Chapter 5

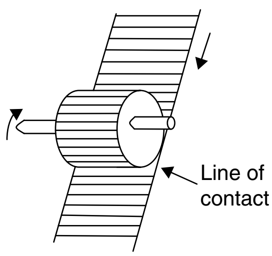
Design of Machine Elements

CHAPTER HIGHLIGHTS

- *Gears*
- *Line of Action or Pressure Line*
- *Path of Contact or Contact Length*
- *Law of Gearing*
- *Conjugate Profile of Gears*
- *Gear Materials and Beam Strength of Gear Teeth*
- *Permissible Working Stress for Gear Teeth in the Lewis Equation*
- *Static Tooth Load*
- *Coefficient of Friction for Journal Bearing*
- *Eccentricity Ratio and Dynamic Load Carrying Capacity*
- *Shafts Subjected to Fluctuating Loads*
- *Design of Shafts on the Basis of Rigidity*
- *Pivoted Block or Shoe Brake*
- *Cone Clutch and Centrifugal Clutch*
- *Eccentric Loading of Bolts*
- *Combination of Parallel and Transverse Fillet Welds*

GEARS

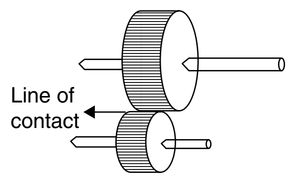
Gears are toothed wheels used for transmission of rotary motion and the power gears fixed to shafts mesh with each other transmitting the rotary motion. The smaller wheel is generally known as **pinion**. Gear drives are also used to convert rotary motion to translating motion as in **rack and pinion** arrangement. **Rack** is a straight piece with teeth (or infinite diameter gear).

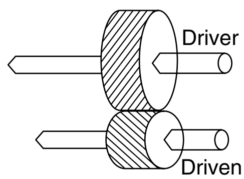


Gears are used with parallel shafts, intersecting shafts or skew shafts (non-parallel, non intersecting).

Gears used with parallel shafts are called **spur gears**. In spur gears, generally, the teeth are parallel to the axis of shaft. Helical gears also belong to the family of spur gears. In **helical gears,** the teeth are at some angle with respect to the axis of the shaft.

When the axes of the shafts are intersecting, **bevel gears** are used. Bevel gear can be straight or spiral depending upon the inclination of the teeth. Generally, straight bevel gears are used to connect shafts at low speeds at right angles.





Commonly Used Terms in Spur Gears

1. **Pitch cylinders:** Pitch cylinders are the imaginary friction cylinders which roll together without slip, giving the same velocity ratio as that of the meshing gears.
2. **Pitch circle:** It is the circle which by pure rolling action gives the same velocities of the gears. It is a section of the pitch cylinder at right angle to the axis.
3. **Pitch diameter:** It is the diameter of the pitch circle or pitch cylinder.
4. **Pitch surface:** It is the surface of the pitch cylinder.
5. **Pitch point:** It is the common point or point of contact of two pitch circles.

6. **Pitch line:** It is a straight line through the teeth of a rack which is tangential to the pitch circle of the pinion.
7. **Circular pitch:** It is the distance between two corresponding points on adjacent teeth along the pitch circle. It is denoted as

$$P_c = \frac{\pi d}{T}$$

Where P_c = Circular pitch
d = Pitch diameter
T = Number of teeth

8. **Diametral pitch:** It is the ratio of the number of teeth to pitch circle diameter or the number of teeth per unit of length of pitch circle diameter. It is denoted as

$$P_d = \frac{T}{d}$$

9. **Module (*m*):** It is the ratio of pitch diameter in millimeter to the number of teeth. It is denoted as

$$m = \frac{d}{T}$$

Therefore, **module is the reciprocal of diametral pitch**.

Also, $P_c = \frac{\pi d}{T} = \pi m$

For mating gears, the module will be the same.

10. **Gear ratio (*G*):** The ration of the number of teeth of the gear to the number of teeth of the pinion is called gear ratio.

$$G = \frac{T}{t}$$

where T = number of teeth on gear and
t = number of teeth on pinion

11. **Velocity ratio (*VR*):** It is the ratio of angular velocities of driven to driving gear.

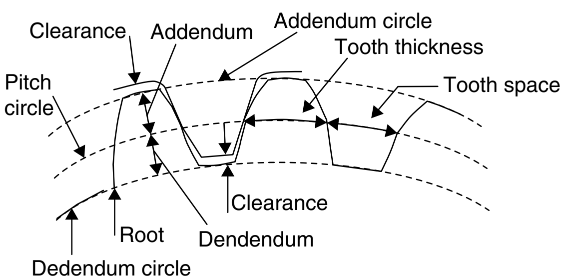
or $$VR = \frac{\omega_2}{\omega_1} = \frac{2\pi N_2}{2\pi N_1} = \frac{N_2}{N_1}$$

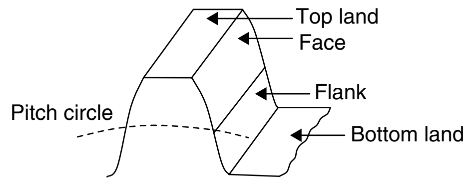
where ω = angular velocity in rad/s
N = angular speed in revolution per minute
As linear velocity along pitch circle is same for both gears.

$$\pi d_1 N_1 = \pi d_2 N_2$$

$$\therefore\ VR = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$$

as $d \propto T$ for same module





12. **Addendum circle:** The circle passing through the tips of teeth is known as addendum circle.
13. **Addendum:** It is the radial distance from pitch circle to the top of the tooth. Its standard value is one module (*m*).
14. **Dedendum circle or root circle:** The circle passing through the roots of the teeth is known as dedendum circle.
15. **Dedendum:** It is the radial distance from the pitch circle to the bottom of the tooth. Its standard value is 1.157 m.
16. **Clearance:** It is the radial distance from the tip of a tooth to the bottom land of the meshing gear. Therefore, it is the difference between addendum and dedendum values. So, the standard clearance is 0.157m

A circle passing through the top of the meshing gear is known as **clearance circle**.

17. **Full depth of teeth:** It is the total radial depth of the tooth.

$\therefore$ Full depth = Addendum + Dedendum
= 2.157 m

18. **Working depth of the teeth:** It is the maximum depth a tooth penetrates into the tooth space of the meshing gear. So it is the sum of the addendum of the meshing gears.

Space Width or Tooth Space

It is the width of the space between adjacent teeth measured along pitch circle.

Tooth Thickness

It is the width of the tooth along the pitch circle.

Backlash

It is the difference between the tooth space and tooth thickness measured along the pitch circle.

Face Width

It is the width of the gear tooth measured parallel to the gear axis.

Face

It is the tooth surface between the pitch circle and the top land, parallel to axis of the gear. It is the working surface of the addendum.

Flank

It is the tooth surface between the face and bottom land. It is the working surface of the Dedendum. It includes the fillet.

Fillet

It is the curved portion of the tooth flank at the root circle.

Top Land

The top surface of the tooth is called top land.

Bottom Land

It is the bottom surface of the tooth space (surface between adjacent fillets)

Line of Action or Pressure Line

The locus of contact point of mating tooth surfaces is a straight line tangential to the base circles and passing through the pitch point. This line is called the line of action or pressure line. This line also is the common normal to the surfaces at the points of contact.

Pressure Angle or Angle of Obliquity (ϕ)

The acute angle ϕ between the line of action and common tangent to pitch circles is called the pressure angle or the angle of obliquity.

Path of Contact or Contact Length

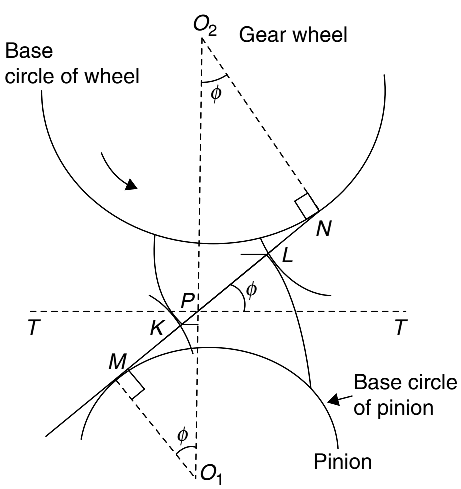
It is the locus of the point of contact from the beginning to the end of engagement of two mating teeth or it is the active portion of the line of action.

Path of Approach

It is the portion of the path of contact from the beginning of engagement to the pitch point.

Path of Recess

It is the portion of the path of contact from the pitch point to the end of engagement.
Therefore, path of contact = path of approach + path of recess.



Line of Action and Path of Contact

P – Pitch point.
MN – Line of action
TT – Common tangent to pitch circles
KP – Path of approach = $KN - PN$
PL – Path of recess = $ML - MP$
KL = Path of contact
M and N are **interference points**.

If interference is to be avoided, the path of contact ≤ the line of action;

i.e. $KL \le MN$,
or $KP \le MP$
and $PL \le PN$.

For a pinion and gear set, the **path of approach**

$$KP = KN - PN$$
$$= \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

and the **path of recess** $PL = ML - MP$

$$= \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

where R and r = radius of pitch circles.
and R_A and r_A = radius of addendum circles.

Arc of Contact

It is the locus of a point on the pitch circle from the beginning to the end of the engagement of a pair of teeth.

Similarly to the path of contact it consists of arc of approach and arc of recess.

$$\text{Length of the arc of contact} = \frac{\text{Length of path of contact}}{\cos\phi}$$

Angle of Action (δ)

The angle turned by a gear from the beginning of engagement to the end of engagement with a pair teeth is called the angle of action. Similarly, the angle of approach (α) and the angle of recess (β) can be defined.

$$\therefore\ \delta = \alpha + \beta$$

Contact Ratio

It is the ratio of the length of the arc of contact to the circular pitch and represents the pairs of teeth in contact at a time.

There should be at least one pair of teeth in contact always for continuous action. The larger the contact ratio is, the more quietly the gears will operate.

Law of Gearing

According to the Law of gearing, the common normal at the point of contact between two teeth always pass through the pitch point at all positions of the gears.

Velocity of Sliding

If the curved surfaces of the two teeth are to remain in contact, one can have a sliding motion relative to the other along the common tangent $T - T$.

Velocity of sliding = sum of angular velocities × distance between the pitch point and the point of contact.

Conjugate Profile of Gears

When the profile of gear teeth is such that a constant angular velocity ratio is maintained, such profiles are called conjugate profiles. The commonly used tooth profiles are

1. Cycloidal profile,
2. Involute profile.

In involute gears, the pressure angle remains constant from the start to the end of engagement, but in cycloidal teeth the pressure angle varies.

Cycloidal teeth are free from interference problem, while interference problem occurs in involute gears.

Interference

In involute gears, the involute profile of the teeth are formed from the base circle. The radius of the base circle should be less than the dedendum circle for the smooth working of the gears. The relative position of the base circle and the dedendum circle depends upon the number of teeth, module and pressure angle. If radius of the base circle is less than the dedendum circle, the tip of the mating gear digs into the flank of the gear. This phenomenon is known as **interference**.

Undercutting

The interference in involute gears is avoided by the undercutting of the teeth base. The undercutting of the teeth during gear manufacture is dependent upon the number of teeth. The smaller the number of teeth is, the higher will be the undercutting.

Minimum Number of Teeth to Avoid Interference

To avoid interference the **minimum number of teeth required on the pinion** is given by

$$t = \frac{2A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$$

where A_p = Addentum coefficient of pinion

$$= \frac{\text{Addentum}}{\text{module}}$$

G = gear ratio

$$= \frac{T}{t} = \frac{D}{d} = \frac{N_P}{N_G}$$

Similarly, the **minimum number of teeth on the gear** is given by

$$T = \frac{2A_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

Where A_w = Addentum coefficient of gear

From the above equation, the **minimum number of teeth for pinion** may be obtained as

$$t = \frac{2A_w}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]}$$

The minimum number of teeth on the pinion **to avoid interference with a rack** is given by

$$t = \frac{2A_R}{\sin^2\phi}$$

where A_R = Addendum coefficient of the rack.

Gear Materials

The materials used for gear manufacture depend upon the service conditions, strength required, etc. Metallic materials like cast iron, steel and bronze and non-metallic materials like wood, rawhide, compressed paper and synthetic resins like nylon are used for gears.

Design Considerations for a Gear Drive

The power to be transmitted, speed of driving gear, speed ratio, centre distance, etc. are the usually available data.

In the design of gears the following requirements should be met.

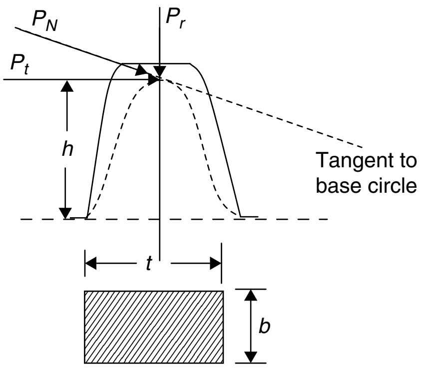
1. Gear teeth should have sufficient strength under static and dynamic load conditions.
2. Wear characteristics should be such that the gear should have sufficient life.
3. Economical use of space and material.
4. Alignment of shafts and gears and deflection should be considered.

Beam Strength of Gear Teeth (Lewis Equation)

The beam strength of toothed gears can be determined by the Lewis equation which gives satisfactory results for the load carrying capacity.

The assumptions made in the equation are

1. Effect of radial force(P_r) which induces compressive stresses is neglected.
2. Tangential force(P_t) is uniformly distributed over the face width.
3. Effect of stress concentration is neglected.
4. Only one pair of tooth is in contact at any time.



Section at xx

Each tooth can be considered as a cantilever. Force P_N acts normal to the surface of the tooth through the line of action (tangent to base circle). P_N can be resolved into tangential component P_t and radial component P_r.

Bending moment at section *xx*;

$$M_b = P_t \times h$$

Moment of inertia of section, $I = \dfrac{bt^3}{12}$

Bending stress, $\sigma_b = \dfrac{M_b y}{I} = \dfrac{(P_t \times h) \times \frac{t}{2}}{\frac{bt^3}{12}}$

$$\Rightarrow P_t = b\sigma_b \left(\frac{t^2}{6h}\right)$$

$$= mb\sigma_b \left(\frac{t^2}{6hm}\right)$$

$$= mb\sigma_b y$$

where y = **Lewis form factor or tooth form factor**

$$= \frac{t^2}{6hm}$$

When σ_b = the permissible bending stress (σ_w), force P_t is called the **beam strength** (S_b)

$\therefore$ Beam strength $S_b = mb\sigma_w y$

where σ_w is the permissible bending stress. This equation is known as **Lewis equation**.

The value for form factor (y) is a constant and depends only on the number of teeth and the system of teeth. For 14.5° composite and full depth involute system,

$$y = 0.124 - \frac{0.684}{T}$$

For 20° full depth involute system,

$$y = 0.154 - \frac{0.912}{T}$$

and for 20° stub system, $y = 0.175 - \dfrac{0.841}{T}$

Permissible Working Stress for Gear Teeth in the Lewis Equation

The permissible working stress is arrived at using the following formula

$$\sigma_w = \sigma_0 \times C_v$$

where σ_0 = allowable static stress

and C_v = velocity factor

The values of velocity factor for various working conditions are as follows.

1. For ordinary cut gears, with operating velocity upto 12.5 m/s

$$C_v = \frac{3}{3+v}$$

2. For carefully cut gears with operating velocities upto 12.5 m/s

$$C_v = \frac{4.5}{4.5+v}$$

3. For very accurately cut and ground metallic gears with operating velocities upto 20 m/s

$$C_v = \frac{6}{6+v}$$

(v = Pitch line velocity in m/s)

Dynamic Tooth Load (W_D)

Dynamic tooth load is obtained using the following formula.

$$W_D = W_T + W_I = W_T + \frac{21v(bc + W_T)}{21v + \sqrt{bc + W_T}}$$

where W_D = Total dynamic load

W_T = steady load due to the torque transmitted.

W_I = increment load due to dynamic action.

v = pitch line velocity (m/s)

b = face width of gear tooth (mm)

c = deformation or dynamic factor (N/mm)

$$c = \frac{k.e}{\frac{1}{E_p} + \frac{1}{E_g}}$$

Where E_p and E_g are Young's modulus of pinion and gear materials, respectively,

e = tooth error action (mm)

k = a factor which depends up on the form of the tooth.

= 0.115 for 20° stub system.

= 0.111 for 20° full depth involute system

= 0.107 for 14.5° composite and full depth involute system.

Static Tooth Load (W_S)

Static tooth load or endurance strength or beam strength at endurance limit is obtained by replacing σ_w in the Lewis equation with σ_e the flexural endurance limit or elastic limit stress.

Thus,

$W_s = mb\sigma_e y$

σ_e is obtained using the following relation.

$\sigma_e = 1.75 \times$ BHN (in MPa)

Buckingham has suggested the following relationship between W_S and W_D.

1. $W_S \geq 1.25\ W_D$ for steady loads.
2. $W_S \geq 1.35\ W_D$ for pulsating loads.
3. $W_S \geq 1.5\ W_D$ for shock loads.

Wear Tooth Load (W_w)

Wear of gears depend upon the load they carry as well as radius of curvature of the tooth profiles, elasticity and surface fatigue limit of the materials.

The maximum limiting load against premature wear failure is given by Buckingham as follows.

$$W_w = D_p.b.Q.k$$

Where D_p = pitch circle diameter of pinion in mm.

b = face width of pinion in mm

Q = Ratio factor.

k = load stress factor in N/mm^2

$$Q = \frac{2V_r}{V_r + 1} = \frac{2T_G}{T_G + T_P} \quad \text{for external gears.}$$

$$Q = \frac{2V_r}{V_r - 1} = \frac{2T_G}{T_G - T_P} \quad \text{for internal gears.}$$

Where V_r = velocity ratio = $\dfrac{T_G}{T_P}$

$$k = \frac{(\sigma_{es})^2 \sin\phi}{1.4}\left[\frac{1}{E_P} + \frac{1}{E_G}\right]$$

Where σ_{es} = surface endurance limit.

ϕ = pressure angle.

E_P, E_G = Young's modulus of pinion and gear materials.

For safe operation, wear tooth gear load must be greater than the dynamic load.

The following points are to be noted in the design of spur gears.

1. Lewis equation is **applied to the weaker of the two** mating gears.
2. If the material of construction is same, the pinion is weaker.
3. If pinion and gear are of different materials, then the product ($\sigma_w \times y$) or ($\sigma_0 \times y$) is the deciding factor. Lewis equation is applied on the wheel for which ($\sigma_w y$) or ($\sigma_0\, y$) is minimum
4. The centre distance between the gear and pinion is given by

$$C_D = \frac{m(T_G + T_P)}{2} \qquad [\because mT = \text{diameter}]$$

where m = module

T_G, T_P = Number of teeth on gear and pinion.

5. Power transmitted and the tangential force is same for both the gear and pinion.

Power transmitted = ωT

$= v \times F_t$

Direction for questions (Examples 1 to 3): A pair of involute spur gears with 16° pressure angle and module 6 mm is in mesh. The number of teeth on pinion is 16 and its rotational speed is 240 rpm. The gear ratio is 1.75.

Solved Examples

Example 1: In order that interference is just avoided, the addendum on pinion is

(A) 10.76 mm (B) 8.26 mm
(C) 12.81 mm (D) 6.85 mm

Solution:

Pitch radius of the pinion

$$r = \frac{mt}{2} = \frac{6 \times 16}{2} = 48 \text{ mm}$$

Pitch radius of wheel

$$R = \frac{mT}{2} = \frac{6 \times 28}{2} = 84 \text{ mm}$$

When interference of the pinion is just avoided, the path of recess is maximum.

i.e. $\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi = R\sin\phi$

or $\sqrt{r_a^2 - 48^2\cos^2 16°}$

$= (84 + 48)\sin 16°$

$\Rightarrow r_a = 58.76$ mm

Addendum of pinion = $r_a - r$

$= 58.76 - 48$

$= 10.76$ mm

Example 2: Addendum of gear is

(A) 3.95 mm (B) 4.56 mm
(C) 5.11 mm (D) 6.23 mm

Solution:

When interference is just avoided, the path of approach is maximum.

i.e. $\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi = r\sin\phi$

or $\sqrt{R_a^2 - 84^2\cos^2 16°}$

$= (48 + 84)\sin 16°$

$\Rightarrow R_a = 88.56$ mm

Addendum of wheel = $R_a - R$

$= 88.56 - 84$

$= 4.56$ mm

Example 3: Length of path of contact is

(A) 35 mm (B) 28.32 mm
(C) 36.38 mm (D) 41.32 mm

Solution:

As interference is just avoided, the length of path of contact

$$= r\sin\phi + R\sin\phi$$
$$= (48 + 84)\sin 16°$$
$$= 36.38 \text{ mm}$$

Bearings

The purpose of a bearing is to support rotating axles and shafts, with parts fitted on them, ensuring free rotation. Bearings also transmit the force acting on them to the machine frame or a foundation. Depending upon the nature of friction in the bearings they are classified as sliding contact plain bearings or rolling contact antifriction bearings.

Sliding Contact Bearings

In sliding contact bearings, the surface of the journal of a shaft slides over the surface of a bearing. A layer of lubricant is always introduced between the rubbing surface to reduce wear and friction losses. This type of bearings in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads is called journal or sleeve bearings.

When the sliding action is guided in a straight line and carries radial loads, these are called slipper or guide bearings.

Depending upon the angle of contact the bearings may be full journal or partial journal bearings.

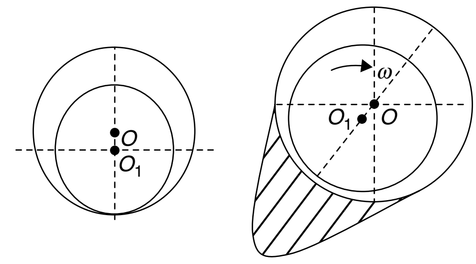
The journal bearings work on the **principle of hydrodynamic lubricating film**. If a lubricant is introduced into a wedge or tapered gap between stationary and moving members, the oil film is drawn into the wedge shaped gap, generating a pressure that can support a load.

Bearing Characteristic Number

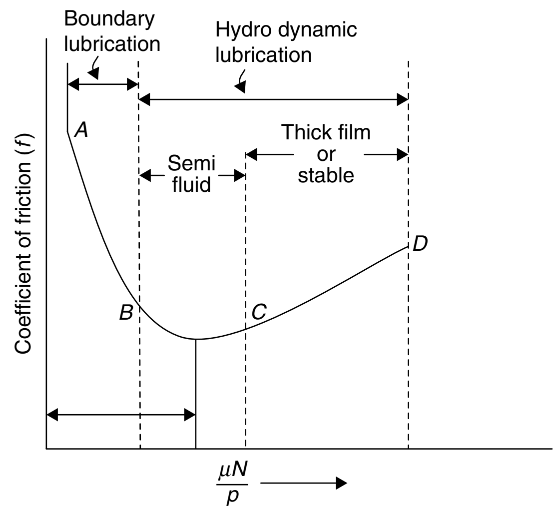
Bearing characteristic number is a dimensionless number given by $\frac{\mu N}{p}$ where,

μ = Absolute viscosity of the lubricant
N = speed of journal.
p = unit bearing pressure
$= \frac{W}{\ell d}$ where W = radial load,
ℓ = length of bearing
d = diameter of bearing

Coefficient of friction (f) varies with the bearing characteristic number as shown in the figure. In the operation of the bearing for low values of bearing characteristic number (left of point B), lubrication is not stable and is called boundary lubrication or thin lubrication. For high values of bearing characteristic number (right of C), lubrication is stable and is called thick film or hydrodynamic lubrication. Between B and C it is semi fluid or partial lubrication. When journal rotates clockwise as shown in the figure, a minimum oil film thickness occurs at the left of the bottom point and the lubricant pressure is maximum at this point.



