SOLVED PAPER VITEEE 2006

PART - I (PHYSICS)

1 A potential difference of 300 V is applied to a combination of 2.0μF and 8.0 μF capacitors connected in series. The charge on the 2.0μF capacitor is

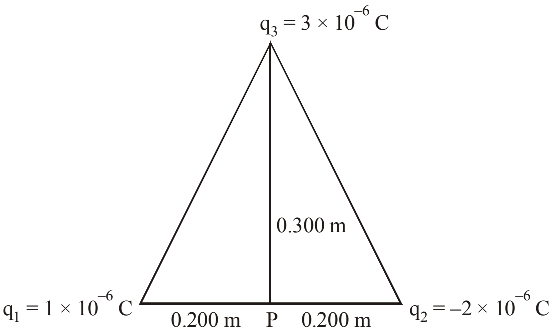
(a) 2.4×10^{-4}C (b) 4.8×10^{-4}C
(c) 7.2×10^{-4} C (d) 9.6×10^{-4} C

2. Two point charges 4μC and – 2μC are separated by a distance of 1 m in air. Then the distance of the point on the line joining the charges, where the resultant electric field is zero, is (in metre)

(a) 0.58 (b) 0.75
(c) 0.67 (d) 0.81

3. Figure shows a triangular array of three point charges. The electric potential V of these source charges at the midpoint P of the base of the triangle is

$$\left[\frac{1}{4\pi \epsilon_0} = 9\times10^9\,\text{Nm}^2\text{C}^{-2}\right]$$

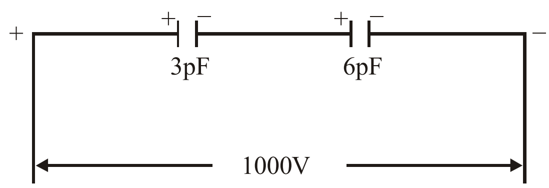


(a) 55kV (b) 45kV
(c) 63kV (d) 49kV

4. A current of 5A is passing through a metallic wire of cross-sectional area $4 \times 10^{-6}\text{m}^2$. If the density of the charge carriers in the wire is $5 \times 10^{26}\text{m}^{-3}$, the drift speed of the electrons will be [$e = 1.602 \times 10^{-19}$C]

(a) $1.56\times10^{-2}\text{ms}^{-1}$ (b) $1.98\times10^{-2}\text{ms}^{-1}$
(c) $2.42\times10^{-2}\text{ms}^{-1}$ (d) $2.84\times10^{-2}\text{ms}^{-1}$

5. The series combination of two capacitors shown in figure is connected across 1000V. The magnitude of the charges on the capacitors will be



(a) 3×10^{-9} C (b) 2×10^{-9} C
(c) 2.5×10^{-9} C (d) 3.5×10^{-9} C

6. Three resistances of values 2Ω, 3Ω and 6Ω are to be connected to produce an effective resistance of 4Ω. This can be done by connecting

(a) 6Ω resistance in series with the parallel combination of 2Ω and 3Ω
(b) 3Ω resistance in series with the parallel combination of 2Ω and 6Ω
(c) 2Ω resistance in series with the parallel combination of 3Ω and 6Ω
(d) 2Ω resistance in parallel with the parallel combination of 3Ω and 6Ω

7. The resistance of a field coil measures 50Ω at 20ºC and 65Ω at 70ºC. The temperature coefficient of resistance is

(a) 0.0086/ºC (b) 0.0068/ºC
(c) 0.0096/ºC (d) 0.0999/ºC

8. The electrolyte used in Lechlanche cell is

(a) copper sulphate solution
(b) ammonium chloride solution
(c) dilute sulphuric acid
(d) zinc sulphate

9. A galvanometer has a resistance of 50Ω. If a resistance of 1Ω is connected across its terminals, the total current flow through the galvanometer is [I_g represents the maximum current that can be passed through the galvanometer]
(a) 42 I_g (b) 53 I_g
(c) 46 I_g (d) 51 I_g

10. In a tangent galvanometer, a current of 1A produces a deflection of 30°. The current required to produce a deflection of 60° is
(a) 3A (b) 2A
(c) 4A (d) 1A

11. In the presence of magnetic field 'B' and electric field 'E', the total force on a moving charged particle is
(a) $\vec{F} = \vec{v}[(\vec{q} \times \vec{B}) + \vec{E}]$
(b) $\vec{F} = q[(\vec{v} \times \vec{E}) + \vec{B}]$
(c) $\vec{F} = q[(\vec{v} \times \vec{B}) + \vec{E}]$
(d) $\vec{F} = \vec{B}[(\vec{q} \times \vec{E}) + \vec{v}]$

12. A circular coil of radius 40 mm consists of 250 turns of wire in which the current is 20mA. The magnetic field in the center of the coil is [$\mu = 4\pi \times 10^{-7}$ Hm^{-1}]
(a) 0.785G (b) 0.525G
(c) 0.629G (d) 0.900G

13. RMS value of AC is _______ of the peak value.
(a) 7% (b) 7.7%
(c) 70% (d) 70.7%

14. Q-factor can be increased by having a coil of
(a) large inductance, small ohmic resistance
(b) large inductance, large ohmic resistance
(c) small inductance, large ohmic resistance
(d) small inductance, small ohmic resistance

15. A small piece of metal wire is dragged across the gap between the pole pieces of a magnet in 0.5 second. The magnetic flux between the pole pieces is known to be 8 × 10^{-4} Wb. The emf induced in the wire is
(a) 16mV (b) 1.6V
(c) 1.6mV (d) 16V

16. Current in the LCR circuit becomes extremely large when
(a) frequency of AC supply is increased
(b) frequency of AC supply is decreased
(c) inductive reactance becomes equal to capacitive reactance
(d) inductance becomes equal to capacitance

17. Our eyes respond to wavelengths ranging from
(a) 400 nm to 700 nm
(b) 700 nm to 800 nm
(c) 0 to ∞
(d) – ∞ to + ∞

18. A new system of units is evolved in which the values of μ_0 and ε_0 are 2 and 8 respectively. Then the speed of light in this system will be
(a) 0.25 (b) 0.5
(c) 0.75 (d) 1

19. A ray of light strikes a piece of glass at an angle of incidence of 60° and the reflected beam is completely plane polarised. The refractive index of glass is
(a) $2\sqrt{3}$ (b) $\sqrt{3}$
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

20. In an experiment on Newton's rings, the diameter of the 20th dark ring was found to be 5.82mm and that of the 10th ring 3.36 mm. If the radius of the plano-convex lens is 1 m, the wavelength of light used is
(a) 5646 A° (b) 5896 A°
(c) 5406 A° (d) 5900 A°

21. What is the angular momentum of an electron in the fourth orbit of Bohr's model of hydrogen atom?
(a) $\frac{h}{2\pi}$ (b) $\frac{2h}{\pi}$
(c) h (d) $\frac{h}{4\pi}$

22. The transition of an electron from n_2 = 5,6, ... to n_1 = 4 gives rise to
(a) Pfund series (b) Lyman series
(c) Paschen series (d) Brackett series

23. The ground state energy of hydrogen atom is – 13.6 eV. What is the potential energy of the electron in this state?
(a) –27.2 eV (b) –13.6 eV
(c) +13.6 eV (d) 0 eV

24. The longest wavelength that can be analysed by a sodium chloride crystal of spacing d = 2.82 A° in the second order is
(a) 2.82 A° (b) 5.64 A°
(c) 8.46 A° (d) 11.28 A°

25. Which is the incorrect statement of the following?
(a) Photon is a particle with zero rest mass
(b) Photon is a particle with zero momentum
(c) Photons travel with velocity of light in vacuum
(d) Photons even feel the pull of gravity

26. The deBroglie wavelength associated with a steel ball of mass 1000 gm moving at a speed of 1 ms^{-1} is $[h = 6.626 \times 10^{-34}$ Js]
(a) 6.626×10^{-31}m (b) 6.626×10^{-37}m
(c) 6.626×10^{-34}m (d) 6.626×10^{34}m

27. The velocity v, at which the mass of a particle is double its rest mass is
(a) $v = c$ (b) $v = \sqrt{\frac{3}{4}}c$
(c) $v = \sqrt{\frac{3}{2}}c$ (d) $v = 2c$

28. How much energy is produced, if 2 kg of a substance is fully converted into energy? $[c = 3 \times 10^8 \ ms^{-1}]$
(a) 9×10^{16} J (b) 11×10^{16} J
(c) 15×10^{16} J (d) 18×10^{16} J

29. The difference between the rest mass of the nucleus and the sum of the masses of the nucleons composing a nucleus is known as
(a) packing fraction (b) mass defect
(c) binding energy (d) isotopic mass

30. The half life period of Radium is 3 minute. Its mean life time is
(a) 1.5 minute (b) $\frac{3}{0.6931}$ minute
(c) 6 minute (d) (3×0.6931) minute

31. 'Pair production' involves conversion of a photon into
(a) a neutron-electron pair
(b) a positron-neutron pair
(c) an electron-proton pair
(d) an electron-positron pair

32. The sub atomic particles proton and neutron fall under the group of
(a) mesons (b) photons
(c) leptons (d) baryons

33. When the conductivity of a semiconductor is only due to the breaking up of the covalent bonds, the semiconductor is known as
(a) donor (b) extrinsic
(c) intrinsic (d) acceptor

34. In a P-type semiconductor, the acceptor impurity produces an energy level
(a) just below the valence band
(b) just above the conduction band
(c) just below the conduction band
(d) just above the valence band

35. An oscillator is essentially
(a) an amplifier with proper negative feedback network circuits
(b) converts alternating current into direct current
(c) an amplifier with no feedback network
(d) an amplifier with proper positive feedback network circuits

36. Which of the following gates can perform perfect binary addition?
(a) AND gate (b) OR gate
(c) EXOR gate (d) NAND gate

37. The frequency of an FM transmitter without signal input is called
(a) the centre frequency
(b) modulation factor
(c) the frequency deviation
(d) the carrier swing

38. The fundamental radio antenna is a metal rod which has a length equal to
(a) λ in free space at the frequency of operation
(b) $\frac{\lambda}{2}$ in free space at the frequency of operation
(c) $\frac{\lambda}{4}$ in free space at the frequency of operation
(d) $\frac{3\lambda}{4}$ in free space at the frequency of operation

39. Vidicon works on the principle of
(a) electrical conductivity
(b) photoconductivity
(c) thermal conductivity
(d) SONAR

40. The maximum range, $d_{max,}$ of radar is
(a) proportional to the cube root of the peak transmitted power
(b) proportional to the fourth root of the peak transmitted power
(c) proportional to the square root of the peak transmitted power
(d) not related to the peak transmitted power at all

PART - II (CHEMISTRY)

