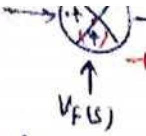
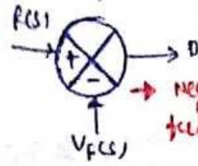
CONTROL SYSTEM



→ Positive feedback system



→ negative feedback system.

1. Basic concept of control system

→ Positive feedback system decreases the stability of the system.

→ Negative feedback system ↑ the stability

→ Neg. f/B System:

$$\boxed{T(s) = \frac{G(s)}{1+G(s)H(s)}}$$

→ Positive f/b System:

$$\boxed{T(s) = \frac{G(s)}{1-G(s)H(s)}}$$

→ $G(s)H(s)$ is known as OLTF [But exam me agar Seedha ki G(s) is Transfer function to manna hi padega usko T(s).]

→ $\boxed{CLTF = \frac{OLTF}{1+OLTF}}$ → only for negative feedback system.

→ Man lo T(s) i.e Transfer function dia hai aur OLTF puch raha hai to

$$\boxed{OLTF = \frac{N^r \text{ of } T(s)}{D^r \text{ of } T(s) - N^r \text{ of } T(s)}}$$

DC gain: D.C gain is the value of CLTF or OLTF when frequency = 0.

→ Remember that DC gain can not be zero or ∞.

Note: (1) For a good control system, O/P should be less sensitive wrt forward path gain (G) and parameter variation in the system and highly sensitive wrt Input command.

→ for Good control system

$$\boxed{S << 1}$$

Sensitivity: (1) $\boxed{S_G^T = \frac{\partial T/T}{\partial G/G}}$ — (A)

Sensitivity of T wrt G.

(2) $\boxed{S_H^T = \frac{\partial T/T}{\partial H/H}}$ — (B)

Sensitivity of T wrt H.

ye nai bhi yad to chalega but (A) aur (B) ko Jarur yad rkhna hai.

For Negative feedback System

$$S_G^T = \frac{1}{1+GH}$$

$$S_H^T = \frac{-HG}{1+GH}$$

→ S_G^T (open loop) = 1.

→ If Given like this Q

$G = 100 \pm 5\%$

then, $G = 100$

$\frac{\partial G}{G} = 5$.

Similarly for H(s) also.

Desensitivity: $\boxed{D = 1+GH}$ — Also know as loop Gain or Return difference.

In control system, we use;

$$\delta(t) = \begin{cases} 1.0 ; & t=0 \\ 0.0 ; & t \neq 0 \end{cases}$$

Impulse function.

→ $A\delta(t)$ → A = area (impulse)

$A\,u(t)$ → A = height / Amplitude (unit)

$A\,t\,u(t)$ → A = Slope (ramp)

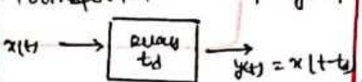
$L[\delta(t)] = 1.0$

$L[u(t)] = \frac{1}{s}$) $\times \frac{1}{s}$

$L[t\,u(t)] = \frac{1}{s^2}$) $\times \frac{1}{s}$

$s \to \frac{d}{dt}$ and $\frac{1}{s} \to \int dt$

Transportation Delay signal.



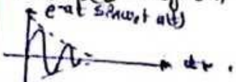
$$\boxed{\frac{Y(s)}{X(s)} = e^{-t_d s}}$$

Parabolic Signal:

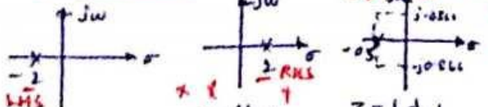
$r(t) = t^2 u(t)$ or $r(t) = \frac{t^2}{2} u(t)$

or $r(t)$ = constant $\times t^2 u(t)$.

Damped signal:



τ: Time constant (τ) is defined as the Reciprocal of Magnitude of negative "real" root or LHS side root.



→ In τ and ω_0 will be always real positive Number

$\tau = \left|-\frac{1}{2}\right| = \frac{1}{2}$

$\tau = \left|\frac{1}{2}\right| = \frac{1}{2}$ wrong

$\tau = \left|\frac{1}{0.5}\right|$, $\tau = 2$

→ τ does not exist for unstable system, It is only valid for Absolute stable system.

$$\boxed{t_{settl} \propto \tau}$$

→ Jo jitna jada jaldi settle hoga, wo utna fast response hoga

$\tau \propto \frac{1}{\ldots} \propto t_{settl}$

Condition for stability:-

(1) If all poles lie in the LHS of s-plane then system will be stable (Absolutely).

(2) If any pole of system lies on RHS of s-plane, then system will be unstable.

(3) If complex conjugate pole lie on Imaginary axis, the system will be marginally stable or oscillatory stable.

(4) If Repeated poles or multiple poles lie on the imaginary axis then system will be unstable.

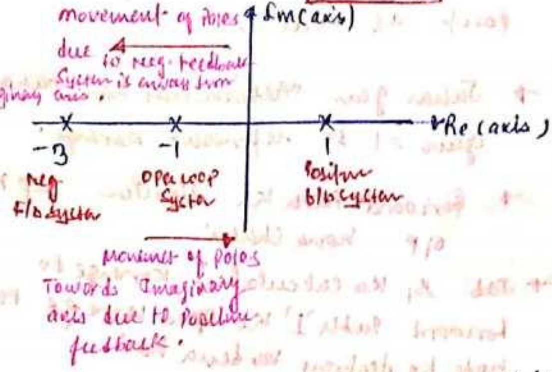
Initial value Theorem: (Applicable for both stable and unstable system).

$$\boxed{\lim_{t\to 0} x(t) = x(0) = \lim_{s\to\infty} sX(s)}$$

Final value Theorem: (Applicable only for Absolutely stable system)

$$\boxed{\lim_{t\to\infty} x(t) = x(\infty) = \lim_{s\to 0} sX(s)}$$

→ In "first-order" system $\boxed{B.W = \frac{1}{\tau}}$



→ Stability order : Neg f/b > open system > Positive f/b system.

→ Due to Neg. fb system, Gain ↓, Energy ↓ or Energy is degenerated.

→ Due to Positive f/b system, Gain ↑, Energy ↑ or Energy is generated.

Responses :

(1) Impulse Response : matlab input me impulse signal apply hua hai or jo usse response aega usko impulse response bolenge.

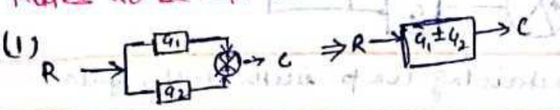
(2) Step response : matlab input me step signal apply hua hai or jo usse response aega usko step response bolenge.

$$\boxed{\text{Step response} = \int_0^t \text{Impulse response}}$$

2. Block Diagram and SFG

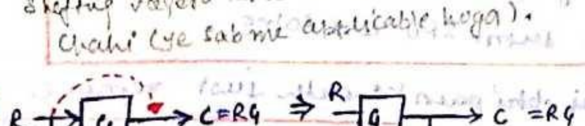
Rules to be kept in mind :

(1)

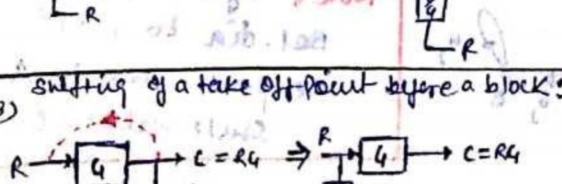


(2) Shifting of a take off point after a Block :

shifting vagera karne ke bad o/p same rehna chahi (ye sab me applicable hoga).

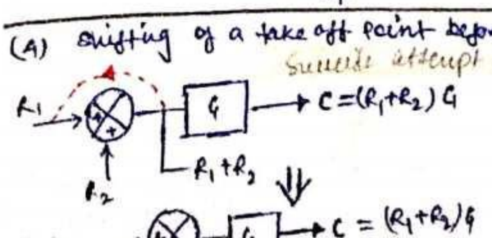


(3) Shifting of a take off point before a block :



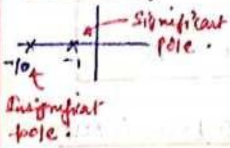
(4) Shifting of a take off point before an adder.

Suicide attempt hai



adder ke phle shift karne pe ek better adder create hoga.

Concept of Dominant pole :-



→ Significant pole ko dominant pole bolte hai

→ LHS me jo origin ke pass hoga wo dominant pole hoga.

→ Speed of Time response due to significant pole is slower than insignificant pole.

Note : For existence of dominant pole, the ratio of insignificant pole to significant pole should be greater or equal to 5

$$\boxed{\frac{\text{Insignificant Pole}}{\text{Significant pole}} \geq 5}$$

→ In case of Dominant pole concept, overall time constant is defined as the reciprocal of magnitude of dominant pole.

→ Third order aega matlab Dominant pole ka concept lagega.

→ DC gain should be same before and after applying Dominant pole concept.

For eg = $X(s) = \frac{100}{(s+10)(s+1)}$

$X(s=0) = 10 = \text{D.C gain}$

∴ After D.P concept, $X(s) = \frac{100}{(s+1)}$ ✗

$X(s) = \frac{10}{(s+1)} = \frac{10}{0+1} = 10$.

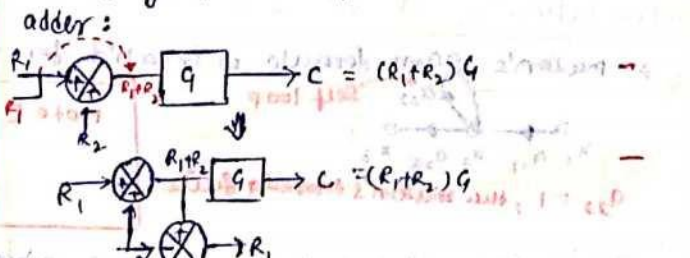
Note : Transfer function is unique. Ek system se Transfer function nikal lenge aur us T.F ko system me use karenge with different I/P and O/P conditions.

