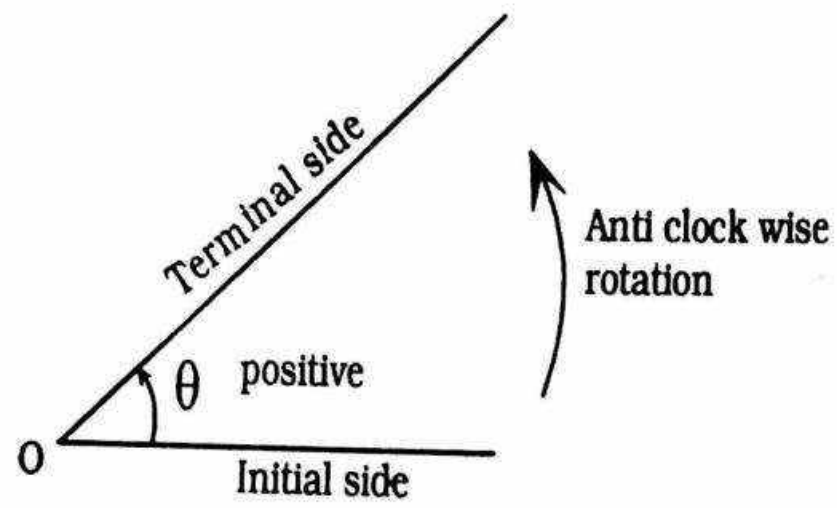
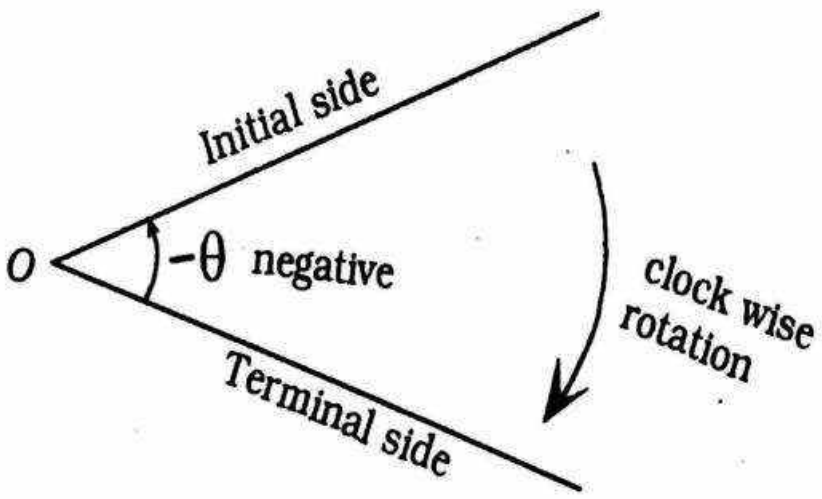
CHAPTER **18**

TRIGNOMETRY
CIRCULAR MEASURE OF ANGLES

Angles :-

When two rays (initial and terminal) meet at a point after rotation in a plane then they are said to have described an angle .





Systems of Measurement of Angles:

1. **Sexagonal System** :-

$1 \text{ degree} = 1^0 = \frac{1}{90}$ right angle

$1 \text{ minute} = 1' = \frac{1}{60}$ degree

$1 \text{ second} = 1'' = \frac{1}{60}$ minute

In other words :- 90^0 = 1 right angle

$60' = 1^0$

$60'' = 1'$

2. Centesimal or French System :-

1 right angle = 100 grades ($=100^g$)

1 grade = 100 minute ($=100'$)

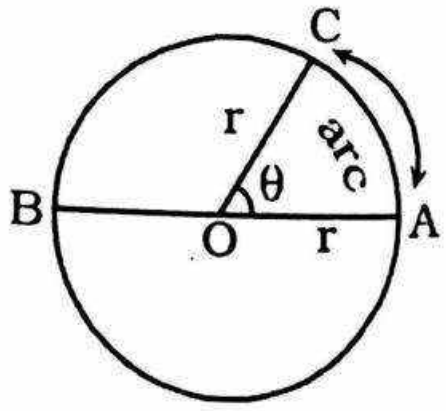
1 minute = 100 seconds ($=100''$)

3. Circular System :-

In this system, the unit of measurement is "radian".

