8. Optics

8.1. At what distance f_1 from a biconvex lens must we place an object for the distance between the object and the real image to be minimal?

8.2. Two biconvex lenses a and b with the same radii of curvature are manufactured from glass samples with different refractive indices. How should we employ the graphs that represent the dependence of the distance f_2 between a lens and the image of an object on the distance f_1 between the lens and the object in order to determine the ratio of the refractive indices?

8.3. When taking a picture of a group of objects that are positioned at different distances from the camera, one must allow for the so-called depth of focus, or the limits of the greatest and the smallest distance between which the image is sharp for a given focus setting of the camera. Why is the depth of focus the greater the smaller the aperture setting?

8.4. A pinhole camera consists of a rectangular (hollow) glass prism whose front base and lateral faces are blackened and whose back base is covered with a photographic plate. A small circular section of the front base is left

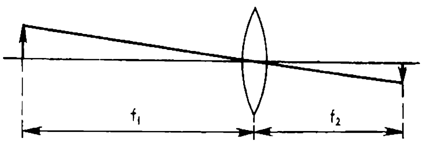

Fig. 8.1

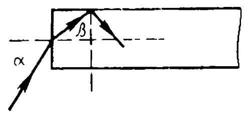

Fig. 8.5

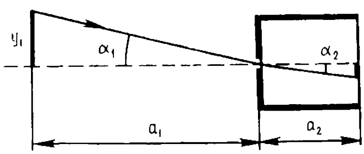

Fig. 8.4

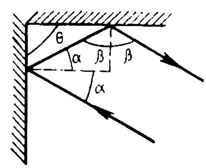

Fig. 8.6

unblackened, and through this "pinhole" the light enters the camera. The refractive index of the glass is n, the distance from the object to the camera is a_1, and the length of the camera is a_2. Determine the ratio of the size of the image, y_2, to the size of the object, y_1, assuming that $y_1 \ll a_1$.

8.5. Light falls on an end face of a glass rod at an angle α. What is the smallest refractive index that the glass may have so that the light after entering the rod cannot leave it through a lateral face irrespective of the values of α?

8.6. At what angle to each other must two flat mirrors be positioned for a beam of light incident on one of the mirrors at an arbitrary angle in a plane that is perpendicular to the mirror surface to be reflected from both mirrors in such a manner that the refracted beam is parallel to the incident beam? Is a prism suitable for this purpose?

8.7. An electric bulb is hanging above the center of a round table whose radius is R. At what height h must it be hung for the intensity of illumination at the edge of the table to be maximal?

8.8. A beam of light propagates through a medium *1* and falls onto another medium, *2*, at an angle α_1. After that it propagates in medium *2* at an angle α_2. The light's wavelength in medium *1* is λ_1. What wavelength has the light in medium *2*?

8.9. Two identical coherent sources of light, S_1 and S_2, separated by a distance a produce an interference pattern on a screen. The wavelength of the monochromatic light

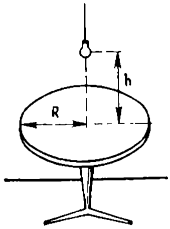

Fig. 8.7

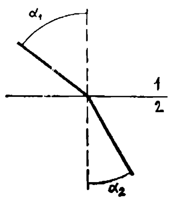

Fig. 8.8

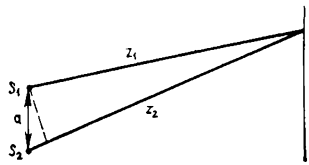

Fig. 8.9

emitted by the sources is λ. Determine the maximal number of interference fringes that can be observed assuming that the screen is infinitely large.

8.10. In an experiment that involves the observation of interference of light via two Fresnel mirrors, the source of light is positioned symmetrically in relation to both mirrors at a distance l from the boundary between them. How does the distance between the first interference fringes on a screen that is positioned far from the mirrors depend on the angle θ between the mirrors?

8.11. When there is interference of light waves emitted by two coherent sources, the geometric locus of points with the same difference in the phases of the oscillations that arrive at that point from the two sources constitutes a surface whose sections with the plane of the drawing are the curves ab and $a'b'$ shown in the figure. What is this surface?

8.12. A transparent dielectric is deposited in the form of a thin film on two substrates made of different dielectrics. Both films form geometrically identical wedge-like layers. The refractive index of the material of the film is n and those of the substrates are n_1 and n_2, with $n_1 < n < n_2$. Suppose that two light beams of similar spectral composition fall on the two systems at the same angle. In what respects do the resulting interference patterns differ?

8.13. An air wedge is illuminated by monochromatic light. The distance between the resulting interference fringes is a. How will the distance between the interference fringes change if the space between the plates that constitutes the wedge is filled with a transparent liquid?

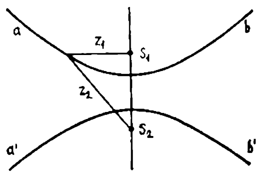

Fig. 8.11

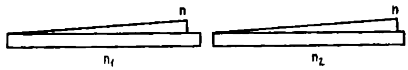

Fig. 8.12

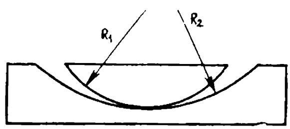

Fig. 8.14

8.14. A plano-convex lens with a radius of curvature R_1 is lying on a reflecting cylindrical surface whose radius of curvature is R_2. The lens is illuminated from above. What shape do the interference fringes have?

8.15. A plano-convex segment of a glass cylinder whose curvature radius is R is lying on a flat plate. A parallel beam of light falls on this segment from above. What shape will the interference fringes have and how will the distance between the fringes change as we move away from the straight line along which the segment touches the plate?

8.16. During observation of Newton rings, a small particle of unknown thickness a got between the lens and the plate. How can one determine the wavelength of monochromatic light incident from above on the lens using only graphical considerations? What scales along the vertical and horizontal axes are preferable?

8.17. On a reflecting substrate there lies a transparent plane-parallel plate that forms an angle α with the substrate. Thus a wedge-like film of air is formed. The substrate has a triangular ledge whose cross section is an isosceles triangle with angles θ at its base. The plate is illuminated with monochromatic light from above. Assuming that the angles α and θ are small, sketch the positions of the interference fringes. The size of the wavelength is shown in the figure.

8.18. In the observation of the interference pattern in an air wedge (Figure (a)) there sometimes appear interference fringes with distortions caused by the presence of a ledge or a dent on the substrate. Which of the two inter-

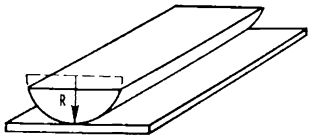

Fig. 8.15

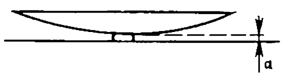

Fig. 8.16

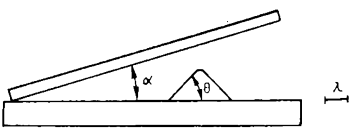

Fig. 8.17

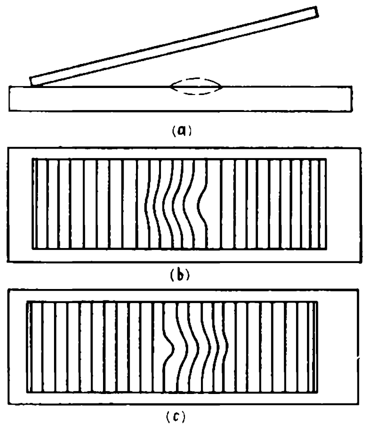

Fig. 8.18

ference patterns in Figures (b) and (c) corresponds to which defect?

8.19. Light from a distant source falls on a screen with a round hole. At a certain distance from this screen an-

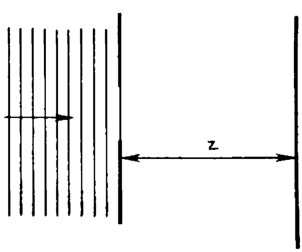

Fig. 8.19

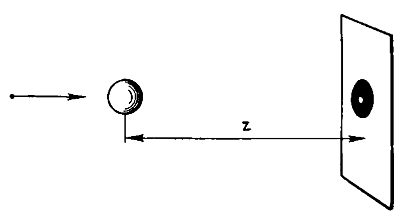

Fig. 8.20

other screen is placed, and it is on this screen that the diffraction pattern is observed. How will the intensity of illumination at the center of the second screen change if the distance between the screens is gradually increased, that is, does the intensity of illumination remain constant or does it monotonically decrease or does it vary periodically?

8.20. Light from a distant monochromatic source, which can be considered point-like, is incident on a small round opaque disk or sphere. A screen is positioned at a certain distance z from the object. This distance, z, is great if compared with the diameter of the object, so that the object covers only several Fresnel zones into which the plane wave can be partitioned. Can it be possible that under such conditions the geometric shadow on on the screen contains a bright spot in its center?

8.21. What maxima in the spectrum obtained through the use of a diffraction grating correspond to the line

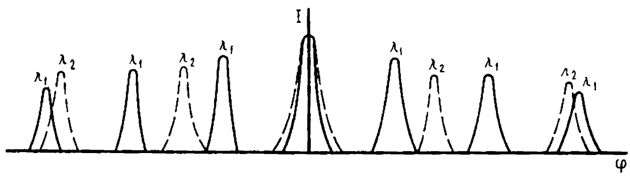

Fig. 8.21

with a longer wavelength and what maxima, to the line with a shorter wavelength? What approximately is the ratio of these wavelengths?

8.22. In a spectrum obtained through the use of a diffraction grating, a spectral line is obtained in the first order at an angle φ_1. Determine the highest order of the spectrum in which this line can be observed by means of the same diffraction grating if the light falls on the grating at right angles to the grating's surface.

8.23. Suppose the wavelength of a spectral line is measured via two diffraction gratings. The spectral maxima in the zeroth and first orders have the shape depicted in the figures. The scales used in both figures are the same. Which grating has a larger period and which, a higher resolving power? Estimate approximately the resolving power of each grating assuming that the natural line width and the Doppler line width are considerably smaller than the one obtained in experiments.