Eg: $\frac{d^2x}{dt^2} + \frac{1}{2}\frac{dx}{dt} + \frac{1}{18}x = \underbrace{10 + 5e^{-4t} + 2e^{-5t}}_{\text{Input part}}$

→ give D.E ke C.F se 'τ' nikalte hai.

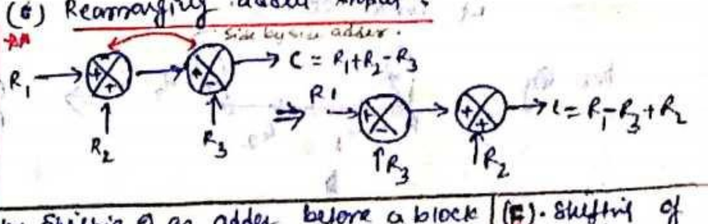
aur humlog 'τ' hamesa o/p ya response part se nikalte hai.

(5) Shifting of a take off point after an adder :

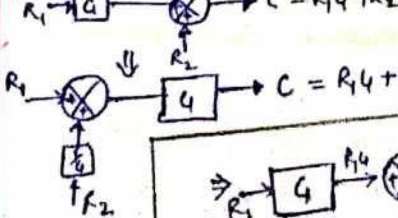


(6) Rearranging adder input :

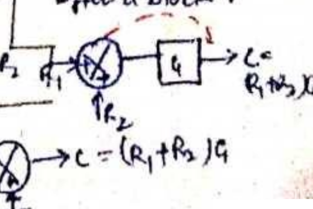
Side by side adder.



(7) Shifting of an adder before a block



(8). Shifting of an adder after a block :



Signal flow graph (SFG) :-

$$\text{Mason's Gain formula} = \sum_{K=1}^{n} \frac{P_K \Delta_K}{\Delta}$$ ***

n = no. of forward path i.e path from input node to o/p node w/o repetition of any node.

P_K = forward path gain of K^{th} path

Δ = determinant of SFG

Δ_K = Associated path factor depends upon forward path and isolated loop gain.

Δ = 1 - [sum of all individual loop gain] + [sum of product of all two non touching loop gain] - [sum of product of all three non touching loop gain] + ...

Δ_K = 1 - [sum of all individual isolated loop gain] + [sum of all pdt of all two non touching isolated loop gain] - [sum of pdt of all three no touching isolated loop gain].

→ for given SFG, Δ is unique. woo nai badalta chahe MISO ya SISO system ho.

→ Take each adder and take off point as node.

→ Jahan gain mention nai hai wahan gain = 1 se represent karenge.

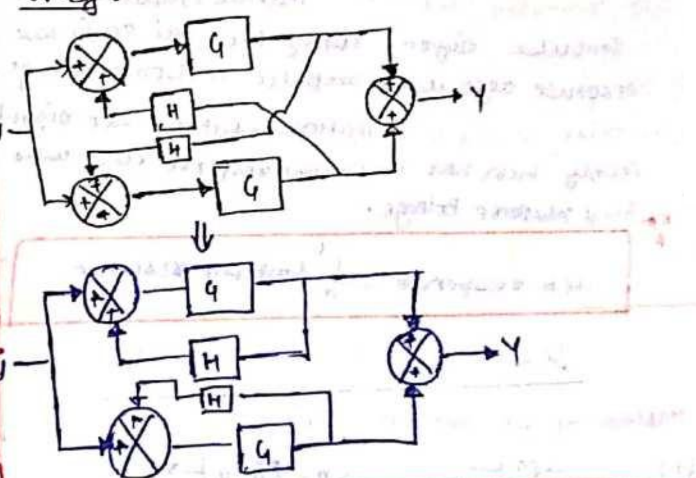
→ Forward path ka direction I/p to o/p hona chahi.

→ Jab Δ_1 ka calculation karenge to forward path '1' ko open karege to node ko destroy kar dena hai.

→ Non touching me node bhi common nai hona chahiye.

Note: Due to symmetry in Block diagram we can interchange the take up point across the I/p of adder at the o/p side.

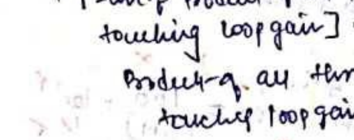
for eg:



Limitation of Mason's gain formula:

Limitation of Mason's gain formula: self sustaining loop with unity gain.

→ mason's gain formula is invalid for self sustaining loop with unity gain.

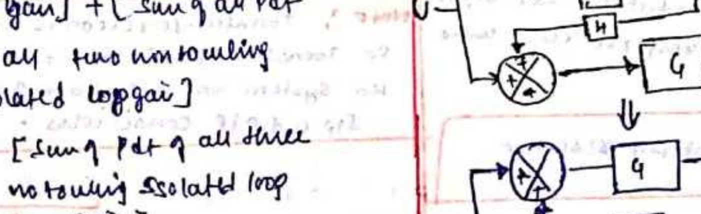


$a_{22} = 1$, then mason's formula fails.

Note: Whenever there is self loop with unity gain just neglect or remove it from SFG and solve. **

Imp: Agar input pe self loop ya kisi bhi gain ke sath just remove it and solve. **

for eg



Note: Agar zero mean Bd. dia to, o/p response ka steady state value = 0

Questions to be done from chapter 1 of control system:

Gate: 13, 19, 33,

Ese: 25,

Questions to be done from chapter 2 of CS:

Gate: 5, 9, 14, 17, 27, 36, 37, 41

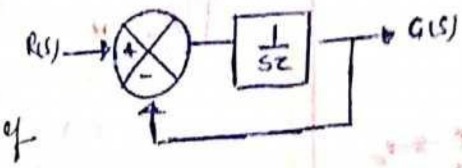
Question to be done from chapter 3:

Gate: 6, 9, 8, 12, 36, 57, 59, 64, 67, 71, 77, 85, 86, 70

Ese: 14, 68, 103, 104, 136, 137

3. Time Response

Analysis of 1st order System:



Here $\boxed{G(s) = \frac{1}{s\tau}}$ *

$H(s) = 1$

$\boxed{T(s) = \frac{1}{1+s\tau}}$ **

Generalise Expression of Transfer function of 1st order System:

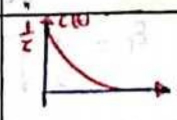
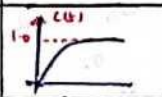
$$\boxed{T(s) = \frac{Ke^{-t_d s}}{1+s\tau}}$$

where, k = DC gain
t_d = Transportation delay time
τ = Time constant.

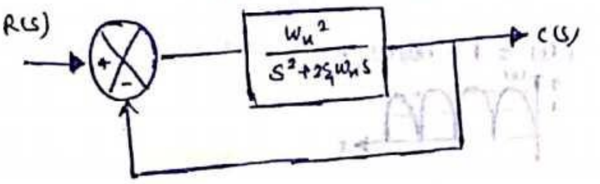
Error:

** $\boxed{\text{Error} = \text{Input} - \text{output} = r(t) - c(t)}$

↓ for first order system.

Input r(t)	Output c(t)	Output c(t) graph	Error e(t)	e_{ss} (ye yad rkhna)
* 1. Impulse $\delta(t)$.	$\frac{1}{\tau}e^{-t/\tau}u(t)$		$\delta(t) - \frac{1}{\tau}e^{-t/\tau}u(t)$	Not possible *
*** 2. Step input $u(t)$	$(1-e^{-t/\tau})u(t)$		$e^{-t/\tau}u(t)$	0
3. Ramp input $tu(t)$	$-\tau u(t) + t u(t) + \tau e^{-t/\tau}u(t)$	-	$\tau u(t) - \tau e^{-t/\tau}u(t)$	τ
4. Parabolic Input $\frac{t^2}{2}u(t)$.	$\tau^2 u(t) - \tau t u(t) + \frac{t^2}{2}u(t) - \tau^2 e^{-t/\tau}u(t)$	-	$-\tau^2 u(t) + \tau t u(t) + \tau^2 e^{-t/\tau}u(t)$	∞

Analysis of 2nd order system:



Where $G(s) = \frac{w_n^2}{s^2+2\xi w_n s}$

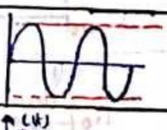
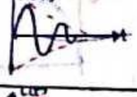
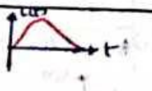
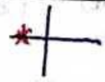
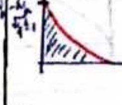
$H(s) = 1$

Where, K = DC gain
w_n = Natural frequency
ξ = Damping ratio α = Damping coefficient $= \xi w_n$.

