Code No. 041

Roll No. ☐☐☐☐☐☐☐

- Please check that this question paper contains **5** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **33** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS–XII
Sample Paper (Solved)

Time allowed: 3 hours *Maximum Marks: 80*

General Instructions:

PART A

Section I

All questions are compulsory. In case of internal choices attempt any one.

1. State the reason why the Relation R = $\{(a, b) : a \leq b^2\}$ on the set R of real numbers is not reflexive.

Or

State the reason for the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive.

2. Find the derivative of $f(e^{\tan x})$ w.r.t to x at $x = 0$. It is given that $f'(1) = 5$.

3. Find the Projection (vector) of $2\hat{i} - \hat{j} + \hat{k}$ on $\hat{i} - 2\hat{j} + \hat{k}$.

Or

For vector $\vec{a}$, if $|\vec{a}| = a$, then write the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$

4. If A is a square matrix of order 3 and $|2A| = k|A|$, then find the value of k.

5. Prove that the diagonal elements of a skew symmetric matrix are all zeroes.

Or

If A = $[a_{ij}]$ is a matrix of order 2 × 2, such that $|A| = -15$ and c_{ij} represents the cofactor of a_{ij}, then find $a_{21}c_{21} + a_{22}c_{22}$.

6. If A and B are invertible matrices of order 3, $|A| = 2$ and $|AB|^{-1} = \frac{-1}{6}$. Find $|B|$.

7. Find: $\int \frac{e^x(x-3)}{(x-1)^3}dx$

Or

Write the value of : $\int \frac{x + \cos 6x}{3x^2 + \sin 6x}dx$

8. Evaluate : $\int_e^{e^2} \frac{dx}{x \log x}$

9. Find the general solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$.

Or

Find the integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$.

10. Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{a} = 2\hat{i} - 3\hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$.

11. Find the angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

12. Find a unit vector parallel to the sum of the vectors $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j} + 5\hat{k}$.

13. If the equations of a line AB are $\frac{3-x}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$, find the direction cosines of a line parallel to AB.

14. What is the distance (in units) between the two planes $3x + 5y + 7z = 3$ and $9x + 15y + 21z = 9$?

(*a*) 0 (*b*) 3 (*c*) $\frac{6}{\sqrt{83}}$ (*d*) 6

15. If P(A) = 0.6, P(B) = 0.5 and P(A | B) = 0.3, then find P(A ∪ B).

16. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

Section II

Both the case-study based questions are compulsory. Attempt any 4 sub parts from each question (17–21) and (22 – 26). Each question carries 1 mark.

17. Case Study—An insurance company insured 2000 scooterdrivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively.

One of the insured persons meets with an accident.

Answer the following questions:

(*i*) **Write the probability the events to insure scooter, car and truck driver respectively.**

(*a*) $\frac{2}{3}, \frac{1}{2}, \frac{4}{6}$ (*b*) $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ (*c*) $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ (*d*) $\frac{2}{6}, \frac{2}{3}, \frac{4}{2}$

(*ii*) **What are the probability of an accident involving a scooter, a car and a truck?**

(*a*) $\frac{1}{100}, \frac{3}{100}, \frac{15}{100}$ (*b*) $\frac{2}{100}, \frac{6}{100}, \frac{3}{100}$ (*c*) $\frac{3}{100}, \frac{9}{100}, \frac{45}{100}$ (*d*) $\frac{6}{10}, \frac{30}{10}, \frac{7}{10}$

(*iii*) **What is the probability that the insured person who met with an accident, is a scooter driver?**

(*a*) $\frac{5}{52}$ (*b*) $\frac{4}{52}$ (*c*) $\frac{2}{52}$ (*d*) $\frac{1}{52}$

(*iv*) **Given that the events A and B are such that P(A) = $\frac{1}{2}$, P(A ∪ B) = $\frac{3}{5}$ and P(B) = *p*. Find *p* if they are mutually exclusive.**

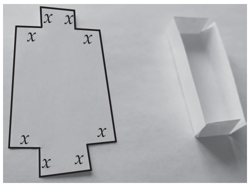
(*a*) $\frac{3}{10}$ (*b*) $\frac{2}{10}$ (*c*) $\frac{1}{10}$ (*d*) $\frac{1}{5}$

(*v*) **In addition to the point (*iv*), find *p* if they are mutually independent.**

(*a*) $\frac{1}{5}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{3}{5}$ (*d*) $\frac{4}{5}$

18. Case Study—*Rectangular Box*

This figure shows the Rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off squares from each corner and folding up the flaps.



Answer the following questions :

(*i*) Find the change in volume of the box w.r.t. the side to be cut *i.e., x*.

(*a*) $x^2 + 23x - 90$ (*b*) $12(x^2 - 23x + 90)$

(*c*) $x^2 - 23x + 90$ (*d*) $6x^2 - 138x + 540$

(*ii*) What should be the side of the square to be cut off so that volume of the box is maximum?

(*a*) 5 (*b*) 12 (*c*) 18 (*d*) 24

(*iii*) What is the double derivative *i.e.,* $\frac{d^2V}{dx^2}$ at the value of *x*?

(*a*) –156 (*b*) –165 (*c*) –110 (*d*) 156

(*iv*) Write the dimensions of the Rectangular Box?

(*a*) 45, 24, 5 (*b*) 40, 19, 5 (*c*) 24, 45, 15 (*d*) 35, 14, 5

(*v*) What is the maximum value of the Box?

(*a*) 5240 (*b*) 4250 (*c*) 2450 (*d*) 2540

PART B

Section III

19. If $4\sin^{-1} x + \cos^{-1} x = \pi$, then find the value of x.

20. Two farmers X and Y cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale in rupees of these varieties of rice by both the farmers in the months of September and October are given by the following matrices A and B.

September sales in rupees **October sales in rupees**

$$A = \begin{array}{c} \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ \begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix} \end{array} \begin{array}{c} \\ X \\ Y \end{array} \qquad B = \begin{array}{c} \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ \begin{bmatrix} 5,000 & 10,000 & 6,000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix} \end{array} \begin{array}{c} \\ X \\ Y \end{array}$$

Find :

(*i*) What were the combined sales in September and October for each farmer in each variety?

(*ii*) If both farmers decided to donate 2% of the gross rupees sales in October, for the welfare of their workers, compute the total amount paid by each farmer for the welfare of the workers.

Or

To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap-books and pastel-sheets made by them using recycled paper at the rate of ₹ 20, ₹ 15 and ₹ 10 per unit respectively. School A sold 25 paper bags, 10 scrap-books and 30 pastel-sheets. School B sold 20 paper bags, 15 scrap-books and 30 pastel-sheets. While school C sold 25 paper bags, 18 scrap-books and 35 pastel-sheets. Using matrices, find the total amount raised by each school.

21. If $y = x^{\sin x} + \sin(x^x)$, find $\frac{dy}{dx}$

22. Find the equation(s) of the tangent(s) to the curve $y = (x^3 - 1)(x - 2)$ at the points where the curve intersects the x-axis.

23. Find $\int \left(\frac{1-x}{1+x^2}\right)^2 e^x \, dx$.

Or

Find: $\int \frac{(x^2 + \sin^2 x)\sec^2 x}{1 + x^2} dx$.

24. Find the area of the region $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$ by using integration.

25. Solve the following differential equation $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + x\,dy = 0$

26. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}\,.\,\vec{b} = \vec{a}\,.\,\vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

27. Find the vector equation of the line joining (1, 2, 3) and (–3, 4, 3) and show that it is perpendicular to the z-axis.

28. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of number of aces.

Or

A family has two children. What is the probability that both the children are boys, given that at least one of them is a boy?

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. Let A = {1, 2, 3, ..., 9} and R be the relation in A × A defined by (a, b) R (c, d) if $a + d = b + c$ for $a, b, c, d \in$ A.
Prove that R is an equivalence relation. Also obtain the equivalence class [(2, 5)].

30. Discuss the differentiability of the function $f(x) = \begin{cases} 2x-1, & x < \frac{1}{2} \\ 3-6x, & x \geq \frac{1}{2} \end{cases}$ at $x = \frac{1}{2}$.

31. For what value of k is the following function continuous at $x = \frac{-\pi}{6}$?

$$f(x) = \begin{cases} \dfrac{\sqrt{3}\sin x + \cos x}{x + \frac{\pi}{6}}, & x \neq -\frac{\pi}{6} \\ k, & x = -\frac{\pi}{6} \end{cases}$$

Or

If $y = \log\left(\sqrt{x} + \frac{1}{x}\right)^2$, then prove that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.

32. Find the intervals in which the function $f(x) = -3\log(1+x) + 4\log(2+x) - \frac{4}{2+x}$ is strictly increasing or strictly decreasing.

33. Find $\int \frac{\sec x}{1 + \operatorname{cosec} x}\, dx$.

34. Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.

