14. Quadratic Equations

Exercise 14.1

1. Question

Solve the following quadratic equations by factorization method

$x^2 + 1 = 0$

Answer

Given $x^2 + 1 = 0$

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$x^2 - i^2 = 0$

$\Rightarrow (x + i)(x - i) = 0$ [$\because a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow x + i = 0$ or $x - i = 0$

$\Rightarrow x = -i$ or $x = i$

$\therefore x = \pm i$

Thus, the roots of the given equation are $\pm i$.

2. Question

Solve the following quadratic equations by factorization method

$9x^2 + 4 = 0$

Answer

Given $9x^2 + 4 = 0$

$\Rightarrow 9x^2 + 4 \times 1 = 0$

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$9x^2 + 4(-i^2) = 0$

$\Rightarrow 9x^2 - 4i^2 = 0$

$\Rightarrow (3x)^2 - (2i)^2 = 0$

$\Rightarrow (3x + 2i)(3x - 2i) = 0$ [$\because a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow 3x + 2i = 0$ or $3x - 2i = 0$

$\Rightarrow 3x = -2i$ or $3x = 2i$

$\Rightarrow x = -\frac{2}{3}i \text{ or } \frac{2}{3}i$

$\therefore x = \pm\frac{2}{3}i$

Thus, the roots of the given equation are $\pm\frac{2}{3}i$.

3. Question

Solve the following quadratic equations by factorization method

$x^2 + 2x + 5 = 0$

Answer

Given $x^2 + 2x + 5 = 0$

$\Rightarrow x^2 + 2x + 1 + 4 = 0$

$\Rightarrow x^2 + 2(x)(1) + 1^2 + 4 = 0$

$\Rightarrow (x + 1)^2 + 4 = 0$ [$\because (a + b)^2 = a^2 + 2ab + b^2$]

$\Rightarrow (x + 1)^2 + 4 \times 1 = 0$

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$(x + 1)^2 + 4(-i^2) = 0$

$\Rightarrow (x + 1)^2 - 4i^2 = 0$

$\Rightarrow (x + 1)^2 - (2i)^2 = 0$

$\Rightarrow (x + 1 + 2i)(x + 1 - 2i) = 0$ [$\because a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow x + 1 + 2i = 0$ or $x + 1 - 2i = 0$

$\Rightarrow x = -1 - 2i$ or $x = -1 + 2i$

$\therefore x = -1 \pm 2i$

Thus, the roots of the given equation are $-1 \pm 2i$.

4. Question

Solve the following quadratic equations by factorization method

$4x^2 - 12x + 25 = 0$

Answer

Given $4x^2 - 12x + 25 = 0$

$\Rightarrow 4x^2 - 12x + 9 + 16 = 0$

$\Rightarrow (2x)^2 - 2(2x)(3) + 3^2 + 16 = 0$

$\Rightarrow (2x - 3)^2 + 16 = 0$ [$\because (a + b)^2 = a^2 + 2ab + b^2$]

$\Rightarrow (2x - 3)^2 + 16 \times 1 = 0$

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$(2x - 3)^2 + 16(-i^2) = 0$

$\Rightarrow (2x - 3)^2 - 16i^2 = 0$

$\Rightarrow (2x - 3)^2 - (4i)^2 = 0$

Since $a^2 - b^2 = (a + b)(a - b)$, we get

$(2x - 3 + 4i)(2x - 3 - 4i) = 0$

$\Rightarrow 2x - 3 + 4i = 0$ or $2x - 3 - 4i = 0$

$\Rightarrow 2x = 3 - 4i$ or $2x = 3 + 4i$

$$\Rightarrow x = \frac{3-4i}{2} \text{ or } \frac{3+4i}{2}$$

$$\Rightarrow x = \frac{3}{2} - 2i \text{ or } \frac{3}{2} + 2i$$

$$\therefore x = \frac{3}{2} \pm 2i$$

Thus, the roots of the given equation are $\frac{3}{2} \pm 2i$.

5. Question

Solve the following quadratic equations by factorization method

$x^2 + x + 1 = 0$

Answer

Given $x^2 + x + 1 = 0$

$$\Rightarrow x^2 + x + \frac{1}{4} + \frac{3}{4} = 0$$

$$\Rightarrow x^2 + 2(x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \frac{3}{4} = 0$$

$\Rightarrow \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} = 0$ [$\because (a + b)^2 = a^2 + 2ab + b^2$]

$$\Rightarrow \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \times 1 = 0$$

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$$\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}(-i^2) = 0$$

$$\Rightarrow \left(x + \frac{1}{2}\right)^2 - \frac{3}{4}i^2 = 0$$

$$\Rightarrow \left(x + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}i\right)^2 = 0$$

$\Rightarrow \left(x + \frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(x + \frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 0$ [$\because a^2 - b^2 = (a + b)(a - b)$]

$$\Rightarrow x + \frac{1}{2} + \frac{\sqrt{3}}{2}i = 0 \text{ or } x + \frac{1}{2} - \frac{\sqrt{3}}{2}i = 0$$

$$\Rightarrow x = -\frac{1}{2} - \frac{\sqrt{3}}{2}i = 0 \text{ or } x = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$\therefore x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

Thus, the roots of the given equation are $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.

6. Question

Solve the following quadratics

$4x^2 + 1 = 0$

Answer

Given $4x^2 + 1 = 0$

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$4x^2 - i^2 = 0$

$\Rightarrow (2x)^2 - i^2 = 0$

$\Rightarrow (2x + i)(2x - i) = 0$ [$\because a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow 2x + i = 0$ or $2x - i = 0$

$\Rightarrow 2x = -i$ or $2x = i$

$\Rightarrow x = -\frac{1}{2}i$ or $\frac{1}{2}i$

$\therefore x = \pm\frac{1}{2}i$

Thus, the roots of the given equation are $\pm\frac{1}{2}i$.

7. Question

Solve the following quadratics

$x^2 - 4x + 7 = 0$

Answer

Given $x^2 - 4x + 7 = 0$

$\Rightarrow x^2 - 4x + 4 + 3 = 0$

$\Rightarrow x^2 - 2(x)(2) + 2^2 + 3 = 0$

$\Rightarrow (x - 2)^2 + 3 = 0$ [$\because (a - b)^2 = a^2 - 2ab + b^2$]

$\Rightarrow (x - 2)^2 + 3 \times 1 = 0$

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$(x - 2)^2 + 3(-i^2) = 0$

$\Rightarrow (x - 2)^2 - 3i^2 = 0$

$\Rightarrow (x-2)^2 - \left(\sqrt{3}i\right)^2 = 0$

Since $a^2 - b^2 = (a + b)(a - b)$, we get

$\left(x - 2 + \sqrt{3}i\right)\left(x - 2 - \sqrt{3}i\right) = 0$

$\Rightarrow x - 2 + \sqrt{3}i = 0$ or $x - 2 - \sqrt{3}i = 0$

$\Rightarrow x = 2 - \sqrt{3}i$ or $x = 2 + \sqrt{3}i$

$\therefore x = 2 \pm \sqrt{3}i$

Thus, the roots of the given equation are $2 \pm \sqrt{3}i$.

8. Question

Solve the following quadratics

$x^2 + 2x + 2 = 0$

Answer

Given $x^2 + 2x + 2 = 0$

$\Rightarrow x^2 + 2x + 1 + 1 = 0$

$\Rightarrow x^2 + 2(x)(1) + 1^2 + 1 = 0$

$\Rightarrow (x + 1)^2 + 1 = 0$ [$\because (a + b)^2 = a^2 + 2ab + b^2$]

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$(x + 1)^2 + (-i^2) = 0$

$\Rightarrow (x + 1)^2 - i^2 = 0$

$\Rightarrow (x + 1)^2 - (i)^2 = 0$

$\Rightarrow (x + 1 + i)(x + 1 - i) = 0$ [$\because a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow x + 1 + i = 0$ or $x + 1 - i = 0$

$\Rightarrow x = -1 - i$ or $x = -1 + i$

$\therefore x = -1 \pm i$

Thus, the roots of the given equation are $-1 \pm i$.

9. Question

Solve the following quadratics

$5x^2 - 6x + 2 = 0$

Answer

Given $5x^2 - 6x + 2 = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, $a = 5$, $b = -6$ and $c = 2$

$$\Rightarrow x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(2)}}{2(5)}$$

$$\Rightarrow x = \frac{6 \pm \sqrt{36 - 40}}{10}$$

$$\Rightarrow x = \frac{6 \pm \sqrt{-4}}{10}$$

$$\Rightarrow x = \frac{6 \pm \sqrt{4(-1)}}{10}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{6 \pm \sqrt{4i^2}}{10}$$

$$\Rightarrow x = \frac{6 \pm \sqrt{(2i)^2}}{10}$$

$$\Rightarrow x = \frac{6 \pm 2i}{10}$$

$$\Rightarrow x = \frac{2(3 \pm i)}{10}$$

$$\Rightarrow x = \frac{3 \pm i}{5}$$

$$\therefore x = \frac{3}{5} \pm \frac{1}{5}i$$

Thus, the roots of the given equation are $\frac{3}{5} \pm \frac{1}{5}i$.

10. Question

Solve the following quadratics

$21x^2 + 9x + 1 = 0$

Answer

Given $21x^2 + 9x + 1 = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, a = 21, b = 9 and c = 1

$$\Rightarrow x = \frac{-9 \pm \sqrt{9^2 - 4(21)(1)}}{2(21)}$$

$$\Rightarrow x = \frac{-9 \pm \sqrt{81 - 84}}{42}$$

$$\Rightarrow x = \frac{-9 \pm \sqrt{-3}}{42}$$

$$\Rightarrow x = \frac{-9 \pm \sqrt{3(-1)}}{42}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{-9 \pm \sqrt{3i^2}}{42}$$

$$\Rightarrow x = \frac{-9 \pm \sqrt{\left(\sqrt{3}i\right)^2}}{42}$$

$$\Rightarrow x = \frac{-9 \pm \sqrt{3}i}{42}$$

$$\Rightarrow x = -\frac{9}{42} \pm \frac{\sqrt{3}}{42}i$$

$$\therefore x = -\frac{3}{14} \pm \frac{\sqrt{3}}{42}i$$

Thus, the roots of the given equation are $-\frac{3}{14} \pm \frac{\sqrt{3}}{42}i$.

11. Question

Solve the following quadratics

$x^2 - x + 1 = 0$

Answer

Given $x^2 - x + 1 = 0$

$\Rightarrow x^2 - x + \frac{1}{4} + \frac{3}{4} = 0$

$\Rightarrow x^2 - 2(x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \frac{3}{4} = 0$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} = 0$ [∵ $(a - b)^2 = a^2 - 2ab + b^2$]

$\Rightarrow \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} \times 1 = 0$

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}(-i^2) = 0$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 - \frac{3}{4}i^2 = 0$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}i\right)^2 = 0$

$\Rightarrow \left(x - \frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(x - \frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 0$ [∵ $a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow x - \frac{1}{2} + \frac{\sqrt{3}}{2}i = 0 \text{ or } x - \frac{1}{2} - \frac{\sqrt{3}}{2}i = 0$

$\Rightarrow x = \frac{1}{2} - \frac{\sqrt{3}}{2}i = 0 \text{ or } x = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$\therefore x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

Thus, the roots of the given equation are $\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.