Journal at rest Journal at full speed



Coefficient of Friction

Experimentally, it has been shown that coefficient of friction for full lubricated journal bearing is a function of three variables.

$$\frac{\mu N}{p}, \frac{d}{c} \text{ and } \frac{\ell}{d}$$

Therefore, coefficient of friction is expressed as

$$f = \phi\left(\frac{\mu N}{p}, \frac{d}{c}, \frac{\ell}{d}\right)$$

where f = coefficient of friction
μ = Absolute viscosity of lubricant in Ns/m^2
N = Speed of journal in rpm
p = Bearing pressure in N/mm^2
d = diameter of journal
c = diametral clearance.

$\frac{\mu N}{p}$ is bearing characteristic number as already mentioned.

$\frac{\mu N}{p}$ value for minimum value of f is known as **bearing modulus** which is denoted by α. It is not desirable to operate the bearing at bearing modulus because a slight decrease in speed or slight increase in pressure will lead to breakage of oil film resulting in metal to metal contact which leads to high friction, wear and heating.

Coefficient of Friction for Journal Bearing

Based on experimental investigation, **McKee** arrived at the following empirical relation for coefficient of friction.

$$f = \frac{0.326}{10^6}\left(\frac{\mu N}{p}\right)\left(\frac{d}{c}\right) + K$$

where K = a factor to correct for end leakage

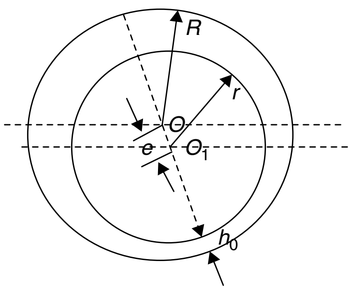
Value of K depends on length to diameter ratio $\left(\frac{\ell}{d}\right)$ of the bearing

For $\frac{\ell}{d}$ ratio 0.75 to 2.8

K is taken as 0.002

Eccentricity Ratio

The ratio of eccentricity to radial clearance is called eccentricity ratio or attitude.



O = centre of bearing

O_1 = Displaced centre of the journal under load.

e = eccentricity.

= Distance between O and O_1

h_0 = Minimum oil film thickness

$= R - r - e$

$= c_1 - e$

where c_1 = radial clearance

$= R - r$

(value of h_0 is taken as $\frac{c_1}{2}$)

Eccentricity ratio, $\varepsilon = \frac{e}{c_1}$

Diametral clearance

$c = D - d = 2(R - r) = 2c_1$

Diametral Clearance Ratio

Ratio of diametral clearance to diameter of the journal is called the diametral clearance ratio.

i.e. Diametral clearance ratio $= \frac{D-d}{d} = \frac{c}{d}$

Critical Pressure

It is the pressure at which oil film breaks and metal to metal contact begins. It is the minimum operating pressure. It is given by the following empirical relation.

$$p = \frac{\mu N}{4.75 \times 10^6}\left(\frac{d}{c}\right)^2\left(\frac{\ell}{\ell + d}\right) \text{N/mm}^2$$

Sommerfield Number

Sommerfield number is a dimensionless parameter used in the design journal bearings. It is given by

$$\frac{\mu N_s}{p}\left(\frac{d}{c}\right)^2$$

where μ = absolute viscosity in $\frac{kg}{ms}$ or $\frac{Ns}{m^2}$

c = diametral clearance.

N_s = revolution per second (rps)

p = bearing pressure in N/m²

Heat Generated in Journal Bearing

The heat generated due to friction in a journal bearing is given by

$$H_g = fwv. \text{ Nm/s or watt}$$

where f = coefficient of friction

w = load on bearing

$= p \times \ell d$

v = the rubbing velocity

$= \frac{\pi dN}{60}$

Heat dissipated by the bearing

$$H_d = CA(t_b - t_a)$$

where C = heat dissipation coefficient in W/m² °C

A = projected area of the bearing

$= \ell d$

t_b = temperature of bearing in °C

t_a = atmospheric temperature.

If t_o = the oil temperature, $t_b - t_a = \frac{1}{2}(t_o - t_a)$

or $$t_b = \frac{1}{2}(t_a + t_o)$$

Example 4: A journal bearing sustains a radial load of 3500 N. Diameter of bearing is 50 mm and its length is 90 mm. Diametral clearance is 0.1mm and shaft rotates at 500 rpm. If oil viscosity is 0.06 kg/ms

Value of Sommer field number is

(A) 9.7×10^6 (B) 0.16×10^6

(C) 15.3×10^6 (D) 0.16

Solution:

$W = 3500$ N, $d = 50$ mm, $\ell = 90$ mm,

$c = 0.1$ mm, $\mu = 0.06$ Ns/m²

$$p = \frac{W}{\ell d} = \frac{3500}{90 \times 50} = 0.778 \text{ N/mm}^2$$

$$= 0.778 \times 10^6 \text{ N/m}^2$$

$$N_s = \frac{500}{60} rps, \quad \frac{d}{c} = \frac{50}{0.1} = 500$$

$$\text{Sommerfield number} = \frac{\mu N_s}{p}\left(\frac{d}{c}\right)^2$$

$$= \frac{0.06}{0.778 \times 10^6} \times \frac{500}{60} \times 500^2 = 0.16$$

Direction for questions (Examples 5 and 6): A journal bearing of 50 mm diameter and 80mm length supports a shaft running at 900 rpm.

Viscosity of lubricating oil is 0.013 Pa *s* at the operating temperature of 132 °C. The diametral clearance is 0.05 mm. The bearing is to operate in still air with a radial load of 1250 N.

Example 5: Value of coefficient of friction is

(A) 0.0142 (B) 0.0284
(C) 0.0344 (D) 0.0122

Solution:

Coefficient of friction

$$f = \frac{0.326}{10^6}\left(\frac{\mu N}{p}\right)\left(\frac{D}{C}\right) + 0.002$$

$$p = \frac{W}{ld} = \frac{1250}{80 \times 50} = 0.3125 \text{ N/mm}^2$$

$$\mu = 0.013 \text{Pa } s = 0.013 \frac{Ns}{m^2}$$

$$N = 900 \text{ rpm}$$

$$\frac{D}{C} = \frac{50}{0.05} = 1000$$

$$\therefore f = \frac{0.326}{10^6} \times \left(\frac{0.013 \times 900}{0.3125} \times 1000\right) + 0.002$$

$$= 0.0142.$$

Example 6: Power loss due to friction is

(A) 32.84 W (B) 41.82 W
(C) 46.72 W (D) 50.33 W

Solution:

Power loss $= f\,W\,v$ watt

$$= fW \times \frac{\pi DN}{60}$$

$$= 0.0142 \times 1250 \times \pi \times \frac{50}{1000} \times \frac{900}{60}$$

$$= 41.82 \text{ W}.$$

Rolling Contact Bearings

In rolling contact bearings or anti friction bearings, the sliding friction is replaced by rolling friction. The main advantages of rolling contact bearings are

1. Smaller moments due to forces of friction
2. Easy maintenance
3. Lower consumption of lubricants
4. High degree of standardization
5. Low cost etc

The bearing consists of an inner race and an outer race with antifriction elements in between.

Depending upon the antifriction elements, these are classified as

1. Ball bearings.
2. Roller bearings.

The bearings are made of high strength hardened steel.

The bearings can be radial, radial thrust or thrust depending upon the type of load.

Static Equivalent Load for Rolling Contact Bearings

Static equivalent load is the static radial load which if applied, would cause the same total permanent deformation at the most heavily stressed element (ball or roller) and race contact as that which occurs under the actual loading condition.

Static equivalent load for radial or roller bearings is given by the following equations. The greater value of the two is selected.

1. $W_{OR} = X_0 R + Y_0 W_A$ and
2. $W_{OR} = W_R$

where W_R = radial load

W_A = axial or thrust load

X_0 = radial load factor.

Y_0 = axial or thrust load factor.

Static Load Carrying Capacity

Static load is the load acting on the bearing when the shaft is at rest static load capacity is given by **Stribecks's equation** as given below.

$$C_0 = \frac{kd^2 z}{5}$$

where C_0 = static load capacity on a ball bearing

d = diameter of ball.

z = number of balls

k = a constant depending upon radius of curvature at the point of contact and Young's modulus of the material.

Dynamic Load Carrying Capacity

Dynamic load carrying capacity of a bearing is the load the bearing can carry (radial load for radial bearings) for a minimum life of one million revolutions. it is denoted by the letter C.

Rating Life or Minimum Life

It is the life that 90% of a group of bearings can attain before fatigue failure. Rating life is denoted as L_{10}.

The life that 50% of the group can attain is the average life. It has been found that the average life is 5 times the rating life. The maximum life of a single bearing is about 30 to 50 times the minimum life.

Dynamic Equivalent Bearing Load

Dynamic equivalent load is the constant radial load in the radial bearing (or thrust load in the thrust bearing) which gives the same bearing life under actual load condition.

Equivalent dynamic load $W = X\,V\,W_R + Y\,W_A$

Where W_R = radial load in newton

W_A = axial load in newton

V = race rotation factor

X and Y are radial and axial load factors.

When outer race is stationary and inner race rotates,

$$V = 1$$

When inner race is stationary and outer race rotates, $V = 1.2$

Load Life Relationship

Rating life $L_{10} = \left(\dfrac{C}{W}\right)^p$ million revolutions (MR)

or
$$C = W\left(L_{10}\right)^{1/p}$$

Where C = dynamic load capacity (newton)

W = equivalent dynamic load.

$p = 3$ for ball bearings $= \dfrac{10}{3}$ for roller bearings

Rated Bearing Life in Hours (L_{10h})

$$L_{10h} = \frac{L_{10} \times 10^6}{N \times 60}$$

or
$$L_{10} = \frac{60 N\, L_{10h}}{10^6}$$

where N = speed of rotation(rpm)

Example 7: In a ball bearing, If the number of balls is reduced to half and diameter of the ball is increased to 4 times, then static load capacity of ball bearing is

(A) reduced 3 times. (B) increased 8 times.
(C) increased 4 times. (D) reduced 2 times.

Solution:

Static load capacity $C_0 = \dfrac{kd^2}{5} \times Z$

$$C'_0 = \frac{k(4d)^2}{5} \times \frac{Z}{2}$$

$$\frac{C'_0}{C_0} = 4^2 \times \frac{1}{2} = 8$$

$$C'_0 = 8C_0$$

Example 8: Life of a ball bearing at a load of 8 kN is 16000 h. Its life in hour, If the load is increased to 16 kN is (all other conditions are same)

(A) 1000 h (B) 2000 h
(C) 8000 h (D) 32000 h

Solution:

$L_{10} = \left(\dfrac{C}{W}\right)^3$ for ball bearing

$$\therefore L \propto \left(\frac{1}{W}\right)^3$$

$$\frac{L_2}{L_1} = \left(\frac{W_1}{W_2}\right)^3 = \left(\frac{8}{16}\right)^3 = \frac{1}{8}$$

$$\Rightarrow L_2 = 16{,}000 \times \frac{1}{8} = 2000 \text{ h}$$

Example 9: Dynamic load bearing capacity of a roller bearing is 32 kN.

The desired life for 90% survival of the bearing is 8000 h. Bearing speed is 600 rpm. Equivalent radial load the bearing can carry is

(A) 4764 N (B) 5173 N
(C) 5852 N (D) 6348 N

Solution:

$C = 32$ kN

$N = 600$ rpm

$L_{10h} = 8000$

$p = \dfrac{10}{3}$ for roller bearing

$$L_{10} = \frac{L_{10h} \times 60N}{10^6} \text{ Million revolutions}$$

$$= \frac{8000 \times 60 \times 600}{10^6} = 288 \text{ Million revolutions}$$

$$L_{10} = \left(\frac{C}{W}\right)^p$$

$$\therefore 288 = \left(\frac{32}{W}\right)^{\frac{10}{3}}$$

$$\Rightarrow W = \frac{32}{(288)^{0.3}} = 5.852 \text{ kN}$$

$$= 5852 \text{ N}$$

Shafts

Shafts are rotating machine elements, usually of circular cross section, solid or hollow, transmitting power. They support transmission elements such as gears, pulleys and sprockets and are supported in bearings. Shafts are subjected to tensile, bending or torsional shear stresses or a combination of these. Shafts are designed on the basis of strength or rigidity or both strength and rigidity.

Shafts Subjected to Axial Force

Tensile stress $\sigma_t = \dfrac{P}{\frac{\pi}{4}d^2} = \dfrac{4P}{\pi d^2}$

where P = tensile force, d = diameter

Shafts Subjected to Pure Bending Moment

Bending stress $\sigma_b = \dfrac{M_b y}{I}$

where M_b = bending moment

y = distance from neutral axis

I = moment of inertia

$= \dfrac{\pi d^4}{64}$ for solid shafts

$\therefore \sigma_b = \dfrac{32M_b}{\pi d^3}$ where $y = \dfrac{d}{2}$

$= \dfrac{M_b}{Z}$ where $Z = \dfrac{\pi d^3}{32}$

= Section modulus

Shafts Subjected to Pure Torsional Moment

$$\frac{M_t}{J} = \frac{\tau}{r}$$

where τ = shear stress

r = radial distance

J = polar moment of inertia $= \dfrac{\pi d^4}{32}$

$\tau = \dfrac{16M_t}{\pi d^3}$ when $r = \dfrac{d}{2}$

When σ_x = axial stress

τ = shear stress

Maximum principal stress

$$\sigma_{\max} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

Maximum shear stress

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

$\sigma_{\max}$ is limited to $\dfrac{S_{yt}}{FOS}$

where S_{yt} = Tensile yield stress

FOS = factor of safety

and $\tau_{\max}$ is limited to $\dfrac{S_{sy}}{FOS} = \dfrac{0.5S_{yt}}{FOS}$

where S_{sy} = yield shear stress

($S_{sy} = 0.5\ S_{yt}$ according to maximum shear stress theory)

From the above, the following equation can be written

$$\sigma_{\max} = \frac{1}{2}\left[\frac{32M_b}{\pi d^3} + \frac{4P}{\pi d^2}\right] + \sqrt{\left[\frac{1}{2}\left(\frac{32M_b}{\pi d^3} + \frac{4P}{\pi d^2}\right)^2\right] + \left(\frac{16M_t}{\pi d^3}\right)^2}$$

When there is no axial load,

$$\sigma_{\max} = \frac{16M_b}{\pi d^3} + \sqrt{\left(\frac{16M_b}{\pi d^3}\right)^2 + \left(\frac{16M_t}{\pi d^3}\right)^2}$$

$$= \frac{16}{\pi d^3}\left[M_b + \sqrt{M_b^2 + M_t^2}\right]$$

or $$\frac{S_{yt}}{FOS} = \frac{16}{\pi d^3}\left[M_b + \sqrt{M_b^2 + M_t^2}\right]$$

Similarly,

$$\tau_{\max} = \frac{0.5S_{yt}}{FOS} = \frac{16}{\pi d^3}\sqrt{M_b^2 + M_t^2}$$

Equivalent Bending Moment (M_e) and Equivalent Torsional Moment (T_e)

$$\sigma_{\max} = \frac{32M_e}{\pi d^3}$$

$$\tau_{\max} = \frac{16T_e}{\pi d^3}$$

So we can write

$$M_e = \frac{1}{2}\left[M_b + \sqrt{M_b^2 + M_t^2}\right]$$

$$T_e = \sqrt{M_b^2 + M_t^2}$$

Hollow Shafts

For hollow shafts with external diameter D and internal diameter d,

Area of cross section $= \dfrac{\pi\left(D^2 - d^2\right)}{4}$

$= \dfrac{\pi}{4}D^2\left(1 - k^2\right)$ where $k = \dfrac{d}{D}$

Moment of inertia

$$I = \frac{\pi}{64}\left(D^4 - d^4\right)$$

$$= \frac{\pi}{64} D^4 \left(1 - k^4\right)$$

So we can write,

$$\sigma_{\max} = \frac{32 M_e}{\pi D^3 \left(1 - k^4\right)}$$

$$\tau_{\max} = \frac{16 T_e}{\pi D^3 \left(1 - k^4\right)}$$

Shafts Subjected to Fluctuating Loads

Bending and torsional moments are to be multiplied by factors K_m and K_T to account for shock and fatigue as per the ASME code. So

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{\left(K_m \times M\right)^2 + \left(K_T \times T\right)^2}\right]$$

and $\quad T_e = \sqrt{\left(K_m \times M\right)^2 + \left(K_T \times T\right)^2}$

Design of Shafts on the Basis of Rigidity

The following relationship is used for the design where torsional rigidity is important

$$\frac{T}{J} = \frac{G\theta}{L}$$

or $$\theta = \frac{TL}{GJ}$$

where θ = angle of twist or torsional deflection in radian.

T = twisting moment or torque on the shaft (M_t)

J = polar moment of inertia

G = modulus of rigidity or shear modulus.

L = length of the shaft.

(GJ = torsional rigidity)

When lateral rigidity is important, deflection equation depending upon loads and support conditions are to be used. For shafts of varying cross section, lateral deflection is to be derived from the fundamental equation.

$$\frac{d^2 y}{dx^2} = \frac{M}{EI}$$

where y = deflection

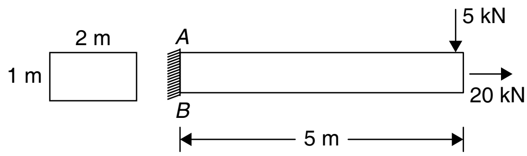
M = bending moment

E = Young's modulus

I = moment of inertia

(EI = flexural rigidity)

Example 10: Calculate the axial stresses at A and B for the cantilever loaded as shown in the figure.



Solution:

Tensile stress due to axial pull $= \dfrac{20}{2 \times 1}$

$= 10$ kN/m²

$$\sigma_{tA} = \sigma_{tB} = 10 \text{ kN/m}^2$$

A tensile stress is produced at point A and a compressive stress is produced at point B due to bending.

Bending moment at $AB = 5 \times 5$

$= 25$ kN m

$$\sigma_b = \frac{M}{I} y = \frac{M}{Z} = \frac{M}{\left(\dfrac{bd^2}{6}\right)}$$

$$= \frac{25 \times 6}{2 \times 1^2} = 75 \text{ kN/m}^2$$

Total stress at point $A = 10 + 75$

$= 85$ kN/m²

Total stress at point $B = 10 - 75$

$= -65$ kN/m²

$= 65$ kN/m²(compressive)

Example 11: A solid uniform shaft of circular cross section is subjected to a maximum bending moment of 3 kNm and a twisting moment of 4 kNm. The shaft material has an alternate tensile stress of 720 N/mm² and ultimate shear atress of 500 N/mm². Determine the shaft diameter required for a factor of safety of 6.

Solution:

$M_b = 3$ kNm, $M_t = 4$ kNm

Equivalent torsional moment

$$T_e = \sqrt{M_b^2 + M_t^2}$$

$$= \sqrt{3^2 + 4^2}$$

$= 5$ kNm $= 5 \times 10^6$ N mm

Equivalent bending moment

$$M_e = \frac{1}{2}\left[M_b + \sqrt{M_b^2 + M_t^2}\right]$$

$$= \frac{1}{2}\left[M_b + T_e\right]$$

$$= \frac{1}{2}[3 + 5]$$

$= 4$ kNm

$$\frac{\tau_{ut}}{\text{FOS}} = \frac{16T_e}{\pi d^3}$$

$$\therefore \frac{500}{6} = \frac{16\times 5\times 10^6}{\pi d^3}$$

$$\Rightarrow d = 67.4 \text{ mm}$$

Also $\dfrac{\sigma_{ut}}{\text{FOS}} = \dfrac{32M_e}{\pi d^3}$

$$\therefore \frac{720}{6} = \frac{32\times 4\times 10^6}{\pi d^3}$$

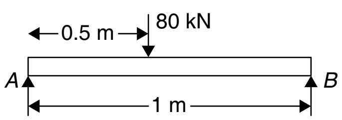
$\Rightarrow d = 69.8$ mm $\simeq 70$ mm

Choosing the highest value, required diameter is 70 mm.

Example 12: Power input to an electric generator rotates at 180 rpm is 200 kW. Equivalent load at the centre of the shaft due to the weight of armature and magnetic pulls is 80 kN. Length of the shaft is 1m between bearings. The shaft material has an ultimate shear strength of 380 MN/m^2. Diameter of the shaft based on equivalent torsional moment is

(A) 122 mm (B) 135 mm
(C) 148 mm (D) 156 mm

Solution:



$$R_A = R_B = \frac{80}{2} = 40 \text{ kN}$$

Maximum bending moment

$$M = 40 \times 0.5$$
$$= 20 \text{ kNm}$$

Power input $P = 200$ kW

$$N = 180 \text{ rpm}$$

$$P = \frac{2\pi NT}{60}$$

$$200 = 2\pi \times \frac{180\times T}{60}$$

$$\Rightarrow T = 10.61 \text{ kN m}$$

Equivalent torsional moment

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(20)^2 + (10.61)^2}$$
$$= 22.64 \text{ kN m}$$

$$\frac{S_{us}}{\text{FOS}} = \frac{16T_e}{\pi d^3}$$

$$\frac{380\times 10^6}{6} = \frac{16\times 22.64\times 10^3}{\pi \times d^3}$$

$$\Rightarrow d = 0.122 \text{ m} = 122 \text{ mm.}$$

Brakes

Brake is a device used for retarding or stopping the motion of a machine by means of frictional resistance. In the braking process, either the kinetic energy or potential energy is absorbed and the energy is dissipated in the form of heat.

Types of Brakes

According to the means used for transforming energy, brakes may be classified as

1. Hydraulic brakes.
2. Electric brakes.
3. Mechanical brakes.

Hydraulic and electric brakes are used where large amount of energy is to be transformed. They cannot bring the member to rest and are used for controlling the speed.

Mechanical brakes may be classified into radial brakes or axial brakes according to the direction of acting force.

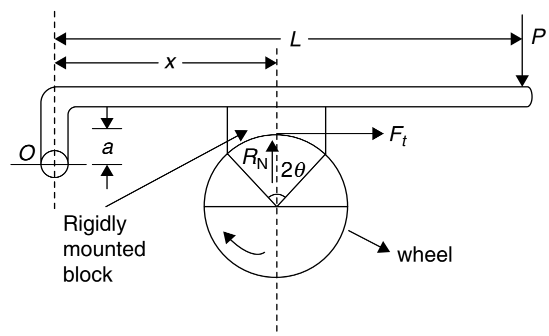
In axial brakes, the force acting on the brake drum is in the axial direction.