8.24. Suppose there are two diffraction gratings with spacings c_1 and c_2 and a total number of lines N_1 and N_2, respectively. Here $c_1 < c_2$ and $N_1 > N_2$, but the product

cN is the same for both gratings. Which of the two gratings has a greater maximal resolving power if the same spectral line is observed at normal incidence of light on the gratings?

8.25. A parallel beam of light falls at an angle θ on a flat diffraction grating with a spacing d. Determine the fundamental grating condition for the wavelength λ, the maximum order of the spectrum in which the appropriate spectral line can be observed, the maximum wavelength for which a line in the spectrum can be resolved, and the maximum dispersive power of the grating?

8.26. A phonograph record can be used as a reflecting diffraction grating. To obtain a clear diffraction pattern,

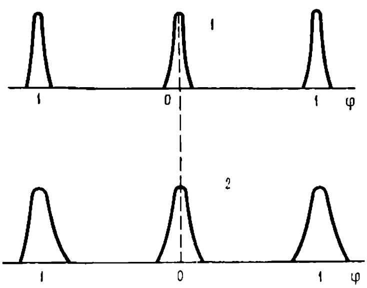

Fig. 8.23

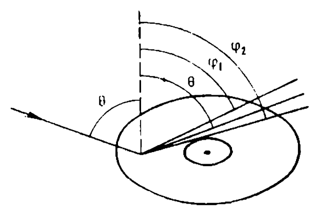

Fig. 8.26

one must direct the light at an angle that is as close to the grating angle to the surface of the record as possible. Why?

8.27. What minimal value can the Brewster angle have when light falls from air onto the surface of any dielectric?

8.28. When light is incident on a transparent dielectric at the Brewster angle ($\tan \alpha = n$), the reflected light proves to be completely polarized. Is the refracted light also completely polarized in this case?

8.29. Natural nonpolarized light is incident on a double-refracting crystal. The normals to the ordinary wave (o) and the extraordinary wave (e) are directed as shown in the figure. Find the ratio of the wavelengths of these waves.

8.30. A T-shaped pipe with blackened walls is filled with a turbid medium. Light falls onto one end of the pipe in the direction designated by *1*. As a result of scattering, a fraction of the light emerges from the pipe in the direction designated by *2*. Prove that this fraction is

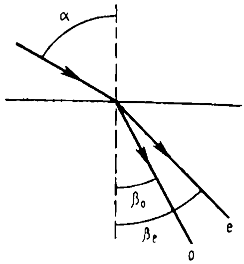

Fig. 8.29

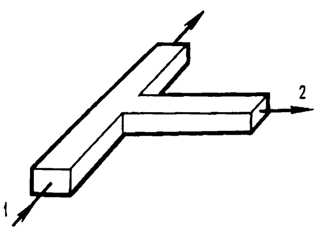

Fig. 8.30

polarized and determine the direction in which the electric field vector oscillates in this fraction.

8.31. Suppose that a ray of light falls on a flat boundary of a double-refracting crystal. In one case the crystal has been cut in such a manner that the wave surfaces of the ordinary and extraordinary rays have the form depicted in Figure (a), while in the other case it has been cut in such a manner that the corresponding wave surfaces have the form shown in Figure (b). How is the optic axis of the crystal directed in each case and is the crystal positive or negative?

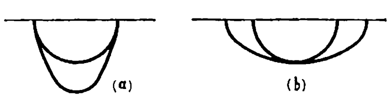

Fig. 8.31

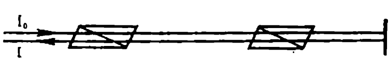

Fig. 8.32

8.32. Natural light with intensity I_0 passes through two Nicol prisms whose transmission planes are at an angle θ to each other. After the light has passed through the second prism, it falls on a mirror, is reflected by the mirror, and passes through the two Nicol prisms once more. What is the intensity I of the light that has travelled this path?

8.33. Polarized light passes through a transparent substance that is placed in a longitudinal magnetic field. The result is the so-called Faraday effect (rotation of the polarization plane in a magnetic field). After passing through the substance (and magnetic field), the light is reflected by a mirror and travels in the opposite direction, whereby it travels through the magnetic field once more but in the opposite direction. Will the angle of rotation

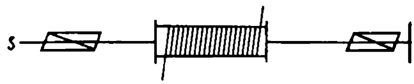

Fig. 8.33

of the polarization plane be doubled or will it cancel itself out?

8.34. When an electric field is applied to a capacitor that is submerged in nitrobenzene, artificial anisotropy emerges in the medium and the nitrobenzene behaves like a

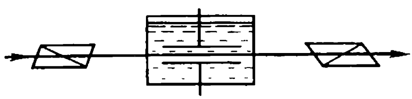

Fig. 8.34

double-refracting crystal in which the reftactive index of the extraordinary ray, n_e, is greater than that of the ordinary wave, n_0. The phenomenon, known as the Kerr effect, can be observed via two crossed Nicol prisms. Does the observed pattern change if the direction of the electric field is reversed?

8.35. When a source of light moves toward the observer, the optical Doppler effect manifests itself. The curves in the figure depict the dependence of the perceptible frequency of the light on the speed of the source of light, with one curve corresponding to the results predicted by classical theory and the other. to the results predicted by the theory of relativity. The ratio of the speed of the source to the speed of light is laid off on the horizontal axis, while the ratio of the perceptible frequency to the frequency of the light emitted by the source (i.e. of a fixed source) is laid off on the vertical axis. Which curve corresponds to which theory?

8.36. To determine the directional velocity of the ions that move in an electric field in a plasma, one commonly measures the wavelength of the waves emitted by the excited ions. The measurements are carried out in two directions, counter to the direction of motion of the ions and "in pursuit" of the ions. The measured wavelengths are λ_1 and λ_2, respectively. Can we employ the classical formulas of the Doppler effect or must we use the relativistic formulas? The ion velocities range from 10^4 to 10^5 m/s.

8.37. The figure depicts the same spectral line emitted by a gas at different temperatures. The wavelength is laid off on the horizontal axis, while the ratio of the inten-

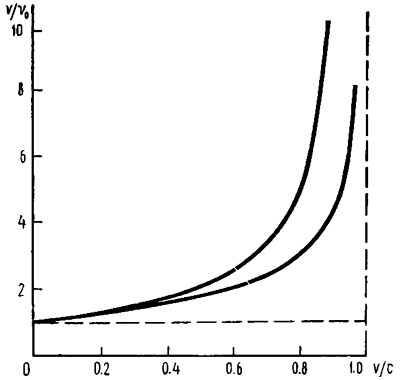

Fig. 8.35

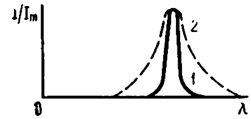

Fig. 8.37

sity at a given wavelength to the maximal intensity at a given temperature is laid off on the vertical axis. Which curve corresponds to a higher temperature?

8.38. An electric current flows through a rarefied gas in a tube *1* (Figure (a)). The radiation emitted by the excited positive ions is analyzed in the transverse direction by a spectrograph *2*. The wavelength distribution of the intensity of the radiation for one spectral line is shown in Figure (b). Can analyzing this distribution yield the temperature of the ions?

8.39. Two objects having the same shape and size but different absorption coefficients (immisivities) are heated to the same temperature and placed in a vacuum. As a result of emission of radiation the objects cool off. The curves in the figure show the change in temperature in the process of cooling. The cooling-off time from the moment the objects were placed in the vacuum is laid off on the horizontal axis, while the temperature of the objects is laid off on the vertical axis. Which curve character-

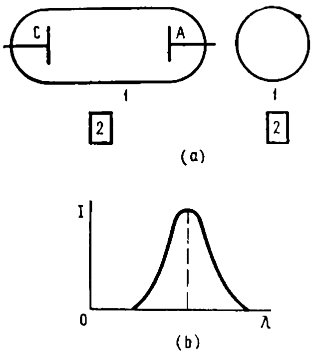

Fig. 8.38

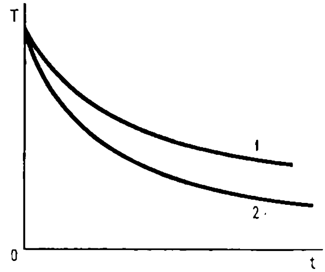

Fig. 8.39

izes the object with a higher absorption coefficient and which, with a lower absorption coefficient?

8.40. An ideal gas is placed inside a closed isolated volume. The concentration of the molecules of the gas is n. At what temperature will the volume density of the kinetic energy of translational molecular motion in the gas be equal to the volume density of the energy of blackbody (electromagnetic) radiation? Illustrate the result with numerical examples.

8.41. Two separate segments of equal area are isolated in the energy distribution of blackbody radiation. Are

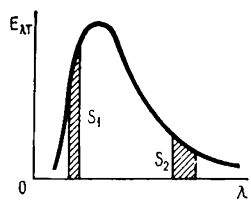

Fig. 8.41

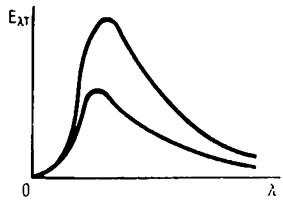

Fig. 8.42

the emissive powers over the respective wavelength intervals the same? What about the number of emitted photons in each segment?

8.42. A student has sketched the curves representing the energy distribution in the emission spectrum of blackbody radiation for two temperatures as shown in the figure. What mistake did the student make?

8.43. Determine the volume density of the energy of blackbody radiation over the frequency range from ν_1 to ν_2. The radiation function is laid off on the vertical axis.

8.44. The figure shows two curves: one corresponding to the energy distribution of blackbody radiation at a certain temperature obtained from theoretical assumptions

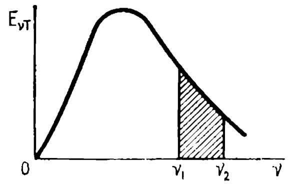

Fig. 8.43

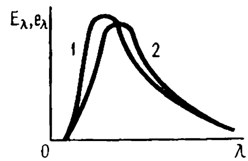

Fig. 8.44

(curve *1*), and the other corresponding to the energy distribution of the radiation emitted by a certain object that has been heated to the same temperature (curve *2*). Why can we be sure that the experimental curve does not give a true picture?