12. Question

Solve the following quadratics

$x^2 + x + 1 = 0$

Answer

Given $x^2 + x + 1 = 0$

$\Rightarrow x^2 + x + \frac{1}{4} + \frac{3}{4} = 0$

$\Rightarrow x^2 + 2(x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \frac{3}{4} = 0$

$\Rightarrow \left(x+\frac{1}{2}\right)^2+\frac{3}{4}=0$ [∵ $(a + b)^2 = a^2 + 2ab + b^2$]

$\Rightarrow \left(x+\frac{1}{2}\right)^2+\frac{3}{4}\times 1=0$

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$\left(x+\frac{1}{2}\right)^2+\frac{3}{4}(-i^2)=0$

$\Rightarrow \left(x+\frac{1}{2}\right)^2-\frac{3}{4}i^2=0$

$\Rightarrow \left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}i\right)^2=0$

$\Rightarrow \left(x+\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(x+\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=0$ [∵ $a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow x+\frac{1}{2}+\frac{\sqrt{3}}{2}i=0 \text{ or } x+\frac{1}{2}-\frac{\sqrt{3}}{2}i=0$

$\Rightarrow x=-\frac{1}{2}-\frac{\sqrt{3}}{2}i=0 \text{ or } x=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$\therefore x=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$

Thus, the roots of the given equation are $-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$.

13. Question

Solve the following quadratics

$17x^2 - 8x + 1 = 0$

Answer

Given $17x^2 - 8x + 1 = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

Here, a = 17, b = –8 and c = 1

$\Rightarrow x=\frac{-(-8)\pm\sqrt{(-8)^2-4(17)(1)}}{2(17)}$

$\Rightarrow x=\frac{8\pm\sqrt{64-68}}{34}$

$\Rightarrow x=\frac{8\pm\sqrt{-4}}{34}$

$\Rightarrow x=\frac{8\pm\sqrt{4(-1)}}{34}$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{8 \pm \sqrt{4i^2}}{34}$$

$$\Rightarrow x = \frac{8 \pm \sqrt{(2i)^2}}{34}$$

$$\Rightarrow x = \frac{8 \pm 2i}{34}$$

$$\Rightarrow x = \frac{2(4 \pm i)}{34}$$

$$\Rightarrow x = \frac{4 \pm i}{17}$$

$$\therefore x = \frac{4}{17} \pm \frac{1}{17}i$$

Thus, the roots of the given equation are $\frac{4}{17} \pm \frac{1}{17}i$.

14. Question

Solve the following quadratics

$27x^2 - 10x + 1 = 0$

Answer

Given $27x^2 - 10x + 1 = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, a = 27, b = –10 and c = 1

$$\Rightarrow x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(27)(1)}}{2(27)}$$

$$\Rightarrow x = \frac{10 \pm \sqrt{100 - 108}}{54}$$

$$\Rightarrow x = \frac{10 \pm \sqrt{-8}}{54}$$

$$\Rightarrow x = \frac{10 \pm \sqrt{8(-1)}}{54}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{10 \pm \sqrt{8i^2}}{54}$$

$$\Rightarrow x = \frac{10 \pm \sqrt{\left(2\sqrt{2}i\right)^2}}{54}$$

$$\Rightarrow x = \frac{10 \pm 2\sqrt{2}i}{54}$$

$$\Rightarrow x = \frac{2\left(5 \pm \sqrt{2}i\right)}{54}$$

$\Rightarrow x = \frac{5 \pm \sqrt{2}i}{27}$

$\therefore x = \frac{5}{27} \pm \frac{\sqrt{2}}{27}i$

Thus, the roots of the given equation are $\frac{5}{27} \pm \frac{\sqrt{2}}{27}i$.

15. Question

Solve the following quadratics

$17x^2 + 28x + 12 = 0$

Answer

Given $17x^2 + 28x + 12 = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

Here, a = 17, b = 28 and c = 12

$\Rightarrow x = \frac{-28 \pm \sqrt{28^2 - 4(17)(12)}}{2(17)}$

$\Rightarrow x = \frac{-28 \pm \sqrt{784 - 816}}{34}$

$\Rightarrow x = \frac{-28 \pm \sqrt{-32}}{34}$

$\Rightarrow x = \frac{-28 \pm \sqrt{32(-1)}}{34}$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$x = \frac{-28 \pm \sqrt{32i^2}}{34}$

$\Rightarrow x = \frac{-28 \pm \sqrt{\left(4\sqrt{2}i\right)^2}}{34}$

$\Rightarrow x = \frac{-28 \pm 4\sqrt{2}i}{34}$

$\Rightarrow x = \frac{2\left(-14 \pm 2\sqrt{2}i\right)}{34}$

$\Rightarrow x = \frac{-14 \pm 2\sqrt{2}i}{17}$

$\therefore x = -\frac{14}{17} \pm \frac{2\sqrt{2}}{17}i$

Thus, the roots of the given equation are $-\frac{14}{17} \pm \frac{2\sqrt{2}}{17}i$.

16. Question

Solve the following quadratics

$21x^2 – 28x + 10 = 0$

Answer

Given $21x^2 – 28x + 10 = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, a = 21, b = –28 and c = 10

$$\Rightarrow x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4(21)(10)}}{2(21)}$$

$$\Rightarrow x = \frac{28 \pm \sqrt{784 - 840}}{42}$$

$$\Rightarrow x = \frac{28 \pm \sqrt{-56}}{42}$$

$$\Rightarrow x = \frac{28 \pm \sqrt{56(-1)}}{42}$$

We have $i^2 = –1$

By substituting $–1 = i^2$ in the above equation, we get

$$x = \frac{28 \pm \sqrt{56i^2}}{42}$$

$$\Rightarrow x = \frac{28 \pm \sqrt{\left(2\sqrt{14}i\right)^2}}{42}$$

$$\Rightarrow x = \frac{28 \pm 2\sqrt{14}i}{42}$$

$$\Rightarrow x = \frac{2\left(14 \pm \sqrt{14}i\right)}{42}$$

$$\Rightarrow x = \frac{14 \pm \sqrt{14}i}{21}$$

$$\Rightarrow x = \frac{14}{21} \pm \frac{\sqrt{14}}{21}i$$

$$\therefore x = \frac{2}{3} \pm \frac{\sqrt{14}}{21}i$$

Thus, the roots of the given equation are $\frac{2}{3} \pm \frac{\sqrt{14}}{21}i$.

17. Question

Solve the following quadratics

$8x^2 – 9x + 3 = 0$

Answer

Given $8x^2 – 9x + 3 = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x} = \frac{-\mathbf{b} \pm \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}$$

Here, a = 8, b = –9 and c = 1

$$\Rightarrow x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(8)(3)}}{2(8)}$$

$$\Rightarrow x = \frac{9 \pm \sqrt{81 - 96}}{16}$$

$$\Rightarrow x = \frac{9 \pm \sqrt{-15}}{16}$$

$$\Rightarrow x = \frac{9 \pm \sqrt{15(-1)}}{16}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{9 \pm \sqrt{15i^2}}{16}$$

$$\Rightarrow x = \frac{9 \pm \sqrt{\left(\sqrt{15}i\right)^2}}{16}$$

$$\Rightarrow x = \frac{9 \pm \sqrt{15}i}{16}$$

$$\therefore x = \frac{9}{16} \pm \frac{\sqrt{15}}{16}i$$

Thus, the roots of the given equation are $\frac{9}{16} \pm \frac{\sqrt{15}}{16}i$.

18. Question

Solve the following quadratics

$13x^2 + 7x + 1 = 0$

Answer

Given $13x^2 + 7x + 1 = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x} = \frac{-\mathbf{b} \pm \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}$$

Here, a = 13, b = 7 and c = 1

$$\Rightarrow x = \frac{-7 \pm \sqrt{7^2 - 4(13)(1)}}{2(13)}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{49 - 52}}{26}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{-3}}{26}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{3(-1)}}{26}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{-7 \pm \sqrt{3i^2}}{26}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{\left(\sqrt{3}i\right)^2}}{26}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{3}i}{26}$$

$$\therefore x = -\frac{7}{26} \pm \frac{\sqrt{3}}{26}i$$

Thus, the roots of the given equation are $-\frac{7}{26} \pm \frac{\sqrt{3}}{26}i$.

19. Question

$2x^2 + x + 1 = 0$

Answer

Given $2x^2 + x + 1 = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, a = 2, b = 1 and c = 1

$$\Rightarrow x = \frac{-1 \pm \sqrt{1^2 - 4(2)(1)}}{2(2)}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 8}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-7}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{7(-1)}}{4}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{-1 \pm \sqrt{7i^2}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{\left(\sqrt{7}i\right)^2}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{7}i}{4}$$

$$\therefore x = -\frac{1}{4} \pm \frac{\sqrt{7}}{4}i$$

Thus, the roots of the given equation are $-\frac{1}{4} \pm \frac{\sqrt{7}}{4}i$.

20. Question

Prove: $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

Answer

Given $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = \sqrt{3}$, $b = -\sqrt{2}$ and $c = 3\sqrt{3}$

$$\Rightarrow x = \frac{-(-\sqrt{2}) \pm \sqrt{(-\sqrt{2})^2 - 4(\sqrt{3})(3\sqrt{3})}}{2(\sqrt{3})}$$

$$\Rightarrow x = \frac{\sqrt{2} \pm \sqrt{2 - 36}}{2\sqrt{3}}$$

$$\Rightarrow x = \frac{\sqrt{2} \pm \sqrt{-34}}{2\sqrt{3}}$$

$$\Rightarrow x = \frac{-\sqrt{2} \pm \sqrt{34(-1)}}{2\sqrt{3}}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{-\sqrt{2} \pm \sqrt{34i^2}}{2\sqrt{3}}$$

$$\Rightarrow x = \frac{-\sqrt{2} \pm \sqrt{(\sqrt{34}i)^2}}{2\sqrt{3}}$$

$$\Rightarrow x = \frac{-\sqrt{2} \pm \sqrt{34}i}{2\sqrt{3}}$$

$$\therefore x = -\frac{\sqrt{2}}{2\sqrt{3}} \pm \frac{\sqrt{34}}{2\sqrt{3}}i$$

Thus, the roots of the given equation are $-\frac{\sqrt{2}}{2\sqrt{3}} \pm \frac{\sqrt{34}}{2\sqrt{3}}i$.

