2010	2000	2010
A	12	14	18	16
B	15	16	20	15
C	14	15	24	20
D	12	12	29	23

17. Using the following data, construct Fisher's Ideal index and show how it satisfies Factor Reversal Test and Time Reversal Test?

Commodity	Price in Rupees per unit		Number of units	
	Base year	Current year	Base year	Current year
A	6	10	50	56
B	2	2	100	120
C	4	6	60	60
D	10	12	50	24
E	8	12	40	36

18. Using Fisher's Ideal Formula, compute price index number for 1999 with 1996 as base year, given the following:

Year	Commodity: A		Commodity: B		Commodity: C	
	Price (Rs.)	Quantity (Kg)	Price (Rs.)	Quantity (Kg)	Price (Rs.)	Quantity (Kg)
1996	5	10	8	6	6	3
1999	4	12	7	7	5	4

19. Calculate Fisher's index number to the following data. Also show that it satisfies Time Reversal Test.

Commodity	2016		2017	
	Price (Rs.)	Quantity (Kg)	Price (Rs.)	Quantity (Kg)
Food	40	12	65	14
Fuel	72	14	78	20
Clothing	36	10	36	15
Wheat	20	6	42	4
Others	46	8	52	6

20. The following are the group index numbers and the group weights of an average working class family's budget. Construct the cost of living index number:

Groups	Food	Fuel and Lighting	Clothing	Rent	Miscellaneous
Index Number	2450	1240	3250	3750	4190
Weight	48	20	12	15	10

21. Construct the cost of living Index number for 2015 on the basis of 2012 from the following data using family budget method.

Commodity	Price		Weights
	2012	2015	
Rice	250	280	10
Wheat	70	85	5
Corn	150	170	6
Oil	25	35	4
Dhal	85	90	3

22. Calculate the cost of living index by aggregate expenditure method:

Commodity	Weights 2010	Price (Rs.)	
		2010	2015
P	80	22	25
Q	30	30	45
R	25	42	50
S	40	25	35
T	50	36	52

9.3 Statistical Quality Control (SQC)

Introduction

It is one of the most important applications of statistical techniques in industry. The term Quality means a level or standard of a product which depends on Material, Manpower, Machines, and Management (4M's). Quality Control ensures the quality specifications all along them from the arrival of raw materials through each of their processing to the final delivery of goods. This technique is used in almost all production industries such as automobile, textile, electrical equipment, biscuits, bath soaps, chemicals, petroleum products etc.

9.3.1 Meaning

Quality Control is a powerful technique used to diagnose the lack of quality in any of the raw materials, processes, machines etc… It is essential that the end products should possess the qualities that the consumer expects from the manufacturer.

9.3.2 Causes of Variation

There are two causes of variation which affects the quality of a product, namely

1. Chance Causes (or) Random causes

2. Assignable Causes

Chance Causes

These are small variations which are natural and inherent in the manufacturing process. The variation occurring due to these causes is beyond the human control and cannot be prevented or eliminated under any circumstances. The minor causes which do not affect the quality of the products to an extent are called as Chance Causes (or) Random causes. For example Rain, floods, power cuts, etc…

Assignable Causes

The second type of variation which is present in any production process is due to non-random causes. The assignable causes may occur in at any stage of the process, right from the arrival of the raw materials to the final delivery of the product. Some of the important factors of assignable causes are defective raw materials, fault in machines, unskilled manpower, worn out tools, new operation, etc.

The main purpose of SQC is to device statistical techniques which would help in elimination of assignable causes and bring the production process under control.

9.3.3 Process Control and Product Control

The main objective in any production process is to control and maintain a satisfactory quality level of the manufactured product. This is done by 'Process Control'. In Process Control the proportion of defective items in the production process is to be minimized and it is achieved through the technique of control charts. Product Control means that controlling the quality of the product by critical examination through sampling inspection plans. Product Control aims at a certain quality level to be guaranteed to the customers. It attempts to ensure that the product sold does not contain a large number of defective items. Thus it is concerned with classification of raw materials, semi-finished goods or finished goods into acceptable or rejectable products.

