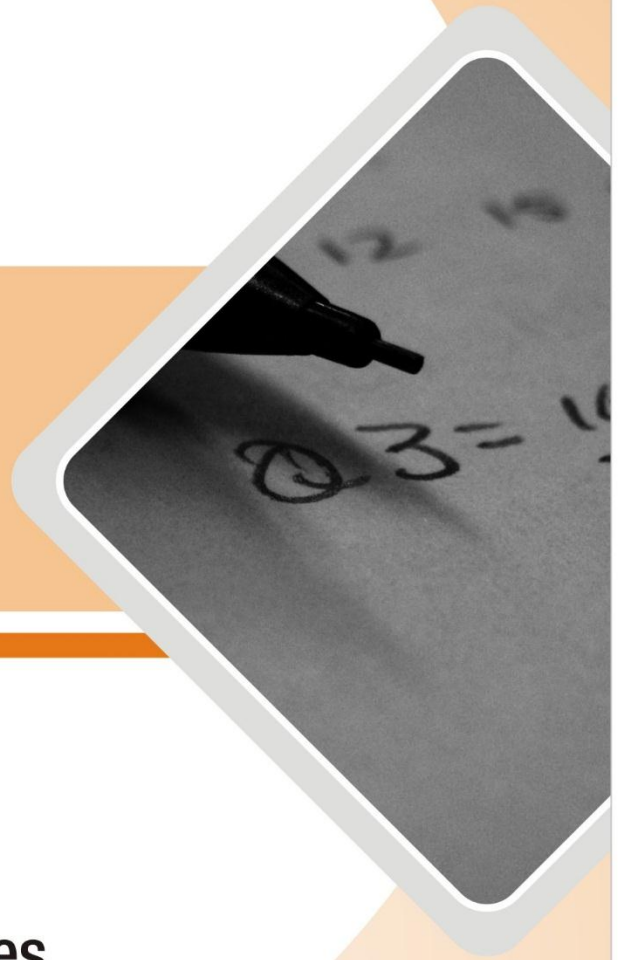


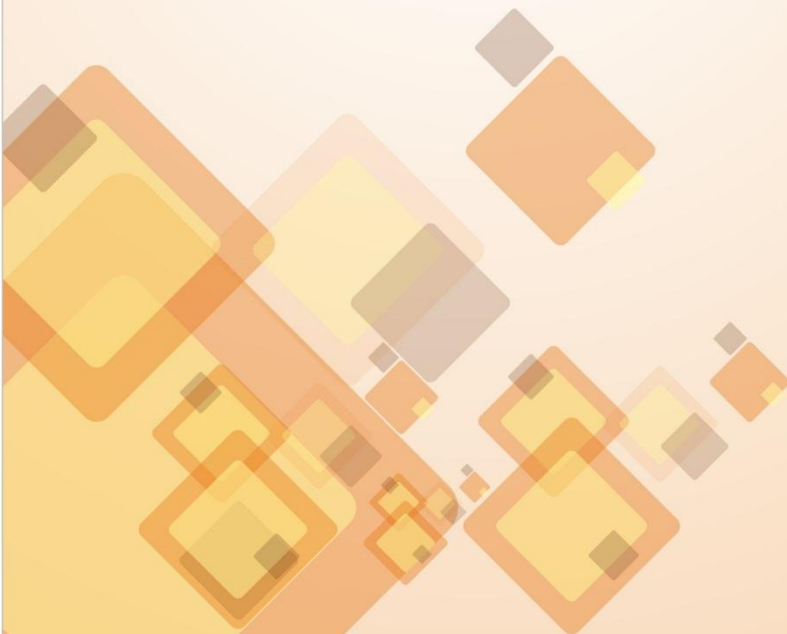
# Chapter

## Line, Angle & Triangle



### Learning Objectives

- ◆ Introduction
- ◆ Geometrical Basic Shapes
- ◆ Triangles





## Introduction

Lines and angles are the main geometrical concept and every geometrical figure is made up of lines and angles. Triangles are constructed by these lines and angles.

In this chapter we will learn about the basic shapes of geometrical figures.



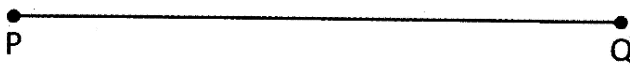
## Geometrical Basic Shapes



### Point

A dot which indicates position but not dimension is called a point.

A point does not have length, breadth and height. P and Q are points in the figure below.



