Chapter : 16. DEFINITE INTEGRALS

Exercise : 16A

Question: 1

Evaluate:

Solution:

$\frac{242}{5}$

Evaluation:

$$\int_1^3 x^4 dx = \left[\frac{x^5}{5}\right]$$

$$= \frac{3^5}{5} - \frac{1}{5}$$

$$= \frac{243 - 1}{5}$$

$$= \frac{242}{5}$$

Question: 2

Evaluate:

Solution:

$\frac{14}{3}$

Evaluation:

$$\int_1^4 \sqrt{x} dx = \left[\frac{2}{3} x^{\frac{3}{2}}\right]$$

$$= \frac{2}{3}\left[4^{\frac{3}{2}} - 1\right]$$

$$= \frac{14}{3}$$

Question: 3

Evaluate:

Solution:

$\frac{15}{64}$

Evaluation:

$$\int_1^2 x^{-5} dx = \left[\frac{x^{-4}}{-4}\right]$$

$$= \frac{2^{-4}}{-4} - \frac{1}{-4}$$

$$= \frac{16 - 1}{64}$$

$$= \frac{15}{64}$$

Question: 4

Evaluate:

Solution:

$\frac{512}{7}$

Evaluation:

$\int_{0}^{16} x^{\frac{3}{4}} dx = \left[\frac{4}{7} x^{\frac{7}{4}}\right]$

$= \frac{4}{7}\left[16^{\frac{7}{4}} - 1\right]$

$= \frac{512}{7}$

Question: 5

Evaluate:

Solution:

$-log4$

Evaluation:

$\int_{-4}^{-1} \frac{dx}{x} = -[logx]$

=[log(-1)-log(-4)]

=-[log(-4)-log(-1)]

$= -\left[log\left(\frac{-4}{-1}\right)\right]$

=-log 4

Question: 6

Evaluate:

Solution:

2

Evaluation:

$\int_{1}^{4} \frac{dx}{\sqrt{x}} = [2\sqrt{x}]$

=[2√4-2]

=[4-2]

=2

Question: 7

Evaluate:

Solution:

$\frac{3}{2}$

Evaluation:

$\int_{0}^{1} \frac{dx}{\sqrt[3]{x}} = \left[\frac{3}{2} x^{\frac{2}{3}}\right]$

$= \left[\frac{3}{2} 1^{\frac{4}{3}} - 0\right]$

$= \frac{3}{2}$

Question: 8

Evaluate:

Solution:

3

Evaluation:

$\int_1^8 \frac{dx}{x^{\frac{2}{3}}} = \left[\frac{3}{1} x^{\frac{1}{3}}\right]$

$= \left[3(8)^{\frac{1}{3}} - 3(1)^{\frac{1}{3}}\right]$

=[6-3]

=3

Question: 9

Evaluate:

Solution:

6

Evaluation:

$\int_2^4 3dx = 3[x]$

=3[4-2]

=6

Question: 10

Evaluate:

Solution:

$\frac{\pi}{4}$

Evaluation:

$\int_0^1 \frac{dx}{1 + x^2} = [\tan^{-1} x]$

$=[\tan^{-1} 1-\tan^{-1} 0]$

$=\pi/4$

Question: 11

Evaluate:

Solution:

$\frac{\pi}{2}$

Evaluation:

$$\int_0^{\infty} \frac{dx}{1+x^2} = [\tan^{-1}x]$$

$=[\tan^{-1} \infty - \tan^{-1} 0]$

$=\pi/2$

Question: 12

Evaluate:

Solution:

$$\frac{\pi}{2}$$

Evaluation:

$$\int_0^{1} \frac{dx}{\sqrt{1-x^2}} = [\sin^{-1}x]$$

$=[\sin^{-1} 1 - \sin^{-1} 0]$

$$=\frac{\pi}{2}$$

Question: 13

Evaluate:

Solution:

$$\frac{1}{\sqrt{3}}$$

Evaluation:

$$\int_0^{\frac{\pi}{6}} \sec^2 x \, dx = [\tan x]$$

$$= \left[\tan\left(\frac{\pi}{6}\right) - \tan 0\right]$$

$$= \frac{1}{\sqrt{3}}$$

Question: 14

Evaluate:

Solution:

-2

Evaluation:

$$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \operatorname{cosec}^2 x dx = [-\cot x]$$

$$= \left[-\cot\left(\frac{\pi}{4}\right) + \cot\left(-\frac{\pi}{4}\right)\right]$$

$$= \left[-\cot\left(\frac{\pi}{4}\right) - \cot\left(\frac{\pi}{4}\right)\right]$$

$=-2$

Question: 15

Evaluate:

Solution:

$$\left(1-\frac{\pi}{4}\right)$$

Evaluation:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^2 x dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\operatorname{cosec}^2 x - 1) dx$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\operatorname{cosec}^2 x - 1) dx = [-\cot x - x]$$

$$= \left[-\cot\left(\frac{\pi}{2}\right) - \frac{\pi}{2} + \cot\left(\frac{\pi}{4}\right) + \frac{\pi}{4}\right]$$

$$= \left[0 - \frac{\pi}{4} + 1\right]$$

$$= \left[1 - \frac{\pi}{4}\right]$$

Question: 16

Evaluate:

Solution:

$$\left(1-\frac{\pi}{4}\right)$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} \tan^2 x dx = \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) dx$$

$$\int_0^{\frac{\pi}{4}} (\sec^2 x - 1) dx = [\tan x - x]$$

$$= \left[\tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4} - \tan(0) - 0\right]$$

$$= \left[1 - \frac{\pi}{4}\right]$$

Question: 17

Evaluate:

Solution:

$$\frac{\pi}{4}$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} \sin^2 x dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}(1 - \cos 2x) dx$$

$$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - \frac{\sin\pi}{2} - 0 + \frac{\sin 0}{2}\right]$$

$$= \frac{\pi}{4}$$

Question: 18

Evaluate:

Solution:

$$\left(\frac{\pi}{8}+\frac{1}{4}\right)$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} \cos^2 x dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}(1+\cos 2x)dx$$

$$= \frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4}+\frac{\sin(\frac{\pi}{2})}{2}-0-\frac{\sin 0}{2}\right]$$

$$= \frac{\pi}{8}+\frac{1}{4}$$

Question: 19

Evaluate:

Solution:

log 2

Evaluation:

$$\int_0^{\frac{\pi}{3}} tan x dx = log|sec x|$$

$$= log\left|sec\left(\frac{\pi}{3}\right)\right| - ln|cos 0|$$

=log|2|-log|1|

=log2

Question: 20

Evaluate:

Solution:

$$\log\left(\sqrt{2}-1\right)+\log\left(2+\sqrt{3}\right)$$

Evaluation:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} cosec x dx = -log|cosec x + cot x|$$

$$= -log\left|cosec\left(\frac{\pi}{4}\right)+cot\left(\frac{\pi}{4}\right)\right| + log|cosec(\frac{\pi}{6}) + cot(\frac{\pi}{6})|$$

=-log|√2+1|+log|2+√3|

Question: 21

Evaluate:

Solution:

$$\frac{3\sqrt{3}}{8}$$

Evaluation:

$$\int_0^{\frac{\pi}{3}} cos^3 x\, dx = \frac{1}{4}\int_0^{\frac{\pi}{3}}(3cosx + cos3x)dx$$

$$\frac{1}{4}\int_0^{\frac{\pi}{3}}(3cosx - cos3x)dx = \frac{1}{4}\left[3sinx + \frac{sin3x}{3}\right]$$

$$= \frac{1}{4}\left[3sin\left(\frac{\pi}{3}\right) + \frac{sin\pi}{3}\right] - \frac{1}{4}\left[3sin0 + \frac{sin0}{3}\right]$$

$$= \frac{1}{4}\left[\frac{3\sqrt{3}}{2}\right]$$

$$= \frac{3\sqrt{3}}{8}$$

Question: 22

Evaluate:

Solution:

$$\frac{2}{3}$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} sin^3 x\, dx = \frac{1}{4}\int_0^{\frac{\pi}{2}}(3sinx - sin3x)dx$$

$$\frac{1}{4}\int_0^{\frac{\pi}{2}}(3sinx - sin3x)dx = \frac{1}{4}\left[-3cosx + \frac{cos3x}{3}\right]$$

$$= \frac{1}{4}\left[-3cos(\frac{\pi}{2}) + \frac{cos(\frac{3\pi}{2})}{3}\right] - \frac{1}{4}\left[-3cos0 + \frac{cos0}{3}\right]$$

$$= \frac{1}{4}\left[\frac{9-1}{3}\right]$$

$$= \frac{2}{3}$$

Question: 23

Evaluate:

Solution:

$$\left(4 - 3\sqrt{2}\right)$$

Evaluation:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(1 - 3cosx)}{sin^2 x} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(cosec^2(x) - 3cosec(x)cot(x))dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(cosec^2(x) - 3cosec(x)cot(x))dx$$

Question: 24

Evaluate:

Solution:

1

Evaluation:

$$\int_0^{\frac{\pi}{4}} \sqrt{1+\cos 2x}\, dx = \int_0^{\frac{\pi}{4}} \sqrt{2\cos^2 x} dx$$

$$= \sqrt{2}[\sin x]$$

$$= \sqrt{2}\left[\sin\left(\frac{\pi}{4}\right) - \sin 0\right]$$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}}\right]$$

=1

Question: 25

Evaluate:

Solution:

$$\left(\sqrt{2}-1\right)$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} \sqrt{1-\sin 2x}\ dx = \int_0^{\frac{\pi}{4}} \sqrt{\sin^2 x + \cos^2 x - 2\sin x\cos x}\, dx$$

$$= \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\, dx$$

=[sin x + cos x]

$$= \left[\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right) - \cos 0 - \sin 0\right]$$

$$= \left[+\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right]$$

=[√2-1]

Question: 26

Evaluate:

Solution:

2

Evaluation:

$$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+\sin x} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\left(\frac{x}{2}\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2} dx$$

Let $u = \left(\tan\left(\frac{x}{2}\right) + 1\right)$

$$dx = \frac{2}{\sec^2\left(\frac{x}{2}\right)} du$$

$$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+\sin x} = 2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{u^2} du$$

$$= -\frac{2}{u}$$

$$= -\frac{2}{tan\left(\frac{x}{2}\right)+1}$$

=2

Question: 27

Evaluate:

Solution:

$$\frac{1}{2}$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} \frac{dx}{1+cos2x} = \int_0^{\frac{\pi}{4}} \frac{dx}{2cos^2 x}$$

$$\int_0^{\frac{\pi}{4}} \frac{dx}{2cos^2 x} = \int_0^{\frac{\pi}{4}} \frac{1}{2} sec^2 x dx$$

$$\int_0^{\frac{\pi}{4}} \frac{1}{2} sec^2 x dx = \frac{1}{2}[tanx]$$

$$= \frac{1}{2}\left[tan\left(\frac{\pi}{4}\right) - tan0\right]$$

$$= \frac{1}{2}[1]$$

$$= \frac{1}{2}$$

Question: 28

Evaluate:

Solution:

$$\frac{1}{2}$$

Evaluation:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{1-cos2x} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{2sin^2 x}$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{2sin^2 x} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} cosec^2 x dx$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} cosec^2 x dx = \frac{1}{2}[cotx]$$

$$= \frac{1}{2}\left[cot(\frac{\pi}{4}) - cot0\right]$$

$$= \frac{1}{2}[1]$$

$$= \frac{1}{2}$$

Question: 29

Evaluate:

Solution:

$$\frac{3}{5\sqrt{2}}$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} sin2xsin3xdx = \frac{1}{2}\int_0^{\frac{\pi}{4}}(cosx - cos5x)dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}}(cosx - cos5x)dx$$

$$= \frac{1}{2}\left[sinx - \frac{sin5x}{5}\right]$$

$$= \frac{1}{2}\left[sin(\frac{\pi}{4}) - \frac{sin\left(\frac{5\pi}{4}\right)}{5}\right] - \frac{1}{2}\left[sin(0) - \frac{sin(0)}{5}\right]$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2}} + \frac{1}{5\sqrt{2}}\right]$$

$$= \frac{3}{5\sqrt{2}}$$

Question: 30

Evaluate:

Solution:

$$\frac{5}{12}$$

Evaluation:

$$\int_0^{\frac{\pi}{6}} cosxcos2xdx = \frac{1}{2}\int_0^{\frac{\pi}{6}}(cos3x + cosx)dx$$

$$= \frac{1}{2}\left[\frac{sin3x}{3} + sinx\right]$$

$$= \frac{1}{2}\left[\frac{sin\left(\frac{\pi}{2}\right)}{3} + sin\left(\frac{\pi}{6}\right)\right] - 0$$

$$= \frac{1}{2}\left[\frac{1}{3} + \frac{1}{2}\right]$$

$$= \frac{5}{12}$$

Question: 31

Evaluate:

Solution:

$$\frac{-4}{5}$$

Evaluation:

$$\int_0^{\pi} sin2xcos3xdx = \frac{1}{2}\int_0^{\pi}(sin5x - sinx)dx$$

$$= \frac{1}{2}\left[\frac{-cos5x}{5} + cosx\right]$$

$$= \frac{1}{2}\left[-\frac{cos(5\pi)}{5} + cos(\pi)\right] - \frac{1}{2}\left[-\frac{cos(0)}{5} + cos(0)\right]$$

$$= \frac{1}{2}\left[\frac{-(-1)}{5} - 1\right] - \frac{1}{2}\left[-\frac{1}{5} + 1\right]$$

$$= \frac{1}{2}\left[\frac{-4}{5} - \frac{4}{5}\right]$$

$$= \frac{1}{2}2\left(-\frac{4}{5}\right)$$

$$= -\frac{4}{5}$$

Question: 32

Evaluate:

Solution:

2

Explanation:

$$\int_0^{\frac{\pi}{2}} \sqrt{1 + sin(x)}dx = \int_0^{\frac{\pi}{2}} \sqrt{2}cos\left(\frac{2x - \pi}{4}\right)dx$$

$$= 2^{\frac{3}{2}} sin\left(\frac{2x - \pi}{4}\right)$$

$$= 2^{\frac{3}{2}}\left(0 - sin(-\frac{\pi}{4}\right)$$

$$= \frac{2\sqrt{2}}{\sqrt{2}}$$

=2

Question: 33

Evaluate:

Solution:

2

Explanation:

$$\int_0^{\frac{\pi}{2}} \sqrt{1 + cos(x)}dx = \int_0^{\frac{\pi}{2}} \sqrt{2}cos\left(\frac{x}{2}\right)dx$$

$$= 2^{\frac{3}{2}} sin\left(\frac{x}{2}\right)$$

$$= 2^{\frac{3}{2}}\left(sin\left(\frac{\pi}{4}\right) - 0\right)$$

$$= \frac{2\sqrt{2}}{\sqrt{2}}$$

=2

Question: 34

Evaluate:

Solution:

$\left(\frac{2}{3}+2\tan^{-1}2\right)$

Explanation:

$$\int_0^2 \left\{\frac{(x^4+1)}{x^2+1}\right\} dx = \int_0^2 \frac{x^4+2-1}{x^2+1}\ dx$$

$$= \int_0^2 \frac{x^4-1}{x^2+1} dx + \int_0^2 \frac{2}{x^2+1} dx$$

$$= \int_0^2 \frac{(x^2-1)(x^2+1)}{x^2+1} dx + \int_0^2 \frac{2}{x^2+1} dx$$

$$= \int_0^2 (x^2-1) dx + 2tan^{-1}x$$

$$= \left[\frac{x^3}{3} - x + 2tan^{-1}x\right]_0^2$$

$$= \frac{2}{3} + 2tan^{-1}2$$

Question: 35

Evaluate:

Solution:

(2 log 3 – 3 log 2)

Explanation:

$$\int_1^2 \frac{dx}{(x+1)(x+2)} = \int_1^2 \frac{(x+2)-(x+1))}{(x+1)(x+2)} dx$$

$$= \int_1^2 \frac{1}{(x+1)} dx - \int_1^2 \frac{1}{(x+2)} dx$$

$$= [log(x+1) - log(x+2)]_1^2$$

=2log3-3log2

Question: 36

Evaluate:

Solution:

$\frac{1}{2}(\log\ 2+\log\ 3)$

Explanation:

$$\int_1^2 \frac{x+3}{x(x+2)} dx = \int_1^2 \frac{3}{2x} dx - \int_1^2 \frac{1}{x+2} dx$$

$$= \frac{3}{2} log\, x - log(x+2)$$

$$= \frac{1}{2}(log2 + log3)$$

Question: 37

Evaluate:

Solution:

$$\frac{1}{4}(\log 5 - \log 3)$$

Evaluation:

$$\int_3^4 \frac{dx}{x^2-4} = \int_3^4 \frac{1}{(x-2)(x+2)} dx$$

$$= \int_3^4 \frac{1}{4(x-2)} dx - \int_3^4 \frac{1}{4(x+2)} dx$$

$$= \frac{1}{4}\log(x-2) - \frac{1}{4}\log(x+2)$$

$$= \frac{1}{4}log3 - \frac{1}{4}log1 - \frac{1}{4}\log 6 + \frac{1}{4}log5$$

$$= \frac{1}{4}\left(log5 - \log\left(\frac{6}{2}\right)\right)$$

$$= \frac{1}{4}(log5 - log3)$$

Question: 38

Evaluate:

Solution:

$$\log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$$

Evaluation:

$$\int \frac{dx}{\sqrt{x^2+2x+3}} = \int \frac{dx}{\sqrt{(x+1)^2+2}}$$

Substitute:

$$\frac{x+1}{\sqrt{2}} = u$$

$$\therefore\ dx = \sqrt{2}du$$

$$= \int \frac{\sqrt{2}du}{\sqrt{2u^2+2}}$$

$$= log(\sqrt{u^2+1} + u)$$

Undo substitution: $u = \frac{x+1}{\sqrt{2}}$

$$\therefore \int_0^4 \frac{dx}{\sqrt{x^2+4x+3}} = log(\sqrt{(x+1)^2+2} + x + 1)$$

$$= log\left(\sqrt{(4+1)^2+2} + 4 + 1\right) - log(\sqrt{(0+1)^2+2} + 0 + 1)$$

$$= log(5+3\sqrt{3}) - log(1+\sqrt{3})$$

$$= \log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$$

Question: 39

Evaluate:

Solution:

$\log\left(4+\sqrt{15}\right)-\log\left(3+\sqrt{8}\right)$

Evaluation:

$$\int \frac{dx}{\sqrt{x^2+4x+3}} = \int \frac{dx}{\sqrt{(x+2)^2-1}}$$

Substitute:

x+2=u

∴ dx=du

$$= \int \frac{du}{\sqrt{u^2-1}}$$

$$= log(\sqrt{u^2-1}+u)$$

Undo substitution: $u = x + 2$

$$\therefore \int_1^2 \frac{dx}{\sqrt{x^2+4x+3}} = log(\sqrt{(x+2)^2-1}+x+2)$$

$$= log\left(\sqrt{(2+2)^2-1}+2+2\right) - log(\sqrt{(1+2)^2-1}+1+2)$$

=log(4+√15)-log(3+√8)

Question: 40

Evaluate:

Solution:

$$\frac{2}{\sqrt{7}}\left\{ran^{-1}\frac{5}{\sqrt{7}} - \tan^{-1}\frac{1}{\sqrt{7}}\right\}$$

Evaluation:

$$\int_0^1 \frac{1}{2x^2+x+1}dx = \int_0^1 \frac{1}{\left(\left(\sqrt{2x}+\frac{1}{2^{\frac{3}{2}}}\right)2+\frac{7}{8}\right)dx}$$

Substitute 4x+1√7=u

$$\therefore dx = \frac{\sqrt{7}}{4}du$$

Now solving:

$$\int \left(\frac{1}{u^2}+ 1\right)du = \tan^{-1}u$$

$$\frac{2}{\sqrt{7}}\int \frac{1}{u^2+1}du = \frac{2}{\sqrt{7}}\tan^{-1}u$$

$$\therefore \int_0^1 \frac{1}{2x^2+x+1}dx = \frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)$$

$$= \frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{4+1}{\sqrt{7}}\right) - \frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)$$

$$= \frac{2}{\sqrt{7}}\{\tan^{-1}\left(\frac{5}{\sqrt{7}}\right) - \tan^{-1}\left(\frac{1}{\sqrt{7}}\right)\}$$

Question: 41

Evaluate:

Solution:

$$\frac{\pi}{4}(a+b)$$

Evaluation:

$$\int_0^{\frac{\pi}{2}}(acos^2x+bsin^2x)dx=\int_0^{\frac{\pi}{2}}\left[\frac{a}{2}(cos2x+1)+\frac{b}{2}(1-cos2x)\right]dx$$

$$=\left[\frac{a}{2}\left(\frac{sin2x}{2}+x\right)+\frac{b}{2}\left(x-\frac{sin2x}{2}\right)\right]$$

$$=\left[\frac{a}{2}\left(\frac{sin\pi}{2}+\frac{\pi}{2}\right)+\frac{b}{2}\left(\frac{\pi}{2}-\frac{sin\pi}{2}\right)-\frac{a}{2}\left(\frac{sin0}{2}+0\right)-\frac{b}{2}\left(0-\frac{sin0}{2}\right)\right]$$

$$=\left[\frac{a}{2}\left(0+\frac{\pi}{2}\right)+\frac{b}{2}\left(\frac{\pi}{2}-0\right)-\frac{a}{2}(0+0)-\frac{b}{2}(0-0)\right]$$

$$=\frac{\pi}{4}(a+b)$$

Question: 42

Evaluate:

Solution:

$$\frac{-2}{\sqrt{3}}$$

Evaluation:

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}(tanx+cotx)^2dx=\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\left(\frac{tan^2x+1}{tanx}\right)^2dx$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\left(\frac{tan^2x+1}{tanx}\right)^2dx=\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\frac{sec^2x(tan^2x+1)}{tan^2x}dx$$

Substitute:

tan(x)=u

$$\therefore dx=\frac{1}{sec^2(x)}du$$

$$\therefore=\int\frac{(u^2+1)}{u^2}du$$

$$\therefore=u-\frac{1}{u}$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\left(\frac{tan^2x+1}{tanx}\right)^2dx=[tan(x)-cot(x)]$$

$$=\left[tan\left(\frac{\pi}{4}\right)-cot\left(\frac{\pi}{4}\right)-tan\left(\frac{\pi}{3}\right)+cot\left(\frac{\pi}{3}\right)\right]$$

$$=\left[1-1-\sqrt{3}+\frac{1}{\sqrt{3}}\right]$$

$$=-\frac{2}{\sqrt{3}}$$

Question: 43

Evaluate:

Solution:

$$\frac{3\pi}{16}$$

Evaluation:

By reduction formula:

$$\int_0^{\frac{\pi}{2}} cos^4 x dx = \frac{cos^3(x)sin(x)}{4} + \frac{3}{4}\int cos^2 x dx$$

We know that,

$$\int cos^2 x\, dx = \frac{1}{2}\left[\frac{sin2x}{2} + x\right]$$

$$\int_0^{\frac{\pi}{2}} cos^4 x dx = \frac{cos^3(x)sin(x)}{4} + \frac{3}{8}\left[\frac{sin2x}{2} + x\right]$$

$$= \frac{cos^3\left(\frac{\pi}{2}\right) sin\left(\frac{\pi}{2}\right)}{4} + \frac{3}{8}\left[\frac{sin\pi}{2} + \frac{\pi}{2}\right] - \frac{cos^3(0)sin(0)}{4} - \frac{3}{8}\left[\frac{sin0}{2} + 0\right]$$

$$= 0 + \frac{3}{8}\left[0 + \frac{\pi}{2}\right] - 0 - \frac{3}{8}[0 + 0]$$

$$= \frac{3\pi}{16}$$

Question: 44

Evaluate:

Solution:

$$\frac{1}{\sqrt{5}a} \log\left\{\frac{7+3\sqrt{5}}{2}\right\}$$

Evaluation:

Assume that a≠0.

$$\int_0^2 \frac{1}{-x^2 + ax + a^2}\ dx = -\int_0^2 \frac{1}{x^2 - ax - a^2}\ dx$$

$$= \int_0^2 \frac{4}{(2x + (-\sqrt{5} - 1)a)(2x + (\sqrt{5} - 1)a)} dx$$

$$= \int_0^2 \left(\frac{2}{\sqrt{5}a(2x + (-\sqrt{5} - 1)a)} - \frac{2}{\sqrt{5}a(2x + (\sqrt{5} - 1)a)}\right) dx$$

