Chapter 7: Probability

EXERCISE 7.1 [PAGES 99 - 100]

Exercise 7.1 | Q 1.1 | Page 99

State the sample space and n(S) for the following random experiments.
A coin is tossed twice. If a second throw results in a tail, a die is thrown.

SOLUTION

A coin is tossed twice.
If the second throw gives tail, a die is thrown.
∴ S = {HH, TH, HT1, HT2, HT3, HT4, HT5, HTG, TT1, TT2, TT3, TT4, TT5, TT6}
∴ n(S) = 14.

Exercise 7.1 | Q 1.2 | Page 99

State the sample space and n(S) for the following random experiments.
A coin is tossed twice. If a second throw results in head, a die thrown, otherwise a coin is tossed.

SOLUTION

A coin is tossed twice.
If a second throw gives head,
a die is thrown, otherwise a coin is tossed again.
∴ S = {HH1, HH2, HH3, HH4, HH5, HH6, TH1, TH2, TH3, TH4, THE, TH6, HTH, HTT, TTH, TTT}
∴ n(S) = 16.

Exercise 7.1 | Q 2 | Page 99

In a bag there are 3 balls; one black, one red and one green. Two balls are drawn one after another with replacement. State sample space and n(S)

SOLUTION

To draw any two coloured balls out of given 3 balls.
(B, R, G) one after the other with replacement.
∴ S = {BB, BR, BG, RR, RB, RG, GG, GB, GR}
∴ n(S) = 9.

Exercise 7.1 | Q 3.1 | Page 99

A coin and a die are tossed. State sample space of following events A: getting a head and an even number.

SOLUTION

A coin and a die are tossed together.
∴ S = {H1, H2, H3, H4, H5, H6, T1, T2. T3, T4; T5, T6}
∴ n(S) = 12
Let A: Coin gives a head and die shows an even number.
A = {H2, H4, H6}
∴ n(A) = 3.

Exercise 7.1 | Q 3.2 | Page 99
A coin and a die are tossed. State sample space of following events B: getting a prime number.

SOLUTION

A coin and a die are tossed together.
∴ S = {H1, H2, H3, H4, H5, H6, T1, T2. T3, T4; T5, T6}
∴ n(S) = 12
Let B: The die gives a prime number.
B = {H2, H3, H5, T2, T3, T5}
∴ n(B) = 6.

Exercise 7.1 | Q 3.3 | Page 99
A coin and a die are tossed. State sample space of following events C: getting a tail and perfect square.

SOLUTION

A coin and a die are tossed together.

∴ S = {H1, H2, H3, H4, H5, H6, T1, T2. T3, T4; T5, T6}

∴ n(S) = 12

Let C: Coin gives a tail and die shows a perfect square.

C = {T1, T4}

∴ n(C) = 2.

Exercise 7.1 | Q 4.1 | Page 99
Find total number of distinct possible outcomes n(S) for each of the following random experiments From a box containing 25 lottery tickets any 3 tickets are drawn at random.

SOLUTION

3 tickets are selected from a box containing 25 tickets at random.
3 tickets can be drawn from 25 tickets in ${}^{25}C_3$ ways.
∴ n(S) = ${}^{25}C_3$

$$= \frac{25 \cdot 24 \cdot 23}{3 \cdot 2 \cdot 1}$$

$= 2300.$

Exercise 7.1 | Q 4.2 | Page 99

Find total number of distinct possible outcomes n(S) for each of the following random experiments From a group of 4 boys and 3 girls, any two students are selected at random.

SOLUTION

Two students are selected at random from a group of 4 boys and 3 girls (total 7)
2 students can be selected from a group of 7 students in ${}^{7}C_{2}$ ways.
∴ n(S) = ${}^{7}C_{2}$

$$= \frac{7 \cdot 6}{2 \cdot 1}$$

$= 21.$

Exercise 7.1 | Q 4.3 | Page 99

Find total number of distinct possible outcomes n(S) for each of the following random experiments 5 balls are randomly placed into 5 cells, such that each cell will be occupied.

SOLUTION

5 balls are arranged in 5 linear cells one in each.
Since the objects are linearly arranged in numbered places, each outcome is a permutation.
∴ n(S) = ${}^{5}P_{5}$
= 5!
= 120.

Exercise 7.1 | Q 4.4 | Page 99

Find total number of distinct possible outcomes n(S) for each of the following random experiments 6 students are arranged in a row for a photograph.

SOLUTION

5 students are arranged in 6 linear places for a photograph.
Since a photograph is a linear arrangement each outcome is a permutation.
∴ n(S) = ${}^{6}P_{6}$
= 6!
= 720.

Exercise 7.1 | Q 5.1 | Page 99

Two dice are thrown. Write favourable Outcomes for the following events P: Sum of the numbers on two dice is divisible by 3 or 4.

SOLUTION

Let S: Two dice are rolled
∴ Sample space is,
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4). (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(S) = 36

Let P: Sum of the numbers on two dice is divisible by 3 or 4
∴ P = {(1, 2), (1, 3), (1, 5), (2, 1), (2, 2), (2,4), (2, 6), (3, 1), (3, 3), (3, 5), (3, 6), (4, 2) (4, 4), (4, 5), (5, 1), (5, 3), (5,4), (6, 2), (6, 3), (6, 6)}
∴ n(P) = 20

Exercise 7.1 | Q 5.2 | Page 99

Two dice are thrown. Write favourable Outcomes for the following events Q: Sum of the numbers on two dice is 7.

SOLUTION

Let S: Two dice are rolled
∴ Sample space is,
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4). (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(S) = 36

Let Q: Sum of the numbers on two dice is 7.
∴ Q = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6,1)}
∴ n(Q) = 6.

Exercise 7.1 | Q 5.3 | Page 100

Two dice are thrown. Write favourable Outcomes for the following events R: Sum of the numbers on two dice is a prime number.
Also check whether Events P and Q are mutually exclusive and exhaustive.

SOLUTION

Let S: Two dice are rolled
∴ Sample space is,
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2),

(5, 3), (5, 4). (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(S) = 36

Let R: Sum of the numbers on two is a prime number.
∴ R = {(1, 1), (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4) (4, 3), (5, 2), (6, 1), (5, 6), (6, 5)}
∴ n(R) = 15

P ∪ Q ≠ S but P ∩ Q ≠ Ø
∴ P and Q are not exhaustive but they are mutually exclusive.

Exercise 7.1 | Q 6.1 | Page 100
A card is drawn at random ' from an ordinary pack of 52 playing cards. State the number of elements in the sample space if consideration of suits Is not taken into account.

SOLUTION

If consideration of suits is not taken into consideration, then a card is selected at random from 52 cards.
i.e. To select one card out of 52.
∴ n(S) = $^{52}C_1$ = 52.

Exercise 7.1 | Q 6.2 | Page 100
A card is drawn at random ' from an ordinary pack of 52 playing cards. State the number of elements in the sample space if consideration of suits Is taken into account.

SOLUTION

If consideration of suits is taken into account then the desired suit is separated and a card is selected from that suit.
i.e. A card is selected at random from the desired suit containing 13 cards. (Either spades or clubs or hearts or diamonds).
∴ n(S) = $^{13}C_1$ = 13.

Exercise 7.1 | Q 7 | Page 100
Box I contains 8 red (R_{11}, R_{12}, R_{13}) and 2 blue (B_{11}, B_{12}) marbles while Box II contains 2 red(R_{21}, R_{22}) and 4 blue (B_{21}, B_{22}, B_{23}, B_{24}) marbles. A fair coin is tossed. If the coin turns up heads, a marble is chosen from Box I; if it turns up tails, a marble is chosen from Box II. Describe the sample space.

SOLUTION

A coin is tossed, if the result is head, a marble is chosen from Box I, for tail, a marble is chosen from Box II.
S = {(H, R_{11}), (H, R_{12}), (H, R_{13}),
(H, B_{11}), (H, B_{12}), (T, R_{21}),
(T, R_{22}), (T, B_{21}), (T, B_{22}),
(T, B_{23}), (T, B_{24})}.
∴ n(S) = 11.

Exercise 7.1 | Q 8.1 | Page 100

Consider an experiment of drawing two cards at random from a bag containing 4 cards marked 5, 6, 7 and 8. Find the sample Space if cards are drawn with replacement.

SOLUTION

Two cards are selected at random one after the other from a bag containing 4 cards (5, 6, 7, 8) with replacement. (i.e. repetition is allowed).
S = {(5,5),(5,6),(5,7),(5,8),(6,5), (6,6), (6, 7), (6, 8), (7, 5), (7, 6), (7, 7), (7, 8), (8, 5), (8, 6), (8, 7), (8, 8)}
∴ n(S) = 16.

Exercise 7.1 | Q 8.2 | Page 100

Consider an experiment of drawing two cards at random from a bag containing 4 cards marked 5, 6, 7 and 8. Find the sample Space if cards are drawn without replacement.

SOLUTION

Two cards are selected at random one after the other from bag without replacement. (i.e. repetition is not allowed).
S = {(5, 6), (5, 7), (5, 8), (6, 5), (6, 1), (6, 8), (7, 5), (7, 6), (7, 8), (8, 5), (8, 6), (8, 7)}
∴ n(S) = 12.

EXERCISE 7.2 [PAGES 102 - 103]

Exercise 7.2 | Q 1.1 | Page 102

A fair die 18 thrown two times. Find the chance that product of the numbers on the upper face is 12.

SOLUTION

A fair die 1s thrown two times.

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4. 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(S) = 36.

Let event A: Product of the numbers on the uppermost face is 12.

A = {(2, 6), (3, 4), (4, 3), (6, 2)}

∴ n(A) = 4.

$$\therefore P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{4}{36}$$

$$= \frac{1}{6}.$$

Exercise 7.2 | Q 1.2 | Page 102

A fair die 18 thrown two times. Find the chance that sum of the numbers on the upper face is 10.

SOLUTION

A fair die 1s thrown two times.
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4. 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(S) = 36.

Let event B: Sum of the numbers on the uppermost face is 10
B = {(4, 6), (5, 5), (6, 4)}
∴ n(B) = 3.

$$\therefore P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{3}{36}$$

$$= \frac{1}{6}.$$

Exercise 7.2 | Q 1.3 | Page 102

A fair die 18 thrown two times. Find the chance that Sum of the numbers on the upper face is at least 10.

SOLUTION

A fair die 1s thrown two times.
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4. 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(S) = 36.

Let event C: Sum of the numbers on the upper most face is at least 10 (i.e. 10, 11 or 12)
C = {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}
∴ n(C) = 6.

$$\therefore P(C) = \frac{n(C)}{n(S)}$$

$$= \frac{6}{36}$$

$$= \frac{1}{6}.$$

Exercise 7.2 | Q 1.4 | Page 102

A fair die 18 thrown two times. Find the chance that sum of the numbers on the upper face is at least 4.

SOLUTION

A fair die 1s thrown two times.

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4. 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(S) = 36.

Let event D: Sum of the numbers on the uppermost face is at least 4.

∴ D': Sum of the numbers on the uppermost face is < 4 (i.e. 2 or 3).

D' = {(1, 1), (1, 2), (2, 1)}

∴ n(D’) = 3

$$\therefore P(D') = \frac{n(D')}{n(S)}$$

$$= \frac{3}{36}$$

$$= \frac{1}{12}$$

Required Probability,

$$\therefore P(D) = 1 - P(D')$$

$$= 1 - \frac{1}{12}$$

$$= \frac{11}{12}.$$

Exercise 7.2 | Q 1.5 | Page 102

A fair die 18 thrown two times. Find the chance that the first throw gives an odd number and second throw gives multiple of 3.