8.45. Curve *1* in the figure depicts the energy distribution in the emission spectrum of a black body. Curve *2* represents, in schematic form, the energy distribution in the emission spectrum of a certain object that has been heated to the same temperature as the black body. Curve *2* consists of three segments: on the segments ranging from $\lambda = 0$ to λ_1 and from λ_2 to $\lambda = \infty$ all ordinates of curve *2* are one-half the respective ordinates of curve *1*, while on the segment from λ_1 to λ_2 the value of e_λ remains constant. Sketch the distribution of the absorption coefficient (immisivity) over the wavelengths for the object in question.

8.46. The radiation emitted by a black body can be represented either by the energy distribution over the wavelengths (Figure (a)) or by the energy distribution over the frequencies (Figure (b)). In the first case the wavelength at which the black body emits a maximum amount of radiation is λ_m, while in the second the frequency at which

the black body emits maximum amount of radiation is ν_m. Is it true that at a fixed temperature the quantities λ_m and ν_m are related through the formula $\nu_m = c/\lambda_m$?
8.47. Represent the volume density of the energy of blackbody radiation in the form of a distribution function for the number of quanta in the energy of one quantum.

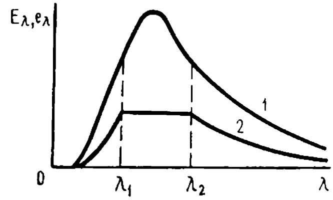

Fig. 8.45

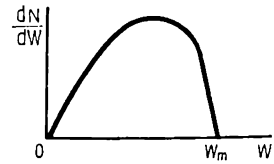

Fig. 8.51

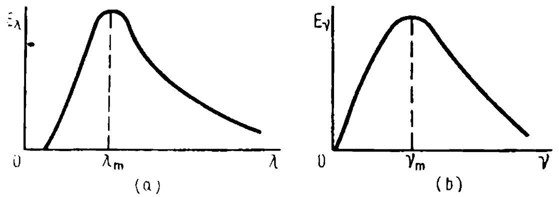

Fig. 8.46

8.48. How does the volume specific heat capacity of the vacuum depend on temperature?
8.49. According to the electromagnetic theory of light, the light incident on a surface always exerts a pressure on that surface equal to

$$p = \frac{I}{c}(1+R), \tag{8.49.1}$$

where I is the intensity of the light, that is, the light energy arriving every second at a unit area of the surface, and R is the reflection coefficient. Can the origin of this pressure be explained in the same manner as is done in the kinetic theory of gases, where the pressure of a gas on the wall of a vessel is interpreted as transfer of momentum from each particle to the wall?
8.50. Are there any practical means by which one can obtain a beam of parallel rays of light in the mathematical sense (using the terminology of wave optics, a stream of strictly plane waves)?

8.51. The energy distribution function for photoelectrons has the form shown in the figure. What determines the maximal energy of the photoelectrons?
8.52. In the Lukirskii-Prilezhaev experiments (also conducted independently by R. A. Millikan), the dependence of the stopping potential U_{stop}, that is, the potential needed to stop the photocurrent in a photocell and the associated electric circuit, on the frequency of the light incident on the surface of the photocell is depicted by straight lines. How to find the Planck constant knowing the

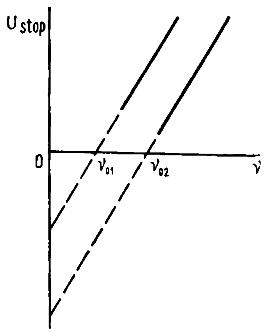

Fig. 8.52

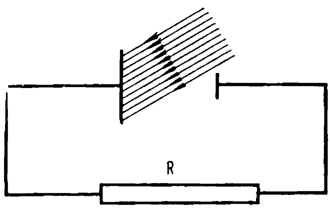

Fig. 8.53

slope of these straight lines? In what respect do the parameters that characterize these two straight lines differ?
8.53. Two electrodes placed in a vacuum at a certain distance from each other are connected electrically by a resistor. One electrode is illuminated with light from a source whose spectrum contains radiation with a wavelength λ that satisfies the condition

$$hc/\lambda > p,$$

where p is the work function of electrons leaving the metal of the illuminated electrode. Will there be any current in this circuit?
8.54. A photocathode can be illuminated by the light from two sources, each of which emits monochromatic radiation. The sources are positioned at equal distances from the photocathode. The dependence of the photocurrent on the voltage between the cathode and the anode is depicted by curve *1* for one source and by curve *2* for the other. In what respect do these sources differ?

8.55. Two photocathodes are illuminated by the light emitted by a single source. The dependence of the photocurrent on the voltage between the cathode and the anode is depicted by curve *1* for one cathode and by curve *2*

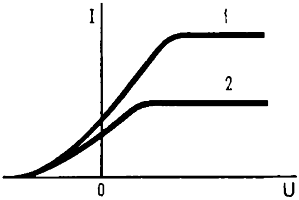

Fig. 8.54

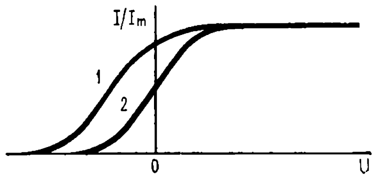

Fig. 8.55

for the other? What photocathode has a higher work function?

8.56. The stopping potential applied between a photocathode and the respective anode is such that the fastest photoelectrons can fly only one-half of the distance between the cathode and the anode. Will the electrons be able to reach the anode if the distance between the cathode and the anode is reduced by half but the voltage is kept constant?

8.57. In one case of Compton scattering a photon flies at an angle θ to the initial direction of the incident photons,

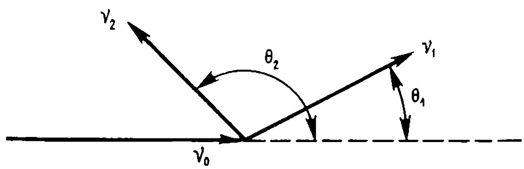

Fig. 8.57

and in other case it flies at an angle θ_2. In which case is the wavelength of the radiation after scattering greater, and in which case does the electron participating in the interaction receive a greater portion of energy?

8. Optics

8.1. If we introduce the notation $l = f_1 + f_2$ in the lens formula

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$$

and perform simple manipulations, we get

$$l = \frac{f_1^2}{(f_1 - F)}.$$

To determine the minimum of l, we nullify the derivative

$$\frac{dl}{df_1} = \frac{2f_1(f_1 - F) - f_1^2}{(f_1 - F)^2},$$

whence $f_1 = 2F$.

8.2. The lens formula that allows for the parameters of the lens is

$$\frac{1}{f_1} + \frac{1}{f_2} = (n-1)\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = \frac{1}{F} \qquad (8.2.1)$$

(the sign of the radius of curvature is determined by the direction from the surface and to the center of curvature). The ratio of the principal focal lengths is

$$\frac{F_a}{F_b} = \frac{n_b - 1}{n_a - 1}, \qquad (8.2.2)$$

where we have allowed for the fact that the radii of curvature of both lenses are the same. Formula (8.2.1) can be transformed thus:

$$f_2 = \frac{f_1 F}{f_1 - F}.$$

On the curve representing the f_2 vs. f_1 dependence, the value of F is determined by the position of the vertical asymptote of each curve. However, a more exact value can be obtained by drawing a straight line that passes through the origin at an angle of 45° to the axes. In this case the coordinates of the points of intersection of this straight line with the curves yield $f_2 = f_1 = 2F$ for both lenses, while the ratios of these coordinates determine, via formula (8.2.2), the ratio of $n_b - 1$ to $n_a - 1$.

8.3. The smaller the aperture, the lower the optical distortions caused by the large width of the beam of light incident on the lenses of the objective. If the aperture is very small, the optical properties of the camera closely resemble those of a pinhole camera, whose aperture, in terms of geometrical optics, can be as small as desired and

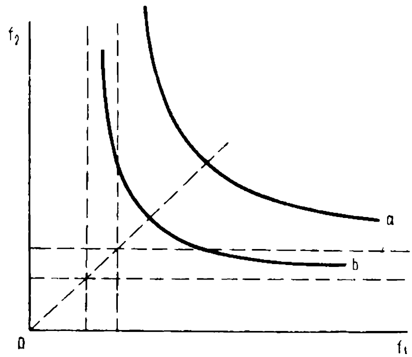

Fig. 8.2

whose depth of focus extends from zero to infinity. Actually, however, diffraction imposes certain restrictions on this ideal case. The limiting value of the diameter of the aperture, D, is determined by the wavelength of the light and by the distance from the aperture to the photographic plate. Theoretical considerations suggest that D must be close to the value for which only one Fresnel zone fits into the aperture:*

$$D \approx 4\sqrt{\lambda f}.$$

For instance, at $\lambda \approx 0.5$ µm and $f \approx 5$ cm, the diameter of the aperture is approximately 0.6 mm. Note that in photography the size of the aperture is characterized by a quantity known as the aperture ratio, or the ratio of the diameter of the aperture to the focal length. Usually the aperture ratio is marked by a fraction whose numerator is unity (1/4.5, 1/5.6, 1/8, 1/11). In the example we are discussing here the aperture ratio is equal to 1/80. In cameras the smallest aperture ratio is practically never less than 1/16, so that diffraction effects play no role in the present problem and need not be taken into account.

* According to Rayleigh, the sharpest focus in a pinhole camera is achieved when the radius of the aperture is 0.95 of the radius of the zeroth Fresnel zone.