Now,

$$\int \frac{1}{2x + (-\sqrt{5} - 1)a} dx$$

Substitute:

u=2x+(-√5-1)a

$$\therefore dx = \frac{1}{2} du$$

$$= \frac{1}{2}\int \frac{1}{u}\ du$$

$$=\frac{1}{2}logu$$

Undo substitution:

$$u=2x+(-\sqrt{5}-1)a$$

$$\therefore\int\frac{1}{2x+\left(-\sqrt{5}-1\right)a}dx=\frac{1}{2}log(2x+\left(-\sqrt{5}-1\right)a)$$

Now,

$$\int\frac{1}{2x+\left(\sqrt{5}-1\right)a}dx$$

Substitute:

$$u=2x+(\sqrt{5}-1)a$$

$$\therefore dx=\frac{1}{2}du$$

$$=\frac{1}{2}\int\frac{1}{u}\,du$$

$$=\frac{1}{2}logu$$

Undo substitution:

$$u=2x+(\sqrt{5}-1)a$$

$$\therefore\int\frac{1}{2x+\left(\sqrt{5}-1\right)a}dx=\frac{1}{2}log(2x+\left(\sqrt{5}-1\right)a)$$

$$\frac{2}{\sqrt{5}a}\int_0^2\frac{1}{\left(2x+\left(-\sqrt{5}-1\right)a\right)}dx-\frac{2}{\sqrt{5}a}\int_0^2\frac{1}{2x+\left(\sqrt{5}-1\right)a}dx$$

$$=\frac{log\left(2x+\left(-\sqrt{5}-1\right)a\right)}{\sqrt{5}a}-\frac{log\left(2x+\left(\sqrt{5}-1\right)a\right)}{\sqrt{5}a}$$

$$-\int_0^2\frac{1}{x^2-ax-a^2}\,dx=\frac{log\left(2x+\left(\sqrt{5}-1\right)a\right)}{\sqrt{5}a}-\frac{log\left(2x+\left(-\sqrt{5}-1\right)a\right)}{\sqrt{5}a}$$

$$=\frac{log\left(4+\left(\sqrt{5}-1\right)a\right)}{\sqrt{5}a}-\frac{log\left(4+\left(-\sqrt{5}-1\right)a\right)}{\sqrt{5}a}-\frac{log\left(0+\left(\sqrt{5}-1\right)a\right)}{\sqrt{5}a}+\frac{log\left(0+\left(-\sqrt{5}-1\right)a\right)}{\sqrt{5}a}$$

$$=\frac{1}{\sqrt{5}a}log\left(\frac{7+3\sqrt{5}}{2}\right)$$

Question: 45

Evaluate:

Solution:

$$\frac{\pi}{6}$$

Evaluation:

$$\int_{\frac{1}{4}}^{\frac{1}{2}}\frac{dx}{\sqrt{x-x^2}}=\int_{\frac{1}{4}}^{\frac{1}{2}}\frac{1}{\sqrt{\frac{1}{4}-\left(x-\frac{1}{2}\right)^2}}$$

Substitute:

2x-1=u

$\therefore dx = \frac{1}{2} du$

$\int \frac{1}{\sqrt{1-u^2}} du = sin^{-1}(u)$

Undo Substitution:

u=2x-1

$\therefore = \sin^{-1}(2x-1)$

$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{dx}{\sqrt{x-x^2}} = sin^{-1}(2x-1)$

$= sin^{-1}(1-1) - sin^{-1}(\frac{1}{2}-1)$

$= \frac{\pi}{6}$

Question: 46

Evaluate:

Solution:

$\frac{\pi}{8}$

Evaluation:

$\int_0^1 \sqrt{x-x^2}dx = \int_0^1 \sqrt{\frac{1}{4} - \left(x - \frac{1}{2}\right)^2}\ dx$

$= \frac{1}{2}\int_0^1 \sqrt{1-(2x-1)^2}\ dx$

Substitute:

2x-1=u

$\therefore dx = \frac{1}{2} du$

$\therefore \frac{1}{2} \int \sqrt{1-u^2} du$

Substitute:

u=sin(v)

$\therefore \sin^{-1}(u)=v$

$\therefore du=\cos(v)dv$

$= \int cos(v)\sqrt{a - sin^2(v)} dv$

$= \int cos^2(v) dv$

We know that,

$\int cos^2(v)\ dv = \frac{1}{2}\left[\frac{sin(2v)}{2} + v\right]$

Undo Substitution:

v=sin^{-1} (u)sin(sin^{-1} (u))=u$cos(sin^{-1}(u)) = \sqrt{1-u^2}$

$= \frac{sin^{-1}(u)}{2} + \frac{u\sqrt{1-u^2}}{2}$

Undo Substitution:

u=2x-1

$\therefore = \frac{sin^{-1}(2x-1)}{4} + \frac{(2x-1)\sqrt{1-(2x-1)^2}}{4}$

$\frac{1}{2}\int_0^1 \sqrt{1-(2x-1)^2}\,dx \ = \frac{sin^{-1}(2x-1)}{8} + \frac{(2x-1)\sqrt{1-(2x-1)^2}}{8}$

$= \frac{sin^{-1}(2-1)}{8} + \frac{(2-1)\sqrt{1-(2-1)^2}}{8} - \frac{sin^{-1}(0-1)}{8} - \frac{(0-1)\sqrt{1-(0-1)^2}}{8}$

$= \frac{\pi}{16} + 0 - \frac{\pi}{8} - 0$

$= \frac{\pi}{8}$

Question: 47

Evaluate:

Solution:

$\log 2 - \log 3 + \frac{2}{3}$

Evaluation:

$\int_1^3 \frac{1}{x^2(x+1)}dx$

Perform partial fraction decomposition:

$\int_1^3 \frac{1}{x^2(x+1)}dx \ = \int_1^3 \left(\frac{1}{x+1} - \frac{1}{x} + \frac{1}{x^2}\right)dx$

$= \left[log(x+1) - log(x) - \frac{1}{x}\right]$

$= \left[log(4) - log(3) - \frac{1}{3} - log(2) + log(1) + \frac{1}{1}\right]$

$= log(2) - log(3) + \frac{2}{3}$

Question: 48

Evaluate:

Solution:

$\log 6 - \log 5 - \frac{2}{15}$

Evaluation:

$\int_1^2 \frac{1}{x(2x+1)^2}dx \ = \int_1^2 \left(-\frac{2}{2x+1} - \frac{2}{(2x+1)^2} + \frac{1}{x}\right)dx$

$= -2\int_1^2 \frac{1}{2x+1}dx - 2\int_1^2 \frac{1}{(2x+1)^2}dx + \int_1^2 \frac{1}{x}dx$

$$= -2\left[\frac{1}{2}log(2x+1)\right] - 2\left[\frac{-1}{2(2x+1)}\right] + [log(x)]$$

$$= -[log(5)] + \left[\frac{1}{(5)}\right] + [log(2)] + [log(3)] - \left[\frac{1}{(3)}\right] + [log(1)]$$

$$= log(6) - log(5) - \frac{2}{15}$$

Question: 49

Evaluate:

Solution:

1

Evaluation:

$$\int_0^1 xe^x dx = \int_0^1 (x-1+1)e^x dx$$

$=[(x-1)e^x]$

$=[(1-1)\ e^1-(0-1)\ e^0]$

$=1$

Question: 50

Evaluate:

Solution:

$$\left(\frac{\pi^2}{4} - 2\right)$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} x^2 cos(x)dx = x^2 sin(x) - \int 2xsin(x)dx$$

$$\int_0^{\frac{\pi}{2}} x^2 cos(x)dx = [x^2 sin(x) - 2sin(x) - 2xcos(x)]$$

$$= [\left(\frac{\pi}{2}\right)^2 sin\left(\frac{\pi}{2}\right) - 2sin\left(\frac{\pi}{2}\right) - \pi cos\left(\frac{\pi}{2}\right) - (0)^2 sin(0) + 2sin(0) + 0]$$

$$= \left[\frac{\pi^2}{4} - 2 - 0 - 0 + 0 + 0\right]$$

$$= \left(\frac{\pi^2}{4} - 2\right)$$

Question: 51

Evaluate:

Solution:

$$\left(\sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2\right)$$

Evaluation:

From integrate by parts:

$$\int_0^{\frac{\pi}{4}} x^2 sin(x)dx = -x^2 cos(x) - \int -2xcos(x)dx$$

From integrate by parts:

$$\int_0^{\frac{\pi}{4}} x^2 cos(x)dx = [-x^2 cos(x) + 2xsin(x) + 2cos(x)]$$

$$= [2xsin(x) + (2-x^2)cos(x)]$$

$$= [\frac{\pi}{2} sin\left(\frac{\pi}{4}\right) + \left(2 - \frac{\pi^2}{16}\right) cos\left(\frac{\pi}{4}\right) - 2(0)sin(0) - (2-0)cos(0)]$$

$$= \left[\frac{\pi}{2\sqrt{2}} + \frac{2}{\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} + 0 - 0 - 2\right]$$

$$= \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

Question: 52

Evaluate:

Solution:

$$\frac{-\pi}{4}$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} x^2 cos(2x)dx = \frac{x^2 sin(2x)}{2} - \int xsin(x)dx$$

$$\int_0^{\frac{\pi}{2}} x^2 cos(x)dx = \left[\frac{x^2 sin(2x)}{2} - \frac{sin(2x)}{4} + \frac{xcos(2x)}{2}\right]$$

$$= \left[\frac{(\frac{\pi}{2})^2 sin(\pi)}{2} - \frac{sin(\pi)}{4} + \frac{(\frac{\pi}{2})cos(\pi)}{2} - \frac{(0)^2 sin(0)}{2} + \frac{sin(0)}{4} - \frac{(0)cos(0)}{2}\right]$$

$$= \left[0 - 0 - \frac{\pi}{4} - 0 + 0 - 0\right]$$

$$= -\frac{\pi}{4}$$

Question: 53

Evaluate:

Solution:

$$\left(\frac{2}{27} - \frac{\pi^2}{12}\right)$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} x^3 sin(3x)dx = -\frac{x^3 cos(3x)}{3} - \int -x^2 cos(3x)dx$$

$$= -\frac{x^3 cos(3x)}{3} + \frac{x^2 sin(3x)}{3} - \int \frac{2xsin(3x)}{3}dx$$

$$= -\frac{x^3 cos(3x)}{3} + \frac{x^2 sin(3x)}{3} + \frac{2xcos(3x)}{9} + \frac{2}{3}\int -\frac{cos(3x)}{3}dx$$

$$= -\frac{x^3cos(3x)}{3} + \frac{x^2sin(3x)}{3} + \frac{2xcos(3x)}{9} - \frac{2sin(3x)}{27}$$

$$= -0 + \frac{\left(\frac{\pi}{2}\right)^2 sin\left(\frac{3\pi}{2}\right)}{3} + 0 - \frac{2sin\left(\frac{3\pi}{2}\right)}{27} + 0 - 0 - 0 + 0$$

$$= \left(\frac{2}{27} - \frac{\pi^2}{12}\right)$$

Question: 54

Evaluate:

Solution:

$$\left(\frac{\pi^3}{48} - \frac{\pi}{8}\right)$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} x^2cos^2xdx = \int_0^{\frac{\pi}{2}} \frac{x^2}{2}(cos(2x)+1)dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{x^2}{2}cos(2x) + \frac{x^2}{2}\right)dx$$

$$\int_0^{\frac{\pi}{2}} \left(\frac{x^2}{2}cos(2x) + \frac{x^2}{2}\right)dx = \frac{x^2sin(2x)}{2} - \int xsin(2x)dx + \frac{x^3}{6}$$

$$= \frac{x^2sin(2x)}{2} + \frac{xcos(2x)}{4} + \int -\frac{cos(2x)}{2}dx + \frac{x^3}{6}$$

$$= \frac{x^2sin(2x)}{2} + \frac{xcos(2x)}{4} - \frac{sin(2x)}{4} + \frac{x^3}{6}$$

$$= \frac{x^2sin(2x)}{2} + \frac{xcos(2x)}{4} - \frac{sin(2x)}{4} + \frac{x^3}{6}$$

$$= 0 + \frac{\frac{\pi}{2}cos(\pi)}{4} - 0 + \frac{\left(\frac{\pi}{2}\right)^3}{6} - 0 - 0 + 0 - 0$$

$$= \left(\frac{\pi^3}{48} - \frac{\pi}{8}\right)$$

Question: 55

Evaluate:

Solution:

(2 log 2 – 1)

Evaluation:

$$\int_1^2 log(x)dx = xlog(x) - (x)$$

$$= 2log(2) - (2) - 1log(1) + (1)$$

$$= 2log(2) - 1$$

Question: 56

Evaluate:

Solution:

$\frac{3}{4}\log 3 - \log 2$

Evaluation:

$$\int_1^3 \frac{log(x)}{(1+x)^2}dx = -\frac{log(x)}{1+x} - \int\left(-\frac{1}{x(1+x)}\right)dx$$

Now,

$$\int\left(-\frac{1}{x(1+x)}\right)dx = -\int\left(\frac{1}{x^2\left(\frac{1}{x}+1\right)}\right)dx$$

Let,

$$\frac{1}{x}+1 = u$$

$\therefore$dx=-x^2 du

$$\therefore -\int\left(\frac{1}{x^2\left(\frac{1}{x}+1\right)}\right)dx = \int\frac{1}{u}du$$

$$= log(u)$$

Undo substitution:

$$u = \frac{1}{x}+1$$

$$\int_1^3 \frac{log(x)}{(1+x)^2}dx = -\frac{log(x)}{1+x} + log\left(\frac{1}{x}+1\right)$$

$$= -\frac{log(3)}{4} + log\left(\frac{4}{3}\right) + \frac{log(1)}{2} - log(2)$$

$$= -\frac{log(3)}{4} + log(4) + log(3) - log2$$

$$= \frac{3}{4}log3 - log2$$

Question: 57

Evaluate:

Solution:

$$\left(\frac{e^2}{2} - e\right)$$

Correct answer is $\frac{e^2}{2}$

Evaluation:

Let,

log(x)=u

$\rightarrow$x=e^u

$\rightarrow$dx=e^u du

$$\int\left\{\frac{1}{u}-\frac{1}{u^2}\right)e^u du = \frac{e^u}{u}$$

Undo substitution:

$u = log(x)$

$$\int_0^{e^2} \left\{\frac{1}{log(x)} - \frac{1}{log(x)^2}\right\} dx = \frac{x}{log(x)}$$

$$= \frac{e^2}{log(e^2)} - 0$$

$$= \frac{e^2}{2}$$

Question: 58

Evaluate:

Solution:

e^e

Evaluation:

$$\int_1^e e^x \left(\frac{(1 + xlog(x))}{x}\right) dx = \int_1^e e^x \left(\frac{1}{x} + log(x)\right) dx$$

$=\log(x)\ e^x$

$=\log(e)\ e^e - \log(1)\ e^1$

$=e^e$

Question: 59

Evaluate:

Solution:

$$\left(\frac{e}{2} - 1\right)$$

Evaluation:

$$\int_0^1 \frac{xe^x}{(1+x)^2} dx$$

From Integrates by parts:

$$= -\frac{xe^x}{x+1} - \int \frac{-xe^x - e^x}{x+1}\ dx$$

$$\therefore \int \frac{-xe^x - e^x}{x+1}\ dx = \int -e^x dx$$

$=-e^x$

$$\int_0^1 \frac{xe^x}{(1+x)^2} dx = \left[-\frac{xe^x}{x+1} - e^x\right]$$

$$= \left[-\frac{1e^1}{1+1} - e^1 - \frac{0}{1+0} + e^0\right]$$

$$= \left[-\frac{e}{2} + e + 0 - 1\right]$$

$$= \left[\frac{e}{2} - 1\right]$$

Question: 60

Evaluate:

Solution:

(1 – log 2)

Evaluation:

$$\int_0^{\frac{\pi}{2}} 2tan^3xdx = 2\int_0^{\frac{\pi}{2}} tan^2xtanxdx$$

$$= 2\int_0^{\frac{\pi}{2}} tan^2xtanxdx$$

$$= 2\int_0^{\frac{\pi}{2}} (sec^2x - 1)tanxdx$$

Substitute:

$$sec(x) = u$$

$$\therefore \mathrm{dx} = \frac{1}{\mathrm{sec(x)tan(x)}}\mathrm{du}$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{(u^2 - 1)}{u} du$$

$$= 2\int_0^{\frac{\pi}{2}} \left(u - \frac{1}{u}\right) du$$

$$= 2\int_0^{\frac{\pi}{2}} \left(u - \frac{1}{u}\right) du$$

$$= 2\left[\frac{\mathrm{u}^2}{2} - \mathrm{logu}\right]$$

Undo substitution:

$$u = sec(x)$$

$$\therefore \int_0^{\frac{\pi}{2}} 2tan^3xdx = 2\left[\frac{\mathrm{sec^2x}}{2} - \mathrm{log(secx)}\right]$$

$$= 2\left[\frac{\sec^2\left(\frac{\pi}{2}\right)}{2} - \log\left(\sec\left(\frac{\pi}{2}\right)\right) - \frac{\sec^2(0)}{2} + \log(\sec(0))\right]$$

$$= 2\left[\frac{1}{2} - \log(1)\right]$$

=1-log2

Question: 61

Evaluate:

Solution:

$$5 - \frac{5}{2}\left(9\log\frac{5}{4} - \log\frac{3}{2}\right)$$

Explanation:

$$\int_1^2 \frac{5x^2}{(x^2 + 4x + 3)} dx = 5[\int_1^2 \frac{x^2}{(x+3)(x+1)} dx]$$

$$= 5[\int_1^2 \left(1 - \frac{9}{2(x+3)} + \frac{1}{2(x+1)}\right) dx]$$

$= 5\left[x - \frac{9}{2}log(x+3) + \frac{1}{2}log(x+1)\right]_1^2$

$= 5[2 - \frac{9}{2}log5 + \frac{1}{2}log3 - 1 + \frac{9}{2}log4 - \frac{1}{2}log2]$

$= 5[1 - \frac{9}{2}\log\left(\frac{5}{4}\right) + \frac{1}{2}\log\left(\frac{3}{2}\right)$

$= 5 - \frac{5}{2}\left(9\log\left(\frac{5}{4}\right) - \log\left(\frac{3}{2}\right)\right)$

Exercise : 16B

Question: 1

Evaluate the foll

Solution:

Let $I = \int_0^1 \frac{1}{2x-3}dx$

Let 2x-3=t

⇒ 2dx=dt.

Hence,

$I = \frac{1}{2}\int_0^1 \frac{1}{t}dt = \frac{1}{2}\log_e|t|$

$= \frac{1}{2}\log_e|2x-3|\Big|_0^1$

$\Rightarrow I = \frac{1}{2}\log_e 1 - \frac{1}{2}\log_e 3 = \frac{1}{2}\log_e \frac{1}{3}$

$= -\frac{1}{2}\log_e 3$

$(Since\ \ log_a \frac{1}{b} = -log_a b)$

Question: 2

Evaluate the foll

Solution:

Let $I = \int_0^1 \frac{2x}{1+x^2}dx$

Let $1+x^2=t$

⇒ 2xdx=dt.

Also,

when x=0, t=1

and

when x=1, t=2

Hence, $I = \int_1^2 \frac{1}{t}dt = \log_e|t|\,\Big|_1^2$

$= \log_e 2 - \log_e 1$

$= \log_e 2$

Question: 3

Evaluate the foll

Solution:

Let $I = \int_1^2 \frac{3x}{9x^2-1} dx$

Let $9x^2-1=t$

$\Rightarrow$ 18xdx=dt.

Also,

when x=1, t=8

and

when x=2, t=35.

Hence,

$I = \frac{1}{6}\int_8^{35}\frac{1}{t} dt = \frac{1}{6}\log_e t \Big|_8^{35} = \frac{1}{6}(\log_e 35 - \log_e 8)$

Question: 4

Evaluate the foll

Solution:

Let $I = \int_0^1 \frac{tan^{-1}x}{1+x^2} dx$

Let $\tan^{-1}x=t$

$\Rightarrow \frac{1}{1+x^2} dx = dt.$

Also, when x=0, t=0

and when x=1, $t = \frac{\pi}{4}$

Hence,

$I = \int_0^{\frac{\pi}{4}} t\, dt = \frac{1}{2}t^2 \Big|_0^{\frac{\pi}{4}} = \frac{\pi^2}{32}$

Question: 5

Evaluate the foll

Solution:

Let $I = \int_0^1 \frac{e^x}{1+e^{2x}} dx$

Let $e^x=t$

$\Rightarrow e^x$ dx=dt.

Also,

when x=0, t=1

and

when x=1, t=e.

Hence,

$I = \int_1^e \frac{1}{1+t^2} dt = tan^{-1}t\Big|_1^e$

$= tan^{-1}e - \frac{\pi}{4}$

Question: 6

Evaluate the foll

Solution:

Let $I = \int_0^1 \frac{2x}{1+x^4} dx$

Let $x^2=t$

$\Rightarrow$ 2xdx=dt.

Also,

when x=0, t=0

and

when x=1, t=1.

Hence,

$I = \int_0^1 \frac{1}{1+t^2} dt$

$= tan^{-1} t \Big|_0^1$

$= \frac{\pi}{4}$

Question: 7

Evaluate the foll

Solution:

Let $I = \int_0^1 xe^{x^2}\, dx$

Let $x^2=t$

$\Rightarrow$ 2xdx=dt.

Also,

when x=0, t=0

and

when x=1, t=1.

Hence,

$I = \frac{1}{2}\int_0^1 e^t\, dt$

$= \frac{1}{2} e^t \Big|_0^1$

$= \frac{1}{2}(e-1)$

Question: 8

Evaluate the foll

Solution:

Let $I = \int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx$

Let $\frac{1}{x} = t$

$\Rightarrow \frac{-1}{x^2} dx = dt.$

Also,

when x=1, t=1

and

when x=2, $t = \frac{1}{2}$.

Hence,

$$I = -\int_{1}^{\frac{1}{2}} e^t \, dt$$

$$= -e^t \Big|_{1}^{\frac{1}{2}}$$

$$= e - \sqrt{e}$$

Question: 9

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{6}} \frac{cosx}{3+4sinx} dx$

Let 3+4sinx=t

⇒ 4cosxdx=dt.

Also,

when x=0, t=3

and

when $x = \frac{\pi}{6}$, t=5.

Hence,

$I = \frac{1}{4}\int_3^5 \frac{1}{t} dt$

$$= \frac{1}{4}\log_e t \Big|_3^5$$

$$= \frac{1}{4}(\log_e 5 - \log_e 3)$$

Question: 10

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \frac{sinx}{1+cos^2 x} dx$

Let cos x=t

⇒ -sin x dx=dt.

Also,

when x=0, t=1

and

when $x = \frac{\pi}{2}$, t=0.

Hence,

$I = -\int_1^0 \frac{1}{1+t^2} dt$

$= -tan^{-1}t\Big|_1^0$

$= \frac{\pi}{4}$

Question: 11

Evaluate the foll

Solution:

Let $I = \int_0^1 \frac{1}{e^x+e^{-x}} dx = \int_0^1 \frac{e^x}{1+e^{2x}} dx$

Let e^x=t

⇒ e^x dx=dt.

Also,

when x=0, t=1

and

when x=1, t=e.

Hence,

$I = \int_1^e \frac{1}{1+t^2} dt$

$= tan^{-1}t\Big|_1^e$

$= tan^{-1}e - \frac{\pi}{4}$

Question: 12

Evaluate the foll

Solution:

Let $I = \int_{\frac{1}{e}}^{e} \frac{1}{x(\log_e x)^{\frac{1}{3}}} dx$

Let $\log_e x = t$

$\Rightarrow \frac{1}{x} dx = dt.$

Also,

when $x = \frac{1}{e}$, t=-1

and

when x=e, t=1.

Hence,

$I = \int_{-1}^{1} \frac{1}{t^{\frac{1}{3}}} dt$

$= \frac{3}{2} t^{\frac{2}{3}} \Big|_{-1}^{1}$

$= \frac{3}{2}(1-1)$

=0

Question: 13

Evaluate the foll

Solution:

Let $I = \int_0^1 \frac{\sqrt{tan^{-1}x}}{1+x^2} dx$

Let $\tan^{-1}x=t$

$\Rightarrow \frac{1}{1+x^2} dx = dt.$

Also,

when x=0, t=0

and

when x=1, $t = \frac{\pi}{4}$

Hence,

$I = \int_0^{\frac{\pi}{4}} \sqrt{t}\, dt$

$= \frac{2}{3} t^{\frac{3}{2}} \Big|_0^{\frac{\pi}{4}}$

$= \frac{\pi^{\frac{3}{2}}}{12}$

Question: 14

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \frac{sinx}{\sqrt{1+cosx}} dx$

Let 1+cos x=t

⇒ -sin x dx=dt.