Generalised Expression of T.F of IInd order system *

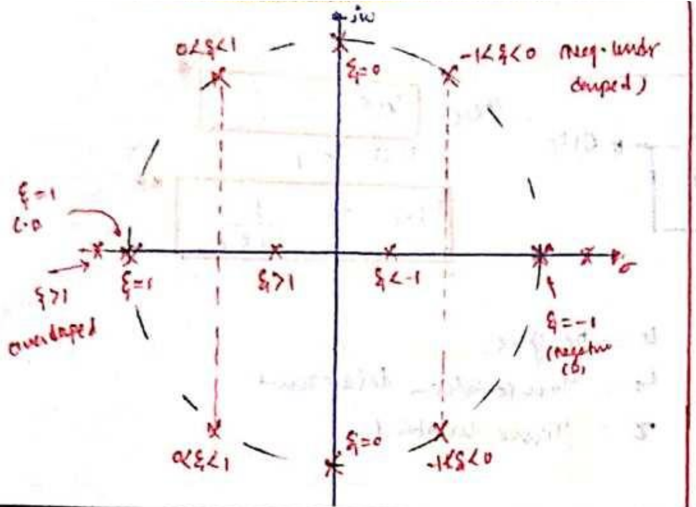
$$\boxed{T(s) = \frac{Kw_n^2}{s^2+2\xi w_n s + w_n^2}}$$

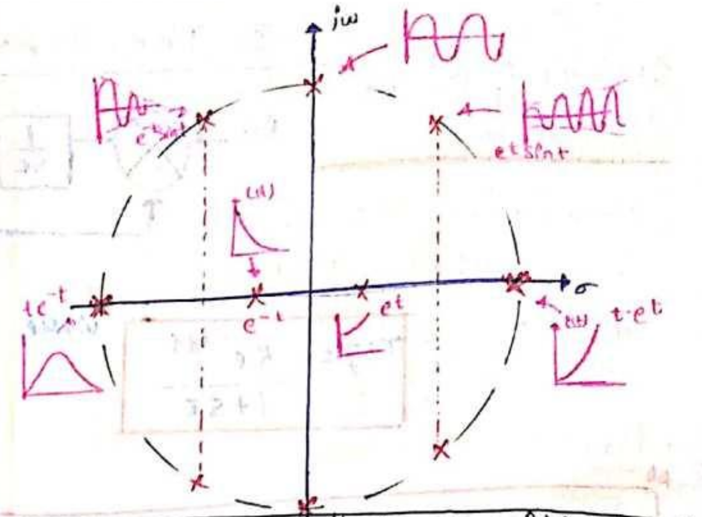
Unit Impulse Response of 2nd order system (20% weightage): -

Oscillation	Output c(t)	o/p graph	ξ	τ	s-plane	Position of Poles
(1) Undamped	$w_n \sin w_n t$		$\xi = 0$	∞		Complex conjugate poles on Imaginary axis.
*** (2) Underdamped	$\frac{w_n}{\sqrt{1-\xi^2}}e^{-\xi w_n t}\sin w_d t$		$0<\xi<1$	$\frac{1}{\xi w_n}$		Complex Conjugate Poles in LHS.
* (3) Critical damped	$w_n^2 t e^{-w_n t}$		$\xi = 1$	$\frac{1}{w_n}$		Negative equal repeated real roots / Poles.
** (4) Overdamped	$\frac{\xi w_n - w_n\sqrt{\xi^2-1}}{s+\xi w_n - w_n\sqrt{\xi^2-1}}$ after apply dominant pole concept.		$\xi > 1$	$\frac{1}{\xi w_n - w_n\sqrt{\xi^2-1}}$ ***		Negative Unequal Real roots / Poles.

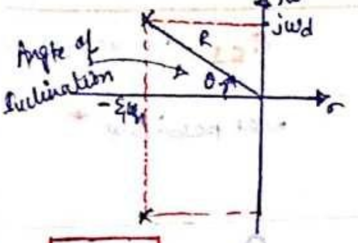
→ w_d = Damped frequency $= w_n\sqrt{1-\xi^2}$

→ α = Actual damping $= \xi w_n$





Concept of Angle of Inclination :



$\boxed{R = \omega_n}$ **

$\boxed{\xi = \cos\theta}$ **

$\boxed{\sin\theta = \sqrt{1-\xi^2}}$ **

$\boxed{\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}}$ **

$\boxed{\theta \propto \frac{1}{\xi}}$ → Damping Ratio

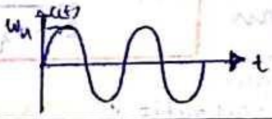
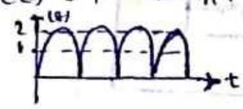
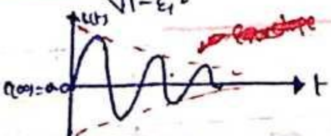
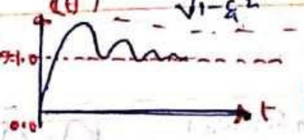
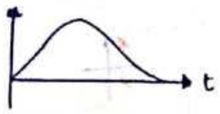
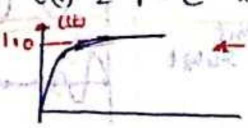
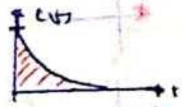
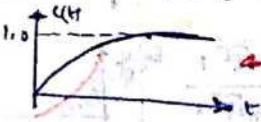
→ Angle of inclination

agar θ ki value 90° se jada hui to system unstable hoga qki $\cos(90+\theta) = -ve$, hence $\xi = -ve$

Series RLC N/W	Parallel RLC N/W
(1) $\omega_n = \frac{1}{\sqrt{LC}}$	(1) $\omega_n = \frac{1}{\sqrt{LC}}$
(2) $\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$	(2) $\xi = \frac{1}{2R}\sqrt{\frac{L}{C}}$
(3) $Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}$ or $Q_s = \frac{1}{2\xi_s}$	(3) $Q_P = R\sqrt{\frac{C}{L}}$ or $Q_P = \frac{1}{2\xi_P}$
(4) C/S eqn: $\boxed{s^2 + \frac{R}{L}s + \frac{1}{LC} = 0}$	(4) C/S eqn: $\boxed{s^2 + \frac{s}{RC} + \frac{1}{LC} = 0}$

Note : agar kabhi question me puch lia ki overdamped condition ke lie 'R' range kya hogi, to hum ξ ko (koi aur bhi damping de skta hai) se relate karke nikal lenge.

~~example~~ $\xi > 1 \Rightarrow \frac{R}{2}\sqrt{\frac{C}{L}} > 1$ se relate karke nikal lenge.

ξ	Impulse Response	Step Response ***
ξ = 0 undamped	$c(t) = h(t) = \omega_n \sin\omega_n t$ 	$c(t) = 1 - \cos\omega_n t$ 
0<ξ<1 *** underdamped	$c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t$ 	$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\theta)$ very imp.  [Envelope: $1 \pm \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}$.]
ξ = 1.0 critical damped	$c(t) = h(t) = \omega_n^2 t e^{-\omega_n t}$ 	$c(t) = 1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t}$  ← fast response.
ξ > 1 overdamped	$c(t) = h(t) = (\xi\omega_n - \omega_n\sqrt{\xi^2-1})e^{-(\xi\omega_n - \omega_n\sqrt{\xi^2-1})t}$ 	$c(t) = 1 - e^{-(\xi\omega_n - \omega_n\sqrt{\xi^2-1})t}$  ← slow response.

Order of time constant : τ (undamped) > τ (overdamped) > τ (underdamped) > τ (critically damped)

Order of Response/Speed : (C.D) > underdamped > (overdamped) > undamped.

Note : agar sine ke sath koi exponential term aya to wo $e^{-\xi\omega_n t}$ hi hoga.

Note: Agar given expression hai o/p ka to sabse phle Tss me D.C gain check kare nikal le.

r(t)	T(s)	Impulse response	r(t)	T(s)	Response Step
$\delta(t)$	$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t$	$u(t)$	$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$1-\frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin(\omega_d t+\theta)$ $c(\infty)=1$
$A\delta(t)$	$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$\frac{A\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t$	$A\,u(t)$	$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$A\left[1-\frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin(\omega_d t+\theta)\right]$ $c(\infty)=A$
$\delta(t)$	$\frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$\frac{K\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t$	$u(t)$	$\frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$K\left[1-\frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin(\omega_d t+\theta)\right]$ $c(\infty)=K$
$A\delta(t)$	$\frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$\frac{AK\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t$	$A\,u(t)$	$\frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$AK\left[1-\frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin(\omega_d t+\theta)\right]$ $c(\infty)=AK$

K = DC gain,
A = Amplitude of input signal.

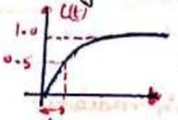
$$\boxed{e_{ss} \text{ of step response } = 0}$$

Generalised Result of error for step response of 2nd order:

$$\boxed{c(t) = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\theta)}$$

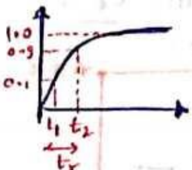
Time Domain Parameter for 1st order system: (Here we have assumed step response) :-

Delay time : time taken by the response to reach from 0 to 50% of its steady state value.



$$\boxed{t_d = 0.693\,\tau}$$

Rise time : taken time by the response to reach from 10 to 90% of its steady state value



$t_r = t_2 - t_1$

$\boxed{t_r = 2.2\,\tau}$ or $\boxed{t_r = \frac{0.35}{B.W}}$ — 3dB Bandwidth.

Settling time :

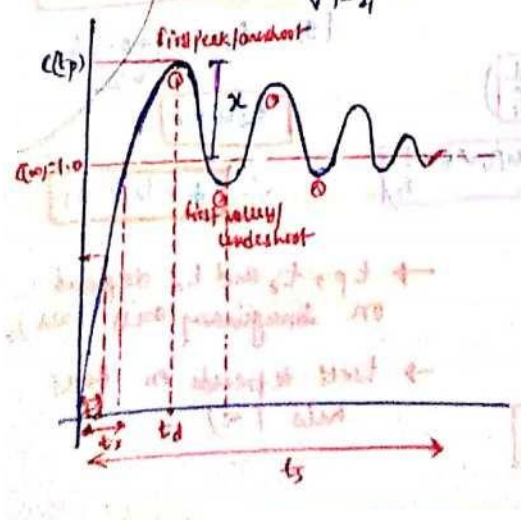
% Error Band	t_s
1%	4.6τ
2%	4τ
3%	3.5τ
4%	3.2τ
5%	3τ

For 1st order system:

$$\boxed{t_s > t_r > t_d}$$

Time domain parameter for 2nd order "Underdamped System".