21. Question

Solve the following quadratics $\sqrt{2}x^2 + x + \sqrt{2} = 0$

Answer

Given $\sqrt{2}x^2 + x + \sqrt{2} = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = \sqrt{2}$, $b = 1$ and $c = \sqrt{2}$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1^2 - 4(\sqrt{2})(\sqrt{2})}}{2(\sqrt{2})}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 8}}{2\sqrt{2}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-7}}{2\sqrt{2}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{7(-1)}}{2\sqrt{2}}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{-1 \pm \sqrt{7i^2}}{2\sqrt{2}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{(\sqrt{7}i)^2}}{2\sqrt{2}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$$

$$\therefore x = -\frac{1}{2\sqrt{2}} \pm \frac{\sqrt{7}}{2\sqrt{2}}i$$

Thus, the roots of the given equation are $-\frac{1}{2\sqrt{2}} \pm \frac{\sqrt{7}}{2\sqrt{2}}i$.

22. Question

Solve the following quadratics $x^2 + x + \frac{1}{\sqrt{2}} = 0$

Answer

Given $x^2 + x + \frac{1}{\sqrt{2}} = 0$

$$\Rightarrow \left(x^2 + x + \frac{1}{\sqrt{2}}\right) \times \sqrt{2} = 0 \times \sqrt{2}$$

$$\Rightarrow \sqrt{2}x^2 + \sqrt{2}x + 1 = 0$$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, $a = \sqrt{2}$, $b = \sqrt{2}$ and $c = 1$

$$\Rightarrow x = \frac{-\sqrt{2} \pm \sqrt{(\sqrt{2})^2 - 4(\sqrt{2})(1)}}{2(\sqrt{2})}$$

$$\Rightarrow x = \frac{-\sqrt{2} \pm \sqrt{2 - 4\sqrt{2}}}{2\sqrt{2}}$$

$$\Rightarrow x = \frac{-\sqrt{2} \pm \sqrt{2(1-2\sqrt{2})}}{2\sqrt{2}}$$

$$\Rightarrow x = \frac{-\sqrt{2} \pm \sqrt{2} \times \sqrt{1-2\sqrt{2}}}{2\sqrt{2}}$$

$$\Rightarrow x = \frac{\sqrt{2}\left(-1 \pm \sqrt{1-2\sqrt{2}}\right)}{2\sqrt{2}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1-2\sqrt{2}}}{2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{(2\sqrt{2}-1)(-1)}}{2}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{-1 \pm \sqrt{(2\sqrt{2}-1)i^2}}{2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{\left(\sqrt{2\sqrt{2}-1}i\right)^2}}{2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{2\sqrt{2}-1}i}{2}$$

$$\therefore x = -\frac{1}{2} \pm \frac{\sqrt{2\sqrt{2}-1}}{2} i$$

Thus, the roots of the given equation are $-\frac{1}{2} \pm \frac{\sqrt{2\sqrt{2}-1}}{2} i$.

23. Question

Solve: $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

Answer

Given $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

$$\Rightarrow \left(x^2 + \frac{x}{\sqrt{2}} + 1\right) \times \sqrt{2} = 0 \times \sqrt{2}$$

$$\Rightarrow \sqrt{2}x^2 + x + \sqrt{2} = 0$$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, $a = \sqrt{2}$, $b = 1$ and $c = \sqrt{2}$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1^2 - 4(\sqrt{2})(\sqrt{2})}}{2(\sqrt{2})}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1-8}}{2\sqrt{2}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-7}}{2\sqrt{2}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{7(-1)}}{2\sqrt{2}}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{-1 \pm \sqrt{7i^2}}{2\sqrt{2}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{\left(\sqrt{7}i\right)^2}}{2\sqrt{2}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$$

$$\therefore x = -\frac{1}{2\sqrt{2}} \pm \frac{\sqrt{7}}{2\sqrt{2}}i$$

Thus, the roots of the given equation are $-\frac{1}{2\sqrt{2}} \pm \frac{\sqrt{7}}{2\sqrt{2}}i$.

24. Question

Solve the following quadratics $\sqrt{5}x^2 + x + \sqrt{5} = 0$

Answer

Given $\sqrt{5}x^2 + x + \sqrt{5} = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, $a = \sqrt{5}$, $b = 1$ and $c = \sqrt{5}$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1^2 - 4\left(\sqrt{5}\right)\left(\sqrt{5}\right)}}{2\left(\sqrt{5}\right)}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1-20}}{2\sqrt{5}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-19}}{2\sqrt{5}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{19(-1)}}{2\sqrt{5}}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{-1 \pm \sqrt{19i^2}}{2\sqrt{5}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{\left(\sqrt{19}i\right)^2}}{2\sqrt{5}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{19}i}{2\sqrt{5}}$$

$$\therefore x = -\frac{1}{2\sqrt{5}} \pm \frac{\sqrt{19}}{2\sqrt{5}}i$$

Thus, the roots of the given equation are $-\frac{1}{2\sqrt{5}} \pm \frac{\sqrt{19}}{2\sqrt{5}}i$.

25. Question

$-x^2 + x - 2 = 0$

Answer

Given $-x^2 + x - 2 = 0$

$\Rightarrow x^2 - x + 2 = 0$

$$\Rightarrow x^2 - x + \frac{1}{4} + \frac{7}{4} = 0$$

$$\Rightarrow x^2 - 2(x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \frac{7}{4} = 0$$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 + \frac{7}{4} = 0$ [$\because (a - b)^2 = a^2 - 2ab + b^2$]

$$\Rightarrow \left(x - \frac{1}{2}\right)^2 + \frac{7}{4} \times 1 = 0$$

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$$\left(x - \frac{1}{2}\right)^2 + \frac{7}{4}(-i^2) = 0$$

$$\Rightarrow \left(x - \frac{1}{2}\right)^2 - \frac{7}{4}i^2 = 0$$

$$\Rightarrow \left(x - \frac{1}{2}\right)^2 - \left(\frac{\sqrt{7}}{2}i\right)^2 = 0$$

$\Rightarrow \left(x - \frac{1}{2} + \frac{\sqrt{7}}{2}i\right)\left(x - \frac{1}{2} - \frac{\sqrt{7}}{2}i\right) = 0$ [$\because a^2 - b^2 = (a + b)(a - b)$]

$$\Rightarrow x - \frac{1}{2} + \frac{\sqrt{7}}{2}i = 0 \text{ or } x - \frac{1}{2} - \frac{\sqrt{7}}{2}i = 0$$

$$\Rightarrow x = \frac{1}{2} - \frac{\sqrt{7}}{2}i = 0 \text{ or } x = \frac{1}{2} + \frac{\sqrt{7}}{2}i$$

$$\therefore x = \frac{1}{2} \pm \frac{\sqrt{7}}{2}i$$

Thus, the roots of the given equation are $\frac{1}{2} \pm \frac{\sqrt{7}}{2}i$.

26. Question

Solve: $x^2 - 2x + \frac{3}{2} = 0$

Answer

Given $x^2 - 2x + \frac{3}{2} = 0$

$\Rightarrow x^2 - 2x + 1 + \frac{1}{2} = 0$

$\Rightarrow x^2 - 2(x)\left(\frac{1}{2}\right) + 1^2 + \frac{1}{2} = 0$

$\Rightarrow (x-1)^2 + \frac{1}{2} = 0$ [∵ $(a - b)^2 = a^2 - 2ab + b^2$]

$\Rightarrow (x-1)^2 + \frac{1}{2} \times 1 = 0$

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$(x-1)^2 + \frac{1}{2}(-i^2) = 0$

$\Rightarrow (x-1)^2 - \frac{1}{2}i^2 = 0$

$\Rightarrow (x-1)^2 - \left(\frac{1}{\sqrt{2}}i\right)^2 = 0$

$\Rightarrow \left(x - 1 + \frac{1}{\sqrt{2}}i\right)\left(x - 1 - \frac{1}{\sqrt{2}}i\right) = 0$ [∵ $a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow x - 1 + \frac{1}{\sqrt{2}}i = 0$ or $x - 1 - \frac{1}{\sqrt{2}}i = 0$

$\Rightarrow x = 1 - \frac{1}{\sqrt{2}}i = 0$ or $x = 1 + \frac{1}{\sqrt{2}}i$

$\therefore x = 1 \pm \frac{1}{\sqrt{2}}i$

Thus, the roots of the given equation are $1 \pm \frac{1}{\sqrt{2}}i$.

27. Question

Solve the following quadratics $3x^2 - 4x + \frac{20}{3} = 0$

Answer

Given $3x^2 - 4x + \frac{20}{3} = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, a = 3, b = –4 and $c = \frac{20}{3}$

$$\Rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)\left(\frac{20}{3}\right)}}{2(3)}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{16 - 80}}{6}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{-64}}{6}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{64(-1)}}{6}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{4 \pm \sqrt{64i^2}}{6}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{(8i)^2}}{6}$$

$$\Rightarrow x = \frac{4 \pm 8i}{6}$$

$$\Rightarrow x = \frac{2(2 \pm 4i)}{6}$$

$$\Rightarrow x = \frac{2 \pm 4i}{3}$$

$$\therefore x = \frac{2}{3} \pm \frac{4}{3}i$$

Thus, the roots of the given equation are $\frac{2}{3} \pm \frac{4}{3}i$.

Exercise 14.2

1 A. Question

Solve the following quadratic equations by factorization method:

$x^2 + 10ix - 21 = 0$

Answer

$x^2 + 10ix - 21 = 0$

Given $x^2 + 10ix - 21 = 0$

$\Rightarrow x^2 + 10ix - 21 \times 1 = 0$

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$x^2 + 10ix - 21(-i^2) = 0$

$\Rightarrow x^2 + 10ix + 21i^2 = 0$

$\Rightarrow x^2 + 3ix + 7ix + 21i^2 = 0$

$\Rightarrow x(x + 3i) + 7i(x + 3i) = 0$

$\Rightarrow (x + 3i)(x + 7i) = 0$

⇒ $x + 3i = 0$ or $x + 7i = 0$

∴ $x = –3i$ or $–7i$

Thus, the roots of the given equation are –3i and –7i.

1 B. Question

Solve the following quadratic equations by factorization method:

$x^2 + (1 – 2i)x – 2i = 0$

Answer

$x^2 + (1 – 2i)x – 2i = 0$

Given $x^2 + (1 – 2i)x – 2i = 0$

⇒ $x^2 + x – 2ix – 2i = 0$

⇒ $x(x + 1) – 2i(x + 1) = 0$

⇒ $(x + 1)(x – 2i) = 0$

⇒ $x + 1 = 0$ or $x – 2i = 0$

∴ $x = –1$ or $2i$

Thus, the roots of the given equation are –1 and 2i.