41. The equivalent weight of potassium permanganate when it acts as oxidising agent in ferrous ion estimation is
(a) 158 (b) 31.6
(c) 79 (d) 39.5

42. The magnetic moment of lanthanide ions is determined from which one of the following relation?
(a) $\mu = \sqrt{n(n+2)}$ (b) $\mu = g\sqrt{J(J+1)}$
(c) $\mu = g\sqrt{n(n+1)}$ (d) $\mu = 2\sqrt{n(n+1)}$

43. Which one of the following has maximum number of unpaired electrons?
(a) Mg^{2+} (b) Ti^{3+}
(c) V^{3+} (d) Fe^{2+}

44. Excess of NaOH reacts with Zn to form
(a) ZnH_2 (b) Na_2ZnO_2
(c) ZnO (d) $Zn(OH)_2$

45. How many isomers does $Co(en)_2Cl_2^+$ have?
(a) 1 (b) 3
(c) 2 (d) 4

46. NH_3 group in a coordination compound is named as
(a) ammonium (b) ammine
(c) amine (d) ammonia

47. Name the complex $Ni(PF_3)_4$
(a) tetrakis (phosphorus (III) fluoride) nickel (0)
(b) tetra (phosphorus (III) fluoride) nickel
(c) Nickel tetrakis phosphorus (III) fluoride
(d) (phosphorus (III) tetrakis fluoride) nickel (0)

48. The purple colour of $KMnO_4$ is due to
(a) charge transfer (b) d-d transition
(c) f-f transition (d) d-f transition

49. How many lattice points belong to a face centered cubic unit cell?
(a) 1 (b) 2
(c) 4 (d) 3

50. Schottky defect in solids is due to
(a) a pair of cation and anion vacancies
(b) occupation of interstitial site by a pair of cation and anion
(c) occupation of interstitial site by a cation
(d) occupation of interstitial site by an anion

51. Which one of the following is amorphous?
(a) Polystyrene (b) Table salt
(c) Silica (d) Diamond

52. The metal that crystallises in simple cubic system is
(a) Po (b) Na
(c) Cu (d) Ag

53. When ideal gas expands in vacuum, the work done by the gas is equal to
(a) PV (b) RT
(c) 0 (d) nRT

54. For a closed system consisting of a reaction $N_2O_4(g) \rightarrow 2NO_2(g)$, the pressure
(a) remains constant (b) decreases
(c) increases (d) becomes zero

55. 6 moles of an ideal gas expand isothermally and reversibly from a volume of 1 litre to a volume of 10 litres at 27°C. What is the maximum work done?
(a) 47 kJ (b) 100 kJ
(c) 0 (d) 34.465 kJ

56. The reaction, $Zn(s) + CuSO_4(aq) \rightarrow ZnSO_4(aq) + Cu(s)$ is an example of a
(a) spontaneous process
(b) isobaric process
(c) non-spontaneous process
(d) reversible process

57. For the reaction, $H_2\,(g) + I_2\,(g) \Leftrightarrow 2HI\,(g)$
(a) $K_p = -K_c$ (b) $K_c = 0$
(c) $K_p = K_c$ (d) $K_p = 0$

58. The increase of pressure on ice $\Leftrightarrow$ water at a constant temperature will cause
(a) water to vaporize (b) water to freeze
(c) no change (d) ice to melt

59. The order of the reaction
$$N_2O_5 \rightarrow N_2O_4(g) + \frac{1}{2}O_2(g) \text{ is}$$
(a) 3 (b) 2
(c) 1 (d) 0

60. The reactions with low activation energy are always
(a) adiabatic
(b) slow
(c) non-spontaneous
(d) fast

61. For a cell reaction to be spontaneous, the standard free energy change of the reaction must be
(a) zero (b) positive
(c) infinite (d) negative

62. Equivalent conductance of an electrolyte containing NaF at infinite dilution is 90.1 $Ohm^{-1}cm^2$. If NaF is replaced by KF what is the value of equivalent conductance?
(a) 90.1 $Ohm^{-1}cm^2$ (b) 111.2 $Ohm^{-1}cm^2$
(c) 0 (d) 222.4 $Ohm^{-1}cm^2$

63. The tendencies of the electrodes made up of Cu, Zn and Ag to release electrons when dipped in their respective salt solutions decrease in the order
(a) Zn > Ag > Cu (b) Cu > Zn > Ag
(c) Zn > Cu > Ag (d) Ag > Cu > Zn

64. The electrode reaction that takes place at the anode of $CH_4 - O_2$ fuel cell is
(a) $2O_2 + 8H^+ + 8e^- \rightarrow 4H_2O$
(b) $CH_4 + 2H_2O \rightarrow CO_2 + 8H^+ + 8e^-$
(c) $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$
(d) $2H^+ + 2e^- \rightarrow H_2$

65. What is the hybridization of oxygen atom in an alcohol molecule?
(a) sp^3 (b) sp
(c) sp^2 (d) p^2

66. $R{-}\overset{\overset{O}{\|}}{C}{-}OH \xrightarrow{LiAlH_4}$?
(a) RCH_2CH_2OH (b) RCHO
(c) RCOR (d) RCH_2OH

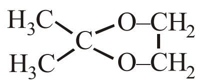
67. Which one of the following is correct?
(a) $RCH_2OH \xrightarrow{KMnO_4}$ No reaction
(b) $CH_3CH_2OH \xrightarrow{Na_2Cr_2O_7, H_2SO_4}$ No reaction
(c) $CH_3CHO \xrightarrow{Na_2Cr_2O_7, H_2SO_4}$ No reaction
(d) $CH_3{-}\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}{-}OH \xrightarrow{\text{alkaline } KMnO_4}$ No reaction

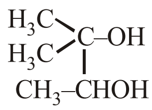
68. Which one of the following products obtained when diethyl ether is boiled with water in presence of dilute acid?
(a) Glycol (b) Ethyl alcohol
(c) Ethylene oxide (d) Peroxide

69. Identify the product for the following reaction

$$CH_3{-}\overset{\overset{O}{\|}}{C}{-}CH_3 + \begin{matrix} CH_2OH \\ | \\ CH_2OH \end{matrix} \overset{HCl}{\rightleftharpoons} ?$$

(a) $CH_3{-}\overset{\overset{CH_3}{|}}{CH}OH + \begin{matrix} COOH \\ | \\ COOH \end{matrix}$

(b) 

(c) 

(d) No reaction

70. What is the reaction of acetaldehyde with concentrated sulphuric acid?
(a) No reaction
(b) Decomposition
(c) Charred to black residue
(d) Polymerisation

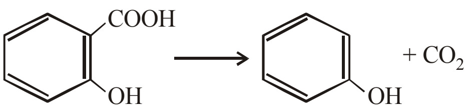
71. Calcium Acetate on heating under distillation gives
(a) Acetaldehyde and Calcium Oxide
(b) Calcium Carbonate and Acetic acid
(c) Acetone and Calcium Carbonate
(d) Calcium Oxide and CO_2

72. Identify the correct statement
(a) Aldehydes on reduction give secondary alcohols
(b) Ketones on reduction give primary alcohols
(c) Ketones reduce Fehling's solution and give red cuprous oxide
(d) Ketones do not react with alcohols

73. The O – H stretching vibration of alcohols absorbs in the region 3700 – 3500 cm^{-1}. The O – H stretching of carboxylic acids absorb in the region
(a) 3900 – 3700 cm^{-1} (b) 3000 – 2500 cm^{-1}
(c) 3700 – 3500 cm^{-1} (d) 1700 – 2000 cm^{-1}

74. Which among the following reduces Fehling's solution?
(a) Acetic acid (b) Formic acid
(c) Benzoic acid (d) Salicylic acid

75. Determine the experimental condition for the following reaction



(a) in presence of KOH
(b) on heating
(c) in presence of NaOH
(d) in presence of HCl

76. Which one of the following is an ingredient of Pthalic acid manufacture by catalytic oxidation
(a) Benzene (b) Salicylic acid
(c) Anthranilic acid (d) naphthalene

77. On comparison with $H-C-H$ bond angle of methane, the $C-N-C$ bond angle of trimethylamine is
(a) higher (b) no change
(c) not comparable (d) lower

78. The treatment of acylazide $(RCON_3)$ with acidic or alkaline medium gives
(a) $RCONH_2$ (b) $R-NH_2$
(c) RCH_2NH_2 (d) RCOCHNH

79. The sequence of basic strength of alky1 amines follows the order
(a) $RNH_2 < R_2NH > R_3N$
(b) $R_2NH_2 < R_2NH < R_3N$
(c) $R_2NH < RNH_2 < R_3N$
(d) $RNH_2 < R_2NH < R_3N$

80. Activation of benzene ring in aniline can be decreased by treating with
(a) dil. HCl (b) ethyl alcohol
(c) acetic acid (d) acetyl chloride

PART - III (MATHEMATICS)

81. The value of x, for which the matrix

$$A = \begin{bmatrix} \frac{2}{x} & -1 & 2 \\ 1 & x & 2x^2 \\ 1 & \frac{1}{x} & 2 \end{bmatrix}$$ is singular, is

(a) ± 1 (b) ± 2
(c) ± 3 (d) ± 4

82. If $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, then other two roots are
(a) 3, 7 (b) 2, 7
(c) 3, 6 (d) 2, 6

83. The values of α for which the system of equation $x+y+z=1$, $x+2y+4z=\alpha$, $x+4y+10z=\alpha^2$ is consistent are given by
(a) 1, –2 (b) –1, 2
(c) 1, 2 (d) 1, 1

84. Let $A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & t \\ 4 & 7-t & -6 \end{pmatrix}$, then the values of t for which inverse of A does not exist
(a) –2, 1 (b) 3, 2
(c) 2, –3 (d) 3, –1

85. The non integer roots of
$x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$
(a) $\frac{1}{2}(3+\sqrt{13}), \frac{1}{2}(3-\sqrt{13})$
(b) $\frac{1}{2}(3-\sqrt{13}), \frac{-1}{2}(3+\sqrt{13})$
(c) $\frac{1}{2}(3+\sqrt{17}), \frac{1}{2}(3-\sqrt{17})$
(d) $\frac{1}{2}(3-\sqrt{17}), \frac{-1}{2}(3+\sqrt{17})$

86. If $e^x = y + \sqrt{1+y^2}$, then the value of y is
(a) $\frac{1}{2}(e^x + e^{-x})$ (b) $\frac{1}{2}(e^x - e^{-x})$
(c) $e^x - e^{\frac{-x}{2}}$ (d) $e^x + e^{\frac{-x}{2}}$