Or

Using integration find the area of the region bounded by the triangle whose vertices are (1, 3), (2, 5) and (3, 4).

35. Find the general solution of the differential equation: $\frac{dx}{dy} = \frac{y\tan y - x\tan y - xy}{y\tan y}$

Section V

All questions are compulsory. In case of internal choices attempt any one.

36. If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations: $x - y = 3$; $2x + 3y + 4z = 17$; $y + 2z = 7$.

Or

If $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, find AB.

Use this to solve the following system of equations :

$x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$

37. Find the distance of the point $3\hat{i} - 2\hat{j} + \hat{k}$ from the plane $3x + y - z + 2 = 0$ measured parallel to the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$. Also, find the foot of the perpendicular from the given point upon the given plane.

Or

Find the equation of the plane through the point (4, −3, 2) and perpendicular to the line of intersection of the planes $x - y + 2z - 3 = 0$ and $2x - y - 3z = 0$. Find the point of intersection of the line $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + 3\hat{j} - 9\hat{k})$ and the plane obtained above.

38. Solve the following graphically and also find the maximum profit.
Maximum Profit, $Z = 7.8x + 7.1y$
Subject to the constraints: $\frac{x}{4} + \frac{y}{3} \leq 90$; $\frac{x}{8} + \frac{y}{3} \leq 80$; $x \leq 200$; $x \geq 0, y \geq 0$.

Or

Solve the following graphically and also find the maximum profit.
Maximum Profit, $Z = 50x + 60y$
Subject to the constraints: $20x + 10y \leq 180$; $10x + 20y \leq 120$; $10x + 30y \leq 150$; $x \geq 0, y \geq 0$.

Answer Sheet

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MATHEMATICS

1. R is not reflexive, as $\frac{1}{2} \leq \left(\frac{1}{2}\right)^2$ is not true, $\frac{1}{2} > \left(\frac{1}{2}\right)^3$

Therefore, $\left(\frac{1}{2}, \frac{1}{2}\right) \notin$ R **Hence, R is not reflexive.**

Or

R = {1, 2, 3}, R = {(1, 2), (2, 1)}
As (1, 2) ∈ R and (2, 1) ∈ R
But (1, 1) ∉ R
So, R is not transitive.

2. Let $y = f(e^{\tan x})$
Differentiating w.r.t. x, we have

$$\frac{dy}{dx} = f'(e^{\tan x}) \cdot \frac{d}{dx}(e^{\tan x})$$

$$\frac{dy}{dx} = f'(e^{\tan x}) \cdot e^{\tan x} \cdot \frac{d}{dx}(\tan x)$$

$$\frac{dy}{dx} = f'(e^{\tan x}) \cdot e^{\tan x} \cdot \sec^2 x$$

$$\left.\frac{dy}{dx}\right]_{\text{at } x=0} = f'(e^{\tan 0}) \cdot e^{\tan 0} \cdot \sec^2 0$$

$$= f'(1) \cdot e^0 \cdot (1)^2 = 5(1)(1) = \mathbf{5}$$ **[∵ tan 0 = 0, $e^0 = 1$ and $f'(1) = 5$**

3. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

∴ $\vec{a} \cdot \vec{b} = (2\hat{i} - \hat{j} + \hat{k}) \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 2 + 2 + 1 = \mathbf{5}$

$|\vec{b}| = \sqrt{(1)^2 + (-2)^2 + (1)} = \sqrt{1 + 4 + 1} = \sqrt{6}$

The required Projection (vector) of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}$

$$= \mathbf{\frac{5}{6} \cdot (\hat{i} - 2\hat{j} + \hat{k})}$$

Or

Given : $|\vec{a}| = a$

Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ $\therefore$ $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

$\Rightarrow$ $a = \sqrt{x^2 + y^2 + z^2}$ **...[$\because |\vec{a}| = a$**

$\Rightarrow$ $a^2 = x^2 + y^2 + z^2$ ...(i)

$$(\vec{a} \times \hat{i}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix} = \hat{i}(0) - \hat{j}(-z) + \hat{k}(-y) = z\hat{j} - y\hat{k}$$

$(\vec{a} \times \hat{i})^2 = (z\hat{j} - y\hat{k})(z\hat{j} - y\hat{k}) = y^2 + z^2$ ...(ii)

Similarly, $(\vec{a} \times \hat{j})^2 = x^2 + z^2$ ...(iii)

$(\vec{a} \times \hat{k})^2 = x^2 + y^2$ ...(iv)

Now, $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$

$= y^2 + z^2 + x^2 + z^2 + x^2 + y^2$ **[From (ii), (iii) & (iv)**

$= 2(x^2 + y^2 + z^2) = \mathbf{2a^2}$ ...**[From (i)**

4. **Given.** $|2A| = k|A|$

$\Rightarrow$ $2^3|A| = k|A|$ $\Rightarrow$ $8|A| = k|A|$ $\therefore$ $\boldsymbol{k = 8}$

5. A square matrix $A = [a_{ij}]$ is said to be a skew symmetric matrix if $A' = -A$, that is $a_{ji} = -a_{ij}$ for all possible values of i and j.

Now, when $i = j$, $a_{ii} = -a_{ii}$

$a_{ii} + a_{ii} = 0$

$2\,a_{ii} = 0$

$\boldsymbol{a_{ii} = 0}$ for all i's.

Hence all the diagonal elements of a skew symmetric matrix are zero.

Or

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, We have $|A| = -15$

$\Rightarrow$ $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = -15$ $\Rightarrow$ $a_{11}a_{22} - a_{21}a_{12} = -15$ ...(i)

As c_{ij} represents the cofactors of a_{ij}

$\therefore$ $c_{21} = -a_{12}$; $c_{22} = a_{11}$

So, $\boldsymbol{a_{21}c_{21} + a_{22}c_{22}} = a_{21}(-a_{12}) + a_{22}a_{11} = -a_{21}a_{12} + a_{22}a_{11}$

$= a_{22}a_{11} - a_{21}a_{12} = \mathbf{-15}$ ...**[From (i)**

6. **Given:** $|AB|^{-1} = \dfrac{-1}{6}$ and $|A| = 2$

$\Rightarrow$ $\dfrac{1}{|AB|} = -\dfrac{1}{6}$ $\Rightarrow$ $\dfrac{1}{|A||B|} = -\dfrac{1}{6}$ **...[$\because |AB| = |A||B|$**

$\Rightarrow$ $\dfrac{1}{2|B|} = \dfrac{-1}{6}$ $\Rightarrow$ $\mathbf{|B| = -3}$

7. $$\int \frac{e^x(x-3)}{(x-1)^3}dx = \int e^x\left[\frac{(x-1)-2}{(x-1)^3}\right]dx$$

$$= \int e^x\left[\frac{1}{(x-1)^2} + \left(\frac{-2}{(x-1)^3}\right)\right]dx\,dx$$

$$= \frac{\boldsymbol{e^x}}{\boldsymbol{(x-1)^2}} + \mathbf{C}$$ ...$\left[\because \int e^x[f(x) + f'(x)]dx = e^x f(x) + c\right.$

Or

$$\int \frac{x+\cos 6x}{3x^2+\sin 6x}\,dx = \frac{1}{6}\int \frac{dp}{p} \quad \ldots\left[\begin{array}{l}\textbf{Let } p = 3x^2+\sin 6x \Rightarrow dp = 6x + 6\cos 6x dx \\ \quad dp = 6(x+\cos 6x)dx\end{array}\right.$$

$$= \frac{1}{6}\log|p| + C$$

$$= \mathbf{\frac{1}{6}\log|3x^2+\sin 6x| + C}$$

8. $$\int_e^{e^2} \frac{dx}{x\log x} = \int_1^2 \frac{dp}{p} \quad \ldots\left[\begin{array}{l}\textbf{Let } p = \log x \Rightarrow dp = \frac{1}{x}dx \\ \textbf{when } x = e^2, p = 2 \textbf{ and } x = e, p = 1\end{array}\right.$$

$$= [\log|p|]_1^2 = \log 2 - \log 1 = \mathbf{\log 2} \quad [\because \log 1 = 0]$$

9. $$\log\left(\frac{dy}{dx}\right) = 3x + 4y$$

$$\Rightarrow \left(\frac{dy}{dx}\right) = e^{3x+4y} \quad \Rightarrow \frac{dy}{dx} = e^{3x}\,.\,e^{4y}$$

$$\Rightarrow \frac{dy}{e^{4y}} = e^{3x}\,dx$$

$$\Rightarrow \int e^{-4y}\,dy = \int e^{3x}\,dx$$

$$\Rightarrow \mathbf{\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c}$$

Or

$$\frac{dy}{dx} + y = \frac{1+y}{x} \quad \Rightarrow \frac{dy}{dx} + y = \frac{1}{x} + \frac{y}{x}$$

$$\frac{dy}{dx} + y - \frac{y}{x} = \frac{1}{x} \quad \Rightarrow \frac{dy}{dx} + \left(1 - \frac{1}{x}\right)y = \frac{1}{x}$$