Also, when x=0, t=2

and

when $x = \frac{\pi}{2}$, t=1

Hence,

$I = -\int_2^1 \frac{1}{\sqrt{t}} dt$

$= -2\sqrt{t} \Big|_2^1$

=2(√2-1)

Question: 15

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \sqrt{sinx}\, cos^5 x dx$

Let sinx=t

⇒ cos x dx=dt.

Also,

when x=0, t=0

and

when $x = \frac{\pi}{2}$, t=1.

Consider $\cos^5 x = \cos^4 x \times \cos x = (1-\sin^2 x)^2 \times \cos x$ (Using $\sin^2 x + \cos^2 x = 1$)

Hence,

$I = \int_0^1 \sqrt{x}\,(1-x^2)^2 dx$

$= \int_0^1 \sqrt{x}\,dx + \int_0^1 x^{\frac{9}{2}}\,dx - 2\int_0^1 x^{\frac{5}{2}}\,dx$

$\Rightarrow I = \frac{2}{3} t^{\frac{3}{2}}\Big|_0^1 + \frac{2}{11} t^{\frac{11}{2}}\Big|_0^1 - \frac{4}{7} t^{\frac{7}{2}}\Big|_0^1$

$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$

$= \frac{64}{231}$

Question: 16

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \frac{sinxcosx}{1+sin^4x} dx$

Let $\sin^2 x$=t

⇒ 2sin x cos x=dt.

Also,

when x=0, t=0

and

when $x = \frac{\pi}{2}$, t=1.

Hence,

$I = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{1}{1+t^2} dt$

$= \frac{1}{2} tan^{-1} t\Big|_0^1$

$= \frac{\pi}{8}$

Question: 17

Evaluate the foll

Solution:

Let $I = \int_0^a \sqrt{a^2 - x^2} dx$

Let x=a sin t

⇒ a cos t dt=dx.

Also,

when x=0, t=0

and

when x=a, $t = \frac{\pi}{2}$.

Hence,

$$I = \int_0^{\frac{\pi}{2}} \sqrt{a^2 - a^2 sin^2 t}\, acost\, dt = a^2 \int_0^{\frac{\pi}{2}} cos^2 tdt$$

Using $cos^2 t = \frac{1+cos2t}{2}$, we get

$$I = \frac{a^2}{2}\int_0^{\frac{\pi}{2}}(1 + cos2t)dt$$

$$= \frac{a^2}{2}\left(t + \frac{sin2t}{2}\right)\Big|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi a^2}{4}$$

Question: 18

Evaluate the foll

Solution:

Let $I = \int_0^{\sqrt{2}} \sqrt{2 - x^2} dx$

Consider, $I = \int_0^{a} \sqrt{a^2 - x^2} dx$

Let x=a sin t

⇒ a cos t dt=dx.

Also, when x=0, t=0

and when x=a, $t = \frac{\pi}{2}$.

Hence,

$$I = \int_0^{\frac{\pi}{2}} \sqrt{a^2 - a^2 sin^2 t}\, acost\, dt = a^2 \int_0^{\frac{\pi}{2}} cos^2 tdt$$

Using $cos^2 t = \frac{1+cos2t}{2}$, we get

$$I = \frac{a^2}{2}\int_0^{\frac{\pi}{2}}(1 + cos2t)dt$$

$$= \frac{a^2}{2}\left(t + \frac{sin2t}{2}\right)\Big|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi a^2}{4}$$

Here $a = \sqrt{2}$, hence $I = \frac{\pi}{2}$

Question: 19

Evaluate the foll

Solution:

Let $I = \int_0^a \frac{x^4}{\sqrt{a^2-x^2}} dx$

Let x=a sin t

⇒ a cos t dt=dx.

Also, when x=0, t=0

and when x=a, $t = \frac{\pi}{2}$.

Hence,

$I = \int_0^{\frac{\pi}{2}} \frac{a^4 sin^4 t}{\sqrt{a^2-a^2 sin^2 t}} acostdt$

$= a^4 \int_0^{\frac{\pi}{2}} sin^4 tdt$

Using $sin^2 t = \frac{1-cos2t}{2}$, we get

$I = a^4 \int_0^{\frac{\pi}{2}} \left(\frac{1-cos2t}{2}\right)^2 dt$

$= \frac{a^4}{4} \int_0^{\frac{\pi}{2}} (1 + cos^2 2t - 2cos2t)dt$

$\Rightarrow I = \frac{a^4}{4} \left(t \Big|_0^{\frac{\pi}{2}} - sin2t \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \left(\frac{1+cos4t}{2}\right) dt \right)$

$\left(Using\ cos^2 t = \frac{1+cos2t}{2} \right)$

Hence,

$I = \frac{\pi a^4}{8} + \frac{a^4}{4} \times \frac{t}{2} \Big|_0^{\frac{\pi}{2}} + \frac{a^4}{32} sin4t \Big|_0^{\frac{\pi}{2}}$

$= \frac{3\pi a^4}{16}$

Question: 20

Evaluate the foll

Solution:

Let $I = \int_0^a \frac{x}{\sqrt{a^2+x^2}} dx$

Let $a^2+x^2=t^2$

⇒ x dx=t dt.

Also, when x=0, t=a

and when x=a, $t = \sqrt{2}a$.

Hence,

$I = \int_a^{\sqrt{2}a} \frac{t}{\sqrt{t^2}} dt$

$= t \Big|_a^{\sqrt{2}a}$

=a(√2-1)

Question: 21

Evaluate the foll

Solution:

Let $I = \int_0^2 x\sqrt{2-x}dx$

Using the property that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$$I = \int_0^2 (2-x)\sqrt{x}dx$$

$$= \int_0^2 2\sqrt{x}dx - \int_0^2 x^{\frac{3}{2}}dx$$

$$= 2 \times \frac{2}{3}x^{\frac{3}{2}}\Big|_0^2 - \frac{2}{5}x^{\frac{5}{2}}\Big|_0^2$$

Hence,

$$I = 2\sqrt{2}\left(\frac{4}{3} - \frac{4}{5}\right)$$

$$= \frac{16}{15}\sqrt{2}$$

Question: 22

Evaluate the foll

Solution:

Let $I = \int_0^1 sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

Let $f(x) = sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Let x=tanθ

⇒ θ=$\tan^{-1}$x

$$\Rightarrow f(x) = sin^{-1}\left(\frac{2tan\theta}{1+tan^2\theta}\right)$$

$$= sin^{-1}\left(\frac{2tan\theta}{sec^2\theta}\right)$$

=$\sin^{-1}$ (2sinθcosθ)

=$\sin^{-1}$ (sin2θ)

Hence f(x)=2θ

=2$\tan^{-1}$x

Hence $I = 2\int_0^1 1 \times tan^{-1}x dx$

Using integration by parts, we get

$$I = 2xtan^{-1}x\Big|_0^1 - \int_0^1 \frac{2x}{1+x^2}dx$$

$= \frac{\pi}{2} - \int_0^1 \frac{2x}{1+x^2}dx$ -(1)

Let $I' = \int_0^1 \frac{2x}{1+x^2}dx$

Let 1+x^2=t

⇒ 2x dx=dt.

Also, when x=0, t=1

and when x=1, t=2

Hence,

$I' = \int_1^2 \frac{1}{t} dt = \log_e |t| \,|_1^2$

$= \log_e 2 - \log_e 1$

$= \log_e 2$ –(2)

Substituting value of (2) in (1), we get

$I = \frac{\pi}{2} - \log_e 2$

Question: 23

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \sqrt{1 + cosx}\ dx$

Using $1 + cosx\ = 2cos^2 \frac{x}{2}$, we get

$I = \sqrt{2} \int_0^{\frac{\pi}{2}} \cos\left(\frac{x}{2}\right)\ dx$

$= 2\sqrt{2} \sin\left(\frac{x}{2}\right) \Big|_0^{\frac{\pi}{2}}$

=2

Question: 24

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \sqrt{1 + sinx}\ dx$

Using $sin^2 \frac{x}{2} + cos \frac{x}{2} = 1$ and $sinx\ = 2sin \frac{x}{2} cos \frac{x}{2}$

$I = \int_0^{\frac{\pi}{2}} \sqrt{\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)^2}\ dx$

$= \int_0^{\frac{\pi}{2}} \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) dx$

$= -2\cos\left(\frac{x}{2}\right) \Big|_0^{\frac{\pi}{2}} + 2\sin\left(\frac{x}{2}\right) \Big|_0^{\frac{\pi}{2}}$

=-(√2-2) +(√2)

=2

Question: 25

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2 cos^2 x + b^2 sin^2 x} dx$

Dividing by $\cos^2 x$ in numerator and denominator, we get

$$I = \int_0^{\frac{\pi}{2}} \frac{sec^2 x}{a^2 + b^2 tan^2 x} dx$$

Let tan x=t

⇒ $\sec^2 x dx = dt$

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2 + b^2 t^2} dt = \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{1}{\frac{a^2}{b^2} + t^2} dt$$

Let $t = \frac{a}{b} tan\theta = tanx$

$$I = \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{\frac{a}{b} sec^2\theta}{\frac{a^2}{b^2} + \frac{a^2}{b^2} tan^2\theta} d\theta$$

$$= \frac{1}{ab} \theta$$

$$= \frac{1}{ab} tan^{-1} \left(\frac{b}{a} tanx\right) \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2ab}$$

Question: 26

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \frac{1}{1+cos^2 x} dx$

Dividing by $\cos^2 x$ in numerator and denominator, we get

$$I = \int_0^{\frac{\pi}{2}} \frac{sec^2 x}{sec^2 x + tan^2 x} dx = \int_0^{\frac{\pi}{2}} \frac{sec^2 x}{1 + 2tan^2 x} dx$$

Consider $I = \int_0^{\frac{\pi}{2}} \frac{sec^2 x}{a^2 + b^2 tan^2 x} dx$

Let tan x=t

⇒ $\sec^2 x dx = dt$

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2 + b^2 t^2} dt$$

$$= \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{1}{\frac{a^2}{b^2} + t^2} dt$$

Let $t = \frac{a}{b} tan\theta$

=tan x

$$I = \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{\frac{a}{b} sec^2\theta}{\frac{a^2}{b^2} + \frac{a^2}{b^2} tan^2\theta} d\theta$$

$= \frac{1}{ab}\theta = \frac{1}{ab}tan^{-1}\left(\frac{b}{a}tanx\right)\Big|_{0}^{\frac{\pi}{2}}$

$= \frac{\pi}{2ab}$

Here, a=1 and b=√2

Hence,

$I = \frac{\pi}{2\sqrt{2}}$

Question: 27

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \frac{1}{4+9cos^2x}dx$

Dividing by cos^2x in numerator and denominator, we get

$I = \int_0^{\frac{\pi}{2}} \frac{sec^2x}{4sec^2x + 9tan^2x}dx$

$= \int_0^{\frac{\pi}{2}} \frac{sec^2x}{4 + 13tan^2x}dx$

Consider $I = \int_0^{\frac{\pi}{2}} \frac{sec^2x}{a^2+b^2tan^2x}dx$

Let tan x=t

⇒ $sec^2xdx=dt$

$I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2 + b^2t^2}dt$

$= \frac{1}{b^2}\int_0^{\frac{\pi}{2}} \frac{1}{\frac{a^2}{b^2} + t^2}dt$

Let $t = \frac{a}{b}tan\theta$

=tan x

$I = \frac{1}{b^2}\int_0^{\frac{\pi}{2}} \frac{\frac{a}{b}sec^2\theta}{\frac{a^2}{b^2} + \frac{a^2}{b^2}tan^2\theta}d\theta$

$= \frac{1}{ab}\theta$

$= \frac{1}{ab}tan^{-1}\left(\frac{b}{a}tanx\right)\Big|_{0}^{\frac{\pi}{2}}$

$= \frac{\pi}{2ab}$

Here, a=2 and b=√13

Hence,

$I = \frac{\pi}{4\sqrt{13}}$

Question: 28

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \frac{1}{5+4sinx} dx$

Using $sinx = \frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$, we get

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{5 + 4\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2\left(\frac{x}{2}\right)}{5 + 5\tan^2\left(\frac{x}{2}\right) + 8\tan\left(\frac{x}{2}\right)} dx$$

Let $tan\left(\frac{x}{2}\right) = t$

$\Rightarrow \frac{1}{2} sec^2\left(\frac{x}{2}\right) dx = dt,$

when x=0, t=0 and when $x = \frac{\pi}{2}$, t=1.

Hence, $I = \int_0^1 \frac{2}{5+5t^2+8t} dt$

$$= \frac{2}{5}\int_0^1 \frac{1}{t^2 + \frac{8}{5}t + \frac{16}{25} + \frac{9}{25}} dt$$

$$= \frac{2}{5}\int_0^1 \frac{1}{\left(t+\frac{4}{5}\right)^2 + \frac{9}{25}} dt$$

Let $t + \frac{4}{5} = u$

$\Rightarrow$ dt=du.

When t=0, $u = \frac{4}{5}$ and when t=1, $u = \frac{9}{5}$.

$$I = \frac{2}{5}\int_{\frac{4}{5}}^{\frac{9}{5}} \frac{1}{(u)^2 + \frac{9}{25}} du$$

$$= \frac{2}{5} \times \frac{5}{3} tan^{-1}\left(\frac{5x}{3}\right)\Bigg|_{\frac{4}{5}}^{\frac{9}{5}}$$

$$= \frac{2}{3}\left(tan^{-1}3 - tan^{-1}\left(\frac{4}{3}\right)\right)$$

$$= \frac{2}{3} \times tan^{-1}\left(\frac{3-\frac{4}{3}}{5}\right)$$

$= \frac{2}{3} tan^{-1}\left(\frac{1}{3}\right)$

$$\left(Using\ tan^{-1}x - tan^{-1}y = tan^{-1}\left(\frac{x-y}{1+xy}\right)\right)$$

Question: 29

Evaluate the foll

Solution:

Let $I=\int_0^{\pi}\frac{1}{6-cosx}dx$

Using $cosx=\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$, we get

$$I=\int_0^{\pi}\frac{1}{6-\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}dx$$

$$=\int_0^{\pi}\frac{\sec^2\left(\frac{x}{2}\right)}{5+7\tan^2\left(\frac{x}{2}\right)}dx$$

Let $tan\left(\frac{x}{2}\right)=t$

$\Rightarrow\ \frac{1}{2}sec^2\left(\frac{x}{2}\right)dx=dt,$

when x=0, t=0 and when x=π, t=∞.

Hence, $I=\int_0^{\infty}\frac{2}{5+7t^2}dt$

$$=\frac{2}{7}\int_0^{\infty}\frac{1}{t^2+\frac{5}{7}}dt$$

$$=\frac{2}{7}\times\sqrt{\frac{7}{5}}tan^{-1}\left(\sqrt{\frac{7}{5}}x\right)\Big|_0^{\infty}$$

$$\Rightarrow\ I=\frac{2}{\sqrt{35}}\left(\frac{\pi}{2}-0\right)$$

$$=\frac{\pi}{\sqrt{35}}$$

Question: 30

Evaluate the foll

Solution:

Let $I=\int_0^{\pi}\frac{1}{5+4cosx}dx$

Using $cosx=\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$, we get

$$I=\int_0^{\pi}\frac{1}{5+4\times\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}dx$$

$$=\int_0^{\pi}\frac{\sec^2\left(\frac{x}{2}\right)}{9+\tan^2\left(\frac{x}{2}\right)}dx$$

Let $tan\left(\frac{x}{2}\right)=t$

$\Rightarrow \frac{1}{2} sec^2\left(\frac{x}{2}\right) dx = dt,$

when x=0, t=0 and when x=π, t=∞.

Hence, $I = \int_0^\infty \frac{2}{9+t^2} dt$

$$= 2\int_0^\infty \frac{1}{9+t^2} dt$$

$$= 2 \times \frac{1}{3} tan^{-1}\left(\frac{x}{3}\right)\Big|_0^\infty$$

$$\Rightarrow\ I = \frac{2}{3}\left(\frac{\pi}{2} - 0\right)$$

$$= \frac{\pi}{3}$$

Question: 31

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \frac{1}{cosx+2sinx} dx$

Using $sinx = \frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$

And

$cosx = \frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)},$

we get

$$\Rightarrow\ I = \int_0^{\frac{\pi}{2}} \frac{1}{\frac{1-tan^2\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)} + 2\frac{2\,tan\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2\left(\frac{x}{2}\right)}{1-\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} dx$$

Let $tan\left(\frac{x}{2}\right) = t$

$\Rightarrow \frac{1}{2} sec^2\left(\frac{x}{2}\right) dx = dt,$

when x=0, t=0

and when $x = \frac{\pi}{2}$, t=1.

Hence,

$I = \int_0^1 \frac{2}{1-t^2+4t} dt$

$$= -2\int_0^1 \frac{1}{t^2-4t+4-5} dt$$

$$= -2\int_0^1 \frac{1}{(t-2)^2-5} dt$$

Let t-2=u

$\Rightarrow$ dt=du.

Also, when t=0, u=-2

and when t=1, u=-1.

$$\Rightarrow\ I = -2\int_{-2}^{-1} \frac{1}{u^2-5}\,dt$$

$$= -2 \times \frac{1}{2\sqrt{5}} \log_e \left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right| \Big|_{-2}^{-1}$$

$$\left(Using\ \int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\,log_e\left|\frac{x-a}{x+a}\right|\right)$$

Hence,

$$I = -\frac{1}{\sqrt{5}}\left(\log_e\left|\frac{-1-\sqrt{5}}{-1+\sqrt{5}}\right| - \log_e\left|\frac{-2-\sqrt{5}}{-2+\sqrt{5}}\right|\right)$$

$$= \frac{-1}{\sqrt{5}}\left(\log_e\left|\frac{\sqrt{5}+1}{\sqrt{5}-1}\right| \times \left|\frac{\sqrt{5}-2}{2+\sqrt{5}}\right|\right)$$

$$\left(Using\ \ log_e\,a - log_e\,b = log_e\frac{a}{b}\right)$$

$$\Rightarrow\ I = \frac{-1}{\sqrt{5}}\left(log_e\left|\frac{3-\sqrt{5}}{3+\sqrt{5}}\right|\right)$$

$$= \frac{-2}{\sqrt{5}}\left(\log_e\left(\frac{3-\sqrt{5}}{2}\right)\right)$$

(Using $\log_e a^b = b\log_e a$)

Question: 32

Evaluate the foll

Solution:

Let $I = \int_0^{\pi} \frac{1}{3+cosx+2sinx}\,dx$

Using $sinx = \frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$

And

$cosx = \frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$,

we get

$$\Rightarrow\ I = \int_0^{\pi} \frac{1}{3+\frac{1-tan^2\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}+2\frac{2\,tan\left(\frac{x}{2}\right)}{1+tan^2\left(\frac{x}{2}\right)}}\,dx$$

$$= \int_0^{\pi} \frac{\sec^2\left(\frac{x}{2}\right)}{4+2\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)}\,dx$$

Let $tan\left(\frac{x}{2}\right) = t$

$$\Rightarrow\ \frac{1}{2}sec^2\left(\frac{x}{2}\right)dx = dt,$$

when x=0, t=0

and when $x = \pi$, t=∞.

Hence,

$$I = \int_0^{\infty} \frac{1}{(t+1)^2+1} dt$$

Let t+1=u

⇒ dt=du.

Also, when t=0, u=1

and when t=∞, u=∞.

$$I = \int_1^{\infty} \frac{1}{u^2+1} dt$$

$$= tan^{-1} u \Big|_1^{\infty}$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{\pi}{4}$$

Question: 33

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{4}} \frac{tan^3 x}{1+cos2x} dx$

Using 1+cos2x=2cos^{2}x, we get

$$I = \frac{1}{2}\int_0^{\frac{\pi}{4}} tan^3 x sec^2 x\, dx$$

Let tan x=t

⇒ sec^{2}xdx=dt.

when x=0, t=0

and when $x = \frac{\pi}{4}$, t=1.

$$= \frac{1}{2}\int_0^1 t^3\, dt = \frac{t^4}{8}\Big|_0^1$$

$$= \frac{1}{8}$$

Question: 34

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \frac{sinxcosx}{cos^2 x+3cosx+2} dx$

Let cos x=t

⇒ -sin x dx=dt.

Also, when x=0, t=1

and when $x = \frac{\pi}{2}$, t=0.

Hence,

$I = -\int_1^0 \frac{t}{t^2+3t+2} dt$

$= -\int_1^0 \frac{2(t+1)-(t+2)}{(t+1)(t+2)} dt$

$= -\int_1^0 \frac{2}{(t+2)} dt + \int_1^0 \frac{1}{(t+1)} dt$

$\Rightarrow\ I = -2\, log_e(t+2) \Big|_1^0 + log_e(t+1) \Big|_1^0$

$= -2\log_e 2 + 2\log_e 3 - \log_e 2$

Hence $I = \log_e 9 - \log_e 8$

$(Using\ b log_e a = log_e a^b\ and\ log_e a + log_e b = log_e ab)$

Question: 35

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \frac{sin2x}{sin^4x+cos^4x} dx$

Using sin 2x =2 sin x cos x, we get

$I = \int_0^{\frac{\pi}{2}} \frac{2sinxcosx}{cos^4x(tan^4x+1)} dx$

$= 2\int_0^{\frac{\pi}{2}} \frac{tanxsec^2x}{(tan^4x+1)} dx$

Let tan x=t

$\Rightarrow$ $sec^2xdx=dt$.

Also, when x=0, t=0

and when $x = \frac{\pi}{2}$, t=∞.

Hence, $2\int_0^{\infty} \frac{t}{(t^4+1)} dt$

Let $x^2=t$

$\Rightarrow$ 2xdx=dt.

Also, when x=0, t=0

and when x=∞, t=∞.

Hence, $I = \int_0^{\infty} \frac{1}{1+t^2} dt$

$= tan^{-1}t \Big|_0^{\infty}$

$= \frac{\pi}{2}$

Question: 36

Evaluate the foll

Solution:

Let $I = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+cosx}}{(1-cosx)^{\frac{5}{2}}} dx$

Using $1 + cosx = 2cos^2\left(\frac{x}{2}\right)$

And

$1 - cosx = 2sin^2\left(\frac{x}{2}\right)$,

we get

$$I = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{2}\cos\left(\frac{x}{2}\right)}{4\sqrt{2}\left(\sin\left(\frac{x}{2}\right)\right)^5} dx$$

$$= \frac{1}{4}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot\left(\frac{x}{2}\right) cosec^4\left(\frac{x}{2}\right) dx$$

Let $\cot\left(\frac{x}{2}\right) = t$

$\Rightarrow -\frac{1}{2} cosec^2\left(\frac{x}{2}\right) dx = dt$.

Also, when $x = \frac{\pi}{3}$, $t = \sqrt{3}$

and when $x = \frac{\pi}{2}$, t=1

Hence,

$I = -\frac{1}{2}\int_{\sqrt{3}}^{1} t\,(1 + t^2)dt$

$$= -\frac{1}{2}\frac{t^2}{2}\Big|_{\sqrt{3}}^{1} - \frac{1}{2}\frac{t^4}{4}\Big|_{\sqrt{3}}^{1}$$

$$= \frac{1}{2} + 1$$

$$= \frac{3}{2}$$

Question: 37

Evaluate the foll

Solution:

Let $I = \int_0^1 (cos^{-1}x)^2\, dx$

Let x=cost ⇒ dx=-sin t dt.

Also, when x=0, $t = \frac{\pi}{2}$

and when x=1, t=0.

Hence, $I = -\int_{\frac{\pi}{2}}^{0} t^2 sint\, dt$

Using integration by parts, we get

$$I = -\left(t^2 \times -cost\,\Big|_{\frac{\pi}{2}}^{0} + 2\int_{\frac{\pi}{2}}^{0} tcost\, dt\right)$$

$$= -\left(0 - 0 + 2t \times sint\,\Big|_{\frac{\pi}{2}}^{0} - 2\int_{\frac{\pi}{2}}^{0} sint\, dt\right)$$

$$= -\left(-\pi + 2cost\,\Big|_{\frac{\pi}{2}}^{0}\right)$$

Hence, I=π-2

Question: 38

Evaluate the foll

Solution:

Let $I = \int_0^1 x(tan^{-1}x)^2\,dx$

Using integration by parts, we get

$$I = \frac{(tan^{-1}x)^2x^2}{2}\Big|_0^1 - \int_0^1 \frac{2tan^{-1}x}{1+x^2} \times \frac{x^2}{2}dx$$

$$= \frac{\pi^2}{32} - 0 - \int_0^1 \frac{tan^{-1}x}{1+x^2} \times (1+x^2-1)\,dx$$

$$= \frac{\pi^2}{32} - \int_0^1 tan^{-1}x dx + \int_0^1 \frac{tan^{-1}x}{1+x^2}dx$$

Let $\tan^{-1}x=t$

$\Rightarrow \frac{1}{1+x^2}dx = dt.$

When x=0, t=0 and when x=1, $t = \frac{\pi}{4}$.