87. Consider an infinite geometric series with the first term a and common ratio r. If its sum is 4 and the second term is $\frac{3}{4}$, then
(a) $a = \frac{4}{7}, r = \frac{3}{7}$ (b) $a = 2, r = \frac{3}{8}$
(c) $a = \frac{3}{2}, r = \frac{1}{2}$ (d) $a = 3, r = \frac{1}{4}$

88. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then the value of $\alpha^3 + \beta^3$ is

(a) $\frac{3abc+b^3}{a^3}$ (b) $\frac{a^3+b^3}{3abc}$

(c) $\frac{3abc-b^3}{a^3}$ (d) $\frac{-(3abc+b^3)}{a^3}$

89. The volume of the tetrahedron with vertices P (–1, 2, 0), Q (2, 1, –3), R (1, 0, 1) and S (3, –2, 3) is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

90. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ then t such that $\vec{a} + t\vec{b}$ is at right angle to $\vec{c}$ will be equal to

(a) 5 (b) 4

(c) 6 (d) 2

91. An equation of the plane passing through the line of intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ and passing through (1, 1, 1) is

(a) $2x+3y+4z=9$ (b) $x+y+z=3$

(c) $x+2y+3z=6$ (d) $20x+23y+26z=69$

92. The length of the shortest distance between the lines $\vec{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda(\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = -\hat{i} - \hat{j} - \hat{k} + \mu(7\hat{i} - 6\hat{j} + \hat{k})$ is

(a) 83 units (b) $\sqrt{6}$ units

(c) $\sqrt{3}$ units (d) $2\sqrt{29}$ units

93. The region of the argand plane defined by $|z-i| + |z+i| \le 4$ is

(a) interior of an ellipse

(b) exterior of a circle

(c) interior and boundary of an ellipse

(d) interior of a parabola

94. The value of the sum $\sum_{n=1}^{13}(i^n + i^{n+1})$ where $i = \sqrt{-1}$ equals

(a) i (b) i –1

(c) – i (d) 0

95. If $\sin\theta$, $\cos\theta$, $\tan\theta$ are in G.P. then $\cos^9\theta + \cos^6\theta + 3\cos^5\theta - 1$ is equal to

(a) –1 (b) 0

(c) 1 (d) 2

96. If in a triangle ABC, $5\cos C + 6\cos B = 4$ and $6\cos A + 4\cos C = 5$, then $\tan\frac{A}{2}\tan\frac{B}{2}$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) $\frac{1}{5}$ (d) 5

97. In a model, it is shown that an arc of a bridge is semielliptical with major axis horizontal. If the length of the base is 9m and the highest part of the bridge is 3m from horizontal; the best approximation of the height of the arch, 2m from the centre of the base is

(a) $\frac{11}{4}$m (b) $\frac{8}{3}$m

(c) $\frac{7}{2}$m (d) 2m

98. The number of real tangents through (3,5) that can be drawn to the ellipses $3x^2 + 5y^2 = 32$ and $25x^2 + 9y^2 = 450$ is

(a) 0 (b) 2

(c) 3 (d) 4

99. If the normal to the rectangular hyperbola $xy = c^2$ at the point $\left(ct, \frac{c}{t}\right)$ meets the curve again at $\left(ct', \frac{c}{t'}\right)$, then

(a) $t^3t' = 1$ (b) $t^3t' = -1$

(c) $tt' = 1$ (d) $tt' = -1$

100. An equilateral triangle is inscribed in the parabola $y^2 = 4x$ one of whose vertex is at the vertex of the parabola, the length of each side of the triangle is

(a) $\frac{\sqrt{3}}{2}$ (b) $4\frac{\sqrt{3}}{2}$

(c) $8\frac{\sqrt{3}}{2}$ (d) $8\sqrt{3}$

101. If $f(2) = 4$ and $f'(2) = 1$,

then $\lim\limits_{x \to 2} \dfrac{xf(2) - 2f(x)}{x - 2}$ is equal to

(a) 0 (b) $\dfrac{1}{2}$

(c) 1 (d) 2

102. What is the least value of k such that the function $x^2 + kx + 1$ is strictly increasing on (1,2)

(a) 1 (b) –1

(c) 2 (d) –2

103. The maximum value of $\left(\dfrac{1}{x}\right)^x$ is

(a) e (b) e^e

(c) $e^{\frac{1}{e}}$ (d) $\left(\dfrac{1}{e}\right)^{\frac{1}{e}}$

104. If $u = \tan^{-1}\left\{\dfrac{x^3 + y^3}{x + y}\right\}$, then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} =$

(a) sin 2u (b) cos 2u

(c) $\sec^2 2u$ (d) tan 2u

105. If $f'(x) = \dfrac{x}{\sqrt{1+x}}$ and $f(0) = 0$, then $f(x) =$

(a) $\dfrac{2}{3}\left\{(1+x)^{\frac{3}{2}} - 6(1+x)^{\frac{1}{2}} + 1\right\}$

(b) $\dfrac{2}{3}\left\{(1+x)^{\frac{3}{2}} - 3(1+x)^{\frac{1}{2}} + 2\right\}$

(c) $\dfrac{2}{3}\left\{(1+x)^{\frac{3}{2}} - 4(1+x)^{\frac{1}{2}} + 2\right\}$

(d) $\dfrac{2}{3}\left\{(1+x)^{\frac{3}{2}} - 3(1+x)^{\frac{1}{2}} + 1\right\}$

106. The value of the integral $\displaystyle\int_0^{\frac{\pi}{2}} \log(\tan x)dx =$

(a) 0 (b) 1

(c) $\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{4}$

107. What is the area of a loop of the curve $r = a\sin 3\theta$?

(a) $\dfrac{\pi a^2}{6}$ (b) $\dfrac{\pi a^2}{8}$

(c) $\dfrac{\pi a^2}{12}$ (d) $\dfrac{\pi a^2}{24}$

108. The value of the integral $\displaystyle\int_1^9 e^{\sqrt{t}}dt =$

(a) e^3 (b) $4e^3$

(c) $4(e^3 - e)$ (d) $4e^3 - 2e$

109. The differential equation that represents all parabolas each of which has a latus rectum 4a and whose axes are parallel to the x – axis is

(a) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} = 0$

(b) $\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^3 = 0$

(c) $a\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^3 = 0$

(d) $2a\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^3 = 0$

110. The solution of $\left(x\,\mathrm{cosec}\left(\dfrac{y}{x}\right) - y\right)dx + xdy = 0$ is

(a) $\log|x| - \cos\left(\dfrac{x}{y}\right) = c$

(b) $\log|x| - \cos\left(\dfrac{y}{x}\right) = c$

(c) $\log|x| - \sin\left(\dfrac{x}{y}\right) = c$

(d) $\log|x| - \sin\left(\dfrac{y}{x}\right) = c$

111. The particular integral of $\frac{d^2y}{dx^2} + 2y = x^2$ is

(a) x^2-1 (b) x^2+1

(c) $\frac{1}{2}(x^2-1)$ (d) $\frac{1}{2}(x^2+1)$

112. The solution of $(D^2 + 16)\, y = \cos 4x$ is

(a) $A\cos 4x + B\sin 4x + \frac{x}{8}\sin 4x$

(b) $A\cos 4x + B\sin 4x - \frac{x}{8}\sin 4x$

(c) $A\cos 4x + B\sin 4x + \frac{x}{4}\sin 4x$

(d) $A\cos 4x + B\sin 4x - \frac{x}{4}\sin 4x$

113. Determine which one of the following relations on $X = \{1,2,3,4\}$ is not transitive.

(a) $R_1 = \phi$, the empty relation

(b) $R_2 = X \times X$, the universal relation

(c) $R_3 = \{(1,3), (2,1)\}$

(d) $R_4 = \{(1,1), (1,2), (2,3), (1,3), (4,4)\}$

114. Find the number of ways in which five large books, four medium-size books, and three small books can be placed on a shelf so that all books of the same size are together.

(a) $5 \times 4 \times 3$ (b) $5! \times 4! \times 3!$

(c) $3 \times 5! \times 4! \times 3!$ (d) $3! \times 5! \times 4! \times 3!$

115. Consider the set Q of rational numbers. Let $*$ be the operation on Q defined by $a * b = a + b - ab$. The identity element under $*$ is

(a) 0 (b) 1

(c) 2 (d) not exist

116. The statement $\sim p \vee q$ is equivalent to

(a) $p \rightarrow q$ (b) $\sim p \rightarrow q$

(c) $\sim p \rightarrow \sim q$ (d) $p \rightarrow \sim q$

117. In rolling two fair dice, what is the probability of obtaining a sum greater than 3 but not exceeding 6 ?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

118. Team A has probability $\frac{2}{3}$ of winning whenever it plays. Suppose A plays four games. What is the probability that A wins more than half of its games?

(a) $\frac{16}{27}$ (b) $\frac{19}{27}$

(c) $\frac{19}{81}$ (d) $\frac{32}{81}$

119. An unprepared student takes five-questions of true-false type quiz and guesses every answer. What is the probability that the student will pass the quiz if at least four correct answers is the passing grade?

(a) $\frac{1}{16}$ (b) $\frac{3}{16}$

(c) $\frac{1}{32}$ (d) $\frac{3}{32}$

120. The probability density f(x) of a continuous random variable is given by $f(x) = Ke^{-|x|}, -\infty < x < \infty$. Then the value of K is

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{1}{4}$ (d) 4

2006 SOLUTIONS

PART - I (PHYSICS)

1. (b) $V = 300\,V$, $C_1 = 2.0\mu F$, $C_2 = 8.0\mu F$,

Net capacitance, $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\Rightarrow C_s = \frac{C_1C_2}{C_1 + C_2}$$

$$\Rightarrow C_s = \frac{2\times 8}{2+8} = \frac{16}{10} = 1.6\,\mu F.$$

Now total charge,

$Q = V_s \times C_s = 300 \times 1.6 \times 10^{-6} = 4.8 \times 10^{-4}\,C.$

$\because$ In series charge is same on capacitors

$\Rightarrow$ Charge on $2\mu F$ capacitor is $4.8 \times 10^{-4}C$

2. (a) $4\mu C \quad P \quad -2\mu C$

x

Let the point P where resultant field is zero be x m from $4\mu C$ charge and $(1-x)$ m distance apart from $-2\mu C$ charge. Since field is zero at this point then,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 0$$

$$\Rightarrow |\vec{E}| = \frac{1}{4\pi \in_0}\frac{q_1}{r_1^2} + \frac{1}{4\pi \in_0}\frac{q_2}{r_2^2}$$

$$\Rightarrow 0 = \frac{1}{4\pi \in_0}\left[\frac{4\mu C}{x^2} + \frac{(-2\mu C)}{(1-x)^2}\right]$$

$$\Rightarrow \frac{4\mu C}{x^2} = \frac{2\mu C}{(1-x)^2} \Rightarrow \frac{2}{x^2} = \frac{1}{(1-x)^2}$$

$$\Rightarrow 2(1-x)^2 = x^2$$

Taking root $\sqrt{2}(1-x) = x$

$$\Rightarrow 1.414(1-x) = x \Rightarrow 1.414 - 1.414x = x$$

$$\Rightarrow 1.414 = (1+1.414)x \Rightarrow x = \frac{1.414}{2.414}$$

$$\Rightarrow x = 0.58m$$

3. (b) The net electric potential is algebraic sum of potential due to individual point charges.