Comparing the above with $\frac{dy}{dx}$ + Py = Q, we have 'P' = $1 - \frac{1}{x}$ and 'Q' = $\frac{1}{x}$

Integrating Factor (I.F.) = $e^{\int Pdx} = e^{\int\left(1-\frac{1}{x}\right)dx} = \mathbf{e^{x-\log|x|}}$ **or** $\mathbf{\frac{e^x}{x}}$

10. $$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ 2 & -1 & 2\end{vmatrix} = (-6+4)\hat{i} - (4-8)\hat{j} + (-2+6)\hat{k} = -2\hat{i} + 4\hat{j} + 4\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{(-2)^2 + (4)^2 + (4)^2} = \sqrt{4+16+16} = 6$$

$\therefore$ **Area of the parallelogram** = $\frac{|\vec{a}\times\vec{b}|}{2} = \frac{6}{2}$ = **3 sq. units.**

11. The angle θ between the vectors $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\Rightarrow \cos\theta = \frac{(\hat{i}+\hat{j}-\hat{k})\cdot(\hat{i}-\hat{j}+\hat{k})}{\sqrt{(1)^2+(1)^2+(-1)^2}\cdot\sqrt{(1)^2+(-1)^2+(1)^2}} \quad \Rightarrow \cos\theta = \frac{1-1-1}{\sqrt{3}\cdot\sqrt{3}}$$

$$\Rightarrow \cos\theta = -\frac{1}{3} \qquad \therefore \theta = \mathbf{\cos^{-1}\left(-\frac{1}{3}\right)}$$

12. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$

Now, $\vec{a} + \vec{b} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

and $|\vec{a} + \vec{b}| = \sqrt{(3)^2 + (-2)^2 + (6)^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7$

$\therefore$ **Required vector** $= \dfrac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \dfrac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7} = \boldsymbol{\dfrac{3}{7}\hat{i} - \dfrac{2}{7}\hat{j} + \dfrac{6}{7}\hat{k}}$

13. Given line is $\dfrac{-(x-3)}{-3} = \dfrac{y+2}{-2} = \dfrac{z+2}{6}$

Direction ratios of given lines are 3, –2, 6.

$\therefore$ $a = 3,\ b = -2,\ c = 6$

Now, $\sqrt{a^2 + b^2 + c^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7$

Let the direction cosines be l, m, n.

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{3}{7},\ m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{-2}{7},\ n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{6}{7}$$

Therefore, direction cosines are $\boldsymbol{\frac{3}{7}, \frac{-2}{7}}$ and $\boldsymbol{\frac{6}{7}}$.

14. $3x + 5y + 7z = 3$ …(*i*) and $9x + 15y + 21z = 9$ …(*ii*)

Dividing (*ii*) both sides by 3, we have $3x + 5y + 7z = 3$ …(*iii*)

Since Planes (*i*) & (*iii*) coincide each other.

$\therefore$ **Required distance = 0**

15. **Given:** P(A | B) = 0.3, P(A) = 0.6, P(B) = 0.5

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = 0.3$$

$\Rightarrow$ $P(A \cap B) = 0.3 \times P(B)$

$\Rightarrow$ $P(A \cap B) = 0.5 \times 0.3 = 0.15$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow$ $P(A \cup B) = 0.6 + 0.5 - 0.15$

$\therefore$ $\boldsymbol{P(A \cup B) = 0.95}$

16

Boys	Girls	Total
3	5	8

$\therefore$ $$P(2\text{ boys}, 2\text{ girls}) = \frac{{}^3C_2 \times {}^5C_2}{{}^8C_4} = \frac{\frac{3!}{2! \times 1!} \times \frac{5!}{2! \times 3!}}{\frac{8!}{4! \times 4!}} = \frac{\frac{3}{1} \times \frac{5.4}{2.1}}{\frac{8.7.6.5}{4.3.2.1}} = \frac{30}{70} \text{ or } \boldsymbol{\frac{3}{7}}$$

17. (***i***) (***b***); Scooter drivers = 2000; Car drivers = 4000; Truck drivers = 6,000

$\therefore$ Total Drivers = 12,000

Let E_1, E_2 and E_3 be the event to insure scooter driver, car driver and truck driver.

$\therefore$ $P(E_1) = \dfrac{2000}{12000} = \boldsymbol{\dfrac{1}{6}}$; $P(E_2) = \dfrac{4000}{12000} = \boldsymbol{\dfrac{1}{3}}$; $P(E_3) = \dfrac{6000}{12000} = \boldsymbol{\dfrac{1}{2}}$

(***ii***) (***a***); Let E be the event of an accident

$P(E \mid E_1) = 0.01 = \boldsymbol{\dfrac{1}{100}}$; $P(E \mid E_2) = 0.03 = \boldsymbol{\dfrac{3}{100}}$; $P(E \mid E_2) = 0.15 = \boldsymbol{\dfrac{15}{100}}$

(iii) **(d)**; Required probability $P(E_1 | E) = \dfrac{P(E_1)P(E|E_1)}{P(E_1)P(E|E_1)+P(E_2)P(E|E_2)+P(E_3)P(E|E_3)}$

$$= \frac{\frac{1}{6}\times\frac{1}{100}}{\frac{1}{6}\times\frac{1}{100}+\frac{1}{3}\times\frac{3}{100}+\frac{1}{2}\times\frac{15}{100}}$$

$$= \frac{\frac{1}{6}}{\frac{1+6+45}{6}} = \frac{1}{6}\times\frac{6}{52} = \mathbf{\frac{1}{52}}$$

(iv) **(c)**; If event are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$

$\Rightarrow \frac{3}{5} = \frac{1}{2} + p \quad \Rightarrow p = \frac{3}{5} - \frac{1}{2} \quad \Rightarrow p = \frac{6-5}{10} = \mathbf{\frac{1}{10}}$

(v) **(a)**; If events are independent, then $P(A \cap B) = P(A) . P(B)$

$P(A) + P(B) - P(A \cup B) = \left(\frac{1}{2}\right)(p)$

$\Rightarrow \frac{1}{2} + p - \frac{3}{5} = \frac{p}{2} \quad \Rightarrow \frac{1}{2} - \frac{3}{5} = \frac{p}{2} - p$

$\Rightarrow \frac{5-6}{10} = \frac{p-2p}{2} \quad \Rightarrow \frac{-1}{10} = \frac{-p}{2} \quad \therefore \ \mathbf{p = \frac{1}{5}}$

18. **(i)** **(b)**; Let the side to be cut = x cm

$\therefore$ Length of box, $l = 45 - 2x$; Breadth of box, $b = 24 - 2x$, Height of box, $h = x$

Volume of box, $V = lbh$

$V = (45 - 2x)(24 - 2x)(x)$

$V = 2(45 - 2x)(12 - x)(x)$

$V = 2[540 - 45x - 24x + 2x^2]x$

$V = 2[2x^3 - 69x^2 + 540x]$

Differentiating the above w.r.t., we get

$\frac{dV}{dx} = 2[6x^2 - 138 + 540]$

$= 2 \times 6[x^2 - 23x + 90] = \mathbf{12(x^2 - 23x + 90)}$

(ii) **(a)**; For maximum volume, we have $\frac{dV}{dx} = 0$

$\Rightarrow 12(x^2 - 23x + 90) = 0$

$\Rightarrow x^2 - 23x + 90 = 0$

$\Rightarrow x^2 - 18x - 5x + 90 = 0$

$\Rightarrow x(x - 18) - 5(x - 18) = 0$

$\Rightarrow (x - 18)(x - 5) = 0$

$\Rightarrow x = 18$ and $x = 5$

$\therefore$ $\mathbf{x = 5}$

(iii) **(a)**; We have, $\frac{dV}{dx} = 12(x^2 - 23x + 90)$ ...[From (*i*)

Now, $\frac{d^2V}{dx^2} = 12(2x - 23)$

$\left[\frac{d^2V}{dx^2}\right]_{x=5} = 12[2(5) - 23] = 12(10 - 23) = -156 < 0$ (maximum)

(iv) **(d)**; Length of the Rectangular Box, $l = (45 - 2x) = 35$ cm

Breath of the Rectangular Box, $b = (24 - 2x) = 14$ cm

Height of the Rectangular Box, $h = x = 5$ cm (*v*) (*c*); Maximum volume of the Box= $l \times b \times h = 35 \times 14 \times 5 =$ **2450 cm²**

19. **Given:** $4 \sin^{-1} x + \cos^{-1} x = \pi$

$$4 \sin^{-1} x + \frac{\pi}{2} - \sin^{-1} x = \pi \qquad \ldots \left[\begin{array}{l} \because \sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2} \\ \Rightarrow \cos^{-1}\theta = \frac{\pi}{2} - \sin^{-1}\theta \end{array}\right.$$

$$\Rightarrow \quad 3 \sin^{-1} x = \pi - \frac{\pi}{2} = \frac{\pi}{2} \qquad \Rightarrow \quad \sin^{-1} x = \frac{\pi}{2 \times 3} = \frac{\pi}{6} \qquad \therefore \quad \boldsymbol{x = \frac{1}{2}}$$