SOLUTION

A fair die 1s thrown two times.

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4. 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

∴ n(S) = 36.

Let event E: First throw gives an odd number (1, 3 or 5) and second throw gives multiple of 3 (3 or 6).

E = {(1, 3), (1, 6), (3, 3), (37 6), (5, 3), (5, 6)}

∴ n(E) = 6

$$\therefore P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{6}{36}$$

$$= \frac{1}{6}.$$

Exercise 7.2 | Q 1.6 | Page 102
A fair die 18 thrown two times. Find the chance that both the times die shows same number (doublet).

SOLUTION

A fair die 1s thrown two times.
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4. 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(S) = 36.

Let event F: Both times die shows the same number (doublet).
F = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)}   n(F) = 6
∴ n(F) = 6

$$\therefore P(F) = \frac{n(F)}{n(S)}$$

$$= \frac{6}{36}$$

$$= \frac{1}{6}.$$

Exercise 7.2 | Q 2.1 | Page 102
Two cards are drawn from a pack of 52 cards. Find the probability that Both are black.

SOLUTION

Two cards are drawn at random from a pack of 52 cards.

$\therefore n(S) = {}^{52}C_2$

Let event A: Both are black. There are 26 black cards

$\therefore n(A) = {}^{26}C_2$

$$\therefore P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{{}^{26}C_2}{{}^{52}C_2}$$

$$= \frac{26.25}{52.51}$$

$= \dfrac{25}{102}$.

Exercise 7.2 | Q 2.2 | Page 102

Two cards are drawn from a pack of 52 cards. Find the probability that Both are diamonds.

SOLUTION

Two cards are drawn at random from a pack of 52 cards.
∴ $n(S) = {}^{52}C_2$
Let event B: Both are diamonds.
There are 13 diamond cards.
∴ $n(B) = {}^{13}C_2$

$\therefore P(B) = \dfrac{n(B)}{n(S)}$

$= \dfrac{{}^{13}C_2}{{}^{52}C_2}$

$= \dfrac{13.12}{52.51}$

$= \dfrac{1}{17}$.

Exercise 7.2 | Q 2.3 | Page 102

Two cards are drawn from a pack of 52 cards. Find the probability that Both are ace cards.

SOLUTION

Two cards are drawn at random from a pack of 52 cards.
∴ $n(S) = {}^{52}C_2$

Let event C: Both are ace cards.

There are 4 ace cards.

∴ $n(C) = {}^{4}C_2$

$$\therefore P(C) = \frac{n(C)}{n(S)}$$

$$= \frac{{}^{4}C_2}{{}^{52}C_2}$$

$$= \frac{4.2}{52.51}$$

$$= \frac{1}{221}.$$

Exercise 7.2 | Q 2.4 | Page 102

Two cards are drawn from a pack of 52 cards. Find the probability that Both are face cards.

SOLUTION

Two cards are drawn at random from a pack of 52 cards.
$\therefore n(S) = {}^{52}C_2$
Let event D: Both are face cards.
There are 12 face cards.
$\therefore n(D) = {}^{12}C_2$

$$\therefore P(D) = \frac{n(D)}{n(S)}$$

$$= \frac{{}^{12}C_2}{{}^{52}C_2}$$

$$= \frac{12.11}{52.51}$$

$$= \frac{11}{221}.$$

Exercise 7.2 | Q 2.5 | Page 102

Two cards are drawn from a pack of 52 cards. Find the probability that One is spade and other is non-spade.

SOLUTION

Two cards are drawn at random from a pack of 52 cards.
∴ n(S) = $^{52}C_2$
Let event E: One is spade and other is non-spade.
There are 13 spades and 39 non-spades.
∴ n(E) = $^{13}C_1 \times {}^{39}C_1$

$$\therefore P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{^{13}C_1 \times {}^{39}C_1}{^{52}C_2}$$

$$= \frac{13 \times 39}{\frac{52 \times 51}{1 \times 2}}$$

$$= \frac{13}{34}.$$

Exercise 7.2 | Q 2.6 | Page 102

Two cards are drawn from a pack of 52 cards. Find the probability that Both are from same suit.

SOLUTION

Two cards are drawn at random from a pack of 52 cards.
∴ n(S) = $^{52}C_2$
Let event F: Both are from the same suit.
A suit (out of 4) is selected then two cards are selected from that suit (13 cards).
∴ n(F) = $^{4}C_1 \times {}^{13}C_1$

$$\therefore P(F) = \frac{n(F)}{n(S)}$$

$$= \frac{^{4}C_1 \times {}^{13}C_1}{^{52}C_2}$$

$$= \frac{4 \times 13 \times 12}{52 \times 51}$$

$$= \frac{4}{17}.$$

Exercise 7.2 | Q 2.7 | Page 102

Two cards are drawn from a pack of 52 cards. Find the probability that Both are from same denomination.

SOLUTION

Two cards are drawn at random from a pack of 52 cards.
∴ n(S) = $^{52}C_2$
Let event G: Both are from same denomination.
A denomination is selected (from 13) and two cards are selected from that- denomination.
∴ n(G) = 13C1 × 14C1

$$\therefore P(G) = \frac{n(G)}{n(S)}$$

$$= \frac{^{13}C_1 \times {}^{14}C_1}{^{52}C_2}$$

$$= \frac{13 \times 4 \times 3}{52 \times 51}$$

$$= \frac{1}{17}.$$

Exercise 7.2 | Q 3.1 | Page 102

Four cards are drawn from a pack of 52 cards. What is the probability that 3 are kings and 1 is jack.

SOLUTION

4 cards are drawn at random from a pack of 52 cards.
∴ n(S) = $^{52}C_4$

Let event A: 3 cards are kings and 1 is jack. There are 4 kings and 4 jacks.
∴ n(A) = $^4C_3 \times {}^4C_1$

$$\therefore P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{^{4}C_5 \times {}^{4}C_1}{^{52}C_4}.$$

Exercise 7.2 | Q 3.2 | Page 102

Four cards are drawn from a pack of 52 cards. What is the probability that All the cards are from different suit.

SOLUTION

4 cards are drawn at random from a pack of 52 cards.

$\therefore n(S) = {}^{52}C_4$

Let event B: All 4 cards are from different suits.

To draw 1 card each from every suit of 13 cards

$\therefore n(B) = {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$\therefore P(B) = \dfrac{n(B)}{n(S)}$

$= \dfrac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_4}.$

Exercise 7.2 | Q 3.3 | Page 102

Four cards are drawn from a pack of 52 cards. What is the probability that At least one heart.

SOLUTION

4 cards are drawn at random from a pack of 52 cards.
$\therefore n(S) = {}^{52}C_4$

Let event C: At least, one heart.

$\therefore$ n(C): All 4 cards are non-hearts

There are 39 non heart cards.

$\therefore n(C) = {}^{39}C_4$

$$\therefore P(C) = \frac{n(C')}{n(S)}$$

$$= \frac{^{39}C4}{^{52}C4}$$

Required Probability,

$\therefore P(C) = 1 - P(C')$

$$= 1 - \frac{^{39}C4}{^{52}C4}.$$

Exercise 7.2 | Q 3.4 | Page 102

Four cards are drawn from a pack of 52 cards. What is the probability that All cards are club and one of them is jack.

SOLUTION

4 cards are drawn at random from a pack of 52 cards.

$\therefore n(S) = {}^{52}C_4$

Let event D: All 4 cards are clubs and one of them is a jack.

There is one jack of club, out of total 13 club cards.

$\therefore n(D) = 1 \times {}^{12}C_3$

$$\therefore P(D) = \frac{n(D)}{n(S)}$$

$$= \frac{^{12}C_3}{^{52}C_4}.$$

Exercise 7.2 | Q 4.1 | Page 102

A bag contains 15 balls of three different colours, Green, Black and Yellow. A ball is drawn at random from the bag. The probability of green ball is 1/3. The probability of yellow ball is 1/5 What is the probability of black ball?

SOLUTION

Let event G = A Green ball is sleeted from the bag.

Given: P(G) = $\frac{1}{3}$

Event B = A Black ball is selected from the bag.

To find: P(B).

Event Y = A Yellow ball is selected from the bag.

Given: P(Y) = $\frac{1}{5}$.

Since the events are mutually exclusive and, exhaustive,

P(G) + P(B) + P(Y) = $1 + \frac{1}{3} + \mathbf{P(B)} + \frac{1}{5}$ = 1

$\therefore$ P(B) = $1 - \frac{1}{3} - \frac{1}{5}$

$\therefore$ Probability of a black ball selected is $\frac{7}{15}$.

Exercise 7.2 | Q 4.2 | Page 102

A bag contains 15 balls of three different colours, Green, Black and Yellow. A ball is drawn at random from the bag. The probability of green ball is 1/3. The probability of yellow ball is 1/5. How many balls are green, black, and yellow?

SOLUTION

Let event G = A Green ball is sleeted from the bag.

Given: P(G) = $\frac{1}{3} \Rightarrow$

Event B = A Black ball is selected from the bag.

To find: P(B).

Event Y = A Yellow ball is selected from the bag.

Given: P(Y) = $\frac{1}{5}$.

Let S: To draw 1 ball from the bag containing 15 balls.

$\therefore$ n(S) = ${}^{15}C_1$ = 15

Let the number of. Green, Black and Yellow balls be x, y, z respectively.

$\therefore$ n(G) = x, n(B) = y, n (Y) = z

Since,

P(G) = $\frac{1}{3} = \frac{\mathrm{n(G)}}{\mathrm{n(S)}} = \frac{x}{15}$ ⇒ x = 5

P(B) = $\frac{1}{3} = \frac{\mathrm{n(B)}}{\mathrm{n(S)}} = \frac{y}{15}$ ⇒ y = 7

P(Y) = $\frac{1}{3} = \frac{\mathrm{n(Y)}}{\mathrm{n(S)}} = \frac{z}{15}$ ⇒ z = 3

i.e. there are 5 Green, 7 Black and 3 Yellow balls in the bag.

Exercise 7.2 | Q 5.1 | Page 102

A box contains 75 tickets numbered 1 to 75. A ticket is drawn at random from the box. What is the probability that Number on ticket is divisible by 6.

SOLUTION

To select one ticket at random out of 75 tickets numbered from 1 to 75.

$\therefore$ n(S) = $^{75}C_1$ = 75.

Let event A: Number on the ticket is divisible by 6. A = {6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72}

$\therefore$ n(A) = 12C1 = 12.

$\therefore$ P(A) = $\frac{\mathrm{n(A)}}{\mathrm{n(S)}}$

= $\frac{12}{75}$

= $\frac{4}{25}$.

Exercise 7.2 | Q 5.2 | Page 102

A box contains 75 tickets numbered 1 to 75. A ticket is drawn at random from the box. What is the probability that Number on ticket is a perfect square.

SOLUTION

To select one ticket at random out of 75 tickets numbered from 1 to 75.

$\therefore$ n(S) = $^{75}C_1$ = 75.

Let event B: Number on the ticket is a perfect square.

B = {1, 4, 9, 16, 25, 36, 49, 64}
∴ $n(B) = {}^{8}C_{1} = 8$.

$$\therefore P(B) = \frac{n(A)}{n(S)}$$

$$= \frac{8}{75}.$$

Exercise 7.2 | Q 5.3 | Page 102

A box contains 75 tickets numbered 1 to 75. A ticket is drawn at random from the box. What is the probability that Number on ticket is prime.