$$V = \frac{1}{4\pi \in_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{1\times 10^{-6}}{0.2} - \frac{2\times 10^{-6}}{0.2} + \frac{3\times 10^{-6}}{0.3}\right]$$

$$= 9\times 10^9\,[5 - 10 + 10]\times 10^{-6}$$

$$= 9\times 10^3\,[5] = 45\times 10^3\,V = 45\,kV$$

4. (a) In a metal, conduction current is due to electrons given by

$I = nAev$

$$\Rightarrow \text{drift velocity, } v = \frac{I}{nAe}$$

$$\Rightarrow v = \frac{5}{5\times 10^{26}\times 4\times 10^{-6}\times 1.602\times 10^{-19}}$$

$$= \frac{1}{4\times 1.602\times 10^1}$$

$$= \frac{10^{-1}}{6.408} = 1.56\times 10^{-2}\,m/s$$

5. (b) In series combination of capacitors, charges on both capacitors will be same.

$$V_s = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\Rightarrow 1000 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\Rightarrow 1000 = Q\left(\frac{C_1 + C_2}{C_1C_2}\right)$$

$$\Rightarrow Q = \frac{1000\times C_1C_2}{C_1 + C_2}$$

$$Q = 1000\times\frac{3\times 10^{-12}\times 6\times 10^{-12}}{(3+6)\times 10^{-12}} = \frac{18}{9}\times 10^{-9}$$

$$= 2\times 10^{-9}\,C$$

6. (c) Parallel combination of 3 Ω and 6 Ω gives effective resistance,

$$R_p = \frac{3\times 6}{3+6} = \frac{18}{9} = 2\Omega$$. This in series with

2 Ω gives net ressistance as 4 Ω.

7. (b) The value of temperature coefficient of resistance is given by

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} = \frac{65-50}{50(70-20)}$$

(t_1 and t_2 are in °C)

$$\Rightarrow \alpha = \frac{15}{50\times 50} = 0.006/°C$$

8. (b) In Leclanche cell a strong solution of ammonium chloride acts as an electrolyte.

9. (d) In the galvanometer, I_g = max. current through galvanometer, S = shunt resistance, G = galvanometer resistance then

$$I_g(\text{galvanometer}) = I\left(\frac{S}{G+S}\right)$$

$$\Rightarrow I = I_g\left(\frac{G+S}{S}\right) = \left(\frac{50+1}{1}\right)I_g = 51 I_g$$

10. (a) Current in tangent galvanometer

$$I = \frac{H}{G}\tan\theta$$

Where G = galvanometer constant
H= earth's horizontal field = constant

$$\therefore \frac{I_1}{I_2} = \frac{\tan\theta_1}{\tan\theta_2} \Rightarrow \frac{1}{I_2} = \frac{\tan 30°}{\tan 60°}$$

$$\Rightarrow I_2 = \frac{\tan 60°}{\tan 30°} = \frac{1.7321}{0.5774} = 2.999 \text{Å}$$

$$\Rightarrow I_2 \simeq 3\text{A}$$

11. (c) Lorentz force on a charged particle in presence of magnetic and electic field is

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

$$\Rightarrow \vec{F} = q\vec{E} + q\left(\vec{v}\times\vec{B}\right)$$

12. (a) In a circular coil of n turns, magnetic field is

$$B = \frac{\mu_0 nI}{2r} = \frac{4\pi\times 10^{-7}\times 250\times 20\times 10^{-3}}{2\times 40\times 10^{-3}}$$

($\because$ n = no. of turns, I = current through coil, r = radius of coil)

$$\Rightarrow B = \frac{4\pi\times 250\times 20\times 10^{-7-3+3}}{2\times 40}$$

$$= 250\times 3.14\times 10^{-7}$$

$$= 785\times 10^{-7} = 0.785\times 10^{-4} \text{ tesla}$$
$$= 0.785 \text{ gauss}$$

13. (d) RMS value of A.C is

$$I_v = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

I_0 = peak value
$\therefore$ it is 70.7% of peak value.

14. (a) Q-factor is given by $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$\therefore$ If resistance R is decreased, Q increases and inductance L is increased, Q increases.

15. (c) Induced emf $e = \frac{-d\phi}{dt}$. Assuming, small change in flux $d\phi = 8\times 10^{-4}$ Wb
change in time dt = 0.5s

$$|e| = \frac{8\times 10^{-4}}{0.5} = \frac{80\times 10^{-4}}{5}$$

$$= 16\times 10^{-4} = 1.6\times 10^{-3}\text{ V} = 1.6\text{ mV}$$

16. (c) Current through an LCR circuit is maximum when impedance is minimum. Now impedance

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$ is minimum at

resonance frequency when $\omega L = \frac{1}{\omega C}$ and

Z= R= minium i.e., inductive reactance (ωL) is equal to capacitive reactance ($1/\omega C$)

17. (a) Our eyes respond to visible range from 400 nm to 700 nm

18. (a) Velocity of electromagnatic wave in space

is $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \Rightarrow c = \frac{1}{\sqrt{2\times 8}}$

$$= \frac{1}{\sqrt{16}} = \frac{1}{4} = 0.25$$

19. (b) According to Brewster's law, reflected light is plane polarised if unpolarised light falls at the interface of air and medium at an angle i_p called polarising angle then

$\mu = \tan i_p \Rightarrow \mu(\text{glass}) = \tan 60° = \sqrt{3}$

20. (a) Newton's ring arrangement is used for determining the wavelength of monochromatic light. For this the diameter of n^{th} dark ring (D_n) and $(n+p)^{th}$ dark ring (D_{n+p}) are measured then

$$D_{(n+p)}^2 = 4(n+p)\lambda R \text{ and } D_n^2 = 4n\lambda R$$

$$\Rightarrow \lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

Here, $n = 10, n + p = 20$;

$\therefore$ $p = 10$; $R = 1$ m, $D_{10} = 3.96 \times 10^{-3}$ m, $D_{20} = 5.82 \times 10^{-3}$ m

$$\therefore \lambda = \frac{D_{20}^2 - D_{10}^2}{4pR}$$

$$= \frac{(5.82\times10^{-3})^2 - (3.36\times10^{-3})^2}{4\times10\times1}$$

= 5646 Å

21. (b) Angular momentum in any stationary orbit

is $mvr = \frac{nh}{2\pi}$ for 4^{th} orbit, $n = 4$

$$\Rightarrow mvr = \frac{4h}{2\pi}$$

22. (d) According to Bohr's, Brackett series is obtained when an electron jumps to 4^{th} orbit from any other outer orbit

23. (a) Total energy of electron

$$E = -\frac{KZe^2}{2r} = K.E. + P.E.$$

Potential energy in the orbit $P.E. = \frac{-KZe^2}{r}$

$\therefore$ P.E. $= 2 \times E \Rightarrow$ P.E.$=2\times(-13.6)=-27.2$ eV

24. (a) Bragg's condition is $2d\sin\theta = n\lambda$, for second order $n = 2$, $\sin\theta = 1$. For longest λ, $2d = 2\lambda$ $\Rightarrow \lambda = d$

25. (b) Photon moves with speed of light ie, v = c and rest mass of a particle is

$$m_0 = m\sqrt{1 - v^2/c^2}$$

hence m_0 (photon)= 0

$\therefore$ photon has zero rest mass.

Momentum of photon $= \frac{h}{\lambda}$

26. (c) de Broglie wavelength is given by

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}}{1000\times10^{-3}\times1} \simeq 6.626\times10^{-34}\,\text{m}$$

27. (b) Let the velocity of a particle be v where mass m is double the rest mass i.e., $m = 2m_0$ then

$$m_0 = m\sqrt{1-\frac{v^2}{c^2}} \Rightarrow m_0 = 2m_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{1}{2} = \sqrt{1-\frac{v^2}{c^2}} \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2}$$

$$\Rightarrow \frac{v^2}{c^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow v = \sqrt{\frac{3}{4}}c$$

28. (d) By Einstein's equation $E= mc^2$ where $m = 2$kg

$\Rightarrow E = 2\times(3\times10^8)^2$

$= 2\times3\times3\times10^{16} = 18\times10^{16}$ J

29. (b) By definition, the difference between the sum of the masses of neutrons and protons forming a nucleus and mass of nucleus is called mass defect

30. (b) Mean life time $\tau = 1.44$ T where T is half life period of an atom

$$\therefore \tau = 1.44\,T = \frac{T}{0.6931} = \frac{3}{0.6931} \text{ minute}$$

31. (d) (by conservation of charge)

32. (d) Baryons are proton, neutron, lamda,

sigma $(\Sigma^+, \Sigma^0, \Sigma^-)$, $X_i\left(\equiv^0, \equiv^-\right)$

33. (c) As donor and acceptor impurities are added to semiconductor to make an extrinsic semi-conductor, intrinsic semiconductor is formed by internal generation of e^- by breaking up of covalent bonds.

34. (d) In p-type semiconductor, valency = 3, thus there is one urY med bond or hole created. This hole is in valence band and is able to cause hole current. The energy levels of acceptor are in forbidden gap just above valence band

35. (d) In an oscillator, L-C circuit is coupled with transistor amplifier in such a way that there is a positive feed back to the LC circuit i.e., proper energy supply to LC at proper timings. So that total energy of LC circuit remains same.

36. (c) The gates AND, OR, NAND do not give binary addition, however in EXOR gate truth table is

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

This shows it gives perfect binary addition

37. (d) In FM, carrier frequency is the constant frequency which is modulated by signal amplitude. It is also called carrier swing. (Centre frequency is f_c in AM wave, frequency deviation $\delta = f_{max} - f_c$,

$\text{mod ulation factor} = \frac{\delta_{max}}{f_c}$)

38. (c) The common antenna is a straight conductor of length $l = \frac{\lambda}{4}$ held vertically with its lower end touching the ground.

39. (b) The vidicon is a storage-type camera tube in which a charge-density pattern is formed by the imaged scene radiation on a photoconductive surface which is then scanned by a beam of low-velocity electrons. The fluctuating voltage coupled out to a video amplifier can be used to reproduce the scene being imaged. The electrical charge produced by an image will remain in the face plate until it is scanned or until the charge dissipates.