Angle (in radian) $\theta = \frac{\text{arc AC}}{\text{radius}} = \frac{\overset{\frown}{AC}}{r}$

1 radian or 1^c is the angle subtended by an arc at the centre of a circle whose length is equal to the radius of the circle.



i.e. if arc=radius = r, then

$\theta = \frac{r}{r} = 1 \text{ read} = 1^c$

when arc = ACB = πr

angle $=180^0 = \frac{\pi r}{r} = \pi$ radian

i.e π rad $=180^0 = 2$ right angle

$\therefore$ 1 rad $= 1^c = \frac{180^0}{\pi} = 57.2958^0$

or $\boxed{1^c = 1\text{rad} = 57^0 16' 22'' \text{ approx.}}$

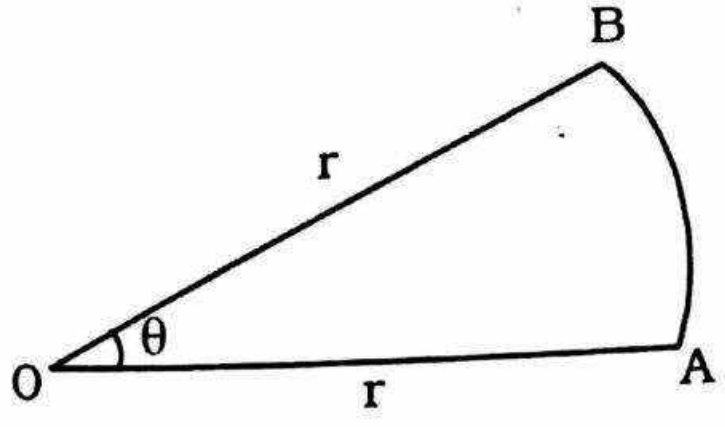
$\left\{\text{and } 1° = \frac{\pi}{180}\text{ radian} = \left(\frac{22}{7\times 180}\right)\text{rad}\right\}$

$\boxed{\text{or } 1° = 0.01746 \text{ radian}}$

⇒ Area of a sector (or Sectorial area) :-

The area of the sector OAB $= \frac{1}{2}r^2\theta$

Here θ is in radian.



Note :- Radian is a constant angle.

Some Useful Points :-

The angle between two consecuive digits in a clock is $30^0 \left(= \frac{\pi}{6}\text{radians}\right)$

- The hour hand rotates through an angle of 30^0 in one hour i.e. $\left(\frac{1}{2}\right)^0$ in one minute.
- The minute hand rotates through an angle of 6^0 in one minute.

Degree	Radian
30°	π/6
45°	π/4
60°	π/3
90°	π/2
120°	2π/3

Degree	Radian
135°	3π/4
150°	5π/6
180°	π
270°	3π/2
360°	2π

LEVEL - I

1. 1 radian is equal to :-

(a) 100° (b) $\left(\frac{\pi}{180}\right)^0$

(c) $\left(\frac{180}{\pi}\right)^0$ (d) 90^0

2. Find the digree measure corresponding to $\left(\frac{4\pi}{15}\right)^c$:-

(a) 48^0 (b) 24^0
(c) 36^0 (d) 72^0

3. Find the degree measure corresponding to $\left(\frac{1}{6}\right)^c$:-

(a) $9^0 32'$ (b) $9^0 32' 43.6''$
(c) 10^0
(d) None of these

4. Find the radian measure corresponding to $13^0 7' 30''$

(a) $\left(\frac{\pi}{48}\right)^c$ (b) $\left(\frac{\pi}{96}\right)^c$

(c) $\left(\frac{5\pi}{96}\right)^c$ (d) $\left(\frac{7\pi}{96}\right)^c$

5. Find the digree measure corresponding to $\left(\frac{5\pi}{6}\right)^c$:-

(a) 140^0 (b) 130^0
(c) 150^0 (d) 145^0

6. The value of 80^0 in radian is :-

(a) $\left(\frac{4\pi}{9}\right)^c$ (b) $\left(\frac{2\pi}{9}\right)^c$

(c) $\left(\frac{2\pi}{3}\right)^c$ (d) $\left(\frac{4\pi}{3}\right)^c$

7. When a pendulam of length 50 cm oscillates, it produces an arc of 16 cm. The angle so formed in degree measure is (approx) :
(a) $18°25'$ (b) $18°35'$
(c) $18°20'$ (d) $18°08'$

8. Three interior angles of a quadrilateral are $60°$, $120°$, $90°$. The remaining angle in circular measure is given by :