Disc brakes and cone brakes are examples of axial brakes. Analysis of these brakes are similar to that of clutches.

Radial brakes include block or shoe brakes, band brakes, internal expanding brakes, etc.

Single Block or Shoe Brake

In this type of brakes, a block attached to a pivoted lever presses against the brake wheel drum as shown in below figure. Braking force is applied at the end of the lever. The tangential frictional force provides the braking torque.



Forces acting on the lever for a clockwise rotation of the wheel is shown in figure. The normal force R_N acts upwards and the force F_t acts to the right. Let the fulcrum of the lever be at a distance a below the line of action of the braking force (frictional force F_t).

Taking moments about the fulcrum O,

$$R_N \times x = F_t \times a + P \times L$$

But $F_t = \mu R_N$ where μ = coefficient of friction

$$\therefore R_N x - \mu R_N \times a = PL$$

$$\Rightarrow R_N = \frac{PL}{x - \mu a}$$

Braking force $F_t = \dfrac{\mu PL}{x - \mu a}$

Braking torque $\tau = F_t \times r$

$$= \frac{\mu PLr}{x - \mu a}$$

where r = radius of the brake wheel drum.

In this case, **the moment due to μR_N and the external force P are in the same direction. So the brake is said to be self energizing**.

Depending upon the position of fulcrum and direction of rotation of the of brake wheel drum, the above equations will be modified.

When the line of action of F_t passes through the fulcrum O of the lever; $a = 0$ and $R_N = \dfrac{PL}{x}$

Braking force $F_t = \dfrac{\mu PL}{x}$

Braking torque $T_B = \dfrac{\mu PLr}{x}$

The **brake becomes self locking when $P = 0$**

And $\quad R_N \times x = \mu R_N \times a$

ie $\quad x = \mu a$

Pivoted Block or Shoe Brake

When the angle of contact 2θ is less than 60° as in the case of single block or shoe brake, the normal pressure between block and wheel can be assumed to be uniform. But when 2θ is greater than 60°, the normal pressure is less towards the ends of the shoe. In such cases, pivoted shoes or brakes are used. This results in uniform wear of the shoe in the direction of the applied force. Breaking torque is given by,

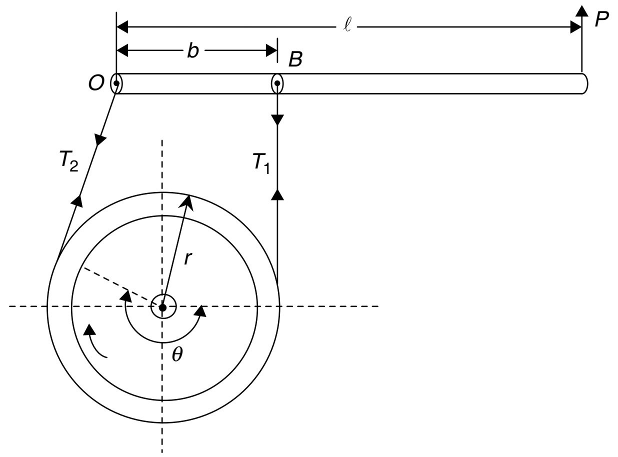
$$T_B = F_t \times r = \mu' R_N \times r$$

Where $\quad \mu' = \dfrac{4\mu \sin\theta}{2\theta + \sin 2\theta}$

Simple Band Brake

In a band brake, a flexible band of leather, one or more ropes or a steel band lined with friction material is used. The band embraces a part of the circumference of the drum and its ends are attached to a pivoted lever.

The following figure shows a simple band brake. One end of the band is fixed at fulcrum O and the other end is attached to the lever at B at right angle.



When the brake drum rotates clockwise as shown in figure, the end attached to B is the tight side and the other end is the slack side.

(For the anticlockwise rotation tight side will be the end connected to fulcrum O)

Ratio of the tensions; $\dfrac{T_1}{T_2} = e^{\mu\theta}$

Where μ = coefficient of friction and

θ = angle of lap

Braking force on the drum = $T_1 - T_2$

Braking torque = $(T_1 - T_2)r_e$

where r_e = effective radius = $r + t$

Taking moments about O,

$$T_1 \times b = P \times \ell$$

$$\therefore T_1 = P \times \frac{\ell}{b}$$

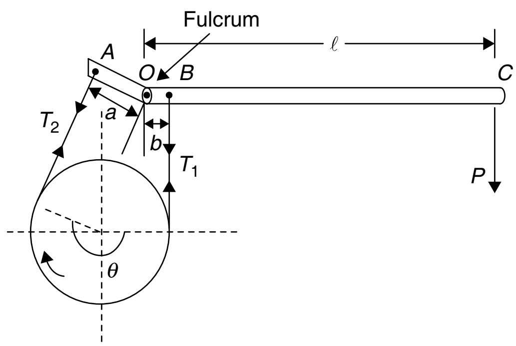
Tensile stress in the band $\sigma_t = \dfrac{T_1}{wt}$

Where w = width of band

and t = thickness of the band.

Differential Band Brake

In differential band brakes, the belt end is not fixed at the fulcrum of the lever. The fulcrum is in between the belt ends.



For a clockwise rotation of the drum arrangement is as shown in figure. T_1 is tension in the tight side and T_2 is tension in the slack side.

Taking moments about fulcrum O.

$$P\ell + T_1 b = T_2 a$$

$$P\ell = T_2 a - T_1 b$$

For self locking, the force P should be zero or negative.

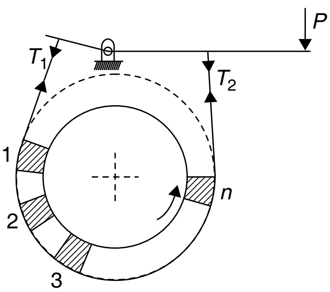
$$\therefore \quad T_1 b \geq T_2 a$$

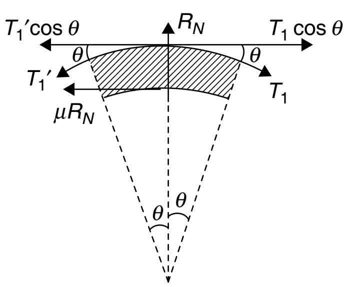
or $\dfrac{T_1}{T_2} \geq \dfrac{a}{b}$ for self locking.

For anticlockwise rotation, end A becomes tight side and the relations will be modified accordingly.

Band and Block Brakes

In band and block brakes, the band is lined with a number of blocks covering the angle of lap. Let there be n blocks and each block subtend an angle 2θ at the centre of the drum.





For the anticlockwise direction of rotation, the forces on the band and blocks will be as shown in figure.

$$(T_1 + T_1')\sin\theta = R_N$$

$$(T_1 - T_1')\cos\pi = \mu R_N$$

$$\Rightarrow \frac{T_1}{T_1'} = \frac{1+\mu\tan\theta}{1-\mu\tan\theta} \text{ for the first block}$$

For 2nd block,

$$\frac{T_1'}{T_2'} = \frac{1+\mu\tan\theta}{1-\mu\tan\theta} \text{ etc}$$

$$\therefore \frac{T_1}{T_2} = \frac{T_1}{T_1'} \times \frac{T_1'}{T_2'} \times \frac{T_2'}{T_3'} \times \ldots\ldots \times \frac{T'_{n-1}}{T_2}$$

$$= \left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right)^n$$

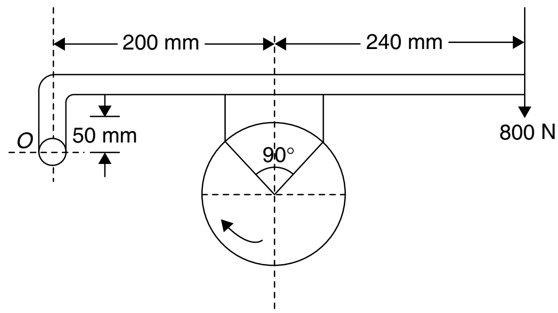
Braking torque = $(T_1 - T_2)\, r_e$

where r_e = effective radius.

Internal Expanding Brakes

In the internal expanding brakes, two shoes lined with friction material are pivoted at one end and actuated by the cam at the other end. The cam side of the shoes are held in position by a spring.

Example 13: In a single block brake, the drum diameter is 420 mm, angle of contact is 90° and dimensions of lever are as shown in the figure. If the operating force applied at the end of the lever is 800 N, and coefficient of friction is 0.3, determine the torque transmitted (in Nm)



Solution:

$$P = 800 \text{ N}$$

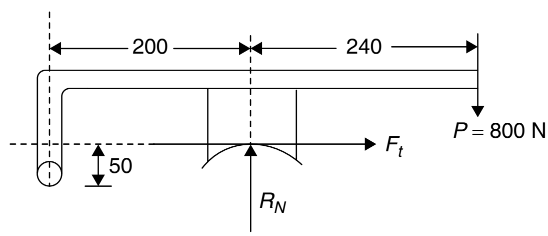
$$D = 420 \text{ mm}, r = 210 \text{ mm}$$

$$2\theta = 90° > 60°$$

$$\mu = 0.3$$

$$\mu' = \frac{4\mu\sin\theta}{2\theta + \sin 2\theta}$$

$$= \frac{4\times 0.3\sin 45°}{\frac{\pi}{2} + \sin 90°} = 0.33$$



Taking moment about O,

$$P\times 440 + F_t \times 50 = R_N \times 200$$

or $R_N \times 200 - \mu' R_N \times 50 = P \times 440$

$\Rightarrow R_N(200 - 0.33 \times 50) = 800 \times 440$

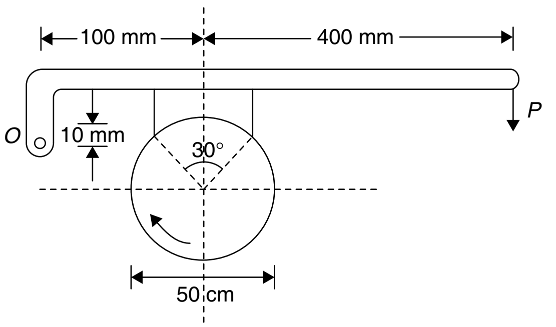
$\Rightarrow R_N = 1918.26$ N

Braking torque $= F_t \times r = \mu' R_N \times r$

$$= 0.33 \times 1918.26 \times \frac{210}{1000}$$

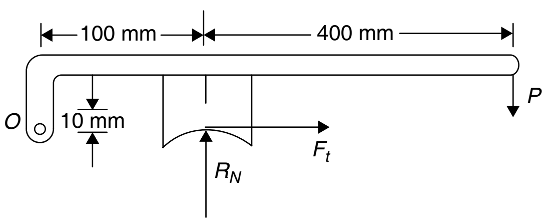
$$= 132.94 \text{ Nm.}$$

Example 14: A single block brake has a brake drum diameter of 50 cm and angle of contact 30°. It takes 200 Nm torque at 100 rpm (coefficient of friction is 0.35). The arrangement of brake is as shown in the figure. If the drum is rotating in clockwise direction, find the force *P* required at the end of the lever.



Solution:

For the clockwise rotation of the drum, the forces on the lever are as shown



Friction force acting on the drum

$$F_t = \mu R_n$$

$$= 0.35\, R_n$$

Torque $= \mu R_n \times r$

$\therefore$ $200 = 0.35 \times 0.25\, R_n$

$\Rightarrow R_n = 2285.714$ N

Taking moments about the fulcrum *O*,

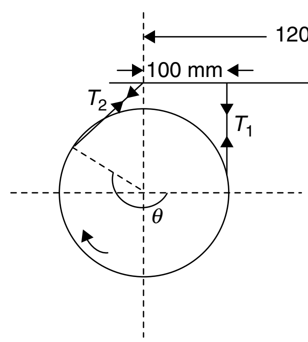
$R_n \times 100 + F_t \times 10 = P \times 500$

$\Rightarrow 2285.714(100 + 0.35 \times 10) = P \times 500$

$\Rightarrow P = 473.143$ N.

Example 15: A band brake acts on $\frac{2}{3}^{\text{rd}}$ circumference of drum of 200 mm diameter. The band brake provides a braking torque of 100 Nm. One end of the band is attached to a fulcrum pin on the end of the lever and the other end at 100 mm from fulcrum. Find the operating force required at a distance 1200 mm from the fulcrum when drum rotates clockwise ($\mu = 0.25$).

Solution:



The brake arrangement is as shown in the figure.

Angle of lap $\theta = \frac{2}{3} \times 2\pi = \frac{4\pi}{3}$ rad

Braking torque = 100 Nm

$$= (T_1 - T_2) \times 0.1$$

$$\Rightarrow T_1 - T_2 = 1000\ N \qquad (1)$$

$$\Rightarrow \frac{T_1}{T_2} = e^{\mu\theta}$$

$$= e^{\frac{4\pi}{3} \times 0.25} = e^{\frac{\pi}{3}} = 2.8497$$

$$\Rightarrow T_1 = 2.8497 T_2 \qquad (2)$$

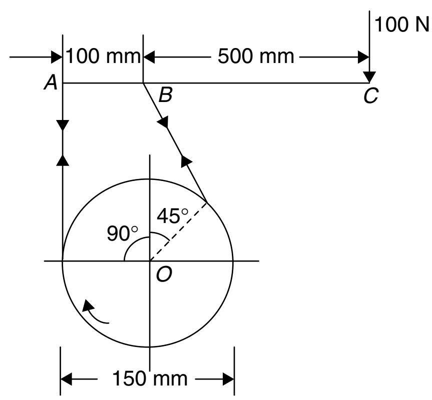
From (1) and (2)

$T_2 + 1000 = 2.8497 T_2$

$\Rightarrow T_2 = 540.63$ N and $T_1 = 1540.63$ N.

Example 16: For the simple band brake shown in the figure, the braking torque due to the applied load, when the wheel rotates in the clockwise direction is (*B* is fulcrum and coefficient of friction is 0.2)

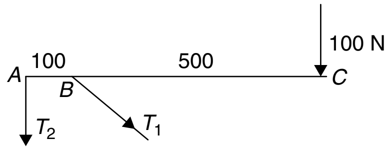
(A) 36.7 Nm (B) 42.6 Nm
(C) 44.8 Nm (D) 48.4 Nm



Solution:

For the clockwise rotation of the wheel, band at *B* is at the light side and band at *A* is at the slack side.

Forces acting on the lever are as follows.



Taking moments about the fulcrum *B*.

$$T_2 \times 100 = 100 \times 500$$
$$\Rightarrow T_2 = 500 \text{ N}$$

Angle of lap $\theta = 360° - (90° + 45°)$

$$= 225°$$
$$= \frac{225 \times \pi}{180}$$
$$= 3.93 \text{ rad}$$
$$\frac{T_1}{T_2} = e^{\mu\theta}$$
$$= e^{0.2 \times 3.93}$$
$$= 2.195$$

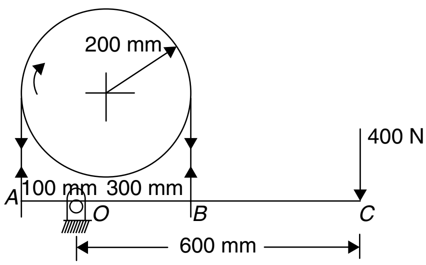
$\therefore\ T_1 = 500 \times 2.195$

$= 1097.5$ N

Braking torque $= (T_1 - T_2) \times r$

$$= (1097.5 - 500) \times \frac{75}{1000}$$
$$= 44.81 \text{Nm}$$

Example 17:



For the differential band brake shown in figure, a clockwise torque of 800 Nm is applied on the drum. Find maximum forces in the band.

Solution:

For the clockwise rotation, tight side tension on the band. T_1 is at the end *A* and slack side tension T_2 is at *B*.

Torque $= (T_1 - T_2)r$

$\therefore\ 800 = (T_1 - T_2) \times 0.2$

$\therefore\ T_1 - T_2 = 4000$ N

or $T_1 = T_2 + 4000$

Taking moments about the fulcrum *O*

$T_1 \times 100 - T_2 \times 300 + 600 \times 400 = 0$

$T_1 \times 100 - T_2 \times 300 = -240000$

$T_1 - 3T_2 = -2400$

$(T_2 + 4000) - 3T_2 = -2400$

$-2T_2 = -6400$

$\Rightarrow T_2 = 3200$ N

$\therefore T_1 = 3200 + 4000 = 7200$ N.

Clutches

Clutches may be divided into two types—positive clutches and friction clutches. Jaw and claw clutches are examples of positive clutches. Electrical, electro magnetic and hydraulic clutches are also used.

A clutch is primarily used to engage and disengage driver and driven shafts according to requirements. Its principal application is in power transmission where shafts and machines are to be started and stopped frequently.

A brake is used for stopping or controlling the speed of a running machine, whereas a clutch is used to disengage a machine before stoppage or engage the driven shaft from rest to proper speed.

In friction clutches, plates or cones with friction lining are used to engage or disengage the driver and driven shafts.

The main types of friction clutches are

1. Disc or plate clutch(single or multiple plate clutches)
2. Cone clutch
3. Centrifugal clutches.

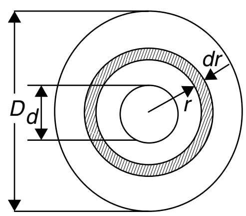
Normal Forces and Frictional Torques on Clutches

Friction surfaces of a plate clutch remain in contact by the application of an axial thrust or load *P*.

Let *D* and *d* be the outer and inner diameter of the friction surface and

μ = coefficient of friction

p = intensity of axial pressure.



Considering an elementary ring at radius *r* with thickness *dr*.

Elemental axial force δw = pressure × area

$$= p \times 2\pi r dr$$

Elemental frictional force $= \mu\delta w = \delta F_r$

$$= \mu p \times 2\pi r dr$$

Elemental frictional torque $\delta Tr = r\delta F_r$

$$= \mu p \times 2\pi r^2 dr$$

Forces and Torques can be calculated based on two theories.

1. Uniform pressure theory and
2. Uniform wear theory.

Considering Uniform Pressure Theory

Normal or Axial force, $W = \int_{d/2}^{D/2} p2\pi r dr$

$$= \left[p \times 2\pi \frac{r^2}{2} \right]_{d/2}^{D/2}$$

$$= \frac{\pi p}{4}\left(D^2 - d^2\right)$$

$$\Rightarrow W = \pi p\left(r_1^2 - r_2^2\right)$$

where $D = 2r_1$ and $d = 2r_2$

Frictional torque $T = \int_{d/2}^{D/2} \mu p 2\pi r^2 dr$

$$= \mu p 2\pi \left[\frac{r^3}{3} \right]_{d/2}^{D/2}$$

$$= \frac{\pi \mu p}{12}\left(D^3 - d^3\right)$$

$$\Rightarrow T = \frac{2}{3}\pi \mu p\left(r_1^3 - r_2^3\right)$$

$T = \mu W \times R_m$ where R_m = mean radius

$$\therefore \ \mu \pi p\left(r_1^2 - r_2^2\right) R_m = \frac{2}{3}\mu \pi p\left(r_1^3 - r_2^3\right)$$

$$\therefore \ R_m = \frac{2}{3}\frac{\left(r_1^3 - r_2^3\right)}{\left(r_1^2 - r_2^2\right)}$$

Considering Uniform Wear Theory

According to uniform wear theory,

$$pr = \text{constant} = C$$

Normal or axial force

$$W = \int_{d/2}^{D/2} 2\pi (pr) dr$$

$$= 2\pi c [r]_{d/2}^{D/2}$$

$$= \pi c[D - d]$$

$$\Rightarrow W = 2\pi c(r_1 - r_2)$$

Frictional torque

$$T = \int_{d/2}^{D/2} \mu p 2\pi r^2 dr$$

$$= \int_{d/2}^{D/2} \mu 2\pi c r dr$$

$$= \mu 2\pi c \left[\frac{r^2}{2} \right]_{d/2}^{D/2}$$

$$= \frac{\mu \pi c}{4}\left(D^2 - d^2\right)$$

$$\Rightarrow T = \mu \pi c\left(r_1^2 - r_2^2\right)$$

$T = \mu W R_m$

$$\therefore \ \mu \pi c\left(r_1^2 - r_2^2\right) = 2\pi c\left(r_1 - r_2\right) R_m$$

$$\Rightarrow (r_1 + r_2) = 2R_m$$

$$\Rightarrow R_m = \frac{\left(r_1 + r_2\right)}{2}$$

As $p \times r$ is constant, the maximum pressure acts at the inner radius and the minimum pressure acts at the outer radius.

$\therefore \ p_{\max} \times r_2 = p_{\max} \dfrac{D}{2} = C$ and

$$p_{\min} \times r_1 = p_{\min} \times \frac{d}{2} = C$$

$$\therefore \ \frac{p_{\max}}{p_{\min}} = \frac{r_1}{r_2}$$

Average pressure on the friction surfaces can be obtained using

$$p_{av} = \frac{W}{\pi\left(r_1^2 - r_2^2\right)}$$

Usually, uniform wear theory is used for old clutches and uniform pressure theory is used for new clutches.

Multiple Disc Clutch

In multiple disc clutch, let n be the number of pairs of contact friction surfaces.

$$n = n_1 + n_2 - 1$$

where n_1 = number of discs on driving shaft and n_2 = number of discs on the driven shaft.
Frictional torque on one pair of surfaces

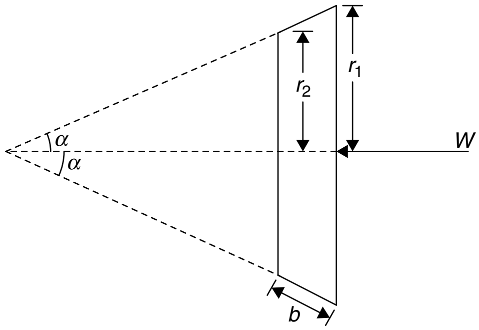
$$= \mu W R_m$$

$\therefore$ Total frictional torque $T = n\mu W R_m$

where W = axial load and
R_m = mean radius.

Cone Clutch

In cone clutch, the mating surfaces are conical. An internal cone is fixed on the driving member and a movable cone is fixed on the driven member.



Let p_n be the normal pressure on the surface.

Axial force $W = p_n \times$ projected area

$$= p_n \times \pi \left(r_1^2 - r_2^2\right)$$

Torque transmitted $T = \mu \dfrac{W}{\sin\alpha} R_m$

where R_m = mean radius

and 2α = cone angle.

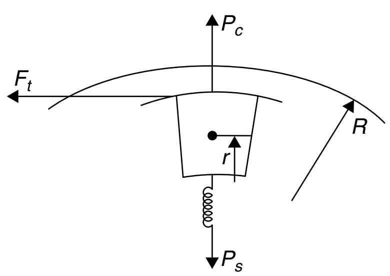
(when α = 90°, the clutch becomes a plate clutch and $T = \mu W R_m$)

If face width is b and mean radius is R_m, W may be obtained using

$$W = p_n(2\pi R_m b) \times \sin\alpha$$

Centrifugal Clutch

In a centrifugal clutch shoes held by springs move radially outward due to centrifugal force and engage the inside surface of the cylindrical drum, in which the shoes are provided.