Control Charts

In an industry, there are two kinds of problems to be faced, namely

(i) To check whether the process is conforming to its standard level.

(ii) To improve the standard level and reduce the variability.

Shewhart's control charts provide an answer to both. It is a simple technique used for detecting patterns of variations in the data. Control charts are simple to construct and easy to interpret. A typical control charts consists of the following three lines.

(i) Centre Line (CL) indicates the desired standard level of the process.

(ii) Upper Control Limit (UCL) indicates the upper limit of tolerance.

(iii) Lower Control Limit (LCL) indicates the lower limit of tolerance.

If the data points fall within the control limits, then we can say that the process is in control, instead if one or more data points fall outside the control limits, then we can say that the process is out of control.

For example, the following diagram shows all the three control lines with the data points plotted, since all the points falls within the control limits, we can say that the process is in control.

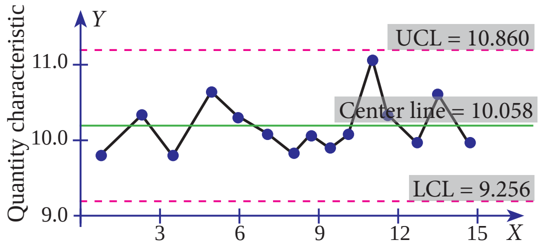

Fig 9.4

Control Charts for Variables

These charts may be applied to any quality characteristic that can be measured quantitatively. A quality characteristic which can be expressed in terms of a numerical value is called as a variable. Many quality characteristics such as dimensions like length, width, temperature, tensile strength etc… of a product are measurable and are expressed in a specific unit of measurements. The variables are of continuous type and are regarded to follow normal probability law. For quality control of such data, there are two types of control charts used. They are as follows :

(i) Charts for Mean ($\overline{X}$)

(ii) Charts for Range (R)

9.3.4 Construction of $\overline{X}$ and R charts

Any production process is not perfect enough to produce all the products exactly the same. Some amount of variation is inherent in any production process. This variation is a total of number of characteristics of the production process such as raw materials, machine setting, operators, handling new operations and new machines, etc. The $\overline{X}$ chart is used to show the quality averages of the samples taken from the given process. The *R* chart is used to show the variability or dispersion of the samples taken from the given process. The control limits of the $\overline{X}$ and R charts shows the presence or absence of assignable causes in the production process. Both $\overline{X}$ and R charts are usually required for decision making to accept or reject the process.

The procedure for constructing $\overline{X}$ and R charts are outlined below.

Procedure for $\overline{X}$

(i) Let X_1,X_2, X_3, etc. be the samples selected, each containing 'n' observations (usually n = 4, 5 or 6)

(ii) Calculate mean for each samples $\overline{X_1}, \overline{X_2}, \overline{X_3}$.... by using

$$\overline{X_i} = \frac{\sum X_i}{n}, i = 1, 2, 3, 4, ...$$

where $\sum X_i$ = total of 'n' values included in the sample X_i.

(iii) Find the mean $\left(\overline{\overline{X}}\right)$ of the sample means.

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{number\ of\ sample\ means}$$

where $\sum \overline{X}$ = total of all the sample means.

Procedure for R -Charts.

Calculate $R = x_{\max} - x_{\min}$

Let R_1, R_2, R_3... be the ranges of the 'n' samples. The average range is given by

$$\overline{R} = \frac{\sum R}{n}$$

The calculation of control limits for $\overline{X}$ chart in two different cases is

Case (i) when $\overline{X}$ and SD are given	Case (ii) when $\overline{X}$ and SD are not given
$UCL = \overline{\overline{X}} + 3\frac{\sigma}{\sqrt{n}}$	$UCL = \overline{\overline{X}} + A_2\overline{R}$
$CL = \overline{\overline{X}}$	$CL = \overline{\overline{X}}$
$LCL = \overline{\overline{X}} - 3\frac{\sigma}{\sqrt{n}}$	$LCL = \overline{\overline{X}} - A_2\overline{R}$

Table 9.18

The calculation of control limits for R chart in two different cases are

Case (i) when SD is given	Case (ii) when SD is not given
$UCL = \overline{R} + 3\sigma_R$	$UCL = D_4\overline{R}$
$CL = \overline{R}$	$CL = \overline{R}$
$LCL = \overline{R} - 3\sigma_R$	$LCL = D_3\overline{R}$

Table 9.19

The values of A_2, D_3 and D_4 are given in the table.