40. (b) Maximum range of radar $d_{max} \propto \sqrt{l}$ and power transmitted by antenna of length l is $p \propto (1/\lambda)^2 \Rightarrow l \propto \sqrt{p}$ and $d \propto (p)^{1/4}$

PART - II (CHEMISTRY)

41. (b) The oxidation of ferrous ion by $KMnO_4$ takes place in acidic medium as per following reaction

$$2KMnO_4 + 8H_2SO_4 + 10FeSO_4$$
$$\rightarrow K_2SO_4 + 2MnSO_4 + 8H_2O + 5Fe_2(SO_4)_3$$

$\therefore$ Eq. mass of $KMnO_4$

$$\frac{\text{Molecular mass}}{\text{change in oxidation number}}$$

$$= \frac{\text{Molecular mass}}{5} = \frac{158}{5} = 31.6$$

42. (b) In case of lanthanoids, 4f orbitals lie too deep and hence the magnetic effect of the motion of the electron in its orbital is not quenched out. Here spin contribution S and orbital contribution L couple together to give a new quantum number J.

Thus magnetic moment of lanthanoids is given by, $\mu = g\sqrt{J(J+1)}$

where J = L – S when the shell is less than half fill

J = L + S when the shell is more than half fill

and $g = 1\frac{1}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$

43. (d) $Mg^{2+} = 1s^2, 2s^2, 2p^6$ (No unpaired electrons)

$Ti^{3+} = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^0, 3d^1$

(One unpaired electrons)

$V^{3+} = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^0, 3d^2$

(Two unpaired electrons)

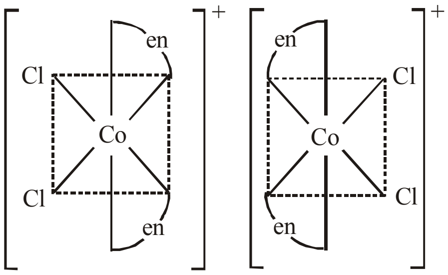
$Fe^{2+} = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^0, 3d^6$

(Four unpaired electrons)

$\therefore$ Fe^{2+} has highest number of unpaired electrons.

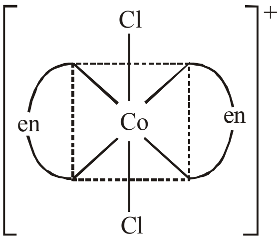
44. (d) $Zn + 2NaOH \rightarrow \underset{\text{Sod. zincate}}{Na_2ZnO_2} + H_2$

45. (c)



d - cis form

l - cis form



trans form (meso)

46. (b) Neutral ligands are given the same names as the neutral molecule. However, two very important exceptions to this rule are: H_2O Aquo (Aqua) NH_3 Ammine.

47. (a) $Ni(PF_3)_4$ – tetrakis phosphours (III) fluoride nickel (0).

48. (a) The colour of $KMnO_4$ is due to charge transfer. The configuration of manganese in permagnate ion is d^0 but it is coloured because its electrons are photo-exited.

49. (c) In face centred cubic lattice, the atoms are present at eight corners of faces and one each at 6 faces.

$\therefore$ Lattice points belonging to face centred cubic unit cell = $8\times\frac{1}{8}+6\times\frac{1}{2}=4$

50. (a) Schottky defect is caused when equal number of cations and anions are missing from their lattice sites.

51. (a) Polystyrene is thermoplastic substance.

52. (a) Po – Simple cubic lattice
Na – bcc
Cu – fcc

53. (c) $W_{irr}=-P_{ext}\frac{R\left[P_1T_2-P_2T_1\right]}{P_1P_2}$

During expansion in vaccum $P_{ext}=0$
$\therefore$ work done = 0.

54. (b) As the system is closed, hence the reaction will be reversible, hence according to Le-chatelier principle pressure decreases since the volume is increasing.

55. (d) $W=-2.303\,nRT\log\frac{V_2}{V_1}$

Given n = 6, T = 27°C = 273 + 27 = 300 K
$V_1 = 1\,L, V_2 = 10\,L$

$\therefore W=-2.303\times 6\times 8.314\times 300\log\frac{10}{1}$

= 34.465 kJ

56. (a) It is spontaneous process because zinc is more reactive than copper, hence can easily repace Cu from $CuSO_4$.

57. (c) $K_p = K_c\,(RT)^{\Delta n}$
$\Delta n = n_{p(g)} - n_{r(g)} = 2 - 2 = 0$
$\therefore\ K_p = K_c$

58. (d) Ice $\rightleftharpoons$ Water

The volume of ice is more than water. Therefore when pressure is increased the equilibrium shifts in forward direction. It favoures melting of ice.

59. (c) It is a first order reaction because rate of reaction $\propto [N_2O_5]$

60. (d) The reactions with low activation energies are always fast whereas the reactions with high activation energy are always slow.

61. (d) For spontaneous reaction free energy change is negative.
$\Delta G = -nFE$

62. (a) Because at infinite dilution the equivalent conductance of strong electrolytes furnishing same number of ions is same.

63. (c) Reducing character i.e tendency to loose electron decreases down the series, hence the correct order is Zn > Cu > Ag.

64. (b) At anode the following reaction takes place
$CH_4 + 2H_2O \rightarrow CO_2 + 8H^+ + 8e^-$

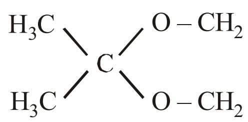
65. (a) Oxygen atom in alcohol molecule is sp^3 hybridised.

66. (d) In this reaction $LiAlH_4$ acts as reducing agent. $R-\overset{\overset{\large O}{\|}}{C}-OH \xrightarrow{LiAlH_4} RCH_2OH$

67. (d) 3° alcohols are resistant to oxidation and are oxidised only by strong oxidising agents like conc. HNO_3. They are resistant to oxidation in neutral or alkaline $KMnO_4$.

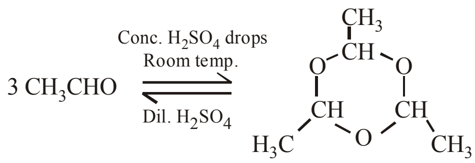
68. (b) $C_2H_5-O-C_2H_5 \xrightarrow[\text{boil, dil. acid}]{+H_2O} 2C_2H_5OH$
ethyl alcohol

69. (b) $CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3 + \begin{matrix} CH_2OH \\ | \\ CH_2OH \end{matrix} \xrightarrow{HCl}$

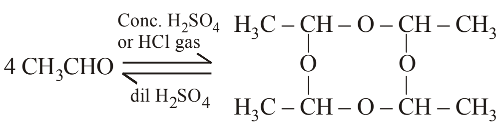


cyclic ketal

70. (b)



Para aldehyde
(pleasant smelling liquid, used as hypnotic and soporofic)
(sleep producing)



Metaldehyde
(used as solid fuel in spirit lamps)

71. (c) $2(CH_3COO)_2Ca \xrightarrow{\text{distillation}} CH_3COCH_3 + 2CaCO_3$
Acetone Calcium carbonate

72. (d) Ketones do not react with alcohol.

(i) Aldehydes $\xrightarrow{\text{reduction}}$ 1° alcohol

e.g., $CH_3CHO \longrightarrow CH_3CH_2OH$

(ii) Ketones $\xrightarrow{\text{reduction}}$ 2° alcohol

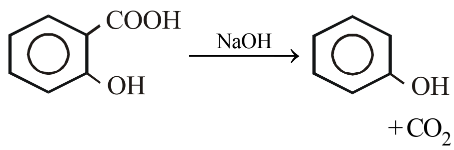
$\begin{matrix} CH_3 \\ CH_3 \end{matrix}\!\!>C=O \qquad \begin{matrix} CH_3 \\ CH_3 \end{matrix}\!\!>CHOH$

(iii) Ketones do not reduce Fehling solution but aldehydes do so.

73. (d) The O – H strecting of carboxylic acid absorb in region of 1700-2000 cm^{-1}

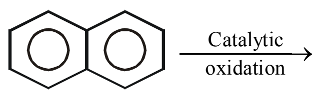
74. (b) Formic acid $\left(H-\overset{\overset{\displaystyle O}{\|}}{C}-OH\right)$ has – CHO group and therefore it reduces Fehling solution.

75. (c)

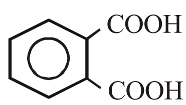


This is decarboxylation reaction.

76. (d)



Napthalene



Phthalic acid

77. (b) In both the cases carbon is sp^3 hybridised and bond angle is 109°28'.

78. (b) This reaction is known as curtius rearrangement.

$RCON_3 \xrightarrow{-N_2} RNCO \xrightarrow{2NaOH} Na_2CO_3 + RNH_2$

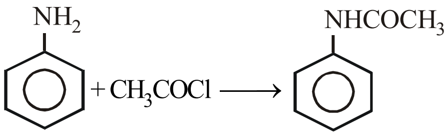
1° amine is formed.

79. (a) It is expected that the basic nature of amines should be in order tertiary > secondary > primary but the observed order in the case of lower members is found to be as secondary > Primary > tetriary. This anomalous behaviour of tetriary amines is due to steric factors i.e crowding of alkyl groups cover nitrogen atom from all sides thus makes the approach and bonding by a proton relatively difficult which results the maximum steric strain in tetiary amines. The electrons are there but the path is blocked, resulting the reduction in basicity.

Thus the correct order is

$R_2NH > RNH_2 > R_3N$.

80. (d) On acetylation aniline is converted into acetamide which is resonance stablised and therefore less reactive.



Acetanilide

PART - III (MATHEMATICS)

81. (a) We know that, A is singular if $|A| = 0$

$$|A| = \begin{vmatrix} \frac{2}{x} & -1 & 2 \\ 1 & x & 2x^2 \\ 1 & \frac{1}{x} & 2 \end{vmatrix} = 0$$

$$\Rightarrow |A| = \frac{2}{x}[2x - 2x] + 1\left[2 - 2x^2\right] + 2\left[\frac{1}{x} - x\right] = 0$$

$$\Rightarrow \frac{2}{x}[0] + \left[2 - 2x^2\right] + \frac{2}{x} - 2x = 0$$

$$\Rightarrow 2x - 2x^3 + 2 - 2x^2 = 0$$

$$\Rightarrow x^3 + x^2 - x - 1 = 0$$

$$\Rightarrow x^2(x+1) - 1(x+1) = 0$$

$$\Rightarrow (x+1)(x^2 - 1) = 0$$

$$\Rightarrow x = \pm 1$$

82. (b) Given $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

$$\Rightarrow x[x^2 - 12] - 3[2x - 14] + 7[12 - 7x] = 0$$

$$\Rightarrow x^3 - 67x + 126 = 0$$

But given (x = 9) is a root of given determinant.