P_s = spring force = $s \times a$

where s = stiffness of springs

a = amount of compression of spring

R = radius of friction surface

r = Radial distance of centre of gravity of shoe

m = mass of each shoe

n = number of shoes.

When engagement starts the centrifugal force is equal to the spring force.

Centrifugal force $P_c = m\omega^2 r$

where ω = angular velocity

Net outword force = $P_c - P_s$

Friction force on each shoe = $\mu(P_c - P_s)$

Frictional torque on each shoe = $\mu(P_c - P_s) \times R$

Total torque transmitted = $n\mu(P_c - P_s)R$

Force exerted on each shoe = $P_c - P_s = p\ell b$

Where p = pressure intensity on each shoe.

ℓ = contact length of the shoe.

b = breadth of the shoe

contact length $\ell = R\theta$

where θ = the subtending angle of the shoe

Example 18: The axial force in a plate clutch is 6 kN. The inside radius of contact surface is 50 mm and outside radius is 90 mm. Assuming uniform wear, find the maximum pressure, minimum pressure and average pressure in the plate.

Solution:

Maximum pressure = p_{max}

= intensity of pressure at the inner radius (r_2)

$\therefore p_{max} r_2 = C$

$\Rightarrow C = p_{max} \times 50$

Axial force $W = 2\pi c(r_1 - r_2)$

$\therefore 6000 = 2\pi \times p_{max} \times 50(90 - 50)$

$\Rightarrow p_{max} = 0.4775$ N/mm^2

$p_{min} \times r_1 = C = p_{max} \times r_2$

ie, $p_{min} \times 90 = 0.4775 \times 40$

$\Rightarrow p_{min} = 0.2122$ N/mm^2.

Average pressure p_{av}

$$= \frac{\text{axial force}}{\text{crosssection area of contact surface}}$$

$$= \frac{6000}{\pi\left(r_1^2 - r_2^2\right)}$$

$$= \frac{6000}{\pi\left(90^2 - 50^2\right)}$$

$$= 0.341 \text{ N/mm}^2$$

Example 19: A power of 6 kW is to be transmitted at 2000 rpm using a disc clutch. The friction lining has a coefficient of friction equal to 0.25. The bore radius of friction lining is 25 mm. Assuming uniform contact pressure of 1MPa, find the outside radius of friction lining.

Solution:

Power transmitted $P = \dfrac{2\pi NT}{60} = 6$ kW

i.e., $\dfrac{2\pi \times 2000 \times T}{60} = 6 \times 10^3$

$\Rightarrow$ Torque $T = 28.65$ Nm

For uniform pressure,

$$T = \frac{2}{3}\mu W\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$W = p \times \pi\left(r_1^2 - r_2^2\right)$$

$$\Rightarrow T = \frac{2}{3}\mu\pi p\left(r_1^3 - r_2^3\right)$$

$\therefore$ 28.65 × 1000

$$= \frac{2}{3}\times 0.25\times\pi\times 1\left(r_1^3 - 25^3\right)$$

$$\Rightarrow r_1 = 41.28 \text{ mm}$$

Direction for questions (Example 20 to 22): The effective diameter of the contact surface of a cone clutch is 80 mm; the semi angle of the cone is 15° and the coefficient of friction is 0.35. If the axial force is 200 N

Example 20: Find the torque required to produce slipping of the clutch.

Solution:

Torque required $T = \dfrac{\mu WR}{\sin\alpha}$

$$= \frac{0.35\times 200\times 40\times 10^{-3}}{\sin 15°}$$

= 10.82 Nm.

Example 21: If the clutch is employed to connect a motor running uniformly at 900 rpm with a fly wheel, find the time required to reach full speed from rest.

(Mass of the flywheel is 15 kg and radius of gyration is 16 cm)

Solution: $T = I\alpha = mk^2 \times \alpha$

where I = moment of inertia

α = angular acceleration

$\therefore$ $10.82 = 15 \times (0.16)^2 \times \alpha$

$\Rightarrow \alpha = 28.18 \text{ rad/s}^2$

Angular speed $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi\times 900}{60}$

= 94.25 rad/s

$\omega = \omega_0 + \alpha t$

$\Rightarrow 94.25 = 0 + 28.18 \times t$

$\Rightarrow t = 3.345$ s

Example 22: Find the energy lost in the slipping of the clutch during engagement.

Solution:

Energy lost in slipping = $T\theta$

where θ = average angular speed × time

$$= \frac{\omega_0 + \omega}{2}\times t$$

$$= \frac{0 + 94.25}{2}\times 3.345$$

= 157.633 rad

$\therefore$ Energy lost = 10.82 × 157.633

= 1705.59 Nm.

Direction for questions (Examples 23 and 24): A centrifugal clutch transmitting 15 kW at 900 rpm have 4 shoes which engage from $\frac{3}{4}$ of running speed. Centre of gravity of the shoes are at 12 cm from the centre of the spider. Inside radius of the drum ring is 150 mm and coefficient of friction for the shoes is 0.3.

Example 23: Find the torque transmitted.

Solution:

Power = $\dfrac{2\pi NT}{60}$

$\therefore$ $15 \times 10^3 = \dfrac{2\pi\times 900\times T}{60}$

$\Rightarrow$ Torque transmitted

$T = 159.15$ Nm

Example 24: Find the mass of a shoe.

Solution: Centrifugal force at the rated speed

$P_c = m\omega^2 r$ where m = mass

$$= m\times\left(\frac{2\pi N}{60}\right)^2\times r$$

$$= m\times\left(\frac{2\pi\times 900}{60}\right)^2\times 0.12$$

$= m \times 1065.9$ N

Centrifugal force at the engagement = spring force

$P_s = m(0.75\omega)^2 r$

$= m \times 1065.9 \times (0.75)^2$

$= m \times 599.57$ N

Torque transmitted = $\mu(P_c - P_s)R$ × number of shoes

159.15 = 0.3m(1065.9 – 599.57) × 0.15 × 4

$\Rightarrow m = 1.896$ kg.

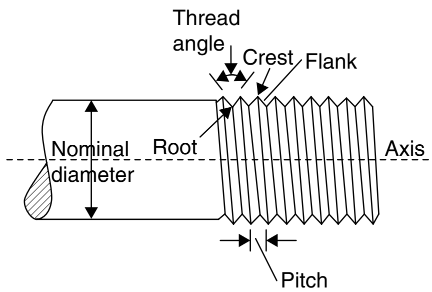
Bolted Joints

Bolted joint is a separable joint of two or more components held together by means of a threaded fastening such as bolt and nut.

Other threaded fastenings are screws, studs, threaded couplings etc.

A bolt is a cylindrical bar with a thread at one end and head at the other. The threaded portion is screwed into a nut. Bolts have external threads and nuts have internal threads

Important Terms used in Screw Threads



Major Diameter

It is the largest diameter of an external or internal screw thread. It is also known as the **nominal diameter**. It is the diameter representing the crest of an external thread.

Minor Diameter

It is the smallest diameter of an external or internal thread. It is also known as **core diameter or root diameter** for external threads.

Pitch Diameter (d_p)

It is defined as the diameter of an imaginary cylinder that pass through pitch points of the thread. **Pitch points** are points on the surface of thread such that width of a thread is equal to the space between the threads.

Pitch (p)

Pitch is the distance between two similar points on adjacent threads, measured parallel to the axis of the thread.

$$\text{Pitch} = \frac{1}{\text{Number of threads per unit length}}$$

Lead (L)

Lead is the distance between two corresponding points on the same helix. It is important in the case of multi start threads.

Lead is equal to pitch in the case of single start threads. It is the distance travelled by the nut in one turn.

$L = n \times p$

where n = number of starts

Thread angle

It is the included angle of flanks of two adjacent threads.

Stresses in Screwed Fastenings due to Static Loading

Following are the important stresses due to static loading.

1. Initial stresses due to screwing up forces.
2. Stresses due to external forces.
3. Stresses due to combination of the above.

Initial tension in a bolt is found using the empirical relation

$$P_i = 2840d \text{ newton}$$

where d = Nominal diameter of bolt in mm.

The above relation **holds good for fluid tight joints** like steam engine cylinder cover joints etc.

If a fluid tight joint is not required, initial tension can be reduced to half of the above value.

i.e. $P_i = 1420d$ newton

Small diameter bolts (less than M_{16} or M_{18}) are not used for fluid tight joints as these fail during tightening.

For bolts not initially stressed, maximum safe axial load is given as

$$P = \text{permissible stress} \times \text{cross sectional area}$$

Cross-sectional area used for the above purpose is

$$= \frac{\pi}{4}\left(\frac{d_p + d_c}{2}\right)^2$$

Where d_p = pitch diameter of the bolt

d_c = core diameter of the bolt

When a bolted joint is subjected tensile force P, tensile stress $\sigma_t = \dfrac{P}{\frac{\pi}{4} d_c^2}$

where d_c = the core diameter (diameter of weakest section)

Torque Required for Bolt Tightening

Two factors considered for tightening the bolt are:

1. Torque to overcome thread friction and induce the pre-load.
2. Torque to overcome collar friction between the nut and the washer.

Torque required to overcome thread friction,

$$M_t = \frac{P_i d_m}{2} \times \frac{(\mu \sec\theta + \tan\theta)}{(1 - \mu \sec\theta + \tan\alpha)}$$

For ISO metric screw threads,

$\theta = 30^o$, $\alpha = 25^o$ and $d_m = 0.9\, d$

d = nominal or major diameter of the bolt

$\mu = 0.12 - 0.2$

taking $\mu = 0.15$

$M_t = 0.098\, P_i\, d$

Collar friction torque

$$(M_t)_C = \left(\frac{\mu P_i}{2}\right)\left(\frac{D_0 + D_i}{2}\right)$$

Taking $\left(\frac{D_0 + D_i}{0}\right) = 1.4\text{d}$ and $\mu = 0.15$ for ISO threads

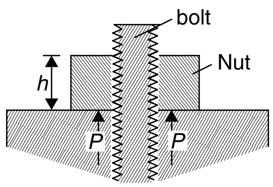
$$(M_t)_C = 0.105\ P_i\,d$$

Total torque (M_t) is given by

$$(M_t) = M_t + (M_t)_C = (0.098 + 0.105)P_i\,d = 0.2\ P_i\,d$$

The above equation gives the wrench torque (M_t) required to create the required pre-load P_i.

Height of Nut (*h*)



Permissible tensile stress of bolt

$$\frac{S_{yt}}{\text{FOS}} = \frac{P}{\frac{\pi}{4}(d_c)^2}$$

where d_c = core diameter

P = axial load

$$\therefore\ P = \left(\frac{S_{yt}}{\text{FOS}}\right)\frac{\pi}{4}(d_c)^2 \rightarrow (1)$$

Permissible shear stress on bolt threads

$$\frac{S_{sy}}{\text{FOS}} = \frac{P}{\pi d_c h}$$

$$\therefore P = \left(\frac{S_{sy}}{\text{FOS}}\right)\times \pi d_c h$$

$$= \left(\frac{S_{yt}}{2\text{FOS}}\right)\times \pi d_c h \quad \rightarrow \quad (2)$$

as $S_{sy} = \frac{S_{yt}}{2}$ by maximum shear stress theory

$\therefore$ From (1) and (2)

$$\frac{d_c^2}{4} = \frac{d_c h}{2}$$

$$\Rightarrow h = \frac{d_c}{2}$$

$$\Rightarrow h = 0.5 d_c$$

Approximate relationship between core diameter and nominal diameter is

$$d_c = 0.8d$$

$$\therefore \qquad h = 0.4d$$

If a bolt is subjected to tensile stress σ_t and torsional shear stress τ

Maximum principal stress

$$\sigma_{\max} = \frac{\sigma_t}{2} + \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$$

Maximum shear stress $\tau_{\max} = \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$

Coarse Series and Fine Series Threads

Coarse series threads are designated by M followed by the nominal diameter in mm (for example, $M16$)

In fine series pitch also is mentioned. For example, $M12 \times 1.25$ means fine series threads of 12 mm nominal diameter and 1.25 mm pitch.

Example 25: An equipment weighing 10 kN is lifted using an eye bolt. The eye bolt is screwed into the frame of the equipment and the equipment is lifted by a crane after connecting the hook of the crane to the eye of the bolt. The yield point stress of the bolt is 400 N/mm^2. If core diameter of the bolt is 0.64 times the nominal diameter, find the nominal diameter (Assume a factor of safety of 5) in mm.

Solution:

Allowable tensile stress of the bolt.

$$\sigma_{\max} = \frac{\sigma_{ut}}{FOS} = \frac{400}{5} = 80\ \text{N/mm}^2$$

Axial load $P = 10 \times 10^3$ N

$= \sigma_{\max} \times$ core area

ie, $10 \times 10^3 = 80 \times \frac{\pi}{4} d_c^2$ v$\Rightarrow d_c = 12.61$ mm

Nominal diameter $d = \frac{12.61}{0.64} = 19.71$

$= 20$ mm.

Example 26: The nominal diameter of $M30$ bolt is 1.12 times the core diameter. If safe tensile stress of the bolt material is 42 MPa, the tensile load it can take is

(A) 44.56 kN (B) 33.48 kN
(C) 23.66 kN (D) 18.76 kN

Solution:

Nominal diameter $d = 30$ mm

Core diameter $d_c = \frac{30}{1.12} = 26.786$ mm

Tensile load $P = \sigma_{\max} \times$ core area

$$= \sigma_{max} \times \frac{\pi}{4} d_c^2$$

$$= 42 \times \frac{\pi}{4} \times (26.786)^2 = 23667 \text{ N}$$

$$= 23.667 \text{ kN}.$$

Direction for questions (Examples 27 and 28): A bolt is subjected to a tensile load of 25 kN and a shear load of 10 kN. Yield point stress in tension is 300 N/mm^2. Factor of safety is 2.5 and Poissons ratio is 0.25.

Example 27: Determine the principal stress in terms of core area a.

Solution:

Given data

Tensile load $P = 25 \text{ kN} = 25 \times 10^3$ N

Shear load $F = 10 \text{ kN} = 10 \times 10^3$ N

FOS = 2.5

Yield stress $\sigma_{yt} = 300$ N/mm^2

Poisson's ratio $\mu = 0.25$

$$\sigma_x = \frac{p}{a} = \frac{25 \times 10^3}{a}$$

$$\tau = \frac{F}{a} = \frac{10 \times 10^3}{a}$$

$$\text{Principal stress} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

$$= \frac{12.5 \times 10^3}{a} \pm \sqrt{\left(\frac{12.5 \times 10^3}{a}\right)^2 + \left(\frac{10 \times 10^3}{a}\right)^2}$$

$$= \frac{12.5 \times 10^3}{a} \pm \frac{10^3}{a}\sqrt{12.5^2 + 10^2}$$

$$= \frac{10^3}{a}[12.5 \pm 16]$$

$\sigma_1 = \dfrac{28.5 \times 10^3}{a}$ and $\sigma_2 = \dfrac{3.5 \times 10^3}{a}$ N/mm^2 where a is in mm^2

Example 28: Determine bolt diameter according to maximum principal stress theory.

Solution:

Maximum principal stress

$$\sigma_1 = \frac{28.5 \times 10^3}{a} = \frac{28.5 \times 10^3}{\pi\left(\frac{d_c}{2}\right)^2}$$

$$= \frac{36.287 \times 10^3}{d_c^2} \text{ N/mm}^2$$

According to maximum principal stress theory,

$$\sigma_1 = \frac{\sigma_{yt}}{\text{FOS}} \text{ i.e., } \frac{36.287 \times 10^3}{d_c^2} = \frac{300}{2.5}$$

$$\Rightarrow d_c = 17.39$$

$$\text{Nominal diameter} = \frac{17.39}{0.8} = 21.74 \text{ mm}$$

$$\simeq 22 \text{ mm}$$

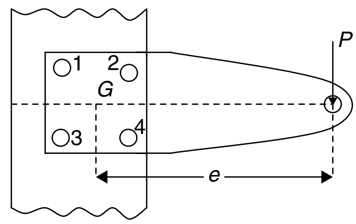
Eccentric Loading of Bolts

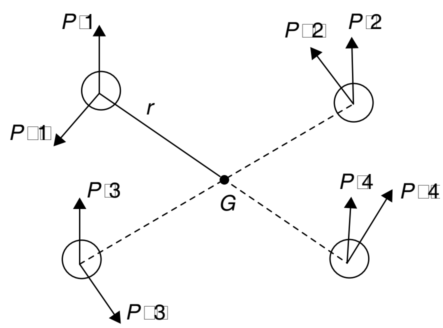
Eccentric loading of bolts can be of two types.

1. Eccentric loading in a plane containing bolt.
2. Eccentric loading perpendicular to the axis of bolts.

In the first case the bolts are subjected to shear.

Figure shows an example of eccentric loading in plain containing shear.





The bolts are subjected to two types of forces causing shear. One is force due to direct loading or the primary force.

$$P_1' = P_2' = P_3' = P_4'$$

$$= \frac{P}{n} \text{ where } n = \text{number of bolts}$$

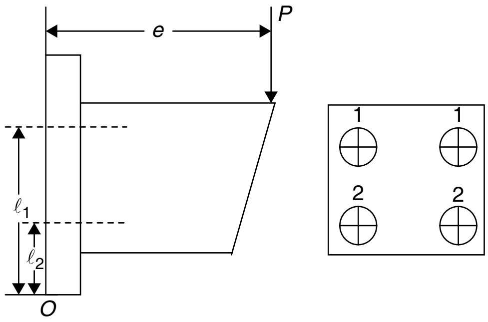
The second is the force due to the moment of the applied force about the centre of gravity of the bolt arrangement. This causes a secondary shear force on each bolt depending upon the distance from the centre of gravity. The secondary forces are P_1'', P_2'', P_3'' ….. etc such that $P_1'' \times r_1 + P_2'' \times r_2 + P_3'' \times r_3 + \ldots = P \times e$ and $P_1'' = Cr_1$, $P_2'' = Cr_2$, $P_3'' = Cr_3$ etc

$$\therefore P \times e = C(r_1^2 + r_2^2 + r_3^2 + \ldots)$$

$$\Rightarrow C = \frac{P \times e}{\sum r^2}$$

The design of the bolt is based on the bolt with maximum resultant force.

An example of eccentric load perpendicular to the axis of bolt is shown in below figure.



Direct shear force on bolts = $P_1' = P_2' = \dfrac{P}{n}$

where n = number of bolts (4 in this case)

A tensile force depending on the distance from the tilting edge O, acts on all bolts

$$P_1'' \propto l_1 \text{ and } P_2'' \propto l_2$$

$\therefore\ P_1'' = cl_1$

and $P_2'' = cl_2$

$$P_e = 2P_1'' \times l_1 + 2P_2'' \times l_2$$

$$= 2c\ell_1^2 + 2c\ell_2^2$$

$$\therefore \qquad C = \frac{Pe}{2\left(\ell_1^2 + \ell_2^2\right)}$$

and $P_1'' = \dfrac{Pel_1}{2\left(l_1^2 + l_2^2\right)}$

$$P_2'' = \frac{Pel_2}{2\left(l_1^2 + l_2^2\right)}$$

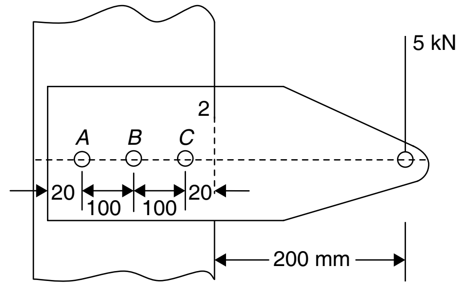
On bolts 1, shear stress $\tau = \dfrac{P_1'}{A}$

Tensile stress $\sigma_t = \dfrac{P_1''}{A}$

Principal stress $\sigma_1 = \dfrac{\sigma_t}{2} + \sqrt{\left(\dfrac{\sigma_t}{2}\right)^2 + \tau^2}$

Maximum shear stress $\tau_{max} = \sqrt{\left(\dfrac{\sigma_t}{2}\right)^2 + \tau^2}$

Direction for questions (Examples 29 and 30): A steel plate subjected to a force of 5 kN is fixed to a frame by means of 3 identical bolts as shown in the figure. The ultimate tensile stress of the bolt material is 400 N/mm^2 and factor of safety is 3.



(Dimensions are in mm)

Example 29: Resultant force on the maximum loaded bolt is

(A) 6525 N (B) 7820 N
(C) 8447 N (D) 9667 N

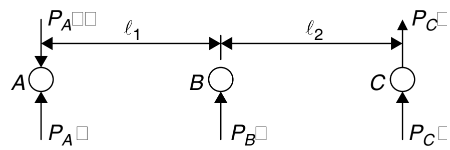
Solution:

Primary shear force

$$P_A' = P_B' = P_C' = \frac{5}{3} \text{ kN}$$

Secondary shear force $P_B' = 0$ as the bolt is at the centre of gravity.

The secondary shear force in A and C are equal but opposite in directions.



The primary and the secondary forces are in same direction in C.

$\therefore$ The maximum loaded bolt is C.

$$P_c'' = \frac{Pel_2}{\left(l_1^2 + l_2^2\right)}$$

$$= \frac{5\left(100 + 20 + 200\right) \times 100}{\left(100^2 + 100^2\right)}$$

$$= 8 \text{ kN}$$

Resultant force on $C = P_c'' + P_c''$

$$= \frac{5}{3} + 8$$

$$= 9\frac{2}{3} \text{ kN} = 9666.67 \text{ N}$$

Example 30: If the standard size of the bolt is 1.25 times the core diameter, the standard size is

(A) M8 (B) M16
(C) M18 (D) M24

Solution:

Permissible shear stress

$$\frac{S_{ut}}{2 \times \text{FOS}} = \frac{400}{2 \times 3} = \frac{200}{3} \text{ N/mm}^2$$

Let d_c be the core diameter of the bolt
Permissible stress × area = resultant force

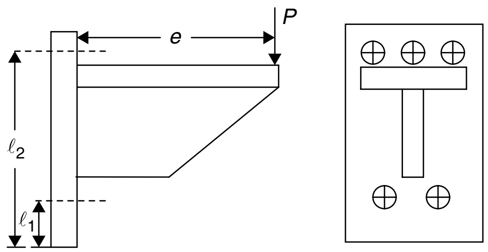
$$\frac{200}{3}\times\frac{\pi d_c^2}{4} = 9666.67$$

$\Rightarrow d_c = 13.59$ mm

Nominal diameter $d = 1.25 d_c$
$= 16.98$ mm

Nearest standard size above the value is M18 with 18 mm diameter.

Example 31: A bracket is fitted to a vertical channel with 5 bolts as shown in the figure.