1 C. Question

Solve the following quadratic equations by factorization method:

$$x^2 - \left(2\sqrt{3} + 3i\right)x + 6\sqrt{3}i = 0$$

Answer

$x^2 - \left(2\sqrt{3} + 3i\right)x + 6\sqrt{3}i = 0$

Given $x^2 - \left(2\sqrt{3} + 3i\right)x + 6\sqrt{3}i = 0$

⇒ $x^2 - \left(2\sqrt{3}x + 3ix\right) + 6\sqrt{3}i = 0$

⇒ $x^2 - 2\sqrt{3}x - 3ix + 6\sqrt{3}i = 0$

⇒ $x\left(x - 2\sqrt{3}\right) - 3i\left(x - 2\sqrt{3}\right) = 0$

⇒ $\left(x - 2\sqrt{3}\right)(x - 3i) = 0$

⇒ $x - 2\sqrt{3} = 0$ or $x - 3i = 0$

∴ $x = 2\sqrt{3}$ or $3i$

Thus, the roots of the given equation are $2\sqrt{3}$ and 3i.

1 D. Question

Solve the following quadratic equations by factorization method:

$6x^2 – 17ix – 12 = 0$

Answer

$6x^2 – 17ix – 12 = 0$

Given $6x^2 – 17ix – 12 = 0$

$\Rightarrow 6x^2 - 17ix - 12 \times 1 = 0$

We have $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$6x^2 - 17ix - 12(-i^2) = 0$

$\Rightarrow 6x^2 - 17ix + 12i^2 = 0$

$\Rightarrow 6x^2 - 9ix - 8ix + 12i^2 = 0$

$\Rightarrow 3x(2x - 3i) - 4i(2x - 3i) = 0$

$\Rightarrow (2x - 3i)(3x - 4i) = 0$

$\Rightarrow 2x - 3i = 0$ or $3x - 4i = 0$

$\Rightarrow 2x = 3i$ or $3x = 4i$

$\therefore x = \frac{3}{2}i \text{ or } \frac{4}{3}i$

Thus, the roots of the given equation are $\frac{3}{2}i$ and $\frac{4}{3}i$.

2 A. Question

Solve the following quadratic equations:

$$x^2 - \left(3\sqrt{2} + 2i\right)x + 6\sqrt{2}i = 0$$

Answer

$x^2 - \left(3\sqrt{2} + 2i\right)x + 6\sqrt{2}i = 0$

Given $x^2 - \left(3\sqrt{2} + 2i\right)x + 6\sqrt{2}i = 0$

$\Rightarrow x^2 - \left(3\sqrt{2}x + 2ix\right) + 6\sqrt{2}i = 0$

$\Rightarrow x^2 - 3\sqrt{2}x - 2ix + 6\sqrt{2}i = 0$

$\Rightarrow x\left(x - 3\sqrt{2}\right) - 2i\left(x - 3\sqrt{2}\right) = 0$

$\Rightarrow \left(x - 3\sqrt{2}\right)(x - 2i) = 0$

$\Rightarrow x - 3\sqrt{2} = 0 \text{ or } x - 2i = 0$

$\therefore x = 3\sqrt{2} \text{ or } 2i$

Thus, the roots of the given equation are $3\sqrt{2}$ and 2i.

2 B. Question

Solve the following quadratic equations:

$x^2 - (5 - i)x + (18 + i) = 0$

Answer

$x^2 - (5 - i)x + (18 + i) = 0$

Given $x^2 - (5 - i)x + (18 + i) = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, a = 1, b = –(5 – i) and c = (18 + i)

$$\Rightarrow x = \frac{-(-(5-i)) \pm \sqrt{(-(5-i))^2 - 4(1)(18+i)}}{2(1)}$$

$$\Rightarrow x = \frac{(5-i) \pm \sqrt{(5-i)^2 - 4(18+i)}}{2}$$

$$\Rightarrow x = \frac{(5-i) \pm \sqrt{25 - 10i + i^2 - 72 - 4i}}{2}$$

$$\Rightarrow x = \frac{(5-i) \pm \sqrt{-47 - 14i + i^2}}{2}$$

By substituting $i^2 = -1$ in the above equation, we get

$$x = \frac{(5-i) \pm \sqrt{-47 - 14i + (-1)}}{2}$$

$$\Rightarrow x = \frac{(5-i) \pm \sqrt{-48 - 14i}}{2}$$

$$\Rightarrow x = \frac{(5-i) \pm \sqrt{(-1)(48 + 14i)}}{2}$$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{(5-i) \pm \sqrt{i^2(48 + 14i)}}{2}$$

$$\Rightarrow x = \frac{(5-i) \pm i\sqrt{48 + 14i}}{2}$$

We can write 48 + 14i = 49 – 1 + 14i

$\Rightarrow 48 + 14i = 49 + i^2 + 14i$ [$\because i^2 = -1$]

$\Rightarrow 48 + 14i = 7^2 + i^2 + 2(7)(i)$

$\Rightarrow 48 + 14i = (7 + i)^2$ [$\because (a + b)^2 = a^2 + b^2 + 2ab$]

By using the result $48 + 14i = (7 + i)^2$, we get

$$x = \frac{(5-i) \pm i\sqrt{(7+i)^2}}{2}$$

$$\Rightarrow x = \frac{(5-i) \pm i(7+i)}{2}$$

$$\Rightarrow x = \frac{(5-i) + i(7+i)}{2} \text{ or } \frac{(5-i) - i(7+i)}{2}$$

$$\Rightarrow x = \frac{5 - i + 7i + i^2}{2} \text{ or } \frac{5 - i - 7i - i^2}{2}$$

$$\Rightarrow x = \frac{5+6i+(-1)}{2} \text{ or } \frac{5-8i-(-1)}{2} \quad [\because i^2 = -1]$$

$$\Rightarrow x = \frac{5 + 6i - 1}{2} \text{ or } \frac{5 - 8i + 1}{2}$$

$$\Rightarrow x = \frac{4 + 6i}{2} \text{ or } \frac{6 - 8i}{2}$$

$$\Rightarrow x = \frac{2(2+3i)}{2} \text{ or } \frac{2(3-4i)}{2}$$

∴ x = 2 + 3i or 3 – 4i

Thus, the roots of the given equation are 2 + 3i and 3 – 4i.

2 C. Question

Solve the following quadratic equations:

$(2 + i)x^2 – (5 – i)x + 2(1 – i) = 0$

Answer

$(2 + i)x^2 – (5 – i)x + 2(1 – i) = 0$

Given $(2 + i)x^2 – (5 – i)x + 2(1 – i) = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, a = (2 + i), b = –(5 – i) and c = 2(1 – i)

$$\Rightarrow x = \frac{-\left(-(5-i)\right) \pm \sqrt{\left(-(5-i)\right)^2 - 4(2+i)\left(2(1-i)\right)}}{2(2+i)}$$

$$\Rightarrow x = \frac{(5-i) \pm \sqrt{(5-i)^2 - 8(2+i)(1-i)}}{2(2+i)}$$

$$\Rightarrow x = \frac{(5-i) \pm \sqrt{25 - 10i + i^2 - 8(2 - 2i + i - i^2)}}{2(2+i)}$$

By substituting $i^2 = –1$ in the above equation, we get

$$x = \frac{(5-i) \pm \sqrt{25 - 10i + (-1) - 8\left(2 - i - (-1)\right)}}{2(2+i)}$$

$$\Rightarrow x = \frac{(5-i) \pm \sqrt{24 - 10i - 8(3-i)}}{2(2+i)}$$

$$\Rightarrow x = \frac{(5-i) \pm \sqrt{24 - 10i - 24 + 8i}}{2(2+i)}$$

$$\Rightarrow x = \frac{(5-i) \pm \sqrt{-2i}}{2(2+i)}$$

We can write –2i = –2i + 1 – 1

$\Rightarrow –2i = –2i + 1 + i^2$ [∵ $i^2 = –1$]

$\Rightarrow –2i = 1 – 2i + i^2$

$\Rightarrow –2i = 1^2 – 2(1)(i) + i^2$

$\Rightarrow –2i = (1 – i)^2$ [∵ $(a – b)^2 = a^2 – 2ab + b^2$]

By using the result $–2i = (1 – i)^2$, we get

$$x = \frac{(5-i) \pm \sqrt{(1-i)^2}}{2(2+i)}$$

$$\Rightarrow x = \frac{(5-i) \pm (1-i)}{2(2+i)}$$

$$\Rightarrow x = \frac{(5-i) + (1-i)}{2(2+i)} \text{ or } \frac{(5-i) - (1-i)}{2(2+i)}$$

$$\Rightarrow x = \frac{5-i+1-i}{2(2+i)} \text{ or } \frac{5-i-1+i}{2(2+i)}$$

$$\Rightarrow x = \frac{6-2i}{2(2+i)} \text{ or } \frac{4}{2(2+i)}$$

$$\Rightarrow x = \frac{3-i}{2+i} \text{ or } \frac{2}{2+i}$$

$$\Rightarrow x = \frac{3-i}{2+i} \times \frac{2-i}{2-i} \text{ or } \frac{2}{2+i} \times \frac{2-i}{2-i}$$

$$\Rightarrow x = \frac{(3-i)(2-i)}{(2+i)(2-i)} \text{ or } \frac{2(2-i)}{(2+i)(2-i)}$$

$$\Rightarrow x = \frac{6-3i-2i+i^2}{2^2-i^2} \text{ or } \frac{4-2i}{2^2-i^2}$$

$$\Rightarrow x = \frac{6-5i+(-1)}{4-(-1)} \text{ or } \frac{4-2i}{4-(-1)} \; [\because i^2 = -1]$$

$$\Rightarrow x = \frac{5-5i}{4+1} \text{ or } \frac{4-2i}{4+1}$$

$$\Rightarrow x = \frac{5(1-i)}{5} \text{ or } \frac{4-2i}{5}$$

$$\therefore x = 1 - i \text{ or } \frac{4}{5} - \frac{2}{5}i$$

Thus, the roots of the given equation are 1 – i and $\frac{4}{5} - \frac{2}{5}i$.

2 D. Question

Solve the following quadratic equations:

$x^2 - (2 + i)x - (1 - 7i) = 0$

Answer

$x^2 - (2 + i)x - (1 - 7i) = 0$

Given $x^2 - (2 + i)x - (1 - 7i) = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, a = 1, b = –(2 + i) and c = –(1 – 7i)

$$\Rightarrow x = \frac{-(-(2+i)) \pm \sqrt{(-(2+i))^2 - 4(1)(-(1-7i))}}{2(1)}$$

$$\Rightarrow x = \frac{(2+i) \pm \sqrt{(2+i)^2 + 4(1-7i)}}{2}$$

$$\Rightarrow x = \frac{(2+i) \pm \sqrt{4+4i+i^2+4-28i}}{2}$$

$\Rightarrow x = \frac{(2+i) \pm \sqrt{8 - 24i + i^2}}{2}$

By substituting $i^2 = -1$ in the above equation, we get

$x = \frac{(2+i) \pm \sqrt{8 - 24i + (-1)}}{2}$

$\Rightarrow x = \frac{(2+i) \pm \sqrt{7 - 24i}}{2}$

We can write 7 – 24i = 16 – 9 – 24i

⇒ 7 – 24i = 16 + 9(–1) – 24i

⇒ 7 – 24i = 16 + $9i^2$ – 24i [∵ $i^2 = -1$]

⇒ 7 – 24i = $4^2 + (3i)^2 - 2(4)(3i)$

⇒ 7 – 24i = $(4 - 3i)^2$ [∵ $(a - b)^2 = a^2 - b^2 + 2ab$]

By using the result 7 – 24i = $(4 - 3i)^2$, we get

$x = \frac{(2+i) \pm \sqrt{(4-3i)^2}}{2}$

$\Rightarrow x = \frac{(2+i) \pm (4-3i)}{2}$

$\Rightarrow x = \frac{(2+i) + (4-3i)}{2} \text{ or } \frac{(2+i) - (4-3i)}{2}$

$\Rightarrow x = \frac{2 + i + 4 - 3i}{2} \text{ or } \frac{2 + i - 4 + 3i}{2}$

$\Rightarrow x = \frac{6 - 2i}{2} \text{ or } \frac{-2 + 4i}{2}$

$\Rightarrow x = \frac{2(3 - i)}{2} \text{ or } \frac{2(-1 + 2i)}{2}$

∴ x = 3 – i or –1 + 2i

Thus, the roots of the given equation are 3 – i and –1 + 2i.