8.4. The solution can be found from simple trigonometric reasoning under common assumptions and approximations:

$$\sin\alpha_1 \approx \tan\alpha_1 \approx \alpha_1, \quad \sin\alpha_2 \approx \tan\alpha_2 \approx \alpha_2,$$

$$\alpha_1/\alpha_2 \approx n, \quad y_1/a_1 \approx \alpha_1, \quad y_2/a_2 \approx \alpha_2.$$

Whence,

$$\frac{y_2}{y_1} = \frac{a_2}{a_1 n}.$$

8.5. A ray that enters the rod at an angle α, travels in the glass after being refracted at an angle β given by Snell's law:

$$\sin\beta = n^{-1}\sin\alpha. \qquad (8.5.1)$$

The ray falls on the lateral face of the rod at an angle that is not smaller than the critical angle. From the figure accompanying the problem it follows that this angle is $\pi/2 - \beta$. According to the critical angle condition,

$$\sin(\pi/2 - \beta) = \cos\beta \geqslant n^{-1}. \qquad (8.5.2)$$

The maximal value of β at $\alpha = \pi/2$, according to (8.5.1), obeys the condition

$$\sin\beta = 1/n. \qquad (8.5.3)$$

Squaring (8.5.2) and (8.5.3) and adding the squares, we get

$$1 \geqslant 2/n^2,$$

whence

$$n \geqslant \sqrt{2}.$$

The phenomenon of light "trapping" in a glass rod is widely used in fiber optics. If the attenuation of light in the glass is low, the ray can travel over great distances. Bundles of such rods (or fibers) form cables over which data can be transmitted with a high accuracy and a low level of noise. Internal organs of human beings can be illuminated with the light transmitted by such fibers, which at present is widely used in medical practice for diagnostic purposes.

8.6. The figure accompanying the problem shows that after reflection from the first mirror the beam changes its direction by an angle of 2α, while after reflection from the second mirror the beam changes its direction by an additional angle of 2β. For the refracted beam to travel in the direction opposite to the direction of the incident beam, the sum $2\alpha + 2\beta$ must be equal to π, or $\alpha + \beta = \pi/2$.

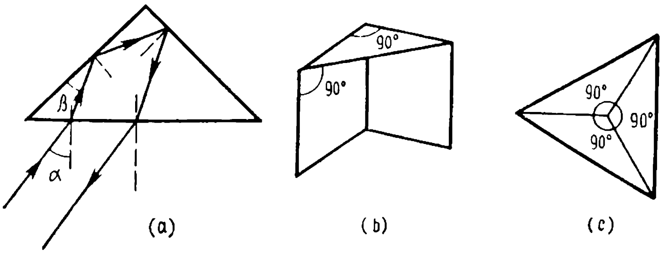

Fig. 8.6

In this case the angle between the normals to the mirrors is

$$\theta = \pi - (\alpha + \beta) = \pi/2.$$

The angle between the mirrors must also be equal to $\pi/2$.

If instead of the two mirrors we take a prism (see Figure (a) accompanying the answer), then a beam incident on the base of the prism at an angle α will enter the prism at an angle β determined by Snell's law. For the refracted beam to leave the prism in the direction opposite to the one of the incident beam after undergoing total internal reflection from the lateral surfaces of the prism, the beam must fall on the base of the prism (after it has been reflected by the second lateral surface) at an angle β. Figure (a) accompanying the answer shows that the beam travels the same path as in the case of two mirrors, whereby the angle at the apex of the prism must be equal to $\pi/2$. We see that a prism may also be used to reverse a beam. For the beam to retain its energy after traveling through the prism practically for all angles of incidence, the lateral surfaces of the prism must be metalized. If three flat mirrors are positioned at right angles, as shown in Figure (b) accompanying the answer, it can be demonstrated that the beam of light may be oriented with respect to the first mirror (on which it is incident) in an arbitrary manner and yet the refracted beam will always be parallel to the incident one. Instead of three mirrors we can use a glass tetrahedron with a right trihedral angle at the apex and identical metalized lateral surfaces in the form of right isosceles triangles (see Figure (c) accompanying the answer). A beam incident on the base of the tetrahedron is reflected by the metalized surfaces and leaves the tetrahedron through the base in the direction opposite but parallel to the one of the incident beam. An optical device of this type is known as a corner reflector.

8.7. The intensity of illumination of a surface that is r distant from the source and forms an angle α with the incident ray is

$$E = \frac{I}{r^2} \sin \alpha, \tag{8.7.1}$$

where I is the intensity of the source. At the edge of the table, according to (8.7.1),

$$E = \frac{Ih}{(R^2+h^2)^{3/2}}.$$

To find the maximum of E we must nullify the derivative:

$$\frac{dE}{dh} = I\,\frac{(R^2+h^2)^{3/2} - 3h^2\,(R^2+h^2)^{1/2}}{(R^2+h^2)^3} = 0,$$

whence

$$h = R/\sqrt{2}.$$

8.8. The ratio of the sines of the angles is equal to the ratio of the speeds of light in the media:

$$\sin \alpha_1/\sin \alpha_2 = c_1/c_2.$$

The ratio of the wavelengths is equal to the ratio of the speeds of light:

$$\lambda_1/\lambda_2 = c_1/c_2.$$

Therefore

$$\lambda_2 = \frac{\sin \alpha_2}{\sin \alpha_1}\,\lambda_1.$$

8.9. The optical path difference, which determines the interference pattern, is $|z_2 - z_1|/\lambda$. Since $|z_2 - z_1|$ cannot be greater than a, the maximal possible number of fringes on each side from the middle of the screen (i.e. for $z_2 > z_1$ and for $z_2 < z_1$) is equal to the ratio a/λ, while the total number of fringes is $2a/\lambda$. Actually the number of fringes that can be observed is considerably lower, since at $z_2 - z_1 = a$ the interference fringes must lie in the plane in which the sources lie.

8.10. As the source of light is positioned symmetrically in relation to the mirrors, its virtual images appear at equal distances d from the source and, as the figure accompanying the answer shows,

$$d = 2l \cos(\theta/2).$$

The source and its virtual images lie at the vertices of an isosceles triangle. The distance between the virtual images is

$$a = 2d \sin(\theta/2),$$

$$\text{or} \quad a \approx 2l \sin \theta.$$

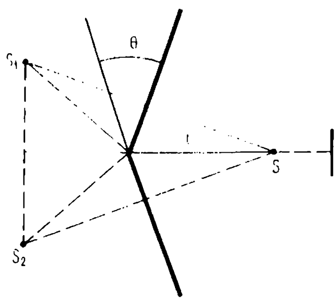

Fig. 8.10

The first interference fringes on a screen that is L distant from the mirrors are separated by a distance of

$$h = \lambda L/a,$$

and, hence, the smaller the value of θ, the greater the distance h.

8.11. Since equal phase differences correspond to equal optical path differences, we can write

$$(z_2 - z_1)/\lambda = \text{const}, \quad \text{or} \quad z_2 - z_1 = n\lambda,$$

where n is an integer. A surface whose points possess the property that the difference in the distances from any point to two fixed points (the foci) is a constant, constitutes a hyperboloid. The section of this hyperboloid by any plane containing these sources results in two branches of a hyperbola. The sections of the hyperboloid by planes that are perpendicular to the straight line which passes through the middle of the segment connecting the sources are also branches of hyperbolas. For this reason, the observed interference fringes have the form of hyperbolas.

8.12. When light is reflected from the upper boundary of each film, the phase of the wave changes to the opposite or, as it is usually said, a half-wave is lost. The light that passes through the film is reflected by the substrate, which in one case has a refractive index greater than that of the film and in the other, smaller than that of the film. When

$n_2 > n$, a new change in the phase of the reflected wave to the opposite one occurs, while when $n_1 < n$, the phase of the reflected wave is retained. For this reason, the places on one film where light is observed correspond to the dark places on the other film, and vice versa.

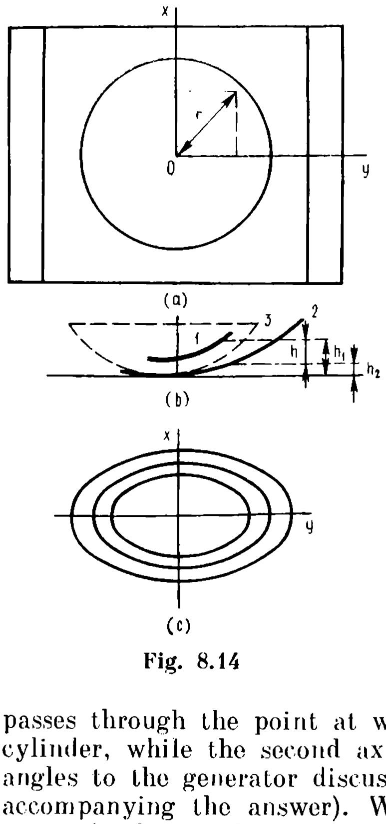

Fig. 8.14

8.13. The difference between neighboring interference fringes in air is determined by the relationship

$$a_0 = \lambda_0/2 \tan \alpha,$$

while for a liquid this relationship is

$$a = \lambda/2 \tan \alpha.$$

Since $\lambda = \lambda_0/n$, we can write $a = a_0/n$.

8.14. Interference is caused by the difference in paths of the light rays that forms in the space between the lens and the cylinder. The interference fringes constitute bands of equal width.

Let us introduce a system of coordinates. One axis, the x axis, is directed along the generator of the cylinder that passes through the point at which the lens touches the cylinder, while the second axis, the y axis, is at right angles to the generator discussed above (see Figure (a) accompanying the answer). We draw a plane that is perpendicular to the x axis and passes at a distance y from the origin. Figure (b) shows the section of the lens by the plane (curve *1*) and the section of the cylinder by the plane (curve *2*). The same figure demonstrates the section of the lens by a plane that is perpendicular to the x axis and intersects the lens along its diameter (dashed curve *3*). From Figure (b) it also follows that the gap between the lens and the cylinder is

$$h = h_1 - h_2 = \frac{r^2}{2R_1} - \frac{y^2}{2R_2} = \frac{x^2+y^2}{2R_1} - \frac{y^2}{2R_2}.$$

Here, as usual, we assume the following approximations to be valid:

$$r = \sqrt{2R_1h}, \quad y = \sqrt{2R_2h}.$$

After carrying out the appropriate transformations we get

$$\frac{x^2}{2R_1h} + \frac{y^2}{2h}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 1. \qquad (8.14.1)$$

If we introduce the notation $a^2 = 2R_1h$ and $b^2 = 2hR_1R_2/(R_2 - R_1)$, then (8.14.1) assumes the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

The interference fringes have the shape of ellipses (see Figure (c) accompanying the answer) in which h is a parameter. In reflected light, $h = (2k + 1)/\lambda$ (with $k = 0, 1, 2, 3, \ldots$) for bright bands and $h = k\lambda$ for dark.