Example 9.19

A machine drills hole in a pipe with a mean diameter of 0.532 cm and a standard deviation of 0.002 cm. Calculate the control limits for mean of samples 5.

Solution:

Given $\overline{\overline{X}} = 0.532$, σ = 0.002, n= 5

The control limits for $\overline{X}$ chart is

$$UCL = \overline{\overline{X}} + 3\frac{\sigma}{\sqrt{n}} = 0.532 + 3\frac{0.002}{\sqrt{5}} = 0.5346$$

$$CL = \overline{\overline{X}} = 0.532$$

$$UCL = \overline{\overline{X}} - 3\frac{\sigma}{\sqrt{n}} = 0.532 - 3\frac{0.002}{\sqrt{5}} = 0.5293$$

Example 9.20

The following data gives the readings for 8 samples of size 6 each in the production of a certain product. Find the control limits using mean chart.

Sample	1	2	3	4	5	6
Mean	300	342	351	319	326	333
Range	25	37	20	28	30	22

Given for $n = 6$, $A_2 = 0.483$,

Solution:

Sample	1	2	3	4	5	6	**Total**
Mean	300	342	351	319	326	333	**1971**
Range	25	37	20	28	30	22	**162**

Table 9.20

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{number\ of\ samples} = \frac{1971}{6} = 328.5 \qquad \overline{R} = \frac{\sum R}{n} = \frac{162}{6} = 27$$

The control limits for $\overline{X}$ chart is

$$UCL = \overline{\overline{X}} + A_2\overline{R} = 328.5 + 0.483(27) = 341.54$$

$$CL = \overline{\overline{X}} = 328.5$$

$$LCL = \overline{\overline{X}} - A_2\overline{R} = 328.5 - 0.483(27) = 315.45$$

Example 9.21

The data shows the sample mean and range for 10 samples for size 5 each. Find the control limits for mean chart and range chart.

Sample	1	2	3	4	5	6	7	8	9	10
Mean	21	26	23	18	19	15	14	20	16	10
Range	5	6	9	7	4	6	8	9	4	7

Solution:

Sample	1	2	3	4	5	6	7	8	9	10	**Total**
Mean	21	26	23	18	19	15	14	20	16	10	**182**
Range	5	6	9	7	4	6	8	9	4	7	**65**

Table 9.21

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{number\ of\ samples} = \frac{182}{10} = 18.2 \qquad \overline{R} = \frac{\sum R}{n} = \frac{65}{10} = 6.5$$

The control limits for $\overline{X}$ chart is

$$UCL = \overline{\overline{X}} + A_2\overline{R} = 18.2 + 0.577(6.5) = 21.95$$
$$CL = \overline{\overline{X}} = 18.2$$
$$LCL = \overline{\overline{X}} - A_2\overline{R} = 18.2 - 0.577(6.5) = 14.5795$$

The control limits for Range chart is

$$UCL = D_4\overline{R} = 2.114(6.5) = 13.741$$
$$CL = \overline{R} = 6.5$$
$$LCL = D_3\overline{R} = 0(6.5) = 0$$

Example 9.22

The following data gives readings of 10 samples of size 6 each in the production of a certain product. Draw control chart for mean and range with its control limits.

Sample	1	2	3	4	5	6	7	8	9	10
Mean	383	508	505	582	557	337	514	614	707	753
Range	95	128	100	91	68	65	148	28	37	80

Solution:

Sample	1	2	3	4	5	6	7	8	9	10	**Total**
Mean	383	508	505	582	557	337	514	614	707	753	**5460**
Range	95	128	100	91	68	65	148	28	37	80	**840**

Table 9.22

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{10} = \frac{5460}{10} = 546$$

$$\overline{R} = \frac{\sum R}{n} = \frac{840}{10} = 84$$

The control limits for $\overline{X}$ chart is

$$UCL = \overline{\overline{X}} + A_2\overline{R} = 546 + 0.483(84) = 586.57$$
$$CL = \overline{\overline{X}} = 546$$
$$LCL = \overline{\overline{X}} - A_2\overline{R} = 546 - 0.483(84) = 505.43$$

The control limits for Range chart is

$$UCL = D_4\overline{R} = 2.004(84) = 168.336$$
$$CL = \overline{R} = 84$$
$$LCL = D_3\overline{R} = 0(84) = 0$$

Example 9.23

You are given below the values of sample mean ($\overline{X}$) and the range (R) for ten samples of size 5 each. Draw mean chart and comment on the state of control of the process.