In the picture above, P and Q are points

In the picture above two points lie on the same line and therefore called collinear points. Points do not lie on the same line are called non collinear points.



### Line

A set of infinite points which can be extended infinite distance in both sides is called a line.



In the picture above, M is a line

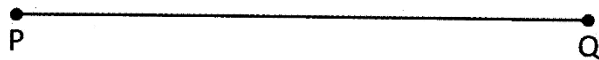
#### ❖ Features of the Line

- (i) The length of a line is infinite.
- (ii) It has no terminal, therefore, can be extended infinite in both directions.
- (iii) It is made up of infinite points.



### Line Segment

A line of fix length is called line segment.



In the picture above, P and Q is a line segment and represented by  $\overline{PQ}$ .



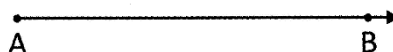
### Features of the Line Segment

- (i) A Line segment has fix length.
- (ii) It has two end points.



### Ray

A ray is defined as the line that can be extended infinite in one direction.



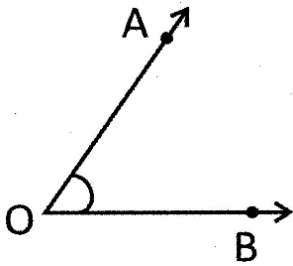
In the picture above, end point of terminal point

Ray AB is represented as  $\overrightarrow{AB}$



### Angle

Angle is formed between two rays which having common end point. Symbol of angle =  $\angle$



Vertex or common end point = O

Arms of angle  $\angle AOB$

= OA and OB the name of the above angle can be  $\angle AOB$  or  $\angle BOA$

The unit of measurement of an angle is degree ( $^{\circ}$ )

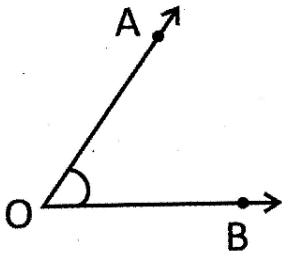


## Types of Angles

There are various types of angles which are the following:

### ❖ Acute Angle

The angle between  $0^{\circ}$  and  $90^{\circ}$  is called an acute angle.

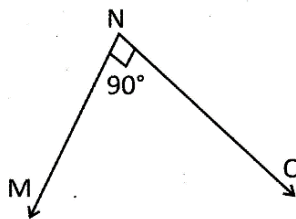
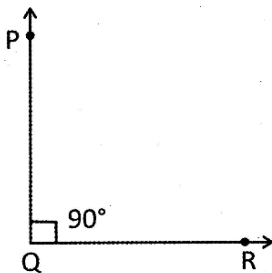


In the picture above  $\angle AOB$  or  $\angle BOA$  is an acute angle. The inclined arm OA on the horizontal is less inclined than vertical line.

$10^{\circ}, 30^{\circ}, 60^{\circ}, 80^{\circ}$  are acute angles.

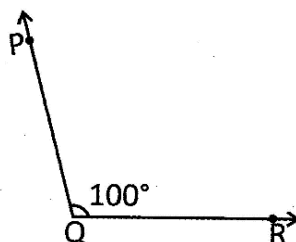
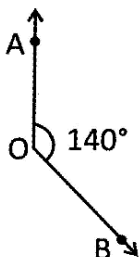
### ❖ Right Angle

An angle of  $90^{\circ}$  is called right angle.



### ❖ Obtuse Angle

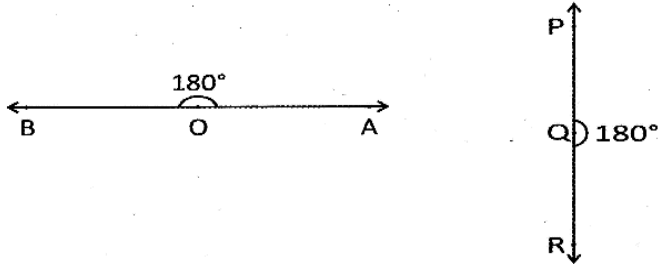
An angle whose measure is between  $90^{\circ}$  and  $180^{\circ}$  is called an obtuse angle.



### ❖ Straight Angle

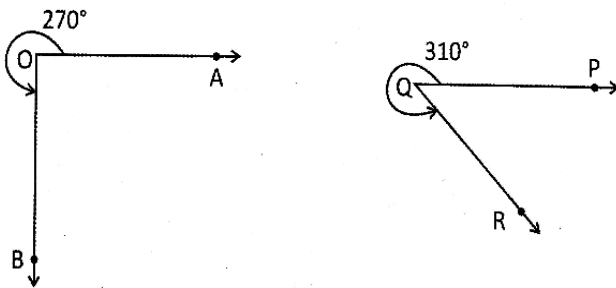
An angle whose measure is  $180^\circ$  is called straight angle.