8.15. The section of the cylinder segment by a plane parallel to the plane of the drawing is everywhere the same. For this reason, all points that have the same path difference for the ray reflected from the lower surface of the cylinder and the ray reflected from the upper surface of the plate lie at the same distance from the cylinder's generator that touches the plate, with the result that the interference fringes are in the form of straight lines parallel to the generator. The method of determining the distances between the sequential fringes closely resembles the method of determining the radii of Newton rings. The distances from the generator that touches the plate satisfy the same conditions as the radii of Newton rings do, namely,

$$h = \sqrt{R\lambda \frac{k+1}{2}}$$

for bright bands in reflected light and dark bands in transmitted light, and

$$h = \sqrt{R\lambda k}$$

for dark bands in reflected light and bright bands in transmitted light. As we move away from the generator, the distances between neighboring bands become smaller, just as the radii of Newton rings do.

8.16. The width of the air gap between the lens and the plate is the sum of the thickness of the lens section and the particle thickness:

$$h = h_0 + a = \frac{r^2}{2R} + a,$$

where r is the radius of the ring being observed. A bright ring whose number is k is observed at

$$h = \frac{2k+1}{2}\,\frac{\lambda}{2}.$$

Thus,

$$r^2 = \frac{2k+1}{2}\lambda R - 2Ra.$$

If the numbers of sequential rings are laid off on the horizontal axis and the square of the radii of the corresponding rings, on the vertical axis, we obtain a straight line (see the figure accompanying the answer) whose slope is equal to the ratio of the difference of squares of radii of two neighboring rings to the product λR, that is, $(r_k^2 - r_{k-1}^2)/\lambda R$. Knowing R, we can find λ. Note that in this method there is no difference between bright and dark rings, and knowing the exact number of a ring is not necessary. For this reason, in the figure accompanying the answer we have assigned a number k to an arbitrary ring, while the numbers $k - 1$ and $k + 1$ are assigned to the neighboring rings.

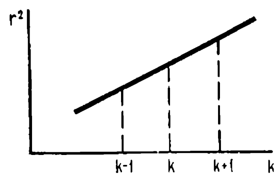

Fig. 8.16

8.17. To construct the interference fringes, we draw a number of straight lines parallel to the plate in such a way that the distances between them along the vertical line are equal to one-half of the wavelength. The points at which these straight lines intersect the substrate (including the surface of the ledge) determine the position of the interference fringes of equal width. Analyzing the position of the fringes obtained here, one can establish that from the wider side of the wedge (in the figure accompanying the problem, on the right) the distance between the fringes, or bands, is smaller (for any value of θ)

than over the flat sections of the substrate. The distance between the fringes from the narrow side of the wedge (on the left) can be either smaller or greater than the distance over the flat sections depending on the relationship between θ and α. For $\theta > \alpha$ (see Figure (a) accompanying the answer),

$$\theta = \alpha + \beta.$$

The left side of the ledge acts as a substrate and forms an angle β with the plate, that is, a wedge. If $\theta > 2\alpha$, then $\beta > \alpha$ and the distance between the fringes is smaller than that between the fringes over the flat section of the plate. This case is depicted in Figure (b) and corresponds to the case depicted in the figure accompanying the problem. But if $\theta < 2\alpha$, we have $\beta < \alpha$ and the fringes above the left side of the wedge are separated by a distance greater than that separating the fringes over the flat section of the plate. For $\theta < \alpha$ (see Figure (c) accompanying the answer), the left side of the wedge also acts as a substrate and forms a wedge with an angle $\beta < \alpha$ with the plate. In this case, too, the distance between the fringes is greater than that between the fringes over the flat section of the plate.

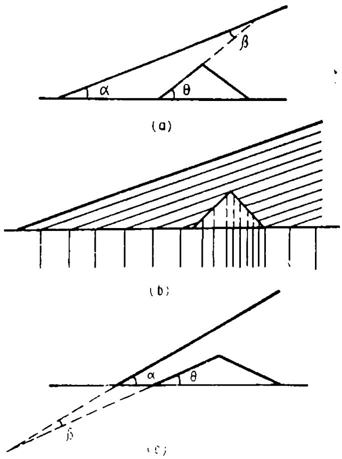

Fig. 8.17

8.18. The interference fringes in the wedge constitute bands of equal width. Ledges diminish, while dents increase the width of the air gap where the path difference of rays is formed. For this reason, at the points of a ledge the path difference is the same as at the points of the wedge closer to the narrow part of the gap, while at the points of a dent the difference is the same as at points closer to the wide part of the gap. For this reason, the interference pattern depicted in Figure (b) accompanying the problem corresponds to a ledge, while that depicted in Figure (c) corresponds to a dent.

8.19. The intensity of illumination at the center of the second screen is determined by the number of Fresnel zones into which the section of the wave surface limited by the hole in the first screen can be partitioned. If this number is not large and is even, the light is practically absent from the center, while if the number is odd, light is observed at the center. If a is the diameter of the hole, λ is the wavelength of the incident light, and z is the distance between the screens, the number of Fresnel zones is determined by the expression

$$k = a^2/4\lambda z.$$

As the distance between the screens is increased, the number of zones assumes alternately odd and even values, and this is accompanied by an increase or a decrease in the illumination at the center of the diffraction pattern. Since the number of zones continuously decreases as z gets larger and larger, the limit distance is the one at which k becomes equal to unity, that is,

$$z = a^2/4\lambda.$$

At a distance greater than this value, the intensity decreases monotonically, and for $z \gg a^2/4\lambda$ the intensity changes in inverse proportion to z^2, that is, just like for a point source.

8.20. When the central Fresnel zone and several neighboring zones are screened, the light intensity at the center of the geometric shadow is exactly the same as if one-half of the first nonscreened zones was acting. The calculation is carried out in the same manner as when there is no obstacle, the only difference being that the calculation of the overall action of the Fresnel zones starts not from the zeroth (or central) zone but from the first nonscreened zone. Therefore, a bright spot is always observed at the center of the screen irrespective of the distance to the obstacle or of the wavelength of the light wave (the only requirement is that the number of zones screened by the obstacle be moderate).

A theoretical description of the formation of a bright spot at the center of the geometric shadow was first carried out by Poisson, who used it as an objection against the wave theory of light, since he assumed that such a spot could simply not exist. But an experiment carried out by Arago proved without doubt that such a spot does indeed exist. Actually, this spot was discovered roughly a hundred years earlier by Maraldi. Curiously enough, the spot was later named the Poisson spot.

8.21. The maximum condition in the spectrum of a diffraction grating is

$$c \sin \varphi = k\lambda.$$

Longer wavelengths correspond to larger angles. The figure accompanying the question shows that the position of the second-order maximum of the line λ_2 is close to that of the third-order maximum of the line λ_1. Therefore, $c \sin \varphi = 2\lambda_2 \approx \lambda_3$. Whence, $\lambda_2/\lambda_1 \approx 1.5$.

8.22. The condition for a first-order diffraction maximum to occur is

$$c \sin \varphi_1 = \lambda.$$

For the highest-order maximum we have $c \sin \varphi_m = k_m\lambda$, whence

$$k_m = \frac{\sin \varphi_m}{\sin \varphi_1}.$$

Since the value of $\sin \varphi_m$ cannot exceed unity,

$$k_m \leqslant \frac{1}{\sin \varphi_1}. \qquad (8.22.1)$$

If k_m contains both an integral part and a fractional part, the latter must be discarded irrespective of its value. For instance, if in the first order the line is observed at an angle of 8.36°, formula (8.22.1) yields $k_m \approx 6.88$. The maximal order, therefore, is $k_m = 6$.

8.23. The angles that determine the position of the first maximum for both gratings are the same, which means that the gratings spacings are the same. To estimate the resolving power, we must find the ratio of the wavelength at the maximum of a line to the difference between this wavelength and the wavelength corresponding to a neighboring minimum. For small angles the sine function may be replaced with the angles, so that

$$\varphi_{max} \approx \lambda_{max}, \quad \varphi_{min} \approx \lambda_{min}.$$

The resolving power,

$$\delta = \frac{\lambda}{\lambda_{max} - \lambda_{min}},$$

is equal approximately to 25 for grating *1* and 10 for grating *2*.

8.24. The resolving power of a grating is

$$\delta = kN, \tag{8.24.1}$$

where N is the general number of lines (or grooves), and k is the order of the spectrum. The maximal resolving power is determined by the maximum possible order of the spectrum:

$$k_{max} = c/\lambda. \tag{8.24.2}$$

Substituting (8.24.2) into (8.24.1) yields

$$\delta = cN/\lambda. \tag{8.24.3}$$

Since the product cN is the same for both gratings and the observed spectral lines are the same, the resolving power of the two gratings must also be the same. A small difference in resolving powers determined via (8.24.3) can be caused by the fact that the exact form of (8.24.2) must be

$$k_{max} \leqslant c/\lambda, \tag{8.24.4}$$

whence

$$\delta_{max} \leqslant cN/\lambda. \tag{8.24.5}$$

Since only the integral parts are taken in (8.24.4) and (8.24.5), the values of δ_{max} of the two gratings may differ somewhat.

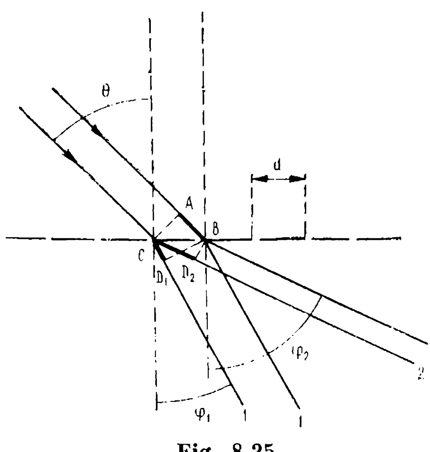

Fig. 8.25

8.25. The path difference between the rays from two neighboring slits is determined, as illustrated by the figure accompanying the answer, for direction *1* by the difference between the segments AB and CD_1 and for direction 2, by the difference between AB and CD_2. Accordingly, the path differences for directions 1 and 2 are

$$\delta_1 = d(\sin\theta - \sin\varphi_1) \quad \text{and} \quad \delta_2 = d(\sin\varphi_2 - \sin\theta),$$

or

$$\delta_1 = 2d\cos\left(\frac{\theta+\varphi_1}{2}\right)\sin\left(\frac{\theta-\varphi_1}{2}\right),$$

$$\delta_2 = 2d\cos\left(\frac{\theta+\varphi_2}{2}\right)\sin\left(\frac{\varphi_1-\theta}{2}\right).$$

Thus, the diffraction maximum conditions can be written thus:

$$2d\cos\left(\frac{\theta+\varphi_1}{2}\right)\sin\left(\frac{\theta-\varphi_1}{2}\right) = k\lambda,$$

$$2d\cos\left(\frac{\varphi_2+\theta}{2}\right)\sin\left(\frac{\varphi_2-\theta}{2}\right) = k\lambda.$$

In the first approximation we can assume that $\theta + \varphi_1 \approx \varphi_2 + \theta \approx 2\theta$. Hence,

$$d\cos\theta \times (\theta - \varphi_1) \approx k\lambda, \quad d\cos\theta \times (\varphi_2 - \theta) \approx k\lambda. \tag{8.25.1}$$

This formulas have the same form as for the case of normal incidence of light on a grating with spacing $d\cos\theta$.