(a) $\frac{\pi^c}{3}$ (b) $\frac{\pi^c}{2}$

(c) $\frac{\pi^c}{4}$ (d) $\frac{3\pi^c}{4}$

9. In Δ ABC, $\angle A = 30°$, $\angle B = 60°$. Find $\angle C$ in circular measure :

(a) $\frac{2\pi^c}{3}$ (b) $\frac{3\pi^c}{4}$

(c) $\frac{\pi^c}{4}$ (d) $\frac{\pi^c}{2}$

10. In circular measure, the value of the angle $11°15'$ is :

(a) $\frac{\pi^c}{16}$ (b) $\frac{\pi^c}{8}$

(c) $\frac{\pi^c}{4}$ (d) $\frac{\pi^c}{12}$

LEVEL - II

1. Find the length of an arc of a circle of radius 10cm subtending a central angle measuring 12^0

(A) $\frac{\pi}{15}$ cm (B) $\frac{4\pi}{3}$ cm

(C) $\frac{2\pi}{3}$ cm (D) $\frac{\pi}{3}$ cm

2. The moon's distance from the earth is 360000 km and its diameter subtends an angle of 30' at the eye of the observer. Find the diameter of the moon.

(a) 100π km (B) 1000π km

(c) 1500π km (D) 2000π km

3. Find in degress the angle through which a pendulum swings if its length is 90cm and its tip describes an arc of length 22 cm .

(a) 14^0 (b) $13^0\,16'$

(c) $14^0\,8'$ (d) 13^0

4. A rail road curve is to be laid out on a circle. What radius should be used if the track is to change direction by 25° in a distance of 40 metres ?

(a) 91.64 metres (b) 90.46 metres

(c) 89.64 metres (d) 93.64 metres

5. Two angles of a triangle are $\frac{1}{2}$ radian and $\frac{1}{3}$ radian. The measure of the third angle in degree (taking $\pi = \frac{22}{7}$

(a) $132\frac{1}{11}^\circ$ (b) $132\frac{2}{11}^\circ$

(c) $132\frac{3}{11}^\circ$ (d) 132°

6. By decreasing 15° of each angle of a triangle, the ratios of their angles are 2 : 3 : 5. The radian measure of greatest angle is :

(a) $\frac{11\pi}{24}$ (b) $\frac{\pi}{12}$

(c) $\frac{\pi}{24}$ (d) $\frac{5\pi}{24}$

7. In a triangle ABC, $\angle ABC = 75°$ and $\angle ACB = \frac{\pi^\circ}{4}$. The circcular measure of $\angle BAC$ is :

(a) $\frac{5\pi}{12}$ radian (b) $\frac{\pi}{3}$ radian

(c) $\frac{\pi}{24}$ (d) $\frac{5\pi}{24}$

8. The minute hand of a big wall-clock is 35 cm long. Taking $\pi = \frac{22}{7}$, length of the arc, its extremity moves in 18 seconds is :

(a) 11 cm (b) 1.1 cm

(c) 6.6 cm (d) 6 cm

LEVEL - III

1. The minute hand of a watch is 3cm long. How far does its tip move in 50 minute ?

(a) 10.32 cm (b) 17.67 cm
(c) 15.71 cm (d) 18.23 cm

2. Find the angle between the hour hand and the minute hand at half past four .

(a) $\frac{\pi}{4}$ radian (b) $\frac{\pi}{6}$ radian

(c) $\frac{2\pi}{3}$ radian (d) $\frac{\pi}{3}$ radian

3. In a circle of diameter 30cm , the length of the chord is 15cm. Find the length of the minor arc corresponding to the chord .

(a) $\frac{5\pi}{3}$ cm (b) 5π cm

(c) $\frac{5\pi}{2}$ cm

(d) None of these

4. If the arcs of same length in two circles subtend angles 60^0 and 75^0 at their centres. Find the ratio of their radii :-

(a) 5 : 3 (b) 5 : 8
(c) 4 : 7 (d) 5 : 4

5. Find the angle between the minute hand of a clock and the hour hand when the time is 5:20 AM.

(a) 50^0 (b) 30^0
(c) 40^0 (d) 45^0

6. A wheel makes 240 revolutions per minute. Through how many radians does it turns in 1 secound ?

(a) 8π (b) 6π
(c) 4π (d) 16π

7. If the angular diameter of the moon be 30' , how far from the eye a coin of diameter 4.4 cm be kept to hide the moon ?