Hence

$$I = \frac{\pi^2}{32} - tan^{-1}x \times x\Big|_0^1 + \int_0^1 \frac{x}{1+x^2}dx + \int_0^{\frac{\pi}{4}} t dt$$

$$= \frac{\pi^2}{32} - \frac{\pi}{4} + \frac{t^2}{2}\Big|_0^{\frac{\pi}{4}} + \int_0^1 \frac{x}{1+x^2}dx$$

Let $1+x^2=y$

⇒ 2xdx=dy.

Also, when x=0, y=1

and when x=1, y=2.

$$I = \frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2}\int_1^2 \frac{1}{y}dy$$

$$= \frac{\pi}{4}\left(\frac{\pi}{4} - 1\right) + \frac{1}{2}\log_e y\Big|_1^2$$

$$= \frac{\pi}{4}\left(\frac{\pi}{4} - 1\right) + \frac{1}{2}\log_e 2.$$

Question: 39

Evaluate the foll

Solution:

Let $I = \int_0^1 sin^{-1}\sqrt{x}\,dx$

Let √x=t

$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

or

dx=2tdt.

When, x=0, t=0

and when x=1, t=1.

Hence,

$I = 2\int_0^1 t\sin^{-1}t\,dt$

Using integration by parts, we get

$$I = 2\left(\sin^{-1}t \times \frac{t^2}{2}\Big|_0^1 - \int_0^1 \frac{1}{\sqrt{1-t^2}} \times \frac{t^2}{2}dt\right)$$

$$= \frac{\pi}{2} - \int_0^1 \frac{t^2}{\sqrt{1-t^2}}dt$$

Let t=sin y

⇒ dt=cos y dy.

When t=0, y=0, when t=1, $y = \frac{\pi}{2}$.

$I = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \sin^2 y dy$ ..... (1)

Using, $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$I = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \cos^2 y dy$ .....(2)

Adding (1) and (2), we get

$$2I = \pi - \int_0^{\frac{\pi}{2}} dy$$

$$= \pi - \frac{\pi}{2}$$

Hence,

$I = \frac{\pi}{4}$

Question: 40

Evaluate the foll

Solution:

Let $I = \int_0^a \sin^{-1}\sqrt{\frac{x}{a+x}}dx$

Let x=a $\tan^2$y

⇒ dx=2a tan y $\sec^2$y dy.

Also, when x=0, y=0

and when x=a, $y = \frac{\pi}{4}$

Hence $I = \int_0^{\frac{\pi}{4}} \sin^{-1}\left(\sqrt{\frac{a\tan^2 y}{a+a\tan^2 y}}\right) 2a\tan y\sec^2 y\,dy = 2a\int_0^{\frac{\pi}{4}} y\tan y\ \sec^2 y dy$

Using integration by parts, we get

$$I = 2a\left(y\int_0^{\frac{\pi}{4}} \tan y \sec^2 y dy - \int_0^{\frac{\pi}{4}}\left(\int \tan y \sec^2 y dy\right)dy\right)$$

Let tan y=t

⇒ $\sec^2$ydy=dt.

Also, when y=0, t=0

and when $y = \frac{\pi}{4}$, t=1.

Also, $y=\tan^{-1}t$

$$\Rightarrow dy = \frac{dt}{1+t^2}$$

$$I = 2a\left(tan^{-1}t\int tdt\,\Big|_0^1 - \int_0^1\left(\int tdt\right)\frac{dt}{1+t^2}\right)$$

$$= 2a\left(\frac{tan^{-1}t\times t^2}{2}\Big|_0^1\right) - 2a\int_0^1 \frac{t^2}{2}\frac{dt}{1+t^2}$$

$$= \frac{a\pi}{4} - a\int_0^1 \frac{t^2}{1+t^2}dt$$

Let $I' = \int_0^1 \frac{t^2}{1+t^2}dt$

$$= \int_0^1 \frac{1+t^2-1}{1+t^2}dt$$

$$= \int_0^1 dt - \int_0^1 \frac{1}{1+t^2}dt$$

$$= t\,\Big|_0^1 - tan^{-1}t\,\Big|_0^1$$

Hence $I' = 1 - \frac{\pi}{4}$

Substituting value of I' in I, we get

$$I = \frac{a\pi}{4} - a\left(1 - \frac{\pi}{4}\right)$$

$$= a\left(\frac{\pi}{2} - 1\right)$$

Question: 41

Evaluate the foll

Solution:

Let $I = \int_0^9 \frac{1}{1+\sqrt{x}}dx$

Let √x=u

$$\Rightarrow \frac{1}{2\sqrt{x}}dx = du$$

$= \frac{1}{2u}dx$ or dx=2udu.

Also, when x=0, u=0 and x=9, u=3.

Hence,

$I = \int_0^3 \frac{2u}{1+u}du$

$$= 2\left(\int_0^3 \frac{u+1-1}{1+u}du\right)$$

$$= 2\left(\int_0^3 du - \int_0^3 \frac{1}{1+u}du\right)$$

$I = 2u \Big|_0^3 - \log_e(1+u)\Big|_0^3$

$= 6 - 2\log_e 4$

$= 6 - 4\log_e 2$

$(Using\ \log_e a^b = b\log_e a)$

Question: 42

Evaluate the foll

Solution:

Let $I = \int_0^1 x^3\sqrt{1+3x^4}\,dx$

Let $1+3x^4=t$

$\Rightarrow 12x^3dx=dt.$

Also, when x=0, t=1 and when x=1, t=4.

$I = \frac{1}{12}\int_1^4 \sqrt{t}\,dt$

$= \frac{1}{12} \times \frac{2}{3} t^{\frac{3}{2}}\Big|_1^4$

$= \frac{7}{18}$

Question: 43

Evaluate the foll

Solution:

Let $I = \int_0^1 \frac{1-x^2}{(1+x^2)^2}dx$

Let $I' = \int_0^1 \frac{1}{(1+x^2)^2}dx$

Let x=tan t

$\Rightarrow dx=\sec^2 t dt.$

Also when x=0, t=0 and when x=1, $t = \frac{\pi}{4}$.

Hence, $I' = \int_0^{\frac{\pi}{4}} \frac{sec^2 t}{(1+tan^2 t)^2}dt$

$= \int_0^{\frac{\pi}{4}} cos^2 t dt$

Using $cos^2 t = \frac{1+cos2t}{2}$, we get

$I' = \int_0^{\frac{\pi}{4}} \left(\frac{1+cos2t}{2}\right)dt$

$= \frac{t}{2}\Big|_0^{\frac{\pi}{4}} + \frac{sin2t}{4}\Big|_0^{\frac{\pi}{4}}$

$= \frac{\pi+2}{8}$

Let $I'' = \int_0^1 \frac{x^2}{(1+x^2)^2}dx$

$$= \int_0^1 x \times \frac{x}{(1+x^2)^2} dx$$

$$= x \int_0^1 \frac{x}{(1+x^2)^2} dx - \int_0^1 \left(\int \frac{x}{(1+x^2)^2} dx \right) dx$$

Let $1+x^2=t \Rightarrow 2xdx=dt$.

When x=0, t=1 and when x=1, t=2.

$$I'' = \sqrt{t-1} \times \frac{1}{2} \int_1^2 \frac{1}{t^2} dt - \int_1^2 \frac{\left(\frac{1}{2} \int \frac{1}{t^2} dt\right) dt}{2\sqrt{t-1}}$$

$$= -\frac{\sqrt{t-1}}{2} \times \frac{1}{t} \Big|_1^2 + \int_1^2 \frac{dt}{4t\sqrt{t-1}}$$

$$= -\frac{1}{4} + \int_1^2 \frac{dt}{4t\sqrt{t-1}}$$

Substituting $t=1+x^2$

$\Rightarrow 2xdx=dt$.

When t=1, x=0 and when t=2, x=1.

$$I'' = -\frac{1}{4} + \int_0^1 \frac{2xdx}{4x(1+x^2)}$$

$$= -\frac{1}{4} + \frac{1}{2} tan^{-1} x \Big|_0^1$$

$$= \frac{\pi - 2}{8}$$

Hence,

$$I = \frac{\pi + 2}{8} - \frac{\pi - 2}{8}$$

$$= \frac{1}{2}$$

Question: 44

Evaluate the foll

Solution:

Let $I = \int_1^2 \frac{1}{(x+1)\sqrt{x^2-1}} dx$

Let x=sect

$\Rightarrow$ dx=sec t tan t dt.

Also,

when x=1, t=0 and when x=2, $t = \frac{\pi}{3}$

Hence,

$$I = \int_0^{\frac{\pi}{3}} \frac{secttant}{(sect+1)\sqrt{sec^2 t-1}} dt$$

$$= \int_0^{\frac{\pi}{3}} \frac{sect}{(sect+1)} dt$$

$$= \int_0^{\frac{\pi}{3}} \frac{1}{(1+cost)} dt$$

Using $1 + cost = 2cos^2\left(\frac{t}{2}\right)$, we get

$$I = \frac{1}{2}\int_0^{\frac{\pi}{3}} sec^2\left(\frac{t}{2}\right) dt$$

$$= \tan\left(\frac{t}{2}\right)\Big|_0^{\frac{\pi}{3}}$$

$$= \frac{1}{\sqrt{3}}$$

Question: 45

Evaluate the foll

Solution:

Let $I = \int_0^{\frac{\pi}{2}}(\sqrt{tanx} + \sqrt{cotx})dx = \int_0^{\frac{\pi}{2}}\frac{sinx+cosx}{\sqrt{sinxcosx}}dx$

Let sin x- cos x=t

⇒ (cos x + sin x)dx=dt.

When x=0, t=-1 and $x = \frac{\pi}{2}$, t=1.

Also, $t^2=(\sin x - \cos x)^2$

$=\sin^2 x+\cos^2 x-2\sin x\cos x$

=1-2sinxcosx

or

$$sincosx = \frac{1 - t^2}{2}$$

Hence $I = \sqrt{2}\int_{-1}^{1}\frac{1}{\sqrt{1-t^2}}dt$

Let t=sin y

⇒ dt=cos y dy.

Also, when t=-1, $y = -\frac{\pi}{2}$

and when t=1, $y = \frac{\pi}{2}$.

$$I = \sqrt{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{cosy}{\sqrt{1 - sin^2 y}}dy$$

$$= \sqrt{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dy = \pi\sqrt{2}$$

Question: 46

Evaluate the foll

Solution:

Let $I = \int_2^3 \frac{2-x}{\sqrt{5x-6-x^2}}dx$

Let,

$2 - x = a\frac{d}{dx}(5x - 6 - x^2) + b$

=-2ax+5a+b

Hence -2a=-1 and 5a+b=2.

Solving these equations,

we get $a = \frac{1}{2}$ and $b = -\frac{1}{2}$.

We get,

$I = \frac{1}{2}\int_2^3 \frac{-2x+5}{\sqrt{5x-6-x^2}}dx - \frac{1}{2}\int_2^3 \frac{1}{\sqrt{5x-6-x^2}}dx$

Let $I' = \int_2^3 \frac{-2x+5}{\sqrt{5x-6-x^2}}dx$

Let $5x-6-x^2=t$

⇒ (5-2x) dx=dt.

When x=2, t=0 and when x=3, y=0.

Hence $I' = \int_0^0 \frac{1}{\sqrt{t}}dt = 0$

$\left(Since \int_a^a f(x)dx = 0\right)$

Let,

$I'' = \int_2^3 \frac{1}{\sqrt{5x-6-x^2}}dx$

$= \int_2^3 \frac{1}{\sqrt{\frac{1}{4} - \left(x - \frac{5}{2}\right)^2}}$

$= sin^{-1}\left(\frac{x - \frac{5}{2}}{\frac{1}{2}}\right)$

$= sin^{-1}(2x-5)\Big|_2^3$

$=\pi$

Hence,

$I = \frac{1}{2} \times 0 - \frac{1}{2} \times \pi$

$= -\frac{\pi}{2}$

Question: 47

Evaluate the foll

Solution:

Let $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{cosx}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3}dx$

Using $cosx = cos^2\left(\frac{x}{2}\right) - sin^2\left(\frac{x}{2}\right)$, we get

$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}{\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)^2}dx$

Let $\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right) = t$

$\Rightarrow \frac{1}{2}\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) dx = dt.$

Also, when $x = \frac{\pi}{4}$, $t = \cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right) = \alpha(Let)$

and when $x = \frac{\pi}{2}$, $t = \sqrt{2}$

$$I = \int_{\alpha}^{\sqrt{2}} \frac{2}{t^2} dt$$

$$= -2 \times \frac{1}{t}\Big|_{\alpha}^{\sqrt{2}}$$

$$= \frac{2}{\cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right)} - \sqrt{2}$$

Question: 48

Evaluate the foll

Solution:

Let $I = \int_0^{\left(\frac{\pi}{2}\right)^{\frac{1}{3}}} x^2 \sin(x^3)\, dx$

Let x^3=t

$\Rightarrow 3x^2$=dt.

Also, when x=0, t=0 and when $x = \left(\frac{\pi}{2}\right)^{\frac{1}{3}}, t = \frac{\pi}{2}.$

Hence, $I = \frac{1}{3}\int_0^{\frac{\pi}{2}} \sin(t)\, dt$

$$= \frac{-1}{3} cost \Big|_0^{\frac{\pi}{2}}$$

$$= -\frac{1}{3}(0 - 1)$$

$$= \frac{1}{3}$$

Question: 49

Evaluate the foll

Solution:

Let $I = \int_1^2 \frac{1}{x(1+\log_e x)^2} dx$

Let $1 + \log_e x = t$

$\Rightarrow \frac{1}{x} dx = dt.$

Also, when x=1, t=1 and when x=2, $t = 1 + \log_e 2$

Hence $I = \int_1^{1+\log_e 2} \frac{1}{t^2} dt$

$$= -\frac{1}{t}\Big|_1^{1+\log_e 2}$$

$$= 1 - \frac{1}{1 + \log_e 2}$$

$= \frac{\log_e 2}{1 + \log_e 2}$

Question: 50

Evaluate the foll

Solution:

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{cosecxcotx}{1+cosec^2 x} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{cosx}{1+sin^2 x} dx$

Let sinx=t

⇒ cos x dx=dt.

Also, when $x = \frac{\pi}{6}, t = \frac{1}{2}$ and when $x = \frac{\pi}{2}$, t=1.

$$I = \int_{\frac{1}{2}}^{1} \frac{1}{1+t^2} dt$$

$$= tan^{-1} t \Big|_{\frac{1}{2}}^{1}$$

$$= tan^{-1} 1 - tan^{-1}\left(\frac{1}{2}\right)$$

$$= tan^{-1}\left(\frac{1-\frac{1}{2}}{1+\frac{1}{2}}\right)$$

$$= tan^{-1}\left(\frac{1}{3}\right)$$

(Using $tan^{-1}a - tan^{-1}b = tan^{-1}\left(\frac{a-b}{1+ab}\right)$)

Exercise : 16C

Question: 1

Prove that

Solution:

$y = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{2\cos x}{\sin x+\cos x} dx$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\cos x + \cos x - \sin x + \sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 1 + \frac{\cos x - \sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2}\left((x)_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{\sin x + \cos x} dx \right)$$

Let, sin x + cos x = t

⇒ (cos x – sin x) dx = dt

At x = 0, t = 1

At x = π/2, t = 1

$$y = \frac{1}{2}\left(\frac{\pi}{2} + \int_{1}^{1} \frac{1}{t} dt\right)$$

$$y = \frac{1}{2}(\frac{\pi}{2} + \ (\ln t)_{1}^{1}$$

$$y = \frac{\pi}{4}$$

Question: 2

Prove that

Solution:

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx \ldots(1)$$

Use King theorem of definite integral

$$\int_{a}^{b} f(x)\,dx = \int_{a}^{b} f(a+b-x)\,dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{2}-x\right)} + \sqrt{\cos\left(\frac{\pi}{2}-x\right)}\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x}+\sqrt{\sin x})} dx \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx$$

$$= \int_0^{\pi/2} 1\,dx$$

$$= (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 3 A

Prove that

Solution:

$$y = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x} dx \ldots(1)$$

Use King theorem of definite integral

$$\int_{a}^{b} f(x)\,dx = \int_{a}^{b} f(a+b-x)\,dx$$

$$y = \int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2}-x\right)}{\sin^3\left(\frac{\pi}{2}-x\right) + \cos^3\left(\frac{\pi}{2}-x\right)} dx$$

$y = \int_0^{\pi/2} \frac{cos^3 x}{sin^3 x + cos^3 x} dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{sin^3 x}{sin^3 x + cos^3 x} dx + \int_0^{\pi/2} \frac{cos^3 x}{sin^3 x + cos^3 x} dx$$

$$= \int_0^{\pi/2} \frac{sin^3 x + cos^3 x}{sin^3 x + cos^3 x} dx$$

$$= \int_0^{\pi/2} 1\, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 3 B

Prove that

Solution:

$y = \int_0^{\pi/2} \frac{cos^3 x}{sin^3 x + cos^3 x} dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

$$y = \int_0^{\pi/2} \frac{cos^3\left(\frac{\pi}{2} - x\right)}{sin^3\left(\frac{\pi}{2} - x\right) + cos^3\left(\frac{\pi}{2} - x\right)} dx$$

$y = \int_0^{\pi/2} \frac{sin^3 x}{sin^3 x + cos^3 x} dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{cos^3 x}{sin^3 x + cos^3 x} dx + \int_0^{\pi/2} \frac{sin^3 x}{sin^3 x + cos^3 x} dx$$

$$2y = \int_0^{\pi/2} \frac{cos^3 x + sin^3 x}{sin^3 x + cos^3 x} dx$$

$$2y = \int_0^{\pi/2} 1\, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 4 A

Prove that

Solution:

$y = \int_0^{\pi/2} \frac{sin^7 x}{sin^7 x + cos^7 x} dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi/2} \frac{\sin^7\left(\frac{\pi}{2}-x\right)}{\sin^7\left(\frac{\pi}{2}-x\right)+\cos^7\left(\frac{\pi}{2}-x\right)}dx$$

$$y = \int_0^{\pi/2} \frac{\cos^7 x}{\sin^7 x+\cos^7 x}dx \quad ...(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin^7 x}{\sin^7 x+\cos^7 x}dx + \int_0^{\pi/2} \frac{\cos^7 x}{\sin^7 x+\cos^7 x}dx$$

$$2y = \int_0^{\pi/2} \frac{\sin^7 x+\cos^7 x}{\sin^7 x+\cos^7 x}dx$$

$$2y = \int_0^{\pi/2} 1\,dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 4 B

Prove that

Solution:

$$y = \int_0^{\pi/2} \frac{\cos^4 x}{\sin^4 x+\cos^4 x}dx \quad ...(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi/2} \frac{\cos^4\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}dx$$

$$y = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x+\cos^4 x}dx \quad ...(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\cos^4 x}{\sin^4 x+\cos^4 x}dx + \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x+\cos^4 x}dx$$

$$2y = \int_0^{\pi/2} \frac{\cos^4 x+\sin^4 x}{\sin^4 x+\cos^4 x}dx$$

$$2y = \int_0^{\pi/2} 1\,dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 5

Prove that

Solution:

$y = \int_0^{\pi/2} \frac{cos^4 x}{sin^4 x+cos^4 x} dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi/2} \frac{cos^4\left(\frac{\pi}{2}-x\right)}{sin^4\left(\frac{\pi}{2}-x\right)+cos^4\left(\frac{\pi}{2}-x\right)} dx$$

$y = \int_0^{\pi/2} \frac{sin^4 x}{sin^4 x+cos^4 x} dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{cos^4 x}{sin^4 x + cos^4 x} dx + \int_0^{\pi/2} \frac{sin^4 x}{sin^4 x + cos^4 x} dx$$

$$2y = \int_0^{\pi/2} \frac{cos^4 x + sin^4 x}{sin^4 x + cos^4 x} dx$$

$$2y = \int_0^{\pi/2} 1\,dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 6

Prove that

Solution:

$y = \int_0^{\pi/2} \frac{cos^{\frac{1}{4}} x}{sin^{\frac{1}{4}} x+cos^{\frac{1}{4}} x} dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi/2} \frac{cos^{\frac{1}{4}}\left(\frac{\pi}{2}-x\right)}{sin^{\frac{1}{4}}\left(\frac{\pi}{2}-x\right)+cos^{\frac{1}{4}}\left(\frac{\pi}{2}-x\right)} dx$$

$y = \int_0^{\pi/2} \frac{sin^{\frac{1}{4}} x}{sin^{\frac{1}{4}} x+cos^{\frac{1}{4}} x} dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{cos^{\frac{1}{4}} x}{sin^{\frac{1}{4}} x + cos^{\frac{1}{4}} x} dx + \int_0^{\pi/2} \frac{sin^{\frac{1}{4}} x}{sin^{\frac{1}{4}} x + cos^{\frac{1}{4}} x} dx$$

$$2y = \int_0^{\pi/2} \frac{cos^{\frac{1}{4}} x + sin^{\frac{1}{4}} x}{sin^{\frac{1}{4}} x + cos^{\frac{1}{4}} x} dx$$

$$2y = \int_0^{\pi/2} 1\,dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 7

Prove that

Solution:

$$y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx + \int_0^{\pi/2} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$2y = \int_0^{\pi/2} 1\,dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 8

Prove that

Solution:

$$y = \int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi/2} \frac{\sin^n\left(\frac{\pi}{2} - x\right)}{\sin^n\left(\frac{\pi}{2} - x\right) + \cos^n\left(\frac{\pi}{2} - x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} dx \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx + \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin^n x + \cos^n x}{\sin^n x + \cos^n x} dx$$

$$2y = \int_0^{\pi/2} 1\, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 9

Prove that

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\frac{\sin x}{\cos x}}}{\sqrt{\frac{\sin x}{\cos x}} + \sqrt{\frac{\cos x}{\sin x}}} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx \quad \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx \quad \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx + \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2y = \int_0^{\pi/2} 1\, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 10

Prove that

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\frac{\cos x}{\sin x}}}{\sqrt{\frac{\sin x}{\cos x}} + \sqrt{\frac{\cos x}{\sin x}}} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx \quad \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$y = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}dx \;\ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x}dx + \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}dx$$

$$2y = \int_0^{\pi/2} \frac{\sin x+\cos x}{\sin x+\cos x}dx$$

$$2y = \int_0^{\pi/2} 1\,dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 11

Prove that

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{\sin x}{\cos x}}dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x}dx \;\ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$y = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}dx \;\ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x}dx + \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}dx$$

$$2y = \int_0^{\pi/2} \frac{\sin x+\cos x}{\sin x+\cos x}dx$$

$$2y = \int_0^{\pi/2} 1\,dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 12

Prove that

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{\cos x}{\sin x}} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x} dx \quad ...(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x} dx \quad ...(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x} dx + \int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin x+\cos x}{\sin x+\cos x} dx$$

$$2y = \int_0^{\pi/2} 1\, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 13

Prove that

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{\sin^3 x}{\cos^3 x}} dx$$

$$y = \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x+\cos^3 x} dx \quad ...(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_0^{\pi/2} \frac{\cos^3\left(\frac{\pi}{2}-x\right)}{\sin^3\left(\frac{\pi}{2}-x\right)+\cos^3\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x} dx \quad ...(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x+\cos^3 x} dx + \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x} dx$$

$$2y = \int_0^{\pi/2} \frac{\cos^3 x + \sin^3 x}{\sin^3 x + \cos^3 x} dx$$

$$2y = \int_0^{\pi/2} 1\, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 14

Prove that

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{\cos^3 x}{\sin^3 x}} dx$$

$$y = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x} dx \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2}-x\right)}{\sin^3\left(\frac{\pi}{2}-x\right)+\cos^3\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x+\cos^3 x} dx \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx + \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x} dx$$

$$2y = \int_0^{\pi/2} 1\, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 15

Prove that

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\frac{\sin x}{\cos x}}} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\left(\sqrt{\sin x}+\sqrt{\cos x}\right)} dx \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx \ ...(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx + \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx$$

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx$$

$$2y = \int_0^{\pi/2} 1 \, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 16