The maximum order of the spectrum in which the wavelength λ is observed is

$$k = d\cos\theta/\lambda,$$

while the longest wavelength ($k = 1$) is

$$\lambda = d\cos\theta.$$

The dispersive power can be conveniently expressed in terms of the angle with respect to the direction of the zeroth maximum, $\theta - \varphi_1$ and $\varphi_2 - \theta$. If by ψ we denote these differences, which are close in absolute value, we find that

$$\frac{d\psi}{d\lambda} \approx \frac{k}{d\cos\theta\cos\psi}.$$

At angles θ close to 90°, the dispersive power of the grating may be considerably higher than for normal incidence of light on the grating. However, the maximum dispersive power is $1/\lambda \cos\psi$, just as for normal incidence.

8.26. If we assume that the diffracted rays are reflected in the plane of the grating just like in a mirror (see the figure accompanying the answer), we arrive at a pattern similar to the one obtained in the answer to Problem 8.25.

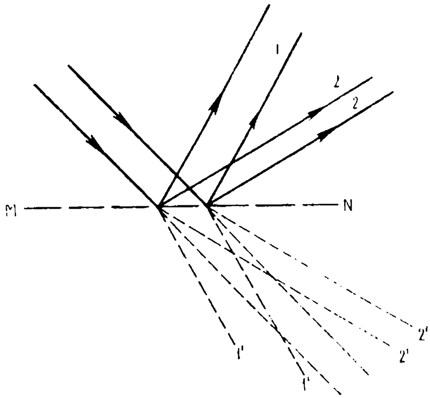

Fig. 8.26

Just like in the case of oblique incidence of the rays on the grating, the dispersive power increases with a coefficient of $(\cos\theta)^{-1}$.

8.27. According to Brewster's law, when light is reflected from a dielectric, complete polarization occurs when the tangent of the angle of incidence is equal to the refractive index of the medium reflecting the light. Since when light propagates in air and falls on a dielectric the refractive index is always greater than unity, we have $\tan\alpha > 1$, or $\alpha > 45°$.

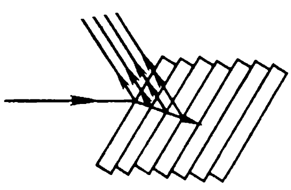

Fig. 8.28

8.28. Refracted light is polarized only partially. Light that is practically completely polarized can be obtained if one uses a stack (see the figure accompanying the answer) of parallel plates whose surfaces are oriented at the Brewster angle to the incident light. Light becomes partially polarized as it is refracted by the first plate, and as it travels from one plate to another, it becomes more and more polarized.

8.29. The ratio of the wavelengths is determined by the ratio of the speeds of propagation of the two waves:

$$\lambda_e/\lambda_o \approx c_e/c_o.$$

At the same time,

$$c_e c_o = \sin \beta_e \sin \beta_o.$$

Hence,

$$\lambda_e/\lambda_o = \sin \beta_e/\sin \beta_o, \quad \lambda_e > \lambda_o.$$

8.30. The figure accompanying the answer shows the directions of the incident and scattered light and the planes in which the oscillations of the electric field vector

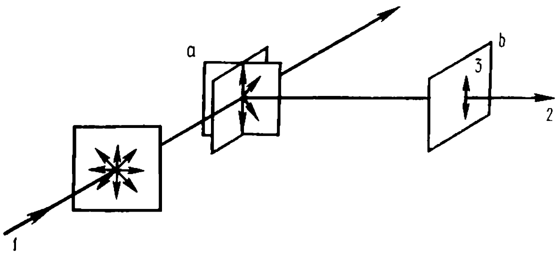

Fig. 8.30

lie. In the scattered light the oscillations must occur simultaneously in plane *a*, which is perpendicular to direction *1*, and in plane *b*, which is perpendicular to direction *2*. This, obviously, may happen only if the oscillations take place in the directions designated by arrow *3*. The blackening of the walls of the pipe, which was mentioned in the statement of the problem, is necessary so that no reflection can occur, since otherwise various directions of propagation of the light might become possible.

8.31. In the direction of the optic axis, the speed of propagation of the extraordinary and ordinary waves is the same and therefore the axis is perpendicular to the plane tangent to both wave surfaces at the point where the surfaces touch. In the first case (see Figure (a) accompanying the problem) the optic axis is parallel to the crystal boundary, while in the other (Figure (b)) it is perpendicular to the boundary. Since in all directions except the optic axis the speed of the extraordinary wave is higher than that of the ordinary, by the common nomenclature the crystal is negative.

8.32. After the light has passed through the first Nicol prism, its intensity becomes $I_1 = (1/2)\, I_0$ (it is assumed that the extraordinary wave loses no intensity when it is reflected and when it travels through a Nicol prism).

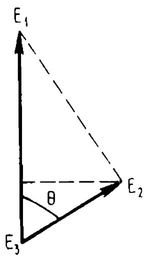

Fig. 8.32

According to Malus' law, after the light has passed through the second Nicol prism the intensity becomes

$$I_2 = I_1 \cos^2 \theta = (1/2)\, I_0 \cos^2 \theta.$$

The figure accompanying the answer shows the direction of oscillations of the electromagnetic field vector in the electromagnetic wave after the wave has passed through the first Nicol prism, E_1, and after the wave has passed through the second Nicol prism, E_2. In the reverse direction the electric field vector will be retained after the reflected wave has passed through the first Nicol prism but will change to $E_2 \cos \theta$ after the wave has passed through the second Nicol prism. Accordingly, the intensity after the light has passed through the two Nicol prisms in both directions will be

$$I_3 = I_2 \cos^2 \theta = I_1 \cos^4 \theta = (1/2)\, I_0 \cos^4 \theta.$$

8.33. The sense of rotation of the polarization plane depends on the direction of propagation of light in relation to the direction of the external magnetic field. For an overwhelming majority of substances ("positive" substances), the rotation is clockwise (looking in the direction of the ray of light) if the direction of propagation of light corresponds with that of the external magnetic field, and counterclockwise if the two directions are opposite. If the directions of the light ray and the external magnetic field coincided when the light passed from the source to the mirror and, therefore, the polarization plane rotated clockwise, after the light is reflected by the mirror the directions of the light ray and the external magnetic field are in opposition and the polarization plane rotates counterclockwise. If one views this process from the mirror, the rotation sense coincides with the clockwise rotation of the polarization plane when light passes in the primary direction. As a result, the two rotations are added and the angle doubles.

8.34. In the Kerr effect, the difference of the refractive indices of the extraordinary and ordinary waves obeys the law

$$n_e - n_o = kE^2, \qquad (8.34.1)$$

where k is a constant characterizing the medium. Since the electric field strength is squared in (8.34.1), the difference $n_e - n_o$ does not depend on the direction of the electric field. The optic axis in nitrobenzene coincides in direction with the electric field vector. The path difference between the ordinary and extraordinary rays,

$$\delta = l\,(n_e - n_o) = kE^2 l$$

(l is the length of the light path in the nitrobenzene), is also independent of the direction of the electric field vector, whereby the optical pattern caused by the emerging elliptical polarization will not change under reversal of direction of electric field.

8.35. According to classical theory, when a source of electromagnetic waves moves toward the observer, the ratio of the perceptible frequency to the frequency of the light emitted by a fixed source is

$$\frac{\nu_{cl}}{\nu_0} = \frac{1}{1-\beta},$$

with β the ratio of the speed of the source to the speed of light. According to the theory of relativity, this frequency ratio does not depend on whether the source or the observer is considered fixed and

$$\frac{\nu_{t.r}}{\nu_0} = \sqrt{\frac{1+\beta}{1-\beta}}.$$

The $\nu_{t.r}$-to-ν_{cl} ratio is given by the formula

$$\frac{\nu_{t.r}}{\nu_{cl}} = \sqrt{1-\beta^2}.$$

Hence, the upper curve corresponds to classical-theory results, while the lower curve corresponds to the theory-of-relativity results. For $\beta \ll 1$ the difference between the two formulas is moderate (e.g. at $\beta = 0.1$ the difference amounts only to 0.5%).

8.36. The ratio of the ion velocity to the speed of light, $\beta = v/c$, in the case at hand is of the order of 10^{-4}. For such values of β the difference between the classical and relativistic formulas for the Doppler effect is negligible. If the source moves with a velocity v, the wavelength of the light measured by the receiver is

$$\lambda = \lambda_0 \pm vT = \lambda_0 (1 \pm v/c).$$

Here λ_0 is the wavelength of the light emitted by a fixed source, and the plus sign corresponds to the case where the source is moving away from the receiver, while the minus sign corresponds to the case where the source is moving toward the receiver. The difference in wavelengths measured from both sides of the tube with the plasma in which the ions move is

$$\Delta\lambda = 2\lambda_0 (v/c),$$

which yields the following formula for the velocity of the ions:

$$v = \frac{\Delta\lambda}{2\lambda_0} c.$$

Since the ions have different velocities, each observed spectral line is blurred, or broadened. The maximal intensity corresponds to the most probable velocity, while the extent to which the line is blurred characterizes the velocity distribution of the directional motion of the ions.