Find the diameter of the bolts required for an applied load P = 15 kN and permissible shear stress 37 N/mm². The dimensions of the figure are

$$e = 200 \text{ mm} \quad \ell_1 = 50 \text{ mm} \quad \text{and} \quad \ell_2 = 250 \text{ mm}$$

Solution:

$$P\times e = 3P_2\,\ell_2 + 2P_1\,\ell_1$$
$$= 3k\ell_2^2 + 2k\ell_1^2$$

$$\therefore\ 15\times 10^3\times 200 = k\,(3\times 250^2 + 2\times 50^2)$$

$\Rightarrow k = 15.5844$

The maximum load is in the top row and

$$= k\ell_2$$
$$= 15.5844\times 250$$
$$= 3896.1 \text{ N}$$

Tensile stress = σ

$$= \frac{3896.1}{A}$$

where A = area of cross section of bolt

$$\text{Shear stress } \tau = \frac{15000}{5\times A}$$
$$= \frac{3000}{A}$$

$$\text{Maximum shear stress} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{3896.1}{2A}\right)^2 + \left(\frac{3000}{A}\right)^2}$$

$$= \frac{3577}{A}$$

= Permissible shear stress

$$\therefore\ \frac{3577}{A} = 37 \text{ N/mm}^2$$

$\Rightarrow A = 96.6757$ mm²

$$= \frac{\pi}{4}d_c^{\,2}$$

where d_c = core diameter

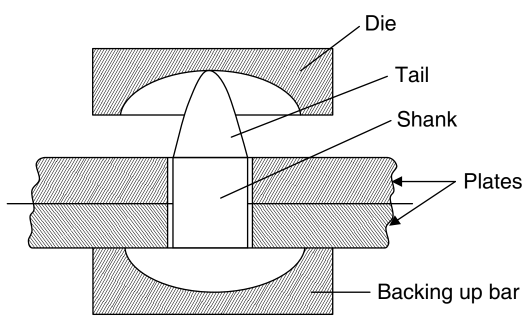
$\Rightarrow d_c = 11.1$ mm; But $d_c = 0.8$ d, where

d = nominal diameter

$$\therefore d = \frac{dc}{0.8} = \frac{11.1}{0.8} = 13.9 \text{ mm}$$
$$\simeq 14 \text{ mm}$$

Riveted Joints

A rivet is a short cylindrical shank with a head used for making permanent joints of metal plates. The shank is inserted into the drilled holes of the plates to be joined and the tail at the end of shank is hammered to form the closing head with the help of a die or snap.



Riveting may be done by hand or riveting machine.

Tough and ductile materials are used for riveting. They are usually made of low carbon steel or nickel steel. Brass, aluminium or copper rivets are also used. For fluid tight joints steel rivets are used.

Important Terminology Used

Pitch (p): It is the centre distance between two adjacent rivets measured parallel to the seam of the joint.

Back pitch: It is the perpendicular distance between centre lines of successive rows of rivets.

Diagonal Pitch (p_d): It is the centre distance between rivets in adjacent rows in zigzag riveting.

Margin or marginal pitch (m): It is the distance between centre of the rivet hole to the nearest edge of the plate.

Failure of a Riveted Joint

Failure of a riveted joint may occur in the following manner.

1. Tearing of the plate at the edge.
2. Tearing of the plate across a row of rivets.
3. Shearing of the rivets.
4. Crushing of the plate or rivet.

Tearing of plate at the edge can be avoided using sufficient marginal pitch. Usually a marginal pitch $m = 1.5\ d$ is provided

where d = diameter of hole

Tearing resistance across a row per pitch length

$$P_t = (p - d)\ t\sigma_t$$

where σ_t = permissible tensile stress

t = thickness of plate.

Shearing resistance of rivets per pitch length

$$P_s = n \times \frac{\pi d^2}{4} \times \tau$$

Where τ = permissible shear stress

n = number of rivets per pitch length.

In the case of double shear where cover plates are used,

$$P_s = n \times 2 \times \frac{d^2}{4} \times \tau$$

According to Indian Boiler Regulations (IBR)

$$P_s = n \times 1.875\ \frac{\pi d^2}{4} \times \tau \text{ for double shear.}$$

Crushing resistance per pitch length

$$P_c = ndt \times \sigma_c$$

Where σ_c = permissible crushing stress for rivet or plate materials.

Strength of a Riveted Joint

A riveted joint will fail if the applied force P is greater than P_t, P_s or P_c.

The strength of a riveted joint is the maximum force it can transmit without failing. So, the strength is the least of P_t, P_s or P_c.

For continuous joints, the strength is calculated per pitch length, but for small joints strength is calculated for the whole width of the plates.

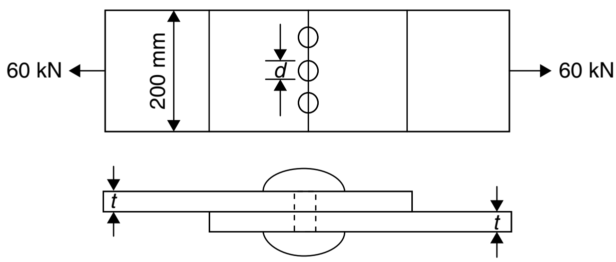
Efficiency of a Riveted Joint

Efficiency of a riveted joint is the ratio of the strength of the joint to the strength of the unriveted or solid plate.

i.e. $$\eta = \frac{\text{Least of } P_t, P_s \text{ or } P_c}{P \times t \times \sigma_t}$$

Example 32: A tensile force of 60 kN is acting on the riveted joint shown. The plates and rivets are made of material having yield tensile stress 260 N/mm^2. If factor of safety is 2.5, then determine the minimum

(a) diameter of the rivets.
(b) Thickness of plate.



Solution:

Given FOS = 2.5

$$P = 60 \text{ kN}$$

$$S_{yt} = 260 \text{ N/mm}^2$$

Permissible tensile stress $\sigma_t = \dfrac{S_{yt}}{\text{FOS}}$

$$= \frac{260}{2.5} = 104 \text{ N/mm}^2$$

Yield stress $S_{sy} = \dfrac{1}{2} S_{yt}$

$$= \frac{260}{2} = 130 \text{ N/mm}^2$$

Permissible shear stress

$$\tau = \frac{S_{sy}}{\text{FOS}} = \frac{130}{2.5} \text{ N/mm}^2 = 52 \text{ N/mm}^2$$

Total shear area $A_s = 3 \times \dfrac{\pi d^2}{4}$

$$\tau \times A_s = P$$

$$\therefore 52 \times \frac{3\pi d^2}{4} = 60 \times 10^3$$

$\Rightarrow d = 22.13$ mm ≈ 22 mm.

Considering tearing resistance of plate

$$(w - 3d)\ t \times \sigma_t = P$$

$$(200 - 3 \times 22) \times t \times 104 = 60{,}000$$

$$\Rightarrow t = 4.3 \text{ mm.}$$

Direction for questions (Examples 33 and 34): A single riveted joint is made from 18 mm thick plates. Rivet diameter and pitch are 22 mm and 70 mm respectively. Ultimate stresses in tension, shear and crushing are 500 MPa, 400 MPa and 600 MPa respectively.

Example 33: Find the minimum force per pitch, which will cause the failure of the joint.

Solution:

$n = 1$

$t = 18$ mm

$d = 22$ mm

$p = 70$ mm

$\sigma_{ut} = 500$ N/mm^2
$\tau_u = 400$ N/mm^2
$\sigma_{cu} = 600$ N/mm^2

Ultimate tearing resistance

$$P_{tu} = (p - d)\, t\, \sigma_{ut}$$
$$= (70 - 22) \times 18 \times 500$$
$$= 432{,}000 \text{ N}$$

Ultimate shearing resistance

$$P_{su} = 1 \times \frac{\pi d^2}{4} \times \tau_u$$
$$= \pi \times \frac{18^2}{4} \times 400$$
$$= 101{,}788 \text{ N}$$

Ultimate crushing resistance

$$P_{cu} = n \times dt \times \sigma_{cu}$$
$$= 1 \times 22 \times 18 \times 600$$
$$= 237{,}600 \text{ N/mm}^2$$

The minimum force which will cause failure is 101788 N or 101.788 kN

Example 34: Find the actual maximum tensile stress in the plate just before failure of the joint.

Solution:

The joint fails

when $P = 101.788$ kN

Let σ_t be the actual maximum tensile stress then,

$101.788 \times 10^3 = (70 - 22) \times 18 \times \sigma_t$

$\Rightarrow \sigma_t = 117.81$ N/mm^2

Welded Joints

Welded joints are permanent joints formed by welding of the metal parts.

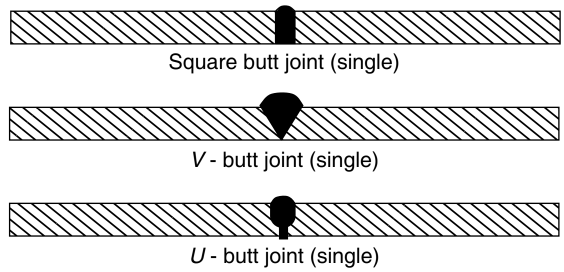
Welding is the process of joining metal parts by heating to plastic or liquid state at the regions where they are to be joined. In the first case, compressive forces are required for welding.

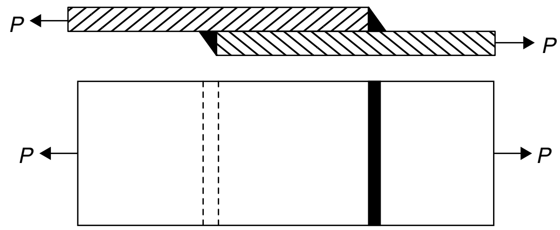
Depending up on the relative position of the parts to be welded, the welded joints can be classified as

1. Butt joint
2. Lap joint
3. T – joint
4. Corner joint
5. Edge joint, etc.

Butt joint is the joint between two components or plates lying in the same plane. In butt welding of thick plates (thickness above 5 mm), the ends of the plates are to be beveled to 'U' or 'V' shape. Single or double butt joints are used depending upon the thickness of plates. Square butt joints are used for plates of thickness less than 5 mm.

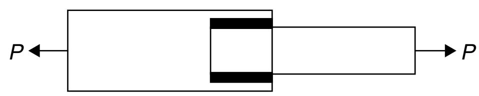
Lap joint is also known as fillet joint as welding forms fillets in this case. In this case, welding is done

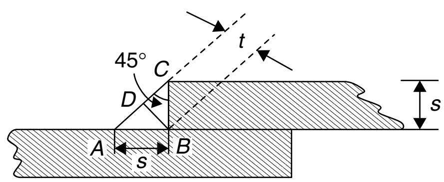




on two overlapping components or plates. Fillet welds can be transverse or longitudinal. In transverse fillet welds, the force is acting perpendicular to the weld directions.

In longitudinal or parallel fillet welds, the direction of force and welding are parallel to each other.





s = leg or size of weld

t = throat thickness (BD)

$= s \times \sin 45°$

$= 0.707$ s

Throat area = minimum area of weld

$= t \times$ length of weld

$= 0.707\, s\, \ell_1$

Tensile Strength of Transverse Welds

= throat area × allowable tensile stress

$= 0.707\, s\, \ell_1\, \sigma_t$

Strength of Parallel Fillet Welds

Parallel fillet welds are designed for shear strength.

Shear strength = throat area × allowable shear stress

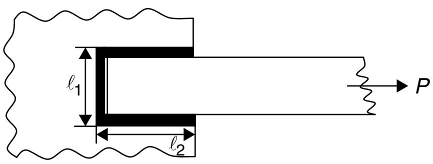
$$= 0.707\, s\, \ell_1\, \tau$$

Shear strength for double parallel fillet weld

$$= 2 \times 0.707\, s\, \ell_1\, \tau$$

$$= 1.414\, s\, \ell_1\, \tau$$

Combination of Parallel and Transverse Fillet Welds



Strength of a combination of transverse and parallel fillet welds as shown above is

$$P = 0.707\, s\, \ell_{t1}\, \sigma_t + 2 \times 0.707\, s\, \ell_{t2} \tau$$

Sometimes, transverse fillet welds are also treated as parallel fillet welds and designed for shear strength. In transverse fillet welds, normal, bending and shear stresses are acting. It is proved that the maximum shear stress is induced in a plane inclined 67.5° to the leg dimension. For any direction of the applied load, shear stress on the throat area can be assumed as the stress for design and the parallel fillet formula can be used. So $P = 0.707\, s \times (\ell_{t1} + 2\,\ell_{t2})\tau$

Welds Subjected to Bending

For welds subjected to bending, stress is calculated using the relation

$\sigma = \dfrac{M}{Z}$ where M = bending moment

Z = section modulus about neutral axis.

Shear stress

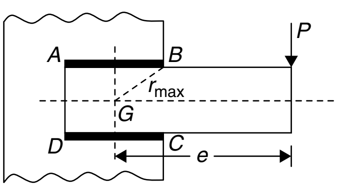
$$\tau = \frac{P}{A}$$

Maximum shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Welds Subjected to Torsion

Consider a welded bracket loaded as shown in below figure.



Owing to the loading the stresses are

$$\tau_1 = \text{direct shear stress}$$

$$= \frac{P}{A}$$

where A = area of weld

τ_2 = torsional shear stress

τ_2 is obtained from the relations

$$\frac{T}{J} = \frac{\tau}{r}$$

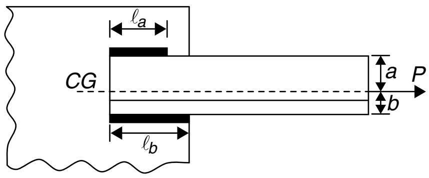
$$\tau_2 = \tau_{max} = \frac{T.r_{max}}{J}$$

where J = polar moment of inertia about G

Here, twisting moment $T = P \times e$

Axially Loaded Unsymmetrical Welded Sections

When unsymmetrical sections such as angles, channels, T – sections, etc. are welded on the flange edges and axially loaded, length of welds should be such that the sum of the resisting moments of the welds about the axial line passing through centre of gravity is zero.



Consider a welded angle loaded as shown above.

$$l_b \times f \times b = l_a \times f \times a = 0$$

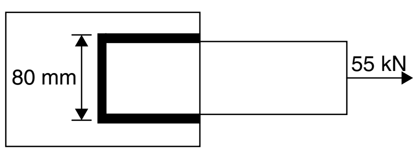
where f = resistance offered by the weld per unit length

$$\therefore\ l_a = l_b \times \frac{b}{a} = \frac{\ell b}{a+b}$$

$$\ell_b = \ell_a \times \frac{a}{b} = \frac{\ell a}{a+b}$$

where $\ell = \ell_a + \ell_b$

Example 35: A plate of 80 mm width and 10 mm thickness is welded to another plate using a single transverse weld and double parallel welds as shown in the figure. The joint is subjected to a maximum tensile force of 55 kN. The permissible tensile and shear stresses in the weld material are 70 MPa and 50 MPa respectively. Calculate the minimum length of each parallel fillet weld.



Solution:

$P = 55 \times 10^3$ N

$\sigma_t = 70$ N/mm²

$\tau = 50$ N/mm²

Plate thickness $h = 10$ mm

Length of transverse fillet weld

$\ell' = 80$ mm

Let the length of each parallel weld be ℓ_1

$0.707\, h\, \ell_1'\, \sigma_t + 2 \times 0.707\, h\, \ell\tau = 55 \times 10^3$

$\therefore 0.707 \times 10 \times 80 \times 70 + 1.414 \times 10 \times \ell_1 \times 50 = 55 \times 10^3$

$\Rightarrow 1.414 \times 500\ \ell_1 = 55 \times 10^3 - 39592$

$\therefore\ \ell = 21.8$ mm

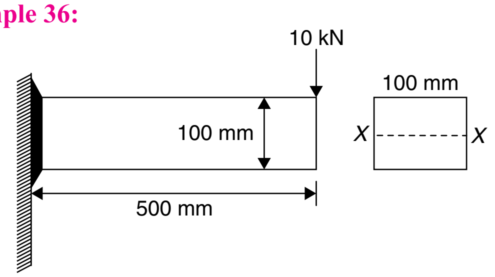
$\Rightarrow\ \ell = 22$ mm

[Assuming the same permissible tensile stress for the plate, tensile strength of plate = 80 × 10 × 70

= 56000 N

= 56 N > 55 kN So safe]

Example 36:



A square bar of size 100 mm × 100 mm is to be welded on 4 sides on a vertical surface and loaded as shown in the figure. Find the leg size of the weld. The permissible shear stress is 75 MPa.

Solution:

$P = 10$ kN

$\tau = 75$ N/mm²

Area of weld $= t \times 100 \times 4 \times 0.707$

$= 400 \times 0.707\, t,$

where t = leg size of weld

Direct shear stress $\tau = \dfrac{10 \times 10^3}{0.707 \times 400t}$

$= 35.4$ N/mm²

Bending moment $M = 10 \times 500 \times 10^3$

$= 5000 \times 10^3$ N mm

Moment of inertia

$$I_{xx} = \left[bt \times \left(\frac{b}{2}\right)^2 \times 2 + \frac{tb^3}{12} \times 2\right] 0.707$$

$$= \left(\frac{tb^3}{2} + \frac{tb^3}{6}\right) 0.707$$

$$= 0.707 \times \frac{2}{3} tb^3 \text{ where}$$

t = size of weld

$$Z_{xx} = \frac{I_{xx}}{\frac{b}{2}}$$

$$= 0.707 \times \frac{2}{3} \times tb^3 \times \frac{2}{b}$$

$$= \frac{4}{3} \times tb^2 \times 0.707$$

$$= \frac{4}{3} \times (100)^2 \times 0.707t$$

$$= 9427\, t$$

Bending stress

$$\sigma_b = \frac{M}{Z}$$

$$= \frac{5000 \times 10^3}{9427t}$$

$$= \frac{530.4}{t} \text{ N/mm}^2$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

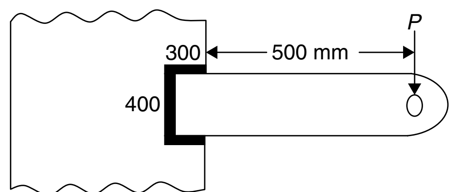
$$= \frac{1}{t}\sqrt{\left(\frac{530.4}{2}\right)^2 + 35.4^2}$$

$$= \frac{267.6}{t}$$

$= 75$ N/mm²

$\Rightarrow t = 3.6$ mm or 4 mm

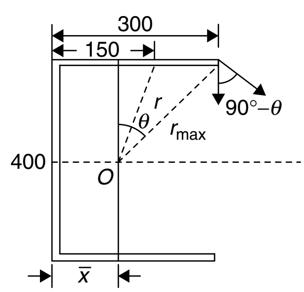
Example 37:



(Dimensions are in mm)

In the figure given above, the throat of the weld is 3 mm and the permissible shear stress is 75 MPa. Find the maximum value of P.

Solution:



$$\bar{x} = \frac{[300\times 2\times 150]}{(300\times 2)+400}$$

$$= 90 \text{ mm}$$

$$r = \sqrt{200^2+(150-90)^2}$$

$$= 208.8 \text{ mm}$$

Polar moment of inertia,

$$J = \left[\frac{1}{12}\times 400^3 + \frac{1}{12}\times 2\times 300^3 + 400\times 90^2 + 2\times 300\times (208.8)^2\right]\times 3$$

$$= 39231797 \times 3 \text{ mm}^4$$

$$r_{\max} = \sqrt{200^2+(300-90)^2}$$

$$= 290 \text{ mm}$$

Torsional shear stress

$$\tau_1 = \frac{T\times r_{\max}}{J}$$

$$= \frac{P\times e\times r_{\max}}{J}$$

$$= P\frac{(800-90)\times 290}{39231797\times 3}$$

$$= 1.7494 \times 10^{-3} \text{ P}$$

Direct shear stress

$$\tau_2 = \frac{P}{3[300+300+400]}$$

$$= \frac{P}{3000}$$

$$= 3.33 \times 10^{-4} \text{ P}$$

$$= 0.333 \times 10^{-3} \text{ P}$$

$$\tan\,\theta = \frac{(300-90)}{200}$$

$\Rightarrow \theta = 46.4°$; $\cos(90° - \theta) = 0.7242$

Resultant shear stress

$$= P\sqrt{\tau_1{}^2+\tau_2{}^2+2\tau_1\tau_2\cos(90°-\theta)}$$

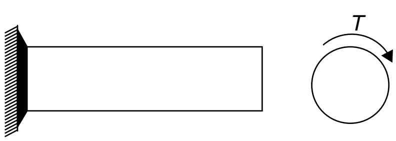
$$= P\sqrt{3.1712\times 10^{-6}+0.844\times 10^{-6}}$$

$$= P\sqrt{4.015\times 10^{-6}}$$

$$= 2.004 \times 10^{-3} \text{ P} = 75 \text{ MPa}$$

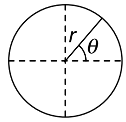
$\Rightarrow P = 37{,}425\ N = 37.425$ kN

Example 38:



A cylindrical beam of diameter d is welded as shown in the figure and a torsional moment T is applied. What is the maximum stress induced in the weld?

Solution:



Consider an element of weld of length $rd\theta$

Area of weld = 0.707 h $rd\theta$

Resistance offered by the element

$$= \tau \times 0.707 \text{ h } r\, d\theta$$

$\therefore$ Torsional moment

$$T = \int_0^{2\pi} \tau\times 0.707\,\text{h}\, r^2 d\theta$$

$$= \tau r^2 \times 0.707 \text{ h} \times 2\pi$$

$$= 1.414\ \pi r^2 h \times \tau$$

$$\Rightarrow \tau = \frac{T}{1.414\pi r^2 h} \text{ i.e. } \tau = \frac{2.83T}{\pi d^2 h}.$$

Example 39: If the beam with circular fillet weld as shown in the example 5.38 is subjected to a bending moment M at the end, what is the maximum bending stress induced in the weld?