SOLUTION

To select one ticket at random out of 75 tickets numbered from 1 to 75.
∴ $n(S) = {}^{75}C_{1} = 75$.
Let event C: Number on the ticket is prime.
C = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 51, 57, 71, 73}

∴ $n(C) = {}^{21}C_{1} = 21$.

$$\therefore P(C) = \frac{n(A)}{n(S)}$$

$$= \frac{21}{75}$$

$$= \frac{7}{25}.$$

Exercise 7.2 | Q 5.4 | Page 102

A box contains 75 tickets numbered 1 to 75. A ticket is drawn at random from the box. What is the probability that Number on ticket is divisible by 3 and 5.

SOLUTION

To select one ticket at random out of 75 tickets numbered from 1 to 75.
∴ $n(S) = {}^{75}C_{1} = 75$.

Let event D: Number on the ticket is divisible by 3 and 5 i.e. by 15

D = {15, 30, 45, 60, 75}
∴ $n(D) = {}^{5}C_{1} = 5$.

$$\therefore P(D) = \frac{n(D)}{n(S)}$$

$$= \frac{5}{75}$$

$$= \frac{1}{15}.$$

Exercise 7.2 | Q 6.1 | Page 102

From a group of 8 boys and 5 girls, 8 committee of 5 is to be formed. Find the Probability that the committee contains 3 boys and 2 girls.

SOLUTION

Douro select 5 from a group of 8 boys, 5 girls (total 13).

$\therefore n(S) = {}^{13}C_5$

Let event A: To select 3 boys from 8 and 2 girls from 5.

$\therefore n(A) = {}^{8}C_3 \times {}^{5}C_2$

$$\therefore P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{{}^{8}C_3 \times {}^{5}C_2}{{}^{13}C_5}.$$

Exercise 7.2 | Q 6.2 | Page 102

From a group of 8 boys and 5 girls, 8 committee of 5 is to be formed. Find the Probability that the committee contains at least 8 boys.

SOLUTION

Douro select 5 from a group of 8 boys, 5 girls (total 13). $\therefore n(S) = {}^{13}C_5$

Let event B: To select 5 consisting of at least 3 boys.

Committee can have 3 boys and 2 girls or 4 boys and 1 girl or all 5 boys.

$\therefore n(B) = {}^{8}C_3 \times {}^{5}C_2 + {}^{8}C_4 \times {}^{5}C_1 + {}^{8}C_5$

$$\therefore P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{{}^{8}C_1 \times {}^{5}C_2 \times {}^{8}C_4 \times {}^{5}C_1 + {}^{8}C_5}{{}^{13}C_5}.$$

Exercise 7.2 | Q 7 | Page 103
A room has 3 sockets for lamps. From a collection of 10 light bulbs of which 6 are defective, a person selects 3 bulbs at random and puts them in socket. What is the probability that the room is lit?

SOLUTION

To select 3 bulbs at random from the collection of 10 bulbs.
∴ n(S) = $^{10}C_3$

$= \frac{10 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1}$
= 120

Let event A: Selected 3 bulbs are fixed in the sockets so that the room gets lit.
∴ A’: The room is not lit.

(i.e.) All 3 bulbs selected are defective (from 6 defective bulbs)
∴ n(A') = $^{6}C_3$

$= \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1}$
= 20

∴ n(A') = n(S')

$= \frac{^{6}C_3}{^{10}C_3}$

$= \frac{20}{120}$

$= \frac{1}{6}$

Required Probability that the room is lit is

∴ P(A') = 1 – P(A')

$= 1 - \frac{1}{6}$

$= \frac{5}{6}.$

Exercise 7.2 | Q 8.1 | Page 103

The letters of the word LOGARITHM are arranged at random. Find the probability that Vowels are always together.

SOLUTION

The word LOGARITHM contains 9 distinct letters consisting of 3 vowels and 6 consonants.

Since the letters are to be rearranged in linear places, each arrangement is a permutation.

To arrange all 9 letters in 9 linear places.

$\therefore$ n(S) = ${}^{9}P_{9}$ = 9!

Let event A: In the arrangement, all 3 vowels are together.

To arrange 1 block of vowels + 6 consonants in 7 places.

Also, 3 vowels are rearranged amongst themselves.

$\therefore$ n(A) = 7! x 3!

$$\therefore P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{7!3!}{9!}$$

$$= \frac{7! \times 3 \times 2 \times 1}{9 \times 8 \times 7!}$$

$$= \frac{1}{12}.$$

Exercise 7.2 | Q 8.2 | Page 103

The letters of the word LOGARITHM are arranged at random. Find the probability that Vowels are never together.

SOLUTION

The word LOGARITHM contains 9 distinct letters consisting of 3 vowels and 6 consonants.

Since the letters are to be rearranged in linear places, each arrangement is a permutation.

To arrange all 9 letters in 9 linear places.

$\therefore$ n(S) = $^{9}P_{9}$ = 9!

Let event B: In the arrangement, no two vowels are together.

6 consonants to be arranged in 6 linear places and

3 consonants are inserted in any 3 of 7 alternate place as shown:

$$\times \boxed{C} \times \boxed{C} \times \boxed{C} \times \boxed{C} \times \boxed{C} \times \boxed{C} \times$$

$\therefore$ n(B) = 6! x $^{7}P_{3}$

$$\therefore P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{6! \times {}^{7}P_{3}}{9!}$$

$$= \frac{6! \times 7 \times 6 \times 5}{9 \times 8 \times 7 \times 6}$$

$$= \frac{5}{12}.$$

Exercise 7.2 | Q 8.3 | Page 103

The letters of the word LOGARITHM are arranged at random. Find the probability that Exactly 4 letters between G and H.

SOLUTION

The word LOGARITHM contains 9 distinct letters consisting of 3 vowels and 6 consonants.
Since the letters are to be rearranged in linear places, each arrangement is a permutation.
To arrange all 9 letters in 9 linear places.

$\therefore$ n(S) = $^{9}P_{9}$ = 9!

Let event C: In the arrangement, there are exactly 4 letters between G and H.

Such an arrangement is possible only if G and H occupy

1st and 6th

OR 2nd and 7th

OR 3rd and 8th

OR 4th and 9th places and remaining,

7 letters can be arranged 1n remaining 7 places.

$\therefore$ n(C) = 4 x 7!

$$\therefore P(C) = \frac{n(C)}{n(S)}$$

$$= \frac{4 \times 7!}{9!}$$

$$= \frac{4 \times 7!}{9 \times 8 \times 7!}$$

$$= \frac{1}{18}.$$

Exercise 7.2 | Q 8.4 | Page 103

The letters of the word LOGARITHM are arranged at random. Find the probability that Begin with O and end with T.

SOLUTION

The word LOGARITHM contains 9 distinct letters consisting of 3 vowels and 6 consonants.

Since the letters are to be rearranged in linear places, each arrangement is a permutation.

To arrange all 9 letters in 9 linear places.

$\therefore$ n(S) = ${}^{9}P_{9}$ = 9!

Let event D: The arrangement begins with O and ends with T.

1st place is occupied by O and 9th place is occupied by T, remaining

7 letters are arranged in 7 remaining places.

$\therefore$ n(D) = 1 x 7! x 1 = 7!

$$\therefore P(D) = \frac{n(D)}{n(S)}$$

$$= \frac{7!}{9!}$$

$$= \frac{7!}{9 \times 8 \times 7!}$$

$$= \frac{1}{72}.$$

Exercise 7.2 | Q 8.5 | Page 103

The letters of the word LOGARITHM are arranged at random. Find the probability that Start with vowel and end with consonant.

SOLUTION

The word LOGARITHM contains 9 distinct letters consisting of 3 vowels and 6 consonants.
Since the letters are to be rearranged in linear places, each arrangement is a permutation.
To arrange all 9 letters in 9 linear places.

$$\therefore n(S) = {}^9P_9 = 9!$$

Let event E: The arrangement starts with a vowel and ends with a consonant.

1st place is occupied by any one of 3 vowels and 9th place is occupied by any one of 6 consonants, remaining 7 places are occupied by remaining 7 letters.
$\therefore$ n(E) = 3 x 7! x 6

$$\therefore P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{3 \times 7! \times 6}{9!}$$

$$= \frac{3 \times 7! \times 6}{9 \times 8 \times 7!}$$

$$= \frac{1}{4}.$$

Exercise 7.2 | Q 9 | Page 103

The letters of the word SAVITA are arranged at random. Find the probability that vowels are always together.

SOLUTION

The word SAVITA contains 6 letters
∴ Let S: b All 6 letters of the word are rearranged in 6 linear places,
Since the letter 'A' is repeated twice.

$\therefore$ n(S) = $\frac{6!}{2!}$

= $\frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$

= 360

Let event A: In the arrangement, the vowels are together

To arrange 1 block of 3 vowels (2 identical) + 3 consonants in 4 linear places.

Also, the 3 Vowels can be arranged among themselves.

Considering the repetition of letter 'A',

$\therefore$ n(A) = $4! \times \frac{3!}{2!}$

= $4 \times 3 \times 2 \times 1 \times 3 \times \frac{2!}{2!}$

= 72

$\therefore$ n(A) = $\frac{n(A)}{n(S)}$

= $\frac{72}{360}$

= $\frac{1}{5}$.

EXERCISE 7.3 [PAGE 104]

Exercise 7.3 | Q 1 | Page 104

Two dice are thrown together. What is the probability that the sum of the numbers on two dice is 5 or number on the second die is greater than or equal to the number-on the first die?

SOLUTION

Let S: Two dice are thrown together.
∴ S = {(1, 1), (1. 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), .(6, 5), (6, 6)}
∴ n(S) = 36.

Let event A: Sum of numbers on two dice is 5.
∴ A = {(1, 4), (2, 3), (3, 2), (4, 1)}
∴ n(A) = 4

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{4}{36}$$

Let event B: Number on the second die ≥ number on the first die.

B = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 5), (5, 6), (6, 6)}Let event B: Number on the second die ≥ number on the first die.

B = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 5), (5, 6), (6, 6)}
∴ n(B) = 21

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{21}{36}$$

Now, (A ∩ B) {(1, 4), (2, 3)}

∴ n(A ∩ B) = 2

$$\therefore n(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{36}$$

Required Probability is P(A ∪ B)

By Addition Theorem:

P(A ∪ B) = P(A) + P(B) − P(A ∩ B)

$$= \frac{4}{36} + \frac{21}{36} - \frac{2}{36}$$

$$P(A \cup B) = \frac{23}{36}.$$

Exercise 7.3 | Q 2.1 | Page 104

A card is drawn from a pack of 52 cards. What is the probability that card is either red or black ?

SOLUTION

Let S: A card is selected at random from a pack of 52 cards.

$\therefore$ n(S) = $^{52}C_1$ = 52.

Let event A: A red card is selected.

There are 26 face cards.

$\therefore$ n(A) = $^{26}C_1$ = 26.

$\therefore$ P(A) = $\frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

Let event B: A black card is selected.

There are 26 black Cards.

$\therefore$ n(B) = $^{26}C_1$ = 26.

$\therefore$ P(B) = $\frac{n(B)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

Let event C: A face card is selected.

There are 12 face cards.

$\therefore$ n(C) = $^{12}C_1$ = 12.

$\therefore$ P(C) = $\frac{n(C)}{n(S)} = \frac{12}{52} = \frac{3}{13}$

A and B are mutually exclusive.