8.37. Since the velocities of atoms are much lower than the speed of light, we can employ the classical formulas for the Doppler effect. As shown in the answer to Problem 8.36, the difference in the wavelengths of the waves emitted by two identical sources that move with velocities of the same absolute value but pointing in opposite directions in relation to the receiver constitutes

$$\Delta\lambda = 2\lambda_0 (v/c),$$

where λ_0 is the wavelength of the wave emitted by a fixed source, and c is the speed of light. In a light-emitting gas, the atoms move with different velocities, in accordance with the Maxwellian distribution law. The higher the temperature, the more extended is the distribution in the direction of higher temperatures, therefore the higher the temperature, the broader the spectral line. Hence, curve *2* corresponds to a higher temperature.

8.38. In accord with the Doppler principle, the distribution in wavelengths of the intensity of the emission lines of excited ions reflects the velocity distribution of the ions (and hence the energy distribution of the ions, too). However, this distribution cannot be associated with the temperature of the gas. The fact is that the motion of ions in the discharge plasma (which is the source of radiation emitted in the tube) is highly anisotropic; this anisotropy is determined by the electric field strength in the tube. The electric field in the tube has a radial component directed from the axis to the wall. On the axis this component is zero; it increases as we approach the wall. This field imparts a directional velocity to the ions. Thus, the left half of the curve in Figure (b) (shorter wavelengths) corresponds to the ions moving away from the axis toward the spectrograph, while the right half corresponds to the ions moving away from the axis in the opposite direction.

8.39. According to Kirchhoff's law, the ratio of the total emissivity of a heat radiator to the absorption coefficient (immissivity) of that same radiator is the same for all objects, constitutes a universal function of the temperature, and is equal to the total emissivity of a black body:

$$e_{\mathrm{T}}/a_{\mathrm{T}} = E_{\mathrm{T}}.$$

Hence, an object with a higher absorption coefficient has a higher emissivity and, therefore, it loses the energy acquired during heating at a higher rate. Curve *1* (see the figure accompanying the problem), therefore, represents the change of temperature in cooling for the object with the lower absorption coefficient or, in other words, curve *2* represents the cooling off of the object with the higher absorption coefficient.

8.40. The average kinetic energy of a molecule of the gas in translational motion is

$$w = \frac{3}{2} kT,$$

where k is the Boltzmann constant. If the concentration of the molecules in the gas is n, the volume density of the energy of the molecules is

$$u_{\mathrm{m}} = \frac{3}{2} nkT.$$

The volume density of the energy of blackbody radiation, according to the Stefan-Boltzmann law, is

$$u_r = \frac{4\sigma}{c} T^4.$$

If we set u_m equal to u_r, we get

$$T = \left(\frac{3nkc}{8\sigma}\right)^{1/3}. \qquad (8.40.1)$$

We will illustrate the above result with two examples. First, suppose that the concentration of the molecules is the same as that at S.T.P. conditions ($T = 273$ K, $p = 101\ 325$ Pa). This concentration (the Loschmidt number) n is equal to 2.686×10^{25} m^{-3}. Substituting into (8.40.1) the values $k = 1.3807 \times 10^{-23}$ J/K, $c = 2.9979 \times 10^8$ m/s, and $\sigma = 5.670 \times 10^{-8}$ W$\cdot$m$^{-2}\cdot$K^{-4}, we find that

$$T = 9.03 \times 10^5 \text{ K}.$$

Under these assumptions, the gas pressure is

$$p = nkT = 3.35 \times 10^8 \text{ Pa} = 3300 \text{ atm}.$$

In the second example, we wish to find the concentration of the molecules of the gas if the temperature at which the energy density of the translational motion of the molecules is equal to the energy density of electromagnetic radiation is to be equal to 0 °C. Equation (8.40.1) yields

$$n = 7.42 \times 10^{14} \text{ m}^{-3}.$$

This concentration yields the following value for the pressure of the gas:

$$p = 2.8 \times 10^{-6} \text{ Pa}.$$

8.41. The emissive power over a definite wavelength interval is

$$\Delta E_T = \int_{\lambda_1}^{\lambda_2} E_{\lambda T}\, d\lambda.$$

Since the integral is the area under the curve limited by the ordinates corresponding to the lower and upper values, the emissive power per each interval is the same. The energy of the quanta corresponding to greater wavelengths is lower, whereby even for the same emissive power there are more quanta of lower energy (i.e. referring to S_2).

8.42. Contrary to Wien's displacement law, the maximum in the blackbody radiation distribution corresponds, for a higher temperature, to a longer wavelength rather than to a shorter wavelength.

8.43. The relationship that exists between the radiation function and the volume radiation density is

$$E_{\nu T} = u_{\nu T}c/4.$$

The radiant emittance over the frequency range from ν_1 to ν_2 is determined by the integral

$$\Delta E_{1,2} = \int_{\nu_1}^{\nu_2} E_{\nu T}\, d\nu,$$

and, hence, the volume radiation density over the same range is

$$\Delta u_{1,2} = \frac{4}{c}\int_{\nu_1}^{\nu_2} E_{\nu T}\, d\nu.$$

8.44. The thermal radiation emitted by a body cannot exceed the blackbody radiation over all possible wavelength intervals. Contrary to this theoretical fact, the experimental curve contains a section that lies above the curve representing blackbody radiation.

8.45. According to Kirchhoff's law,

$$e_\lambda/a_\lambda = E_\lambda,$$

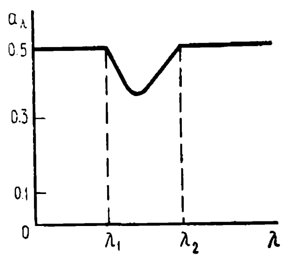

Fig. 8.45

where E_λ and e_λ are the respective radiant emittance of a black body and a given object (which is not a black body), and a_λ is the absorption coefficient of the object. Therefore, the ratio of the ordinates of curve *2* to those of curve *1* yields the value of a_λ for each wavelength. On the segment from $\lambda = 0$ to λ_1 the value of a_λ remains constant and equal to 0.5. The same happens on the segment from λ_2 to $\lambda = \infty$. On the segment from λ_1 to λ_2

the value of a_λ passes through a minimum, as shown in the figure accompanying the answer.

8.46. Since

$$E_\nu = \frac{dE}{d\nu}, \quad E_\lambda = \frac{dE}{d\lambda}, \quad \text{and} \quad \frac{d\lambda}{d\nu} = -\frac{c}{\nu^2},$$

we have

$$E_\nu = \frac{dE}{d\lambda}\frac{d\lambda}{d\nu} = -\frac{c}{\nu^2}\frac{dE}{d\lambda} = -\frac{c}{\nu^2}E_\lambda.$$

To compare the maximal values of E_ν and E_λ, we take the derivative

$$\frac{dE_\nu}{d\nu} = \frac{2c}{\nu^3}E_\lambda - \frac{c}{\nu^2}\frac{dE_\lambda}{d\nu} = \frac{c}{\nu^3}\left(2E_\lambda + \frac{c}{\nu}\frac{dE_\lambda}{d\lambda}\right).$$

At the maximum of E_λ the second term is zero while the first is not. Thus, at the wavelength λ_m the frequency

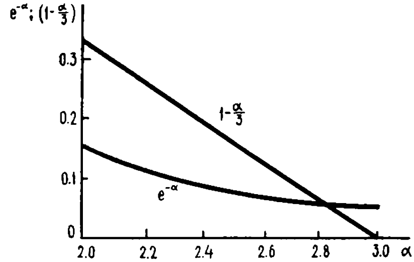

Fig. 8.46

does not correspond to the one at which E_ν is maximal. The maximum occurs at $dE_\lambda/d\lambda$ negative, that is, in the section where E_λ is falling off.

To find the frequency ν_m at which E_ν has its maximum, we must take the derivative of the Planck function with respect to ν, or

$$\frac{dE_\nu}{d\nu} = \frac{2\pi h}{c^2}\left\{\frac{3\nu^2\left[\exp\left(\frac{h\nu}{kT}\right)-1\right] - \frac{h\nu^3}{kT}\exp\left(\frac{h\nu}{kT}\right)}{\left[\exp\left(\frac{h\nu}{kT}\right)-1\right]^2}\right\}.$$

Nullifying this derivative, we arrive at a transcendental equation for $h\nu/kT$:

$$e^{h\nu/kT}\left(3 - \frac{h\nu}{kT}\right) = 3.$$

This equation can be solved graphically by constructing two functions,

$$y_1 = e^{-h\nu/kT} \quad \text{and} \quad y_2 = 1 - h\nu/3kT.$$

An approximate determination (via the intersection point of the two curves) yields a value of 2.82 for $h\nu_m/kT$. A more exact calculation yields

$$h\nu_m/kT = 2.8214,$$

or

$$h\nu_m = 3.896 \times 10^{-23}\ T. \qquad (8.46.1)$$

From (8.46.1) it follows that Wien's displacement law can be written in the form

$$\nu_m = 5.879 \times 10^{10}\ T.$$

The frequency ν_m corresponds to the wavelength (we denote it by $\lambda\ (\nu_m)$)

$$\lambda\ (\nu_m) = 5.10 \times 10^{-2}\ T^{-1}.$$

Thus,

$$\lambda\ (\nu_m)/\lambda_m = 1.760.$$

8.47. The volume density of the energy of blackbody radiation over the frequency range from ν to $\nu + d\nu$ is determined from the Planck formula

$$du = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1} d\nu.$$

The energy of each quantum in this range is $h\nu$. Thus, the distribution function for the number of quanta over the energy of one quantum has the form

$$\frac{dn}{dh\nu} = \frac{8\pi (h\nu)^2}{c^2h^3} \frac{1}{\exp[h\nu/(kT)] - 1}. \qquad (8.47.1)$$

Introducing the dimensionless parameter $\alpha = h\nu/kT$, we can represent (8.47.1) in the form

$$n_\alpha = \frac{dn}{d\alpha} = \frac{8\pi k^3}{c^3h^3} T^3 \frac{\alpha^2}{e^\alpha - 1}. \qquad (8.47.2)$$

The total "concentration" of the quanta can be obtained by integrating (8.47.2) with respect to α from zero to infinity, and the result is

$$n=\frac{8\pi k^3}{c^3h^3}T^3\int_0^\infty \frac{\alpha^2\,d\alpha}{e^\alpha-1}. \qquad (8.47.3)$$

The integral in (8.47.3) can be reduced to tabulated functions (it can also be evaluated by expanding it in a power series). The value of the integral is 2.404, with the result that

$$n=\frac{8\times 2.404\pi k^3}{c^3h^3}T^3=2.028\times 10^7\,T^3.$$

In relative units of $(1/n)\,dn/d\alpha$, the energy distribution function for the quanta is presented in Figure (a).