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\theta)$$



for 2nd order underdamped system;

$$\boxed{t_s > t_p > t_r > t_d}$$

(1) **Peak time:** $\boxed{t_p = \frac{n\pi}{\omega_d}}$

n=1, 1st overshoot
n=2, 1st undershoot
n=3, 2nd overshoot

$$\boxed{c(t_p) = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}}$$ → value of 1st peak overshoot.

$$\boxed{c\left(t_p = \frac{2\pi}{\omega_d}\right) = 1 - e^{-2\pi\xi/\sqrt{1-\xi^2}}}$$ → value of 1st undershoot.

$$\boxed{x = e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

→ Overshoot Peak time is given by

$$\boxed{t_p(OM) = \frac{(2m-1)\pi}{\omega_d}}$$ m = no. of oscillation cycle.

→ Undershoot Peak time is given by

$$\boxed{t_p[UM] = \frac{2m\pi}{\omega_d}}$$

→ Peak time is periodic in nature.
Time period of oscillation, $T_0 = \frac{2\pi}{\omega_d}$.

→ Note: % MPO of undamped = 100%.
and critical damped and overdamped = 0 %

2. Settling time :-

% Error	t_{sett} $(0<\xi_1<1)$	t_{sett} $(\xi_1>1)$
2%	$4\tau = \frac{4}{\xi\omega_n}$	$4\tau = \frac{4}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}$
5%	$3\tau = \frac{3}{\xi\omega_n}$	$3\tau = \frac{3}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}$

→ exam me $\xi_1 > 1$ ka tsett ka question puch skta hai.

4. Delay time (t_d):

$$\boxed{t_d = \frac{1+0.7\xi_1}{\omega_n}}$$ → only valid for underdamped system.

For overdamped system,

$$\boxed{t_d = 0.693\tau = \frac{0.693}{\xi\omega_n - \omega_n\sqrt{\xi_1^2-1}}}$$

Maximum peak overshoot:

$$\% MPO = \frac{c(t)|_{max} - c(t)|_{desired}}{c(t)|_{desired}} \times 100$$

$$MPO = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

$$\boxed{\% MPO = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100}$$

$$\boxed{MPO \propto \theta \propto \frac{1}{\xi_1}}$$

For ξ	% MPO
0	100%
0.5	16.3%
0.707	4.22%
1	0%

Maximum Peak undershoot:

$$\% MPU = \frac{c(t)|_{desired} - c(t)_{min}}{c(t)\ desired} \times 100$$

$$\% MPU = \frac{c(\infty) - c(t)|_{min}}{c(\infty)} \times 100$$

$$\% MPU = e^{-2\pi\xi/\sqrt{1-\xi^2}} \times 100$$

3. Rise time: (only valid for underdamped system).

$$\boxed{t_r = \frac{\pi-\theta}{\omega_d}}$$ → must be in radian.

Note: For overdamped system, t_r is same as 1st order system, ***

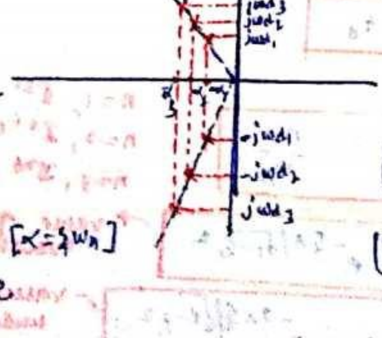
i.e
$$\boxed{t_r = 2.2\tau \qquad t_r = \frac{2.2}{\xi\omega_n - \omega_n\sqrt{\xi_1^2-1}}}$$

Keypoint: No. of oscillation before settled the output is given by **

$$\boxed{N_0 = \frac{t_{sett}}{T_0 (\text{time period of oscillation})}}$$

Case 1: Diagonal Movement :- (of underdamped system).

$s = -\alpha \pm j\omega_d$
$\alpha = \xi\omega_n$
$\omega_d = \omega_n\sqrt{1-\xi_1^2}$



Concept:
θ = constant
α = variable [α = ξωn]
ωd = variable

(1) $\theta_1 = \theta_2 = \theta_3$

(2) $\xi_1 = \xi_2 = \xi_3$ $\left(\because \theta \propto \frac{1}{\xi_1}\right)$

(3) $MPO_1 = MPO_2 = MPO_3$ $\left(\because MPO \propto \theta \propto \frac{1}{\xi_1}\right)$

(4) $\alpha_3 > \alpha_2 > \alpha_1$

(5)** $t_{sett} = 4\tau = \frac{4}{\alpha}$

$t_{sett} \propto \frac{1}{\alpha}$

$\therefore \boxed{t_{sett_1} > t_{sett_2} > t_{sett_3}}$

(6) $\omega_{d_3} > \omega_{d_2} > \omega_{d_1}$

(7)** $t_p \propto \frac{1}{\omega_d}$

$\therefore \boxed{t_{p_1} > t_{p_2} > t_{p_3}}$

(8) $t_r = \frac{\pi-\theta}{\omega_d}$

θ = constant

$\therefore t_r \propto \frac{1}{\omega_d}$

$\boxed{t_{r_1} > t_{r_2} > t_{r_3}}$

(9) $t_d = \frac{1+0.7\xi}{\omega_n}$

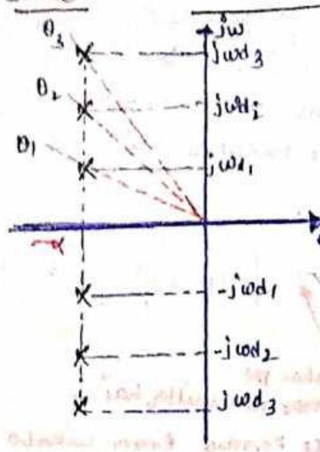
$\boxed{t_d \propto t_r}$

$\therefore \boxed{t_{d_1} > t_{d_2} > t_{d_3}}$

→ t_p, t_r and t_d depend on imaginary axis (ω_d).

→ t_{sett} depends on Real axis (α).

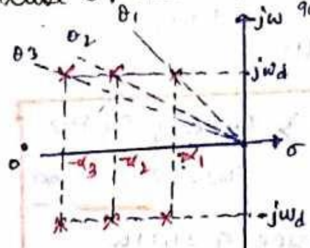
Case 2: Vertical Movement:



concept:
α = constant
θ = variable
ω_d = variable

(1) $\theta_1 < \theta_2 < \theta_3$
(2) $\xi_1 > \xi_2 > \xi_3$ ($\because \theta \propto MPO \propto \frac{1}{\xi}$).
(3) $MPO_1 < MPO_2 < MPO_3$
(4) $t_{sett_1} = t_{sett_2} = t_{sett_3}$ ($\because \alpha$ is constant)
(5) $\omega_{d_3} > \omega_{d_2} > \omega_{d_1}$
(6) $t_{p_1} > t_{p_2} > t_{p_3}$
(7)** $t_{r_1} > t_{r_2} > t_{r_3}$ (isko yad rakhna padega ya phir lekh ke nikalo exam me).
(8) $t_{d_1} > t_{d_2} > t_{d_3}$ ($\because t_d \propto t_r$)
(yad rakhna, t_d, t_r ke proportional hota hai, na ki t_r t_d ke).

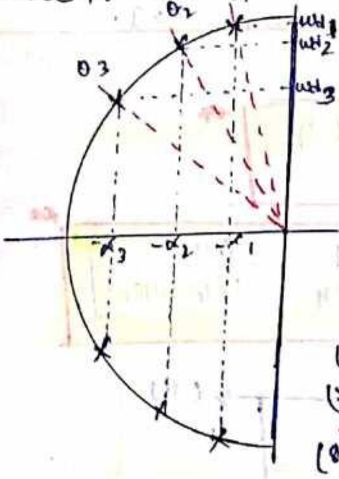
Case 3: Horizontal Movement



Concept:
ω_d = constant
θ = variable
α = variable

(1) $\theta_1 > \theta_2 > \theta_3$
(2) $\xi_3 > \xi_2 > \xi_1$
(3) $MPO_1 > MPO_2 > MPO_3$
(4) $t_{p_1} = t_{p_2} = t_{p_3}$
(5) $t_d = \frac{1 + 0.7\xi}{\omega_n \leftarrow \text{constant}}$
$\therefore t_d \propto \xi$
$\therefore t_{d_3} > t_{d_2} > t_{d_1}$
(6) $t_r = \frac{\pi - \theta}{\omega_d \leftarrow \text{constant}}$
$\therefore$ as $\theta\uparrow$ $\pi - \theta \downarrow$ and $t_r\downarrow$
$\therefore t_r \propto \frac{1}{\theta}$
$\therefore t_{r_3} > t_{r_2} > t_{r_1}$
(7) $t_{sett} \propto \frac{1}{\alpha}$
$\therefore t_{sett_1} > t_{sett_2} > t_{sett_3}$
(8) $\omega d_1 = \omega d_2 = \omega d_3$.