$\therefore$ Probability that the card is either red or black is

P(A ∪ B) = P(A) + P(B)

$\therefore$ P(A ∪ B) = $\frac{1}{2} + \frac{1}{2}$ = 1.

Exercise 7.3 | Q 2.2 | Page 104

A card is drawn from a pack of 52 cards. What is the probability that card is either red or face card?

SOLUTION

Let S: A card is selected at random from a pack of 52 cards.

∴ n(S) = ${}^{52}C_1$ = 52.

Let event A: A red card is selected.

There are 26 face cards.

∴ n(A) = ${}^{26}C_1$ = 26.

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

Let event B: A black card is selected.

There are 26 black Cards.

∴ n(B) = ${}^{26}C_1$ = 26.

∴ P(B) = $\frac{n(B)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

Let event C: A face card is selected.

There are 12 face cards.

∴ n(C) = ${}^{12}C_1$ = 12.

∴ P(C) = $\frac{n(C)}{n(S)} = \frac{12}{52} = \frac{3}{13}$

A ∩ C: A red face card is selected.

There are 6 red face cards

∴ n(A ∩ C) = ${}^{6}C_1$ = 6

∴ P(A ∩ C) = $\frac{n(A \cap C)}{n(S)}$

= $\frac{6}{52}$

= $\frac{3}{26}$

Probability that the card is a red card or face card is,

P(A ∪ C) = P(A) + P(C) − P(A ∩ C)

$= \frac{1}{2} + \frac{3}{13} - \frac{3}{26}$

$= \frac{26}{52} + \frac{12}{52} - \frac{6}{52}$

$= \frac{32}{52}$

$\therefore$ P(A ∪ C) = $\frac{8}{13}$.

Exercise 7.3 | Q 3 | Page 104

Two cards are drawn from a pack of 52 cards. What is the probability that
(a) both the cards are of the same colour?
(b) both the cards are either black or queens?

SOLUTION

Let S: Two cards are selected at random from the pack of 52 cards.

$\therefore$ n(S) = ${}^{52}C_2 = \frac{52 \times 51}{2 \times 1}$ = 1326

Let event A: Both cards are red. There are 26 red cards.

$\therefore$ n(A) = ${}^{26}C_2 = \frac{26 \times 25}{2 \times 1}$ = 325

$\therefore$ P(A) = $\frac{n(A)}{n(S)} = \frac{325}{1326} = \frac{25}{102}$

Let event B: Both cards are black. There are 26 black cards.

$\therefore$ P(B) = $\frac{n(B)}{n(S)} = \frac{25}{102}$

Since A and B are mutually exclusive, probability that both cards are of the Same colour is

P(A ∪ B) = P(A) + P(C)

$= \frac{25}{102} + \frac{25}{102} = \frac{50}{102}$

P(A ∪ B) = $\frac{25}{51}$

Let event C: Both cumin are queens there are 4 queens.

$\therefore$ n(B) = ${}^4C_2 = \dfrac{4 \times 3}{2 \times 1}$ = 6

$\therefore$ P(B) = $\dfrac{n(B)}{n(S)} = \dfrac{6}{1326} = \dfrac{1}{221}$

Event B ∩ C: Both cards are black queens.

There are 2 black queens.

$\therefore$ n(B ∩ C) = ${}^2C_2$ = 1

$\therefore$ P(B ∩ C) = $\dfrac{n(B \cap C)}{n(S)} = \dfrac{1}{1326}$

$\therefore$ Probability that both cards are black or queens

P(B ∪ C) = P(A) + P(C) – P(A ∩ C)

= $\dfrac{25}{102} + \dfrac{1}{221} - \dfrac{1}{1326}$

= $\dfrac{325}{1326} + \dfrac{6}{1236} - \dfrac{1}{1326}$

= $\dfrac{330}{1326}$

$\therefore$ P(B ∪ C) = $\dfrac{55}{221}$.

Exercise 7.3 | Q 4.1 | Page 104

A bag contains 50 tickets, numbered from 1 to 50. One ticket is drawn at random. What is the probability that number on the ticket is a perfect square or divisible by 4?

SOLUTION

Let S: One ticket is selected at random from a bag containing 50 tickets numbered from 1 to 50.

$\therefore$ n(S) = ${}^{50}C_1$ = 50

Let event A: Number on the selected ticket is a perfect square

A = {1, 4, 9, 16, 25, 36, 49}

$\therefore$ n(A) = ${}^{7}C_1$ = 7

$\therefore$ P(A) = $\frac{n(A)}{n(S)} = \frac{7}{50}$

Let event B: Number on the selected ticket is divisible by 4

B = {4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48}

$\therefore$ n(B) = ${}^{12}C_1$ = 12

$\therefore$ P(B) = $\frac{n(B)}{n(S)} = \frac{6}{25}$

A ∩ B: Number on the ticket is a perfect square divisible by 4

A ∩ B = {4, 16, 36}

$\therefore$ n(A ∩ B) = ${}^{3}C_1$ = 3

$\therefore$ P(A ∩ B) = $\frac{n(A \cap B)}{n(S)} = \frac{3}{50}$

Probability that the number on the selected ticket is either a perfect square or divisible by 4 is

P(A ∪ B) = P(A) + P(C) – P(A ∩ B)

$= \frac{7}{50} + \frac{12}{50} - \frac{3}{50}$

$= \frac{16}{50}$

$\therefore$ P(A ∪ B) = $\frac{8}{25}$.

Exercise 7.3 | Q 4.2 | Page 104

A bag contains 50 tickets, numbered from 1 to 50. One ticket is drawn at random. What is the probability that number on the ticket is prime number or greater than 30?

SOLUTION

Let S: One ticket is selected at random from a bag containing 50 tickets numbered from 1 to 50.

$\therefore$ n(S) = ${}^{50}C_1$ = 50

Let event C: Number on the selected ticket is a prime number

C = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47}

$\therefore$ n(C) = ${}^{15}C_1$ = 15

$\therefore$ P(C) = $\frac{n(C)}{n(S)} = \frac{15}{50} = \frac{3}{10}$

Let event D: Number on the ticket is greater than 30

D = {31, 32, ..., 50}

$\therefore$ n(D) = ${}^{20}C_1$ = 20

$\therefore$ P(D) = $\frac{n(D)}{n(S)} = \frac{20}{50} = \frac{2}{5}$

C ∩ D: Number on the selected ticket is a prime number > 30.

C ∩ D = {31, 37,41, 43,47}

$\therefore$ n(C ∩ D) = ${}^{5}C_1$ = 5

$\therefore$ P(C ∩ D) = $\frac{n(C \cap D)}{n(S)} = \frac{5}{50} = \frac{1}{10}$

$\therefore$ Probability that the number on the selected ticket is either a prime number or greater than 30 is

P(C ∪ D) = P(C) + P(D) – P(C ∩ D)

= $\frac{3}{10} + \frac{2}{5} - \frac{1}{10}$

= $\frac{15}{50} + \frac{20}{50} - \frac{5}{10}$

= $\frac{30}{50}$

$\therefore$ P(C ∪ D) = $\frac{3}{5}$.

Exercise 7.3 | Q 5.1 | Page 104
Hundred students appeared for two examinations.
60 passed the first, 50 passed the second and 30 passed in both. Find the probability that student selected at random has passed in at least one examination.

SOLUTION

Let S: A student is selected at random from 100 students appeared in two examinations.

$\therefore$ n(S) = $^{10}\mathrm{C}_1$ = 100

Let event A: The student selected has passed in first examination.

60 students passed in first examination.

$\therefore$ n(A) = $^{60}\mathrm{C}_1$ = 60

$\therefore$ P(A) = $\frac{\mathrm{n(A)}}{\mathrm{n}(S)} = \frac{60}{100} = \frac{6}{10}$

Let event B ': The student selected has passed in second examination.

50 students passed in second examination.

$\therefore$ n(B) = $^{50}\mathrm{C}_1$ = 50

$\therefore$ P(B) = $\frac{\mathrm{n(B)}}{\mathrm{n}(S)} = \frac{50}{100} = \frac{5}{10}$|

Event A ∩ B: The student has passed in both examinations

30 students passed 'both examinations.

n(A ∩ B) = $^{30}\mathrm{C}_1$ = 30

$\therefore$ P(A ∩ B) = $\frac{\mathrm{n(A \cap B)}}{\mathrm{n(S)}} = \frac{30}{100} = \frac{3}{10}$

Probability that a student has passed in at least one of the examinations is P(A ∪ B) = P(C) ++ P((D) – P(A ∩ B)

$= \frac{6}{10} + \frac{5}{10} - \frac{3}{10}$

$= \frac{8}{10}$

$\therefore$ P(A ∩ B) = $\frac{4}{5}$.

Exercise 7.3 | Q 5.2 | Page 104

Hundred students appeared for two examinations.
60 passed the first, 50 passed the second and 30 passed in both. Find the probability that student selected at random has passed in exactly one examination.

SOLUTION

Let S: A student is selected at random from 100 students appeared in two examinations.

$\therefore$ n(S) = ${}^{10}C_1$ = 100

Let event A: The student selected has passed in first examination.

60 students passed in first examination.

$\therefore$ n(A) = ${}^{60}C_1$ = 60

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{60}{100} = \frac{6}{10}$$

Let event B ': The student selected has passed in second examination.

50 students passed in second examination.

$\therefore$ n(B) = ${}^{50}C_1$ = 50

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{50}{100} = \frac{5}{10}$$

Event A ∩ B: The student has passed in both examinations

30 students passed 'both examinations.

n(A ∩ B) = ${}^{30}C_1$ = 30

$$\therefore P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{30}{100} = \frac{3}{10}$$

Probability that a student has passed in exactly one of the examination is P (A but not B or B but not A)

$= P(A \cap B') + P(A' \cap B)$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= \frac{6}{10} - \frac{3}{10} + \frac{5}{10} - \frac{3}{10}$

$= \frac{5}{10}$

$= \frac{1}{2}$.

Exercise 7.3 | Q 5.3 | Page 104

Hundred students appeared for two examinations.
60 passed the first, 50 passed the second and 30 passed in both. Find the probability that student selected at random has failed in both the examinations.

SOLUTION

Let S: A student is selected at random from 100 students appeared in two examinations.

$\therefore$ n(S) = ${}^{10}C_1$ = 100

Let event A: The student selected has passed in first examination.

60 students passed in first examination.

$\therefore$ n(A) = ${}^{60}C_1$ = 60

$\therefore$ P(A) = $\frac{n(A)}{n(S)} = \frac{60}{100} = \frac{6}{10}$

Let event B ': The student selected has passed in second examination.

50 students passed in second examination.

$\therefore$ n(B) = ${}^{50}C_1$ = 50

$\therefore$ P(B) = $\frac{n(B)}{n(S)} = \frac{50}{100} = \frac{5}{10}$|

Event A ∩ B: The student has passed in both examinations

30 students passed 'both examinations.

n(A ∩ B) = ${}^{30}C_1$ = 30

$\therefore$ P(A ∩ B) = $\frac{n(A \cap B)}{n(S)} = \frac{30}{100} = \frac{3}{10}$

Probability that the student has failed in bath the examinations.

i. e. P (Neither A nor B)

= P(A' ∩ B')

= P(A ∪ B) ...D' Morgan's Rule

= 1 – P(A ∪ B) ...P(A) = 1 – P(A')

= $1 - \frac{4}{5}$

= $\frac{1}{5}$.