Sample number	1	2	3	4	5	6	7	8	9	10
$\overline{X}$	43	49	37	44	45	37	51	46	43	47
R	5	6	5	7	7	4	8	6	4	6

Given the following control chart constraint for : $n = 5,\ A_2 = 0.58$, $D_3 = 0$ and $D_4 = 2.115$

Solution:

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{10} = \frac{442}{10} = 44.2$$

$$\overline{R} = \frac{\sum R}{n} = \frac{58}{10} = 5.8$$

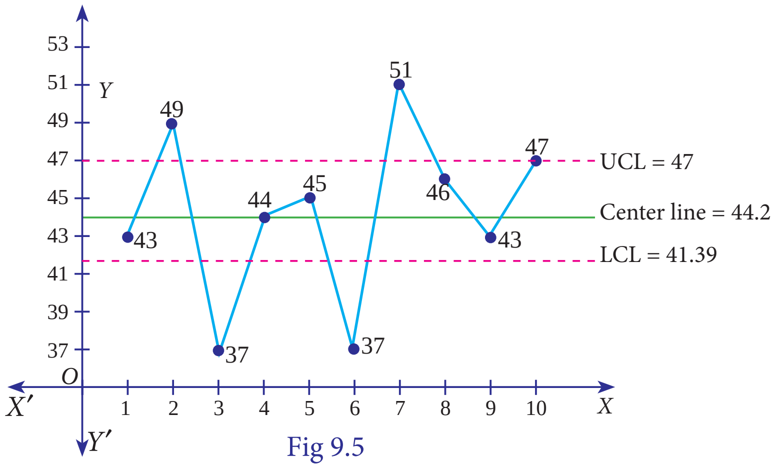

Fig 9.5

$$UCL = \overline{\overline{X}} + A_2\overline{R}$$

$$= 44.2 + 0.483(5.8) = 47.00$$

$$CL = \overline{\overline{X}} = 44.2$$

$$LCL = \overline{\overline{X}} - A_2\overline{R} = 44.2 - 0.483(5.8) = 41.39$$

The above diagram shows all the three control lines with the data points plotted, since four points falls out of the control limits, we can say that the process is out of control.

Exercise 9.3

1. Define Statistical Quality Control.
2. Mention the types of causes for variation in a production process.
3. Define Chance Cause.
4. Define Assignable Cause.
5. What do you mean by product control?
6. What do you mean by process control?
7. Define a control chart.
8. Name the control charts for variables.
9. Define mean chart.
10. Define R Chart.
11. What are the uses of statistical quality control?
12. Write the control limits for the mean chart.
13. Write the control limits for the R chart.
14. A machine is set to deliver packets of a given weight. Ten samples of size five each were recorded. Below are given relevant data:

Sample number	1	2	3	4	5	6	7	8	9	10
$\overline{X}$	15	17	15	18	17	14	18	15	17	16
R	7	7	4	9	8	7	12	4	11	5

Calculate the control limits for mean chart and the range chart and then comment on the state of control. (conversion factors for $n=5,\ A_2=0.58$, $D_3=0$ and $D_4=2.115$)

15. Ten samples each of size five are drawn at regular intervals from a manufacturing process. The sample means ($\overline{X}$) and their ranges (R) are given below:

Sample number	1	2	3	4	5	6	7	8	9	10
$\overline{X}$	49	45	48	53	39	47	46	39	51	45
R	7	5	7	9	5	8	8	6	7	6

Calculate the control limits in respect of $\overline{X}$ chart. (Given $A_2=0.58$, $D_3=0$ and $D_4=2.115$) Comment on the state of control.