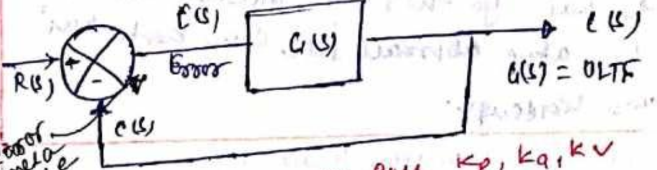
Case 4: Circular Movement:



concept:
ω_d = variable
θ = variable
α = variable

(1) $\theta_1 > \theta_2 > \theta_3$
(2) $MPO_1 > MPO_2 > MPO_3$
(3) $\xi_3 > \xi_2 > \xi_1$
(4) $\alpha_3 > \alpha_2 > \alpha_1$
(5) $t_{sett_1} > t_{sett_2} > t_{sett_3}$
(6) $\omega d_1 > \omega d_2 > \omega d_3$
(7) $t_{p_3} > t_{p_2} > t_{p_1}$
(8) $t_{r_3} > t_{r_2} > t_{r_1}$ (yad rakhna padega).
(9) $t_d \propto t_r$
$\therefore t_{d_3} > t_{d_2} > t_{d_1}$.

Analysis of Steady State Error:



$G(s)$ = OLTF

Error hamesha yahi se nikalega chahe input kahi bhi de do.

→ Isi figure ke liye bus K_p, K_a, K_v wala formula applicable hoga.

$$E(s) = \frac{R(s)}{1 + G(s)} = \frac{I/P}{1 + OLTF}$$ → Basic Result.

→ $E(s)$ is function of I/P and OLTF. ***

$$e_{ss} = \lim_{s \to 0} s\,E(s) = \lim_{s \to 0} s\left[\frac{R(s)}{1 + G(s)}\right]$$

***Note: Hamesa humlog type of system, ya type of OLTF, ya type of CLTF, sirf aur sirf OLTF se bataenge. aur unity negative hona chahiye tabhi G(s) ke type se system ka type bata paenge warna convert karo unity negative feedback me.

Input	error coefficient	ess
(1) $r(t) = u(t)$	$K_p = \lim_{s \to 0} G(s)$	$e_{ss} = \frac{1}{1 + K_p}$
(2) $r(t) = t\,u(t)$	$K_v = \lim_{s \to 0} s\,G(s)$	$e_{ss} = \frac{1}{K_v}$
(3) $r(t) = \frac{t^2}{2} u(t)$	$K_a = \lim_{s \to 0} s^2 G(s)$	$e_{ss} = \frac{1}{K_a}$

Note: (1) for type '0' system, K_p represents the dc gain of OLTF.
(2) For type '1' system, K_v represent the DC gain of OLTF.
(3) For type '2' system, K_a represents the DC gain of OLTF.

Type	error coefficient	ess: $u(t)$	$t\,u(t)$	$\frac{t^2}{2} u(t)$
0	$K_p = \lim_{s \to 0} G(s) = K$ $K_v = \lim_{s \to 0} s\,G(s) = 0$ $K_a = \lim_{s \to 0} s^2 G(s) = 0$	finite	∞	∞
1	$K_p = \infty$ $K_v = K$ $K_a = 0$	0	finite	∞
2	$K_p = \infty$ $K_v = \infty$ $K_a = K$	0	0	finite

Note:- Kabhi bhi steady state position response bola Position control of DC motor me, to samajh jana ki, o/p ka steady state value puch raha.

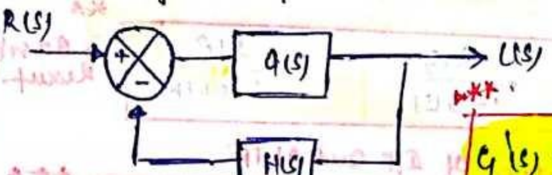
(2) Ramp Input $= t' u(t)$

(3) Parabolic Input $= \frac{t^2}{2} u(t)$

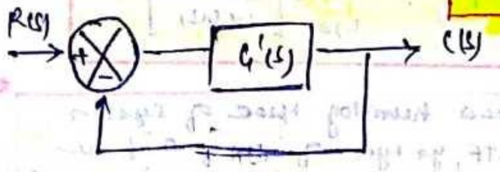
Note:
(1) If order of I/P > type of system (i.e. degree of 't'), then $e_{ss} = \infty$
(2) If order of I/P < type of system then $e_{ss} = 0$
(3) order of I/P = type of system, $e_{ss} =$ finite

Special Note: Kabhi bhi steady state error ka question aya to $1 + G(s) = 0$ karke stability check karenge. qki unstable system ke liye steady state error exist nai karta. Aur man lo unstable agya hai aur opt. me 4 option die hai ya phir NAT question hai to hum apne approach jese solve karte hai vese kardenge.

Conversion from Non unity F/b system to unity F/b system :-



$$G'(s) = \frac{G(s)}{1 + G(s)H(s) - G(s)}$$



→ From conversion, Non unity feed back system to unity feedback system :

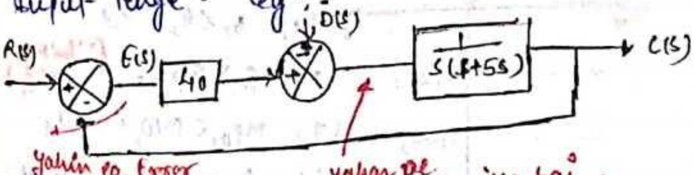
(1) D.C gain of OLTF changes
(2) Type of system changes
(3) Transfer function of CLTF remain same so D.C gain of CLTF also remain same.

Decay Ratio $= \frac{MPO_1}{MPO_2} = \frac{e^{-\pi\xi/\sqrt{1-\xi^2}}}{e^{-3\pi\xi/\sqrt{1-\xi^2}}}$

for no overshoot, $\xi = 1$.

aur agar step response, ke case me bol dia ki minimum settling time to samajh jana $\xi = 1$.

Note: Kabhi bhi hamara standard Negative unity feedback wala fig. nai dikha aur hum SFG bana ke solve karne chah rahe hai to E(s) ko O/P ... or jo bhi input hai uske hoga ya jiske karand error nikalna hai usko input lenge. Eg:



aur agar bola ki D(s) ke karand error nikalo to hum R(s) = 0 karenge aur block diagram se hi E(s) nikal sakte hai aur agar SFG banane gye to R(s) = 0 hogya aur D(s) ko I/P lenge aur E(s) ko O/P lenge.

eg. Find the error of $r(t) = A u(t) + Bt u(t) + Ct^2 u(t)$

$$e_{ss} = \frac{A}{1+K_p} + \frac{B}{K_v} + \frac{2C}{K_a}$$

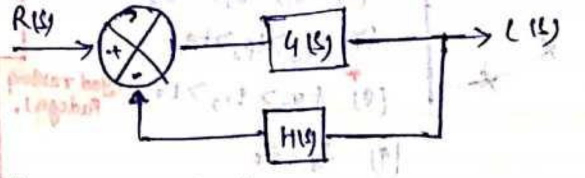
[2C is lie qki $\frac{t^2}{2}u(t)$ ka $\frac{1}{K_a}$ hota hai, na ki $t^2 u(t)$ ka].

Steady State Error for Non-unity F/b system:-

Jis din tough paper banega yahi se question dega.

$$\boxed{E(s) = \frac{R(s)}{K_H} - C(s)}$$

$$\boxed{e_{ss} = \lim_{s \to 0} sR(s)\left[\frac{1}{K_H} - \frac{G(s)}{1 + G(s)H(s)}\right]}$$



Case 1 : H(s) with no poles and zeroes at origin.

$$\boxed{K_H = \lim_{s \to 0} H(s) = \text{DC gain of } H(s)}$$

Case 2: DC gain of H(s) with zeroes of origin of order 'N'

$$\boxed{K_H = \lim_{s \to 0} \frac{H(s)}{s^N} = \text{constant.}}$$

Case 3: DC gain of H(s) with poles at origin of order 'N'.

$$\boxed{K_H = \lim_{s \to 0} s^N H(s) = \text{constant.}}$$

Effect of Adding poles and Zeroes to Transfer Function :

Addition of a pole to the forward Path Transfer function :- (OLTF)

(1) Increases the order of the system
(2)* Increase the overshoot and reduce the stability.
(3) Increase the rise time of the step responses, reduces the B.W.

Addition of a Pole to CLTF :

(1) Increase the rise time
(2) Decrease the overshoot.

Addition of zero to CLTF : (Just opp. of Addition of pole to CLTF)

(1) Decrease the rise time
(2) Increase the overshoot

Addition of a zero to/OLTF : (1) when added zero is very far away from the Im. axis, the overshoot is large and the damping is poor.

(2) The overshoot is reduced and damping improves when the zero moves to the right of its original position

(3) When zero move closer to the origin, the overshoot increases but damping improves.

Note: agar ek system 'K' gain ke sath critically damped to,

(1) If we ↑ K, then system becomes underdamped.
(2) If we ↓ K, then system becomes overdamped.

Decay Ratio $= \frac{MPO_1}{MPO_2} = \frac{e^{-\pi\xi/\sqrt{1-\xi^2}}}{e^{-3\pi\xi/\sqrt{1-\xi^2}}}$

---

4. Routh Hurwitz Criteria

{Sabse Imp Bat is chapter me jo K hoga wo hame K > 0}

Some Important Points :

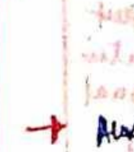
→ For a system to be stable, 1st column of its C/S Equation, should be of same sign; there should not be change of sign.

→ $\boxed{\text{No. of sign change in 1st colomn} = \text{No. of RHS Poles.}}$

→ If any Power of 's' is missing in C/s Equation that represent the presence of atleast one pole at RHS of S-plane and system will be unstable.

