

Continuity And Differentiability

Que 1:

Marks :(4)

If $2^x + 2^y = 2^{x+y}$. find $\frac{dy}{dx}$

Ans:

$$2^x + 2^y = 2^{x+y}$$

diff. wrt x

$$2^x \log 2 + 2^y \log 2 \frac{dy}{dx} = 2^{x+y} \log 2 \left(1 + \frac{dy}{dx}\right)$$

$$2^x + 2^y \frac{dy}{dx} = 2^{x+y} \left(1 + \frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y - 2^{x+y}}$$

$$= \frac{2^x(2^y - 1)}{2^y(1 - 2^x)} = 2^{x-y} \left(\frac{2^y - 1}{1 - 2^x}\right)$$

Que 2:

Marks :(1)

The value of x for which $|x|$ is continuous but not differentiable is.....

Ans: $x = 0$

Que 3:

Marks :(2)

if $f(x) = \frac{3x + \tan^2 x}{x}$ is continuous at $x = 0$. find $f(0)$

Ans:

$$f(x) = \frac{3x + \tan^2 x}{x}$$

$$f(0) = \lim_{x \rightarrow 0} \left(\frac{3x + \tan^2 x}{x}\right) = 3 + 1 \times 0 = 3$$

Que 4:

Marks :(3)

If $\sec\left(\frac{x+y}{x-y}\right) = a$. Find $\frac{dy}{dx}$

Ans:

$$\frac{x+y}{x-y} = \sec^{-1}a$$

$$\frac{(x+y)-(x-y)}{(x+y)+(x-y)} = \frac{\sec^{-1}a-1}{\sec^{-1}a+1}$$

$$\frac{2y}{2x} = k, a \text{ constant}$$

$$diff$$

$$\frac{xy_1-y}{x^2}=0$$

$$xy_1=y$$

$$y_1=\frac{y}{x}$$