16. Construct $\overline{X}$ and R charts for the following data:

Sample Number	Observations		
1	32	36	42
2	28	32	40
3	39	52	28
4	50	42	31
5	42	45	34
6	50	29	21
7	44	52	35
8	22	35	44

(Given for $n=3,\ A_2=0.58$, $D_3=0$ and $D_4=2.115$)

17. The following data show the values of sample mean ($\overline{X}$) and its range (R) for the samples of size five each. Calculate the values for control limits for mean , range chart and determine whether the process is in control.

Sample Number	1	2	3	4	5	6	7	8	9	10
Mean	11.2	11.8	10.8	11.6	11.0	9.6	10.4	9.6	10.6	10.0
Range	7	4	8	5	7	4	8	4	7	9

(conversion factors for $n=5,\ A_2=0.58$, $D_3=0$ and $D_4=2.115$)

18. A quality control inspector has taken ten samples of size four packets each from a potato chips company.The contents of the sample are given below, Calculate the control limits for mean and range chart.

Sample Number	Observations			
	1	2	3	4
1	12.5	12.3	12.6	12.7
2	12.8	12.4	12.4	12.8
3	12.1	12.6	12.5	12.4
4	12.2	12.6	12.5	12.3
5	12.4	12.5	12.5	12.5
6	12.3	12.4	12.6	12.6
7	12.6	12.7	12.5	12.8
8	12.4	12.3	12.6	12.5
9	12.6	12.5	12.3	12.6
10	12.1	12.7	12.5	12.8

(Given for $n=5,\ A_2=0.58\ ,D_3=0$ and $D_4=2.115$)

19. The following data show the values of sample means and the ranges for ten samples of size 4 each. Construct the control chart for mean and range chart and determine whether the process is in control.

Sample Number	1	2	3	4	5	6	7	8	9	10
$\overline{X}$	29	26	37	34	14	45	39	20	34	23
R	39	10	39	17	12	20	05	21	23	15

20. In a production process, eight samples of size 4 are collected and their means and ranges are given below. Construct mean chart and range chart with control limits.

Sample number	1	2	3	4	5	6	7	8
$\overline{X}$	12	13	11	12	14	13	16	15
R	2	5	4	2	3	2	4	3

21. In a certain bottling industry the quality control inspector recorded the weight of each of the 5 bottles selected at random during each hour of four hours in the morning.

Time	Weights in ml				
8:00 AM	43	41	42	43	41
9:00 AM	40	39	40	39	44
10:00 AM	42	42	43	38	40
11:00 AM	39	43	40	39	42

Exercise 9.4

Choose the correct Answer

1. A time series is a set of data recorded
   (a) Periodically
   (b) Weekly
   (c) successive points of time
   (d) all the above

2. A time series consists of
   (a) Five components
   (b) Four components
   (c) Three components
   (d) Two components

3. The components of a time series which is attached to short term fluctuation is
   (a) Secular trend
   (b) Seasonal variations
   (c) Cyclic variation
   (d) Irregular variation

4. Factors responsible for seasonal variations are
   (a) Weather
   (b) Festivals
   (c) Social customs
   (d) All the above

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5. The additive model of the time series with the components T, S, C and I is
   (a) $y=T+S+C\times I$ (b) $y=T+S\times C\times I$
   (c) $y=T+S+C+I$ (d) $y=T+S\times C+I$

6. Least square method of fitting a trend is
   (a) Most exact
   (b) Least exact
   (c) Full of subjectivity
   (d) Mathematically unsolved

7. The value of 'b' in the trend line $y=a+bx$ is
   (a) Always positive
   (b) Always negative
   (c) Either positive or negative
   (d) Zero

8. The component of a time series attached to long term variation is trended as
   (a) Cyclic variation
   (b) Secular variations
   (c) Irregular variation
   (d) Seasonal variations

9. The seasonal variation means the variations occurring with in
   (a) A number of years
   (b) within a year
   (c) within a month
   (d) within a week

10. Another name of consumer's price index number is:
    (a) Whole-sale price index number
    (b) Cost of living index
    (c) Sensitive
    (d) Composite