20. (*i*) $A + B = \begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix} + \begin{bmatrix} 5,000 & 10,000 & 6,000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix}$

$$= \begin{array}{c} \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ \begin{bmatrix} 15,000 & 30,000 & 36,000 \\ 70,000 & 40,000 & 20,000 \end{bmatrix} \begin{array}{c} X \\ Y \end{array} \end{array}$$

Farmer X

Combined sales of Basmati = ₹ **15,000** Combined sales of Permal = ₹ **30,000**

Combined sales of Naura = ₹ **36,000**

Farmer Y

Combined sales of Basmati = ₹ **70,000** Combined sales of Permal = ₹ **40,000**

Combined sales of Naura = ₹ **20,000**

(*ii*) Donate amount $= \begin{bmatrix} 5,000 & 10,000 & 6,000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix} \begin{bmatrix} 2\% \\ 2\% \\ 2\% \end{bmatrix}$

$$= \begin{bmatrix} 5,000 \times \frac{2}{100} + 10,000 \times \frac{2}{100} + 6,000 \times \frac{2}{100} \\ 20,000 \times \frac{2}{100} + 10,000 \times \frac{2}{100} + 10,000 \times \frac{2}{100} \end{bmatrix}$$

$$= \begin{bmatrix} 100 + 200 + 120 \\ 400 + 200 + 200 \end{bmatrix} \begin{array}{c} X \\ Y \end{array} = \begin{bmatrix} 420 \\ 800 \end{bmatrix} \begin{array}{c} X \\ Y \end{array}$$

$\therefore$ Total amount donated by X = ₹ **420** and Total amount donated by Y = ₹ **800**

Or

Sale matrix for A, B and C is

$$\begin{array}{c} \begin{array}{ccc} \text{Paper bags} & \text{Scrap-books} & \text{Pastel-sheets} \end{array} \\ \begin{bmatrix} 25 & 10 & 30 \\ 20 & 15 & 30 \\ 25 & 18 & 35 \end{bmatrix} \end{array} \text{(With the help of quantities)}$$

Price martix is $\begin{bmatrix} 20 \\ 15 \\ 10 \end{bmatrix} \begin{array}{l} \text{Paper bags} \\ \text{Scrap-book} \\ \text{Pastel-sheets} \end{array}$

$\therefore$ Amount raised = Sale matrix × Price matrix $= \begin{bmatrix} 25 & 10 & 30 \\ 20 & 15 & 30 \\ 25 & 18 & 35 \end{bmatrix} \begin{bmatrix} 20 \\ 15 \\ 10 \end{bmatrix}$

$$= \begin{bmatrix} 500 + 150 + 300 \\ 400 + 225 + 300 \\ 500 + 270 + 350 \end{bmatrix} = \begin{bmatrix} 950 \\ 925 \\ 1120 \end{bmatrix}$$

Therefore, **Amount raised by School A = ₹ 950**
Amount raised by School B = ₹ 925
Amount raised by School C = ₹ 1125

21. Let $u = x^{\sin x}$

Taking log on both sides, we get

$\log u = \log x^{\sin x}$

$\log u = \sin x \,.\, \log x$ ...[$\because$ **$\log x^n = n \log x$**

Differentiating the above w.r.t. x,

$$\frac{1}{u}\left(\frac{du}{dx}\right) = \cos x \,.\, \log x + \frac{\sin x}{x}$$

$$\frac{du}{dx} = u\left(\cos x \log x + \frac{\sin x}{x}\right)$$

$$\frac{du}{dx} = x^{\sin x}\left(\cos x \log x + \frac{\sin x}{x}\right) \qquad \ldots(i)$$

Let $v = x^x$

Taking log on both sides, we get

$\log v = \log x^x$

$\log v = x \,.\, \log x$ ...[$\because$ **$\log x^n = n \log x$**

Differentiating the above w.r.t. x,

$$\frac{1}{v}\left(\frac{dv}{dx}\right) = \log x + 1$$

$$\frac{dv}{dx} = v(\log x + 1)$$

$$\frac{dv}{dx} = x^x(\log x + 1) \qquad \ldots(ii)$$

Now, $y = u + \sin v$

Differentiating the above w.r.t x, we get

$$\frac{dy}{dx} = \frac{du}{dx} + \cos v\left(\frac{dv}{dx}\right)$$

From (i) & (ii),

$$\boldsymbol{\frac{dy}{dx} = x^{\sin x}\left(\cos x \log x + \frac{\sin x}{x}\right) + \cos x^x . x^x (\log x + 1)}$$

22. We have, $y = (x^3 - 1)(x - 2)$

$= (x - 1)(x^2 + x + 1)(x - 2)$ ...[$\because$ **$(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$**)

Since the above curve intersects at x-axis,

So, we have $y = 0$

$\Rightarrow \quad (x - 1)(x^2 + x + 1)(x - 2) = 0$

$\therefore \quad x = 1$ or $x = 2$

Now, $\left(\frac{dy}{dx}\right) = x^3 - 1 + (x - 2)(3x^2) = 4x^3 - 6x^2 - 1$

At $x = 1$, $\left(\frac{dy}{dx}\right) = 4(1)^3 - 6(1)^2 - 1 = 4 - 6 - 1 = -3$

At $x = 2$, $\left(\frac{dy}{dx}\right) = 4(2)^3 - 6(2)^2 - 1 = 32 - 24 - 1 = 7$

The required equations of tangents:

At point (1, 0) and slope = –3

$(y - y_1) = m(x - x_1)$

$(y - 0) = -3(x - 1)$

$\therefore \quad \boldsymbol{y = -3x + 3}$

At point (2, 0) and slope = 7

$(y - y_1) = m(x - x_1)$

$(y - 0) = 7(x - 2)$

$\therefore \quad \boldsymbol{y = 7x - 14}$

23.
$$\int\left(\frac{1-x}{1+x}\right)^2 e^x\,dx = \int\left(\frac{1-x}{1+x}\right)^2 e^x\,dx = \int\left(\frac{1+x^2-2x}{(1+x^2)^2}\right)e^x\,dx$$

$$= \int\left[\frac{(1+x^2)}{(1+x^2)^2} - \frac{2x}{(1+x^2)^2}\right]e^x\,dx$$

$$= \int\left[\frac{1}{1+x^2} + \frac{-2x}{(1+x^2)^2}\right]e^x\,dx$$

$$= \int [f(x) + f'(x)]\, e^x\, dx \qquad \ldots \left[\begin{aligned} &\textbf{Where } f(x) = \frac{1}{(1+x^2)} \\ &\therefore\ f'(x) = \frac{-1}{(1+x^2)^2} \times 2x \end{aligned} \right.$$

$$= e^x f(x) + C$$

$$= e^x \cdot \frac{\mathbf{1}}{\mathbf{1 + x^2}} + \mathbf{C}$$

Or

$$I = \int \frac{(x^2 + \sin^2 x)\sec^2 x}{1+x^2}\, dx$$

$$I = \int \frac{(x^2 + \sin^2 x + 1 - 1)\sec^2 x}{1+x^2}\, dx$$

$$= \int \left\{ \frac{(1+x^2) + (\sin^2 x - 1)}{1+x^2} \right\} \sec^2 x\, dx$$

$$= \int \left\{ \frac{1+x^2}{1+x^2} + \frac{(\sin^2 x - 1)}{1+x^2} \right\} \sec^2 x\, dx = \int \left[1 - \frac{\cos^2 x}{1+x^2} \right] \sec^2 x\, dx \qquad \ldots[\because \sin^2 x + \cos^2 x = 1$$

$$= \int \left[\sec^2 x - \frac{1}{1+x^2} \right] dx$$

$\therefore$ **I = tan x – tan^{-1}x + c**

24. Given equation of circle is

$x^2 + y^2 \le 1$

Let $x^2 + y^2 = 1$

$\Rightarrow$ $y = \sqrt{1 - x^2}$

x	0	±1
y	±1	0

Given equation of line is

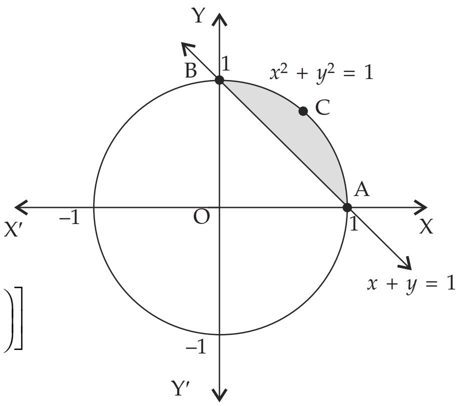
$x + y \ge 1$

Let $x + y = 1$

$y = 1 - x$

x	1	0
y	0	1

Area of bounded region ACBA = [Ar(OACBO) – Ar(OABO)]