Prove that

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\frac{\cos x}{\sin x}}}{1 + \sqrt{\frac{\cos x}{\sin x}}} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx \ ...(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx \ ...(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx + \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx$$

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx$$

$$2y = \int_0^{\pi/2} 1 \, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 17

Prove that

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\frac{\sin x}{\cos x}}}{1+\sqrt{\frac{\sin x}{\cos x}}} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx \text{ ...(1)}$$

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x}+\sqrt{\sin x})} dx \text{ ...(2)}$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x}+\sqrt{\sin x})} dx$$

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx$$

$$2y = \int_0^{\pi/2} 1\,dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 18

Prove that

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin x-\cos x}{1+\sin x\cos x} dx \text{ ...(1)}$$

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos x-\sin x}{1+\cos x\sin x} dx \text{ ...(2)}$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx + \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx$$

$$2y = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x + \cos x - \sin x}{1 + \cos x \sin x} dx$$

$$2y = \int_0^{\frac{\pi}{2}} 0\, dx$$

y = 0

Question: 19

Prove that

Solution:

$y = \int_0^1 x(1-x)^5\, dx$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_0^1 (1-x)x^5\, dx$$

$$y = \int_0^1 x^5 - x^6\, dx$$

$$y = \left(\frac{x^6}{6} - \frac{x^7}{7}\right)_0^1$$

$$y = \frac{1}{6} - \frac{1}{7}$$

$$= \frac{1}{42}$$

Question: 20

Prove that

Solution:

$y = \int_0^2 x\sqrt{2-x}\, dx$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_0^2 (2-x)\sqrt{x}\, dx$$

$$y = \int_0^2 2x^{\frac{1}{2}} - x^{\frac{3}{2}}\, dx$$

$$y = \left(2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right)_0^2$$

$$y = \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5} = \frac{16\sqrt{2}}{15}$$

Question: 21

Prove that

Solution:

$y = \int_0^{\pi} x\, cos^2 x\, dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_0^{\pi} (\pi - x) cos^2(\pi - x)\, dx$$

$y = \int_0^{\pi} \pi\, cos^2 x - x\, cos^2 x\, dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} x\, cos^2 x\, dx + \int_0^{\pi} \pi\, cos^2 x - x\, cos^2 x\, dx$$

$$2y = \int_0^{\pi} \pi\, cos^2 x\, dx$$

$$y = \frac{\pi}{2}\int_0^{\pi} \frac{1+\cos 2x}{2} dx$$

$$y = \frac{\pi}{2}\left(\frac{x}{2} + \frac{\sin 2x}{4}\right)_0^{\pi}$$

$$y = \frac{\pi}{2}\left(\frac{\pi}{2} + \frac{\sin 2\pi}{4}\right) = \frac{\pi^2}{4}$$

Question: 22

Prove that

Solution:

$y = \int_0^{\pi} \frac{x \tan x}{\sec x\, cosec\, x} dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_0^{\pi} \frac{(\pi - x) \tan(\pi - x)}{\sec(\pi - x)\ cosec\ (\pi - x)} dx$$

$$y = \int_0^{\pi} \frac{-(\pi - x) \tan x}{-\sec x\ cosec\ x} dx$$

$$y = \int_0^{\pi} \frac{\pi \tan x - x \tan x}{\sec x \ cosec\, x} dx \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} \frac{x \tan x}{\sec x \ cosec\ x} dx + \int_0^{\pi} \frac{\pi \tan x - x \tan x}{\sec x \ cosec\ x} dx$$

$$2y = \int_0^{\pi} \frac{\pi \tan x}{\sec x \ cosec\ x} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} \times \frac{1}{\sin x}} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{1 - \cos 2x}{2} dx$$

$$y = \frac{\pi}{2} \left(\frac{x}{2} - \frac{\sin 2x}{4} \right)_0^{\pi}$$

$$y = \frac{\pi}{2} \left(\frac{\pi}{2} - \frac{\sin 2\pi}{4} \right) = \frac{\pi^2}{4}$$

Question: 23

Prove that

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{cos^2 x}{\sin x + \cos x} dx \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{cos^2 \left(\frac{\pi}{2} - x \right)}{\sin \left(\frac{\pi}{2} - x \right) + \cos \left(\frac{\pi}{2} - x \right)} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{sin^2 x}{\sin x + \cos x} dx \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\frac{\pi}{2}} \frac{cos^2 x}{\sin x + \cos x} dx + \int_0^{\frac{\pi}{2}} \frac{sin^2 x}{\sin x + \cos x} dx$$

$$2y = \int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx$$

$$2y = \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{1}{\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x} dx$$

$$2y = \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{1}{\sin \left(x + \frac{\pi}{4} \right)} dx$$

$$y = \frac{1}{2\sqrt{2}}\int_0^{\frac{\pi}{2}} cosec\left(x+\frac{\pi}{4}\right)dx$$

$$y = \frac{1}{2\sqrt{2}}\left(\ln\left(\text{cosec}\left(x+\frac{\pi}{4}\right)-\cot\left(x+\frac{\pi}{4}\right)\right)\right)_0^{\frac{\pi}{2}}$$

$$y = \frac{1}{2\sqrt{2}}\left(\ln\left(\text{cosec}\frac{3\pi}{4}-\cot\frac{3\pi}{4}\right)-\ln\left(\text{cosec}\frac{\pi}{4}-\cot\frac{\pi}{4}\right)\right)$$

$$y = \frac{1}{2\sqrt{2}}\ln\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$y = \frac{1}{2\sqrt{2}}\ln(\sqrt{2}+1)^2 = \frac{1}{\sqrt{2}}\ln(\sqrt{2}+1)$$

Question: 24

Prove that

Solution:

$$y = \int_0^{\pi}\frac{x\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+\cos x}dx$$

$$y = \int_0^{\pi}\frac{x\sin x}{1+cos^2 x}dx \ ...(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+cos^2(\pi-x)}dx$$

$$y = \int_0^{\pi}\frac{\pi\sin x - x\sin x}{1+cos^2 x}dx \ ...(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi}\frac{x\sin x}{1+cos^2 x}dx + \int_0^{\pi}\frac{\pi\sin x - x\sin x}{1+cos^2 x}dx$$

$$2y = \int_0^{\pi}\frac{\pi\sin x}{1+cos^2 x}dx$$

Let, cos x = t

⇒ -sin x dx = dt

At x = 0, t = 1

At x = π, t = -1

$$y = -\frac{\pi}{2}\int_1^{-1}\frac{1}{1+t^2}dt$$

$$y = -\frac{\pi}{2}(\tan^{-1}t)_1^{-1}$$

$$y = -\frac{\pi}{2}(\tan^{-1}(-1)-\tan^{-1}1)$$

$$y = \frac{\pi^2}{4}$$

Question: 25

Prove that

Solution:

$y = \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\sin(\pi - x)} dx$$

$y = \int_0^{\pi} \frac{\pi \sin x}{1+\sin x} - \frac{x \sin x}{1+\sin x} dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx + \int_0^{\pi} \frac{\pi \sin x}{1+\sin x} - \frac{x \sin x}{1+\sin x} dx$$

$$2y = \int_0^{\pi} \frac{\pi(\sin x + 1 - 1)}{1+\sin x} dx$$

$$y = \frac{\pi}{2}\int_0^{\pi} 1 - \frac{1}{1+\sin x} dx$$

$$y = \frac{\pi}{2}\int_0^{\pi} 1 - \frac{1-\sin x}{cos^2 x} dx$$

$$y = \frac{\pi}{2}\int_0^{\pi} 1 - sec^2 x + \frac{\sin x}{cos^2 x} dx$$

Let, cos x = t

⇒ -sin x dx = dt

At x = 0, t = 1

At x = π, t = -1

$$y = \frac{\pi}{2}\left((x - \tan x)_0^{\pi} - \int_1^{-1} \frac{1}{t^2} dt\right)$$

$$y = \frac{\pi}{2}\left(\pi - \tan \pi - \left(\frac{-1}{t}\right)_1^{-1}\right)$$

$$y = \frac{\pi}{2}(\pi - 2) = \pi\left(\frac{\pi}{2} - 1\right)$$

Question: 26

Prove that

Solution:

$y = \int_0^{\pi} \frac{x}{1+sin^2 x} dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi} \frac{(\pi - x)}{1 + sin^2(\pi - x)} dx$$

$$y = \int_0^{\pi} \frac{\pi}{1+sin^2 x} - \frac{x}{1+sin^2 x} dx \text{ ...(2)}$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} \frac{x}{1 + sin^2 x} dx + \int_0^{\pi} \frac{\pi}{1 + sin^2 x} - \frac{x}{1 + sin^2 x} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{1}{1 + sin^2 x} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{\frac{1}{cos^2 x}}{\frac{1 + sin^2 x}{cos^2 x}} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{sec^2 x}{sec^2 x + tan^2 x} dx$$

We break it in two parts

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{sec^2 x}{sec^2 x + tan^2 x} dx$$

Let, tan x = t

⇒ $sec^2 x$ dx = dt

At x = 0, t = 0

At x = π, t = 0

$$y = \frac{\pi}{2} \int_0^{0} \frac{1}{1 + 2t^2} dt$$

We know that when upper and lower limit is same in definite integral then value of integration is 0.

So, y = 0

Question: 27

Prove that

Solution:

$$y = \int_0^{\frac{\pi}{2}} \log \frac{cos^2 x}{\sin 2x} dx$$

$$y = \int_0^{\frac{\pi}{2}} \log \frac{cos^2 x}{2 \sin x \cos x} dx$$

$$y = \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{2} \cot x\right) dx \text{ ...(1)}$$

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{2}\cot\left(\frac{\pi}{2} - x\right)\right) dx$$

$y = \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{2}\tan x\right) dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{2}\cot x\right) dx + \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{2}\tan x\right) dx$$

$y = \frac{1}{2}\int_0^{\frac{\pi}{2}} \log\left(\frac{1}{4}\cot x \tan x\right) dx$ [Use cot x tan x = 1]

$$y = \frac{1}{2}\int_0^{\frac{\pi}{2}} \log\left(\frac{1}{4}\right) dx$$

$$y = \frac{1}{2}\log\left(\frac{1}{4}\right)(x)_0^{\frac{\pi}{2}}$$

$$y = -\frac{\pi}{4}\log 4$$

Question: 28

Prove that

Solution:

$$y = \int_0^{\infty} \frac{x}{(1+x)(1+x^2)} dx$$

Let, x = tan t

⇒ dx = $\sec^2$t dt

At x = 0, t = 0

At x = ∞, t = π/2

$$y = \int_0^{\frac{\pi}{2}} \frac{\tan t}{(1+\tan t)(1+tan^2 t)} sec^2 t\, dt$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\tan t}{(1+\tan t)}\, dt$$

$y = \int_0^{\frac{\pi}{2}} \frac{\sin t}{(\cos t+\sin t)}\, dt$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi/2} \frac{sin\left(\frac{\pi}{2}-t\right)}{sin\left(\frac{\pi}{2}-t\right) + cos\left(\frac{\pi}{2}-t\right)} dt$$

$$y = \int_0^{\pi/2} \frac{\cos t}{\sin t + \cos t} dt \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin t}{\sin t + \cos t} dx + \int_0^{\pi/2} \frac{\cos t}{\sin t + \cos t} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin t + \cos t}{\sin t + \cos t} dx$$

$$2y = \int_0^{\pi/2} 1\, dx$$

$$2y = (x)_0^{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 29

Prove that

Solution:

Let, x = a sin t

⇒ dx = a cos t dt

At x = 0, t = 0

At x = a, t = π/2

$$y = \int_0^{\frac{\pi}{2}} \frac{a \cos t}{a \sin t + \sqrt{a^2 - a^2 sin^2 t}} dt$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos t}{\sin t + \cos t} dt$$

$$y = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{\cos t + \cos t - \sin t + \sin t}{\sin t + \cos t} dt$$

$$y = \frac{1}{2} \int_0^{\frac{\pi}{2}} 1 + \frac{\cos t - \sin t}{\sin t + \cos t} dt$$

$$y = \frac{1}{2}\left((t)_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{\cos t - \sin t}{\sin t + \cos t} dt \right)$$

Again, sin t + cos t = z

⇒ (cos t – sin t) dt = dz

At t = 0, z = 1

At t = π/2, z = 1

$$y = \frac{1}{2}\left(\frac{\pi}{2} + \int_1^1 \frac{1}{z} dz \right)$$

$$y = \frac{1}{2}\left(\frac{\pi}{2} + (\ln z)_1^1\right.$$

$y = \frac{\pi}{4}$

Question: 30

$y = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$y = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx + \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} dx$$

$$2y = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} dx$$

$$y = \frac{1}{2}\int_0^a dx$$

$$y = \frac{1}{2}(x)_0^a$$

$$y = \frac{a}{2}$$

Question: 31

Prove that

Solution:

$y = \int_0^\pi sin^2 x\, cos^3 x\, dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^\pi sin^2(\pi - x)\, cos^3(\pi - x)\, dx$$

$y = -\int_0^\pi sin^2 x\, cos^3 x\, dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^\pi sin^2 x\, cos^3 x\, dx + \left(-\int_0^\pi sin^2 x\, cos^3 x\, dx\right)$$

y = 0

Question: 32

Prove that

Solution:

$y = \int_0^{\pi} sin^{2m}x\, cos^{2m+1}x\, dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi} sin^{2m}(\pi - x)\, cos^{2m+1}(\pi - x)\, dx$$

$y = -\int_0^{\pi} sin^{2m}x\, cos^{2m+1}x\, dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} sin^{2m}x\, cos^{2m+1}x\, dx + \left(-\int_0^{\pi} sin^{2m}x\, cos^{2m+1}x\, dx\right)$$

y = 0

Question: 33

Prove that

Solution:

Let, sin x + cos x = t

⇒ cos x – sin x dx = dt

At x = 0, t = 1

At x = π/2, t = 1

$$y = \int_1^1 -\log t\, dt$$

We know that when upper and lower limit in definite integral is

equal then value of integration is zero.

So, y = 0

Question: 34

Prove that

Solution:

$y = \int_0^{\frac{\pi}{2}} \log(2 \sin x \cos x)\, dx$

$$y = \int_0^{\frac{\pi}{2}} \log 2 + \log \sin x + \log \cos x\, dx$$

Let, $I = \int_0^{\frac{\pi}{2}} \log \sin x\, dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx$$

$I = \int_0^{\frac{\pi}{2}} \log\cos x\, dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log\sin x\, dx + \int_0^{\frac{\pi}{2}} \log\cos x\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log\frac{2\sin x\cos x}{2} dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log\sin 2x - \log 2\, dx$$

Let, 2x = t

⇒ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$$2I = \frac{1}{2}\int_0^{\pi} \log\sin t\, dt - \frac{\pi}{2}\log 2$$

$$2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} \log\sin x\, dx - \frac{\pi}{2}\log 2$$

$$2I = I - \frac{\pi}{2}\log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log\sin x\, dx = -\frac{\pi}{2}\log 2$$

Similarly, $\int_0^{\frac{\pi}{2}} \log\cos x\, dx = -\frac{\pi}{2}\log 2$

$$y = \int_0^{\frac{\pi}{2}} \log 2\, dx + \int_0^{\frac{\pi}{2}} \log\sin x\, dx + \int_0^{\frac{\pi}{2}} \log\cos x\, dx$$

$$y = \frac{\pi}{2}\log 2 - \frac{\pi}{2}\log 2 - \frac{\pi}{2}\log 2$$

$$y = -\frac{\pi}{2}\log 2$$

Question: 35

Prove that

Solution:

$y = \int_0^{\pi} x\log\sin x\, dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_0^{\pi} (\pi - x) \log \sin(\pi - x)\, dx$$

$$y = \int_0^{\pi} \pi \log \sin x - x \log \sin x\, dx \;\ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} x \log \sin x\, dx + \int_0^{\pi} \pi \log \sin x - x \log \sin x\, dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \log \sin x\, dx$$

$$y = \frac{2\pi}{2} \int_0^{\frac{\pi}{2}} \log \sin x\, dx \;\ldots(3)$$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

$$y = \pi \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx$$

$$y = \pi \int_0^{\frac{\pi}{2}} \log \cos x\, dx \;\ldots(4)$$

Adding eq.(3) and eq.(4)

$$2y = \pi \left(\int_0^{\frac{\pi}{2}} \log \sin x\, dx + \int_0^{\frac{\pi}{2}} \log \cos x\, dx \right)$$

$$2y = \pi \left(\int_0^{\frac{\pi}{2}} \log \frac{2 \sin x \cos x}{2}\, dx \right)$$

$$2y = \pi \left(\int_0^{\frac{\pi}{2}} \log \sin 2x - \log 2\, dx \right)$$

Let, 2x = t

⇒ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$$2y = \frac{\pi}{2} \int_0^{\pi} \log \sin t\, dt - \frac{\pi^2}{2} \log 2$$

$$2y = \frac{2\pi}{2} \int_0^{\frac{\pi}{2}} \log \sin x\, dx - \frac{\pi^2}{2} \log 2$$

$$2y = y - \frac{\pi^2}{2} \log 2$$

$y = -\frac{\pi^2}{2}\log 2$

Question: 36

Prove that

Solution:

$y = \int_0^{\pi} \log(1+\cos x)\, dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_0^{\pi} \log(1+\cos(\pi - x))\, dx$$

$y = \int_0^{\pi} \log(1-\cos x)\, dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} \log(1+\cos x)\, dx + \int_0^{\pi} \log(1-\cos x)\, dx$$

$$2y = \int_0^{\pi} \log sin^2 x\, dx$$

$y = 2\int_0^{\frac{\pi}{2}} \log \sin x\, dx$ ...(3)

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = 2\int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx$$

$y = 2\int_0^{\frac{\pi}{2}} \log \cos x\, dx$ ...(4)

Adding eq.(3) and eq.(4)

$$2y = 2\left(\int_0^{\frac{\pi}{2}} \log \sin x\, dx + \int_0^{\frac{\pi}{2}} \log \cos x\, dx\right)$$

$$2y = 2\left(\int_0^{\frac{\pi}{2}} \log \frac{2\sin x \cos x}{2}\, dx\right)$$

$$2y = 2\left(\int_0^{\frac{\pi}{2}} \log \sin 2x - \log 2\, dx\right)$$

Let, 2x = t

⇒ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$$2y = \frac{2}{2}\int_0^{\pi} \log\sin t\, dt - \frac{2\pi}{2}\log 2$$

$$2y = \frac{4}{2}\int_0^{\frac{\pi}{2}} \log\sin x\, dx - \frac{2\pi}{2}\log 2$$

$$2y = y - \pi\log 2$$

$$y = -\pi\log 2$$

Question: 37

Prove that

Solution:

$$y = \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) dx$$

$$y = \int_0^{\frac{\pi}{2}} \log\frac{1}{\sin x\cos x}\, dx$$

$$y = -\left(\int_0^{\frac{\pi}{2}} \log\sin x\, dx + \int_0^{\frac{\pi}{2}} \log\cos x\, dx\right)$$

Let, $I = \int_0^{\frac{\pi}{2}} \log\sin x\, dx \ldots(1)$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$I = \int_0^{\frac{\pi}{2}} \log\sin\left(\frac{\pi}{2} - x\right) dx$$

$$I = \int_0^{\frac{\pi}{2}} \log\cos x\, dx \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log\sin x\, dx + \int_0^{\frac{\pi}{2}} \log\cos x\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log\frac{2\sin x\cos x}{2}\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log\sin 2x - \log 2\, dx$$

Let, 2x = t

⇒ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$$2I = \frac{1}{2}\int_0^{\pi} \log\sin t\, dt - \frac{\pi}{2}\log 2$$

$$2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} \log\sin x\, dx - \frac{\pi}{2}\log 2$$

$$2I = I - \frac{\pi}{2}\log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log\sin x\, dx = -\frac{\pi}{2}\log 2$$

Similarly, $\int_0^{\frac{\pi}{2}}\log\cos x\, dx = -\frac{\pi}{2}\log 2$

$$y = -\left(\int_0^{\frac{\pi}{2}} \log\sin x\, dx + \int_0^{\frac{\pi}{2}} \log\cos x\, dx\right)$$

$$y = \frac{\pi}{2}\log 2 + \frac{\pi}{2}\log 2$$

$$y = \pi\log 2$$

Question: 38

Prove that

Solution:

$y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\cos x}{\cos x + \sin x}\, dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\cos\left(\frac{3\pi}{8}+\frac{\pi}{8}-x\right)}{\sin\left(\frac{3\pi}{8}+\frac{\pi}{8}-x\right)+\cos\left(\frac{3\pi}{8}+\frac{\pi}{8}-x\right)} dx$$

$y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sin x}{\sin x + \cos x}\, dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\cos x}{\sin x + \cos x} dx + \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sin x}{\sin x + \cos x} dx$$

$$2y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} 1\, dx$$

$$2y = (x)_{\frac{\pi}{8}}^{\frac{3\pi}{8}}$$

$$2y = \frac{3\pi}{8} - \frac{\pi}{8}$$

$$y = \frac{\pi}{8}$$

Question: 39

Prove that

Solution:

$$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{cos\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}}{\left(\sqrt{sin\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)} + \sqrt{cos\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}\right)} dx$$

$$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{sin\, x}}{\left(\sqrt{cos x} + \sqrt{sin\, x}\right)} dx \quad ...(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{cos\, x}}{\left(\sqrt{sin\, x} + \sqrt{cos\, x}\right)} dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{sin\, x}}{\left(\sqrt{cos\, x} + \sqrt{sin\, x}\right)} dx$$

$$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{sin\, x} + \sqrt{cos\, x}}{\left(\sqrt{sin\, x} + \sqrt{cos\, x}\right)} dx$$

$$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1\, dx$$

$$2y = (x)_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$y = \frac{\pi}{12}$$

Question: 40

Prove that

Solution:

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{2cos^2\frac{x}{2}} dx$$

$$y = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} sec^2\frac{x}{2} dx$$

$$y = \frac{1}{2}\left(\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right)_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$y = \tan\frac{3\pi}{8} - \tan\frac{\pi}{8}$

$y = \left(\sqrt{2}+1\right) - \left(\sqrt{2}-1\right) = 2$

Question: 41

Prove that

Solution:

$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x} dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}{1+\sin\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)} dx$$

$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi - x}{1+\sin x} dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x} dx + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi - x}{1+\sin x} dx$$

$$y = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{1+\sin x} dx$$

$$y = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx$$

$$y = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1-\sin x}{cos^2 x} dx$$

$$y = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} sec^2 x - \frac{\sin x}{cos^2 x} dx$$

Let, cos x = t

⇒ -sin x dx = dt

At x = π/4, t = $\frac{1}{\sqrt{2}}$

At x = 3π/4, t =$\frac{-1}{\sqrt{2}}$

$$y = \frac{\pi}{2}\left((\tan x)_{\frac{\pi}{4}}^{\frac{3\pi}{4}} + \int_{\frac{1}{\sqrt{2}}}^{\frac{-1}{\sqrt{2}}} \frac{1}{t^2}\,dt\right)$$

$$y = \frac{\pi}{2}(\tan\frac{3\pi}{4} - \tan\frac{\pi}{4} + \left(\frac{-1}{t}\right)_{\frac{1}{\sqrt{2}}}^{\frac{-1}{\sqrt{2}}}$$

$$y = \frac{\pi}{2}\left(-1 - 1 + \sqrt{2} + \sqrt{2}\right) = \pi\left(\sqrt{2} - 1\right)$$

Question: 42

Prove that

Solution:

$$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{\frac{3a}{4} + \frac{a}{4} - x}}{\sqrt{\frac{3a}{4} + \frac{a}{4} - x} + \sqrt{x}}\,dx$$

$$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}}\,dx$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}}\,dx + \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}}\,dx$$

$$2y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x} + \sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}}\,dx$$

$$y = \frac{1}{2}\int_{\frac{a}{4}}^{\frac{3a}{4}} 1\,dx$$

$$y = \frac{1}{2}(x)_{\frac{a}{4}}^{\frac{3a}{4}}$$

$$y = \frac{a}{4}$$

Question: 43

Prove that

Solution:

$$y = \int_1^4 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$$

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_1^4 \frac{\sqrt{4+1-x}}{\sqrt{4+1-x}+\sqrt{x}}dx$$

$$y = \int_1^4 \frac{\sqrt{5-x}}{\sqrt{5-x}+\sqrt{x}}dx$$

Adding eq.(1) and eq.(2)

$$2y = \int_1^4 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}dx + \int_1^4 \frac{\sqrt{5-x}}{\sqrt{5-x}+\sqrt{x}}dx$$

$$2y = \int_1^4 \frac{\sqrt{x}+\sqrt{5-x}}{\sqrt{5-x}+\sqrt{x}}dx$$

$$y = \frac{1}{2}\int_1^4 1\,dx$$

$$y = \frac{1}{2}(x)_1^4$$

$$y = \frac{3}{2}$$

Question: 44

Prove that

Solution:

Use integration by parts

$$\int I \times II\,dx = I\int II\,dx - \int \frac{d}{dx}I\left(\int II\,dx\right)dx$$

$$y = x\int \cot x\,dx - \int \frac{d}{dx}x\left(\int \cot x\,dx\right)dx$$

$$y = (x\log\sin x)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log\sin x\,dx$$

Let, $I = \int_0^{\frac{\pi}{2}} \log\sin x\,dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$I = \int_0^{\frac{\pi}{2}} \log\sin\left(\frac{\pi}{2}-x\right)dx$$

$I = \int_0^{\frac{\pi}{2}} \log\cos x\,dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log \sin x \, dx + \int_0^{\frac{\pi}{2}} \log \cos x \, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log \frac{2 \sin x \cos x}{2} dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log \sin 2x - \log 2 \, dx$$

Let, 2x = t

⇒ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$$2I = \frac{1}{2}\int_0^{\pi} \log \sin t \, dt - \frac{\pi}{2} \log 2$$

$$2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} \log \sin x \, dx - \frac{\pi}{2} \log 2$$

$$2I = I - \frac{\pi}{2} \log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin x \, dx \ = \ -\frac{\pi}{2} \log 2$$

$$y = (x \log \sin x)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log \sin x \, dx$$

$$y = \frac{\pi}{2} \log \sin \frac{\pi}{2} - \left(-\frac{\pi}{2} \log 2\right)$$

$$y = \frac{\pi}{2} \log 2$$

Question: 45

Prove that

Solution:

Let, x = sin t

⇒ dx = cos t dt

At x = 0, t = 0

At x = 1, t = π/2

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin^{-1} \sin t}{\sin t} \cos t \, dt$$

$$y = \int_0^{\frac{\pi}{2}} \frac{t \cos t}{\sin t} dt$$

$$y = \int_0^{\frac{\pi}{2}} t \cot t \, dt$$

Use integration by parts

$$\int I \times II \, dt = I \int II \, dt - \int \frac{d}{dt} I \left(\int II \, dt \right) dt$$

$$y = t \int \cot t \, dt - \int \frac{d}{dt} t \left(\int \cot t \, dt \right) dt$$

$$y = (t \log \sin t)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log \sin t \, dt$$

Let, $I = \int_0^{\frac{\pi}{2}} \log \sin t \, dt$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(t) \, dt = \int_a^b f(a + b - t) \, dt$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin \left(\frac{\pi}{2} - t \right) dt$$

$I = \int_0^{\frac{\pi}{2}} \log \cos t \, dt$ ...(2)

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log \sin t \, dt + \int_0^{\frac{\pi}{2}} \log \cos t \, dt$$

$$2I = \int_0^{\frac{\pi}{2}} \log \frac{2 \sin t \cos t}{2} \, dt$$

$$2I = \int_0^{\frac{\pi}{2}} \log \sin 2t - \log 2 \, dt$$

Let, 2t = z

⇒ 2 dt = dz

At t = 0, z = 0

At t = π/2, z = π

$$2I = \frac{1}{2} \int_0^{\pi} \log \sin z \, dz - \frac{\pi}{2} \log 2$$

$$2I = \frac{2}{2} \int_0^{\frac{\pi}{2}} \log \sin z \, dz - \frac{\pi}{2} \log 2$$

$$2I = I - \frac{\pi}{2} \log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin z \, dz \; = \; -\frac{\pi}{2} \log 2$$

$$y = (t \log \sin t)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log t \, dt$$

$$y = \frac{\pi}{2} \log \sin \frac{\pi}{2} - \left(-\frac{\pi}{2} \log 2\right)$$

$$y = \frac{\pi}{2} \log 2$$

Question: 46

Prove that

Solution:

Use integration by parts

$$\int I \times II \, dx = I \int II \, dx - \int \frac{d}{dx} I \left(\int II \, dx\right) dx$$

$$y = \log x \int \frac{1}{\sqrt{1-x^2}} dx - \int \frac{d}{dx} \log x \left(\int \frac{1}{\sqrt{1-x^2}} dx\right) dx$$

$$y = \; (\log x \sin^{-1} x)_0^1 - \int_0^1 \frac{\sin^{-1} x}{x} dx$$

$$y = \; -\int_0^1 \frac{\sin^{-1} x}{x} dx$$

Let, x = sin t

⇒ dx = cos t dt

At x = 0, t = 0

At x = 1, t = π/2

$$y = \; -\int_0^{\frac{\pi}{2}} \frac{\sin^{-1} \sin t}{\sin t} \cos t \, dt$$

$$y = \; -\int_0^{\frac{\pi}{2}} \frac{t \cos t}{\sin t} dt$$

$$y = \; -\int_0^{\frac{\pi}{2}} t \cot t \, dt$$

Use integration by parts

$$\int I \times II \, dt = I \int II \, dt - \int \frac{d}{dt} I \left(\int II \, dt\right) dt$$

$$y = -\left(t \int \cot t \, dt - \int \frac{d}{dt} t \left(\int \cot t \, dt\right) dt\right)$$

$$y = -\left((t \log \sin t)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log \sin t \, dt \right)$$

Let, $I = \int_0^{\frac{\pi}{2}} \log \sin t \, dt$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(t)\, dt = \int_a^b f(a+b-t)\, dt$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - t\right) dt$$

$I = \int_0^{\frac{\pi}{2}} \log \cos t \, dt$ ...(2)

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log \sin t \, dt + \int_0^{\frac{\pi}{2}} \log \cos t \, dt$$

$$2I = \int_0^{\frac{\pi}{2}} \log \frac{2 \sin t \cos t}{2} dt$$

$$2I = \int_0^{\frac{\pi}{2}} \log \sin 2t - \log 2 \, dt$$

Let, 2t = z

⇒ 2 dt = dz

At t = 0, z = 0

At t = π/2, z = π

$$2I = \frac{1}{2} \int_0^{\pi} \log \sin z \, dz - \frac{\pi}{2} \log 2$$

$$2I = \frac{2}{2} \int_0^{\frac{\pi}{2}} \log \sin z \, dz - \frac{\pi}{2} \log 2$$

$$2I = I - \frac{\pi}{2} \log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin z \, dz = -\frac{\pi}{2} \log 2$$

$$y = -\left((t \log \sin t)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log t \, dt \right)$$

$$y = \frac{-\pi}{2} \log \sin \frac{\pi}{2} + \left(-\frac{\pi}{2} \log 2\right)$$

$y = \frac{-\pi}{2}\log 2$

Question: 47

Prove that

Solution:

Let x = tan t

⇒ dx = $\sec^2$t dt

At x = 0, t = 0

At x = 1, t = π/4

$$y = \int_0^{\frac{\pi}{4}} \frac{\log(1+\tan t)}{1+tan^2 t} sec^2 t\, dt$$

$y = \int_0^{\frac{\pi}{4}} \log(1+\tan t)\, dt$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(t)\, dt = \int_a^b f(a+b-t)\, dt$$

$$y = \int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\frac{\pi}{4}-t\right)\right) dt$$

$$y = \int_0^{\frac{\pi}{4}} \log\left(1+\frac{1-\tan t}{1+\tan t}\right) dt$$

$y = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan t}\right) dt$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\frac{\pi}{4}} \log(1+\tan t)\, dt + \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan t}\right) dt$$

$$2y = \int_0^{\frac{\pi}{4}} \log(1+\tan t)\left(\frac{2}{1+\tan t}\right) dt$$

$$2y = \int_0^{\frac{\pi}{4}} \log 2\, dt$$

$$y = \frac{\pi}{8}\log 2$$

Question: 48

Prove that

Solution:

$y = \int_{-a}^{a} x^3\sqrt{a^2-x^2}\, dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(t)\,dt = \int_a^b f(a+b-t)\,dt$$

$$y = \int_{-a}^{a} (a-a-x)^3\sqrt{a^2-(a-a-x)^2}\,dx$$

$y = \int_{-a}^{a} -x^3\sqrt{a^2-x^2}\,dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_{-a}^{a} x^3\sqrt{a^2-x^2}\,dx + \left(-\int_{-a}^{a} x^3\sqrt{a^2-x^2}\,dx\right)$$

y = 0

Question: 49

Prove that

Solution:

$y = \int_{-\pi}^{\pi} sin^{75}x + x^{125}\,dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(t)\,dt = \int_a^b f(a+b-t)\,dt$$

$$y = \int_{-\pi}^{\pi} sin^{75}(\pi-\pi-x) + (\pi-\pi-x)^{125}\,dx$$

$y = \int_{-\pi}^{\pi} -sin^{75}x - x^{125}\,dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_{-\pi}^{\pi} sin^{75}x + x^{125}\,dx + \left(-\int_{-\pi}^{\pi} sin^{75}x + x^{125}\,dx\right)$$

y = 0

Question: 50

Prove that

Solution:

$y = \int_{-\pi}^{\pi} x^{12}sin^9x\,dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(t)\,dt = \int_a^b f(a+b-t)\,dt$$

$$y = \int_{-\pi}^{\pi} (\pi-\pi-x)^{12}sin^9(\pi-\pi-x)\,dx$$

$y = \int_{-\pi}^{\pi} -x^{12}sin^9x\,dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_{-\pi}^{\pi} x^{12} sin^9 x\, dx + \left(- \int_{-\pi}^{\pi} x^{12} sin^9 x\, dx \right)$$

y = 0

Question: 51

Prove that

Solution:

We know that

|x| = -x in [-1, 0)

|x| = x in [0, 1]

$$y = \int_{-1}^{0} e^{|x|}\, dx + \int_{0}^{1} e^{|x|}\, dx$$

$$y = \int_{-1}^{0} e^{-x}\, dx + \int_{0}^{1} e^{x}\, dx$$

$$y = (-e^{-x})_{-1}^{0} + (e^{x})_{0}^{1}$$

y= -(1-e)+(e-1)

y = 2(e – 1)

Question: 52

|x+1| = -(x+1) in [-2, -1)

|x+1| = (x+1) in [-1, 2]

$$y = \int_{-2}^{-1} |x+1|\, dx + \int_{-1}^{2} |x+1|\, dx$$

$$= -\int_{-2}^{-1} (x+1)\, dx + \int_{-1}^{2} (x+1)\, dx$$

$$= -\left(\frac{x^2}{2} + x\right)_{-2}^{-1} + \left(\frac{x^2}{2} + x\right)_{-1}^{2}$$

$$= -\left(\frac{1}{2} - 1 - 2 + 2\right) + \left(2 + 2 - \frac{1}{2} + 1\right)$$

=5

Question: 53

Prove that

Solution:

We know that

|x – 5| = -(x – 5) in [0, 5)

|x – 5| = (x – 5) in [5, 8]

$$y = \int_{0}^{5} |x-5|\, dx + \int_{5}^{8} |x-5|\, dx$$

$$y = -\int_0^5 (x-5)\,dx + \int_5^8 (x-5)\,dx$$

$$y = -\left(\frac{x^2}{2} - 5x\right)_0^5 + \left(\frac{x^2}{2} - 5x\right)_5^8$$

$$y = -\left(\frac{25}{2} - 25\right) + \left(32 - 40 - \frac{25}{2} + 25\right)$$

=17

Question: 54

Prove that

Solution:

We know that

|cos x| = cos x in [0, π/2)

|cos x| = -cos x in [π/2, 3π/2)

|cos x| = cos x in [3π/2, 2π]

$$y = \int_0^{\frac{\pi}{2}} |\cos x|\,dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} |\cos x|\,dx + \int_{\frac{3\pi}{2}}^{2\pi} |\cos x|\,dx$$

$$y = \int_0^{\frac{\pi}{2}} \cos x\,dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x\,dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x\,dx$$

$$y = (\sin x)_0^{\frac{\pi}{2}} - (\sin x)_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + (\sin x)_{\frac{3\pi}{2}}^{2\pi}$$

y=(1-0)—1-1+(0+1)

=4

Question: 55

Prove that

Solution:

We know that

|sin x| = -sin x in [-π/4, 0)

|sin x| = sin x in [0, π/4]

$$y = \int_{\frac{-\pi}{4}}^{0} |\sin x|\,dx + \int_0^{\frac{\pi}{4}} |\sin x|\,dx$$

$$y = -\int_{\frac{-\pi}{4}}^{0} \sin x\,dx + \int_0^{\frac{\pi}{4}} \sin x\,dx$$

$$y = -(-\cos x)_{\frac{-\pi}{4}}^{0} + (-\cos x)_0^{\frac{\pi}{4}}$$

$$y = \left(1 - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} - 1\right)$$

$= 2 - \frac{1}{\sqrt{2}}$

Question: 56

Prove that

Solution:

$y = \int_1^3 f(x)\,dx$

$$y = \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx$$

$$y = \int_1^2 2x + 1\,dx + \int_2^3 x^2 + 1\,dx$$

$$y = (x^2 + x)_1^2 + \left(\frac{x^3}{3} + x\right)_2^3$$

$$y = (4 + 2 - 1 - 1) + \left(9 + 3 - \frac{8}{3} - 2\right)$$

$$= \frac{34}{3}$$

Question: 57

Prove that

Solution:

$y = \int_0^4 f(x)\,dx$

$$y = \int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx$$

$$y = \int_0^2 3x^2 + 4\,dx + \int_2^4 9x - 2\,dx$$

$$y = (x^3 + 4x)_0^2 + \left(\frac{9x^2}{2} - 2x\right)_2^4$$

y=(8+8)+(72-8-18+4)

=66

Question: 58

Prove that

Solution:

$y = \int_0^4 |x| + |x - 2| + |x - 4|\,dx$

$$y = \int_0^2 |x| + |x - 2| + |x - 4|\,dx + \int_2^4 |x| + |x - 2| + |x - 4|\,dx$$

$$y = \int_0^2 x - (x - 2) - (x - 4)\,dx + \int_2^4 x + (x - 2) - (x - 4)\,dx$$

$$y=\left(-\frac{x^2}{2}+6x\right)_0^2+\left(\frac{x^2}{2}+2x\right)_2^4$$

y=(-2+12)+(8+8-2-4)

=20

Exercise : 16D

Question: 1

Evaluate each of

Solution:

f(x) is continuous in [0,2]

$$\int_a^b f(x)dx=\lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=2/n

$$\int_0^2 (x+4)dx=\lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f(2r/n)$$

$$=\lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{2r}{n}\right)+4$$

$$=\lim_{n\to\infty}\left(\frac{2}{n}\right)\left(\frac{(n-1)(n)}{n}+4(n-1)\right)$$

$$=\lim_{n\to\infty}\frac{2}{n}\frac{n^2-n+4n^2-4n}{n}$$

$$=\lim_{n\to\infty}\frac{2}{n}\frac{5n^2-5n}{n}$$

$$=\lim_{n\to\infty}\frac{10n^2-10n}{n^2}$$

$$=\lim_{n\to\infty} 10-(10/n)$$

=10

Question: 2

Evaluate each of

Solution:

f(x) is continuous in [1,2]

$$\int_a^b f(x)dx=\lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=1/n

$$\int_1^2 (3x-2)dx = \lim_{n\to\infty}\left(\frac{1}{n}\right)\sum_{r=0}^{n-1} f\left(1+\left(\frac{r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{n}\right)\sum_{r=0}^{n-1}(3+3\frac{r}{n}-2)$$

$$\lim_{n\to\infty}\left(\frac{1}{n}\right)\left(n+\frac{3(n-1)(n)}{2n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{n}\right)\left(\frac{2n^2+3n^2-3n}{2n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{5n^2-3n}{2n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{5}{2}\right)-\left(\frac{3}{2n}\right)$$

=5/2

Question: 3

Evaluate each of

Solution:

f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=2/n

$$\int_1^3 (x^2)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(1+\left(\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(1+\left(\frac{2r}{n}\right)\right)^2$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}(\frac{4r^2}{n^2}+1+\frac{4r}{n})$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4(n-1)(n)(2n-1)}{6n^2}+n+\frac{4(n-1)(n)}{2n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4(2n^3-2n^2-n^2+n)}{6n^2}+n+\frac{2(n^2-n)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{(8n^3-12n^2+4n)+(6n^3)+(12n^3-12n^2)}{6n^2}\right)$$

$$=\lim_{n\to\infty}\frac{2}{n}\left(\frac{26n^3-24n^2+4n}{6n^2}\right)$$

$$=\lim_{n\to\infty}\left(\frac{52n^3-48n^2+8n}{6n^3}\right)$$

$$=\lim_{n\to\infty}\left(\frac{52}{6}\right)-\left(\frac{26}{6n}\right)+\left(\frac{8}{6n^2}\right)$$

=26/3

Question: 4

Evaluate each of

Solution:

f(x) is continuous in [0,3]

$$\int_a^b f(x)dx=\lim_{n\to\infty}h\sum_{r=0}^{n-1}f(a+rh), where\ h=(b-a)/n$$

here h=3/n

$$\int_0^3 (x^2+1)dx=\lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}f\left(\left(\frac{3r}{n}\right)\right)$$

$$=\lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(\left(\frac{3r}{n}\right)^2+1\right)$$

$$=\lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(\frac{9r^2}{n^2}+1\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n-1)(n)(2n-1)}{6n^2}+n\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n^2-n)(2n-1)}{6n^2}+n\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}(\frac{9(2n^3-2n^2-n^2+n)}{6n^2}+n)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{(18n^3-27n^2+9n)+(6n^3)}{6n^2}\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{24n^3-27n^2+9n}{6n^2}\right)$$

$$=\lim_{n\to\infty}\left(\frac{72n^3-81n^2+27n}{6n^3}\right)$$

$$=\lim_{n\to\infty}\left(\frac{72}{6}\right)-\left(\frac{81}{6n}\right)+\left(\frac{27}{6n^2}\right)$$

=12

Question: 5

Evaluate each of

Solution:

f(x) is continuous in [2,5]

$$\int_a^b f(x)dx=\lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=3/n

$$\int_2^5 (3x^2-5)dx=\lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\left(2+\frac{3r}{n}\right)\right)$$

$$=\lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}(3\left(2+\frac{3r}{n}\right)^2-5)$$

$$=\lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}3\left(\frac{9r^2}{n^2}+4+\frac{12r}{n}\right)-5$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{27(n-1)(n)(2n-1)}{6n^2}+12n+\frac{18n(n-1)}{n}-5n\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{27(n^2-n)(2n-1)}{6n^2}+12n+\frac{18n(n-1)}{n}-5n\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{27(2n^3-2n^2-n^2+n)}{6n^2}+12n+\frac{18n(n-1)}{n}-5n\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{(54n^3-81n^2+27n)+(42n^3)+(108n^3-108n^2)}{6n^2}\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{204n^3-189n^2+27n}{6n^2}\right)$$

$$=\lim_{n\to\infty}\left(\frac{612n^3-567n^2+27n}{6n^3}\right)$$

$$=\lim_{n\to\infty}\left(\frac{612}{6}\right)-\left(\frac{567}{6n}\right)+\left(\frac{27}{6n^2}\right)$$

=102

Question: 6

Evaluate each of

Solution:

f(x) is continuous in [2,5]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=3/n

$$\int_0^3 (x^2+2x)dx = \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\left(\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}(\left(\frac{3r}{n}\right)^2 + \frac{6r}{n})$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(\frac{9r^2}{n^2} + \frac{6r}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n-1)(n)(2n-1)}{6n^2} + \frac{3n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n^2-n)(2n-1)}{6n^2} + \frac{3n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{9(2n^3-2n^2-n^2+n)}{6n^2} + \frac{3n(n-1)}{n})$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{(18n^3-27n^2+9n)+(18n^3-18n^2)}{6n^2}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{36n^3-45n^2+9n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{108n^3-135n^2+27n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{108}{6}\right)-\left(\frac{135}{6n}\right)+\left(\frac{27}{6n^2}\right)$$

=18

Question: 7

Evaluate each of

Solution:

f(x) is continuous in [1,4]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=3/n

$$\int_1^4 (3x^2+2x)dx = \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\left(1+\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(3\left(1+\frac{3r}{n}\right)^2+2\left(1+\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} 3\left(\frac{9r^2}{n^2}+1+\frac{6r}{n}\right)+2\left(1+\frac{3r}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{27(n-1)(n)(2n-1)}{6n^2}+3n+\frac{9n(n-1)}{n}+2n+\frac{3n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{27(n^2-n)(2n-1)}{6n^2}+5n+\frac{12n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{27(2n^3-2n^2-n^2+n)}{6n^2}+5n+\frac{12n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{(54n^3-81n^2+27n)+(30n^3)+(72n^3-72n^2)}{6n^2}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{156n^3-153n^2+27n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{468n^3-459n^2+81n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{468}{6}\right)-\left(\frac{459}{6n}\right)+\left(\frac{81}{6n^2}\right)$$

=78

Question: 8

Evaluate each of

Solution:

f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=3/n

$$\int_1^3 (x^2+5x)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\left(1+\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}(\left(1+\frac{2r}{n}\right)^2+5\left(1+\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} (1 + \frac{4r^2}{n^2} + \frac{4r}{n} + 5 + \frac{10r}{n})$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} (1 + \frac{4r^2}{n^2} + \frac{4r}{n} + 5 + \frac{10r}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{4(n-1)(n)(2n-1)}{6n^2} + 6n + \frac{7n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{4(n^2-n)(2n-1)}{6n^2} + 6n + \frac{7n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{4(2n^3-2n^2-n^2+n)}{6n^2} + 6n + \frac{7n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{(8n^3-12n^2+4n)+(42n^3-42n^2)+(36n^3)}{6n^2})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{86n^3-54n^2+4n}{6n^2})$$

$$= \lim_{n\to\infty} (\frac{172n^3-108n^2+8n}{6n^3})$$

$$= \lim_{n\to\infty} \left(\frac{172}{6}\right) - \left(\frac{108}{6n}\right) + \left(\frac{8}{6n^2}\right)$$

=86/3

Question: 9

Evaluate each of

Solution:

f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=2/n

$$\int_1^3 (2x^2+5x)dx = \lim_{n\to\infty} (\frac{2}{n}) \sum_{r=0}^{n-1} f\left((1+\frac{2r}{n})\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} (2\left(1+\frac{2r}{n}\right)^2 + 5\left(1+\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} (2 + \frac{8r^2}{n^2} + \frac{8r}{n} + 5 + \frac{10r}{n})$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} \left(7 + \frac{8r^2}{n^2} + \frac{18r}{n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{8(n-1)(n)(2n-1)}{6n^2} + 7n + \frac{9n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{8(n^2-n)(2n-1)}{6n^2} + 7n + \frac{9n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{8(2n^3-2n^2-n^2+n)}{6n^2} + 7n + \frac{9n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{(16n^3-24n^2+8n)+(54n^3-54n^2)+(42n^3)}{6n^2}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{112n^3-78n^2+8n}{6n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{224n^3-156n^2+8n}{6n^3}\right)$$

$$= \lim_{n\to\infty} \left(\frac{224}{6}\right) - \left(\frac{156}{6n}\right) + \left(\frac{8}{6n^2}\right)$$

=112/3

Question: 10

Evaluate each of

Solution:

f(x) is continuous in [0,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=2/n

$$\int_0^2 (x^3)dx = \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} \left(\frac{2r}{n}\right)^3$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} \left(\frac{8r^3}{n^3}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{8(n-1)^2(n)^2}{4n^3}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{8(n^2-2n+1)(n^2)}{4n^3})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{8(n^4-2n^3+n^2)}{4n^3})$$

$$= \lim_{n\to\infty} (\frac{16n^4-32n^3+16n^2}{4n^4})$$

$$= \lim_{n\to\infty} \left(\frac{16}{4}\right) - \left(\frac{32}{4n}\right) + \left(\frac{16}{4n^2}\right)$$

=4

Question: 11

Evaluate each of

Solution:

f(x) is continuous in [2,4]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=3/n

$$\int_2^4 (x^2-3x+2)dx = \lim_{n\to\infty} (\frac{2}{n}) \sum_{r=0}^{n-1} f\left((2+\frac{2r}{n})\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} (\left(2+\frac{2r}{n}\right)^2 - 3\left(2+\frac{2r}{n}\right) + 2)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} (\frac{4r^2}{n^2} + \frac{8r}{n} + 4 - 6 - \frac{6r}{n} + 2)$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{4(n-1)(n)(2n-1)}{6n^2} + \frac{n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{4(n^2-n)(2n-1)}{6n^2} + \frac{n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{4(2n^3-2n^2-n^2+n)}{6n^2} + \frac{n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{(8n^3-12n^2+4n)+(6n^3-6n^2)}{6n^2})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{14n^3-18n^2+4n}{6n^2})$$

$$= \lim_{n\to\infty} (\frac{28n^3-36n^2+8n}{6n^3})$$

$$=\lim_{n\to\infty}\left(\frac{28}{6}\right)-\left(\frac{36}{6n}\right)+\left(\frac{8}{6n^2}\right)$$