2 E. Question

Solve the following quadratic equations:

$ix^2 - 4x - 4i = 0$

Answer

$ix^2 - 4x - 4i = 0$

Given $ix^2 - 4x - 4i = 0$

$\Rightarrow ix^2 + 4x(-1) - 4i = 0$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$ix^2 + 4xi^2 - 4i = 0$

$\Rightarrow i(x^2 + 4ix - 4) = 0$

$\Rightarrow x^2 + 4ix - 4 = 0$

$\Rightarrow x^2 + 4ix + 4(-1) = 0$

$\Rightarrow x^2 + 4ix + 4i^2 = 0$ [$\because i^2 = -1$]

$\Rightarrow x^2 + 2ix + 2ix + 4i^2 = 0$

$\Rightarrow x(x + 2i) + 2i(x + 2i) = 0$

$\Rightarrow (x + 2i)(x + 2i) = 0$

$\Rightarrow (x + 2i)^2 = 0$

$\Rightarrow x + 2i = 0$

$\therefore x = -2i$ (double root)

Thus, the roots of the given equation are –2i and –2i.

2 F. Question

Solve the following quadratic equations:

$x^2 + 4ix - 4 = 0$

Answer

$x^2 + 4ix - 4 = 0$

Given $x^2 + 4ix - 4 = 0$

$\Rightarrow x^2 + 4ix + 4(-1) = 0$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$\Rightarrow x^2 + 4ix + 4i^2 = 0$

$\Rightarrow x^2 + 2ix + 2ix + 4i^2 = 0$

$\Rightarrow x(x + 2i) + 2i(x + 2i) = 0$

$\Rightarrow (x + 2i)(x + 2i) = 0$

$\Rightarrow (x + 2i)^2 = 0$

$\Rightarrow x + 2i = 0$

$\therefore x = -2i$ (double root)

Thus, the roots of the given equation are –2i and –2i.

2 G. Question

Solve the following quadratic equations:

$2x^2 + \sqrt{15}ix - i = 0$

Answer

$2x^2 + \sqrt{15}ix - i = 0$

Given $2x^2 + \sqrt{15}ix - i = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, a = 2, $b = \sqrt{15}i$ and c = –i

$$\Rightarrow x=\frac{-(\sqrt{15}i)\pm\sqrt{(\sqrt{15}i)^2-4(2)(-i)}}{2(2)}$$

$$\Rightarrow x=\frac{-\sqrt{15}i\pm\sqrt{15i^2+8i}}{4}$$

By substituting i^2 = –1 in the above equation, we get

$$x=\frac{-\sqrt{15}i\pm\sqrt{15(-1)+8i}}{4}$$

$$\Rightarrow x=\frac{-\sqrt{15}i\pm\sqrt{8i-15}}{4}$$

$$\Rightarrow x=\frac{-\sqrt{15}i\pm\sqrt{(-1)(15-8i)}}{4}$$

By substituting –1 = i^2 in the above equation, we get

$$x=\frac{-\sqrt{15}i\pm\sqrt{i^2(15-8i)}}{4}$$

$$\Rightarrow x=\frac{-\sqrt{15}i\pm i\sqrt{15-8i}}{4}$$

We can write 15 – 8i = 16 – 1 – 8i

⇒ 15 – 8i = 16 + (–1) – 8i

⇒ 15 – 8i = 16 + i^2 – 8i [∵ i^2 = –1]

⇒ 15 – 8i = $4^2 + (i)^2 - 2(4)(i)$

⇒ 15 – 8i = $(4 - i)^2$ [∵ $(a - b)^2 = a^2 - b^2 + 2ab$]

By using the result 15 – 8i = $(4 – i)^2$, we get

$$x=\frac{-\sqrt{15}i\pm i\sqrt{(4-i)^2}}{4}$$

$$\Rightarrow x=\frac{-\sqrt{15}i\pm i(4-i)}{4}$$

$$\Rightarrow x=\frac{-\sqrt{15}i+i(4-i)}{4}\text{ or }\frac{-\sqrt{15}i-i(4-i)}{4}$$

$$\Rightarrow x=\frac{-\sqrt{15}i+4i-i^2}{4}\text{ or }\frac{-\sqrt{15}i-4i+i^2}{4}$$

$\Rightarrow x=\frac{-\sqrt{15}i+4i-(-1)}{4}\text{ or }\frac{-\sqrt{15}i-4i+(-1)}{4}$ [∵ i^2 = –1]

$$\Rightarrow x=\frac{-\sqrt{15}i+4i+1}{4}\text{ or }\frac{-\sqrt{15}i-4i-1}{4}$$

$$\Rightarrow x=\frac{1+(4-\sqrt{15})i}{4}\text{ or }\frac{-1-(4+\sqrt{15})i}{4}$$

$$\therefore x=\frac{1}{4}+\left(\frac{4-\sqrt{15}}{4}\right)i\text{ or }-\frac{1}{4}-\left(\frac{4+\sqrt{15}}{4}\right)i$$

Thus, the roots of the given equation are $\frac{1}{4}+\left(\frac{4-\sqrt{15}}{4}\right)i$ and $-\frac{1}{4}-\left(\frac{4+\sqrt{15}}{4}\right)i$.

2 H. Question

Solve the following quadratic equations:

$x^2 – x + (1 + i) = 0$

Answer

$x^2 – x + (1 + i) = 0$

Given $x^2 – x + (1 + i) = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, a = 1, b = –1 and c = (1 + i)

$$\Rightarrow x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1+i)}}{2(1)}$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1 - 4(1+i)}}{2}$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1 - 4 - 4i}}{2}$$

$$\Rightarrow x = \frac{1 \pm \sqrt{-3 - 4i}}{2}$$

$$\Rightarrow x = \frac{1 \pm \sqrt{(-1)(3 + 4i)}}{2}$$

By substituting $–1 = i^2$ in the above equation, we get

$$x = \frac{1 \pm \sqrt{i^2(3 + 4i)}}{2}$$

$$\Rightarrow x = \frac{1 \pm i\sqrt{3 + 4i}}{2}$$

We can write 3 + 4i = 4 – 1 + 4i

$\Rightarrow 3 + 4i = 4 + i^2 + 4i$ [$\because i^2 = –1$]

$\Rightarrow 3 + 4i = 2^2 + i^2 + 2(2)(i)$

$\Rightarrow 3 + 4i = (2 + i)^2$ [$\because (a + b)^2 = a^2 + b^2 + 2ab$]

By using the result $3 + 4i = (2 + i)^2$, we get

$$x = \frac{1 \pm i\sqrt{(2 - i)^2}}{2}$$

$$\Rightarrow x = \frac{1 \pm i(2 + i)}{2}$$

$$\Rightarrow x = \frac{1 + i(2 + i)}{2} \text{ or } \frac{1 - i(2 + i)}{2}$$

$$\Rightarrow x = \frac{1 + 2i + i^2}{2} \text{ or } \frac{1 - 2i - i^2}{2}$$

$\Rightarrow x = \frac{1+2i+(-1)}{2} \text{ or } \frac{1-2i-(-1)}{2}$ [$\because i^2 = –1$]

$\Rightarrow x = \frac{1 + 2i - 1}{2}$ or $\frac{1 - 2i + 1}{2}$

$\Rightarrow x = \frac{2i}{2}$ or $\frac{2 - 2i}{2}$

$\Rightarrow x = i$ or $\frac{2(1 - i)}{2}$

∴ x = i or 1 – i

Thus, the roots of the given equation are i and 1 – i.

2 I. Question

Solve the following quadratic equations:

$ix^2 - x + 12i = 0$

Answer

$ix^2 - x + 12i = 0$

Given $ix^2 - x + 12i = 0$

$\Rightarrow ix^2 + x(-1) + 12i = 0$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$ix^2 + xi^2 + 12i = 0$

$\Rightarrow i(x^2 + ix + 12) = 0$

$\Rightarrow x^2 + ix + 12 = 0$

$\Rightarrow x^2 + ix - 12(-1) = 0$

$\Rightarrow x^2 + ix - 12i^2 = 0$ [∵ $i^2 = -1$]

$\Rightarrow x^2 - 3ix + 4ix - 12i^2 = 0$

$\Rightarrow x(x - 3i) + 4i(x - 3i) = 0$

$\Rightarrow (x - 3i)(x + 4i) = 0$

$\Rightarrow x - 3i = 0$ or $x + 4i = 0$

∴ x = 3i or –4i

Thus, the roots of the given equation are 3i and –4i.