11. Cost of living at two different cities can be compared with the help of
    (a) Consumer price index
    (b) Value index
    (c) Volume index
    (d) Un-weighted index

12. Laspeyre's index = 110, Paasche's index = 108, then Fisher's Ideal index is equal to:
    (a) 110
    (b) 108
    (c) 100
    (d) 109

13. Most commonly used index number is:

(a) Volume index number

(b) Value index number

(c) Price index number

(d) Simple index number

14. Consumer price index are obtained by:

(a) Paasche's formula

(b) Fisher's ideal formula

(c) Marshall Edgeworth formula

(d) Family budget method formula

15. Which of the following Index number satisfy the time reversal test?

(a)Laspeyre's Index number

(b) Paasche's Index number

(c) Fisher Index number

(d) All of them.

16. While computing a weighted index, the current period quantities are used in the:

(a) Laspeyre's method

(b) Paasche's method

(c) Marshall Edgeworth method

(d) Fisher's ideal method

17. The quantities that can be numerically measured can be plotted on a

(a) p - chart (b) c – chart

(c) x bar chart (d) np – chart

18. How many causes of variation will affect the quality of a product?

(a) 4 (b) 3 (c) 2 (d) 1

19. Variations due to natural disorder is known as

(a) random cause

(b) non-random cause

(c) human cause

(d) all of them

20. The assignable causes can occur due to

(a) poor raw materials

(b) unskilled labour

(c) faulty machines

(d) all of them

21. A typical control charts consists of

(a) CL, UCL (b) CL, LCL

(c) CL, LCL, UCL (d) UCL, LCL

22. $\overline{X}$ chart is a

(a) attribute control chart

(b) variable control chart

(c) neither Attribute nor variable control chart

(d) both Attribute and variable control chart

23. R is calculated using

(a) $x_{max} - x_{min}$ (b) $x_{min} - x_{max}$

(c) $\bar{x}_{max} - \bar{x}_{min}$ (d) $\bar{\bar{x}}_{max} - \bar{\bar{x}}_{min}$

24. The upper control limit for $\overline{X}$ chart is given by

(a) $\overline{X} + A_2\overline{R}$ (b) $\overline{\overline{X}} + A_2 R$

(c) $\overline{\overline{X}} + A_2\overline{R}$ (d) $\overline{\overline{X}} + A_2\overline{\overline{R}}$

25. The LCL for R chart is given by

(a) $D_2\overline{R}$ (b) $D_2\overline{\overline{R}}$

(c) $D_3\overline{\overline{R}}$ (d) $D_3\overline{R}$

Miscellaneous Problems

1. Using three yearly moving averages, Determine the trend values from the following data.

Year	Profit	Year	Profit
2001	142	2007	241
2002	148	2008	263
2003	154	2009	280
2004	146	2010	302
2005	157	2011	326
2006	202	2012	353

2. From the following data, calculate the trend values using fourly moving averages.

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998
Sales	506	620	1036	673	588	696	1116	738	663

3. Fit a straight line trend by the method of least squares to the following data.

Year	1980	1981	1982	1983	1984	1985	1986	1987
Sales	50.3	52.7	49.3	57.3	56.8	60.7	62.1	58.7

4. Calculate the Laspeyre's, Paasche's and Fisher's price index number for the following data. Interpret on the data.

Commodities	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	170	562	72	632
B	192	535	70	756
C	195	639	95	926
D	187	128	92	255
E	185	542	92	632
F	150	217	180	314
7	12.6	12.7	12.5	12.8
8	12.4	12.3	12.6	12.5
9	12.6	12.5	12.3	12.6
10	12.1	12.7	12.5	12.8

5. Using the following data, construct Fisher's Ideal Index Number and Show that it satisfies Factor Reversal Test and Time Reversal Test?