→ Coefficient of C/E should be only Real for Routh Hurwitz Stability.

→ If C/E contain only even power of 's' that represents the presence of poles on the Imaginary axis and system will become Marginal Stable.



unstable (repeated roots in Im. axis)

→ If repeated roots in Imaginary axis, then system becomes unstable.

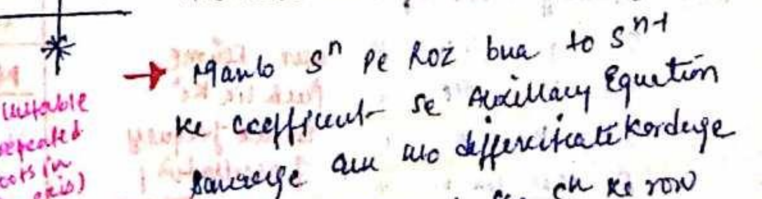
Routh table gives info about No. of Right hand poles in the S-plane.

→ In Routh Hurwitz criteria, we find determinant as follow

$$\begin{vmatrix} a_0 & a_1 \\ a_2 & a_3 \end{vmatrix} = a_2a_1 - a_0a_3$$

Row of zeroes (ROZ) :

→ ROZ tabhi occur karega jab poles origin se symmetrical way me located rahega.



unstable | marginal stable | unstable | marginal stable

All these are symmetric to origin.

→ Manlo s^n pe ROZ hua to s^{n+1} ke coefficient se Auxiliary Equation banayenge aur usko differentiate kardenge aur use coefficient ko s^n ke row me rakh denge.

→ Auxillary Equation hamesa hi even power of 's' ka hoga and roots of A.E are symmetrical about origin.

Root of Auxillary Equations are poles of closed loop system.

→ If only one ROZ occur and 1st coloum are +ve then System will be marginal stable.

→ If more than one ROZ occurs that represents Presence of repeated Poles and no. of Repeated Poles = No. of ROZ.

Note: (1) Hurwitz Improper polynomial are always unstable (i.e missing power of s or -ve coeffient in C/E).

(2) Hurwitz Proper Polynomial can be stable / unstable or marginal stable.

Note: (1) RH table me jitne ROZ ayge utne Poles repeat honge, ye condition tab hai jab ROZ > 1 i.e agar ROZ = 2, to Imaginary axis me ±j, ±j type ke poles dikhenge.

(2) Kabhi kabhi hum Auxillary Equation se hi roots nikal dete hai aur hamko 3 roots repeated milgata hai in. one jabki ROZ = 2 hi tha, ese me hum Dhyan rakhenge aur C/E ko A(s) se divide deke Bache hue roots nikalenge Jarurat Padne par.

→ order of Polynomial, if we have no. of Rows in Routh table:

$$\boxed{\text{Order} = \text{No. of rows} - 1}$$

Special Note: Kabhi agar K marginal puche ya bole ki kis value ke liye system marginal stable hoga to hum ROZ banaenge. For eg:

s^4	1	44	K
s^3	12	48	0
s^2	40	K	0
s^1	$\frac{1920-12K}{40}$	0	0
s^0	K	0	0

aur same puch lia ki Batao frequency of oscillation, to hum ROZ ke upar wale coefficient se Auxillary Equation banaenge aur Equal to zero karenge, uske jo ω aega wo hi frequency of oscillation hoga.

$\frac{1920-12K}{40} = 0$ karenge tab s^1 me ROZ banega.

→ auxillary equation se C/E ko divide deke bache roots nikal sakte hai Jarurat Padne Pe.

Trick to check stability of 3rd order polynomial directly:-

Polynomial P(s)	Internal Pdt	External Pdt	Condition	Stability
$s^3+2s^2+8s+10$	IP = 2×8 = 16	EP = 1×10 = 10	IP > EP	Stable
$s^3+2s^2+8s+20$	IP = 16	EP = 20	EP > IP	unstable
$s^3+2s^2+8s+16$	IP = 16	EP = 16	EP = IP	marginal stable

conclusion:

$$\boxed{\begin{aligned} IP > EP &\Rightarrow \text{stable} \\ IP < EP &\Rightarrow \text{unstable} \\ IP = EP &\Rightarrow \text{marginal stable} \end{aligned}}$$

Above conditions are only valid for 3rd order proper polynomial i.e no power of s term should be missing.

Special Case of RH table:

agar kabhi kisi row me 1st element hi 0 ban gya (Non ROZ case) to usko ε se Replace karenge aur pura solve karenge to $\lim_{\varepsilon \to 0}$ ε se uske value put karenge.

→ Agar sirf stability bola hai to By default system stability batana hai.

→ There is no significance of ROZ in last Row of RH table.

→ Kabhi open loop ka expression deke puch lega ki Bato open loop ke kitne Poles RHS me hai to hum 1+G(s) = 0 nai karenge. But Given G(s) ke poles ka expression = 0 hoga.

→ Kabhi question me ω_n (frequency of oscillation) de dia hai to turant dimag Presence of ROZ taraf jana chahiye.

Note: R-H method is not valid for sine, cosine and exponential function because this function consist infinite no. of terms, so we use linear Approximation for Analysis of these terms.

$$\boxed{e^{-sT} = 1 - sT}$$

Note: (1) Jab Proper Polynomial me ROZ Banega tab 100% poles Imaginary me jaenge But agar Improper Polynomial ROZ Banega tab poles Imaginary, kahin bhi Ja sakte hai i.e Imaginary, RHS, LHS.

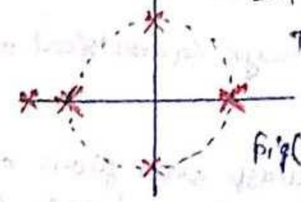
(2) Agar sign change Auxillary equation ke Bad hora to Symmetric pole hoga LHS me wrt RHS pole. (valid only in case of unstable Polynomial).

(3) Agar koi sign change nai hora Auxillary Equation ke niche to koi symmetric pole nai hoga LHS me wrt that of in RHS. (valid only in case of unstable Polynomial).

**.

$s^7 + s^6 + s^5 + s^4 + s^3 ...$

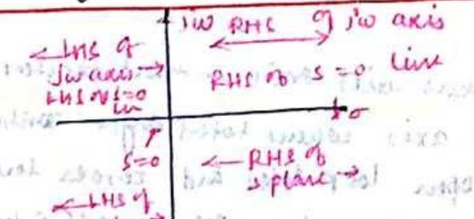
Total roots = 7.



fig(A)

→ Improper Polynomial me ROZ aya to Jaruri nai hai poles Imaginary me hi repeat honge, kahin bhi repeat ho sakte hai LHS, RHS ya Im. me.
fig (A) me agar Auxillary Equation ke Badla hai (given tha Actual question me) aur Improper Bhi is lie right-side poles symmetric hone chahi LHS pole of Auxillary Equation se pole aenge usse.

***Concept of Mapping in Routh Hurwitz :-**
Hum RH table, s=0 line ke lie use karte hai.
Pur kabhi question me bol dia ki RH table se s = a ke lie nikalo
tab hum $S = Z + a$ put karenge give C/s Equation me aur usse jo range ayega wahi Answer hoga.



$Z = U + jV$

→ agar kabhi RH table karte K = -ve aya to us value ko ni lenge qki K > 0. (range of K is 0 to ∞)

Note: sustained Oscillation bola matlab Marginable stable ki bat hori hai.

ROOT LOCUS (only valid for Negative f/b System).

→ In root locus, root represents pole of closed Loop Transfer function and locus represent path of closed loop poles for different values of gain of OLTF (K).

→ For Neg/Fb System, range of K is $0 < K < \infty$.

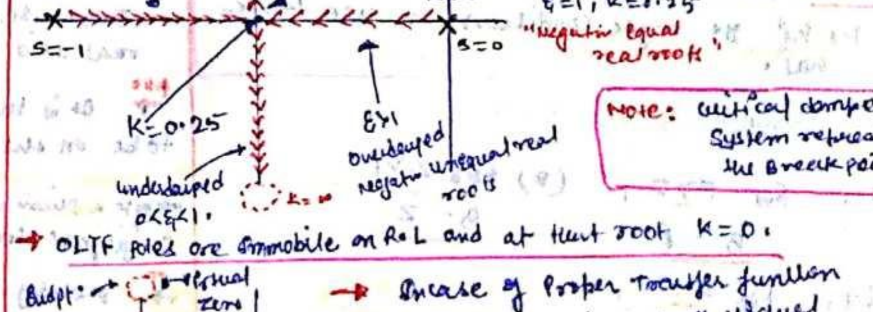
→ Inverse Root locus or complementary root locus is valid for positive f/b system.

→ For Positive f/b system, range of K is $-\infty < K < 0$.

→ If RoL lie completely in the LHS of s plane, then CLTF is Stable.

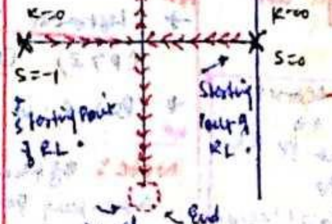
→ If some portion of RL lies in RHS, then CLTF is unstable.

$G(s) = \frac{K}{s(s+1)}$



Note: critical damped System represent the Break point.

→ OLTF poles are Immobile on RoL and at that root K = 0.



→ Incase of Proper Transfer function (poles > zeroes), No. of virtual zeroes, $\boxed{Z^* = P - Z}$

→ Root locus hamesa pole se chalu hoga aur Zero pe Khatam hota hai (In case of proper transfer function).