$$= \int_0^1 \sqrt{1 - x^2}\, dx - \int_0^1 (1 - x)\, dx$$

$$= \frac{1}{2}\left[x\sqrt{1-x^2} + 1\sin^{-1}\frac{x}{1} \right]_0^1 - \left[x - \frac{x^2}{2} \right]_0^1$$

$$\left\{ \because \int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\frac{x}{a} \right] + c \right.$$

$$= \frac{1}{2}\{[1\sqrt{1-1} + \sin^{-1} 1] - [0 + \sin^{-1} 0]\} - \left[\left(1 - \frac{1}{2}\right) - \left(0 - \frac{0}{2}\right) \right]$$

$$= \frac{1}{2}\left[\left(0 + \frac{\pi}{2}\right) - (0 + 0) \right] - \left(\frac{2-1}{2} \right)$$

$$= \left(\frac{\pi}{4} - \frac{1}{2} \right) \textbf{ sq. units}$$

25. $$\left[x\sin^2\left(\frac{y}{x}\right) - y \right] dx + x dy = 0$$

$$x dy = -\left[x\sin^2\left(\frac{y}{x}\right) - y \right] dx$$

$$\frac{dy}{dx} = \frac{-x\sin^2\left(\frac{y}{x}\right)}{x} + \frac{y}{x}$$

$$v + x\frac{dv}{dx} = -\sin^2 v + v \qquad \ldots\left[\begin{array}{l}\textbf{Let } y = vx \Rightarrow \frac{y}{x} = v \\ \frac{dy}{dx} = v.1 + x.\frac{dv}{dx}\end{array}\right.$$

$$x\frac{dv}{dx} = -\sin^2 v$$

$$\frac{dv}{-\sin^2 v} = \frac{dx}{x}$$

Integrating both sides, we get

$$-\int \text{cosec}^2 v\, dv = \int \frac{dx}{x}$$

$$-(-\cot v) = \log|x| + c$$

$$\therefore \quad \boldsymbol{\cot\left(\frac{y}{x}\right) = \log|x| + c}$$

26. $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors

$\therefore \quad |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ $\ldots(i)$

$\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ **(given)**

$\therefore \quad \vec{a}.\vec{b} = 0, \quad \vec{a}.\vec{c} = 0$

$\therefore \quad \vec{a} \perp \vec{b}, \quad \vec{a} \perp \vec{c}$

$\therefore \quad \vec{a}$ is perpendicular to the plane of $\vec{b}$ and $\vec{c}$.

$\Rightarrow \quad \vec{a}$ is parallel to $(\vec{b} \times \vec{c})$

$\Rightarrow \quad \vec{a} = \lambda(\vec{b} \times \vec{c})$ $\ldots(ii)$

where λ is a scalar

$|\vec{a}| = |\lambda(\vec{b} \times \vec{c})|$

$|\vec{a}| = |\lambda|\,|(\vec{b} \times \vec{c})|$

$1 = |\lambda|\left(\frac{1}{2}\right)$

$|\lambda| = 2$

$\lambda = \pm 2$

Putting the value of λ in (ii), we get

$\boldsymbol{\vec{a} = \pm 2(\vec{b} \times \vec{c})}$ **(Hence proved)**

27. Vector equation of the line passing through (1, 2, 3) and (–3, 4, 3) is

$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$ where $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = -3\hat{i} + 4\hat{j} + 3\hat{k}$

$\Rightarrow \quad \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda[(-3-1)\hat{i} + (4-2)\hat{j} + (3-3)\hat{k}]$

$\Rightarrow \quad \boldsymbol{\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-4\hat{i} + 2\hat{j})}$ $\ldots(i)$

Equation of z-axis is $\vec{r} = \mu \hat{k}$ ...(*ii*)

Since $(-4\hat{i} + 2\hat{j}) \cdot \hat{k} = 0$

$\therefore$ **Line (*i*) is $\perp$ to z-axis.**

28. Total number of cards = 52

Total number of ace cards = 4

Let p = P(an ace card) = $\frac{4}{52} = \frac{1}{13}$ $\qquad \therefore \; q = 1 - p = 1 - \frac{1}{13} = \frac{12}{13}$

Let X be the number of ace cards.

Here X can take values 0, 1, 2.

$$P(X = 0) = q^2 = \left(\frac{12}{13}\right)^2 = \frac{144}{169}$$

$$P(X = 1) = 2qp = 2\left(\frac{12}{13}\right)\left(\frac{1}{13}\right) = \frac{24}{169}$$

$$P(X = 2) = p^2 = \left(\frac{1}{13}\right)^2 = \frac{1}{169}$$

$\therefore$ **Probability distribution is**

X	**0**	**1**	**2**
P(X)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Or

Let A : Both the children are boys

B : Atleast one of them is a boy

Now, S (Sample space) = {*bb, bg, gb, gg*},

A = {*bb*}, then P(A) = $\frac{1}{4}$; B = {*bb, bg, gb*}, then P(B) = $\frac{3}{4}$; $A \cap B$ = {*bb*}, then $P(A \cap B) = \frac{1}{4}$

$\therefore$ **Required probability, P(A | B) =** $\frac{P(A \mid B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \mathbf{\frac{1}{3}}$

29. (*i*) For all $a, b \in A$

(a, b) R (a, b)

$a + b = b + a$, $\qquad \therefore$ **R is reflexive**

(*ii*) For $a, b, c, d \in A$

Let (a, b) R (c, d) $\qquad \therefore \; a + d = b + c$

$\Rightarrow \; c + b = d + a \;\Rightarrow\; (c, d)$ R (a, b) $\qquad \therefore$ **R is symmetric**

(*iii*) For $a, b, c, d, e, f \in A$

(a, b) R (c, d) and (c, d) R (e, f)

$\therefore \; a + d = b + c$...(*i*) and $c + f = d + e$ ...(*ii*)

Adding (*i*) and (*ii*),

$a + d + c + f = b + c + d + e$

$\Rightarrow \; a + f = b + c \;\Rightarrow\; (a, b)$ R (e, f) $\qquad \therefore$ **R is transitive**

Hence R is an **equivalence relation** and equivalence class [(2, 5)] is

{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

30. L.H.L. $f'\left(\frac{1}{2}\right) = \lim_{x \to \frac{1}{2}^-} \dfrac{f(x) - f\left(\frac{1}{2}\right)}{x - \frac{1}{2}}$

$= \lim_{x \to \frac{1}{2}^-} \dfrac{2x - 1 - 0}{\frac{(2x-1)}{2}}$ ...$\left[\begin{array}{l} f(x) = 3 - 6x \\ \text{When } x = \frac{1}{2},\ f\left(\frac{1}{2}\right) = 3 - 6x = 3 - 3 = 0 \end{array}\right.$

$= 2 \lim_{x \to \frac{1}{2}^-} \dfrac{(2x-1)}{(2x-1)} = 2(1) = 2$

R.H.L. $f'\left(\frac{1}{2}\right) = \lim_{x \to \frac{1}{2}^+} \dfrac{f(x) - f\left(\frac{1}{2}\right)}{x - \frac{1}{2}} = \lim_{x \to \frac{1}{2}^+} \dfrac{3 - 6x - 0}{\frac{2x - 1}{2}}$

$= \lim_{x \to \frac{1}{2}^+} \dfrac{-3(2x-1)}{(2x-1)} \times \dfrac{2}{1} = -6 \lim_{x \to \frac{1}{2}^+} \dfrac{(2x-1)}{(2x-1)} = -6(1) = -6$

Since L.H.L. $f'\left(\frac{1}{2}\right) \neq$ RHL $f'\left(\frac{1}{2}\right)$

Therefore, f is not differentiable at $x = \frac{1}{2}$.