$\therefore (x+9)$ is a factor

$$\Rightarrow x^3 + 9x^2 - 9x^2 - 81x + 14x + 126 = 0$$

$$\Rightarrow x^2(x+9) - 9x(x+9) + 14(x+9) = 0$$

$$\Rightarrow (x+9)(x^2 - 9x + 14) = 0$$

$$\Rightarrow (x+9)(x^2 - 7x - 2x + 14) = 0$$

$$\Rightarrow (x+9)(x-7)(x-2) = 0$$

$$\Rightarrow x = -9, 7, 2$$

83. (c) We have

$$A:B = \begin{vmatrix} 1 & 1 & 1 & : & 1 \\ 1 & 2 & 4 & : & \alpha \\ 1 & 4 & 10 & : & \alpha^2 \end{vmatrix}$$

$$\sim \begin{vmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 3 & : & \alpha - 1 \\ 0 & 3 & 9 & : & \alpha^2 - 1 \end{vmatrix}$$

$$\begin{bmatrix} \text{applying } R_2 \to R_2 - R_1 \\ \& \ R_3 \to R_3 - R_1 \end{bmatrix}$$

$$\sim \begin{vmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 3 & : & \alpha - 1 \\ 0 & 0 & 0 & : & \alpha^2 - 3\alpha + 2 \end{vmatrix}$$

$$[\text{applying } R_3 \to R_3 - 3R_2]$$

But the system is consistent

$$\therefore \alpha^2 - 3\alpha + 2 = 0$$

$$\Rightarrow (\alpha - 2)(\alpha - 1) = 0 \Rightarrow \alpha = 2 \text{ or } \alpha = 1$$

84. (c) We know that inverse of A does not exist only when $|A| = 0$

$$\therefore \begin{vmatrix} 1 & 3 & 2 \\ 2 & 5 & t \\ 4 & 7-t & -6 \end{vmatrix} = 0$$

$$\Rightarrow (-30 - 7t + t^2) - 3(-12 - 4t) + 2(14 - 2t - 20) = 0$$

$$\Rightarrow -30 - 7t + t^2 + 36 + 12t - 12 - 4t = 0$$

$$\Rightarrow t^2 + t - 6 = 0 \Rightarrow t^2 + 3t - 2t - 6 = 0$$

$$\Rightarrow t(t+3) - 2(t+3) = 0$$

$$\Rightarrow (t+3)(t-2) = 0 \Rightarrow t = 2, -3$$

85. (a) Given $x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$
By using Hit & trial method, we have
$(x - 1)$ is a factor of given equation
$\therefore\ (x-1)(x^3 - 2x^2 - 4x - 1) = 0$

$\Rightarrow\ (x-1)[x^3 + x^2 - 3x^2 - 3x - x - 1] = 0$

$\Rightarrow\ (x-1)\left[x^2(x+1) - 3x(x+1) - 1(x+1)\right] = 0$

$\Rightarrow\ (x-1)(x+1)(x^2 - 3x - 1) = 0$

$\therefore\ x = 1,\ -1$ or $x^2 - 3x - 1 = 0$
Now $x^2 - 3x - 1 = 0$

$$\Rightarrow x = \frac{3 \pm \sqrt{9+4}}{2}$$

$$\left[\because\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$$\Rightarrow x = \frac{3 \pm \sqrt{13}}{2}$$

$\therefore$ non-integer roots of given equation are

$\frac{1}{2}(3+\sqrt{13}), \frac{1}{2}(3-\sqrt{13})$

86. (b) Given $e^x = y + \sqrt{1+y^2}$

$\Rightarrow e^x - y = \sqrt{1+y^2}$
Squaring both side, we have
$e^{2x} + y^2 - 2e^x y = 1 + y^2$
$\Rightarrow\ 2e^x y = e^{2x} - 1$

$$\Rightarrow\ y = \frac{e^{2x} - 1}{2e^x} \Rightarrow y = \frac{1}{2}\left[e^x - e^{-x}\right]$$

87. (d) First term = a & common ratio = r

Given $S_\infty = 4$ & $a_2 = \frac{3}{4}$

$$\Rightarrow\ \frac{a}{1-r} = 4 \qquad ...(1)$$

$$\& \ ar = \frac{3}{4} \quad \left[\because\ S_\infty = \frac{a}{1-r} \ \& \ a_n = ar^{n-1}\right]$$

$$\Rightarrow\ a = \frac{3}{4r}$$

$\therefore$ Equation (1) becomes $\frac{3}{4r(1-r)} = 4$

$\Rightarrow\ 16r^2 - 16r + 3 = 0$

$\Rightarrow\ (4r-3)(4r-1) = 0$

$\Rightarrow\ r = \frac{3}{4}$ or $r = \frac{1}{4}$

when $r = \frac{1}{4}$ then $a = \frac{3}{4 \times \frac{1}{4}} = 3$

$\therefore\ a = 3$ & $r = \frac{1}{4}$

88. (c) Given : α & β are roots of equation $ax^2 + bx + c = 0$

$\therefore\ \alpha + \beta = -\frac{b}{a}$ & $\alpha\beta = \frac{c}{a}$

Now, $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$$\Rightarrow\ \alpha^3 + \beta^3 = \left(-\frac{b}{a}\right)^3 - 3\frac{c}{a}.\left(-\frac{b}{a}\right)$$

$$\Rightarrow\ \alpha^3 + \beta^3 = -\frac{b^3}{a^3} + \frac{3bc}{a^2}$$

$$\Rightarrow\ \alpha^3 + \beta^3 = \frac{-b^3 + 3abc}{a^3}$$

89. (b) Given : The vertices of tetrahedron are $P(-1, 2, 0), Q(2, 1, -3), R(1, 0, 1) \ \& \ S(3, -2, 3)$

$\therefore$ Volume of tetrahedron $= \frac{1}{6}\left[\overrightarrow{PQ}\ \overrightarrow{PR}\ \overrightarrow{PS}\right]$

Now,

$\overrightarrow{PQ} = (2+1)\hat{i} + (1-2)\hat{j} + (-3)\hat{k} = 3\hat{i} - \hat{j} - 3\hat{k}$

Similarly, $\overrightarrow{PR} = 2\hat{i} - 2\hat{j} + \hat{k}$

& $\overrightarrow{PS} = 4\hat{i} - 4\hat{j} + 3\hat{k}$

$\therefore$ Volume of tetrahedron

$$= \frac{1}{6}\begin{vmatrix} 3 & -1 & -3 \\ 2 & -2 & 1 \\ 4 & -4 & 3 \end{vmatrix} = \frac{2}{3}$$

90. (a) We have, $\vec{a}+t\vec{b}=(\hat{i}+2\hat{j}+3\hat{k})+t(-\hat{i}+2\hat{j}+\hat{k})$

$=(1-t)\hat{i}+(2+2t)\hat{j}+(3+t)\hat{k}$

It is $\perp$ to $c=3\hat{i}+\hat{j}$

If $3(1-t)+(2+2t)+(3+t)\,(0)=0$

$\Rightarrow\ 3-3t+2+2t=0\ \Rightarrow t=5$

91. (d) The equation of the plane through the line of intersection of the given planes is

$(x+y+z-6)+\lambda(2x+3y+4z+5)=0$ ... (1)

If equation (1) passes through (1, 1, 1), we have

$-3+14\lambda=0\Rightarrow\lambda=\dfrac{3}{14}$

Putting $\lambda=\dfrac{3}{14}$ in (1), we obtain the equation of the required plane as

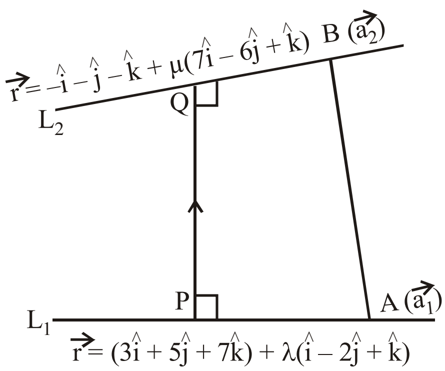
$(x+y+z-6)+\dfrac{3}{14}(2x+3y+4z+5)=0$

$\Rightarrow\ 20x+23y+26z-69=0$

92. (d) Shortest distance $PQ=\left|\dfrac{(\vec{b}_1\times\vec{b}_2)\,.\,(\vec{a}_2-\vec{a}_1)}{|(\vec{b}_1\times\vec{b}_2)|}\right|$

Now, $\vec{a}_2-\vec{a}_1=-\hat{i}-\hat{j}-\hat{k}-3\hat{i}-5\hat{j}-7\hat{k}$

$\Rightarrow\ \vec{a}_2-\vec{a}_1=-4\hat{i}-6\hat{j}-8\hat{k}$



And $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 1\\ 7 & -6 & 1\end{vmatrix}$

$\Rightarrow\ \vec{b}_1\times\vec{b}_2=\hat{i}(-2+6)-\hat{j}(1-7)+\hat{k}(-6+14)$

$\Rightarrow\ \vec{b}_1\times\vec{b}_2=4\hat{i}+6\hat{j}+8\hat{k}$

$\therefore$ Shortest distance

$PQ=\left|\dfrac{(4\hat{i}+6\hat{j}+8\hat{k})\,.\,(-4\hat{i}-6\hat{j}-8\hat{k})}{\sqrt{16+36+64}}\right|$

$\Rightarrow\ PQ=\left|\dfrac{-16-36-64}{\sqrt{116}}\right|$

$=\left|\dfrac{-116}{\sqrt{116}}\right|=\sqrt{116}=2\sqrt{29}$

$\therefore\ PQ=2\sqrt{29}$ units

93. (c) Given, $|z-i|+|z+i|\le 4$

$\Rightarrow\ |z-(0+i)|+|z-(0-i)|\le 0$

This equation represent the interior and boundary of ellipse with foci at (0, 1) & (0, –1), whose major axis is along the y-axis.

94. (b) $\sum_{n=1}^{13}\left(i^n+i^{n+1}\right)=\sum_{n=1}^{13}i^n+\sum_{n=1}^{13}i^{n+1}$

$=i\left(\dfrac{1-i^{13}}{1-i}\right)+i^2\left(\dfrac{1-i^{13}}{1-i}\right)$

$=i\dfrac{(1-i)}{(1-i)}-\dfrac{(1-i)}{(1-i)}=i-1$

95. (b) Given : $\sin\theta,\cos\theta,\tan\theta$ are in G.P.