Solution:

h = size of the weld

t = throat thickness

Section modulus of the weld section

$$= \frac{\pi t d^2}{4}$$

Bending stress

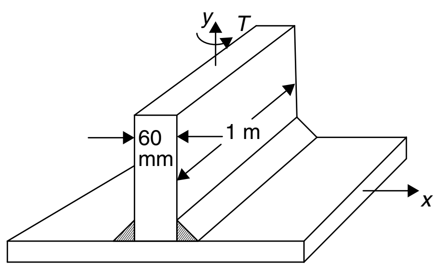
$$\sigma_b = \frac{M}{Z} = \frac{M}{\left(\pi\frac{td^2}{4}\right)} = \frac{4M}{\pi t d^2}$$

$$= \frac{4M}{\pi(0.707h)d^2}$$

i.e.

$$\sigma_b = \frac{5.66\,M}{\pi h d^2}.$$

Example 40: A plate 1 m long 60 mm thick is welded to another plate at right angles to each other by 12 mm fillet weld as shown in figure. If the permissible shear stress intensity in the weld material is 75 N/mm^2, the maximum torque τ the weld can sustain is (kN m)



Solution:

Polar moment of inertia of the weld about CG

$$J = \frac{2 \times tL^3}{12} = \frac{tL^3}{6} \text{ (} t = \text{throat thickness)}$$

Maximum shear stress

$$\tau = \frac{T \times \left(\frac{L}{2}\right)}{J} \text{ where } T = \text{Torque}$$

$$= \frac{T \times \frac{L}{2}}{t\frac{L^3}{6}}$$

$$= \frac{3T}{tL^2}$$

$$= \frac{3T}{(0.707s)L^2}$$

i.e. $\tau = \dfrac{4.242T}{sL^2}$ where s = size of weld

$\therefore 75 = \dfrac{4.242T}{12 \times (1000)^2}$ where T is in N mm

$\Rightarrow T = 212$ kN m

Exercises

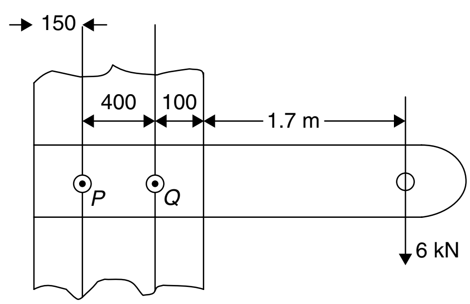
Practice Problems I

1. If load on a ball bearing is reduced $\frac{1}{4}$ times, its life will increase to
   (A) 8 times (B) 16 times
   (C) 32 times (D) 64 times
2. A natural feed journal bearing of diameter 55 mm and length 55 mm operating at 20 revaluations/sec. carries a load of 2.5 kN.
   The lubricant used has a viscosity of 20 MPas. The radial clearance is 50 μm. The Sommerfeld number for the bearing is
   (A) 0.62 (B) 0.146
   (C) 0.250 (D) 0.785
3. Two parallel shafts are 330 mm apart. Power is transmitted from one shaft to the other by gear drive. The pinion runs $\frac{8}{3}$ times as fast as the wheel. If the module of the meshing gears is 6 mm, then the number of teeth in the pinion is
   (A) 30 (B) 45
   (C) 60 (D) 80

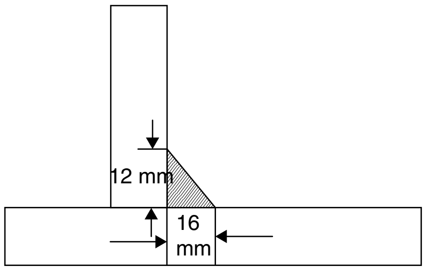
Direction for questions 4 and 5: A 20° full depth involute spur pinion of 4 mm module and 21 teeth is to transmit 15 kW at 1000 rpm. Its face width is 25 mm.

4. The tangential force transmitted (in *N*) is
   (A) 3552 (B) 2611
   (C) 4550 (D) 1305
5. The tooth geometry factor is 0.32 and combined effect of dynamic load and allied factors intensifying the stress is 1.5. The minimum allowable stress (in MPa) for the gear material is
   (A) 242.0 (B) 166.5
   (C) 213.2 (D) 74.0
6. A band brake having band width of 80 mm, drum diameter of 300 mm, coefficient of friction 0.25 and angle of wrap of 270 degrees is required to exert a friction torque of 1500 Nm. The maximum tension (in kN) developed in the band is
   (A) 1.88 kN (B) 14.45 kN
   (C) 6.12 kN (D) 17.56 kN
7. The dynamic load carrying capacity of a roller bearing is 20 kN. The desired life for 90% survival of the bearing is 8000 hours at a speed of 600 rpm. The equivalent radial load the bearing can carry is
   (A) 3.658 kN (B) 4.923 kN
   (C) 5.168 kN (D) 6.734 kN

Direction for questions 8 and 9: A steel bar of 10 × 50 mm is cantilevered with two M12 bolts (*P* and *Q*) to support static load of 6 kN as shown in the figure.



8. The secondary shear load on bolt *P* is
   (A) 25 kN (B) 15 kN
   (C) 30 kN (D) 20 kN
9. The resultant shear stress on bolt *P* is closest to
   (A) 247.6 MPa (B) 159 MPa
   (C) 238.7 MPa (D) 195 MPa
10. In a plate clutch, the axial force is 4 kN. The inside radius of contact surface is 50 mm and the outside radius is 100 mm. For uniform pressure, the mean radius of friction surface will be
   (A) 78 mm (B) 60 mm
   (C) 75 mm (D) 80 mm
11. What is the throat length in this fillet weld joint?.



(A) 10 mm (B) 20 mm
(C) 9.6 mm (D) 12 mm

12. A disc clutch is required to transmit 6 kW at 2500 rpm. The disk has a friction lining with coefficient of friction equal to 0.25. The bore radius of friction lining is equal to 25 mm. Assume uniform contact pressure of 2 MPa. The value of outside radius of friction lining is
(A) 39.4 mm (B) 33.47 mm
(C) 97 mm (D) 142.9 mm

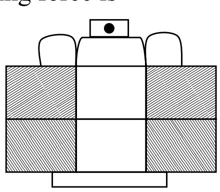
13. Square key of side '$d/4$' each and length $2\,\ell_1$ is used to transmit torque 'S' from the shaft of diameter d to the hub of a pulley. Assuming the length of the key to be equal to the thickness of the pulley, the average shear stress developed in the key is given by.
(A) $\dfrac{4S}{2\ell d}$ (B) $\dfrac{16S}{2\ell d^2}$
(C) $\dfrac{8S}{2\ell d^2}$ (D) $\dfrac{16S}{\pi d^3}$

14. In a band brake the ratio of tight side band tension to the tension on the slack side is 3. If the angle of overlap of band on the drum is 180°, the coefficient of friction required between drum and the band is.
(A) 0.20 (B) 0.25 (C) 0.30 (D) 0.35

15. Which one of the following is the criterion in the design of hydrodynamic journal bearings?
(A) Sommerfeld number
(B) Rating life
(C) Specific dynamic capacity
(D) Rotation factor

16. Two mating spur gears have 50 and 120 teeth respectively. The pinion rotates at 1100 rpm and transmit a torque of 20 N m. The torque transmitted by gear is.
(A) 6.6 N m (B) 48 N m
(C) 40 N m (D) 60 N m

17. The joint shows two members connected with an axial tightening force of 2500 N. If the bolt used has metric threads of 4 mm pitch, the torque required for achieving the tightening force is



(A) 0.7 N m (B) 1.59 N m
(C) 1.4 N m (D) 2.8 N m

18. A 70 mm long and 8 mm size fillet weld carries a steady load of 15 kN along the weld. The shear strength of the weld material is equal to 200 MPa. The factor of safety is
(A) 2.4 (B) 5.28
(C) 4.8 (D) 6.8

19. A cylindrical shaft is subjected to an alternating stress of 100 MPa. The fatigue strength to sustain 1000 cycles is 500 MPa. If the corrected endurance strength is 80 MPa, the estimated shaft life in cycles will be
(A) 426579 (B) 1500
(C) 281914 (D) 928642

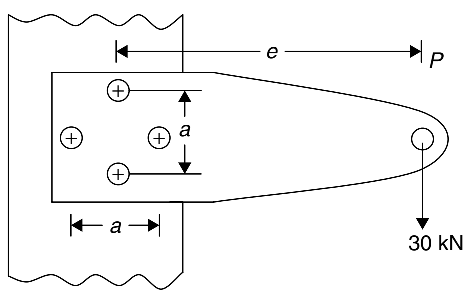
20. A wire rope is designated as 6 × 19 standard hoisting. The numbers 6 × 19 represent
(A) Diameter is mm × length mm
(B) Diameter is cm × length mm
(C) No. of strands × no of wires in strand
(D) No. of wires in each strand × no of strands

21. A cast iron pulley is used to transmit power of 30 kW at 300 rpm through a shaft of 55 mm diameter. Assuming the width of the key as $\dfrac{d}{4}$, find (in mm) the length and thickness of the key (The yield strength and the crushing strength of the key material are 330 MPa and 630 MPa, respectively. Assume FOS = 4)

22. A shaft is to be used for transmitting a power of 25 kW at 1500 rpm. A bending moment of 100 Nm is also expected on the shaft. Shock and fatigue factors are $k_m = 1.5$ and $k_t = 1.2$. Determine (in mm) the minimum diameter of the shaft. The tensile yield strength for the material is 300 MPa. A factor of safety of 3 may be assumed.

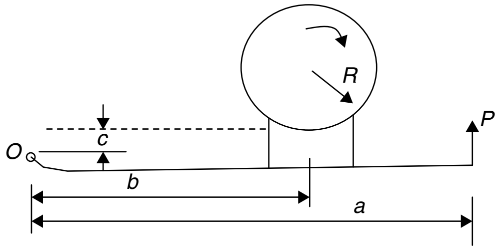
23. A 50 mm diameter shaft is subjected to a shear stress of 40 MPa. Find the length of the shaft (in mm) for an angle of twist of 0.01 radian. Modulus of rigidity for shaft material is 0.8×10^5 MPa.

24.



A bracket is fitted to the channel and loaded as shown in the figure. Dimensions a and e are 150 mm and 300 mm, respectively. Determine the minimum diameter of the bolts (in mm).
(Permissible shear stress = 37.5 MPa)

Direction for questions 25 and 26:



The drum of a block brake rotates clockwise at 600 rpm. The torque capacity is 250 Nm and coefficient of friction is 0.3. The dimensions are as given below:

$R = 400$ mm

$a = 1$ m

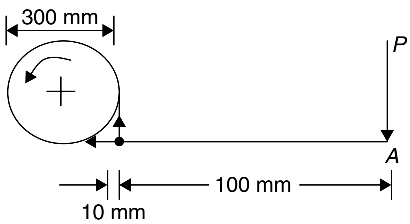
$b = 400$ mm

$c = 50$ mm

25. Find the force P (in N) to be applied at the end of the lever.

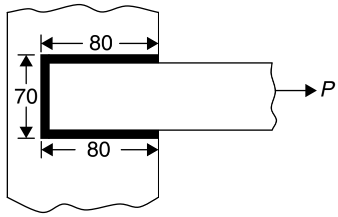
26. If the direction of rotation of the drum is anticlockwise, find the force P (in N) required.

27.



For the band brake shown in the figure, determine the maximum force P in N required for anticlockwise rotation of the drum at 500 rpm absorbing 30 kW power. ($\mu = 0.3$)

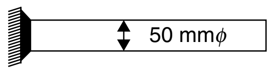
28.



(All dimensions are in mm)

For the welded joint as shown in the figure, determine the minimum leg size of the weld. (Load $P = 60$ kN and allowable shear stress = 80 MPa)

Direction for questions 29 and 30: A cylindrical beam of 50 mm diameter is welded at one end to form a cantilever as shown in the figure.



The size of the weld is 4 mm.

29. If allowable tensile stress of the weld is 80 N/mm^2, the maximum bending moment (in Nm) that can be applied is

(A) 298 Nm (B) 354 Nm

(C) 444 Nm (D) 486 Nm

30. If the allowable shear stress is 70 N/mm^2, the maximum torque (in Nm) that can be applied is

(A) 777 Nm

(B) 628 Nm

(C) 582 Nm

(D) 546 Nm

Practice Problems 2

1. Two shafts A and B are made of the same material. The diameter of shaft B is 1.5 times that of shaft A. The ratio of power that can be transmitted by shaft A to that of shaft B is

(A) $\frac{2}{3}$ (B) $\frac{8}{27}$

(C) $\frac{4}{9}$ (D) $\frac{4}{18}$

2. A cotter joint is used to transmit

(A) Axial tensile force.

(B) Axial tensile or compressive force.

(C) Axial compressive force.

(D) Combined bending and torsional moments.

3. The dynamic load bearing capacity of a ball bearing is 20 kN. The maximum radial load it can sustain for operating at 600 rpm for 2200 hours is

(A) 4.66 kN (B) 4.92 kN

(C) 5.17 kN (D) 5.84 kN

4. A stress that varies in sinusoidal manner with respect to time from zero to maximum value and which has same values of amplitude about the mean value is

(A) Reversed stress (B) Fluctuating stress

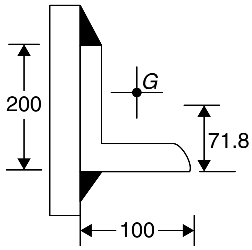
(C) Repeated stress (D) Varying stress

5. The shaft diameter and the length of a journal bearing are 50 mm and 50 mm, respectively. The shaft has an angular velocity of 20 rad/s and the viscosity of lubricant is 20 mPa s. If radial clearance is 0.020 mm, the torque loss due to viscosity of the lubricant is approximately,

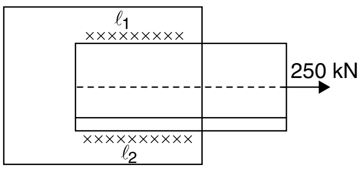
(A) 0.042 Nm (B) 0.067 Nm

(C) 0.098 Nm (D) 0.083 Nm

6. An ISL $200 \times 100 \times 10$ angle is welded to a steel plate by means of fillet welds as shown in the figure. The angle is subjected to a static force of 250 kN and the permissible shear stress for the weld is 70 N/mm^2. The lengths of the weld at the top and bottom are (ℓ_1 and ℓ_2) in mm are respectively



(All dimensions in mm)

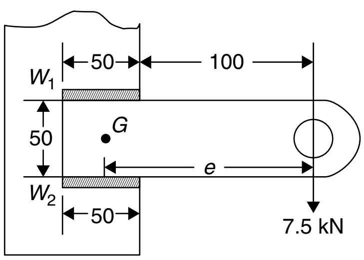


(A) 108.8 and 194.3 (B) 120 and 180
(C) 130 and 185 (D) 181.35 and 323.8

7. A 65 mm diameter solid shaft is to be welded to a flat plate by fillet weld around the circumferences of the shaft. If the torque on the shaft is 3 kN m and allowable shear stress in the weld is 70 MPa, the size of the weld is nearly

(A) 10 mm (B) 15 mm (C) 12.5 mm (D) 18 mm

8. An eccentric force of 7.5 kN is acting as shown below. The permissible shear stress for the weld is 100 N/mm^2. The resultant shear stress is $\tau = \left(\dfrac{454}{t}\right)$ N/mm^2, where 't' is the thickness of the plate. Assume static condition. Then, the size of the weld is nearly



(A) 7 mm (B) 8 mm (C) 6 mm (D) 5 mm

9. In spur gears, the circle on which the involute is generated is called the

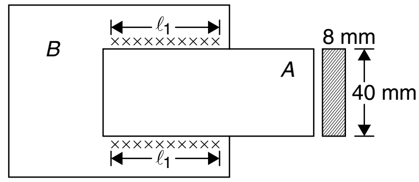
(A) pitch circle (B) base circle
(C) addendum circle (D) dedendum circle

10. A double fillet welded joint with parallel fillet weld of length 'ℓ_1' and size 's' is subjected to a tensile load 'P'. Assuming uniform stress distribution, the shear stress induced in the weld is

(A) $\dfrac{\sqrt{2}P}{s\ell}$ (B) $\dfrac{P}{2s\ell}$

(C) $\dfrac{P}{\sqrt{2}s\ell}$ (D) $\dfrac{2P}{s\ell}$

Direction for questions 11 and 12: A plate A is welded on a plate B as shown in the figure using full size longitudinal fillet welds. The ultimate tensile stress of the plate material is 400 N/mm^2. The permissible shear stress of the weld material 75 MPa. The cross section of the plate A is 40 mm × 8 mm



11. The length of weld $\ell_{1\,1}$ for maximum loading on B is

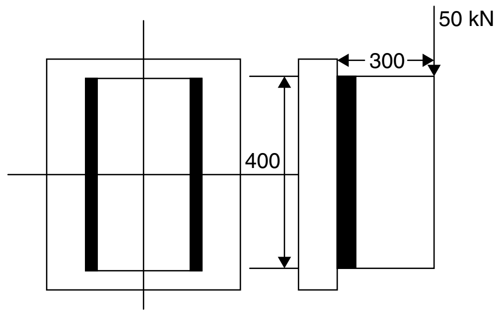
(A) 141.5 mm (B) 150.3 mm
(C) 159.3 mm (D) 163.5 mm

12. If a transverse weld alone is provided along the 40 mm side, the load plate A can take is

(A) 16.97 kN (B) 14.82 kN
(C) 17.35 kN (D) 15.39 kN

Direction for questions 13 to 16: A bracket is welded with two fillet welds as shown. If the permissible shear stress is limited to 100 N/mm^2, determine (given t = throat size of weld in mm).

13. The primary shear stress



(All dimensional are in mm)

(A) $86.1/t$ N/mm^2 (B) $62.5/t$ N/mm^2
(C) $75.0/t$ N/mm^2 (D) $50.1/t$ N/mm^2

14. The bending stress

(A) $281.16/t$ N/mm^2 (B) $201/t$ N/mm^2
(C) $181/t$ N/mm^2 (D) $262/t$ N/mm^2

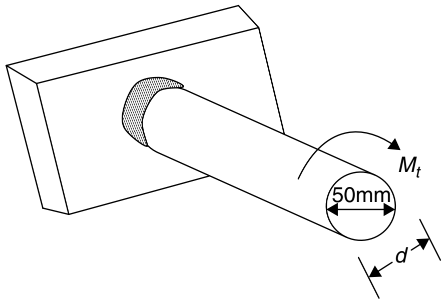
15. The maximum shear stress

(A) $163.1/t$ N/mm^2 (B) $123.1/t$ N/mm^2
(C) $182.1/t$ N/mm^2 (D) $153.85/t$ N/mm^2

16. The size of the weld

(A) 5 mm (B) 2.5 mm
(C) 4 mm (D) 3 mm

Direction for questions 17 and 18: A 50 mm diamter circular shaft is welded by means of circumferential fillet weld and joined to the plate as in figure. It is subjected to torsional moment of 2500 Nm. The permissible shear stress in the weld is limited to 140 N/mm^2. (t = throat size of weld in mm)



17. Torsional shear stress is

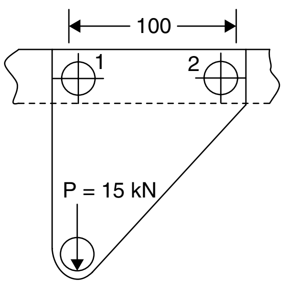
(A) $600/t$ (B) $\frac{637}{t}$

(C) $\frac{555}{t}$ (D) $737/t$

18. Size of weld is (leg size)

(A) 7 mm (B) 8 mm (C) 9 mm (D) 6 mm

Direction for questions 19 to 22: A riveted joint, consisting of two identical rivets is subjected to an eccentric force of 15 kN as shown. The permissible shear stress is 60 N/mm^2. Then,



(All dimensions are in mm)

19. Primary shear force is

(A) 7500 N (B) 8200 N

(C) 6580 N (D) 5580 N

20. Secondary shear force is

(A) 7500 (B) 5580

(C) 6580 (D) 8200 N

21. Resultant shear force is

(A) 14000 N (B) 15000 N

(C) 16000 N (D) 18000 N

22. Diameter of rivets is

(A) 13 mm (B) 15 mm

(C) 18 mm (D) 70 mm

23. A circular steel bar 50 mm diameter and 200 mm long is welded perpendicularly to a steel plate to form a cantilever to be loaded with 5 kN at the free end. Assuming the allowable stress in the weld to be 100 MPa, the size of the weld required is

(A) 10 mm (B) 7.2 mm

(C) 6 mm (D) 4.8 mm

24. The size of gear is usually specified by

(A) Pressure angle

(B) Pitch circle diameter

(C) Diametral pitch

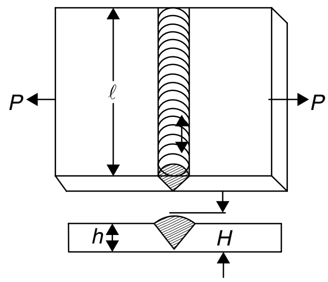
(D) Circular pitch

25. A full penetration butt welded joint, subjected to tensile force P is shown in the given figure,

ℓ_1 = length of the weld (mm) h = thickness of plate in (mm) and H is the total 'height' of the weld including reinforcement The average tensile stress in the weld is.

(A) $\sigma_1 = \frac{P}{H\ell}$ (B) $\sigma_1 = \frac{P}{h\ell}$

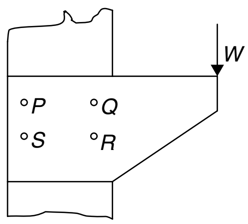
(C) $\sigma_1 = \frac{P}{2hL}$ (D) $\sigma_1 = \frac{2P}{h\ell}$



26. An eccentrically loaded rivet joint is shown in figure with 4 rivets at P, Q, R and S. Which of the rivets is the most loaded

(A) P and Q (B) Q and R

(C) R and S (D) S and P

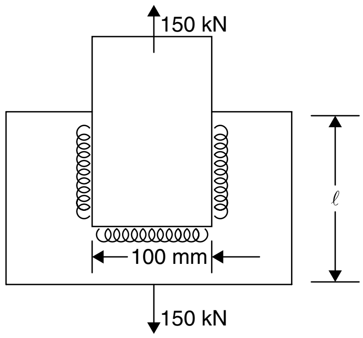


27. Double fillet welding joint with parallel fillet weld of total length L and leg size B is subjected to a tensile force P. Assuming uniform stress distribution, the shear stress of welding is given by

(A) $\frac{\sqrt{2}P}{BL}$ (B) $\frac{P}{2BL}$

(C) $\frac{P}{\sqrt{2}BL}$ (D) $\frac{2P}{BL}$

28. Two plates are joined together by means of single transverse and double parallel fillet weld as shown. The size of the fillet is 5 mm and the allowable shear load is 300 N/mm. What is the approximate length of each parallel fillet?



(A) 150 mm (B) 200 mm
(C) 250 mm (D) 300 mm

29. The following figures show welded joints for the same load and the same dimension of plate and weld.

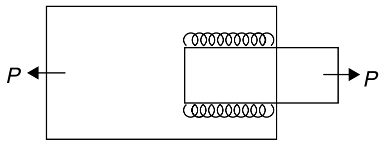


Figure (i)

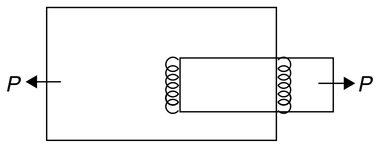
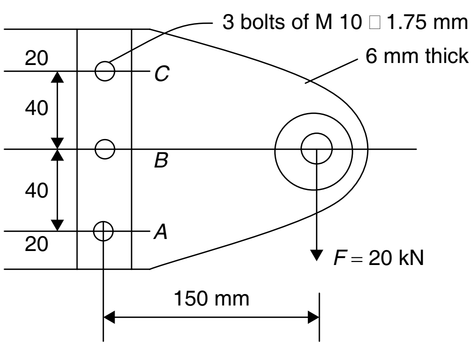


Figure (ii)

The joint shown in
(A) Fig. (i) is better because the weld is not in line with P
(B) Fig. (i) is better because the load transfer from the bar to the plate is not direct.
(C) Fig. (ii) is better because the strength of the weld in tension is greater than in shear.
(D) Fig. (ii) is better because length of weld is less

30.