Or an angle formed between two opposite rays is called straight angle.



### ❖ Reflex Angle

An angle whose measure is more than  $180^\circ$  and less than  $360^\circ$  is called reflex angle.



### ❖ Complementary

Two angles whose sum is  $90^\circ$  is called the complimentary of each other  $60^\circ + 30^\circ = 90^\circ$

Complementary angle of  $60^\circ = 90^\circ - 60^\circ = 30^\circ$

Complementary angle of  $30^\circ = 90^\circ - 30^\circ = 60^\circ$

Complementary angle of  $\theta = 90 - \theta$

The complementary angle of a given angle  $= 90^\circ -$  the given angle.

Therefore, if the sum of two angles is  $90^\circ$  then they are called complementary angles to each other.

### ❖ Supplementary Angle

Two angles whose sum is  $180^\circ$  is called supplementary angles.  $120^\circ + 60^\circ = 180^\circ$

Supplementary Angle of

$120^\circ = 180^\circ - 120^\circ = 60^\circ$

Supplementary Angle of

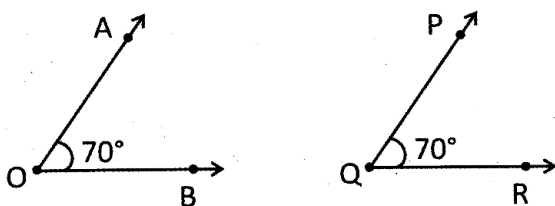
$60^\circ = 180^\circ - 60^\circ = 120^\circ$

The supplementary angle of the given angle  $= 180^\circ -$  the given angle

Supplementary angle of  $\theta = 180^\circ - \theta$

### ❖ Equal Angles

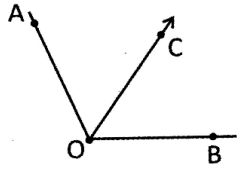
If the measurement of two angles is equal then they are called equal angles.



In the picture above, the measurement of the angles is equal. Therefore,  $\angle AOB$  and  $\angle PQR$  are equal angles.

### ❖ Adjacent Angle

Two angles are said to be adjacent even if they have common vertex and one common arm. In  $\angle AOC$



Vertex = O

Arms = OA, OC

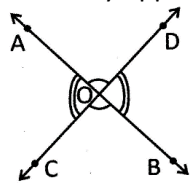
Arms = OC, OB

Hence vertex O and arm OC is common so these two angles AOC and COB are adjacent angles.

### ❖ Vertically Opposite Angle

Two angles are said to be vertically opposite angles if they are formed by opposite rays having common vertex.

Vertically opposite angles are formed by two lines which intersect each other.



In the picture above,  $\angle AOD$  and  $\angle BOC$  are vertically opposite angles. They are formed by opposite rays and they have same or common end point O.

Similarly,  $\angle AOC$  and  $\angle BOD$  are vertically opposite angles.

## Commonly Asked

### QUESTIONS



The measurement of an angle is  $180^\circ$ . Which one of the following types of angle is this?

- (a) Right angle
- (b) Straight angle
- (c) Reflex angle
- (d) All of these
- (e) None of these

**Answer: (b)**

**Explanation**



A straight angle has a measurement of  $180^\circ$ . Which one of the following is the complementary angle of  $30^\circ$ ?

- (a)  $30^\circ$
- (b)  $60^\circ$
- (c)  $90^\circ$
- (d) All of these
- (e) None of these

**Answer: (b)**

**Explanation**

Complementary angle of  $30^\circ = 90^\circ - 30^\circ = 60^\circ$



In the options given below the pair of angles are given. Find the complementary pair.

- (a)  $51^\circ, 25^\circ$
- (b)  $61^\circ, 29^\circ$
- (c)  $45^\circ, 55^\circ$
- (d) All of these
- (e) None of these

**Answer: (b)**

**Explanation**

If the sum of pair of angles is  $90^\circ$  then the pair is called complementary pair of angles. The sum of angles  $61^\circ + 29^\circ = 90^\circ$ . Hence the pair of angles  $61^\circ, 29^\circ$  is a complementary pair of angles.



Choose the pair of supplementary angle from the options given below?