Exercise 7.3 | Q 6.1 | Page 104

If P(A) = $\frac{1}{4}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{1}{2}$ Find the values of the following: P(A ∩ B)

SOLUTION

Given probabilities are:

P(A) = $\frac{1}{4}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{1}{2}$

By Addition Theorem

P(A ∩ B) = P(A) + P(B) – P(A ∩ B)

$\therefore$ P(A ∩ B) = P(A) + P(B) – P(A ∩ B)

= $\frac{1}{4} + \frac{2}{5} - \frac{1}{2}$

= $\frac{5+8-10}{20}$

$\therefore$ P(A ∩ B) = $\frac{1}{20}$.

Exercise 7.3 | Q 6.2 | Page 104

If P(A) = $\frac{1}{4}, \mathrm{P(B)} = \frac{2}{5} \mathrm{and} \mathrm{P(A \cup B)} = \frac{1}{2}$ Find the values of the following: P(A ∩ B')

SOLUTION

Given probabilities are:

P(A) = $\frac{1}{4}, \mathrm{P(B)} = \frac{2}{5} \mathrm{and} \mathrm{P(A \cup B)} = \frac{1}{2}$

P(A ∩ B') = P(A) – P(A ∩ B)

= $\frac{1}{4} - \frac{3}{20}$

= $\frac{5-3}{20}$

= $\frac{2}{20}$

∴ P(A ∩ B') = $\frac{1}{4}$.

Exercise 7.3 | Q 6.3 | Page 104

If P(A) = $\frac{1}{4}, \mathrm{P(B)} = \frac{2}{5} \mathrm{and} \mathrm{P(A \cup B)} = \frac{1}{2}$ Find the values of the following: P(A' ∩ B)

SOLUTION

Given probabilities are:

P(A) = $\frac{1}{4}, \mathrm{P(B)} = \frac{2}{5} \mathrm{and} \mathrm{P(A \cup B)} = \frac{1}{2}$

P(A' ∩ B) = P(B) – P(A ∩ B)∩ B)

= $\frac{2}{5} - \frac{3}{20}$

= $\frac{8-3}{20}$

= $\frac{}{}$20

$= \frac{5}{20}$

$\therefore$ P(A' ∩ B) = $\frac{1}{4}$.

Exercise 7.3 | Q 6.4 | Page 104

If P(A) = $\frac{1}{4}, \mathbf{P(B)} = \frac{2}{5} \mathbf{and P(A \cup B)} = \frac{1}{2}$ Find the values of the following: P(A' ∪ B')

SOLUTION

Given probabilities are:

P(A) = $\frac{1}{4}, \mathbf{P(B)} = \frac{2}{5} \mathbf{and P(A \cup B)} = \frac{1}{2}$

P(A' ∪ B') = P(A ∩ B)\` ...D'Morgan's Rule

= 1 – P(A ∩ B)   ...P(A) = 1 – P(A')

$= 1 - \frac{3}{20}$

$\therefore$ P(A' ∪ B') $\frac{17}{20}$.

Exercise 7.3 | Q 6.5 | Page 104

If P(A) = $\frac{1}{4}, \mathbf{P(B)} = \frac{2}{5} \text{ and } \mathbf{P(A \cup B)} = \frac{1}{2}$ Find the values of the following: P(A' ∩ B')

SOLUTION

Given probabilities are:

P(A) = $\frac{1}{4}, \mathbf{P(B)} = \frac{2}{5} \text{ and } \mathbf{P(A \cup B)} = \frac{1}{2}$

P(A' ∩ B') = P(A ∪ B)'   ...D'Morgan's Rule

= 1 – P(A ∪ B)   ...P(A) = 1 – P(A')

$= 1 - \frac{1}{2}$

$P(A' \cap B') = \frac{1}{2}$.

Exercise 7.3 | Q 7 | Page 104

A computer software company is bidding for computer programs A and B. The probability that. the company will get software A is 3/5, the probability that the company will get software B is 1/3 and the company will get both A and B is 1/8. What is the probability that the company will get at least one software?

SOLUTION

Let event
A: The company gets software A
B: The company gets software B
Then A ∩ B: The company gets both software.

Now, $\mathrm{P(A)} = \frac{1}{3}, \mathrm{P(B)} = \frac{1}{3}\ \mathrm{and} \mathrm{P(A \cap B)} = \frac{1}{8}$

Probability that the company gets at least one software is P(A ∪ B) = P(A) + P( ) – P(A ∩ B)

$= \frac{3}{5} + \frac{1}{3} - \frac{1}{8}$

$= \frac{72 + 40 - 15}{120}$

$\therefore P(A \cup B) = \frac{97}{120}$.

Exercise 7.3 | Q 8 | Page 104

A card is drawn from a well shuffled pack of 52 cards. Find the probability of it being a heart or a queen.

SOLUTION

Let S: A card is selected at random from a well shuffled pack of 52 cards.

$\therefore$ n(S) = ${}^{52}C_1$ = 52

Let event A: The card drawn is a heart.

There are 13 hearts.

$\therefore$ n(A) = ${}^{13}C_1$ = 13

$\therefore$ P(A) = $\frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

Event B: The card drawn is a queen.

There are 4 queens.

$\therefore$ n(B) = ${}^{4}C_1$ = 4

$\therefore$ P(B) = $\frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

Event A ∩ B: The card drawn is a queen of hearts

$\therefore$ n(A ∩ B) = 1

$\therefore$ P(A ∩ B) = $\frac{n(A \cap B)}{n(S)} = \frac{1}{52}$

Probability that the card selected is either heart or a queen is given as,

P(A ∪ B) = P(A) + P(B) − P(A ∩ B)

$= \frac{1}{4} + \frac{1}{13} - \frac{1}{52}$

$= \frac{13 + 4 - 1}{52}$

$= \frac{16}{52}$

$\therefore$ P(A ∪ B) = $\frac{4}{13}$.

Exercise 7.3 | Q 9 | Page 104

In a group of students, there are 3 boys and 4 girls. Four students are to be selected at random from the group. Find the probability that either 3 boys and 1 girl or 8 girls and 1 boy are selected.

SOLUTION

Let S: 4 students are selected at random from a group of 7 students (3 boys, 4 girls)

∴ n(S) = $^7C_4 = \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1}$ = 35

Let A: Event that the selected group has boys and 1 girl.

3 boys can be selected out of 3 in $^3C_3$ ways and 1 girl can be selected out of 4 in $^4C_1$ ways.

∴ n(A) = $^3C_3 \times {^4C_1}$ = 1 x 4 = 4

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{4}{35}$

Event B: The selected group has 1 boy and 3 girls.

1 boy can be selected out of 3 in $^3C_1$ ways and 3 girls can be selected out of 4 in $^4C_1$ ways.

∴ n(B) = $^3C_1 \times {^4C_3} = {^3C_1} \times {^4C_1}$ = 3 x 4 = 12

∴ P(B) = $\frac{n(B)}{n(S)} = \frac{12}{35}$

Since events A and B are mutually exclusive

Required probability,

P(A ∪ B) = P(A) + P(B)

= $\frac{4}{35} + \frac{12}{35}$

∴ P(A ∪ B) = $\frac{14}{35}$.

EXERCISE 7.4, EXERCISE 7.4EXERCISE 7.4 [PAGES 107 - 108]

Exercise 7.4 | Q 1 | Page 107

Two dice are thrown simultaneously. It at least one of the dice shows a number 5, what is the probability that, sum of the numbers on two dice is 9?

SOLUTION

Two dice are thrown together.
S = = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 3), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(S) = 36
Let A: Event that one of the dice shows the number 5

∴ A = {(1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 5)}
∴ n(A) = 11

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{11}{36}$$

Let B: Event that the sum of the numbers of the dice is 9

B = {(3, 6), (4, 5), (5, 4), (6, 3)}

∴ n(B) = 4

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{4}{36}$$

∴ A ∩ B is the event that one dice shows a 5 and the sum of the numbers is 9.

∴ A ∩ B = {(4, 5), (5, 4)}

∴ n(A ∩ B) = 2

$$\therefore n(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{36}$$

Now, probability that sum of the numbers on the dice is 9 given that one dice shows 5 is given by,

$$P\left(\frac{B}{A}\right) = \frac{p(A \cap B)}{P(A)}$$

$$= \frac{\frac{2}{36}}{\frac{11}{36}}$$

$$P\left(\frac{B}{A}\right) = \frac{2}{11}.$$

Exercise 7.4 | Q 2 | Page 107

A pair of dice is thrown. If sum of the numbers is an even number, what is the probability that it is a perfect square?

SOLUTION

A pair of dice is thrown,
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 3), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

∴ n(S) = 36
Let A: Event that sum of the numbers is even
∴ A = {(1, 1), (1, 3), (1, 5) (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)}
∴ n(A) = 18

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

Let B: Event that sum of the numbers is a perfect square

B = {(1, 3), (2, 2), (3, 1), (3, 6), (4, 5), (5, 4), (6, 3)}

∴ n(B) = 7

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{7}{36}$$

∴ A ∩ B is the event that the sum of the numbers is an even perfect square.

∴ A ∩ B = {(1, 3), (2, 2), (3, 1)}

$$\therefore n(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36} = \frac{1}{2}$$

Now, Probability that the sum of the numbers is a Perfect square given that it is even is given by,

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{1}{12}}{\frac{1}{2}}$$

$$P\left(\frac{B}{A}\right) = \frac{1}{6}.$$

Exercise 7.4 | Q 3 | Page 107

A box contains 11 tickets numbered from 1 to 11. Two tickets are drawn at random with replacement. If the sum is even, find the probability that both the numbers are odd.

SOLUTION

Two tickets are drawn from 11 tickets numbered 1 to 11 with replacement
∴ n(S) = $^{11}C_1$ x $^{11}C_1$ = 121.
Let A: Event that sum of the numbers on the tickets is even.
For this both tickets need to be even numbered or odd numbered.
Out of 5 even numbered tickets, 2 can be drawn in 5 · 5 = 25 ways.
Similarly, out of 6 odd numbered tickets, 2 can be drawn in 6 · 6
= 36 ways. ...(I)
∴ n(A) = 25 + 36 = 61

$\therefore$ P(A) = $\dfrac{n(A)}{n(S)} = \dfrac{61}{121}$

Let B: Event that both tickets drawn are odd numbered.

$\therefore$ n(B) = 36

$\therefore$ P(B) = $\dfrac{n(B)}{n(S)} = \dfrac{36}{121}$

$\therefore$ A ∩ B is the event that the tickets drawn are odd numbered and their sum is even.

$\therefore$ n(A ∩ B) = 36

$\therefore$ n(A ∩ B) = $\dfrac{n(A \cap B)}{n(S)} = \dfrac{36}{121}$

Now, probability that the tickets drawn are odd numbered given that their sum is even is,

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{36}{121}}{\frac{61}{121}}$$

$$P\left(\frac{B}{A}\right) = \frac{36}{61}.$$

Exercise 7.4 | Q 4 | Page 107

A card is drawn from a well shuffled pack of 52 cards. Consider two events A and B. A: a club 6 card is drawn. B: an ace card 18 drawn. Determine whether the events A and B are independent or not.

SOLUTION

Let S: A card is drawn at random from a deck of 52 cards.
$\therefore$ n(S) = 52

Event A: A club card is drawn.

There are 13 club cards.
$\therefore$ n(A) = 13

$\therefore$ P(A) = $\dfrac{n(A)}{n(S)} = \dfrac{13}{52} = \dfrac{1}{4}$

Event B: An ace card is drawn.

There are 4 aces.

$\therefore$ n(B) = 4

$\therefore$ P(B) = $\dfrac{n(B)}{n(S)} = \dfrac{4}{121} = \dfrac{1}{13}$

Event A ∩ B: An ace of clubs is drawn.