(All dimensions are in mm)

The maximum shear stress in MPa in the bolts at A and B are (core diameter = 0.84 times nominal diameter)
(A) 242.6, 42.5
(B) 42.5, 242.6
(C) 42.5, 42.5
(D) 687.3, 120.3

31. Module of a gear is

(A) $\dfrac{1}{\text{Pitch circle diameter (in mm)}}$

(B) $\dfrac{\text{Number of teeth}}{\text{Pitch circle diameter (in mm)}}$

(C) Pitch circle diameter × number of teeth

(D) $\dfrac{\text{Pitch circle diameter (in mm)}}{\text{Number of teeth}}$

32. When the equivalent radial load on a ball bearing is doubled, its life will become.

(A) $\frac{1}{8}$ times (B) $\frac{1}{4}$ times

(C) $\frac{1}{2}$ times (D) $\frac{1}{16}$ times

33. A differential band brake becomes self locking if the band tensions ratio $\frac{T_1}{T_2}$ is equal to

[Given: T_1 = Tension on tight side
T_2 = Tension on slack side
a = lever arm length on tight side
b = lever arm length on slack side]

(A) $\frac{a}{b}$ (B) $\frac{b}{a}$

(C) $\frac{2b}{a}$ (D) $\frac{a}{2b}$

34. Design of welds is generally based on
(A) shear strength
(B) tensile strength
(C) compressive strength
(D) combination of tensile and shear strength

35. The tearing efficiency of a riveted joint is 70%. Then, the ratio of diameter of rivet hole to pitch of rivets is
(A) 0.15 (B) 0.2
(C) 0.3 (D) 0.7

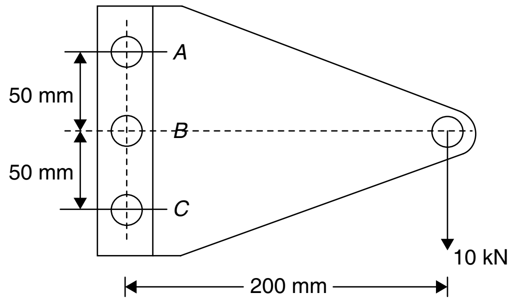
36. The inside diameter of a hollow shaft is $\frac{1}{3}$ of its outside diameter. The ratio of its torque carrying capacity to that of a solid shaft of the same outside diameter and the same material is

(A) $\frac{15}{16}$ (B) $\frac{31}{32}$

(C) $\frac{1}{16}$ (D) $\frac{80}{81}$

37. A fillet weld, 50 mm long and 6 mm size carries a steady load of 12 kN along the weld. If the shear strength of the weld material is 200 MPa, the factor of safety is
(A) 4.2 (B) 3.8
(C) 3.5 (D) 3.2

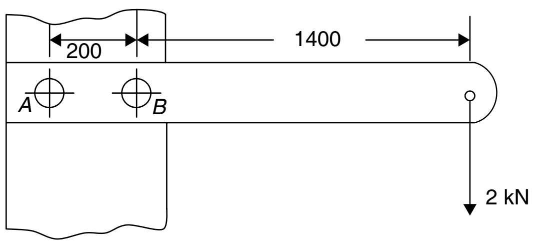
38.



A bracket is fitted with 3 bolts of nominal diameter 10 mm and core diameter 8.4 mm as shown in figure. The maximum shear stress induced in bolt *A* and *B* due to the applied load 10 kN, respectively are (in MPa)

(A) 366, 60.16
(B) 382, 68.32
(C) 403, 78.48
(D) 425, 93.64

Direction for questions 39 and 40:



(All dimensions in mm)

A cantilever beam of 10 mm thickness is bolted at the end by using two 12 mm diameter bolts *A* and *B* as shown in the figure and a load of 2 kN is applied at the free end.

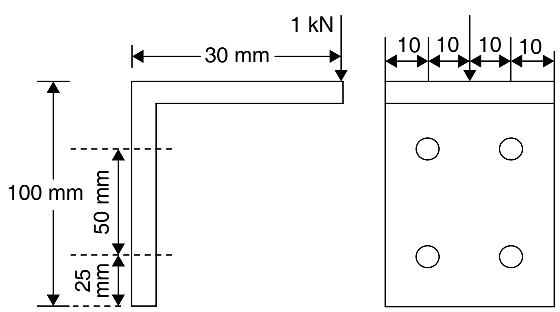
39. Value of secondary shear load on bolt *A* is

(A) 12 kN (B) 15 kN
(C) 18 kN (D) 22 kN

40. Resultant shear stress on bolt *A* in MPa is

(A) 98.7 (B) 110.3
(C) 123.8 (D) 135.6

Direction for questions 41 to 43:



A bracket is rigidly mounted on a wall using 4 numbers 10 mm diameter bolts as shown in the figure.

41. Direct shear stress (in MPa) in the most heavily loaded bolt is

(A) 3.183 (B) 4.623
(C) 6.336 (D) 8.424

42. Tensile stress induced in the most heavily loaded bolt (in MPa) is

(A) 82.76 (B) 96.48
(C) 112.36 (D) 143.24

43. Maximum shear stress induced in the most heavily loaded bolt (in MPa) is

(A) 3.183 (B) 6.336
(C) 35.85 (D) 71.69

44. Two bolts of same material and equal length are subjected to identical tensile load. The strain energy stored in the first bolt is three times the strain energy stored in the second bolt. If the mean diameter of the first bolt is 10 mm, determine the mean diameter of the second bolt (in mm)

Direction for questions 45 and 47: Two plates, each of thickness 6 mm and width 260 mm, respectively are joined by 4 rivets as shown in the figure. (single row lap joint). Other details of the riveted joint are

Diameter of rivet = 10 mm

Diameter of hole = 11 mm

Allowable tensile stress of the plate

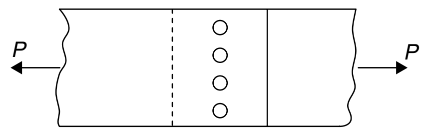
= 200 MPa

Allowable shear stress of the rivet

= 100 MPa

Allowable bearing stress of the rivet

= 150 MPa



45. If the rivets are to be designed to avoid shearing failure, the maximum permissible load *P* in kN is

(A) 26.6 (B) 31.4
(C) 36.8 (D) 42.4

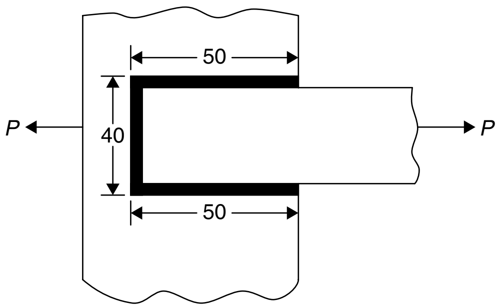
46. If the rivets are to be designed to avoid crushing failure, maximum permissible load *P* in kN is

(A) 36 (B) 32
(C) 28 (D) 26

47. If the plates are to be designed to avoid tearing failure, maximum permissible load *P* in kN is

(A) 143.7 (B) 152.4
(C) 202.6 (D) 259.2

48.



(All dimensional are in mm)

For the welded joint shown in figure, size of the weld is 4 mm and the allowable shear stress of the weld material is 60 MPa. Determine the strength of the weld joint in kN.

49. A pinion of 18 teeth is in mesh with a 38 teeth gear. If the pinion and gear are 20° full depth profiled with module 5 mm, the centre distance between the gear and pinion is

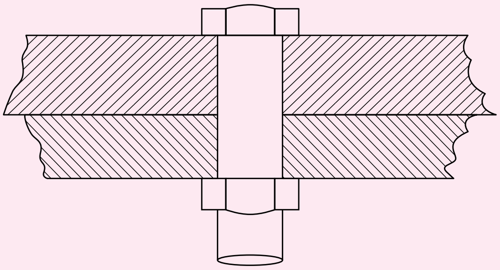
(A) 120 mm (B) 130 mm
(C) 140 mm (D) 150 mm

50. The journal diameter of a full journal bearing is 60 mm. The diameter of the bush bore is 60.06 mm and bush length is 30 mm. If the journal rotates at 1500 rpm and the average viscosity of the lubricant is 0.03 Ns/m^2, the power loss will be

(A) 60.2 W (B) 90.7 W
(C) 102.8 W (D) 125.5 W

Previous Years' Questions

1. In a bolted joint, two members are connected with an axial tightening force of 2200 N. If the bolt used has metric threads of 4 mm pitch, the torque required for achieving the tightening force is **[2004]**



(A) 0.7 Nm (B) 1.0 Nm
(C) 1.4 Nm (D) 2.8 Nm

2. A solid circular shaft of 60 mm diameter transmits a torque of 1600 N.m. The value of maximum shear stress developed is **[2004]**

(A) 37.72 MPa (B) 47.72 MPa
(C) 57.72 MPa (D) 67.72 MPa

3. Match the following:

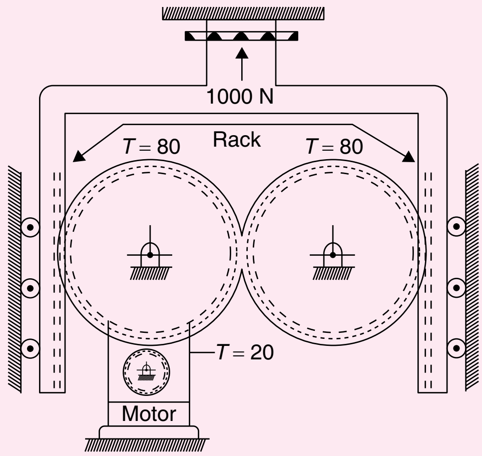
Type of gears	Arrangement of shafts
P. Bevel gears	**1.** Non–parallel off-set shafts
Q. Worm gears	**2.** Non-parallel Intersecting shafts
R. Herringbone gears	**3.** Non – Parallel, non – intersecting shafts
S. Hypoid gears	**4.** Parallel shafts

[2004]

(A) P-4 Q-2 R-1 S-3 (B) P-2 Q-3 R-4 S-1
(C) P-3 Q-2 R-1 S-4 (D) P-1 Q-3 R-4 S-2

Direction for questions 4 and 5: Solve the problems and choose the correct answers.

A compacting machine shown in the figure below is used to create a desired thrust force by using a rack and pinion arrangement. The input gear is mounted on the motor shaft. The gears have involutes teeth of 22 mm module.



4. If the drive efficiency is 80%, the torque required on the input shaft to create 1000 N output thrust is **[2004]**

(A) 20 Nm (B) 25 Nm
(C) 32 Nm (D) 50 Nm

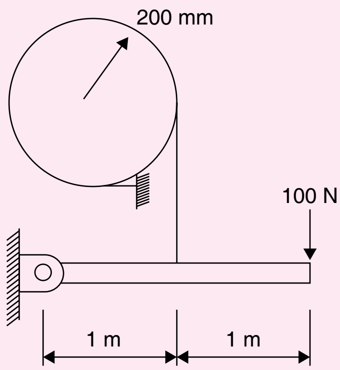
5. If the pressure angle of the rack is 20°, the force acting along the line of action between the rack and the gear teeth is **[2004]**

(A) 250 N (B) 342 N
(C) 532 N (D) 600 N

6. Which one of the following is a criterion in the design of hydrodynamic journal bearings? **[2005]**

(A) Sommerfeld number
(B) Rating life
(C) Specific dynamic capacity
(D) Rotation factor

Direction for questions 7 and 8: A band brake consists of a lever attached to one end of the band. The other end of the band is fixed to the ground. The wheel has a radius of 200 mm and the wrap angle of the band is 270°. The braking force applied to the lever is limited to 100 N, and the coefficient of friction between the band and the wheel is 0.5. No other information is given.



7. The maximum tension that can be generated in the band during braking is **[2005]**

(A) 1200 N (B) 2110 N
(C) 3224 N (D) 4420 N

8. The maximum wheel torque that can be completely braked is **[2005]**

(A) 200 N.m (B) 382 N.m
(C) 604 N.m (D) 844 N.m

9. For a circular shaft of diameter d subjected to torque T, the maximum value of the shear stress is: **[2006]**

(A) $\frac{64T}{\pi d^3}$ (B) $\frac{32T}{\pi d^3}$
(C) $\frac{16T}{\pi d^3}$ (D) $\frac{8T}{\pi d^3}$

10. A disk clutch is required to transmit 5 kW at 2000 rpm. The disk has a friction lining with coefficient of friction equal to 0.25. Bore radius of friction lining is equal to 25 mm. Assume uniform contact pressure of 1 MPa. The value of outside radius of the friction lining is: **[2006]**

(A) 39.4 mm (B) 49.5 mm
(C) 97.9 mm (D) 142.9 mm

11. Twenty degree full depth involute profiled 19-tooth pinion and 37-tooth gear are in mesh. If the module is 5 mm, the centre distance between the gear pair will be **[2006]**

(A) 140 mm (B) 150 mm
(C) 280 mm (D) 300 mm

12. A 60 mm long and 6 mm thick fillet weld carries a steady load of 15 kN along the weld. The shear strength of the weld material is equal to 200 MPa. The factor of safety is **[2006]**

(A) 2.4 (B) 3.4 (C) 4.8 (D) 6.8

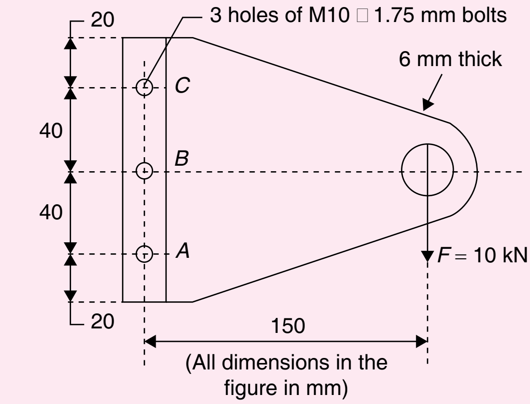
13. A ball bearing operating at a load F has 8000 hours of life. The life of the bearing, in hours, when the load is doubled to $2F$ is **[2007]**

(A) 8000 (B) 6000
(D) 4000 (D) 1000

14. A natural feed journal bearing of diameter 50 mm and length 50 mm operating at 20 revolution/second carries a load of 2.0 kN. The lubricant used has a viscosity of 20 mPa s. The radial clearance is 50 μm. The Sommerfeld number for the bearing is **[2007]**

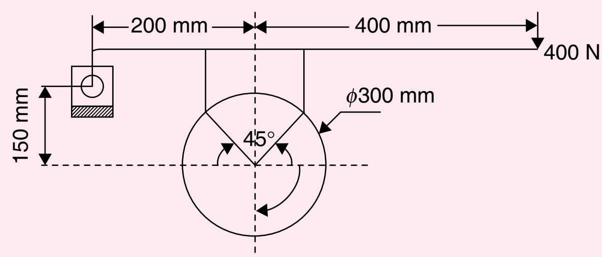
(A) 0.062 (B) 0.125
(C) 0.250 (D) 0.785

15. A bolted joint is shown below. The maximum shear stress, in MPa, in the bolts at A and B, respectively are **[2007]**



(A) 242.6, 42.5 (B) 42.5, 242.6
(C) 42.5, 42.5 (D) 242.6, 242.6

16. A block-brake shown below has a face width of 300 mm and a mean coefficient of friction of 0.25. For an activating force of 400 N, the braking torque in Nm is **[2007]**



(A) 30 (B) 40
(C) 45 (D) 60

17. The piston rod of diameter 20 mm and length 700 mm in a hydraulic cylinder subjected to a compressive force of 10 kN due to the internal pressure. The end condition for the rod may be assumed as guided at the piston end and hinged at the other end. The Young's modulus is 200 GPa. The factor of safety for the piston rod is **[2007]**

(A) 0.68 (B) 2.75
(C) 5.62 (D) 11.0

Direction for questions 18 to 20: A gear set has a pinion with 20 teeth and a gear with 40 teeth. The pinion runs at 30 rev/s and transmits a power of 20 kW. The teeth are on the 20° full-depth system and have a module of 5 mm. The length of the line of action is 19 mm.

18. The centre distance for the above gear set in mm is **[2007]**

(A) 140 (B) 150 (C) 160 (D) 170

19. The contact ratio of the contacting tooth is **[2007]**

(A) 1.21 (B) 1.25 (C) 1.29 (D) 1.33

20. The resultant force on the contacting gear tooth in *N* is **[2007]**

(A) 77.23 (B) 212.20
(C) 2258.1 (D) 289.43

21. A solid circular shaft of diameter 100 mm is subjected to an axial stress of 50 MPa. It is further subjected to a torque of 10 kNm. The maximum principal stress experienced on the shaft is closest to **[2008]**

(A) 41 MPa (B) 82 MPa
(C) 164 MPa (D) 204 MPa

22. A journal bearing has shaft diameter of 40 mm and a length of 40 mm. The shaft is rotating at 20 rad/s and the viscosity of the lubricant is 20 MPa.s. The clearance is 0.020 mm. The loss of torque due to the viscosity of the lubricant is approximately **[2008]**

(A) 0.040 Nm (B) 0.252 Nm
(C) 0.400 Nm (D) 0.652 Nm

23. A clutch has outer and inner diameters 100 mm and 40 mm, respectively. Assuming a uniform pressure of 2 MPa and the coefficient of friction of liner material 0.4, the torque carrying capacity of the clutch is **[2008]**

(A) 148 Nm (B) 196 Nm
(C) 372 Nm (D) 490 Nm

24. A spur gear has a module of 3 mm, number of teeth 16, a face width of 36 mm and a pressure angle of 20°. It is transmitting a power of 3 kW at 20 rev/s. Taking a velocity factor of 1.5, and a form factor of 0.3, the stress in the gear tooth is about **[2008]**

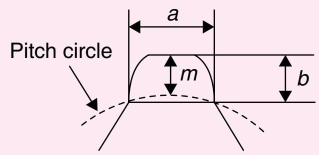
(A) 32 MPa (B) 46 MPa
(C) 58 MPa (D) 70 MPa

25. Match the type of gears with their most appropriate description. **[2008]**

Type of gears	Description
P. Helical	**1.** Axes non parallel and non intersecting
Q. Spiral Bevel	**2.** Axes parallel and teeth are inclined to the axis
R. Hypoid	**3.** Axes parallel and teeth are parallel to the axis
S. Rack and pinion	**4.** Axes are perpendicular and intersecting, and teeth are inclined to the axis.
	5. Axes are perpendicular and used for large speed reduction
	6. Axes parallel and one of the gears has infinite radius

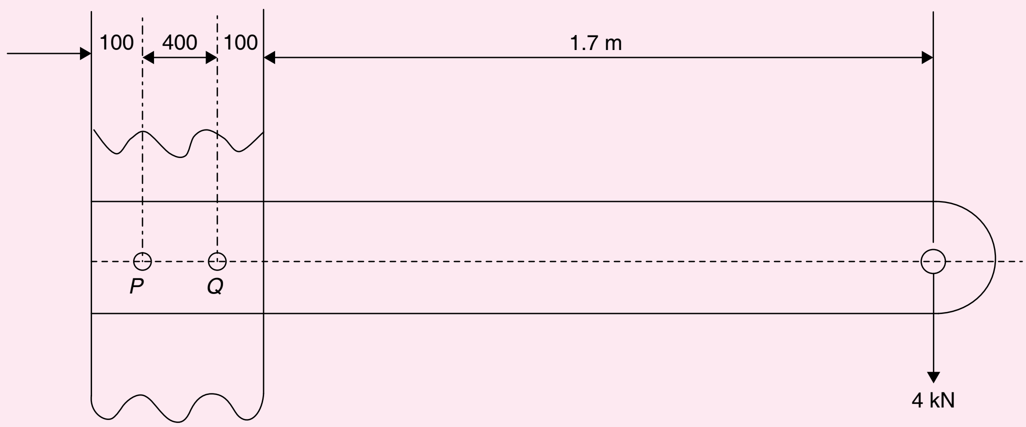
(A) P-2, Q-4, R-1, S-6 (B) P-1, Q-4, R-5, S-6
(C) P-2, Q-6, R-4, S-2 (D) P-6, Q-3, R-1, S-5

26. One tooth of a gear having 4 module and 32 teeth is shown in the figure. Assume that the gear tooth and the corresponding tooth space make equal intercepts on the pitch circumference. The dimensions '*a*' and '*b*', respectively, are closest to **[2008]**



(A) 6.08 mm, 4 mm (B) 6.48 mm, 4.2 mm
(C) 6.28 mm, 4.3 mm (D) 6.28 mm, 4.1 mm

Direction for questions 27 and 28: A steel bar of 10 × 50 mm is cantilevered with two *M* 12 bolts (*P* and *Q*) to support a static load of 4 kN as shown in the figure



27. The primary and secondary shear loads on bolt P, respectively, are **[2008]**
(A) 2 kN, 20 kN
(B) 20 kN, 2 kN
(C) 20 kN, 0 kN
(D) 0 kN, 20 kN

28. The resultant shear stress on bolt P is closest to **[2008]**
(A) 132 MPa
(B) 159 MPa
(C) 178 MPa
(D) 195 MPa

29. A solid circular shaft of diameter d is subjected to a combined bending moment M and torque, T. The material property to be used for designing the shaft using the relation $\frac{16}{\pi d^3}\sqrt{M^2+T^2}$ is **[2009]**
(A) ultimate tensile strength (S_u)
(B) tensile yield strength (S_y)
(C) torsional yield strength (S_{sy})
(D) endurance strength (S_e)

Direction for questions 30 and 31: A 20º full depth involute spur pinion of 4 mm module and 21 teeth is to transmit 15 kW at 960 rpm. Its face width is 25 mm.

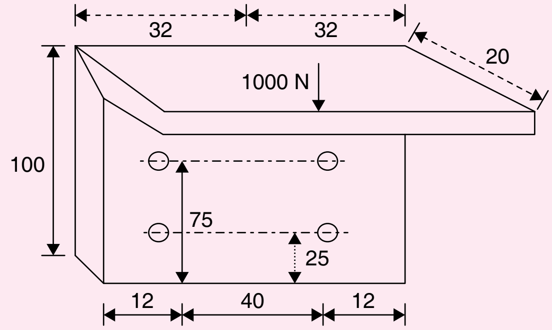
30. The tangential force transmitted (in N) is **[2009]**
(A) 3552 (B) 2611
(C) 1776 (D) 1305

31. Given that the tooth geometry factor is 0.32 and the combined effect of dynamic load and allied factors intensifying the stress is 1.5; the minimum allowable stress (in MPa) for the gear material is **[2009]**
(A) 242.0 (B) 166.5
(C) 121.0 (D) 74.0

32. Tooth interference in an external involute spur gear pair can be reduced by **[2010]**
(A) decreasing centre distance between gear pair
(B) decreasing module
(C) decreasing pressure angle
(D) increasing number of gear teeth

33. A band brake having band-width of 80 mm, drum diameter of 250 mm, coefficient of friction of 0.25 and angle of wrap of 270 degrees is required to exert a friction torque of 1000 N-m. The maximum tension (in kN) developed in the band is **[2010]**
(A) 1.88 (B) 3.56
(C) 6.12 (D) 11.56

34. A bracket (shown in figure) is rigidly mounted on wall using four rivets. Each rivet is 6 mm in diameter and has an effective length of 12 mm.