2 J. Question

Solve the following quadratic equations:

$$x^2 - \left(3\sqrt{2} - 2i\right)x - \sqrt{2}i = 0$$

Answer

$x^2 - \left(3\sqrt{2} - 2i\right)x - \sqrt{2}i = 0$

Given $x^2 - \left(3\sqrt{2} - 2i\right)x - \sqrt{2}i = 0 = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where a ≠ 0, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, a = 1, $b = -(3\sqrt{2} - 2i)$ and $c = -\sqrt{2}i$

$$\Rightarrow x = \frac{-\left(-(3\sqrt{2} - 2i)\right) \pm \sqrt{\left(-(3\sqrt{2} - 2i)\right)^2 - 4(1)(-\sqrt{2}i)}}{2(1)}$$

$$\Rightarrow x = \frac{(3\sqrt{2} - 2i) \pm \sqrt{(3\sqrt{2} - 2i)^2 + 4\sqrt{2}i}}{2}$$

$$\Rightarrow x = \frac{(3\sqrt{2} - 2i) \pm \sqrt{18 - 12\sqrt{2}i + 4i^2 + 4\sqrt{2}i}}{2}$$

$$\Rightarrow x = \frac{(3\sqrt{2} - 2i) \pm \sqrt{18 - 8\sqrt{2}i + 4i^2}}{2}$$

By substituting i^2 = –1 in the above equation, we get

$$x = \frac{(3\sqrt{2} - 2i) \pm \sqrt{18 - 8\sqrt{2}i + 4(-1)}}{2}$$

$$\Rightarrow x = \frac{(3\sqrt{2} - 2i) \pm \sqrt{18 - 8\sqrt{2}i - 4}}{2}$$

$$\Rightarrow x = \frac{(3\sqrt{2} - 2i) \pm \sqrt{14 - 8\sqrt{2}i}}{2}$$

We can write $14 - 8\sqrt{2}i = 16 - 2 - 8\sqrt{2}i$

$\Rightarrow 14 - 8\sqrt{2}i = 16 + 2(-1) - 8\sqrt{2}i$

$\Rightarrow 14 - 8\sqrt{2}i = 16 + 2i^2 - 8\sqrt{2}i$ [$\because i^2 = -1$]

$\Rightarrow 14 - 8\sqrt{2}i = 4^2 + (\sqrt{2}i)^2 - 2(4)(\sqrt{2}i)$

$\Rightarrow 14 - 8\sqrt{2}i = (4 - \sqrt{2}i)^2$ [$\because (a - b)^2 = a^2 - 2ab + b^2$]

By using the result $14 - 8\sqrt{2}i = (4 - \sqrt{2}i)^2$, we get

$$x = \frac{(3\sqrt{2} - 2i) \pm \sqrt{(4 - \sqrt{2}i)^2}}{2}$$

$$\Rightarrow x = \frac{(3\sqrt{2} - 2i) \pm (4 - \sqrt{2}i)}{2}$$

$$\Rightarrow x = \frac{(3\sqrt{2} - 2i) + (4 - \sqrt{2}i)}{2} \text{ or } \frac{(3\sqrt{2} - 2i) - (4 - \sqrt{2}i)}{2}$$

$$\Rightarrow x = \frac{3\sqrt{2} - 2i + 4 - \sqrt{2}i}{2} \text{ or } \frac{3\sqrt{2} - 2i - 4 + \sqrt{2}i}{2}$$

$\Rightarrow x = \frac{3\sqrt{2}+4-(2+\sqrt{2})i}{2}$ or $\frac{3\sqrt{2}-4-(2-\sqrt{2})i}{2}$ [$\because i^2 = -1$]

$$\therefore x = \frac{3\sqrt{2} + 4}{2} - \left(\frac{2 + \sqrt{2}}{2}\right)i \text{ or } \frac{3\sqrt{2} - 4}{2} - \left(\frac{2 - \sqrt{2}}{2}\right)i$$

Thus, the roots of the given equation are $\frac{3\sqrt{2}+4}{2} - \left(\frac{2+\sqrt{2}}{2}\right)i$ and $\frac{3\sqrt{2}-4}{2} - \left(\frac{2-\sqrt{2}}{2}\right)i$.

2 K. Question

Solve the following quadratic equations:

$$x^2 - \left(\sqrt{2} + i\right)x + \sqrt{2}i = 0$$

Answer

xi. $x^2 - \left(\sqrt{2} + i\right)x + \sqrt{2}i = 0$

Given $x^2 - \left(\sqrt{2} + i\right)x + \sqrt{2}i = 0$

$\Rightarrow x^2 - \left(\sqrt{2}x + ix\right) + \sqrt{2}i = 0$

$\Rightarrow x^2 - \sqrt{2}x - ix + \sqrt{2}i = 0$

$\Rightarrow x\left(x - \sqrt{2}\right) - i\left(x - \sqrt{2}\right) = 0$

$\Rightarrow \left(x - \sqrt{2}\right)(x - i) = 0$

$\Rightarrow x - \sqrt{2} = 0 \text{ or } x - i = 0$

$\therefore x = \sqrt{2} \text{ or } i$

Thus, the roots of the given equation are $\sqrt{2}$ and i.

2 L. Question

Solve the following quadratic equations:

$2x^2 - (3 + 7i)x + (9i - 3) = 0$

Answer

$2x^2 - (3 + 7i)x + (9i - 3) = 0$

Given $2x^2 - (3 + 7i)x + (9i - 3) = 0$

Recall that the roots of quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Here, a = 2, b = –(3 + 7i) and c = (9i – 3)

$$\Rightarrow x = \frac{-\left(-(3 + 7i)\right) \pm \sqrt{\left(-(3 + 7i)\right)^2 - 4(2)(9i - 3)}}{2(2)}$$

$$\Rightarrow x = \frac{(3 + 7i) \pm \sqrt{(3 + 7i)^2 - 8(9i - 3)}}{4}$$

$$\Rightarrow x = \frac{(3 + 7i) \pm \sqrt{9 + 42i + 49i^2 - 72i + 24}}{4}$$

$$\Rightarrow x = \frac{(3 + 7i) \pm \sqrt{33 - 30i + 49i^2}}{4}$$

By substituting $i^2 = -1$ in the above equation, we get

$$x = \frac{(3 + 7i) \pm \sqrt{33 - 30i + 49(-1)}}{4}$$

$$\Rightarrow x = \frac{(3 + 7i) \pm \sqrt{33 - 30i - 49}}{4}$$

$$\Rightarrow x=\frac{(3+7i)\pm\sqrt{-16-30i}}{4}$$

$$\Rightarrow x=\frac{(3+7i)\pm\sqrt{(-1)(16+30i)}}{4}$$

By substituting –1 = i^2 in the above equation, we get

$$x=\frac{(3+7i)\pm\sqrt{i^2(16+30i)}}{4}$$

$$\Rightarrow x=\frac{(3+7i)\pm i\sqrt{16+30i}}{4}$$

We can write 16 + 30i = 25 – 9 + 30i

⇒ 16 + 30i = 25 + 9(–1) + 30i

⇒ 16 + 30i = 25 + $9i^2$ + 30i [∵ i^2 = –1]

⇒ 16 + 30i = 5^2 + $(3i)^2$ + 2(5)(3i)

⇒ 16 + 30i = $(5 + 3i)^2$ [∵ $(a + b)^2 = a^2 + b^2 + 2ab$]

By using the result 16 + 30i = $(5 + 3i)^2$, we get

$$x=\frac{(3+7i)\pm i\sqrt{(5+3i)^2}}{4}$$

$$\Rightarrow x=\frac{(3+7i)\pm i(5+3i)}{4}$$

$$\Rightarrow x=\frac{(3+7i)+i(5+3i)}{4}\text{ or }\frac{(3+7i)-i(5+3i)}{4}$$

$$\Rightarrow x=\frac{3+7i+5i+3i^2}{4}\text{ or }\frac{3+7i-5i-3i^2}{4}$$

$$\Rightarrow x=\frac{3+12i+3i^2}{4}\text{ or }\frac{3+2i-3i^2}{4}$$

$$\Rightarrow x=\frac{3+12i+3(-1)}{4}\text{ or }\frac{3+2i-3(-1)}{4}$$ [∵ i^2 = –1]

$$\Rightarrow x=\frac{3+12i-3}{4}\text{ or }\frac{3+2i+3}{4}$$

$$\Rightarrow x=\frac{12}{4}i\text{ or }\frac{6+2i}{4}$$

$$\Rightarrow x=3i\text{ or }\frac{6}{4}+\frac{2}{4}i$$

$$\therefore x=3i\text{ or }\frac{3}{2}+\frac{1}{2}i$$

Thus, the roots of the given equation are 3i and $\frac{3}{2}+\frac{1}{2}i$.

Very Short Answer

1. Question

Write the number of real roots of the equation $(x-1)^2+(x-2)^2+(x-3)^2=0$.

Answer

given $(x-1)^2+(x-2)^2+(x-3)^2=0$

$x^2 + 1 - 2x + X^2 + 4 - 4x + X^2 + 9 - 6x = 0$

$3X^2 - 12x+ 14 = 0$

Comparing it with $aX^2 + bx+ c = 0$ and substituting them in $b^2 – 4ac$, we get

$= (-12)^2-4(3)(14)$

= 144 – 168

= - 24 < 0 .

Hence the given equation do not have real roots. It has imaginary roots.

2. Question

If a and b are roots of the equation $x^2 - px + q = 0$, then write the value of $\frac{1}{a}+\frac{1}{b}$.

Answer

given $x^2- px + q = 0$

We know sum of the roots = p

Product of the roots = q

As given that a and b are roots then,

a + b = p

a b = q

given $\frac{1}{a}+\frac{1}{b}$

$= \frac{a+b}{ab}$

$= \frac{p}{q}$.

3. Question

If roots α, β of equation $x^2 - px + 16 = 0$ satisfy the relation $\alpha^2 + \beta^2 = 9$, then write the value of p.

Answer

given $\alpha^2 + \beta^2 = 9$

$(\alpha + \beta)^2 - 2\alpha\beta = 9$

Given $x^2- px + 16 = 0$ and α, β are roots of the equation then

Sum of roots α + β = p

Product of roots α β = 16

Substituting these in $(\alpha + \beta)^2-2\alpha\beta=9$ we get,

$p^2 – 2(16) = 9$

$p^2 = 41$

$P = \pm\sqrt{41}$

4. Question

If $2+\sqrt{3}$ is a root of the equation $x^2+px+q=0$, then write the values of p and q.

Answer

we know irrational roots always exists in pair hence if $2+\sqrt{3}$ is one root then $2-\sqrt{3}$ is another root.

Given $x^2 + px + q = 0$

Sum of roots = -p

$2+\sqrt{3}+ 2-\sqrt{3} = -p$

P = -4

Product of roots = q

$(2+\sqrt{3})(2-\sqrt{3})=q$

4 – 3 = q

q = 1.

5. Question

If the difference between the roots of the equation $x^2+ax+8=0$ is 2 write the values of a.

Answer

given $x^2 + ax + 8 = 0$ and $\alpha - \beta = 2$

Also from given equation $\alpha\beta = 8$

As $\alpha - \beta = 2$

Then $\alpha - \frac{8}{\alpha} = 2$

$\alpha^2 - 2\alpha - 8 = 0$

$(\alpha-4)(\alpha + 2)=0$

$\alpha = 4$ and $\alpha = -2$

if $\alpha = 4$ then substituting it in $\alpha - \beta = 2$ we get ,

$\beta = 2$

from the given equation,

sum of roots = -a

$\alpha + \beta = -a$

- a = 4+2

a = -6

if $\alpha = -2$ then substituting it in $\alpha - \beta = 2$ we get ,

$\beta = -4$

then sum of roots $\alpha + \beta = -a$

a = 6

therefore a = ± 6.

6. Question

Write the roots of the equation $(a-b)x^2+(b-c)x+(c-a)=0$

Answer

roots of a quadratic equation is $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

From the given equation we get,

$$x = \frac{-(b-c) \pm \sqrt{(b-c)^2 - 4(a-b)(c-a)}}{2(a-b)}$$

$$x = \frac{-(b-c) \pm \sqrt{b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab}}{2(a-b)}$$

$$x = \frac{-(b-c) \pm \sqrt{(-2a+b+c)^2}}{2(a-b)}$$

$$= \frac{-b+c+(-2a+b+c)}{2(a-b)} \text{ and } \frac{-b+c-(-2a+b+c)}{2(a-b)}$$

$$= \frac{2(c-a)}{2(a-b)} \text{ and } \frac{2(a-b)}{2(a-b)}$$

$$= \frac{(c-a)}{(a-b)} \text{ and } 1$$

Therefore $x = 1, \frac{(c-a)}{(a-b)}$.