$\therefore$ n(A ∩ B) = 1

$\therefore$ n(A ∩ B) = $\dfrac{n(A \cap B)}{n(S)} = \dfrac{1}{52}$

Now, P(A) · P(B) = $\dfrac{1}{4} \times \dfrac{1}{13}$

= $\dfrac{1}{52}$

i.e. P(A ∩ B) = P(A) · P(B)

Hence, events A and B are independent.

Exercise 7.4 | Q 5.1 | Page 107

A problem in statistics; is given to three students A, B and C. Their chances of solving the problem are $\dfrac{1}{3}, \dfrac{1}{4}$ and $\dfrac{1}{5}$ respectively.

If all of them try independently, what is the probability that Problem is not solved?

SOLUTION

The probabilities that the students A, B and C solve the problem are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively.

∴ Let $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{5}$

Hence, the probabilities the it the Students do not solve .the problem can be given as

$P(A) = \frac{2}{3}, P(B') = \frac{3}{4}$ and $P(C') = \frac{4}{5}$

Since all the three students independently, events A, B, C, A', B', C are mutually independent.

Probability that the problem is solved

= P ...(At least one of the three Students solve the problem)

= 1 – P ...(None of the students solve)

= 1 – P (A' ∩ B' ∩ C')

= 1 – P(A') · P(B') · P(C')

$= 1 - \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5}$

$= 1 - \frac{2}{5}$

$= \frac{3}{5}$.

Exercise 7.4 | Q 5.2 | Page 107

A problem in statistics; is given to three students A, B and C. Their chances of solving the problem are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively. If all of them try independently, what is the probability that Problem is solved?

SOLUTION

The probabilities that the students A, B and C solve the problem are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively.

$\therefore$ Let $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{5}$

Hence, the probabilities the it the Students do not solve .the problem can be given as

$P(A) = \frac{2}{3}, P(B') = \frac{3}{4}$ and $P(C') = \frac{4}{5}$

Since all the three students independently, events A, B, C, A', B', C are mutually independent.

Probability that the problem is not solved

= P (None of A, B and C solve the problem)

= $P(A' \cap B' \cap C')$

= $P(A') \cdot P(B') \cdot P(C')$

= $\frac{2}{3} \cdot \frac{3}{4} \frac{4}{5}$

= $\frac{2}{5}$.

Exercise 7.4 | Q 6 | Page 107

The probability that a 50-year old man will be alive till age 60 is 0.83 and the probability that a 45-year old woman will be alive till age 55 is 0.97. What is the probability that a man whose age is 50 and his wife whose age is 45 will both be alive after 10 years.

SOLUTION

Let Event A: A 50 year old man is alive at 60.
Event B: A 45 year old woman is alive at 55.
Given: P(A) = 0.83 and P(B) = 0.97
Required probability
= P (a man at 50 and a woman at 45 are alive after 10 years)
= P(A and B)
= $P(A \cap B)$
= $P(A) \cdot P(B)$ ...(events A and B are independent)
= 0.83 x 0.97
= 0.8051

Exercise 7.4 | Q 7.1 | Page 108

In an examination 30% of students have failed in subject I, 20% of the students have failed in subject II and 10% have failed in both subject I and subject II. A student is selected at random, what is the probability that the' student has failed in subject I, if it is known that he has failed in subject II?

SOLUTION

A: A student selected has failed in subject I.
B: A student selected has failed in subject II.
∴ A ∩ B: A student selected has failed in both subjects I and II.

P(A) = 30% = $\frac{30}{100} = \frac{3}{10}$

P(B) = 20% = $\frac{20}{100} = \frac{2}{10}$

And P(A ∩ B) = 10% = $\frac{10}{100} = \frac{1}{10}$

Probability that a selected student has failed in subject I, knowing that he has failed in subject II

P (event A given that B has occurred)

i.e. $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{1}{10}}{\frac{2}{10}}$

$= \frac{1}{2}$

$\therefore P\left(\frac{A}{B}\right) = \frac{1}{2}$ or 0.5

Exercise 7.4 | Q 7.2 | Page 108

In an examination 30% of students have failed in subject I, 20% of the students have failed in subject II and 10% have failed in both subject I and subject II. A student is selected at random, what is the probability that the' student has failed in at least one subject?

SOLUTION

A: A student selected has failed in subject I.

B: A student selected has failed in subject II.

∴ A ∩ B: A student selected has failed in both subjects I and II.

P(A) = 30% = $\frac{30}{100} = \frac{3}{10}$

P(B) = 20% = $\frac{20}{100} = \frac{2}{10}$

And P(A ∩ B) = 10% = $\frac{10}{100} = \frac{1}{10}$

Probability that a selected student has failed in at least one subject

i.e. P(A ∪ B) = P(A) ++ P(B) – P(A∩ B)

= $\frac{3}{10} + \frac{2}{10} - \frac{1}{2}$

= $\frac{4}{10}$

= $\frac{2}{5}$ or 0.4

∴ P(A ∪ B) = $\frac{2}{5}$ or 0.4

Exercise 7.4 | Q 7.3 | Page 108

In an examination 30% of students have failed in subject I, 20% of the students have failed in subject II and 10% have failed in both subject I and subject II. A student is selected at random, what is the probability that the' student has failed in exactly one subject?

SOLUTION

A: A student selected has failed in subject I.

B: A student selected has failed in subject II.

∴ A ∩ B: A student selected has failed in both subjects I and II.

P(A) = 30% = $\frac{30}{100} = \frac{3}{10}$

P(B) = 20% = $\frac{20}{100} = \frac{2}{10}$

And P(A ∩ B) = 10% = $\frac{10}{100} = \frac{1}{10}$

Probability that a selected student has failed in exactly one subject.

i.e. P(A but not B. or B but not A)

= $P(A \cap B') + P(A' \cap B)$

= $P(A) - P(A \cap B) + P(B) - P(A \cap B)$

= $\frac{3}{10} - \frac{1}{10} + \frac{2}{10} - \frac{1}{10}$

= $\frac{3}{10}$ or 0.3

Exercise 7.4 | Q 8 | Page 108

One shot is fired from each of the three guns. Let A, B and C denote the events that the target is hit by the first, second and third gun respectively. Assuming that A, B and C are independent events and that P(A) = 0.5, P(B) = 0.6 and P(C) = 0.8, then find the probability that at least one hit is registered.

SOLUTION

A: The target is hit by first gun.
B: The target is hit by Second gun.
C: The target is hit by third gun.
P(A) = 0.5, P(B) = 0.6 and P(C) = 0.8

Hence, probabilities that the target is not hit are given as
P(A') = 0.5, P(B') = 0.4, and P(C') = 0.2
Since A, B, C are independent
∴ A', B', C' are also independent.

Required Probability that at least one hit is registered
= P (At least one gun hits the target)
1 – P (None of the guns hits the target)
1 – $P(A' \cap B' \cap C')$
1 – $P(A') \cdot P(B') \cdot P(C')$
1 – 0.5 x 0.4 x 0.2
1 – 0.4
= 0.96

Exercise 7.4 | Q 9.1 | Page 108

A bag contains 10 white balls and {5 black balls. Two balls are drawn in succession without replacement. What is the probability that first is white and second is black.

SOLUTION

The bag contains 25 balls. Since two balls are drawn in succession without replacement, for the conditional second event a ball IS reduced in the bag.
Let event A: A white ball is selected.
event B: A black ball is selected.

$$\text{P(A)} = \frac{10}{25}; \mathbf{P(B)} = \frac{15}{12}$$

$\therefore \frac{A}{B}$:Second ball is black, first bell being white.

$\frac{B}{A}$: Second ball is white, first ball being black.

$$P\left(\frac{B}{A}\right) = \frac{15}{24}; P\left(\frac{A}{B}\right) = \frac{10}{24}.$$

Probability4 that first ball 18 white and second ' is black

$= \text{P (event A and event } \left(\frac{B}{A}\right))$

$= \text{P(A)} \cdot P\left(\frac{B}{A}\right)$

$= \frac{10}{25} \times \frac{15}{24}$

$= \frac{1}{4}.$

Exercise 7.4 | Q 9.2 | Page 108

A bag contains 10 white balls and {5 black balls. Two balls are drawn in succession without replacement. What is the probability that one is white and other is black.

SOLUTION

The bag contains 25 balls. Since two balls are drawn in succession without replacement, for the conditional second event a ball IS reduced in the bag.

Let event A: A white ball is selected.
event B: A black ball is selected.

P(A) = $\frac{10}{25}; \mathrm{P(B)} = \frac{15}{12}$

$\therefore\ \frac{\mathrm{A}}{\mathrm{B}}$:Second ball is black, first bell being white.

$\frac{\mathrm{B}}{\mathrm{A}}$: Second ball is white, first ball being black.

$$\mathrm{P}\left(\frac{\mathrm{B}}{\mathrm{A}}\right) = \frac{15}{24}; \mathrm{P}\left(\frac{\mathrm{A}}{\mathrm{B}}\right) = \frac{10}{24}.$$

Probability that one ball is white and the other is black.

= P (First ball is white, second is black or First ball is black, second is white)

$$= \mathrm{P}\left(\mathrm{A\ \ and\ \ }\frac{\mathrm{B}}{\mathrm{A}}\mathrm{\ \ or\ \ B\ \ and\ \ }\frac{\mathrm{A}}{\mathrm{B}}\right)$$

$$= \mathrm{P(A)} \cdot \mathrm{P}\left(\frac{\mathrm{B}}{\mathrm{A}}\right) + \mathrm{P(B)} \cdot \mathrm{P}\left(\frac{\mathrm{A}}{\mathrm{B}}\right)$$

$$= \frac{10}{25} \times \frac{15}{24} + \frac{15}{25} \times \frac{10}{24}$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2}.$$

Exercise 7.4 | Q 10 | Page 108

An urn contains 4 black, 5 white and 6 red balls. Two balls are drawn one after the other without replacement. What is the probability that at least one ball is black?

SOLUTION

To select two balls from the bag containing 15 balls.

$\therefore$ n(S) = $^{15}C_2$

= $\dfrac{15 \times 4}{2 \times 1}$

= 105

Let event A: Selecting at least one black ball.

Balls drawn can be both black or one black and other non-black.

$\therefore$ n(A) = $^{4}C_1 \times {}^{11}C_1 + {}^{4}C_2$

= $4 \times 11 + \dfrac{4 \times 3}{2 \times 1}$

= 44 + 6

= 50

$\therefore$ P(A) = $\dfrac{n(A)}{n(S)}$

= $\dfrac{50}{105}$

= $\dfrac{10}{21}$.

Exercise 7.4 | Q 11 | Page 108

Two balls are drawn from an urn containing 5 green, 3 blue, 7 yellow balls one by one without replacement. What is the probability that at least one ball is blue?

SOLUTION

Let S be the sample space-for two balls drawn from an urn one by one without replacement

There are total 15 balls,

$\therefore$ n(S) = ${}^{15}C_2$

Let A: Event that at least one ball is blue.

$\therefore$ A': Event that none of the balls is blue.

Out of 15 balls, 3 balls are blue

$\therefore$ 2 balls can be drawn from 12 non - blue ball in ${}^{12}C_2$ ways

$\therefore$ n(A') = ${}^{12}C_2$

$\therefore$ P(A') = $\dfrac{n(A)}{n(S)}$

$= \dfrac{{}^{12}C_2}{{}^{15}C_2}$

$= \dfrac{12 \times 11}{15 \times 14}$

$= \dfrac{22}{35}$

$\therefore$ P(A) = 1 – P(A')

$= 1 - \dfrac{22}{35}$

$\therefore$ P(A) = $\dfrac{13}{35}$.