- (a)  $100^\circ, 200^\circ$
- (b)  $105^\circ, 75^\circ$
- (c)  $30^\circ, 180^\circ$
- (d) All of these
- (e) None of these

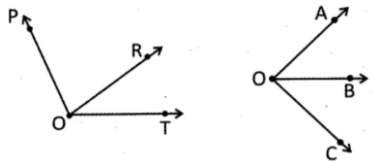
**Answer: (b)**

**Explanation**

The sum of angles of the pair  $105^\circ, 75^\circ = 105^\circ + 75^\circ = 180^\circ$ . Hence, the pair of angles  $105^\circ, 75^\circ$  is a supplementary pair of angles.



Two figures are given below. Which one of the following options is correct for the adjacent angles if the line segment OB divides the AOC into two equal parts?



- (a) AOB adjacent  $\angle BOC$  and  $\angle TOR$  adjacent  $\angle ROP$
- (b)  $\angle TOR$  adjacent  $\angle AOB$  and  $\angle ROP$  adjacent  $\angle AOB$
- (c)  $\angle POT$  adjacent  $\angle AOC$
- (d) All of these
- (e) None of these

**Answer: (a)**

**Explanation**

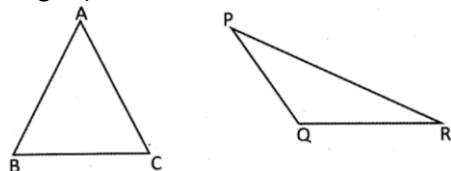
Two angles said to be adjacent even if they have one common arm and one common vertex. Hence,  $\angle AOB$  adjacent  $\angle BOC$  and  $\angle TOR$  adjacent  $\angle ROP$ .



## Triangles

A triangle has three sides and three vertices.

The sum of the three angles of a triangle is equal to  $180^\circ$  and one angle of a triangle  $= 180^\circ - (\text{Sum of two angles})$ .



In  $\triangle ABC$ ,  $\angle A + \angle B + \angle C = 180^\circ$

In  $\triangle PQR$ ,  $\angle P + \angle Q + \angle R = 180^\circ$

### Illustrative EXAMPLE



Find the measurement of  $A$  if  $B$  and  $C$  are given when  $A$ ,  $B$  and  $C$  are the angles of the triangle.

- (a)  $A = 180^\circ - (B + C)$       (b)  $A = 180^\circ + (B - C)$   
(c)  $A = 180^\circ + C - B$       (d) All of these  
(e) None of these

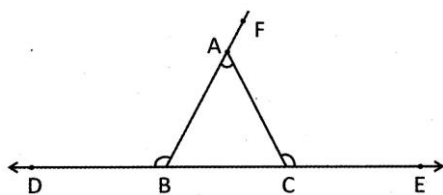
**Answer: (a)**

**Explanation** The sum of angles of the triangle  $= 180^\circ$ . Hence, the angle  $A = 180^\circ - (B + C)$ .



### Exterior Angle of a Triangle

The angle between the produced side and its adjacent side of triangle is called exterior angle. The exterior angle is equal to the sum of two opposite interior angles of that triangle. In the picture given below the angles  $\angle ABD$  and  $\angle ACE$  are the exterior angles of the triangle.



Thus,

$$\angle ACE = \angle BAC + \angle ABC$$

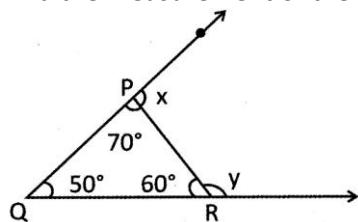
$$\angle ABD = \angle BAC + \angle ACB$$

$$\angle CAF = \angle ABC + \angle ACB$$

### Illustrative EXAMPLE



Find the measurement of the angles  $x$  and  $y$  from the picture given below.



- (a)  $x = 80^\circ$  and  $y = 10^\circ$       (b)  $x = 110^\circ$  and  $y = 120^\circ$   
(c)  $x = 70^\circ$  and  $y = 90^\circ$       (d) All of these  
(e) None of these

**Answer: (b)**

**Explanation**

$x$  and  $y$  are the exterior angle therefore,

$$x = \angle PRQ + \angle PQR = 60^\circ + 50^\circ = 110^\circ$$

$$y = \angle QPR + \angle PQR = 70^\circ + 50^\circ = 120^\circ$$



## Area of the Triangle

The area of a triangle is the area bounded by the sides of the triangle.

