14. PERMUTATION & COMBINNATION

1. Arrangement : number of permutations of n different things taken r at a time =

$^nP_r = n\,(n-1)\,(n-2)\ldots(n-r+1) = \dfrac{n!}{(n-r)!}$

2. Circular Permutation :

The number of circular permutations of n different things taken all at a time is; (n – 1)!

3. Selection : Number of combinations of n different things taken r at a time = $^nC_r = \dfrac{n!}{r!\,(n-r)!} = \dfrac{^nP_r}{r!}$

4. The number of permutations of 'n' things, taken all at a time, when 'p' of them are similar & of one type, q of them are similar & of another type, 'r' of them are similar & of a third type & the remaining $n-(p+q+r)$ are all different is $\dfrac{n!}{p!\,q!\,r!}$.

5. Selection of one or more objects

(a) Number of ways in which atleast one object be selected out of 'n' distinct objects is

$$^nC_1 + {^nC_2} + {^nC_3} + \ldots\ldots\ldots\ldots\ldots + {^nC_n} = 2^n - 1$$

(b) Number of ways in which atleast one object may be selected out of 'p' alike objects of one type 'q' alike objects of second type and 'r' alike of third type is

$$(p+1)\,(q+1)\,(r+1) - 1$$

(c) Number of ways in which atleast one object may be selected from 'n' objects where 'p' alike of one type 'q' alike of second type and 'r' alike of third type and rest $n-(p+q+r)$ are different, is

$$(p+1)\,(q+1)\,(r+1)\,2^{n-(p+q+r)} - 1$$

6. Multinomial Theorem :

Coefficient of x^r in expansion of $(1-x)^{-n} = {^{n+r-1}C_r}$ $(n \in N)$

7. Let $N = p^a\, q^b\, r^c \ldots\ldots$ where p, q, r...... are distinct primes & a, b, c..... are natural numbers then :

(a) The total numbers of divisors of N including 1 & N is $= (a+1)\,(b+1)\,(c+1)\ldots\ldots\ldots$

(b) The sum of these divisors is =

$(p^0 + p^1 + p^2 + \ldots. + p^a)\,(q^0 + q^1 + q^2 + \ldots. + q^b)\,(r^0 + r^1 + r^2 + \ldots. + r^c)\ldots\ldots\ldots$

(c) Number of ways in which N can be resolved as a product of two factors is

$$= \begin{cases} \frac{1}{2}(a+1)(b+1)(c+1)\ldots. & \text{if N is not a perfect square} \\ \frac{1}{2}\left[(a+1)(b+1)(c+1)\ldots.+1\right] & \text{if N is a perfect square} \end{cases}$$

(d) Number of ways in which a composite number N can be resolved into two factors which are relatively prime (or coprime) to each other is equal to 2^{n-1} where n is the number of different prime factors in N.

8. Dearrangement :

Number of ways in which 'n' letters can be put in 'n' corresponding envelopes such that no letter goes to correct envelope is $n!\left(1 - \dfrac{1}{1!} + \dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!} \ldots\ldots\ldots\ldots + (-1)^n \dfrac{1}{n!}\right)$