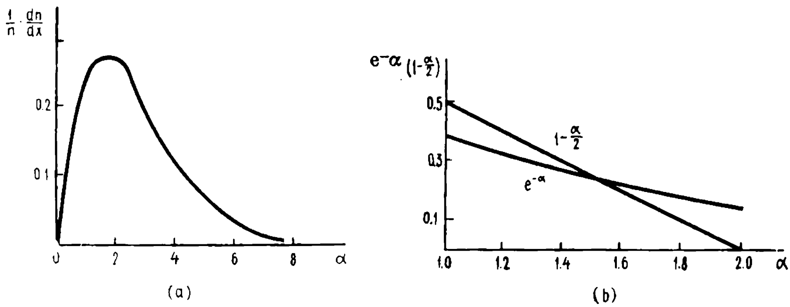

Fig. 8.47

Since the total energy density of blackbody radiation energy is

$$u=\frac{4\sigma}{c}T^4=7.57\times 10^{-16}\,T^4$$

(σ is the constant in the Stefan-Boltzmann law), knowing the total number of quanta (see formula (8.47.3)) we can determine the average energy of a single quantum:

$$h\nu_{av}=\frac{7.57}{2.028}\times 10^{-3}T=3.73\times 10^{-23}T=2.70kT.$$

The distribution function given by (8.47.2) enables finding the energy of the "most probable" quantum, that is, the quantum whose energy corresponds to the maximum in the distribution function. To this end one must nullify the derivative $dn_\alpha/d\alpha$. This leads to the transcendental equation

$$(2 - \alpha)\, e^\alpha = 2.$$

An approximate graphical solution (Figure (b)) yields $\alpha = 1.6$. A more exact value is $\alpha = 1.594$. Hence,

$$h\nu_p = 1.594kT.$$

In the answer to Problem 8.46 it was shown that the energy of the quantum corresponding to the maximum of the function $E_{\nu T}$ is $h\nu_m = 2.8214kT$. Wien's displacement law can then be used to determine the energy of the quantum corresponding to the maximum of the function:

$$h\nu(\lambda_m) = \frac{ch}{b} T = 6.855kT.$$

Note that the average kinetic energy per one degree of freedom of an ideal gas is $w = 0.5kT$.

8.48. At first glance it appears that the question is meaningless. Just think, how can one heat something that does not exist? Actually, however, space is always filled with electromagnetic radiation, whose energy is determined by the Stefan-Boltzmann law:

$$u = \frac{4\sigma}{c} T^4. \tag{8.48.1}$$

If we imagine a region in space bounded by a shell that radiation cannot penetrate either from the outside or from within (and inside the shell a perfect vacuum is maintained), then the electromagnetic radiation inside the shell must be in thermodynamic equilibrium with the shell. To raise the temperature of the shell, we must supply an amount of heat determined not only by the heat capacity of the shell but also by the necessary increase in the density of energy of the electromagnetic radiation inside the shell. If we define the volume specific heat capacity as

$$c_{vol} = \frac{1}{V}\frac{dQ}{dT} = \frac{du}{dT}$$

and use formula (8.48.1) to find the derivative, we get

$$c_{vol} = \frac{16\sigma}{c} T^3.$$

8.49. If the intensity of the light is I, the number of photons of monochromatic light incident every second on a surface of unit area is

$$N = I/h\nu.$$

The momentum of each photon is $h\nu/c$. When hitting the surface, a photon transfers a momentum $h\nu/c$ to the surface if it is totally absorbed or a momentum $2h\nu/c$ if it is totally reflected. The pressure exerted on the surface is equal to the sum of all momenta transferred to the surface per unit time. In the case of absorption,

$$p = \frac{I}{h\nu} \frac{h\nu}{c} = \frac{I}{c},$$

while in the case of reflection,

$$p = 2\frac{I}{h\nu} \frac{h\nu}{c} = 2\frac{I}{c}.$$

If a fraction of the photons are absorbed and the rest are reflected, the latter process being characterized by a reflection coefficient R, then the pressure exerted by the light on the surface is

$$p = \frac{I}{c}(1 + R).$$

This formula coincides with (8.49.1), which was obtained on the basis of the electromagnetic theory of light.

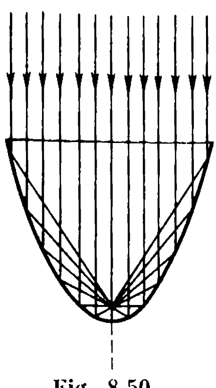

Fig. 8.50

8.50. Let us assume that such radiation has been obtained and is directed onto a mirror that is a paraboloid of revolution, with the rays of light being strictly parallel to the axis of the paraboloid (see the figure accompanying the answer). Since planes that are perpendicular to the rays are wave surfaces, all points in a single plane are in the same phase of oscillation (irrespective of the nature of the oscillation). All rays parallel to the axis converge (after being reflected) at a geometric point that is the focus of the paraboloid. The geometrical properties of a parabola imply that the sum of distances from any point in a plane that is perpendicular to the axis to the parabola and from the parabola to the focus is a constant. This means that the oscillations that arrive at the focus from all points in a wave surface are in phase. Hence, all radiation that travels to the paraboloid will be concentrated at a single point and the volume energy density of the radiation will become infinite at that point. This would make it possible to obtain (theoretically) infinite local temperatures at a finite temperature of the radiation source that provides the flow of plane waves.

The picture can be reversed, that is, we may ask ourselves: what requirements must a source meet for it to produce a stream of plane waves? Taking into account the reversibility of light rays, we conclude that such a source must be concentrated at a geometric point. At present quantum electronics can produce radiation with extremely low angular divergence, something on the order of 10^{-2} or even 10^{-3} of one second of the arc and, respectively, with colossal local power outputs. But even in this case the rays in such radiation cannot be considered strictly parallel.

8.51. The photon energy transferred to an electron in the metal is used to overcome the potential barrier at the boundary of the metal (the work function P) and part of it is lost inside the metal. In addition, one must bear in mind that not only the electrons that occupy levels lying near the Fermi level participate in the photoeffect. In addition to these, there are electrons that move somewhat slower and, hence, require for their liberation energies greater than the external work function. Therefore, Einstein's equation can be written in the form

$$h\nu = A + P + W,$$

where A is the term characterizing the energy losses inside the metal and the additional energy necessary for the electrons lying below the Fermi level to become liberated. The photoelectrons that escape from the surface of the metal have the maximal energy ($A = 0$); the initial energy of such electrons corresponds to the Fermi level:

$$W_{\mathrm{m}} = h\nu - P.$$

8.52. According to Einstein's equation,

$$h\nu = P + mv_{\mathrm{m}}^2/2,$$

where v_m is the maximal energy of the photoelectrons, and P is the work function of electrons ejected by the cathode. To stop the photoelectron current, we must apply a stopping potential no smaller than U_{stop}, which is determined from the equation

$$mv_m^2/2 = eU_{stop},$$

where e is the electron charge. Thus,

$$h\nu = P + eU_{stop}.$$

For a known value of e, the slope of the straight lines, $dU_{stop}/d\nu = h/e$, determines the Planck constant. The straight lines are different because they correspond to cathodes with different work functions. The work function can be determined either by the point of intersection of a straight line (for a particular cathode) with the horizontal axis,

$$P = h\nu_0$$

(with ν_0 the photoelectric threshold), or by the point of intersection of the straight line with the vertical axis,

$$P = -eU_{stop\,0}.$$

8.53. According to the hypothesis, the illuminated electrode emits photoelectrons whose maximal energy is

$$W_m = hc/\lambda - P,$$

which makes it possible to think of the system as an emf source, with the maximal value of the emf being

$$\mathscr{E} = W_m/e. \qquad (8.53.1)$$

This source can generate a current in the circuit; the current is determined by the intensity of illumination of the electrode but cannot exceed a value of

$$I_m = \mathscr{E}/R.$$

At the same time, the current cannot exceed the value

$$I = Ne,$$

where N is the number of electrons ejected by the cathode per unit time due to illumination of the cathode with light. Since according to (8.53.1) the emf is constant and so is the value of R, the interelectrode gap may be considered as a resistance r_{vac} whose value is the smaller the greater the intensity of the light. In darkness this resistance is infinite. Bearing all this in mind, we can write

$$I = \mathscr{E}/(R + r_{\text{vac}}).$$

8.54. The stopping potential difference, that is the voltage at which the photocurrent ceases, is the same for both cases. This potential difference determines the maximal photoelectron energy and equals the difference between the photon energy and the work function; hence, the emission frequency for the two sources is the same, and the sources differ only in the intensity of the radiation they emit.

8.55. According to Einstein's formula, the work function is equal to the difference between the photon energy and the maximal kinetic energy of the photoelectrons:

$$P = h\nu - mv_{\text{m}}^2/2.$$

The higher the maximal energy of the photoelectrons, which energy is equal to the maximal stopping potential, the lower the work function. In the case at hand, the cathode whose current-voltage characteristic is represented by curve *2* has a higher work function.

8.56. The point that an electron can reach thanks to their initial kinetic energy is determined only by the value of the stopping potential difference. Irrespective of the distance between the electrodes, the point is always at the middle of the interelectrode gap, and only such a distance can the fastest electrons leaving the cathode cover.

8.57. In Compton scattering, the photon wavelength changes by

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta).$$

We see that in the case of angle θ_2 the wavelength increases by a larger quantity. Hence, $h\nu_2 < h\nu_1$. As a result of scattering, the photon transfers a fraction of its energy to the electron, and the energy that the electron receives is the greater, the smaller the energy of the photon after scattering, and hence the greater the value of θ is.