31. $\lim_{x \to -\frac{\pi}{6}} f(x) = \lim_{x \to -\frac{\pi}{6}} \dfrac{\sqrt{3}\sin x + \cos x}{x + \frac{\pi}{6}}$

$= \lim_{x \to -\frac{\pi}{6}} \dfrac{2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right)}{x + \frac{\pi}{6}}$

$= \lim_{x \to -\frac{\pi}{6}} \dfrac{2\left(\sin x.\cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6}\right)}{x + \frac{\pi}{6}}$ ...[$\because$ **sin A cos B + cos A sin B = sin (A + B)**

$= 2\lim_{\left(x \to \frac{\pi}{6}\right) \to 0} \dfrac{\sin\left(x + \frac{\pi}{6}\right)}{x + \frac{\pi}{6}}$

$= 2 \times 1$ ...$\left[\because \lim_{\theta \to 0} \frac{\sin\theta}{0} = 1\right.$

$= 2$

At $x = -\frac{\pi}{6},\ f(x) = k \Rightarrow f\left(-\frac{\pi}{6}\right) = k$

Since $f(x)$ is continuous at $x = -\frac{\pi}{6}$ **...[Given**

$\lim_{x \to -\frac{\pi}{6}} f(x) = f\left(-\frac{\pi}{6}\right)$

$\therefore$ $\boldsymbol{k = 2}$

Or

$$y = \log\left[\sqrt{x} + \frac{1}{\sqrt{x}}\right]^2$$

$$y = 2\log\left[\sqrt{x} + \frac{1}{\sqrt{x}}\right] \qquad \ldots[\because \log x^n = n \log x$$

$$y = 2\log\left[\frac{x+1}{\sqrt{x}}\right]$$

$$y = 2[\log(x+1) - \log\sqrt{x}\,]$$

$$y = 2[\log(x+1) - \frac{1}{2}\log x]$$

Differentiating both sides w.r.t. x, we have

$$y_1 = 2\left[\frac{1}{x+1} - \frac{1}{2x}\right] = 2\left[\frac{2x - x - 1}{2x(x+1)}\right]$$

$$y_1 = \frac{x-1}{x(x+1)} \qquad \ldots(i)$$

Again differentiating both sides w.r.t. x, we get

$$y_2 = \frac{x(x+1) - (x-1)(2x+1)}{x^2(x+1)^2}$$

$$y_2 = \frac{x^2 + x - 2x^2 - x + 2x + 1}{x^2(x+1)^2}$$

$$y_2 = \frac{-x^2 + 2x + 1}{x^2(x+1)^2}$$

$$x(x+1)^2 y_2 = \frac{-x^2 + 2x + 1}{x}$$

$$x(x+1)^2 y_2 = \frac{2x - (x^2 - 1)}{x} = \frac{2x - (x-1)(x-1)}{x}$$

$$\Rightarrow \quad x(x+1)^2 y_2 = 2 - \frac{(x+1)(x-1)}{x}$$

$$\Rightarrow \quad x(x+1)^2 y_2 = 2 - (x+1)^2 y_1 \qquad \ldots[\text{Using } (i)$$

$\therefore$ $\boldsymbol{x(x+1)^2 y_2 + (x+1)^2 y_1 = 2}$ **(Hence proved)**

32. We have $f(x) = -3\log(1+x) + 4\log(2+x) - \dfrac{4}{2+x}$

$$\therefore \quad f'(x) = -\frac{3}{1+x} + \frac{4}{2+x} + \frac{4}{(2+x)^2} = \frac{-3(2+x)^2 + 4(1+x)(2+x) + 4(1+x)}{(1+x)(2+x)^2}$$

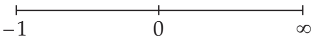
$$= \frac{-3(4 + x^2 + 4x) + 4(2 + x + 2x + x^2) + 4(1+x)}{(1+x)(2+x)^2}$$

$$= \frac{-12 - 3x^2 - 12x + 4x^2 + 12x + 8 + 4 + 4x}{(1+x)(2+x)^2}$$

$$= \frac{x^2 + 4x}{(1+x)(2+x)^2} = \frac{x(x+4)}{(1+x)(2+x)^2}$$

For stationary points, $f'(x) = 0$

$$x(x+4) = 0$$



$\boldsymbol{x = 0}$ **[$\boldsymbol{x \neq -4}$, as $\boldsymbol{-4 \notin (-1, \infty)}$]**

Interval	*Test value*	*Sign of $f'(x)$*	*Nature*
$(-1, 0)$	$x = -0.5$	$\dfrac{(-ve)(+ve)}{(+ve)(+ve)} = -\text{ve}$	Decreasing
$(0, \infty)$	$x = 1$	$\dfrac{(+ve)(+ve)}{(+ve)(+ve)} = +\text{ve}$	Increasing

$\therefore$ f is **Strictly Decreasing** in **(–1, 0]** and f is **Strictly Increasing** in **[0, ∞).**

33. Let $\text{I} = \displaystyle\int \frac{\sec x}{1+\operatorname{cosec} x}\,dx = \int \frac{\frac{1}{\cos x}}{1+\frac{1}{\sin x}}\,dx = \int \frac{\sin x}{\cos x\,(1+\sin x)}\,dx$

$$= \int \frac{\sin x \cos x}{\cos^2 x\,(1+\sin x)}\,dx = \int \frac{\sin x \cos x}{(1-\sin^2 x)(1+\sin x)}\,dx$$

$$= \int \frac{\sin x \cos x}{(1+\sin x)(1-\sin x)(1+\sin x)}\,dx = \int \frac{\sin x \cos x}{(1+\sin x)^2(1-\sin x)}\,dx$$

$\therefore$ $\quad \text{I} = \displaystyle\int \frac{t\,dt}{(1+t)^2(1-t)}$ **…[Let $\boldsymbol{\sin x = t;\ \cos x\,dx = dt}$**

Now, $\displaystyle\int \frac{t}{(1+t)^2(1-t)} = \frac{\text{A}}{(1+t)} + \frac{\text{B}}{(1+t)^2} + \frac{\text{C}}{(1-t)}$

$$t = \text{A}(1+t)(1-t) + \text{B}(1-t) + \text{C}(1+t)^2$$

Put $t = 0$

$0 = \text{A} + \text{B} + \text{C}$

$0 = \text{A} - \dfrac{1}{2} + \dfrac{1}{4}$

…[From (*i*) and (*ii*)

$\dfrac{1}{2} - \dfrac{1}{4} = \text{A}$

$\dfrac{1}{4} = \text{A}$

Put $t = 1$

$1 = \text{C}(2)^2$

$\dfrac{1}{4} = \text{C} \quad \ldots(i)$

Put $t = -1$

$-1 = \text{B}(2)$

$-\dfrac{1}{2} = \text{B} \quad \ldots(ii)$

$\therefore$ $\quad \displaystyle\int \frac{t}{(1+t)^2(1-t)}\,dt = \frac{1}{4}\int \frac{dt}{1+t} + \frac{-1}{2}\int \frac{1}{(1+t)^2}\,dt + \frac{1}{4}\int \frac{1}{(1-t)}\,dt$

$$= \frac{1}{4}\log|1+t| + \frac{-1}{2} \times \frac{-1}{1+t} - \frac{1}{4}\log|1-t| + \text{C}$$

$$= \boldsymbol{\frac{1}{4}\log|1+\sin x| + \frac{1}{2(1+\sin x)} - \frac{1}{4}\log|1-\sin x| + \text{C}}$$

34. Given equations of parabola and line are

$y^2 = x$

$\Rightarrow \quad y = \sqrt{x} \quad \ldots(i)$

x	0	4
y	0	±2

$x + y = 2$

$\Rightarrow \quad y = 2 - x \quad \ldots(ii)$

x	0	2
y	2	0

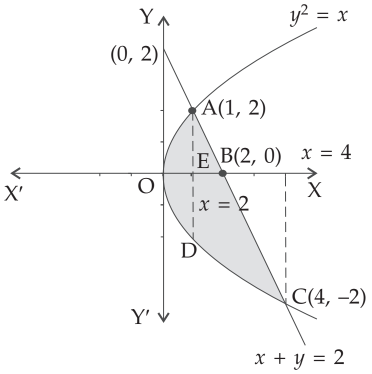
Solving (*i*) and (*ii*), we get

$(2-x)^2 = x \quad \Rightarrow 4 + x^2 - 4x = x$

$\Rightarrow \quad x^2 - 5x + 4 = 0 \quad \Rightarrow x^2 - 4x - x + 4 = 0$

$\Rightarrow \quad x(x-4)-1(x-4)=0$

$\Rightarrow \quad (x-1)(x-4)=0 \qquad\qquad \therefore\ x=1, x=4$



Area of the bounded region OABCDO

= Area of region OAE + Area of region ABE

+ Area of region OBCDO

$= A_1 + A_2 + A_3$

$$\therefore \quad A_1 = \int_0^1 \sqrt{x}\,dx = \left[\frac{x^{3/2}}{3/2}\right]_0^1 = \frac{2}{3}\left[(1)^{3/2}-0\right] = \frac{2}{3} \qquad \ldots(i)$$

$$A_2 = \int_1^2 (2-x)\,dx = \left[2x-\frac{x^2}{2}\right]_1^2 = \left(4-\frac{4}{2}\right)-\left(2-\frac{1}{2}\right)$$

$$= 2-\frac{3}{2}=\frac{1}{2} \qquad \ldots(ii)$$

$$A_3 = \left|\int_0^4 \sqrt{x}\,dx\right| - \left|\int_2^4 (2-x)\,dx\right| = \left|\left[\frac{x^{3/2}}{3/2}\right]_0^4\right| - \left|\left[2x-\frac{x^2}{2}\right]_2^4\right|$$

$$= \left|\left[\frac{2}{3}(4)^{3/2}-0\right]\right| \left|\left[(8-8)-(4-2)\right]\right| = \frac{2}{3}(8)-2 = \frac{16}{3}-2 = \frac{10}{3} \qquad \ldots(iii)$$