Rules for sketching Root Locus :-

(1) Root locus is always symmetrical about real axis.

(2) Root locus always start from OLTF poles ($K=0$) and terminates either finite open loop zero or infinity that means virtual zero ($K=\infty$).

(3.) Existence of pt on real axis :-

agar us point ke right-hand side pe OLTF poles + OLTF zero ka sum odd hua to wo point valid hoga.

" all OLTF poles and zeroes are valid pt. for RL diagram !

(4). Existence of Root locus on Real Axis :-

Root locus will exist in that section of real axis where total angle contributed by open loop poles and zeroes towards that observation point is odd integer multiple of $180°$.

Magnitude condition :

$$\boxed{|GH(s)| = 1}$$

Angle condition :

$$\boxed{\angle GH(s) = 180°}$$

or

$$\boxed{\angle GH(s) = \pm(2n+1)\,180°}$$

For Positive feedback, $\angle GH$ should be even integer multiple of $180°$.

→ magnitude condition for positive f/b system is same as negative f/b system.

→ $\angle GH(s) = 0°$ [Angle condition for positive f/b system].

Note : Hum complex point ko check karne ke liye hi ye angle condition ka use karte hai.

(5.) Number of Branches :-

(1) For $P \geq Z$, $B = P$

(2) For $Z > P$, $B = Z$

(3) For $Z = P$, $B = Z = P$

* Branch hamesa $K=0$ se shuru hoga aur $K=\infty$ pe khatam hota hai.

Note : agar kisi complex point ko check karna hai ki wahan pe R.L hoga ki nahi to hum angle condition lagayenge. Agr point pe RL exist kia to 'K' ki value us point pe magnitude condition laga ke nikal lenge.

Special Note : for existence of complex point, first we apply angle condition after that we apply magnitude condition for calculation of 'K'.

(6). Breakpoint : Brk pt. is that location in s-plane where two branches of closed loop system will coincide.

Steps for calculation of Break point :

(i) $1+G(s)H(s) = 0$

(2) $K = f(s)$

(3) $\frac{dK}{ds}$

(4) Roots of $\frac{dK}{ds}$ [$\frac{dK}{ds}=0$]

(5) valid roots = Break points

(a) Break away point :-

(1) It is maximum value of 'K' for Root locus to be on the real axis.

(2) When 'K' exceeds from this maximum value real root locus will break into two parts real and complex conjugate.

(3)* Break away point matlab wo point jahan pe R.L real axis ko chod ke complex axis me jane lagta hai.

(4)** mostly two adjacent poles consist Break away point.

(5)** mostly multiple poles consist Breakaway points.

(6)*** Break away point is referred as maxima point because at this point the value of K will be maximum on the real axis.

agar kabhi koi puche real axis me 'K' ki maximum value batao to hum Break away point pe 'K' ki value nikal denge.

(B) Break In point : ye wo point hota hai jahan se R.L complex plane chode ke real axis me entry marta hai.

*** → It is the minimum value of K for root locus to be on the real axis.

→ If K < minimum value, the R.L will remain complex conjugate location.

** → mostly two adjacent zeroes consist B.I.P

→ mostly multiple zeroes consist BIP.

*** → Leftmost zero with proper transfer function (P>Z) consist Break In point.

→ B.I.P is referred as "Minima point".

Note : If there is no information of Break or Breaking point, then we consider Break away point.

(7) No. of Asymptotes : Asymptotes are root locus branches which approches to infinity.

(1) for $P>Z$, $A = P-Z$

(2) for $P<Z$, $A = Z-P$

(3) for $P=Z$, $A=0$

In case of Proper transfer function,
No. of Virtual zeroes = No. of Asymptote.

(8) Angle of Asymptote :-

(1) for $P>Z$ (P.T.F)

$$\boxed{\angle A = \frac{(2\alpha+1)180^\circ}{P-Z}} \quad \alpha = 0,1,2,\cdots P-Z-1$$

$$\boxed{\alpha_{max} = P-Z-1}$$

α ki utni hi value hogi, jitna Asymptote honge.

(2) for $Z>P$ (I.T.F)

$$\boxed{\angle A = \frac{(2\alpha+1)180^\circ}{Z-P} \quad \alpha_{max} = Z-P-1}, \quad \alpha = 0,1,2\cdots Z-P-1$$

(3) for $P=Z$ (S.T.F)
(No asymptote, so no angle of asymptote).

No. of Asympt	Angle of Asymptote
1	180°
2	$90^\circ, 270^\circ$
3	$60^\circ, 180^\circ, 300^\circ$
4	$45^\circ, 135^\circ, 225^\circ, 315^\circ$

(9) Centroid :

→ Origin of Asymptote is called centroid
→ Also referred as centre of gravity.

$P>Z$ (P.T.F)

$$\sigma = \frac{\Sigma \text{Real part of poles} - \Sigma \text{Real parts of zeroes}}{P-Z}$$

$Z>P$ (I.T.F)

$$\sigma = \frac{\Sigma \text{Real part of zeroes} - \Sigma \text{Real part of Poles}}{Z-P}$$

$z = p$,
σ = doesnot exist.

Note : (1) Jahan se Asymptote nikalte hai wahi centroid hoga
(2) agar centroid and Break point coincide kre to Asymptotic line se hi Root Locus Diagram Banega.

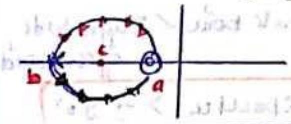
→ Calculation of Kmax and frequency of oscillation Routh Hurwitz methods se karenge.

→ Note : BAP pe 'k' ki value wo hoti jiske lie system critical damped hoti hai, aur wo hi Kmax hota hai for overdamped condition, use jada kara to underdamped hojata hai.

Use bhi hum padhe hai agar ek function 'k' gain ke sath C.D hai to agar k↑ to wo U.D Banega aur K↓ to wo O.D banega.

Root locus diagram describe conditional stability of CLTF.

Eg $GH(s) = \frac{K(s+a)^2}{(s+b)^2}$ $a,b>0$. Let $b>a$



→ agar physical zero and virtual zero agal bagal hai to wo adjacent rehlaenge.

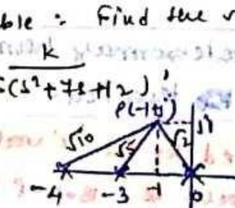
Transfer function of type :
$\frac{K(s+a)}{(s+b)(s+c)}$ for circle as root locus and $-a$ is the centre of that circle locus

a, b and c can be less than zero or greater than zero. (Por Bana ke hi dekhlena Behtar hai).

Note : If transfer function consist only poles at origin then asymptotic plot represent root locus. Eg $GH(s) = \frac{k}{s}$ or $\frac{k}{s^2}$.

Graphical Calculation of 'k' :-
(Tab use karna hai Jab valid point ho)
For Example : Find the value of k at $s = -1+j$

$GH(s) = \frac{K}{s(s^2+7s+12)}$



$$K = \frac{\sqrt{2}\times\sqrt{5}\times\sqrt{10}}{1} = 10$$

Note : $$\boxed{K = \frac{\text{Product of length from given point to poles}}{\text{Product of length from given point to zeros}}}$$

If transfer function does not contain any zero, then we will consider unity distance or length.

$GH(s) = \frac{K}{(s+a)^n}$ type OLTF Bhi Asymptotic line pe Banega R.L qki unka B.A.P aur centroid coincide karte hai.

Angle of Departure :

$$\boxed{\phi_d = 180° - \phi}$$ **

$\phi = \Sigma\phi_{pole} - \Sigma\phi_{zeroes}$

$\Sigma\phi_p$ = Sum of all angles contributed by remaining poles.

$\Sigma\phi_z$ = Sum of all angles contributed by zeroes.

*** Angle of departure is only valid for "complex conjugate poles".

- magnitude of Real Break Point > magnitude of centrode, then Angle of departure $< \pm 90°$
- magnitude of Real Break-point < magnitude of centroid then Angle of departure $> \pm 90°$
- magnitude of Real Break point = magnitude of centroid (σ) then Angle of departures $= \pm 90°$

Note: agar humko kabhi bhi niche wale conjugate pair pe Angle nikalne bola to bhi hum upar wale pe nikalenge aur bus sign reverse kardenge.

Angle of Arrival :

$$\boxed{\phi_a = 180 + \phi}$$ *

$\phi = \Sigma\phi_{poles} - \Sigma\phi_{zero}$

- Angle of arrival is only valid for "complex conjugate zeroes".

Note: Angle contribute kerate samay hamesa anti-clockwise angle lena hai.

→ kabhi critical denoted ke lie 'k' value pucha to root locus ke B.A.P pe 'k' ka value nikalenge.

- kabhi bhi root locus ke diagram me koi bhi branch apne asymptotic line ko cross nai karega but koi dusra branch dusre ke asymptotic line ko cross kar skta hai.

Inverse Root Locus : (Jo chiz yaha likhunga wahi differ karega baki sab Root-locus type hoga).

(1) Angle of Asymptote :-

$\angle A = \dfrac{2\alpha \times 180°}{P-Z}$ $(P>Z)$

$\angle A = \dfrac{2\alpha \times 180°}{Z-P}$ $(Z>P)$

$\angle A$ = does not exist $(P=Z)$.

(2) Condition for existence of Real Point in Inverse Root locus. Addition of Poles and Zeroes should be even number in the RHS of that point.