Exercise 7.4 | Q 12 | Page 108

A bag contains 4 blue and 5 green balls. Another bag contains 3 blue and 7 green balls. If one ball is drawn from each bag, what is the probability that two balls are of the same colour?

SOLUTION

First bag contains 4 blue + 5 green = 9 balls.

Let event A: A blue ball is drawn from bag I.

event A': A green ball is drawn from bag I.

$\therefore \mathbf{P(A) = \frac{4}{9} \text{ and } P(A') = \frac{5}{9}}$

Second bag contains 3 blue + 7 green 10 balls.

Let event B: A blue ball is drawn from bag II.

event B': A green ball is drawn from bag II.

$\therefore \mathbf{P(B) = \frac{3}{10} \text{ and } P(B') = \frac{7}{10}}$

Required probability

= P (both balls are of same colour)

= P (both are blue or both are green)

= P(A and B or A and B')

= P(A ∩ B) + or P(A' ∩ B')

Now, A and B are independent .

∴ A' and B' are independent.

∴ P(A ∩ B) = P(A) · P(B) = $\mathbf{\frac{4}{9} \cdot \frac{3}{10} = \frac{12}{90}}$

and P(A' ∩ B') = P(A') · P(B') = $\mathbf{\frac{5}{9} \cdot \frac{7}{10} = \frac{35}{90}}$

Required probability

= P(A ∩ B) + P(A' ∩ B')

= $\mathbf{\frac{12}{90} + \frac{35}{90}}$

= $\mathbf{\frac{47}{90}}$.

Exercise 7.4 | Q 13 | Page 108

Two cards are drawn one after the other from a pack of 52 cards with replacement. What is the probability that both the cards drawn' ere face cards?

SOLUTION

For 2 cards to be drawn from a pack of 52 cards one after the other with replacement,

$$\therefore P(A \cap B) = P(A) \cdot P(B) = \frac{4}{9} \cdot \frac{3}{10} = \frac{12}{90}$$

$$\text{and } P(A' \cap B') = P(A') \cdot P(B') = \frac{5}{9} \cdot \frac{7}{10} = \frac{35}{90}$$

Required probability

$$= P(A \cap B) + P(A' \cap B')$$

$$= \frac{12}{90} + \frac{35}{90}$$

$$= \frac{47}{90}.$$

MISCELLANEOUS EXERCISE 7 [PAGES 109 - 110]

Miscellaneous Exercise 7 | Q 1 | Page 109

From a group of 2 men (M_1, M_2) and three women (W_1, W_2, W_3), two persons are selected. Describe the sample space of the experiment. If E is the event in which one man and one woman are selected, then which are the cases favourable to E?

SOLUTION

Let S be the sample space of given event.
$\therefore$ S = { (M_1, M_2), (M_1, W_1), (M_1, W_2), (M_1, W_3), (M_2, W_1), (M_2, W_2), (M_2, W_3), (W_1, W_2) (W_1, W_3), (W_2, W_3)}
Let E be the event that one man and one woman are selected.
$\therefore$ E = {(M_1, W_1), (M_1, W_2), (M_1, W_3), (M_2, W_1), (M_2, W_2), (M_2, W_3)}

Miscellaneous Exercise 7 | Q 2 | Page 110

Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys and 1 girl and 3 boys. One child is selected at random from each group. What is the chance that three selected consists of 1 girl and 2 boys?

SOLUTION

Group I		Group - II		Group - II	
Girls	Boys	Girls	Boys	Girls	Boys
3	1	2	2	1	3

Let G_1, G_2, G_3 denote events for selecting a girl and B_1, B_2, B_3 denote events for selecting a boy from 1st, 2nd and 3rd groups respectively.

Then $P(G_1)=\frac{3}{4}, P(G_2)=\frac{2}{4}, P(G_3)=\frac{1}{4}$

$P(B_1)=\frac{1}{4}, P(B_2)=\frac{2}{4}, P(B_3)=\frac{3}{4}$

Where G_1, G_2, G_3, B_1, B_2 and B_3 are mutually exclusive events.

Let E be the event that 1 girl and 2 boys are selected

$\therefore E = (G_1 \cap B_2 \cap B_3) \cup (B_1 \cap G_2 \cap B_3) \cup (B_1 \cap B_2 \cap G_3)$

$\therefore P(E) = P(G_1 \cap B_2 \cap B_3) + P(B_1 \cap G_2 \cap B_3) + P(B_1 \cap B_2 \cap G_3)$

$= P(G_1)\cdot P(B_2)\cdot P(B_3) + P(B_1)\cdot P(G_2)\cdot P(B_3) + P(B_1)\cdot P(B_2)\cdot P(G_3)$

$= \left(\frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}\right)+\left(\frac{1}{4}\times\frac{2}{4}\times\frac{3}{4}\right)+\left(\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}\right)$

$= \frac{18+6+2}{64}$

$= \frac{26}{64}$

$= \frac{13}{32}$.

Miscellaneous Exercise 7 | Q 3 | Page 110

A room has 3 sockets for lamps. From a collection of 10 light bulbs of which 6 are defective, a person selects 3 bulbs at random and puts them in socket. What is the probability that the room is lit?

SOLUTION

To select 3 bulbs at random from the collection of 10 bulbs.

∴ n(S) = $^{10}C_3$

$$= \frac{10 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1}$$

= 120

Let event A: Selected 3 bulbs are fixed in the sockets so that the room gets lit.

∴ A′: The room is not lit.

(i.e.) All 3 bulbs selected are defective (from 6 defective bulbs)

∴ n(A') = $^6C_3$

$$= \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1}$$

= 20

∴ n(A') = n(S')

$$= \frac{^6C_3}{^{10}C_3}$$

$$= \frac{20}{120}$$

$$= \frac{1}{6}$$

Required Probability that the room is lit is

∴ P(A') = 1 – P(A')

$$= 1 - \frac{1}{6}$$

$$= \frac{5}{6}.$$

Miscellaneous Exercise 7 | Q 4 | Page 110

There are 2 red and 3 black balls in a bag. 3 balls are taken out at random from the bag. Find the probability of getting 2 red and 1 black ball or 1 red and 2 black balls.

SOLUTION

There are 2 + 3 = 5 balls in the bag and 3 balls can be drawn out of these in

$$^5C - 3 = \frac{5 \times 4 \times 3}{1 \times 2 \times 3} = 10 \text{ ways.}$$

$\therefore$ n(S) = 10

Let A be the event that 2 balls are red and 1 ball is black

2 red balls can be drawn out of 2 red balls in $^2C_2$ = 1 way and 1 black ball can be drawn out of 3 black balls in $^3C_1$ = 3 ways.

$\therefore$ n(A) = $^2C_2 \times {}^3C_1 = 1 \times 3 = 3$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{10}$$

Let B be the event that 1 ball is red and 2 balls are black 1 red ball out of 2 red balls can be drawn in $^2C_1$ = 2 ways and 2 black balls out of 3 black balls can be drawn in

$$^3C_2 = \frac{3 \times 2}{1 \times 2} = 3 \text{ ways}$$

$\therefore$ n(B) = $^2C_1 \times {}^3C_2 = 2 \times 3 = 6$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{6}{10}$$

Since A and B are mutually exclusive and exhaustive events

$\therefore P(A \cap B) = 0$

$\therefore$ Required probability

$= P(A \cup B)$

$= P(A) + P(B)$

$$= \frac{3}{10} + \frac{6}{10}$$

$$= \frac{9}{10}.$$

Miscellaneous Exercise 7 | Q 5 | Page 110

A box contains 25 tickets numbered 1 to 25. Two tickets are drawn at random. What is the probability that the product of the numbers is even?

SOLUTION

Two tickets can be drawn out of 25 tickets

in $^{25}C_2 = \frac{25 \times 24}{1 \times 2}$ = 300 ways.

∴ n(S) = 300

Let A be the event that product of two numbers is even.

This is possible if both numbers are even, or one number is even and other is odd.

As there are 13 odd numbers and 12 even numbers from 1 to 25.

∴ n(A) = $^{12}C_1 + {}^{12}C_1 \times {}^{13}C_1$

$= \frac{12 \times 11}{1 \times 2} + 12 \times 13$

= 66 + 156

= 222

∴ Required probability = P(A)

$= \frac{n(A)}{n(S)}$

$= \frac{222}{300}$

$= \frac{37}{50}$.

Miscellaneous Exercise 7 | Q 6 | Page 110

A, B and C are mutually exclusive and exhaustive events associated with the random experiment. Find P(A), given that

$$P(B) = \frac{3}{2} \text{ and } P(C) = \frac{1}{2}P(B).$$

SOLUTION

$$P(B) = \frac{3}{2} \text{ and } P(C) = \frac{1}{2}P(B)$$

Since A, B, C are mutually exclusive and exhaustive events,

$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C) = 1$

$$\therefore P(A) + \frac{3}{2}(A) + \frac{1}{2}P(B) = 1$$

$$\therefore P(A) + \frac{3}{2}P(A) + \frac{1}{2} \times \frac{3}{2}P(A) = 1$$

$$\therefore P(A) + \frac{3}{2}P(A) + \frac{3}{4}P(A) = 1$$

$$\therefore P(A) \times \left(1 + \frac{3}{2} + \frac{3}{4}\right) = 1$$

$$\therefore P(A) \times \left(\frac{13}{4}\right) = 1$$

$$\therefore P(A) = \frac{4}{13}.$$

Miscellaneous Exercise 7 | Q 7 | Page 110

An urn contains four tickets marked with numbers 112, 121, 122, 222 and one ticket is drawn at random. Let Ai (i = 1, 2, 3) be the event that i^{th} digit of the number of the ticket drawn is 1. Discuss the independence of the events A_1, A_2, and A_3.

SOLUTION

One ticket can be drawn out of 4 tickets in $4 = {}^4C_1$ ways.
$\therefore n(S) = 4$

According to given information, Let A_1 be the event that 1st digit of the number of ticket is 1

A_2 be the event that 2nd digit of the number of ticket is 1.

A_3 be the event that 3^{rd} digit of the number of ticket is 1.

$\therefore A_1 = \{112, 121, 122\}$, $A_2 = \{112\}$, $A3 = \{121\}$

$$\therefore P(A_1) = \frac{n(A_1)}{n(S)} = \frac{3}{4},$$

$$P(A_2) = \frac{n(A_2)}{n(S)} = \frac{1}{4}$$

$$P(A_3) = \frac{n(A_3)}{n(S)} = \frac{1}{4}$$

$$\left.\begin{array}{l} P(A_1)P(A_2) = \frac{3}{16} \\ P(A_2)P(A_3) = \frac{1}{16} \\ P(A_1)P(A_3) = \frac{3}{16} \end{array}\right\} \quad \text{...(i)}$$

$A_1 \cap A_2 = \{112\}$, $A_2 \cap A_3 = \Phi$, $A_1 \cap A_3 = \{121\}$

$$\left\{\begin{array}{l} P(A_1 \cap A_2) = \frac{n(A_1 \cap A_2)}{n(S)} = \frac{1}{4} \\ P(A_2 \cap A_3) = 0 \\ P(A_1 \cap A_3) = \frac{1}{4} \end{array}\right\} \quad \text{...(ii)}$$

$\therefore$ From (i) and (ii),

$$\left.\begin{array}{l} P(A_1) \cdot P(A_2) \neq P(A_1 \cap A_2) \\ P(A_2) \cdot P(A_3) \neq P(A_2 \cap A_3) \\ P(A_1) \cdot P(A_3) \neq P(A_1 \cap A_3) \end{array}\right\} \quad \text{...(iii)}$$

$\therefore$ A_1, A_2, A_3 are not pairwise independent For mutual independent of events A_1, A_2, A_3 We require to have

$P(A_1 \cap A_2 \cap A_3) = P(A_1)\ P(A_2)\ P(A_3)$

and $P(A_1)\ P(A_2) = P(A_1 \cap A_2)$,

$P(A_2)\ P(A_3) = P(A_2 \cap A_3)$,

$P(A_1)\ P(A_3) = P(A_1 \cap A_3)$

$\therefore$ From (iii),

A_1, A_2, A_3 are not mutually independent.