Direct shear stress (in MPa) in the most heavily loaded rivet is **[2010]**
(A) 4.4 (B) 8.8 (C) 17.6 (D) 35.2

35. A lightly loaded full journal bearing has a journal of 50 mm, bush bore of 50.05 mm and bush length of 20 mm. If the rotational speed of journal is 1200 rpm and average viscosity of liquid lubricant is 0.03 Pa s, the power loss (in W) will be **[2010]**
(A) 37 (B) 74 (C) 118 (D) 237

36. Two identical ball bearings P and Q are operating at loads 30 kN and 45 kN respectively. The ratio of the life of bearing P to the life of bearing Q is. **[2011]**
(A) 81/16 (B) 27/8 (C) 9/4 (D) 3/2

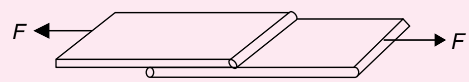
37. The following are the data for two crossed helical gears used for speed reduction:

Gear I: Pitch circle diameter in the plane of rotation 80 mm and helix angle 30º

Gear II: Pitch circle diameter in the plane of rotation 120 mm and helix angle 22.5º

If the input speed is 1440 rpm, the output speed in rpm is **[2012]**
(A) 1200 (B) 900 (C) 875 (D) 720

38. A fillet welded joint is subjected to transverse loading F as shown in the figure. Both legs of the fillets are of 10 mm size and the weld length is 30 mm. If the allowable shear stress of the weld is 94 MPa, considering the minimum throat area of the weld, the maximum allowable transverse load in kN is **[2012]**



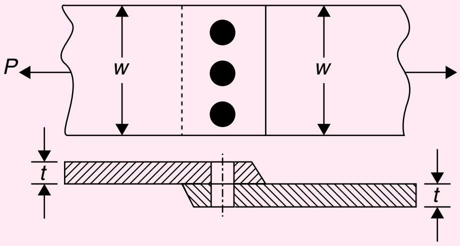
(A) 14.44 (B) 17.92
(C) 19.93 (D) 22.16

39. A solid circular shaft needs to be designed to transmit a torque of 50 N.m. If the allowable shear stress of the material is 140 MPa, assuming a factor of safety of 2, the minimum allowable design diameter in mm is **[2012]**
(A) 8 (B) 16 (C) 24 (D) 32

40. Two threaded bolts A and B of same material and length are subjected to identical tensile load. If the elastic strain energy stored in bolt A is 4 times that of bolt B and the mean diameter of bolt A is 12 mm, the mean diameter of bolt B in mm is **[2013]**

(A) 16 (B) 24
(C) 36 (D) 48

Direction for questions 41 and 42: A single riveted lap joint of two similar plates as shown in the figure below has the following geometrical and material details.



Width of the plate $w = 200$ mm, thickness of the plate $t = 5$ mm, number of rivets $n = 3$, diameter of the rivet $d_r = 10$ mm, diameter of the rivet hole $d_h = 11$ mm, allowable tensile stress of the plate $\sigma_p = 200$ MPa, allowable shear stress of the rivet $\sigma_s = 100$ MPa and allowable bearing stress of the rivet $\sigma_c = 150$ MPa.

41. If the rivets are to be designed to avoid crushing failure, the maximum permissible load P in kN is **[2013]**

(A) 7.50 (B) 15.00
(C) 22.50 (D) 30.00

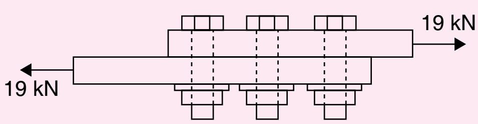
42. If the plates are to be designed to avoid tearing failure, the maximum permissible load P in kN is **[2013]**

(A) 83 (B) 125 (C) 167 (D) 501

43. A hydrodynamic journal bearing is subject to 2000 N load at a rotational speed of 2000 rpm. Both bearing bore diameter and length are 40 mm. If radial clearance is 20 μm and bearing is lubricated with an oil having viscosity 0.03 Pa.s, the Sommerfeld number of the bearing is ______ **[2014]**

44. In a structure subjected to fatigue loading, the minimum and maximum stresses developed in a cycle are 200 MPa and 400 MPa respectively. The value of stress amplitude (in MPa) is ______ **[2014]**

45. For the three bolt system shown in the figure, the bolt material has a shear yield strength of 200 MPa. For a factor of safety of 2, the minimum metric specification required for the bolt is **[2014]**



(A) $M8$ (B) $M10$
(C) $M12$ (D) $M16$

46. A disc clutch with a single friction surface has coefficient of friction equal to 0.3. The maximum pressure which can be imposed on the friction material is 1.5 MPa. The outer diameter of the clutch plate is 200 mm and its internal diameter is 100 mm. Assuming uniform wear theory for the clutch plate, the maximum torque (in N.m) that can be transmitted is ______ **[2014]**

47. A spur pinion of pitch diameter 50 mm rotates at 200 rad/s and transmits 3 kW power. The pressure angle of the tooth of the pinion is 20°. Assuming that only one pair of the teeth is in contact, the total force (in newton) exerted by a tooth of the pinion on the tooth on a mating gear is _____ **[2014]**

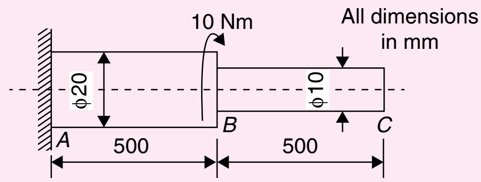
48. Which one of the following is used to convert a rotational motion into a translational motion? **[2014]**

(A) Bevel gears
(B) Double helical gears
(C) Worm gears
(D) Rack and pinion gears

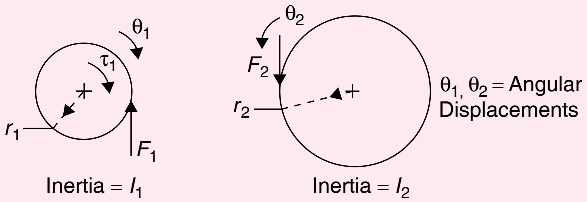
49. Ball bearings are rated by a manufacturer for a life of 10^6 revolutions. The catalogue rating of a particular bearing is 16 kN. If the design load is 2 kN, the life of the bearing will be $p \times 10^6$ revolutions, where p is equal to ____

50. A bolt of major diameter 12 mm is required to clamp two steel plates. Cross-sectional area of the threaded portion of the bolt is 84.3 mm^2. The length of the threaded portion in grip is 30 mm, while the length of the unthreaded portion in grip is 8 mm. Young's modulus of material is 200 GPa. The effective stiffness (in MN/m) of the bolt in the clamped zone is __________ **[2014]**

51. Consider a stepped shaft subjected to a twisting moment applied at B as shown in the figure. Assume shear modulus, $G = 77$ GPa. The angle of twist at C (in degrees) is ______. **[2015]**



52. A pinion with radius r_1, and inertia I_1 is driving a gear with radius r_2 and inertia I_2. Torque τ_1 is applied on pinion. The following are free body diagrams of pinion and gear showing important forces (F_1 and F_2) of interaction. Which of the following relations hold true? **[2015]**



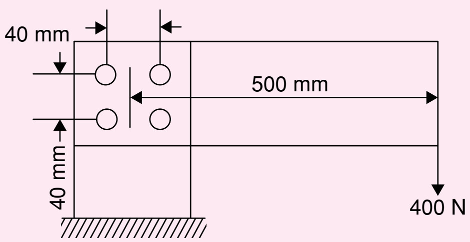
(A) $F_1 \neq F_2;\ \tau_1 = I_1\,\ddot{\theta}_1;\ F_2 = I_2 \dfrac{r_1}{r_2^2}\ddot{\theta}_1$

(B) $F_1 = F_2;\ \tau_1 = \left[I_1 + I_2\left(\dfrac{r_1}{r_2}\right)^2\right]\ddot{\theta}_1; F_2 = I_2 \dfrac{r_1}{r_2^2}\ddot{\theta}_1$

(C) $F_1 = F_2;\ \tau_1 = I_1\,\ddot{\theta}_1; F_2 = I_2 \dfrac{1}{r_2}\ddot{\theta}_2$

(D) $F_1 \neq F_2;\ \tau_1 = \left[I_1 + I_2\left(\dfrac{r_1}{r_2}\right)^2\right]\ddot{\theta}_1; F_2 = I_2 \dfrac{1}{2}\ddot{\theta}_2$

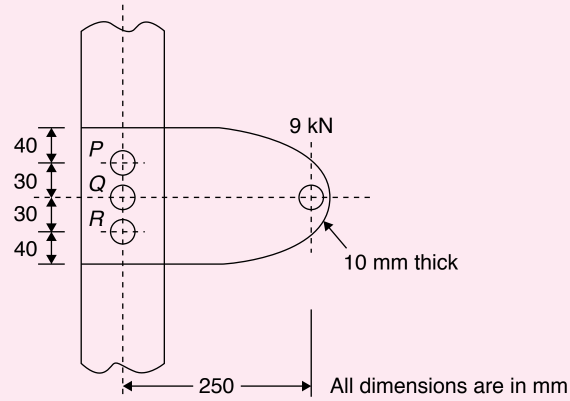
53. A horizontal plate has been joined to a vertical post using four rivets arranged as shown in the figure. The magnitude of the load on the worst loaded rivet (in N) is ______. **[2015]**



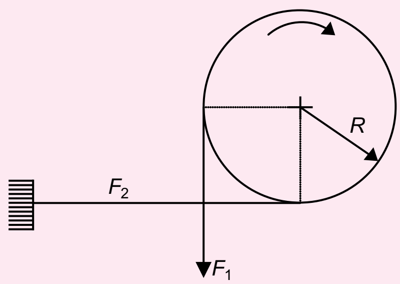
54. A rope-brake dynamometer attached to the crank shaft of an I.C. engine measures a brake power ofv 10 kW when the speed of rotation of the shaft is 400 rad/s. The shaft torque (in N-m) sensed by the dynamometer is ________. **[2015]**

55. For ball bearings, the fatigue life L measured in number of revolutions and the radial load F are related by $FL^{1/3} = K$, where K is a constant. It withstands a radial load of 2 kN for a life of 540 million revolutions. The load (in kN) for a life of one million revolutions is ________. **[2015]**

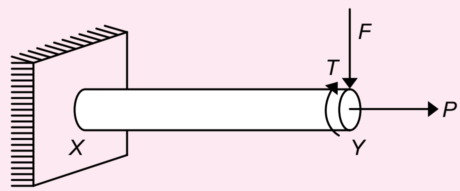
56. A cantilever bracket is bolted to a column using three M12 × 1.75 bolts P, Q and R. The value of maximum shear stress developed in the bolt P(in MPa) is ______. **[2015]**



57. The forces F_1 and F_2 in a brake band and the direction of rotation of the drum are as shown in the figure. The coefficient of friction is 0.25. The angle of wrap is $3\pi/2$ radians. It is given that $R = 1$ m and $F_2 = 1$ N. The torque (in N-m) exerted on the drum is ______. **[2016]**

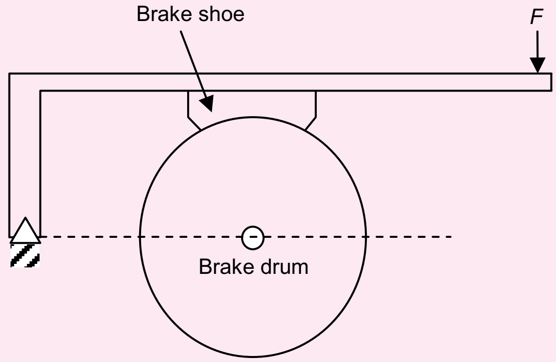


58. A machine element XY, fixed at end X, is subjected to an axial load P, transverse load F, and a twisting moment T at its free and Y. The most critical point from the strength point of view is: **[2016]**



(A) A point on the circumference at location Y
(B) A point at the center at location Y
(C) A point on the circumference at location X
(D) A point at the center at location X

59. For the brake shown in the figure, which one of the following is TRUE? **[2016]**

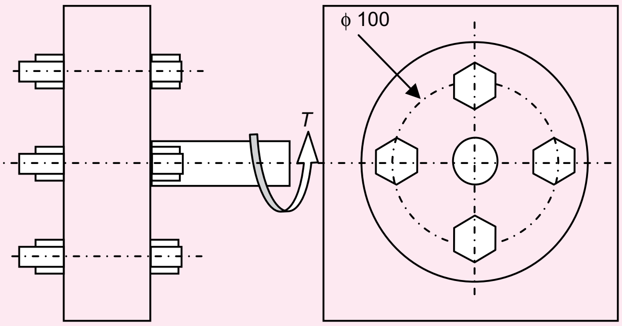


(A) Self-energizing for clockwise rotation of the drum
(B) Self-energizing for anti-clockwise rotation of the drum
(C) Self-energizing for rotation in either direction of the drum
(D) Not of the self-energizing type

60. Which of the following bearings SHOULD NOT be subjected to a thrust load? **[2016]**

(A) Deep groove ball bearing
(B) Angular contact ball bearing
(C) Cylindrical (straight) roller bearing
(D) Single row tapered roller bearing

61. A bolted joint has four bolts arranged as shown in figure. The cross sectional area of each bolt is 25 mm^2. A torque $T = 200$ N-m is acting on the joint. Neglecting friction due to clamping force, maximum shear stress in a bolt is _________ MPa. **[2016]**



Answer Keys

Exercises

Practice Problems I

1. D **2.** B **3.** A **4.** C **5.** C **6.** B **7.** A **8.** C **9.** C **10.** A
11. C **12.** B **13.** C **14.** D **15.** A **16.** B **17.** B **18.** B **19.** A **20.** C
21. $\ell = 60$ mm $t = 8$ mm **22.** 30 mm **23.** 500 mm **24.** 35.7 mm
25. 802.08 N **26.** 864.58 N **27.** 458.9 N **28.** 4.612 mm **29.** C **30.** A

Practice Problems 2

1. B **2.** B **3.** A **4.** A **5.** C **6.** D **7.** A **8.** D **9.** B **10.** C
11. D **12.** A **13.** B **14.** A **15.** D **16.** B **17.** B **18.** A **19.** A **20.** A
21. B **22.** C **23.** B **24.** B **25.** B **26.** B **27.** A **28.** B **29.** C **30.** D
31. D **32.** A **33.** B **34.** A **35.** C **36.** D **37.** C **38.** A **39.** B **40.** C
41. A **42.** D **43.** D **44.** 17.32 mm **45.** B **46.** A **47.** D **48.** 23.755 kN
49. C **50.** D

Previous Years' Questions

1. C **2.** A **3.** B **4.** B **5.** C **6.** A **7.** B **8.** B **9.** C **10.** A
11. A **12.** B **13.** D **14.** B **15.** A **16.** C **17.** C **18.** B **19.** C **20.** C
21. B **22.** A **23.** B **24.** B **25.** A **26.** D **27.** A **28.** B **29.** C **30.** A
31. B **32.** D **33.** D **34.** B **35.** A **36.** B **37.** B **38.** C **39.** B **40.** B
41. C **42.** C **43.** 0.75 to 0.85 **44.** 99 to 101 **45.** B **46.** 529 to 532
47. 638 to 639 **48.** B **49.** 500 to 540 **50.** 460 to 470 **51.** 0.22 to 0.25 **52.** B
53. 1835 to 1845 **54.** 24 to 26 **55.** 15 to 17 **56.** 332 to 494 **57.** 2.2 to 2.3
58. C **59.** A **60.** C **61.** 40

Test

Theory of Machines, Vibrations and Design

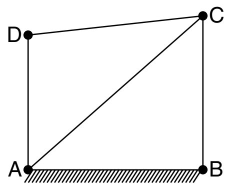
Time: 60 Minutes

Direction for questions 1 to 25: Select the correct alternative from the given choices.

1. Scotch Yoke and Oldham couplings are inversions of
(A) Four bar chain
(B) Crossed slider crank chain
(C) Single slider crank chain
(D) Double slider crank chain

2. A rim type fly wheel is used in a system to store energy. This flywheel is replaced by another rim type flywheel rotating in the same speed, whose mean radius is half of the original one. The energy stored in the second one is
(A) four times that in the first
(B) two times that in the first
(C) $\frac{1}{2}$ of that in the first case
(D) $\frac{1}{4}th$ of that in the first case.

3. According to Kutzbach criterion, the number of mobility of a five bar mechanism shown in figure is



(A) zero (B) one (C) two (D) three.

4. Grubblers criterion is applicable to plane mechanism with mobility
(A) zero (B) one
(C) two (D) more than 2.

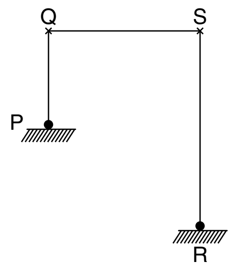
5. Locus of all instantaneous centres of a moving rigid body is known as
(A) axode
(B) centroid
(C) centrode
(D) instantaneous locus.

6. When two links have a sliding contact, the instantaneous centre will lie along
(A) the line of centres
(B) the common normal at the point of contact
(C) the common normal at the point of contact
(D) along the line of approach.

7. A flywheel fitted to an engine has a coefficient of fluctuation of speed C_s and coefficient of fluctuation of energy C_e. If E is the indicated power per cycle of operation, the kinetic energy of the flywheel is
(A) $\frac{2C_sE}{C_e}$ (B) $\frac{C_sE}{2C_e}$
(C) $\frac{2C_eE}{C_s}$ (D) $\frac{C_eE}{2C_s}$.

8. In a $14\frac{1}{2}^\circ$ involute system teeth, when a pinion meshes with a rack, to avoid interference the minimum number of teeth in the pinion shall be
(A) 32 (B) 25 (C) 20 (D) 16.

9. Consider the following system: It is a four-bar mechanism. PQ and RS are both vertical. RS is 30 cm more than PQ. QS is horizontal. PQ rotates with 3^C/s and RS rotates with 1^C/s. The length RS is



(A) 60 cm (B) 45 cm
(C) 35 cm (D) 30 cm.

10. A punch which is motor driven has a flywheel. It punches 25 mm holes on 32 mm plates. The punching operation requires 8 N-m energy per sq. mm of sheared area. The punch makes a hole in every 10 seconds; the power the motor requires is
(A) 1.3 KW (B) 1.6 KW
(C) 1.8 KW (D) 2 KW.

11. A raker in a tank is attached to a motor of 2.5 KW running at 1440 rpm. The raker is running at a speed of 24 rpm. The type of gearing arrangement suitable for this is
(A) Helical gear (B) Spur gear
(C) Worm gear (D) Differential gear.

12. The efficiency of a self locking screw jack is
(A) 50% (B) more than 50%
(C) less than 50% (D) 68.75%.

13. A carburised component is found to have high endurance limit because
(A) carbon diffusion improves the compressive strength
(B) carburised component has a higher hardness
(C) carburised component has a higher density
(D) carburised component has better surface finish.

14. A column of circular section is hinged at both the ends. It is required to evaluate the crippling load. Length of the column is 5 m. Given that, when the column is used as a simply supported beam with a central load of 10 N, the defection produced is 10 mm. The crippling load will be
(A) 850 N (B) 925 N
(C) 1028 N (D) 1210 N.

15. A shaft of 200 mm diameter, holding a load (transverse) of 15 KN and rotating at 1400 spm is supported by a bearing whose length is twice the diameter of the shaft. The coefficient of friction = 0.02. The bearing pressure will be
(A) 0.39 N/mm^2 (B) 0.31 N/mm^2
(C) 0.29 N/mm^2 (D) 0.19 N/mm^2.

16. The ratio of pitch of riveting to diameter of a rivet hole is given as 4. The tearing efficiency of the rivet joint is
(A) 60% (B) 70% (C) 75% (D) 80%.

17. A mass of 1Kg is attached to the free end of a hanging spring of stiffness IN/mm. The critical damping coefficient of the system is
(A) $63.25\dfrac{N}{m/s}$ (B) $56.25\dfrac{N}{m/S}$
(C) $50.5\dfrac{N}{m/S}$ (D) $46.85\dfrac{N}{m/S}$.

18. A truck weighing 400KN moving with a velocity of $5\dfrac{m}{s}$ impinges on a compression spring which compresses 2 mm for every 10 KN. The compression occurring to the spring in 'cms' because of the impingement is
(A) 23 cms (B) 28 cms
(C) 38 cms (D) 45 cms.

19. An equipment is mounted on a coil spring of stiffness $K\dfrac{N}{m}$. The spring is cut into 4 pieces and kept parallel over which the equipment which is now mounted. If t, is the period of vibration in the first case and t_2 the period of vibration in the second case, $\dfrac{t_2}{t_1}$ is
(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{8}$ (D) $\dfrac{1}{16}$.

20. Two heavy rotors are mounted on a shaft. The natural frequency of vibration when each rotor is considered separately are $150\,{}^{C}\!/_{S}$ and $250\,{}^{C}\!/_{S}$ respectively. The lowest critical speed is
(A) $400\dfrac{C}{S}$ (B) $229\,{}^{C}\!/_{S}$
(C) $129\,{}^{C}\!/_{S}$ (D) $100\,{}^{C}\!/_{S}$.

21. A system with viscous damping has a mass of 4 kg. The spring constant is $0.5\dfrac{N}{mm}$. The amplitude decreases to $\dfrac{1}{4}$ th of the original value in five consecutive oscillations. Viscous damping coefficient of the system is
(A) $16.52\dfrac{N}{m/s}$ (B) $12.45\dfrac{N}{m/s}$
(C) $10.5\dfrac{N}{m/s}$ (D) $8.75\dfrac{N}{m/s}$.

22. An undamped vibrating system is represented by the equation $\ddot{x}+\dfrac{64\pi^2}{9}x=$
The natural frequency of vibration is
(A) 5.33 Hz (B) 3.33 Hz
(C) 2.33 Hz (D) 1.33 Hz.

23. A horizontal solid shaft of diameter 150 mm is rotating within a bush bearing at 2990 spm. The shaft is under a vertical load of 40 KN. If the average coefficient of friction between the shaft and bearing is 0.008, the heat generated in walts is
(A) 8315 walts (B) 7515 walts
(C) 6995 walts (D) 5218 walts.

Direction for questions 24 and 25: A journal bearing (square) with 200 ? 200 mm supports a radial load of 50 KN at an operating speed of 1020 rpm. The radial clearance of the bearing is 0.2 mm. The Sommerfield Number is 0.08.

24. The absolute viscosity of lule oil is
(A) 0.0325 Pa.S (B) 0.0235 Pa.S
(C) 0.0428 Pa.S (D) 0.0478 Pa.S.

25. The bearing pressure is
(A) 1.25 MPa (B) 2.25 MPa
(C) 2.75 MPa (D) 3.25 MPa.

Answer Keys

1. D	**2.** D	**3.** A	**4.** B	**5.** C	**6.** C	**7.** D	**8.** A	**9.** B	**10.** D
11. C	**12.** C	**13.** D	**14.** C	**15.** D	**16.** C	**17.** A	**18.** D	**19.** B	**20.** C
21. B	**22.** D	**23.** B	**24.** B	**25.** A					