(a) 252 cm (B) 504 cm
(c) 300 cm (D) 500 cm

8. The angles of a triangle are in Arithmetic Progression. The ratio of the least angle in degrees to the number of radians in the greatest anlgle is 60 : π . The angles in degrees are:

(a) 30°, 60°, 90°
(b) 35°, 55°, 90°
(c) 40°, 50°, 90°
(d) 40°, 55°, 85°

Hints and Solutions

Level -I

1.(c) $\because \pi$ rad $= 180^0$

$\therefore 1$ rad $= \left(\frac{180}{\pi}\right)^\circ$

2.(a) π radian $= 180^0$

$\Rightarrow 1^c = \left(\frac{180}{\pi}\right)^0$

$\therefore \left(\frac{4\pi}{15}\right)^c = \left(\frac{4\pi}{15} \times \frac{180}{\pi}\right)^0 = 48^0$

3.(h) $\left(\frac{1}{6}\right)^c \left(\frac{1}{6} \frac{180}{\pi}\right)^0 = \left(\frac{1}{6} \times \frac{180}{22} \times 7\right)^0$

$= \left(\frac{105}{8} \times \frac{\pi}{180}\right)^c = \left(\frac{7\pi}{96}\right)^c$

5.(c) $\because \pi^c = 180^\circ$

$\therefore \left(\frac{5\pi}{6}\right)^c = \left(\frac{180}{\pi} \times \frac{5\pi}{6}\right)^\circ = 150^\circ$

6.(a) $180^0 = \pi^c$

$\therefore 80^0 = \left(\frac{\pi}{180} \times 80\right)^c = \left(\frac{4\pi}{9}\right)^c$

7.(c) s = 16 cm
r = 50 cm

$\therefore \theta = \frac{s}{r} = \frac{16}{50} = \frac{8}{25}$ radian

8 180° 8 180 - 1008

Level - II

1.(c) $r = 10,$

$$\theta = 12^0 = \left(12 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{15}\right)^c$$

$$\therefore \theta = \frac{\text{arc}}{\text{radius}} \Rightarrow \text{arc} = r\theta = 10 \times \frac{\pi}{15} = \frac{2\pi}{3}$$

2.(b) Diameter d = Arc AB
as the distance between moon and the earth is very large

$$\theta = 30 = \left(\frac{30}{60} \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{360}\right)^c$$

$$\therefore \theta = \frac{\text{arc}}{\text{radius}}$$

4.(a) $\theta = 25° = \frac{25 \times \pi}{180}$ radians

$$= \frac{5\pi}{36} \text{ radians} \quad \theta = \frac{s}{r}$$

$$\Rightarrow r = \frac{s}{\theta} = \frac{40}{\frac{5\pi}{36}} = \frac{40 \times 36}{5\pi} = \frac{40 \times 36 \times 7}{5 \times 22} \text{ metre}$$

$= 91.64$ metre

5.(c) Sum of two angles =

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6} \text{ radian}$$

$$\therefore \text{ Third angle} = \left(\pi - \frac{5}{6}\right) \text{radian}$$

$$= \frac{22}{7} - \frac{5}{6} = \frac{132 - 35}{42} = \frac{97}{42} \text{ radians}$$

7.(b) $\angle ABC = 75°$

$\because 180° = \pi$ radian

$$\therefore 75° = \frac{\pi}{180} \times 75 = \frac{5\pi}{12} \text{ radian}$$

$$= \frac{12\pi - 3\pi - 5\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3} \text{ radian}$$

8.(b) Traced arc length by minute hand in 60×60 seconds $= 2\pi r$

$\therefore$ Length of arc made in 18 seconds

$$= \frac{2\pi r}{60 \times 60} \times 18$$

$$= 2 \times \frac{22}{7} \times \frac{35 \times 18}{60 \times 60} = 1.1 \text{ cm}$$

Level -III

1.(c) The minute hand complete one revolution in 60 minute.

$\therefore$ In 50 minute it will cover $\frac{50}{60} = \frac{5}{6}$ of the revolution.

$\because$ 1 revolution $= 2\pi$ radian.

$$\therefore \frac{5}{6} \text{ revolution} = 2\pi \times \frac{5}{6} = \frac{5\pi}{3} \text{ radian}$$

$$\therefore \text{Distance moved by tip} = 3 \times \frac{5\pi}{3} \text{ cm}$$

$$= 5\pi \text{ cm}$$

$$= 5 \times \frac{22}{7} \text{cm} = 15.71 \text{ cm}$$

2.(a) Angle traced by the hour hand in 12 hours = 360^0

$\therefore$ Angle traced by the hour hand in 4 hrs 30 min. $\left(= \frac{9}{2} \text{hrs}\right)$