### ❖ Heron's Formula

To find the area of a triangle by Heron's Formula, First find the semi perimeter of a triangle by the following formula:

$s = \frac{a+b+c}{2}$  (Here "s" stands for half of perimeter or semi perimeter and a, b and c refers to 3 sides of the triangle)

$Area = \sqrt{s(s-a)(s-b)(s-c)}$  by heron's formulae.

## Illustrative EXAMPLE



If the sides of a triangle are 18 cm, 24 cm and 30 cm then find the area of the triangle.

- (a)  $216cm^2$
- (b)  $210cm^2$
- (c)  $21cm^2$
- (d) All of these
- (e) None of these

**Answer: (a)**

**Explanation**

Let sides

A = 18 cm, B = 24 cm, C = 30 cm

$$s = \frac{a+b+c}{2} = \frac{18+24+30}{2} = \frac{72}{2} = 36$$

$Area = \sqrt{s(s-a)(s-b)(s-c)}$  by heron's formulae

$$= \sqrt{36(36-18)(36-24)(36-30)}$$

$$= \sqrt{36 \times 18 \times 12 \times 6}$$

$$= \sqrt{6^2 \times 9 \times 2 \times 6 \times 2 \times 6}$$

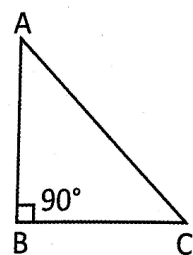
$$= \sqrt{6^2 \times 3^2 \times 2^2 \times 6^2}$$

$$= \sqrt{(6 \times 3 \times 2 \times 6)^2} = 6 \times 3 \times 2 \times 6 = 216cm^2$$

### ❖ Area of the Right Angled Triangle

The area of a right angle triangle is the half of the Product of height and base.

Therefore the area of a right angle triangle  $= \frac{1}{2} \times \text{base} \times \text{height}$ .



In the figure above AB is the height or altitude of the triangle ABC, AC is the hypotenuse and BC is the base of the triangle. The angle ABC is  $90^\circ$ . Therefore triangle ABC is called a right angled triangle.



## Illustrative EXAMPLE



If the base of a triangle is 18 cm and distance between base and opposite vertex is 10 cm, what will be the area of triangle?

- (a)  $206\text{cm}^2$  (b)  $10\text{cm}^2$   
(c)  $90\text{cm}^2$  (d) All of these (e) None of these

**Answer: (c)**

**Explanation**

The distance between base and opposite vertex is height.

$$\begin{aligned}\text{The area} &= \frac{1}{2} \times \text{base} \times \text{distance of base from its opposite vertex (height attitude)} \\ &= \frac{1}{2} \times 18 \times 10 = 90\text{cm}^2\end{aligned}$$

### ❖ Area of the Equilateral Triangle

All three sides of the equilateral triangle are equal. Therefore the area of an equilateral triangle

$$= \frac{\sqrt{3}}{4} \times (\text{side})^2$$

## Illustrative EXAMPLE



Find the area of an equilateral triangle whose each side is 6 cm?

- (a)  $9\sqrt{3}\text{cm}^2$  (b)  $10\sqrt{2}\text{cm}^2$   
(c)  $90\text{cm}^2$  (d) All of these (e) None of these

**Answer: (a)**

**Explanation**

Area of an equilateral triangle

$$= \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times 6^2 = \frac{\sqrt{3}}{4} \times 36 = 9\sqrt{3}\text{cm}^2$$

## You Must KNOW

- ❖ Arrow marks at the far end point on the line represent the direction of the ray.
- ❖ A line is the collection of points.
- ❖ A line segment has finite length.
- ❖ Angles are measured in degrees.
- ❖ A line which is passing through two points A and B is denoted by  $\overleftrightarrow{AB}$

## SUMMARY



- ❖ There are infinite number of points on a line.
- ❖ The figure which does not have length, width and height is called dot or point.
- ❖ The finite length of the line is called line segment.
- ❖ The angle less than  $90^\circ$  is called acute angle.
- ❖ The angle greater than  $90^\circ$  but less than  $180^\circ$  is called obtuse angle.