7. Question

If a and b are roots of the equation $x^2 - x + 1 = 0$, then write the value of $a^2 + b^2$.

Answer

from the given equation sum of roots a + b = 1

Product of roots ab = 1

Now $a^2 + b^2 = (a + b)^2 - 2ab$

= 1 – 2

= - 1.

8. Question

Write the number of quadratic equations, with real roots, which do not change by squaring their roots.

Answer

from the given condition roots remain unchanged only when they are equal to 1 and 0.

Hence the roots may be (0,1) or (1,0) and (1,1) and (0,0).

Hence 3 equations can be formed by substituting these points in (x-a) (x-b) = 0

Where a, b are roots or points.

9. Question

If α, β are roots of the equation $x^2 + lx + m = 0$, write an equation whose roots are $-\frac{1}{\alpha}$ and $-\frac{1}{\beta}$.

Answer

from the given equation sum of the roots α + β = - l

Product of roots αβ = m

Formula to form a quadratic equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Where α, β are roots of equation.

Given $\frac{-1}{\alpha}, \frac{-1}{\beta}$ are roots, then required quadratic equation is

$$x^2 - \left(\frac{-1}{\alpha} + \frac{-1}{\beta}\right)x + \frac{1}{\alpha\beta} = 0$$

$$x^2 + \left(\frac{\alpha+\beta}{\alpha\beta}\right)x + \frac{1}{\alpha\beta} = 0$$

$$x^2 + \left(\frac{-l}{m}\right)x + \frac{1}{m} = 0$$

$mx^2 - lx + 1 = 0$.

10. Question

If α, β are roots of the equation $x^2 - a(x+1) - c = 0$, then write the value of $(1+\alpha)(1+\beta)$.

Answer

given x^2-a(x+1)-c=0

x^2-ax-a-c=0

x^2-ax-(a+c)=0

as α, β are roots of equation, we get

sum of the roots α + β = a

Product of roots αβ = - (a + c)

Given (1+ α) (1+ β)

= 1 + (α + β) + (α β)

= 1 + a – a – c

= 1 – c.

MCQ

1. Question

Mark the Correct alternative in the following:

The complete set of values of k, for which the quadratic equation $x^2 - kx + k + 2 = 0$ has equal rots, consists of

A. $2 + \sqrt{12}$

B. $2 \pm \sqrt{12}$

C. $2 - \sqrt{12}$

D. $-2 - \sqrt{12}$

Answer

Since roots are equal then $b^2 - 4ac = 0$

From the given equation we get,

K^2 – 4 (1) (k+2) = 0

K^2 – 4 k - 8 = 0

$$k = \frac{4 \pm \sqrt{4^2 - 4(-8)}}{2}$$

$$= \frac{4 \pm \sqrt{16 + 32}}{2}$$

$$= \frac{4 \pm \sqrt{48}}{2}$$

$$= \frac{4 \pm 2\sqrt{12}}{2}$$

$=2\pm\sqrt{12}$

2. Question

Mark the Correct alternative in the following:

For the equation $|x|^2 + |x| - 6 = 0$, the sum of the real roots is

A. 1

B. 0

C. 2

D. none of these

Answer

given $|x|^2+|x|- 6 = 0$

When $x > 0$

It can be written as $x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$X = 2$

When $x < 0$

It can be written as $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$X = - 2$

Therefore $x = \pm 2$

Hence sum of the roots = 0.

3. Question

Mark the Correct alternative in the following:

If a, b are the roots of the equation $x^2 + x + 1 = 0$, then $a^2 + b^2 =$

A. 1

B. 2

C. −1

D. 3

Answer

from the given equation sum of roots $a + b = - 1$

Product of roots ab = 1

Given $a^2 + b^2$

$(a + b)^2 - 2ab$

= 1 – 2

= - 1.

4. Question

Mark the Correct alternative in the following:

If α, β are roots of the equation $4x^2 + 3x + 7 = 0$, then 1/α + 1/β is equal to

A. 7/3

B. −7/3

C. 3/7

D. −3/7

Answer

given $4x^2 + 3x + 7 = 0$

We know sum of the roots $= \frac{-3}{4}$

Product of the roots $= \frac{7}{4}$

As given that α and β are roots then,

$$\alpha + \beta = \frac{-3}{4}$$

$$\alpha\beta = \frac{7}{4}$$

given $\frac{1}{\alpha} + \frac{1}{\beta}$

$$= \frac{\alpha + \beta}{\alpha\beta}$$

$$= \frac{-\frac{3}{4}}{\frac{7}{4}}$$

$$= \frac{-3}{7}$$

5. Question

Mark the Correct alternative in the following:

The values of x satisfying $\log_3(x^2 + 4x + 12) = 2$ are

A. 2, −4

B. 1, −3

C. −1, 3

D. −1, −3

Answer

given $\log_3(x^2 + 4x + 12) = 2$

It can be written as

$\log_3(x^2 + 4x + 12) = 2\log_3 3$

$= \log_3 3^2$

$\log_3(x^2 + 4x + 12) = \log_3 9$

$x^2 + 4x + 12 = 9$

$x^2 + 4x + 3 = 0$

$(x + 1)(x + 3) = 0$

$X = -1, -3.$

6. Question

Mark the Correct alternative in the following:

The number of real roots of the equation $(x^2 + 2x)^2 - (x + 1)^2 - 55 = 0$ is

A. 2

B. 1

C. 4

D. none of these

Answer

given $(x^2+2x)^2-(x+1)^2-55=0$

$[(x^2 +2x +1) -1]^2 -(x +1)^2 -55 =0$

$(x +1)^4 -2(x +1)^2 +1 -(x +1)^2 -55 =0$

$(x+1)^4 -3(x +1)^2 -54 =0$

let $(x +1)^2= r$

$r^2 -3r -54 =0$

$r^2 -9r +6r -54 =0$

$r(r -9) +6(r -9) =0$

$r = -6 , 9$

but $(x+1)^2 \geq 0$ so, $(x+1)^2 \neq -6$

so, $(x +1)^2 = 9$

$x + 1 = \pm 3$

$x = -1 \pm 3$

$x = -4$, and 2 .

7. Question

Mark the Correct alternative in the following:

If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\frac{1}{a\alpha + b} + \frac{1}{a\beta + b} =$

A. c/ab

B. a/bc

C. b/ac

D. none of these

Answer

given $\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}$

$$=\frac{a\alpha+b+a\beta+b}{(a\alpha+b)(a\beta+b)}$$

$$=\frac{a(\alpha+\beta)+2b}{a^2(\alpha\beta)+ab(\alpha+\beta)+b^2}$$

$$=\frac{a\left(\frac{-b}{a}\right)+2b}{a^2\left(\frac{c}{a}\right)+ab\left(\frac{-b}{a}\right)+b^2}$$

$$=\frac{b}{ac-b^2+b^2}$$

$=\frac{b}{ac}$.

8. Question

Mark the Correct alternative in the following:

If α, β are the roots of the equation $x^2+px+1=0;\ \gamma,\delta$ the roots of the equation $x^2+qx+1=0$, then $(\alpha-\gamma)(\alpha+\delta)(\beta-\gamma)(\beta+\delta)=$

A. q^2-p^2

B. p^2-q^2

C. p^2+q^2

D. none of these

Answer

$\alpha^2+p\alpha+1=0,\ \beta^2+p\beta+1=0$

$\alpha+\beta=-p,\ \alpha\beta=1$

$\gamma^2+q\gamma+1=0,\ \delta^2+q\delta+1=0$

$\delta-\gamma=\sqrt{q^2-4},\ \gamma\delta=1$

$(\alpha-\gamma)(\alpha+\delta)(\beta-\gamma)(\beta+\delta)$

$=(\alpha^2+\alpha(\delta-\gamma)-\gamma\delta)(\beta^2+\beta(\delta-\gamma)-\delta\gamma)$

$$=\left(\alpha^2+\alpha\sqrt{q^2-4}-1\right)\left(\beta^2+\beta\sqrt{q^2-4}-1\right)$$

$$=\left(-2-p\alpha+\alpha\sqrt{q^2-4}\right)\left(-2-p\beta+\beta\sqrt{q^2-4}\right)$$

$$=4+2p\beta-2\beta\sqrt{q^2-4}+2p\alpha+p^2\alpha\beta-p\alpha\beta\sqrt{q^2-4}-2\alpha\sqrt{q^2-4}$$

$$-p\beta\alpha\sqrt{q^2-4}+\ \alpha^2\beta^2(q^2-4)$$

$= 4 + 2p(\alpha + \beta) - 2\sqrt{q^2 - 4}(\alpha + \beta) + p^2\alpha\beta - 2p\alpha\beta\sqrt{q^2 - 4}$

$+\alpha^2\beta^2(q^2 - 4)$

$= 4 - 2p^2 + 2p\sqrt{q^2 - 4} + p^2 - 2p\sqrt{q^2 - 4} + (q^2 - 4)$

$= 4 - 2p^2 + p^2 + q^2 - 4$

$= q^2 - p^2$

9. Question

Mark the Correct alternative in the following:

The number of real solutions of $|2x - x^2 - 3| = 1$ is

A. 0

B. 2

C. 3

D. 4

Answer

given $|2x-x^2-3| = 1$

$2x-x^2-3 = \pm 1$

When $2x-x^2-3 = 1$

$\Rightarrow 2x-x^2-3 -1 = 0$

$\Rightarrow 2x-x^2-4 = 0$

$= x^2 - 2x +4 = 0$

Discriminant, D = 4 - 16

= -12 < 0

Hence the roots are unreal.

When $2x-x^2-3 = -1$

$= x^2 - 2x -2 = 0$

Discriminant, D = 4 – 8 = - 4 < 0

Hence the roots are unreal.

Hence the given equation has no real roots.

10. Question

Mark the Correct alternative in the following:

The number of solutions of $x^2 + |x - 1| = 1$ is

A. 0

B. 1

C. 2

D. 3

Answer

when $x > 0$

$x^2 + x - 1 = 1$

$x^2 + x - 2 = 0$

$(x-1)(x+2) = 0$

$x = 1, -2$

when $x < 0$

$x^2 - x + 1 = 1$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

hence the given equation has 3 solutions and they are x = 0, 1, -1.