Commodities	Price		Quantity	
	Base Year	Current Year	Base Year	Current Year
Wheat	6	10	50	56
Ghee	2	2	100	120
Firewood	4	6	60	60
Sugar	10	12	30	24
Cloth	8	12	40	36

6. Compute the consumer price index for 2015 on the basis of 2014 from the following data.

Commodities	Quantities	Prices in 2015	Prices in 2016
A	6	5.75	6.00
B	6	5.00	8.00
C	1	6.00	9.00
D	6	8.00	10.00
E	4	2.00	1.50
F	1	20.00	15.00

7. An Enquiry was made into the budgets of the middle class families in a city gave the following information.

Expenditure	Food	Rent	Clothing	Fuel	Rice
Price(2010)	150	50	100	20	60
Price(2011)	174	60	125	25	90
Weights	35	15	20	10	20

What changes in the cost of living have taken place in the middle class families of a city?

8. From the following data, calculate the control limits for the mean and range chart.

Sample No.	1	2	3	4	5	6	7	8	9	10
Sample Observations	50	51	50	48	46	55	45	50	47	56
	55	50	53	53	50	51	48	56	53	53
	52	53	48	50	44	56	53	54	49	55
	49	50	52	51	48	47	48	53	52	54
	54	46	47	53	47	51	51	57	54	52

9. The following data gives the average life(in hours) and range of 12 samples of 5lamps each. The data are

Sample No	1	2	3	4	5	6
Sample Mean	1080	1390	1460	1380	1230	1370
Sample Range	410	670	180	320	690	450
Sample No	7	8	9	10	11	12
Sample Mean	1310	1630	1580	1510	1270	1200
Sample Range	380	350	270	660	440	310

Construct control charts for mean and range. Comment on the control limits.

10. The following are the sample means and ranges for 10 samples, each of size 5. Calculate the control limits for the mean chart and range chart and state whether the process is in control or not.

Sample number	1	2	3	4	5	6	7	8	9	10
Mean	5.10	4.98	5.02	4.96	4.96	5.04	4.94	4.92	4.92	4.98
Range	0.3	0.4	0.2	0.4	0.1	0.1	0.8	0.5	0.3	0.5

Table of Control Chart Constants

Sample Size	A2	D3	D4
2	1.880	0	3.267
3	1.023	0	2.574
4	0.729	0	2.282
5	0.577	0	2.114
6	0.483	0	2.004
7	0.419	0.076	1.924
8	0.373	0.136	1.864
9	0.337	0.184	1.816
10	0.308	0.223	1.777
11	0.285	0.256	1.744
12	0.266	0.283	1.717
13	0.249	0.307	1.693
14	0.235	0.328	1.672
15	0.223	0.347	1.653
16	0.212	0.363	1.637
17	0.203	0.378	1.622

18	0.194	0.391	1.608
19	0.187	0.403	1.597
20	0.180	0.415	1.585
21	0.173	0.425	1.575
22	0.167	0.434	1.566
23	0.162	0.443	1.557
24	0.157	0.451	1.548
25	0.153	0.459	1.541

Table 9.23

Summary

In this chapter we have acquired the knowledge of

- **Method of Moving Averages**

  Three year moving averages: $\frac{a+b+c}{3}, \frac{b+c+d}{3}, \frac{c+d+e}{3}, \frac{d+e+f}{3}, \ldots$

  Four year moving averages: $\frac{a+b+c+d}{4}, \frac{b+c+d+e}{4}, \frac{c+d+e+f}{4}, \frac{d+e+f+g}{4}, \ldots$

- **Method of Least Squares**

  The straight line equation, $Y = a + bX$

  Two Normal Equations, $\Sigma Y = n a + b \Sigma X$; $\Sigma XY = a \Sigma X + b \Sigma X2$

- **Methods of measuring Seasonal Variations-Method of Simple Averages:**

  Seasonal Index (S.I) = $\frac{Seasonal\ Average}{Grand\ average} X 100$

- **If the data is given in months**

  S.I for Jan = $\frac{Monthly\ Average\ (for\ Jan)}{Grand\ average} X 100$

- **If the data is given in quarter**

  S.I for Kth Quarter = $\frac{Average\ of\ \mathrm{K}^{\mathrm{th}}\ quarter}{Grand\ average} X 100$

- Continuous distribution function

  If X is a continuous random variable with the probability density function $f_X(x)$, then the function $F_X(x)$ is defined by