# Self Evaluation TEST



Duration  
10 Minutes

- 
1. Area of a triangle is  $20 \text{ cm}^2$  and base is 20 cm. Find the height of the triangle.
- (a) 2cm
  - (b) 8cm
  - (c) 5cm
  - (d) 6cm
  - (e) None of these
- 
2. Which of the following is an acute angle?
- (a)  $0^\circ$
  - (b)  $90^\circ$
  - (c)  $70^\circ$
  - (d)  $180^\circ$
  - (e) None of these
- 
3. Complementary angle of half of  $60^\circ$  is?
- (a)  $60^\circ$
  - (b)  $30^\circ$
  - (c) An obtuse angle
  - (d) A right angle
  - (e) None of these
- 
4. Which one of the following has only one end point?
- (a) A ray
  - (b) A line
  - (c) A line segment
  - (d) A point
  - (e) None of these
- 
5. The measurement of two angles other than right angle of a right angled triangle is ?
- (a) Obtuse angles
  - (b) Acute angles
  - (c) One acute angle and one obtuse angle
  - (d) One acute angle and one right angle
  - (e) None of these
- 
6. Angles of a triangle are in the ratio of 1 : 2 : 3. Find the measurement of the largest angle?
- (a)  $95^\circ$
  - (b)  $94^\circ$
  - (c)  $90^\circ$
  - (d)  $100^\circ$
  - (e) None of these
-

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**7. Maximum number of obtuse angles in a triangle may be?**

- (a) Three (b) Two  
(c) One (d) Zero  
(e) None of these
- 

**8. Find the area of an equilateral triangle whose side is 4 cm long?**

- (a)  $4\sqrt{3} \text{ cm}^2$  (b)  $2 \text{ cm}^2$   
(c)  $16 \text{ cm}^2$  (d)  $16 \text{ cm}^2$   
(e) None of these
- 

**9. Base and height of a right angled triangle are in the ratio of 5 : 7 and area is  $70 \text{ cm}^2$ . Find the base and height of the triangle.**

- (a) 12 cm and 14 cm (b) 10 cm and 14 cm  
(c) 7 cm and 14 cm (d) 9 cm and 14 cm  
(e) None of these
- 

**10. Find the height of the right angled triangle whose area is  $30 \text{ cm}^2$  and base is 5 cm.**

- (a) 20cm (b) 10cm  
(c) 12cm (d) 40cm  
(e) None of these
- 

**Answers – Self Evaluation Test**

1.	A	2.	C	3.	A	4.	A	5.	B	6.	C	7.	C	8.	A	9.	B	10.	C
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# Self Evaluation TEST



Duration  
10 Minutes

1. Let the height of the triangle is  $x$ .

According to the question  $20 = \frac{1}{2} \times 20 \times x \Rightarrow 20 = 10x \Rightarrow x = \frac{20}{10} = 2 \text{ cm}$ .

Hence, the height of the triangle = 2 cm.

2. If the measurement of an angle is less than  $90^\circ$  then the angle is called an acute angle.

3. Complementary angle of half of  $60^\circ = 90^\circ - \frac{60^\circ}{2} = 90^\circ - 30^\circ = 60^\circ$

4. A ray has only one end point.

5. The sum of measurement of other than right angle of a right angled triangle =  $180^\circ - 90^\circ = 90^\circ$ . Hence, the angle of  $90^\circ$  will be divided into two parts. Thus each angle will be an acute angle.

6. The ratio of the angles is 1 : 2 : 3.

Hence,  $x + 2x + 3x = 180^\circ \Rightarrow 6x = 180^\circ \Rightarrow x = \frac{180^\circ}{6} = 30^\circ$ . Thus, the measurement of the largest angle  $30^\circ \times 3 = 90^\circ$

7. The angle which is greater than a right angle but less than a straight angle is called obtuse angle. Hence, if one angle of a triangle is greater than  $90^\circ$  then the measurement of each remaining angle will be less than  $90^\circ$ . Thus one obtuse angle is possible in a triangle.

8.  $\frac{\sqrt{3}}{4} \times 4 \times 4 = 4\sqrt{3} \text{ cm}^2$

9. Let consider the common multiplier  $x$ . According to the question  $70 = \frac{1}{2} \times 5x \times 7x$   
 $\Rightarrow 70 = \frac{35x^2}{2} \Rightarrow 140 = 35x^2 \Rightarrow x^2 = \frac{140}{35} = 4 \Rightarrow x = \sqrt{4} = 2 \text{ cm}$ . Hence, base of the triangle =  $5 \times 2 = 10 \text{ cm}$  and height =  $7 \times 2 = 14 \text{ cm}$ .

10. According to the question  $30 = \frac{1}{2} \times 5 \times h \Rightarrow 30 = \frac{5h}{2} \Rightarrow 60 = 5h \Rightarrow h = \frac{60}{5} = 12 \text{ cm}$ .