=14/3

Question: 12

Evaluate each of

Solution:

f(x) is continuous in [0,2]

$$\int_a^b f(x)dx=\lim_{n\to\infty}h\sum_{r=0}^{n-1}f(a+rh), where\ h=(b-a)/n$$

here h=2/n

$$\int_0^2 (x^2+x)dx=\lim_{n\to\infty}(\frac{2}{n})\sum_{r=0}^{n-1}f\left(\frac{2r}{n}\right)$$

$$=\lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}(\left(\frac{2r}{n}\right)^2+\left(\frac{2r}{n}\right))$$

$$=\lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}(\frac{4r^2}{n^2}+\frac{2r}{n})$$

$$=\lim_{n\to\infty}\frac{2}{n}(\frac{4(n-1)(n)(2n-1)}{6n^2}+\frac{n(n-1)}{n})$$

$$=\lim_{n\to\infty}\frac{2}{n}(\frac{4(n^2-n)(2n-1)}{6n^2}+\frac{n(n-1)}{n})$$

$$=\lim_{n\to\infty}\frac{2}{n}(\frac{4(2n^3-2n^2-n^2+n)}{6n^2}+\frac{n(n-1)}{n})$$

$$=\lim_{n\to\infty}\frac{2}{n}(\frac{(8n^3-12n^2+4n)+(6n^3-6n^2)}{6n^2})$$

$$=\lim_{n\to\infty}\frac{2}{n}(\frac{14n^3-18n^2+4n}{6n^2})$$

$$=\lim_{n\to\infty}(\frac{28n^3-36n^2+8n}{6n^3})$$

$$=\lim_{n\to\infty}\left(\frac{28}{6}\right)-\left(\frac{36}{6n}\right)+\left(\frac{8}{6n^2}\right)$$

=14/3

Question: 13

Evaluate each of

Solution:

f(x) is continuous in [0,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=3/n

$$\int_0^3 (2x^2+3x+5)dx = \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\frac{3r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}(2\left(\frac{3r}{n}\right)^2 + 3\left(\frac{3r}{n}\right) + 5)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}(\frac{18r^2}{n^2} + \frac{9r}{n} + 5)$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{18(n-1)(n)(2n-1)}{6n^2} + \frac{9n(n-1)}{2n} + 5n)$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{18(n^2-n)(2n-1)}{6n^2} + \frac{9n(n-1)}{2n} + 5n)$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{18(2n^3-2n^2-n^2+n)}{6n^2} + \frac{9n(n-1)}{2n} + 5n)$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{(36n^3-54n^2+18n)+(27n^3-27n^2)+30n^3}{6n^2})$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{93n^3-81n^2+18n}{6n^2})$$

$$= \lim_{n\to\infty}(\frac{279n^3-243n^2+54n}{6n^3})$$

$$= \lim_{n\to\infty}\left(\frac{279}{6}\right) - \left(\frac{243}{6n}\right) + \left(\frac{54}{6n^2}\right)$$

=93/2

Question: 14

Evaluate each of

Solution:

Since it is modulus function so we need to break the function and then solve it

$$f(x) = \int_0^{\frac{1}{3}}(1-3x)dx + \int_{\frac{1}{3}}^{1}(3x-1)dx$$

it is continuous in [0,1]

let $g(x) = \int_0^{\frac{1}{3}}(1-3x)dx$ and $h(x) = \int_{\frac{1}{3}}^{1}(3x-1)dx$

$$g(x) = \int_0^{\frac{1}{3}} (1-3x)dx$$

here h=1/3n

$$\int_0^{\frac{1}{3}} (1-3x)dx = \lim_{n\to\infty} (\frac{1}{3n}) \sum_{r=0}^{n-1} f(r/3n)$$

$$= \lim_{n\to\infty} \left(\frac{1}{3n}\right) \sum_{r=0}^{n-1} \left(1 - 3\left(\frac{r}{3n}\right)\right)$$

$$= \lim_{n\to\infty} \left(\frac{1}{3n}\right) \left(n - \frac{3(n-1)(n)}{6n}\right)$$

$$= \lim_{n\to\infty} \frac{1}{3n} \frac{6n^2 - 3n^2 + 3n}{3n}$$

$$= \lim_{n\to\infty} \frac{1}{3n} \frac{3n^2 + 3n}{3n}$$

$$= \lim_{n\to\infty} \frac{3n^2 + 3n}{9n^2}$$

$$= \lim_{n\to\infty} \frac{1}{3} + \left(\frac{3}{9n}\right)$$

=1/3

$$h(x) = \int_{\frac{1}{3}}^{1} (3x-1)dx$$

here h=2/3n

$$\int_{\frac{1}{3}}^{1} (3x-1)dx = \lim_{n\to\infty} \left(\frac{2}{3n}\right) \sum_{r=0}^{n-1} f\left(\left(\frac{1}{3}\right) + \left(\frac{2r}{3n}\right)\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{3n}\right) \sum_{r=0}^{n-1} (3\left(\frac{1}{3} + \frac{2r}{3n}\right) - 1)$$

$$= \lim_{n\to\infty} \left(\frac{2}{3n}\right) \left(\frac{(n-1)(n)}{n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{3n} . \frac{n^2 - n}{n}$$

$$= \lim_{n\to\infty} \frac{2}{3n} . \frac{n^2 - n}{n}$$

$$= \lim_{n\to\infty} \frac{2n^2 - 2n}{3n^2}$$

$$= \lim_{n\to\infty} \frac{2}{3} - \left(\frac{2}{3n}\right)$$

=2/3

f(x)=g(x)+h(x)

=(1/3)+(2/3)

=3/3

=1

Question: 15

Evaluate each of

Solution:

f(x) is continuous in [0,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a + rh), where\ h = (b - a)/n$$

here h=2/n

$$\int_0^2 (e^x)dx = \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} e^{\frac{2r}{n}}$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) (e^0 + e^h + e^{2h} + \cdots \ldots \ldots \ldots \ldots . + e^{nh}$$

$$sum\ of e^0 + e^h + e^{2h} + \cdots \ldots \ldots \ldots \ldots . + e^{nh}$$

Which is g.p with common ratio $e^{1/n}$

Whose sum is $= \frac{e^h(1-e^{nh})}{1-e^h}$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \left(\frac{e^h(1 - e^{nh})}{1 - e^h}\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \left(\frac{e^h(1 - e^{nh})}{\frac{1 - e^h . h}{h}}\right)$$

$$\lim_{h\to 0} \frac{1 - e^h}{h} = -1$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) . \frac{e^h(1 - e^{nh})}{-h}$$

As h=2/n

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) . \frac{e^{\left(\frac{2}{n}\right)}(1 - e^{n*(2/n)})}{-2/n}$$

$=e^2-1$

Question: 16

Evaluate each of

Solution:

f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=2/n

$$\int_1^3 (e^{-x})dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(1+\left(\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} e^{-(1+\frac{2r}{n})}$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} e^{-1}.e^{-\frac{2r}{n}}$$

Common ratio is $h=-2/n$

$sum = e^{-1}(e^0+e^h+e^{2h}+\cdots\dots\dots\dots\dots.+e^{nh})$

$$=\lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)(e^0+e^h+e^{2h}+\cdots\dots\dots\dots\dots.+e^{nh}$$

$sum\ of = e^0+e^h+e^{2h}+\cdots\dots\dots\dots\dots.+e^{nh}$

Which is g.p. with common ratio $e^{1/n}$

Whose sum is $=\frac{e^h(1-e^{nh})}{1-e^h}$

$$=\lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^h(1-e^{nh})}{1-e^h}\right)$$

$$=\lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^h(1-e^{nh})}{\frac{1-e^h.h}{h}}\right.$$

$$\lim_{h\to 0}\frac{1-e^h}{h} = -1$$

$$=\lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^h(1-e^{nh})}{-h}\right.$$

As h=-2/n

$$=\lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^{(-\frac{2}{n})}(1-e^{n*(-2/n)})}{2/n}\right.$$

$$=\frac{(1-e^{-2})}{e}$$

$$=\frac{(e^2-1)}{e^3}$$

Question: 17

Evaluate each of

Solution:

f(x) is continuous in [a,b]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=(b-a)/n

$$\int_a^b (cosx)dx = \lim_{n\to\infty}\left(\frac{b-a}{n}\right)\sum_{r=0}^{n-1} f(a+rh)$$

$$= \lim_{n\to\infty}\left(\frac{b-a}{n}\right)\sum_{r=0}^{n-1} \cos(a+rh)$$

S=cos(a)+ cos(a+h)+ cos(a+2h)+ cos(a+3h)+......................+ cos(a+(n-1)h)= $\frac{\sin\left(\frac{nh}{2}\right)\cos\left(a+\frac{(n-1)h}{2}\right)}{\sin\left(\frac{h}{2}\right)}$

Putting h=(b-a)/n

$$= \lim_{n\to\infty}\left(\frac{b-a}{n}\right)\frac{\sin\left(\frac{n(b-a)}{2n}\right)\cos\left(a+\frac{(n-1)(b-a)}{2n}\right)}{\frac{\sin\left(\frac{b-a}{2n}\right)}{\frac{b-a}{2n}}.\frac{b-a}{2n}}$$

As we know

$$\lim_{h\to 0}\left(\frac{sinh}{h}\right) = 1$$

$$= \lim_{n\to\infty} 2\sin\left(\frac{(b-a)}{2}\right)\cos\left(a+\left(\frac{1}{2}-\frac{1}{2n}\right)(b-a)\right)$$

$$= 2\sin\left(\frac{b-a}{2}\right)\cos\left(\frac{b+a}{2}\right)$$

Which is trigonometry formula of sin(b)-sin(a)

Final answer is sin(b)-sin(a)

Exercise : OBJECTIVE QUESTIONS

Question: 1

Mark (√) against

Solution:

$$y = \int_1^4 x\sqrt{x}\,dx$$

$$= \int_1^4 x^{\frac{3}{2}}dx$$

$$= \left(\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right)_1^4$$

$$= \frac{2}{5}\left(4^{\frac{5}{2}} - 1^{\frac{5}{2}}\right)$$

$= \frac{2}{5}(32 - 1)$

$= \frac{62}{5}$

=12.4

Question: 2

Mark (√) against

Solution:

$y = \int_0^2 \sqrt{6x + 4}\, dx$

$= \left(\frac{(6x + 4)^{\frac{1}{2}+1}}{6\left(\frac{1}{2} + 1\right)} \right)_0^2$

$= \frac{2}{6 \times 3}\left(16^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)$

$= \frac{2}{6 \times 3}(64 - 8)$

$= \frac{56}{9}$

Question: 3

Mark (√) against

Solution:

$y = \int_0^1 \frac{dx}{\sqrt{5x+3}}$

$= \left(\frac{(5x + 3)^{\frac{-1}{2}+1}}{5\left(\frac{-1}{2} + 1\right)} \right)_0^1$

$= \frac{2}{5}\left(8^{\frac{1}{2}} - 3^{\frac{1}{2}}\right)$

$= \frac{2}{5}(\sqrt{8} - \sqrt{3})$

Question: 4

Mark (√) against

Solution:

$y = \int_0^1 \frac{1}{1+x^2}\, dx$

$= (\tan^{-1} x)_0^1$

$= \tan^{-1} 1 - \tan^{-1} 0$

$= \frac{\pi}{4} - 0$

$= \frac{\pi}{4}$

Question: 5

Mark (√) against

Solution:

$y = \int_0^2 \frac{dx}{\sqrt{4-x^2}}$

Use formula $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$

$y = \left(\sin^{-1}\frac{x}{2}\right)_0^2$

$= \sin^{-1} 1 - \sin^{-1} 0$

$= \frac{\pi}{2}$

Question: 6

Mark (√) against

Solution:

$y = \int_{\sqrt{3}}^{\sqrt{8}} x\sqrt{1+x^2}\,dx$

Let, $x^2 = t$

Differentiating both side with respect to t

$2x\frac{dx}{dt} = 1$

$\Rightarrow xdx = \frac{1}{2}dt$

At $x = \sqrt{3}$, $t = 3$

At $x = \sqrt{8}$, $t = 8$

$y = \frac{1}{2}\int_3^8 \sqrt{1+t}\,dt$

$= \frac{1}{2}\left(\frac{(1+t)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)}\right)_3^8$

$= \frac{1}{3}\left(9^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)$

$= \frac{1}{3}(27-8)$

$= \frac{19}{3}$

Question: 7

Mark (√) against

Solution:

Let, $x^4 = t$

Differentiating both side with respect to t

$4x^3\frac{dx}{dt} = 1$

$\Rightarrow x^3dx = \frac{1}{4}dt$

At $x = 0$, $t = 0$

At $x = 1$, $t = 1$

$$y = \frac{1}{4}\int_0^1 \frac{1}{1+t^2}\,dt$$

$$= \frac{1}{4}(\tan^{-1} t)_0^1$$

$$= \frac{1}{4}(\tan^{-1} 1 - \tan^{-1} 0)$$

$$= \frac{\pi}{16}$$

Question: 8

Mark (√) against

Solution:

Let, log x = t

Differentiating both side with respect to t

$$\frac{1}{x}\frac{dx}{dt} = 1$$

$$\Rightarrow \frac{1}{x}\,dx = dt$$

At x = 1, t = 0

At x = e, t = 1

$$y = \int_0^1 t^2\,dt$$

$$= \left(\frac{t^3}{3}\right)_0^1$$

$$= \frac{1}{3}$$

Question: 9

Mark (√) against

Solution:

$$y = (\ln(\sin x))_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \ln(\sin\frac{\pi}{2}) - \ln(\sin\frac{\pi}{4})$$

$$= \ln 1 - \ln\frac{1}{\sqrt{2}}$$

$$= \frac{1}{2}\ln 2$$

Question: 10

Mark (√) against

Solution:

$$y = \int_0^{\frac{\pi}{4}}(\sec^2 x - 1)\,dx$$

$$= (\tan x - x)_0^{\frac{\pi}{4}}$$

$= \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - (\tan 0 - 0)$

$= 1 - \frac{\pi}{4}$

Question: 11

Mark (√) against

Solution:

$y = \int_0^{\frac{\pi}{2}} \frac{1+\cos 2x}{2} dx$

$= \left(\frac{x}{2} + \frac{\sin 2x}{4}\right)_0^{\frac{\pi}{2}}$

$= \left(\frac{\frac{\pi}{2}}{2} + \frac{\sin\pi}{4}\right) - \left(\frac{0}{2} + \frac{\sin 0}{4}\right)$

$= \frac{\pi}{4}$

Question: 12

Mark (√) against

Solution:

$y = (\ln(\operatorname{cosec} x - \cot x))_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$= \ln\left(\operatorname{cosec}\frac{\pi}{2} - \cot\frac{\pi}{2}\right) - \ln\left(\operatorname{cosec}\frac{\pi}{3} - \cot\frac{\pi}{3}\right)$

$= \ln(1-0) - \ln\left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)$

$= \frac{1}{2}\log 3$

Question: 13

Mark (√) against

Solution:

$y = \int_0^{\frac{\pi}{2}} \cos x\,(1 - \sin^2 x)\, dx$

Let, sin x = t

Differentiating both side with respect to t

$\operatorname{Cos} x \frac{dx}{dt} = 1$

$\Rightarrow \cos x\, dx = dt$

At x = 0, t = 0

At x = $\frac{\pi}{2}$, t = 1

$y = \int_0^1 1 - t^2\ dt$

$= \left(t - \frac{t^3}{3}\right)_0^1$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

Question: 14

Mark (√) against

Solution:

$y = \int_0^{\frac{\pi}{4}} e^{tan\,x} sec^2 x\, dx$

Let, tan x = t

Differentiating both side with respect to t

$sec^2 x \frac{dx}{dt} = 1$

$\Rightarrow \; sec^2 x\; dx = dt$

At x = 0, t = 0

At x =$\frac{\pi}{4}$, t = 1

$y = \int\limits_0^1 e^t\, dt$

$= \; e^t{}_0^1$

= $e^1 - e^0$

= e – 1

Question: 15

Mark (√) against

Solution:

Let, sin x = t

Differentiating both side with respect to t

$Cos\, x \frac{dx}{dt} = \; 1$

$\Rightarrow cos\, x\, dx \; = dt$

At x = 0, t = 0

At x = $\frac{\pi}{2}$, t = 1

$y = \int\limits_0^1 \frac{1}{1+t^2} dt$

$= \; (tan^{-1} t)_0^1$

= $\tan^{-1}1 - \tan^{-1}0$

= π/4

Question: 16

Mark (√) against

Solution:

Let, 1/x = t

Differentiating both side with respect to t

$$\frac{-1}{x^2}\frac{dx}{dt} = 1$$

$$\Rightarrow \frac{1}{x^2}dx = -dt$$

At x = 1/π, t = π

At x = 2/π, t = π/2

$$y = \int_{\pi}^{\frac{\pi}{2}} \sin t \; dt$$

$$= (-\cos t)_{\pi}^{\frac{\pi}{2}}$$

= 1

Question: 17

Mark (√) against

Solution:

$$y = \int_0^{\pi} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx$$

$$= \int_0^{\pi} \frac{1-\sin x}{\cos^2 x} dx$$

$$= \int_0^{\pi} \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} dx$$

$$= \int_0^{\pi} \sec^2 x \, dx - \int_0^{\pi} \frac{\sin x}{\cos^2 x} dx$$

Let, cos x = t

Differentiating both side with respect to t

$$-\sin x \frac{dx}{dt} = 1$$

$$\Rightarrow \sin x \, dx = -dt$$

At x = 0, t = 1

At x = π, t = -1

$$y = (\tan x)_0^{\pi} + \int_1^{-1} \frac{1}{t^2} dt$$

$$= (\tan \pi - \tan 0) + \left(\frac{t^{-1}}{-1}\right)_1^{-1}$$

=2

Question: 18

Mark (√) against

Solution:

$y = \int_0^{\frac{\pi}{2}} sin^{\frac{3}{2}} x cos^3 x\, dx$

$$y = \int_0^{\frac{\pi}{2}} sin^{\frac{3}{2}} x\, cos\, x\, (1 - sin^2 x)\, dx$$

Let, sin x = t

Differentiating both side with respect to t

$$Cos\, x \frac{dx}{dt} = 1$$

⇒cos x dx=dt

At x = 0, t = 0

At x = π/2, t = 1

$$y = \int_0^1 t^{\frac{3}{2}} - t^{\frac{7}{2}}\, dt$$

$$= \left(\frac{t^{\frac{5}{2}}}{\frac{5}{2}} - \frac{t^{\frac{9}{2}}}{\frac{9}{2}}\right)_0^1$$

$$= \frac{2}{5} - \frac{2}{9}$$

$$= \frac{8}{45}$$

Question: 19

Mark (√) against

Solution:

$y = \int_0^1 \frac{e^x (x+1-1)}{(1+x)^2} dx$

$$= \int_0^1 e^x \left(\frac{1}{1+x} - \frac{1}{(1+x)^2}\right) dx$$

Use formula $\int e^x(f(x) + f'(x))dx = e^x f(x)$

If $f(x) = \frac{1}{1+x}$

then $f'(x) = -\frac{1}{(1+x)^2}$

$$y = \left(\frac{e^x}{1+x}\right)_0^1$$

$$y = \frac{e}{2} - 1$$

Question: 20

Mark (√) against

Solution:

$y = \int_0^{\frac{\pi}{2}} e^x \left(\frac{1+\sin x}{2\cos^2\frac{x}{2}}\right) dx$

$$= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2\cos^2\frac{x}{2}} + \frac{\sin x}{2\cos^2\frac{x}{2}} \right) dx$$

$$= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2\cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} \right) dx$$

$$= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2} \right) dx$$

Use formula $\int e^x(f(x) + f'(x))dx = e^x f(x)$

If $f(x) = \tan\frac{x}{2}$ then $f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$

$$y = \left(e^x \tan\frac{x}{2} \right)_0^{\frac{\pi}{2}}$$

$$= e^{\frac{\pi}{2}} \tan\frac{\frac{\pi}{2}}{2} - e^0 \tan\frac{0}{2}$$

$$= e^{\frac{\pi}{2}}$$

Question: 21

Mark (√) against

Solution:

$$y = \int_0^{\frac{\pi}{4}} \sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x}\, dx$$

$$= \int_0^{\frac{\pi}{4}} \sin x + \cos x\, dx$$

$$= (-\cos x + \sin x)_0^{\frac{\pi}{4}}$$

$$= \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - (-1 + 0)$$

y = 1

Question: 22

Mark (√) against

Solution:

$$y = \int_0^{\frac{\pi}{2}} \sqrt{2\cos^2 x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2}\cos x\, dx$$

$$= \sqrt{2}(\sin x)_0^{\frac{\pi}{2}}$$

$=\sqrt{2}$

Question: 23

Mark (√) against

Solution:

$y = \int_0^1 \frac{1-x-1+1}{1+x} dx$

$= \int_0^1 \frac{2}{1+x} - 1\, dx$

$= (2\ln(1+x) - x)_0^1$

= 2 ln 2 – 1

Question: 24

Mark (√) against

Solution:

$y = \int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} dx$

$= \left(\frac{x}{2} - \frac{\sin 2x}{4}\right)_0^{\frac{\pi}{2}}$

$= \frac{\pi}{4} - \frac{\sin\pi}{4}$

$= \frac{\pi}{4}$

Question: 25

Mark (√) against

Solution:

$y = \int_0^{\frac{\pi}{6}} \cos x\,(1 - 2\sin^2 x)\, dx$

$= \int_0^{\frac{\pi}{6}} \cos x - 2\cos x \sin^2 x\, dx$

$= (\sin x)_0^{\frac{\pi}{6}} - 2\int_0^{\frac{\pi}{6}} \cos x \sin^2 x\, dx$

Let, sin x = t

Differentiating both side with respect to t

$\text{Cos}\, x \frac{dx}{dt} = 1$

$\Rightarrow \cos x\, dx = dt$

At x = 0, t = 0

At x = π/6, t = 1/2

$y = \sin\frac{\pi}{6} - \sin 0 - 2\int_0^{\frac{1}{2}} t^2\, dt$

$= \frac{1}{2} - 2\left(\frac{t^3}{3}\right)_0^{\frac{1}{2}}$

$= \frac{1}{2} - \frac{1}{12}$

$= \frac{5}{12}$

Question: 26

Mark (√) against

Solution:

$y = \int_0^{\frac{\pi}{2}} \sin x\,(2\sin x\cos x)\,dx$

$= 2\int_0^{\frac{\pi}{2}} \sin^2 x\cos x\,dx$

Let, sin x = t

Differentiating both side with respect to t

$\mathrm{Cos}\,x\frac{dx}{dt} = 1$

$\Rightarrow \cos x\,dx = dt$

At x = 0, t = 0

At x = π/2, t = 1

$y = 2\int_0^1 t^2\,dt$

$= 2\left(\frac{t^3}{3}\right)_0^1$

$= \frac{2}{3}$

Question: 27

Mark (√) against

Solution:

$y = \int_0^{\pi} (2\sin x\cos x)(4\cos^3 x - 3\cos x)\,dx$

Let, cos x = t

Differentiating both side with respect to t

$-\sin x\frac{dx}{dt} = 1$

$\Rightarrow \sin x\,dx = -dt$

At x = 0, t = 1

At x = π, t = -1

$y = -\int_1^{-1} 8t^4 - 6t^2\,dt$

$= -\left(8\frac{t^5}{5} - 6\frac{t^3}{3}\right)_1^{-1}$

$= -\left[\left(\frac{-8}{5} + 2\right) - \left(\frac{8}{5} - 2\right)\right]$

$= -\frac{4}{5}$

Question: 28

Mark (√) against

Solution:

$y = \int_0^1 \frac{e^x}{1+e^{2x}} dx$

Let $e^x = t$

Differentiating both side with respect to t

$e^x \frac{dx}{dt} = 1$

$\Rightarrow e^x dx = dt$

At x = 0, t = 1

At x = 1, t = e

$y = \int_1^e \frac{1}{1+t^2} dt$

$= (\tan^{-1} t)_1^e$

$= \tan^{-1}e - \tan^{-1}1$

$= \tan^{-1}e - \pi/4$

Question: 29

Mark (√) against

Solution:

Let, $x = t^2$

Differentiating both side with respect to t

$\frac{dx}{dt} = 2t$

$\Rightarrow dx = 2t\,dt$

At x = 0, t = 0

At x = 9, t = 3

$y = \int_0^3 \frac{2t}{1+t} dt$

$= 2\int_0^3 \frac{t+1-1}{1+t} dt$

$= 2\int_0^3 1 - \frac{1}{1+t} dt$

$= 2(t - \ln(1+t))_0^3$

y = 2[(3 – ln 4) – (0 – ln 1)]

= 6 – 2 log 4

Question: 30

Mark (√) against

Solution:

Use integration by parts

$$\int I \times II\, dx = I \times \int II\ dx - \int \frac{d}{dx} I\left(\int II\, dx\right) dx$$

$$y = x\int_0^{\frac{\pi}{2}} \cos x\, dx - \int_0^{\frac{\pi}{2}} \frac{d}{dx} x\left(\int \cos x\, dx\right) dx$$

$$= (x \sin x)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x\, dx$$

$$= \frac{\pi}{2} - (-\cos x)_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} + (0 - 1)$$

$$= \frac{\pi}{2} - 1$$

Question: 31

Mark (√) against

Solution:

We have to convert denominator into perfect square

$1 + x + x2 = x^2 + 2(x)\left(\frac{1}{2}\right) + \frac{1}{4} - \frac{1}{4} + 1$

$$= \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$= \left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$y = \int_0^1 \frac{1}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx$$

Use formula $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$

$$y = \left(\frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1} \frac{x + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)_0^1$$

$$= \frac{2}{\sqrt{3}}\left(\tan^{-1} \frac{2}{\sqrt{3}}\left(\frac{3}{2}\right) - \tan^{-1} \frac{2}{\sqrt{3}}\left(\frac{1}{2}\right)\right)$$

$$= \frac{2}{\sqrt{3}}\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$= \frac{\pi}{3\sqrt{3}}$$

Question: 32

Mark (√) against

Solution:

Let, x = sin t

Differentiating both side with respect to t

$$\frac{dx}{dt} = \cos t \Rightarrow dx = \cos t \; dt$$

At x = 0, t = 0

At x = 1, t = π/2

$$y = \int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\sin t}{1+\sin t}} \cos t \, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\sin t}{1+\sin t} \times \frac{1-\sin t}{1-\sin t}} \cos t \, dt$$

$$= \int_0^{\frac{\pi}{2}} \frac{1-\sin t}{\cos t} \cos t \, dt$$

$$= \int_0^{\frac{\pi}{2}} 1 - \sin t \, dt$$

$$= (t + \cos t)_0^{\frac{\pi}{2}}$$

$$= \left(\frac{\pi}{2} + 0\right) - (0 + 1)$$

$$= \frac{\pi}{2} - 1$$

Question: 33

Mark (√) against

Solution:

$$y = \int_0^1 \frac{1-x+1-1}{1+x} dx$$

$$= \int_0^1 \frac{2}{1+x} - 1 \, dx$$

$$= (2\ln(1+x) - x)_0^1$$

= 2 log 2 – 1

Question: 34

Mark (√) against

Solution:

Let, x = a sin t

Differentiating both side with respect to t

$$\frac{dx}{dt} = a\cos t \Rightarrow dx = a\cos t \; dt$$

At x = -a, t = - π/2

At x = a, t = π/2

$$y = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{a - a\sin t}{a + a\sin t}} a\cos t\, dt$$

$$= a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{1 - \sin t}{1 + \sin t} \times \frac{1 - \sin t}{1 - \sin t}} \cos t\, dt$$

$$= a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1 - \sin t}{\cos t} \cos t\, dt$$

$$= a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 - \sin t\, dt$$

$$= a(t + \cos t)_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= a\left[\left(\frac{\pi}{2} + 0\right) - \frac{\pi}{2} + 0\right)$$

$= a\pi$

Question: 35

Mark (√) against

Solution:

Use formula $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

$$y = \int_0^{\sqrt{2}} \sqrt{\left(\sqrt{2}\right)^2 - x^2}\, dx$$

$$= \left(\frac{x}{2}\sqrt{2 - x^2} + \frac{2}{2}\sin^{-1}\frac{x}{\sqrt{2}}\right)_0^{\sqrt{2}}$$

$$= \left(\frac{\sqrt{2}}{2}\sqrt{2 - 2} + \sin^{-1}\frac{\sqrt{2}}{\sqrt{2}}\right) - (0 + \sin^{-1} 0)$$

$$= \frac{\pi}{2}$$

Question: 36

Mark (√) against

Solution:

We know that

|x| = -x in [-2, 0)

|x| = x in [0, 2]

$$y = \int_{-2}^{0} |x|\, dx + \int_{0}^{2} |x|\, dx$$

$$= \int_{-2}^{0} -x\, dx + \int_{0}^{2} x\, dx$$

$= \left(-\frac{x^2}{2}\right)_{-2}^{0} + \left(\frac{x^2}{2}\right)_{0}^{2}$

y = 0 – (-2) + 2 – 0

= 4

Question: 37

Mark (√) against

Solution:

We know that

|2x – 1| = -(2x – 1) in [0, 1/2)

|2x – 1| = (2x – 1) in [1/2, 1]

$$y = \int_{0}^{\frac{1}{2}} |2x - 1|\,dx + \int_{\frac{1}{2}}^{1} |2x - 1|\,dx$$

$$= \int_{0}^{\frac{1}{2}} -(2x - 1)\,dx + \int_{\frac{1}{2}}^{1} 2x - 1\,dx$$

$$= -(x^2 - x)_{0}^{\frac{1}{2}} + (x^2 - x)_{\frac{1}{2}}^{1}$$

$$= -\left[\left(\frac{1}{4} - \frac{1}{2}\right) - (0 - 0)\right] + \left[(1 - 1) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$y = \frac{1}{2}$$

Question: 38

Mark (√) against

Solution:

We know that

|2x + 1| = -(2x + 1) in [-2, -1/2)

|2x + 1| = (2x + 1) in [-1/2, 1]

$$y = \int_{-2}^{-\frac{1}{2}} |2x + 1|\,dx + \int_{-\frac{1}{2}}^{1} |2x + 1|\,dx$$

$$= \int_{-2}^{-\frac{1}{2}} -(2x + 1)\,dx + \int_{-\frac{1}{2}}^{1} 2x + 1\,dx$$

$$= -(x^2 + x)_{-2}^{-\frac{1}{2}} + (x^2 + x)_{-\frac{1}{2}}^{1}$$

$$= -\left[\left(\frac{1}{4} - \frac{1}{2}\right) - (4 - 2)\right] + \left[(1 + 1) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$y = \frac{9}{2}$$

Question: 39

Mark (√) against

Solution:

We know that

|x| = -x in [-2, 0)

|x| = x in [0, 1]

$$y = \int_{-2}^{0} \frac{|x|}{x} dx + \int_{0}^{1} \frac{|x|}{x} dx$$

$$= \int_{-2}^{0} \frac{-x}{x} dx + \int_{0}^{1} \frac{x}{x} dx$$

$$= \int_{-2}^{0} -1\, dx + \int_{0}^{1} 1\, dx$$

$$= (-x)_{-2}^{0} + (x)_{0}^{1}$$

= -(0 – (-2)) + (1 – 0)

= -1

Question: 40

Mark (√) against

Solution:

We know that

|x| = -x in [-a, 0) where a > 0

|x| = x in [0, a] where a > 0

$$y = \int_{-a}^{0} x|x|\, dx + \int_{0}^{a} x|x|\, dx$$

$$= \int_{-a}^{0} x(-x)\, dx + \int_{0}^{a} x(x)\, dx$$

$$= -\int_{-a}^{0} x^2\, dx + \int_{0}^{a} x^2\, dx$$

$$= -\left(\frac{x^3}{3}\right)_{-a}^{0} + \left(\frac{x^3}{3}\right)_{0}^{a}$$

$$= -\left(0 - \left(\frac{-a^3}{3}\right)\right) + \left(\frac{a^3}{3} - 0\right)$$

= 0

Question: 41

Mark (√) against

Solution:

Find the equivalent expression to |cos x| at $0 \leq x \leq \pi$

In $0 \leq x \leq \frac{\pi}{2}$

=cos x

In $\frac{\pi}{2} \leq x \leq \pi$

=-cos x

$$\Rightarrow \int_0^{\frac{\pi}{2}} \cos x \, dx + \int_{\frac{\pi}{2}}^{\pi} -\cos x \, dx$$

$$\Rightarrow \sin\frac{\pi}{2} - \sin 0 - \cos\pi + \cos\frac{\pi}{2}$$

⇒1-0-(-1) +0=2

Question: 42

Mark (√) against

Solution:

Find the equivalent expression to |sin x| at $0 \leq x \leq 2\pi$

In $0 \leq x \leq \pi$

|sin x| = sin x

In $\pi \leq x \leq 2\pi$

|sin x| = -sin x

$\Rightarrow \int_0^{\pi} \sin x \, dx + \int_{\pi}^{2\pi} -\sin x dx$ =-cos π-(-cos 0)+cos 2π-cos π

=-(-1)+1+1-(-1)

=2+2

=4

Question: 43

Mark (√) against

Solution:

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I \ldots$(let)

$\therefore$ Here,$a = \frac{\pi}{2}$

$$f(x) = \frac{\sin x}{(\sin x + \cos x)}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$\frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos(\frac{\pi}{2} - x)} = \frac{\cos x}{\cos x + \sin x}$$

$$\therefore 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

Question: 44

Mark (√) against

Solution:

We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$\therefore$ Here,

$a = \frac{\pi}{2}$;

$f(x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$

$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$$\frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos(\frac{\pi}{2} - x)} + \sqrt{\sin(\frac{\pi}{2} - x)}} = \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a-x)$

$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$

$= \int_0^{\frac{\pi}{2}} 1 dx$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

Question: 45

Mark (√) against

Solution:

We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$\therefore$ Here,

$a = \frac{\pi}{2}$;

$f(x) = \frac{\sin^4 x}{\sin^4 x + \cos^4 x}$

$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$$\frac{\sin^4\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}=\frac{\cos^4 x}{\sin^4 x+\cos^4 x}$$

$$\therefore\ 2I=\int_0^a f(x)+\int_0^a f(a-x)$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin^4 x+\cos^4 x}{\sin^4 x+\cos^4 x}dx$$

$$=\int_0^{\frac{\pi}{2}}1dx$$

$$\therefore\ 2I=\frac{\pi}{2}$$

$$\therefore\ I=\frac{\pi}{2.2}$$

$$=\frac{\pi}{4}$$

Question: 46

Mark (√) against

Solution:

We know that,

$\therefore\ \int_0^a f(x)=\int_0^a f(a-x)=I$ ...(let)

$\therefore$ Here,

$a=\frac{\pi}{2}$;

$$f(x)=\frac{\cos^{\frac{1}{4}}x}{\sin^{\frac{1}{4}}x+\cos^{\frac{1}{4}}x}$$

$$\therefore\ f(a-x)=f\left(\frac{\pi}{2}-x\right)$$

$$\frac{\cos^{\frac{1}{4}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{1}{4}}}\left(\frac{\pi}{2}-x\right)\cos^{\frac{1}{4}}\left(\frac{\pi}{2}-x\right)=\sin^{\frac{1}{4}}x\sin^{\frac{1}{4}}x+\cos^{\frac{1}{4}}x$$

$$\therefore\ 2I=\int_0^a f(x)+\int_0^a f(a-x)$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{1}{4}}x+\cos^{\frac{1}{4}}x}{\sin^{\frac{1}{4}}x+\cos^{\frac{1}{4}}x}dx$$

$$=\int_0^{\frac{\pi}{2}}1dx$$

$$\therefore\ 2I=\frac{\pi}{2}$$

$$\therefore\ I=\frac{\pi}{2.2}$$

$$=\frac{\pi}{4}$$

Question: 47

Mark (√) against

Solution:

We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$\therefore$ Here,

$a = \frac{\pi}{2}$;

$f(x) = \frac{\sin^n x}{\cos^n x + \sin^n x}$

$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$= \frac{\cos^n x}{\cos^n x + \sin^n x}$

$\therefore\ 2I = \int_0^{\frac{\pi}{2}} 1\,dx$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

Question: 48

Mark (√) against

Solution:

We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$\therefore$ Here,

$a = \frac{\pi}{2}$;

$f(x) = \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}}$

$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$= \frac{\sqrt{\tan x}}{\sqrt{\cot x} + \sqrt{\tan x}}$

$\therefore\ 2I = \int_0^{\frac{\pi}{2}} 1\,dx$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

Question: 49

Mark (√) against

Solution:

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$$= \frac{\sqrt[3]{\tan x}}{\sqrt[3]{\cot x} + \sqrt[3]{\tan x}}$$

$$= \frac{\sqrt[3]{\frac{\sin x}{\cos x}}}{\sqrt[3]{\frac{\sin x}{\cos x}} + \sqrt[3]{\frac{\cos x}{\sin x}}}$$

$$= \frac{\sqrt[3]{\frac{\sin x}{\cos x}} * \left(\sqrt[3]{\sin x}\ \sqrt[3]{\cos x}\right)}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$$= \frac{\sin^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$\therefore$ Here,

$a = \frac{\pi}{2}$;

$$f(x) = \frac{\sin^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\cos^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$$\therefore\ 2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore\ 2I = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question: 50

Mark (√) against

Solution:

$$\frac{1}{1+\tan x} = \frac{1}{1 + \frac{\sin x}{\cos x}}$$

$$= \frac{1}{(\cos x + \sin x)\frac{1}{\cos x}}$$

$$= \frac{\cos x}{\cos x + \sin x}$$

So our integral becomes, $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$

We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I \ldots(\text{let})$

∴ Here,

$$a = \frac{\pi}{2}$$

$$f(x) = \frac{\sin x}{(\sin x + \cos x)}$$

$$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}$$

$$= \frac{\cos x}{\cos x + \sin x}$$

$$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore\ 2I = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question: 51

Mark (√) against

Solution:

So our integral becomes

$$\frac{1}{\sqrt[1]{\cot x} + 1} = \frac{1}{\sqrt{\frac{\cos x}{\sin x}} + 1}$$

$$= \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$$

∴ Here,

$$a = \frac{\pi}{2};$$

$$f(x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$$

$$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}}$$

$$= \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a - x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore\ 2I = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question: 52

Mark (√) against

Solution:

$$\frac{1}{1 + \tan^3 x} = \frac{\cos^3 x}{\sin^3 x + \cos^3 x}$$

$\therefore$ Here,

$$a = \frac{\pi}{2}\ ;$$

$$f(x) = \frac{\cos^3 x}{\sin^3 x + \cos^3 x}$$

We know that,

$$\therefore\ \int_0^a f(x) = \int_0^a f(a - x) = I \ldots\text{(let)}$$

$$f(a - x) = \frac{\sin^3 x}{\sin^3 x + \cos^3 x}$$

$$\therefore\ 2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore\ 2I = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question: 53

Mark (√) against

Solution:

so our integral becomes,

$$\frac{\sec^5 x}{\sec^5 x + \mathrm{cosec}^5 x} = \frac{\frac{1}{\cos^5 x}}{\frac{1}{\cos^5 x} + \frac{1}{\sin^5 x}}$$

$$= \frac{\sin^5 x}{\sin^5 x + \cos^5 x}$$

Here $a = \frac{\pi}{2}$ and $f(x) = \frac{\sin^5 x}{\sin^5 x + \cos^5 x}$

$$f(a-x) = \frac{\cos^5 x}{\sin^5 x + \cos^5 x}$$

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$$\therefore\ 2I == \int_0^{\frac{\pi}{2}} 1dx$$

$$\therefore\ 2I = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question: 54

Mark (√) against

Solution:

So our integral becomes,

$$\frac{\sqrt{\cot x}}{1 + \sqrt{\cot x}} = \frac{\sqrt{\frac{\cos x}{\sin x}}}{1 + \sqrt{\frac{\cos x}{\sin x}}}$$

$$= \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

so, we know that,

$\therefore$ Here,

$$a = \frac{\pi}{2};$$

$$f(a-x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$$

$$\therefore\ f(x) = \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$= \int_0^{\frac{\pi}{2}} 1dx$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

Question: 55

Mark (√) against

Solution:

So our integral becomes,

$$\frac{\tan x}{1+\tan x} = \frac{\sin x}{\cos x}\left(\frac{1}{1+\frac{\sin x}{\cos x}}\right)$$

$$= \frac{\sin x}{\sin x + \cos x}$$

We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I\ \ldots(\text{let})$

$\therefore$ Here,

$a = \frac{\pi}{2}$

$$f(x) = \frac{\sin x}{(\sin x + \cos x)}$$

$$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right) + \cos\left(\frac{\pi}{2}-x\right)}$$

$$= \frac{\cos x}{\cos x + \sin x}$$

$$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1dx$$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

Question: 56

Mark (√) against

Solution:

If f is an odd function,

$$\int_{-a}^{a} f(x)dx = 0$$

as, $\int_0^a f(x)\,dx = -\int_{-a}^0 f(x)dx$

here $f(x)=x^4\sin x$

we will see $f(-x)=(-x)^4\sin(-x)$

$=- x^4\sin x$

Therefore, f(x) is a odd function,

$$\int_{-\pi}^{\pi} x^4 \sin x dx = 0$$

Question: 57

Mark (√) against

Solution:

If f is an odd function,

$$\int_{-a}^{a} f(x)dx = 0$$

as, $\int_0^a f(x)\,dx = -\int_{-a}^0 f(x)dx$

here $f(x)=x^3 \cos^3 x$

we will see $f(-x)= (-x)^3 \cos^3(-x)$

$=-x^3 \cos^3 x$

Therefore, f(x) is a odd function,

$$\int_{-\pi}^{\pi} x^3 \cos^3 x = 0$$

Question: 58

Mark (√) against

Solution:

If f is an odd function,

$$\int_{-a}^{a} f(x)dx = 0$$

as, $\int_0^a f(x)\,dx = -\int_{-a}^0 f(x)dx$

$f(x)=\sin^5 x$

$f(-x)=\sin^5(-x)$

$=-\sin^5 x$

Therefore, f(x) is a odd function,

$$\int_{-\pi}^{\pi} \sin^5 x\, dx = 0$$

Question: 59

Mark (√) against

Solution:

$$\int_{-1}^{-2} x^3(1-x^2)dx = \int_{-1}^{-2}(x^3-x^5)dx$$

$$= \left[\frac{x^4}{4}-\frac{x^6}{6}\right]$$

$$= \left[\frac{2^4}{4}-\frac{1^6}{4}-\frac{2^6}{6}+\frac{1^6}{6}\right]$$

$$= -\frac{27}{4}$$

Question: 60

Mark (√) against

Solution:

If f is an odd function,

$$\int_{-a}^{a} f(x)dx = 0$$

as, $\int_0^a f(x)dx = -\int_{-a}^0 f(x)dx$

$$f(x) = \log\left(\frac{a-x}{a+x}\right)$$

$$f(-x) = \log\frac{a-(-x)}{a-x}$$

$$= \log\frac{a+x}{a-x}$$

$$= -\log\frac{a-x}{a+x}$$

Hence it is a odd function

$$\int_{-a}^{a}\log\frac{a-x}{a+x} = 0$$

Question: 61

Mark (√) against

Solution:

If f is an odd function,

$$\int_{-a}^{a} f(x)dx = 0$$

as, $\int_0^a f(x)dx = -\int_{-a}^0 f(x)dx$

$\sin^{61}x$ and $x^{123}$is an odd function,

so there integral is zero.

Question: 62

Mark (√) against

Solution:

f(x)=tan x

f(-x) =tan(-x)

=-tan x

hence the function is odd,

therefore, I=0

Question: 63

Mark (√) against

Solution:

By by parts,

$$\int \log\left(x+\sqrt{x^2+1}\right) = x\log\left(x+\sqrt{x^2+1}\right) - \int \frac{x}{\left(x+\sqrt{x^2+1}\right)\left(1+\frac{x}{\sqrt{x^2+1}}\right)}$$

$= x\log(x+\sqrt{x^2+1})$-$\int \frac{x}{\sqrt{x^2+1}}$= $x\log(x+\sqrt{x^2+1})$-$\sqrt{x^2+1}$

Question: 64

Mark (√) against

Solution:

cosx is an even function so,

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = 2\int_{0}^{\frac{\pi}{2}} \cos x dx$$

=2(1-0)

=2

Question: 65

Mark (√) against

Solution:

Here,

$$f(x) = \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}$$

$$f(a-x) = \frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}$$

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}} dx$$

$$= \int_0^a dx$$

$$I = \frac{a}{2}$$

Question: 66

Mark (√) against

Solution:

let $I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$$

$$\therefore f(a-x) = \log(1+\tan(\frac{\pi}{4}-x))$$

$$= \log\left(1+\frac{\left(\tan\frac{\pi}{4}-\tan x\right)}{1+\tan\frac{\pi}{4}\tan x}\right) = \log(1+1(1-\tan x)\frac{1}{1+\tan x}$$

$$= \log\frac{2}{1+\tan x}$$

$$\therefore \int_0^a f(a-x) = I$$

$$= \int_0^{\frac{\pi}{4}} \log\frac{2}{1+\tan x}\,dx$$

$$= \int_0^{\frac{\pi}{4}} \log 2dx - \int_0^{\frac{\pi}{4}}(1+\tan x)dx$$

$$\therefore I = \int_0^{\frac{\pi}{4}} \log 2\,dx - I$$

$$\therefore 2I = \frac{\pi}{4}\log 2$$

$$\therefore I = \frac{\pi}{8}\log 2$$

Question: 67

Mark (√) against

Solution:

$$\therefore \int_{-a}^a f(x)dx$$

$$\therefore \int_{-a}^0 f(x)dx + \int_0^a f(x)dx$$

$$\therefore \int_0^a f(-x)dx = \int_{-a}^0 f(x)dx$$

$$\therefore \int_0^a f(-x)dx + \int_0^a f(x)dx$$

Question: 68

Mark (√) against

Solution:

$$\therefore \int_0^{1.5} [x]dx$$

$$= \int_0^1 [x]dx + \int_1^{1.5} [x]dx$$

$$= \int_0^1 0dx + \int_1^{1.5} 1.dx$$

$= \frac{3}{2} - 1$

$= \frac{1}{2}$

Question: 69

Mark (√) against

Solution:

$\int_{-1}^{1}[x]dx = \int_{-1}^{0}[x]dx + \int_{0}^{1}[x]dx$

$= \int_{-1}^{0} -1dx + \int_{0}^{1} 0dx$

=-1-0+0

=-1

Question: 70

Mark (√) against

Solution:

$\int_{1}^{2}|x^2 - 3x + 2|dx$

∴ x^2-3x+2=0

(x-2)(x-1)=0

so, 2, and 1 itself are the limits so no breaking points for the integral,

$\therefore \int_{1}^{2}(-x^2 + 3x - 2)dx$

$= \left[\frac{-x^3}{3} + \frac{3x^2}{2} - 2x\right](1 to 2)$

$\therefore = \frac{1}{6}$

Question: 71

Mark (√) against

Solution:

∴ sin x=0

∴ x=0,π,2π....

So $\pi, 2\pi$ are the limits so no breaking points for the integral,

$\therefore \int_{\pi}^{2\pi} -\sin x dx = -\cos x(\pi \text{ to } 2\pi)$

=2

Question: 72

Mark (√) against

Solution:

put $\sin^{-1} x = t$;

$dt = \frac{dx}{\sqrt{1-x^2}}$;

x=sin t

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

=t;

and $\sin^{-1} 0=0$

=t

Limit changes to,

$$\int_0^{\frac{\pi}{4}} \frac{tdt}{1-\sin^2 t} = \int_0^{\frac{\pi}{4}} tsec^2tdt$$

$$= t\tan t - \int_0^{\frac{\pi}{4}} \tan tdt$$

$$= [t\tan t + \log cost]\left(0 \text{ to } \frac{\pi}{4}\right)$$

$$= \frac{\pi}{4} - \frac{1}{2}\log 2$$

Question: 73

Mark (√) against

Solution:

put x=tan y

$dx=sec^2ydy$

$$\int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2y)\sec^2 ydy$$

$$= 2\int_0^{\frac{\pi}{4}} ysec^2ydy$$

$$= 2[y\tan y - \int_0^{\frac{\pi}{4}} \tan ydy]$$

$$= 2[y\tan y + \log cosy]\left(0 \text{ to } \frac{\pi}{4}\right)$$

$$= 2[\frac{\pi}{4} - \frac{1}{2}\log 2]$$

$$= \frac{\pi}{2} - \log 2$$