$$= \frac{360}{12} \times \frac{9}{2} = 135^0$$

Angle traced by the minute hand in 60 min. $= 360^0$

$\therefore$ Angle traced by the minute hand

$$\text{in 30 min} = \left(\frac{360}{60} \times 30\right)^0 = 180^0$$

Thus, the angle between two hands

$$= 180^0 - 135^0$$

$$= 45^0$$

$$= \frac{\pi}{4} \text{ radian}$$

Short - cut :-

$$\theta = \left|\frac{11}{2}M - 30H\right|$$

Where θ = angle
M = minute
H = hour

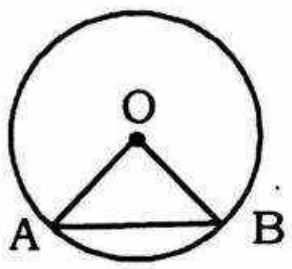
$$\theta = \left|\frac{11}{2}\times 30 - 30\times 4\right|$$

$$= |165° - 120°|$$

$$= 45º$$

3.(b) OA = OB = 15 cm (radius)
and chord AB = 15 cm

$\therefore$ ΔOAB is an equilateral triangle.



$$\therefore \angle AOB = 60^0 = \left(\frac{\pi}{3}\right)^c$$

$$\therefore \theta = \frac{\text{arc}}{\text{radius}} \Rightarrow \text{arc} = \theta \times r = \frac{\pi}{3}\times 15 = 5\pi$$

4.(d) lentgth of arc (l) = θ × radius (r)

$r = l/\theta$

$$\therefore \frac{r_1}{r_2} = \frac{l_1}{l_2}\times\frac{\theta_2}{\theta_1} = \frac{\theta_2}{\theta_1}$$

$$= \frac{75}{60} = \frac{5}{4}$$

5.(c) $$\theta = \left|\frac{11}{2}M - 30H\right|$$

Where θ = angle
M = minute
H = hour

$$\theta = \left|\frac{11}{2}\times 20 - 30\times 5\right|$$

$$= |110° - 150°|$$

$$= 40º$$

6.(a) Number of revolution per second

$$= \frac{240}{60} = 4$$

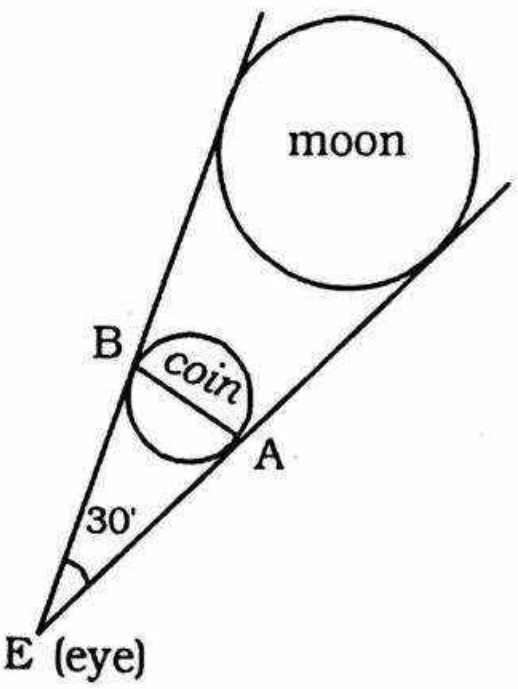
complete circles.

A circle subtends an angle of $2\pi^c$ at its centre it 1 revolution

$\therefore$ Number of radians in 4 revolution

$$= 4\times 2\pi = 8\pi^c$$

7.(b) arc AB = diameter AB = 4.4 cm



$$\theta = 30' = \left(\frac{30}{60}\right)^0 = \left(\frac{1}{2}\right)^0$$

$$= \left(\frac{1}{2}\times\frac{\pi}{180}\right)^c = \left(\frac{\pi}{360}\right)^c$$

$$\theta = \frac{\text{arc}}{\text{radius}} \Rightarrow \frac{\pi}{360} = \frac{4.4}{r}$$

$$\Rightarrow r = \frac{4.4\times 360}{\pi}\text{cm} = \frac{4.4\times 360}{22}\times 7$$

$\Rightarrow r = 504$ cm

8.(a) Angles of triangle

$(a - d)°, a°, (a + d)°$

$a - d + a + a + d = 180°$

$\Rightarrow 3a = 180° \Rightarrow a = 60°$

$$\therefore \frac{a-d}{a+d} = \frac{60}{\pi} = \frac{60}{180} = \frac{1}{3}$$

$$\Rightarrow \frac{60-d}{60+d} = \frac{1}{3}$$

$\Rightarrow 180 - 3d = 60 + d$

$\Rightarrow 4d = 120° \Rightarrow d = 30°$

$\therefore$ Angles of triangle :

$a - d = 60° - 30°$

$a = 60°$

$a + d = 60 + 30 = 90°$