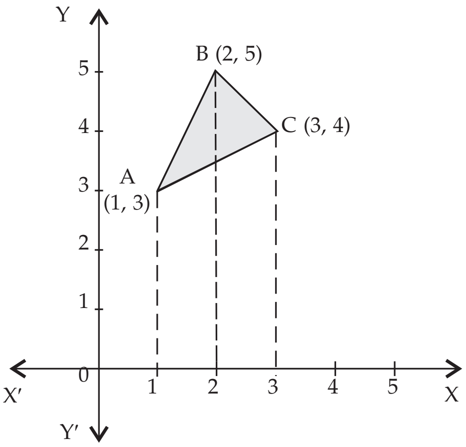
$\therefore$ **Area of shaded region** $= \frac{2}{3}+\frac{1}{2}+\frac{10}{3} = \frac{4+3+20}{6} = \frac{27}{6}$ **…[From (*i*), (*ii*) and (*iii*)**

$= \mathbf{\frac{9}{2}}$ **sq. units**

Or

We have, A (1, 3), B (2, 5) and C (3, 4)

Using equation of line $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$



Equation of AB, $\begin{vmatrix} x & y & 1 \\ 1 & 3 & 1 \\ 2 & 5 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad x(3-5)-y(1-2)+1(5-6)=0$

$\Rightarrow \quad -2x+y-1=0 \quad \Rightarrow y=2x+1$

Equation of BC, $\begin{vmatrix} x & y & 1 \\ 2 & 5 & 1 \\ 3 & 4 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad x(5-4)-y(2-3)+1(8-15)=0$

$\Rightarrow \quad x+y-7=0 \quad \Rightarrow y=7-x$

Equation of CA, $\begin{vmatrix} x & y & 1 \\ 3 & 4 & 1 \\ 1 & 3 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad x(4-3)-y(3-1)+1(9-4)=0 \quad \Rightarrow \quad x-2y+5=0 \quad \Rightarrow \quad y=\frac{x+5}{2}$

Area of $\triangle$ABC $= \int_1^2 (2x+1)\,dx + \int_2^3 (7-x)\,dx - \int_1^3 \frac{x+5}{2}\,dx$

$$= \left(\frac{2x^2}{2}+x\right)_1^2 + \left(7x-\frac{x^2}{2}\right)_2^3 - \frac{1}{2}\left(\frac{x^2}{2}+5x\right)_1^3$$

$$= \left[((2)^2+2)-((1)^2+1)\right] + \left[\left(7(3)-\frac{9}{2}\right)-\left(7(2)-\frac{2^2}{2}\right)\right] - \frac{1}{2}\left[\left(\frac{3^2}{2}+5(3)\right)-\left(\frac{1}{2}+5(1)\right)\right]$$

$$= (6)-(2)+\left(\frac{42-9}{2}\right)-\left(\frac{28-4}{2}\right)-\frac{1}{2}\left[\left(\frac{9+30}{2}\right)-\left(\frac{1+10}{2}\right)\right]$$

$$= 4+\frac{33}{2}-\frac{24}{2}-\frac{1}{2}\left(\frac{39}{2}-\frac{11}{2}\right) = 4+\frac{9}{2}-\frac{1}{2}\left(\frac{28}{2}\right)$$

$$= 4+\frac{9}{2}-\frac{7}{1} = \frac{8+9-14}{2} = \mathbf{\frac{3}{2}}\ \textbf{sq. units}$$

35. $\frac{dx}{dy} = \frac{y\tan y - x\tan y - xy}{y\tan y}$

$\Rightarrow \quad \frac{dx}{dy} = \frac{y\tan y}{y\tan y} - \frac{x\tan y}{y\tan y} - \frac{xy}{y\tan y} \qquad \Rightarrow \quad \frac{dx}{dy} + \left(\frac{1}{y}+\frac{1}{\tan y}\right)x = 1 \qquad \ldots(i)$

Equation (*i*) is a linear differential equation of the form

$\frac{dx}{dy} + \mathrm{P}x = \mathrm{Q}$, where 'P' $= \frac{1}{y}+\frac{1}{\tan y}$ and 'Q' $= 1$

$\therefore \quad$ I.F. $= e^{\int\left(\frac{1}{y}+\cot y\right)dy} = e^{[\log y + \log \sin y]} = e^{\log(y\sin y)} = y\sin y$

Now, $x \times \text{I.F.} = \int (\mathrm{Q}\times \text{I.F.})dy$

$\Rightarrow \quad xy\sin y = \int y\sin y\, dy$

$\Rightarrow \quad xy\sin y = y\int \sin y\, dy - \left[\int \frac{dy}{dy}(y)\times \int \sin y\, dy\right]$

$\Rightarrow \quad xy\sin y = -y\cos y - \int -\cos y\, dy$

$\Rightarrow \quad xy\sin y = -y\cos y + \sin y + \mathrm{C}$

$\therefore \quad \boldsymbol{x = \frac{\sin y - y\cos y + \mathrm{C}}{y\sin y}}$ **is the general solution of differential equation.**

36. We have, $\mathrm{A} = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

$|\mathrm{A}| = 2(-2-0) - 3(2-0) + 4(1) = -4-6+4 = -6 \neq 0 \quad \therefore \mathrm{A}^{-1}$ exists.

Now, Cofactors of A are

$\mathrm{A}_{11} = -2-0 = -2$; $\quad \mathrm{A}_{12} = -(2-0) = -2$; $\quad \mathrm{A}_{13} = 1-0 = 1$

$\mathrm{A}_{21} = -(6-4) = -2$; $\quad \mathrm{A}_{22} = (4-0) = 4$; $\quad \mathrm{A}_{23} = -(2-0) = -2$

$\mathrm{A}_{31} = (0-(-4)) = 4$; $\quad \mathrm{A}_{32} = -(0-4) = 4$; $\quad \mathrm{A}_{33} = -2-3 = -5$

$$\text{Adj A} = \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{Adj } A) = \frac{-1}{6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix} \qquad \ldots(i)$$

Rearranging the given equations: $2x + 3y + 4z = 17$; $x - y = 3$; $y + 2z = 7$

Writing in matrix form, $\underset{A}{\begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}}$

$X = A^{-1}B$ **...Where** $\left[\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 17 \\ 3 \\ 7 \end{pmatrix} \right.$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}\begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix} \qquad \text{...[From } (i)$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{6}\begin{bmatrix} -34 - 6 + 28 \\ -34 + 12 + 28 \\ 17 - 6 - 35 \end{bmatrix} = \frac{-1}{6}\begin{bmatrix} -12 \\ 6 \\ -24 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \qquad \therefore \ \boldsymbol{x = 2, y = -1, z = 4}$$

Or

(*i*) $AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 2+4+0 & 2-2-0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here, $AB = 6I$

$A^{-1}(AB) = A^{-1}(6I) \quad \Rightarrow \quad B = 6A^{-1} \quad \Rightarrow \quad \frac{1}{6}B = A^{-1} \qquad \ldots(i)$

(*ii*) Writing in matrix form

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\begin{aligned} A \quad X &= C \\ A^{-1}(AX) &= A^{-1}C \\ X &= A^{-1}C \end{aligned} \qquad \left[\textbf{where } \mathbf{C} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} \right.$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}BC = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix} \qquad \text{[from } (i)$$

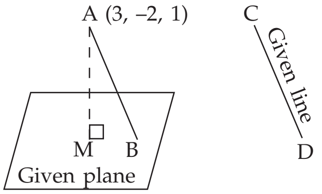
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\therefore$ $\mathbf{x = 2, y = -1, z = 4}$

37. **To Find :** (I) AB, such that AB || CD

(II) Point M (coordinate of point M)

Part I : Given line : $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$



Direction ratios of given line CD are 2, –3, 1

$\therefore$ Direction ratios of line AB are 2, –3, 1 **[∵ AB || CD]**

Equation of line AB using $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$,

$\Rightarrow \quad \frac{x-3}{2} = \frac{y+2}{-3} = \frac{z-1}{1} = \lambda$ (Let)

$x-3 = 2\lambda$	$y+2 = -3\lambda$	$z-1 = \lambda$
$x = 2\lambda + 3$	$y = -3\lambda - 2$	$z = \lambda + 1$

Let B$(2\lambda + 3, -3\lambda - 2, \lambda + 1)$ **...(*i*)** be any point on the line AB

Then, it lies on the given plane, $3x + y - z + 2 = 0$

$3(2\lambda + 3) + (-3\lambda - 2) - (\lambda + 1) + 2 = 0$

$\Rightarrow \quad 6\lambda + 9 - 3\lambda - 2 - \lambda - 1 + 2 = 0$

$\Rightarrow \quad 2\lambda + 8 = 0 \qquad \Rightarrow \mathbf{\lambda = -4}$

By putting, $\lambda = -4$ in equation (*i*), we get, Point B $(-8 + 3, 12 - 2, -4 + 1)$