$\Rightarrow\ \cos^2\theta=\sin\theta\tan\theta\Rightarrow\cos^3\theta=\sin^2\theta$

$\Rightarrow\ \cos^3\theta=1-\cos^2\theta$

$\Rightarrow(\cos^3\theta+\cos^2\theta)=1$ ...(1)

Cubic both sides, we have

$\cos^9\theta+\cos^6\theta+3\cos^5\theta\,.\,(\cos^3\theta+\cos^2\theta)=1$

$\Rightarrow\ \cos^9\theta+\cos^6\theta+3\cos^5\theta=1$

[Using equation (1)]

$\Rightarrow\ \cos^9\theta+\cos^6\theta+3\cos^5\theta-1=0$

96. (c) Given : $5\cos C + 6\cos B = 4$ ...(1)

$6\cos A + 4\cos C = 5$ ...(2)

Adding eq. (1) & (2), we have

$9\cos C + 6(\cos A + \cos B) = 9$

$$\Rightarrow 9\cos C + 6\left[2\cos\frac{A+B}{2}\,.\,\cos\frac{A-B}{2}\right] = 9$$

$$\Rightarrow 9\cos C - 9 + 12\cos\left(\frac{\pi}{2} - \frac{C}{2}\right).\cos\frac{A-B}{2} = 0$$

$$\Rightarrow 9(\cos C - 1) + 12\sin\frac{C}{2}\,.\cos\frac{A-B}{2} = 0$$

$$\Rightarrow 9\left[1 - 2\sin^2\frac{C}{2} - 1\right] + 12\sin\frac{C}{2}.\cos\frac{A-B}{2} = 0$$

$$\Rightarrow -18\sin^2\frac{C}{2} + 12\sin\frac{C}{2}.\cos\frac{A-B}{2} = 0$$

$$\Rightarrow 3\sin\frac{C}{2} = 2\cos\frac{A-B}{2}$$

$$\Rightarrow 3\cos\frac{A+B}{2} = 2\cos\frac{A-B}{2}$$

$$\Rightarrow 3\left(\cos\frac{A}{2}.\cos\frac{B}{2} - \sin\frac{A}{2}.\sin\frac{B}{2}\right)$$

$$= 2\left[\cos\frac{A}{2}.\cos\frac{B}{2} + \sin\frac{A}{2}.\sin\frac{B}{2}\right]$$

$$\Rightarrow 5\sin\frac{A}{2}\,.\,\sin\frac{B}{2} = \cos\frac{A}{2}\,.\,\cos\frac{B}{2}$$

$$\Rightarrow 5\tan\frac{A}{2}\,.\,\tan\frac{B}{2} = 1$$

$$\Rightarrow \tan\frac{A}{2}.\tan\frac{B}{2} = \frac{1}{5}$$

97. (b) Equation of the semielliptical bridge

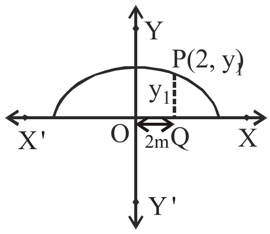
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad .... (1)$$

Here, $2a = 9$

$$\therefore\ a = \frac{9}{2}, \quad b = 3$$

$$\therefore\ \frac{x^2}{\frac{81}{4}} + \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{4x^2}{81} + \frac{y^2}{9} = 1 \quad .... (2)$$



Here, $OQ = 2$ m, let $PQ = y_1$

$\therefore\ P(2, y_1)$

Since point P lies on the ellipse (2)

$$\therefore\ \frac{4\times 4}{81} + \frac{y_1^2}{9} = 1$$

$$\Rightarrow \frac{y_1^2}{9} = 1 - \frac{16}{81} = \frac{81-16}{81} = \frac{65}{81}$$

$$\Rightarrow y_1^2 = \frac{65}{9} \Rightarrow y_1 = \frac{\sqrt{65}}{3} \simeq \frac{8}{3}\text{ m}$$

Hence, best approximation of the height of the arch $= \frac{8}{3}$ m.

98. (c) Given : Equations of ellipses

$3x^2 + 5y^2 = 32$ ...(1)

& $25x^2 + 9y^2 = 450$ ...(2)

Tangents to the ellipse (1) & (2) are passing through the point (3, 5)

$\therefore\ 3(3)^2 + 5(5)^2 - 32 = 27 + 75 - 32 > 0$

So the given point lies outsides the ellipse. Hence, two real tangents can be drawn from the point to the ellipse,

& $25(3)^2 + 9(5)^2 - 450 = 225 + 225 - 450 = 0$

$\therefore$ The point lie on the ellipse. Hence one real tangent can be drawn.

$\therefore$ No. of real tangents = 3

99. (b) The equation of tangent at $\left(ct, \frac{c}{t}\right)$ is

$ty = t^3x - ct^4 + c$

If it passes through $\left(ct', \frac{c}{t'}\right)$ then

$\Rightarrow \frac{tc}{t'} = t^3ct' - ct^4 + c$

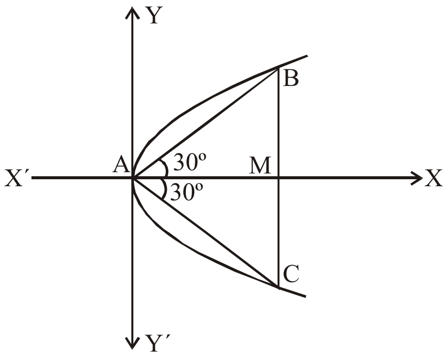
$\Rightarrow t = t^3t'^2 - t^4t' + t'$

$\Rightarrow t.t' = t^3t'(t'.t) \Rightarrow t^3t' = -1$

Note : If we take the co-ordinate axes along the asymptotes of a rectangular hyperbola, then the general equation $x^2 - y^2 = a^2$ becomes $xy = c^2$, where c is a constant.

100. (d) Let AB = ℓ, then $AM = \ell\cos 30° = \frac{\ell\sqrt{3}}{2}$

& $BM = \ell \sin 30° = \frac{\ell}{2}$



So, the coordinates of B are $\left(\frac{\ell\sqrt{3}}{2}, \frac{\ell}{2}\right)$

Since, B lies on $y^2 = 4x$

$\therefore \frac{\ell^2}{4} = 4\left(\frac{\ell\sqrt{3}}{2}\right)$

$\Rightarrow \ell^2 = \frac{16}{2}.\sqrt{3}\ell \Rightarrow \ell = 8\sqrt{3}$

101. (d) Let $f(x) = ax + b$

Given $f(2) = 4$ & $f'(2) = 1$

$\therefore f(2) = a.2 + b = 4 \Rightarrow 2a + b = 4$ ...(1)

& $f'(x) = a \Rightarrow f'(2) = a = 1 \Rightarrow a = 1$

$\therefore 2 \times 1 + b = 4 \Rightarrow b = 2$ [using equation (1)]

$\therefore f(x) = x + 2$

Now, $\lim_{x\to 2} \frac{xf(2) - 2f(x)}{x - 2}$

$= \lim_{x\to 2} \frac{4x - 2(x+2)}{x - 2} = \lim_{x\to 2} \frac{2x - 4}{x - 2}$

$= \lim_{x\to 2} \frac{2(x-2)}{(x-2)} = 2$

102. (d) Let $f(x) = x^2 + kx + 1$

$f'(x) = 2x + k$

f(x) is strictly increasing on (1, 2)

if $f'(x) > 0$ for $x \in (1, 2)$

$\Rightarrow 2x + k > 0$ for $x \in (1, 2)$

$\Rightarrow k > -2x$ for $x \in (1, 2)$

Now, $1 < x < 2 \Rightarrow 2 < 2x < 4$

$\Rightarrow -2 > -2x > -4$

$\Rightarrow -4 < -2x < -2$

$\Rightarrow k \geq -2$

Hence least value of $k = -2$.

103. (c) Let $y = \left(\frac{1}{x}\right)^x \Rightarrow y = x^{-x}$

Then $\log y = -x \log x$

$\therefore \frac{1}{y}\frac{dy}{dx} = -(1 + \log x)$

or $\frac{dy}{dx} = -y(1 + \log x)$

& $\frac{d^2y}{dx^2} = -y.\frac{1}{x} - (1 + \log x).\frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = -\left(\frac{1}{x}\right)^{x+1} - (1 + \log x).\frac{dy}{dx}$

Now, $\frac{dy}{dx} = 0 \Rightarrow 1 + \log x = 0$

$\Rightarrow \log x = -1 = -\log e = \log\left(\frac{1}{e}\right)$

$\Rightarrow x = \left(\frac{1}{e}\right)$

Also, $\frac{d^2y}{dx^2}$ at $x = \left(\frac{1}{e}\right)$ is $-e^{1+\frac{1}{e}} < 0$

$$\left[\because \frac{dy}{dx} = 0\right]$$

So, $x = \left(\frac{1}{e}\right)$ is a point of local maxima.

$\therefore$ Maximum value

$$= \left(\text{value of } y \text{ when } x = \frac{1}{e}\right) = e^{\frac{1}{e}}$$

104. (a) Euler's theorem $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$

Given : $U = \tan^{-1}\frac{x^3 + y^3}{x + y}$

$$\Rightarrow \tan U = \frac{x^3 + y^3}{x + y} = z \text{ (let)}$$

$n = 3 - 1 = 2$

$$\therefore\ x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z$$

$$\Rightarrow\ x\frac{\partial}{\partial x}\tan U + y\frac{\partial}{\partial y}.\tan U = 2\tan U$$

$$\Rightarrow\ x\,.\sec^2 U\,.\frac{\partial U}{\partial x} + y.\sec^2 U\,.\frac{\partial U}{\partial y} = 2\tan U$$

$$\Rightarrow\ \sec^2 U\,.\left[x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y}\right] = 2\tan U$$

$$\Rightarrow\ x\,.\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = 2\,.\frac{\sin U}{\cos U}\,.\cos^2 U$$

$$\Rightarrow\ x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = \sin 2U$$

105. (b) Given : $f'(x) = \frac{x}{\sqrt{1+x}}$, $f(0) = 0$

$$\Rightarrow \int f'(x)dx = \int \frac{x}{\sqrt{1+x}}dx$$

$$\Rightarrow f(x) = \int \frac{x}{\sqrt{1+x}}dx$$

Let $1 + x = t^2 \Rightarrow x = t^2 - 1$

$\Rightarrow dx = 2t\,.\,dt$

$$\therefore\ f(x) = \int \frac{t^2 - 1}{t}.\,2t\,dt = 2\int (t^2 - 1)dt$$

$$\Rightarrow\ f(x) = 2\left[\frac{t^3}{3} - t\right] + c$$

$$\Rightarrow\ f(x) = 2\left[\frac{(1+x)^{3/2}}{3} - (1+x)^{1/2}\right] + c \quad \text{..........(1)}$$