Miscellaneous Exercise 7 | Q 8 | Page 110

The odds against a certain event are 5 : 2 and odds in favour of another independent event are 6 : 5. Find the chance that at least one of the events will happen.

SOLUTION

Let A and B be two independent events.

Odds against A are 5 : 2

∴ the probability of occurrence of event A is given by

P(A) = $\frac{2}{5+2} = \frac{2}{7}$

Odds in favour of B are 6 : 5

∴ the probability of occurrence of event B is given by

P(B) = $\frac{6}{6+5} = \frac{6}{11}$

∴ P(at least one event will happen)

= P(A ∪ B)

= P(A) + P(B) – P(A ∩ B)

= P(A) + P(B) – P(A) P(B) ...[∵ A and B are independent events]

= $\frac{2}{7} + \frac{6}{11} - \frac{2}{7} \times \frac{6}{11}$

= $\frac{2}{7} + \frac{6}{11} - \frac{12}{77}$

= $\frac{22+42-12}{77}$

= $\frac{52}{77}$.

Miscellaneous Exercise 7 | Q 9.1 | Page 110

The odds against a husband who is 55 years old living till he is 75 is 8 : 5 and it is 4 : 3 against his wife who is now 48, living till she is 68. Find the probability that the couple will be alive 20 years hence

SOLUTION

Let A be the event that husband would be alive after 20 years.
Odds against A are 8 : 5
∴ the probability of occurrence of event A is given by

P(A) = $\frac{5}{8+5} = \frac{5}{13}$

∴ P(A') = 1 – (A) = $1 - \frac{5}{13} = \frac{8}{13}$

Let B be the event that wife would be alive after 20 years.

Odds against B are 4 : 3

∴ the probability of occurrence of event B is given by

P(B) = $\frac{3}{4+3} = \frac{3}{7}$

∴ P(B') = 1 – (B) = $1 - \frac{3}{7} = \frac{4}{7}$

Since A and B are independent events

∴ A' and B' are also independent events

Let X be the event that both will be alive after 20 years.

∴ P(X) = (A ∩ B)

∴ P(X) = P(A).P(B) = $\frac{5}{13} \times \frac{3}{7} = \frac{15}{91}$.

Miscellaneous Exercise 7 | Q 9.2 | Page 110

The odds against a husband who is 55 years old living till he is 75 is 8 : 5 and it is 4 : 3 against his wife who is now 48, living till she is 68. Find the probability that at least one of them will be alive 20 years hence.

SOLUTION

Let A be the event that husband would be alive after 20 years.

Odds against A are 8 : 5

∴ the probability of occurrence of event A is given by

$$P(A) = \frac{5}{8+5} = \frac{5}{13}$$

$$\therefore P(A') = 1 - (A) = 1 - \frac{5}{13} = \frac{8}{13}$$

Let B be the event that wife would be alive after 20 years.

Odds against B are 4 : 3

∴ the probability of occurrence of event B is given by

$$P(B) = \frac{3}{4+3} = \frac{3}{7}$$

$$\therefore P(B') = 1 - (B) = 1 - \frac{3}{7} = \frac{4}{7}$$

Since A and B are independent events

∴ A' and B' are also independent events

Let Y be the event that at least one will be alive after 20 years.

∴ P(Y) = P(at least one would be alive)

= 1 – P(both would not be alive)

= 1 – $P(A' \cap B')$

= 1 – P(A'). P(B')

$$= 1 - \frac{8}{13} \times \frac{4}{7}$$

$$= 1 - \frac{32}{91} = \frac{59}{91}.$$

Miscellaneous Exercise 7 | Q 10 | Page 110

Two throws are made, the first with 3 dice and the second with 2 dice. The faces of each die are marked with the number 1 to 6. What is the probability that the total in first throw is not less than 15 and at the same time the total in the second throw is not less than 8?

SOLUTION

When 3 dice are thrown, then the sample space S_1 has 6 x 6 x 6 = 216 sample points.
∴ $n(S_1) = 216$
Let A be the event that the sum of the numbers is not less than 15.
∴ A = {(3,6,6), (4,5,6), (4,6,5), (4,6,6), (5,4,6), (5,5,5), (5,5,6), (5,6,4), (5,6,5), (5,6,6),

(6,3,6), (6,4,5), (6,4,6), (6,5,4), (6,5,5), (6,5,6), (6,6,3), (6,6,4), (6,6,5), (6,6,6)}
∴ n(A) = 20

∴ P(A) = $\frac{n(A)}{n(S_1)} = \frac{20}{216} = \frac{5}{54}$

When 2 dice are thrown, the sample space S_2 has 6 x 6 = 36 sample points.

∴ $n(S_2) = 36$

Let B be the event that sum of numbers is not less than 8.

∴ B = {(2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

∴ n(B) = 15

∴ P(B) = $\frac{n(B)}{n(S_2)} = \frac{15}{36} = \frac{5}{12}$

A ∩ B = Event that the total in the first throw is not less than 15 and at the same time the total in the second throw is not less than 8

∵ A and B are independent events

∴ P(A ∩ B) = P(A).P(B) = $\frac{5}{54} \times \frac{5}{12} = \frac{25}{648}$.

Miscellaneous Exercise 7 | Q 11 | Page 110

Two-third of the students in a class are boys and rest are girls. It is known that the probability of a girl getting first class is 0.25 and that of boy getting is 0.28. Find the probability that a student chosen at random will get first class.

SOLUTION

Let A be the event that student chosen is a boy
B be the event that student chosen is a girl
C be the event that student gets first class

∴ P(A) = $\frac{2}{3}, P(B) = \frac{1}{3}$

Probability of student getting first class, given that student is boy

Probability of student getting first class given that student is a girl, is

$$P\left(\frac{C}{A}\right) = 0.28 = \frac{28}{100} \text{ and } P\left(\frac{C}{B}\right) = 0.25 = \frac{25}{100}$$

∴ Required probability = P((A ∩ C) ∪ (B ∩ C))

Since A ∩ C and B ∩ C are mutually exclusive events

∴ Required probability = P(A ∩ C) + P(B ∩ C)

$$= P(A) \cdot P\left(\frac{C}{A}\right) + P(B) \cdot P\left(\frac{C}{B}\right)$$

$$= \left(\frac{2}{3} \times \frac{28}{100}\right) + \left(\frac{1}{3} \times \frac{25}{100}\right)$$

$$= \frac{56 + 25}{300}$$

$$= \frac{81}{300}$$

$$= \frac{27}{100}$$

= 0.27

Miscellaneous Exercise 7 | Q 12 | Page 110

A number of two digits is formed using the digits 1, 2, 3, …, 9. What is the probability that the number so chosen is even and less than 60?

SOLUTION

The number of two digits can be formed from the given 9 digits in 9 x 9 = 81 different ways.

∴ n(S) = 81.

Let A be the event that the number is even and less than 60.

Since the number is even, the unit place of two digits can be filled in ${}^4P_1 = 4$ different ways by any one of the digits 2, 4, 6, 8.

Also the number is less than 60, so tenth place can be filled in ${}^5P_1 = 5$ different ways by any one of the digits 1, 2, 3, 4, 5.

∴ n(A) = 4 x 5 = 20

$$\therefore \text{Required probability} = P(A) = \frac{n(A)}{n(S)} = \frac{20}{81}.$$

Miscellaneous Exercise 7 | Q 13 | Page 110

A bag contains 8 red balls and 5 white balls. Two successive draws of 3 balls each are made without replacement. Find the probability that the first drawing will give 3 white balls and second drawing will give 3 red balls.

SOLUTION

Total number of balls = 8 + 5 = 13.
3 balls can be drawn out of 13 balls in $^{13}C_3$ ways.
∴ n(S) = $^{13}C3$

Let A be the event that all 3 balls drawn are white.

3 white balls can be drawn out of 5 white balls in $^5C_3$ ways.
∴ n(A) = $^5C_3$

$$\therefore \text{P(A)} = \frac{n(A)}{n(S)} = \frac{^5C_3}{^{13}C_3} = \frac{5 \times 4 \times 3}{13 \times 12 \times 1} = \frac{5}{143}$$

After drawing 3 white balls which are not replaced in the bag, there are 10 balls left in the bag out of which 8 are red balls.

Let B be the event that the second draw of 3 balls are red.

∴ Probability of drawing 3 red balls, given that 3 white balls have been already drawn, is given by

$$P\left(\frac{B}{A}\right) = \frac{^8C_3}{^{10}C_3} = \frac{8 \times 7 \times 6}{10 \times 9 \times 8} = \frac{7}{15}$$

∴ Required probability = P(A ∩ B)

$$= P(A).P\left(\frac{B}{A}\right)$$

$$= \frac{5}{143} \times \frac{7}{15}$$

$$= \frac{7}{429}.$$

Miscellaneous Exercise 7 | Q 14.1 | Page 110

The odds against student X solving a business statistics problem are 8 : 6 and odds in favour of student Y solving the same problem are 14 : 16 What is the chance that the problem will be solved, if they try independently?

SOLUTION

Let A be the event that X solves the problem B be the event that Y solves the problem.
Since the odds against student X solving the problem are 8 : 6
∴ Probability of occurrence of event A is given by.

$$P(A) = \frac{6}{8+6} = \frac{6}{14} \text{ and}$$

$$P(A') = 1 - P(A) = 1 - \frac{6}{14} = \frac{8}{14}$$

Also, the odds in favour of student Y solving the problem are 14 : 16

∴ Probability of occurrence of event B is given by

$$P(B) = \frac{14}{14+16} = \frac{14}{30} \text{ and}$$

$$P(B') = 1 - P(B) = 1 - \frac{14}{30} = \frac{16}{30}$$

Now A and B are independent events.

∴ A' and B' are independent events.

$A' \cap B'$ = Event that neither solves the problem

$= P(A' \cap B') = P(A')\cdot P(B')$

$$= \frac{8}{14} \times \frac{16}{30}$$

$$= \frac{32}{105}$$

$A \cup B$ = the event that the problem is solved

∴ P (problem will be solved) = $P(A \cup B)$

$= 1 - P(A \cup B)`$

$= 1 - P(A' \cap B')$

$$= 1 - \frac{32}{105}$$

$$= \frac{73}{105}.$$

Miscellaneous Exercise 7 | Q 14.2 | Page 110

The odds against student X solving a business statistics problem are 8 : 6 and odds in favour of student Y solving the same problem are 14 : 16 What is the probability that neither solves the problem?

SOLUTION

P (neither solves the problem) = $P(A' \cap B')$

$= P(A')\, P(B')$

$= \frac{8}{14} \times \frac{16}{30}$

$= \frac{32}{105}$.