11. Question

Mark the Correct alternative in the following:

If x is real and $k = \frac{x^2 - x + 1}{x^2 + x + 1}$, then

A. $k \in [1/3, 3]$

B. $k \geq 3$

C. $k \leq 1/3$

D. none of these

Answer

$(x^2 + x + 1)k = (x^2 - x + 1)$

$(k - 1)x^2 + (k + 1)x + (k - 1) = 0$

For roots of quadratic equation real

Case I : $a \neq 0$ and $D \geq 0$

$k - 1 \neq 0 \Rightarrow k \neq 1$

$\sqrt{(k+1)^2 - 4(k-1)(k-1)} \geq 0$

$-3k^2 + 10k - 3 \geq 0$

$3k^2 - 10k + 3 \leq 0$

$3\left(k^2 - \frac{10}{3}k + 1\right) \leq 0$

$k^2 - 2\left(\frac{5}{3}\right)(k) + \frac{25}{9} - \frac{25}{9} + 1 \leq 0$

$\left(k - \frac{5}{3}\right)^2 \leq \frac{16}{9}$

$k - \frac{5}{3} \geq \frac{-4}{3}$ or $k - \frac{5}{3} \leq \frac{4}{3}$

$k \geq \frac{1}{3}$ or $k \leq 3$

Case II : a = 0

k – 1 = 0 ⇒ k = 1

At k = 1, 2x = 0 ⇒ x = 0 is real

So, k = 1 is also count in answer.

Then, final answer is k ∈ [1/3, 3]

12. Question

Mark the Correct alternative in the following:

If the roots of $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c$ is

A. 0

B. 1

C. 2

D. None of these

Answer

given that roots are consecutive, let they be a, a+1

From the formula for quadratic equation,

$(x - a)(x - a - 1) = x^2 - (a + 1)x - ax + a(a + 1) = x^2 - (2a + 1)x + a(a + 1)$ then $b^2 - 4c = (2a + 1)^2 - 4a(a + 1) = 4a^2 + 1 + 4a - 4a^2 - 4a = 1$.

13. Question

Mark the Correct alternative in the following:

The value of a such that $x^2 - 11x + a = 0$ and $x^2 - 14x + 2a = 0$ may have a common root is

A. 0

B. 12

C. 24

D. 32

Answer

subtracting both the equations we get,

$x^2 – 11x + a - x^2 + 14x + 2a = 0$

$3x – a = 0$

$x = \frac{a}{3}$

Substituting it in first equation we get,

$\left(\frac{a}{3}\right)^2 - 11\frac{a}{3} + a = 0$

$\frac{a^2}{9} - \frac{8a}{3} = 0$

$a^2 – 24a = 0$

a = 24.

14. Question

Mark the Correct alternative in the following:

The values of k for which is quadratic equation $kx^2+1=kx+3x-11x^2$ has real and equal roots are

A. -11, -3

B. 5, 7

C. 5, -7

D. None of these

Answer

given $kx^2 + 1 = kx + 3x - 11x^2$

$x^2(k+11) - x(k + 3) + 1 = 0$

as the roots are real and equal then the discriminant is equal to zero.

$D = b^2 - 4ac = 0$

$(k+3)^2 - 4(k+11)(1) = 0$

$K^2 + 9 + 6k - 4k - 44 = 0$

$K^2 + 2k - 35 = 0$

$(k-5)(k+7) = 0$

K = 5, -7.

15. Question

Mark the Correct alternative in the following:

If one root of the equation $x^2+px+12=0$ is 4, while the equation $x^2+px+q=0$ has equal roots, the value of q is

A. 46/4

B. 4/49

C. 4

D. none of these

Answer

multiplying first equation and subtracting both the equations we get,

$2x^2 + 4x + 6\lambda - 2x^2 - 3x - 5\lambda = 0$

$x + \lambda = 0$

$x = -\lambda$

Substituting it in first equation we get,

$(-\lambda)^2 + 2(-\lambda) + 3\lambda = 0$

$\lambda^2 + \lambda = 0$

$\lambda = -1.$

16. Question

Mark the Correct alternative in the following:

If one root of the equation $x^2+px+12=0$ is 4, while the equation $x^2+px+q=0$ has equal roots, the value of q is

A. 46/4

B. 4/49

C. 4

D. none of these

Answer

given 4 is the root of $x^2 + px + 12 = 0$

$= 16 + 4p + 12 = 0$

$4p = -28$

$P = -7$

Given $x^2 + px + q = 0$ has equal roots, then discriminant is 0.

$D = b^2 - 4ac = 0$

$p^2 - 4q = 0$

$4q = 49$

$q = \frac{49}{4}$.

17. Question

Mark the Correct alternative in the following:

The value of p and q$\left(p \neq 0, q \neq 0\right)$ for which p, q are the roots of the equation $x^2 + px + q\ \ ab = 0$ are

A. p = 1, q = −2

B. p = −1, q = −2

C. p = −1, q = 2

D. p = 1, q = 2

Answer

Sum of the roots = $\frac{-b}{a}$ = -p

⇒ p + q = -p ...(1)

Product of the roots = $\frac{c}{a}$ = q

⇒ pq = q

⇒ p = 1

Put value of p in eq.(1)

⇒ 1 + q = -1

⇒ q = -2

18. Question

Mark the Correct alternative in the following:

The set of all vales of m for which both the roots of the equation $x^2 - \left(m+1\right) x + m + 4 = 0$are real and negative, is

A. (−,−3] [5, ∞)

B. [−3, 5]

C. (−4, −3]

D. (−3, −1]

Answer

For roots to be real its D ≥ 0

$\sqrt{(m+1)^2 - 4(1)(m+4)} \geq 0$

$(m + 1)^2 - 4(m + 4) \geq 0$

$m^2 - 2m - 15 \geq 0$

$(m - 1)^2 - 16 \geq 0$

$(m - 1)^2 \geq 16$

$m - 1 \leq -4$ or $m - 1 \geq 4$

$m \leq -3$ or $m \geq 5$

For both roots to be negative product of roots should be

positive and sum of roots should be negative.

Product of roots = $m + 4 > 0 \Rightarrow m > -4$

Sum of roots = $m + 1 < 0 \Rightarrow m < -1$

After taking intersection of D ≥ 0, Product of roots > 0 and

sum of roots < 0. We can say that the final answer is

$m \in (-4, -3]$

19. Question

Mark the Correct alternative in the following:

The number of roots of the equation $\frac{(x+2)(x-5)}{(x-3)(x+6)} = \frac{x-2}{x+4}$ is

A. 0

B. 1

C. 2

D. 3

Answer

given $\frac{(x+2)(x-5)}{(x-3)(x+6)} = \frac{x-2}{x+4}$

$(x+2)(x-5)(x+4) = (x-2)(x-3)(x+6)$

$x^3+4x^2-5x^2-20x+2x^2+8x-10x-40 = x^3+6x^2-3x^2-18x-2x^2-12x+6x+36$

$x^2-22x-40 = x^2-24x+36$

$4x = 76$

$x = 19$

hence the given equation has only one solution.

20. Question

Mark the Correct alternative in the following:

If α and β are the roots of $4x^2 + 3x + 7 = 0$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is

A. $\frac{4}{7}$

B. $-\frac{3}{7}$

C. $\frac{3}{7}$

D. $-\frac{3}{4}$

Answer

given $4x^2 + 3x + 7 = 0$

We know sum of the roots $= \frac{-3}{4}$

Product of the roots $= \frac{7}{4}$

As given that α and β are roots then,

$\alpha + \beta = \frac{-3}{4}$

$\alpha\beta = \frac{7}{4}$

given $\frac{1}{\alpha} + \frac{1}{\beta}$

$= \frac{\alpha + \beta}{\alpha\beta}$

$= \frac{-\frac{3}{4}}{\frac{7}{4}}$

$= \frac{-3}{7}$

21. Question

Mark the Correct alternative in the following:

If α, β are the roots of the equation $x^2 + px + q = 0$, then $-\frac{1}{\alpha}, -\frac{1}{\beta}$ are the roots of the equation

A. $x^2 - px + q = 0$

B. $x^2 + px + q = 0$

C. $qx^2 + px + 1 = 0$

D. $qx^2 - px + 1 = 0$

Answer

from the given equation sum of the roots α + β = - p

Product of roots αβ = q

Formula to form a quadratic equation is x^2- (α + β) x+ αβ = 0

Where α, β are roots of equation.

Given $\frac{-1}{\alpha}, \frac{-1}{\beta}$ are roots, then required quadratic equation is

$$x^2 - \left(\frac{-1}{\alpha} + \frac{-1}{\beta}\right)x + \frac{1}{\alpha\beta} = 0$$

$$x^2 + \left(\frac{\alpha+\beta}{\alpha\beta}\right)x + \frac{1}{\alpha\beta} = 0$$

$$x^2 + \left(\frac{-p}{q}\right)x + \frac{1}{q} = 0$$

$qx^2 - px + 1 = 0$.

22. Question

Mark the Correct alternative in the following:

If the difference of the roots of $x^2 - px + q = 0$ is unity, then

A. $p^2 + 4q = 1$

B. $p^2 - 4q = 1$

C. $p^2 + 4q^2 = (1+2q)^2$

D. $4p^2 + q^2 = (1+2p)^2$

Answer

Difference of the roots $= \frac{\sqrt{D}}{|a|}$

$1 = \frac{\sqrt{b^2-4ac}}{|a|}$

$1 = \frac{\sqrt{(-p)^2-4(1)(q)}}{|1|}$

$p^2 - 4q = 1$

$p^2 - 4q + 4q^2 - 4q^2 = 1$

$p^2 + 4q^2 = 1 + 2(2)(q) + (2q)^2$

$p^2 + 4q^2 = (1 + 2q)^2$

23. Question

Mark the Correct alternative in the following:

If α, β are the roots of the equation $x^2 - p(x+1) - c = 0$, then $(\alpha+1)(\beta+1) =$

A. c

B. c − 1

C. 1 − c

D. none of these

Answer

given x^2-p(x+1)-c=0

x^2-px-p-c=0

x^2-px-(p+c)=0

as α, β are roots of equation, we get

sum of the roots α + β = p

Product of roots αβ = - (p + c)

Given (1+ α) (1+ β)

= 1 + (α + β) + (α β)

= 1 + p – p – c

= 1 – c.

24. Question

Mark the Correct alternative in the following:

The least value of k which makes the roots of the equation $x^2 + 5x + k = 0$ imaginary is

A. 4

B. 5

C. 6

D. 7

Answer

given that the equation has imaginary roots, hence the discriminant is less than 0.

= 25-4k<0

When we submit 7 in k the condition above will be satisfied and when we replace 6 the condition will be false.

So the least value of k is 7.

25. Question

Mark the Correct alternative in the following:

The equation of the smallest degree with real coefficients having 1 + i as one of the roots is

A. $x^2 + x + 1 = 0$

B. $x^2 - 2x + 2 = 0$

C. $x^2 + 2x + 2 = 0$

D. $x^2 + 2x - 2 = 0$

Answer

for the complex roots it will exists in pair.

Hence the roots are 1+i and 1-i

Formula for quadratic equation is (x-a) (x-b) = 0

$(x-1-i)(x-1+i) = 0$

$x^2-x+ix-x+1-i-ix+i-i^2=0$

$x^2-2x+2 = 0.$