But $f(0) = 0 \Rightarrow 2\left[\frac{1}{3} - 1\right] + c = 0$

$$\Rightarrow\ -\frac{4}{3} + c = 0 \Rightarrow c = \frac{4}{3}$$

$\therefore$ Equation (1) becomes

$$f(x) = 2\left[\frac{(1+x)^{3/2}}{3} - (1+x)^{1/2}\right] + \frac{4}{3}$$

$$\Rightarrow f(x) = \frac{2}{3}\left[(1+x)^{3/2} - 3(1+x)^{1/2} + 2\right]$$

106. (a) Let $I = \int_0^{\frac{\pi}{2}} \log(\tan x)dx$ ...(1)

Then, $I = \int_0^{\frac{\pi}{2}} \log\left[\tan\left(\frac{\pi}{2} - x\right)\right]dx$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$\Rightarrow\ I = \int_0^{\frac{\pi}{2}} \log(\cot x)dx$$

$$\Rightarrow\ I = \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{\tan x}\right)dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log(\tan x)^{-1} dx = -\int_0^{\frac{\pi}{2}} \log(\tan x)dx$$

$$\Rightarrow I = -I \Rightarrow 2I = 0 \Rightarrow I = 0$$

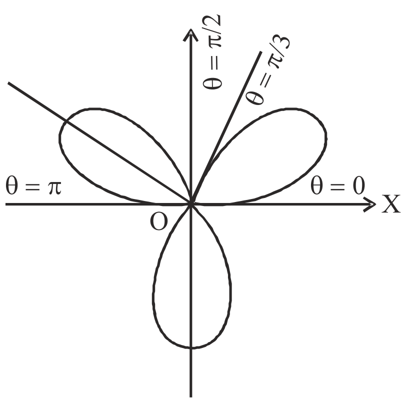
[Using eq. (1)]

107. (d) If curve $r = a \sin 3\theta$

To trace the curve, we consider the following table :

$3\theta =$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
$\theta =$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r =$	0	a	0	$-a$	0	a	0

Thus there is a loop between $\theta = 0$ & $\theta = \frac{\pi}{3}$ as r varies from $r = 0$ to $r = 0$.



Hence, the area of the loop lying in the positive quadrant $= \frac{1}{2}\int_0^{\frac{\pi}{3}} r^2 d\theta$

$$= \frac{1}{2}\int_0^{\frac{\pi}{3}} \sin^2\phi \cdot \frac{1}{3} d\phi$$

[On putting, $3\theta = \phi \Rightarrow d\theta = \frac{1}{3}d\phi$]

$$= \frac{a^2}{6}\int_0^{\frac{\pi}{2}} \sin^2\phi \, d\phi$$

$$= \frac{a^2}{6} \cdot \int_0^{\frac{\pi}{2}} \frac{1-\cos 2\phi}{2} d\phi$$

$$\left[\because \cos 2\theta = 1 - 2\sin^2\theta\right]$$

$$= \frac{a^2}{12} \cdot \left[\phi + \frac{\sin 2\phi}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{a^2}{12} \cdot \left[\frac{\pi}{2} + \sin\pi\right] = \frac{a^2\pi}{24}$$

108. (b) Let $I = \int_1^9 e^{\sqrt{t}} dt$

Put $t = x^2 \Rightarrow dt = 2x \, . \, dx$

For limit : $x = 1$ & $x = 3$

$$I = \int_1^3 e^x \, . \, 2x \, dx = 2\int_1^3 e^x \, . \, x \, dx$$

$$\Rightarrow I = 2\left[\left\{x \, . \, e^x\right\}_1^3 - \int_1^3 e^x dx\right]$$

$$= 2\left[\left\{xe^x\right\}_1^3 - \left\{e^x\right\}_1^3\right]$$

$$\Rightarrow I = 2\left[3e^3 - e^1 - e^3 + e^1\right] = \left[2e^3\right].2 = 4e^3$$

109. (d) Equation of the family of such parabolas is

$(y-k)^2 = 4a(x-h)$ ...(1)

where h & k are arbitrary constants

Differentiating w.r.t. x, we get

$$(y-k)\frac{dy}{dx} = 2a \qquad ...(2)$$

Differentiating again

$$(y-k)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0 \qquad ...(3)$$

Putting value of $(y-k)$ from (2) in (3), we get

$$2a\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3=0$$, which is required equation.

110. (b) Given : $\left(x\operatorname{cosec}\left(\frac{y}{x}\right)-y\right)dx+x\,dy=0$

$$\Rightarrow \left(\frac{x}{\sin\left(\frac{y}{x}\right)}-y\right)dx+x\,dy=0$$

$$\Rightarrow \left[x-y\,.\sin\left(\frac{y}{x}\right)\right]dx+x\sin\left(\frac{y}{x}\right)dy=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{y\sin\frac{y}{x}-x}{x\sin\left(\frac{y}{x}\right)} \qquad ...(1)$$

Put $\frac{y}{x}=z \Rightarrow y=zx$

$$\therefore \frac{dy}{dx}=z.1+x.\frac{dz}{dx}=z+x\frac{dz}{dx}$$

$\therefore$ Equation (1) becomes

$$z+x\frac{dz}{dx}=\frac{zx.\sin z-x}{x\sin z}=z-\operatorname{cosec} z$$

$$\Rightarrow x\frac{dz}{dx}=-\operatorname{cosec} z \Rightarrow -\sin z\,dz=\frac{dx}{x}$$

$$\Rightarrow \log|x|=\cos z+c$$

$$\Rightarrow \log|x|-\cos\left(\frac{y}{x}\right)=c$$

111. (c) If $\frac{d^2y}{dx^2}+2y=x^2$

$$\Rightarrow (D^2+2)y=x^2 \qquad \left[D\equiv\frac{d}{dx}\right]$$

Particular integral (P.I.) $=\frac{1}{D^2+2}.x^2$

$$=\frac{1}{2\left(1+\frac{D^2}{2}\right)}.\,x^2=\frac{1}{2}.\left(1+\frac{D^2}{2}\right)^{-1}.\,(x^2)$$

$$\because\ (1+D)^{-1}=1-D+D^2-D^3+......$$

$$\therefore\ \text{P.I.}=\frac{1}{2}.\left[1-\left(\frac{D^2}{2}\right)+\left(\frac{D^2}{2}\right)^2-.....\right](x^2)$$

$$\Rightarrow\ \text{P.I.}=\frac{1}{2}.\left[x^2-\frac{D^2}{2}(x^2)\right]$$

$$\Rightarrow\ \text{P.I.}=\frac{1}{2}.\left[x^2-1\right]$$

112. (a) If $(D^2+16)y=\cos 4x$

Here the auxiliary equation is $m^2+16=0$

$\Rightarrow m=\pm 4$

$\therefore$ Complementary function

$=(A\cos 4x+B\sin 4x)$

& Particular Integral (P.I.)

$$=\frac{1}{D^2+16}.\cos 4x$$

But $\frac{1}{D^2+a^2}\cos ax=\frac{x}{2a}\sin ax$

$$\therefore\ \text{P.I.}=\frac{x}{2\times 4}.\sin 4x=\frac{x}{8}\sin 4x$$

$\therefore$ Solution y = Complementary function + Particular Integral

$$\Rightarrow y=A\cos 4x+B\sin 4x+\frac{x}{8}\sin 4x$$

113. (c)

114. (d) Let us make one packet for each of the books on the same size. Now, 3 packets can be arranged in $P(3,3)=3!$ ways

5 large books can be arranged in 5! ways

4 medium size books can be arranged in 4! ways

3 small books can be arranged in 3! ways

$\therefore$ Required number of ways

$=3!\times 5!\times 4!\times 3!$ ways

115. (a) An identity relation is one in which every element of a set is related to itself only.

$\mathbf{a * b = a + b - ab}$

As in identity relation 'a' is related to 'a', so the correct option will be the one which gives the value of the relation = 'a'. So, equating a + b – ab = a, we get b(1 – a) = 0. Now putting the values of a, we find b and the option in which a = b, will be the answer.

For a = 0, b = 0, so the correct option.

For a = 1, b(1 – 1) = 0 ⇒ b can have multiple values.

For a = 2, b(1 – 2) = 0 ⇒ b = 0 but a = 2.

116. (a)

p	q	~ p	~ pvq	p → q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

117. (b) Let S be the sample space

$\because n(S) = 36$

Events

[sum greater than 3 but not exceeding 6]

= {(2, 2), (3, 1), (1, 3), (4, 1), (1, 4), (5, 1) (1, 5), (3, 2), (2, 3), (4, 2), (2,4), (3, 3)}

$\Rightarrow n(E) = 12$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{12}{36} = \dfrac{1}{3}$

118. (a) Let 'p' denote the probability of winning of team A whenever it plays

$\therefore\ p = \dfrac{2}{3}\ \ \&\ q = 1 - \dfrac{2}{3} = \dfrac{1}{3}$

Let X denotes the number of winning games out of 4 games i.e. n = 4

$\therefore$ The probability of r success

$P(X = r) = {}^{n}c_{r}\, p^{r} q^{n-r}$, r = 0, 1, 2, 3, 4

$\therefore$ Probability of winning more than half games = P(X > 2)

= P (X = 3) + P(X = 4)

$$= {}^{4}C_{3}\left(\frac{2}{3}\right)^{3}\cdot\left(\frac{1}{3}\right)^{4-3} + {}^{4}C_{4}\left(\frac{2}{3}\right)^{4}\cdot\left(\frac{1}{3}\right)^{4-4}$$

$$= \frac{4!}{3!1!}\cdot\frac{8}{27}\cdot\frac{1}{3} + \frac{4!}{4!0!}\cdot\frac{16}{81}$$

$$= \frac{32}{81} + \frac{16}{81} = \frac{48}{81} = \frac{16}{27}$$

119. (b) n = total number of ways = $2^5 = 32$

Since each answer can be true or false

& m = favourable number of ways

$= {}^{5}C_{4} + {}^{5}C_{5}$

$$= \frac{5!}{4!1!} + \frac{5!}{5!0!} = 5 + 1 = 6 \Rightarrow m = 6$$

Since to pass the quiz, student must give 4 or 5 true answers.

Hence, $p = \dfrac{m}{n} \Rightarrow p = \dfrac{6}{32} \Rightarrow p = \dfrac{3}{16}$

120. (a) Since f(x) is the probability density function of random variable X.

$$\therefore \int_{-\infty}^{\infty} f(x) = 1$$

Now we have

$$\int_{-\infty}^{\infty} Ke^{-|x|}dx = 1 \Rightarrow 2\int_{0}^{\infty} K\,.\,e^{-|x|}dx = 1$$

$$\Rightarrow 2\int_{0}^{\infty} K\,.\,e^{-x}dx = 1$$

$$\Rightarrow -2K\,.\left[e^{-x}\right]_{0}^{\infty} = 1 \Rightarrow 2K = 1$$

$$\Rightarrow K = \frac{1}{2}$$