$(-5, \quad 10, \quad -3)$

Hence, the required distance, AB $= \sqrt{(-5-3)^2 + (10+2)^2 + (-3-1)^2}$

$= \sqrt{64 + 144 + 16} = \sqrt{224}$

$= \sqrt{4 \times 4 \times 14} = \mathbf{4\sqrt{14}}$ **units**

Part II : Direction ratios of the normal to the given plane 3, 1, –1

$\therefore$ Direction ratios of line AM are 3, 1, –1

Equation of line AM is using $\frac{x-3}{3} = \frac{y+2}{1} = \frac{z-1}{-1} = \beta$ (Let)

$x-3 = 3\beta$	$y+2 = \beta$	$z-1 = -\beta$
$x = 3\beta + 3$	$y = \beta - 2$	$z = -\beta + 1$

Let M$(3\beta + 3, \beta - 2, -\beta + 1)$ **...(*ii*)** be any point on the line AM

Then, it lies on the given plane, $3x + y - z + 2 = 0$

$3(3\beta + 3) + \beta - 2 + \beta - 1 + 2 = 0$

$\Rightarrow \quad 9\beta + 9 + 2\beta - 1 = 0$

$\Rightarrow \quad 11\beta + 8 = 0 \qquad \Rightarrow \beta = \frac{-8}{11}$

By putting $\beta = \frac{-8}{11}$ in (*ii*), we get, Point M $\left(\frac{-24}{11} + 3, \frac{-8}{11} - 2, \frac{8}{11} + 1\right) = \left(\frac{9}{11}, \frac{-30}{11}, \frac{19}{11}\right)$

Hence, foot of the perpendicular from the given point upon the given plane is $\left(\mathbf{\frac{9}{11}, \frac{-30}{11}, \frac{19}{11}}\right)$**.**

Or

Given planes are $x - y + 2z - 3 = 0$ ...(*i*)

$2x - y - 3z = 0$ ...(*ii*)

Direction ratios of the normal to the plane (*i*) are 1, –1, 2

Direction ratios of the normal to the plane (*ii*) are 2, –1, –3

$\therefore \quad \vec{b}_1 = \hat{i} - \hat{j} + 2\hat{k}, \ \vec{b}_2 = 2\hat{i} - \hat{j} - 3\hat{k}$

$$\vec{n} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & -1 & -3 \end{vmatrix} = \hat{i}(3+2) - \hat{j}(-3-4) + \hat{k}(-1+2)$$

$$= 5\hat{i} + 7\hat{j} + \hat{k}$$

Point (4, –3, 2) $\quad\quad \therefore \ \vec{a} = 4\hat{i} - 3\hat{j} + 2\hat{k}$

The vector equation of the plane is $(\vec{r} - \vec{a}).\vec{N} = 0$

$\Rightarrow \quad \vec{r}.\vec{N} - \vec{a}.\vec{N} = 0$

$\therefore \quad \vec{r}.\vec{N} = \vec{a}.\vec{N}$

$$\vec{r}.(5\hat{i} + 7\hat{j} + \hat{k}) = (4\hat{i} - 3\hat{j} + 2\hat{k})(5\hat{i} + 7\hat{j} + \hat{k})$$

$$\vec{r}.(5\hat{i} + 7\hat{j} + \hat{k}) = 20 - 21 + 2$$

Therefore, $\boldsymbol{\vec{r}.(5\hat{i} + 7\hat{j} + \hat{k}) = 1}$**, which is the required equation of the plane.** …(*iii*)

Given line is

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} + 3\hat{j} - 9\hat{k}) \quad \ldots(iv)$$

$$\vec{r} = (1+\lambda)\hat{i} + (2+3\lambda)\hat{j} + (-1-9\lambda)\hat{k}$$

The position vector of any points on the given line (*iv*) is,

$$(1+\lambda)\hat{i} + (2+3\lambda)\hat{j} + (-1-9\lambda)\hat{k} \quad \ldots(v)$$

From (*iii*) and (*v*), $(1+\lambda)5 + (2+3\lambda)7 + (-1-9\lambda)1 = 1$

$\Rightarrow \quad 5 + 5\lambda + 14 + 21\lambda - 1 - 9\lambda = 1$

$\Rightarrow \quad 17\lambda + 18 = 1 \quad\quad \Rightarrow \quad 17\lambda = -17 \quad\quad \Rightarrow \quad \lambda = -1$

Putting the value of $\lambda = -1$ in (*v*), we get

$$(1-1)\hat{i} + (2-3)\hat{j} + (-1+9)\hat{k} = 0\hat{i} - \hat{j} + 8\hat{k} \text{ or } -\hat{j} + 8\hat{k}$$

$\therefore$ **The position vector of the required point of intersection is** $\boldsymbol{-\hat{j} + 8\hat{k}}$**.**

38. To maximise: $P = (9 - 1.2)x + (8 - 0.9)y = 7.8x + 7.1y$

Subject to the constraints:

$$\frac{x}{4} + \frac{y}{3} \le 90 \text{ or } 3x + 4y \le 1080 \quad \ldots(i)$$

$$\frac{x}{8} + \frac{y}{3} \le 80 \text{ or } 3x + 8y \le 1920 \quad \ldots(ii)$$

$$x \le 200 \quad \ldots(iii)$$

$x \ge 0, y \ge 0$

Let $3x + 4y = 1080$

x	360	0
y	0	270

Test: When $x = 0, y = 0$

$0 \le 1080$ (True)

Let $3x + 8y = 1920$

x	640	0
y	0	240

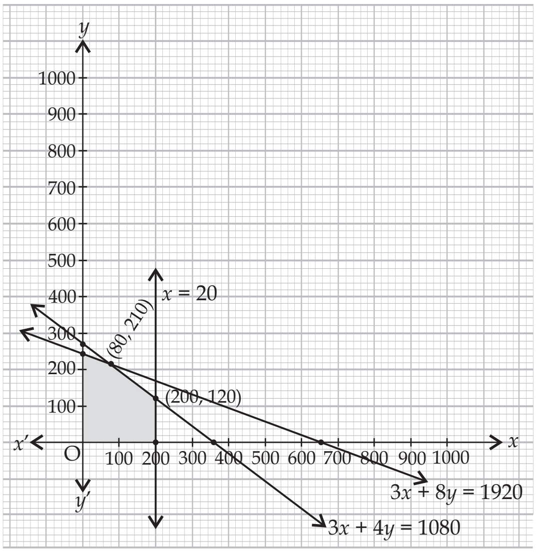
Test: When $x = 0, y = 0$

$0 \le 1920$ (True)

Let $x = 200$

Test: When $x = 0, y = 0$

$0 \le 200$ (True)



Corner Points	*Profit*	
(0, 0)	0	
(200, 120)	**2412**	**(Maximum)**
(0, 240)	1704	
(200, 0)	1560	
(80, 210)	2115	

$\therefore$ **$x = 200$, $y = 120$**

Maximum profit = 2412

Or

Maximize, $Z = 50x + 60y$

Subject to the constraints,

$20x + 10y \leq 180$ ...(*i*)

$10x + 20y \leq 120$ ...(*ii*)

$10x + 30y \leq 150$ ...(*iii*)

$x, y \geq 0$

Let $20x + 10y = 180$

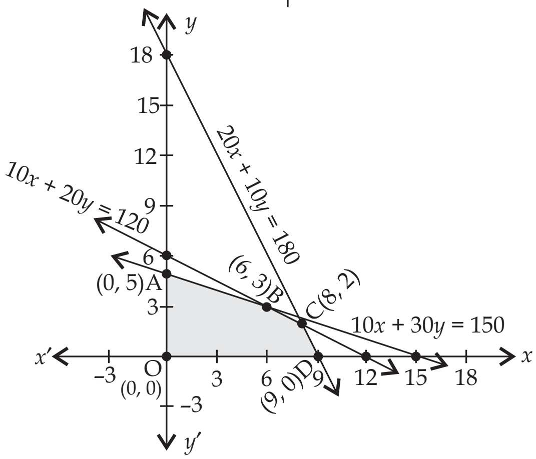
x	0	9
y	18	0

Let $10x + 20y = 120$

x	0	12
y	6	0

Let $10x + 30y = 150$

x	0	15
y	5	0



Corner Points	*z = ₹(50x + 60y)*	
O(0, 0)	0 + 0 = 0	
A(0, 5)	0 + 300 = 300	
B(6, 3)	300 + 180 = 480	
C(8, 2)	400 + 120 = **520**	**(Maximum)**
D(9, 0)	450 + 0 = 450	

$\therefore$ **$x = 8$ and $y = 2$**

Maximum profit = 520

❒❒❒❒