- Weighted Index Number

  Price Index (P_{01}) = $\frac{\sum p_1 w}{\sum p_0 w} x 100$

Laspeyre's price index number $P_{01}^{L} = \dfrac{\sum p_1 q_0}{\sum p_0 q_0} X 100$

Paasche's price index number $P_{01}^{P} = \dfrac{\sum p_1 q_1}{\sum p_0 q_1} X 100$

Fisher's price index number $P_{01}^{F} = \sqrt{P_{01}^{L} x\ P_{01}^{P}} = \sqrt{\dfrac{\sum p_1 q_0\ X \sum p_1 q_1}{\sum p_0 q_0\ X \sum p_0 q_1}}\ X 100$

- Time Reversal Test : $P_{01} \times P_{10} = 1$. Factor Reversal Test : $P_{01}\ X\ Q_{01} = \dfrac{\sum p_1 q_1}{\sum p_0 q_0}$

- Cost of Living Index Number

  Aggregate Expenditure Method = $\dfrac{\sum p_1 q_0}{\sum p_0 q_0} X 100$. Family Budget Method = $\dfrac{\sum PV}{\sum V}$

- Causes of Variation

  1. Chance Causes (or) Random causes 2. Assignable Causes

- Control Charts

  (i) Centre Line (CL) indicates the desired standard level of the process.

  (ii) Upper Control Limit (UCL) indicates the upper limit of tolerance.

  (iii) Lower Control Limit (LCL) indicates the lower limit of tolerance.

The control limits for $\overline{X}$ chart in two different cases are

case (i) when $\overline{X}$ and SD are given	case (i) when $\overline{X}$ and SD are not given
$UCL = \overline{\overline{X}} + 3\dfrac{\sigma}{\sqrt{n}}$	$UCL = \overline{\overline{X}} + A_2 \overline{R}$
$CL = \overline{\overline{X}}$	$CL = \overline{\overline{X}}$
$LCL = \overline{\overline{X}} - 3\dfrac{\sigma}{\sqrt{n}}$	$LCL = \overline{\overline{X}} - A_2 \overline{R}$

The control limits for R chart in two different cases are

case (i) when SD are given	case (i) when SD are not given
$UCL = \overline{R} + 3\sigma_R$	$UCL = D_4 \overline{R}$
$CL = \overline{R}$	$CL = \overline{R}$
$LCL = \overline{R} - 3\sigma_R$	$LCL = D_3 \overline{R}$

GLOSSARY (கலைச்சொற்கள்)

Control charts	கட்டுப்பாட்டு விளக்கப்படங்கள்
Control limit	கட்டுப்பாட்டு எல்லை
Cyclical variation	சுழல் மாறுபாடு
Factor reversal test	காரணி மாற்று சோதனை
Family budget	குடும்ப வரவு– செலவுத் திட்டம்
Fisher's index	பிஷரின் குறியீடு
Index number	குறியீட்டெண்கள்
Irregular variation	ஒழுங்கற்ற மாறுபாடு
Laspeyre's index	லாஸ்பியரின் குறியீடு
Least square	மீச்சிறு வர்க்கம்
Lower control limit	கீழ்க் கட்டுப்பாட்டு எல்லை
Mean charts	சராசரி வரைவுகள்
Moving average	நகரும் சராசரி
Observation	கண்டறிபதிவு / கூர்நோக்கு
Paasche's index	பாசியின் குறியீடு
Process control	செயல்பாட்டு கட்டுப்பாடு (அல்லது) செயலாக்கக் கட்டுபாடு
Product control	உற்பத்தி கட்டுப்பாடு
Range charts	வீச்சு வரைகள்
Seasonal variation	பருவகால மாறுபாடு
Seasonal Index	பருவகால குறியீடு
Secular trend	நீள் காலப்போக்கு
Semi-vverage	பகுதி சராசரி
Statistical quality control	புள்ளியியல் தரக்கட்டுப்பாடு
Time reversal test	காலமாற்று சோதனை
Time series	காலம்சார் தொடர்வரிசை
Trend	போக்கு
Unweighted Index	நிறையிடா குறியீடு
Upper control limit	மேல் கட்டுப்பாட்டு எல்லை
Weighted index	நிறையிட்ட குறியீடு