(3) condition for existence of complex point :- Even multiple of 180°

(4) Angle of Departure :

$$\boxed{\phi_d = -\phi}$$

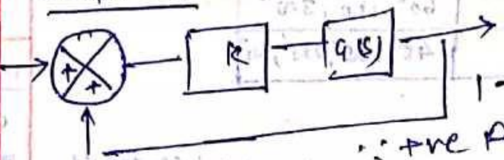
$\phi = \Sigma\phi_P - \Sigma\phi_Z$

(5) Angle of Arrival :

$$\boxed{\phi_a = \phi.}$$

Range of K :
$-\infty < K < 0$

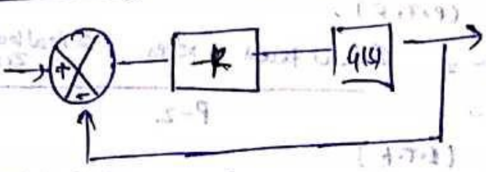
Concept 1 :



$1 - KG(s) = 0$

∴ +ve Feedback.

Concept 2 : $0 < K < \infty$



CE : $1 + G'(s) = 0$

$1 - KG(s) = 0$.

Note: Root locus ke question me $G(s) = \frac{N}{D-N}$ wale tarike se G(s) nai nikalenge agar T(s) diya rahega to. T(s) ko D^r c/s $CE = 1 + G(s)$ hota hai to T(s) ko D^r C/E hota hai usko $1 + G(s)$ ke form me likhenge aur phir G(s) nikalenge.

For eg : $T(s) = \dfrac{K}{s^2 + (4-K)s + 3}$

$CE = s^2 + 4s - Ks + 3 \Rightarrow 1 - \dfrac{Ks}{s^2+4s+3}$

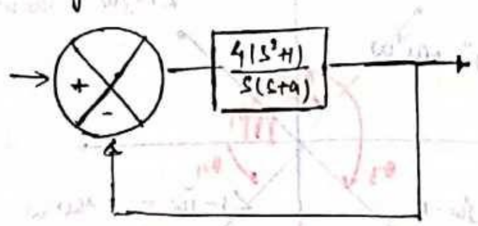
$G(s) = \dfrac{-Ks}{s^2+4s+3}$

Note: In Inverse Root locus, Right most zero with P.T.F consist Break in Point.

Keypoint: In RL and IRL are approximate Exponential functions like Routh Hurwitz Stability Criteria. For linear approximation $\boxed{e^{-s} \simeq 1-s}$.

Special Note: Root locus me hamesa OLTF ke Poles Immobile hone chahi, na ki variable. Jabbhi OLTF me pole variable aye to hum CLTF nikalenge aur uske D^r ko $1+G(s)$ se equate karenge aur $G(s)$ nikalenge.

For eg:-



$$1+G(s)=0$$

$$1+\frac{4(s^2+1)}{s(s+a)}=0$$

$$s^2+as+4s^2+4=0$$

$$5s^2+as+4=0$$

$$1+\frac{as}{5s^2+4}=0$$

$$G(s)=\frac{as}{5s^2+4}.$$

Non Minimum Phase System:-
It consist at least one pole or zero in RHS of s-plane. Mostly Right hand zero system is considered as NMPS.

Minimum phase system:-
Minimum Phase System (MPS) consist all poles and zeroes in the left hand side of s-plane.

Effect of Addition of Poles in Root Locus:-

(1) R.L shift towards the imaginary axis
(2) Relative stability of system decreases.
(3) system becomes more oscillatory in nature.
(4) Range of 'K' for stability decreases.
(5) Damping ratio decreases.
(6) MPO ↑, Hence system becoms less stable.
(7) ω_d ↑
(8) t_r, t_p and t_d ↓, transient Response is improved.

Effect of addition of zeroes of Root locus:-

(1) Result of Effect of Addition of zeroes is Exactly opp. to addition of poles.
(2) Relatively increases
(3) System becoms less oscillatory in nature.

Keypoint: addition of poles and zeros should be in LHS of s plane.

Note: (1) kabhi bhi root locus me $(1-s)$ jese terms aye to usko $(s-1)$ me convert karna hai. Inverse Root locus ke tricky questions me kai use hota hai.

(2) agar kabhi Inverse root locus dike, T(s) pucha to $T(s)=\frac{G(s)}{1-G(s)}$ hoga kyuki Positive feedback me hota hai I.R.L agar K ko −K nai banaya hai question me.

Question to be done from chapter 4:-
Gate: 11, 37, 47, 48, 52
ESE: 7, 64,

Questions to be done from chapter 5: (Root locus):-
Gate: 3, 17, 22, 23, 25, 27, 29, 31, 32, 34, 41, 42, 43

Question to be done from chapter 6 (Polar and Nyquist)
Gate:- 1, 5, 8, 9, 13, 14, 15, 21, 22, 23, 38, 39, 26, 32, 42, 40, 43, 44, 45, 49,
55, 56, 58, 52, 63, 68,
IES; 3, 4, 21, 29, 34, 37

Question to be done from Bode plot:-
Gate:- 7, 10, 13, 16, 17, 26, 28, 29, 30, 34, 33,
IES:- 7, 10, 32,

Question to be done from chapter 8
Gate: 2, 3
IES:- 2, 3, 11

Polor and Nyquist Plot

→ Nyquist plot determine the absolute stability.

→ Polar plot determine the Relative stability.

→ maximum Angle of a pole is $-90°$.

→ maximum angle of a zero is $+90°$.

Plot	Variable	Range
RL	K	$0<K<\infty$
CRL	K	$-\infty<K<0$
Root Contour	K	$-\infty<K<\infty$
Polar Plot	ω	$0<\omega<\infty$
Nyquist Plot	ω	$-\infty<\omega<\infty$

→ Jahan se bhi plot suru hoga wo point $\omega=0$ hoga aur jahan end hoga wo point $\omega=\infty$ hoga.

→ agar Im axis me ω ki value chahi to Given $GH(s)$ me D^r me Real part ko zero kardo, but ye tabhi tak karna Jabtak $GH(s) = \frac{\text{constant wale form me}}{D^r}$ hai agar $GH(s) = \frac{N^r}{D^r}$; $N_r \neq$ constant aya to bhi rationalize karenge aur fir Real Part = 0 karenge. aur ue ω ki value pe kitna value aega $GH(s)$ ke lie Real part ko Zero karne ke Bad jo $GH(s)$ bachega usme ω ki value dalenge.

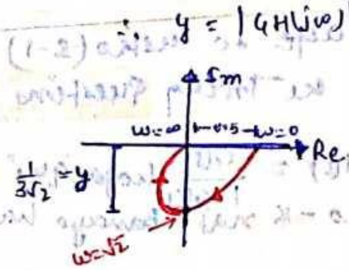
For Eg: $GH(s) = \frac{1}{(s+1)(s+2)}$

$$GH(j\omega) = \frac{1}{(j\omega)^2+3j\omega+2} = \frac{1}{-\omega^2+3j\omega+2}$$

$Re\,[GH(j\omega)=0]$

$\omega = +\sqrt{2}$ ✓, $-\sqrt{2}$ ✗ ∴ for find y: $GH(j\omega)|_{\omega=\sqrt{2}} = \frac{1}{j3\sqrt{2}} = \frac{-j}{3\sqrt{2}}$

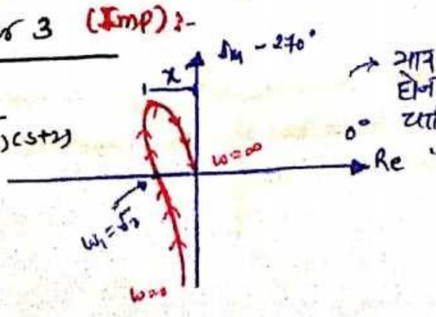
∴ $y = |GH(j\omega)| = \frac{1}{3\sqrt{2}}$ Ans.



→ Jab tak zero nai hai $GH(s)$ me tab tak to 100%. Type 0 system $0°$ se start hoga, Type 1 $-90°$ se start hoga, Type 2 $-180°$ se start hoga aur Type 3 $-270°$ se chalu hoga but zero innclue hue to 99% Jagah ye valid hoga But 1% Jagah valid nai hoga, wo age dekhoge.

Type 1, order 3 **(Imp)**:-

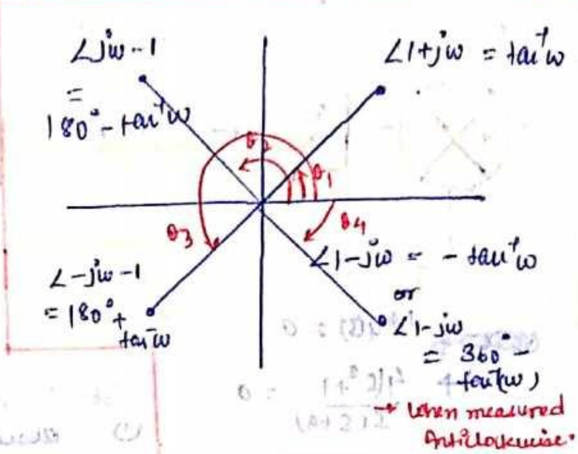
$$GH(s) = \frac{10}{s(s+1)(s+2)}$$



Note: (1) Imaginary zero karte samay j ke sath wale ko zero karta hai $-j$ common nai lena hai.

(2) order ≥ Type of system.

(3) agar kabhi $(s-1)$, $(-s-1)$ type wale pole agye to humara normal approach ki Type 0, 0 se start hoga vagera wala fail hojaega, isko solve karke hi bata paenge ki kahan se start hoga.



Special Note: "Representation of polar plot is not unique because we can obtain same polar plot for different OLTF."

Relative Stability Parameters:

1. Phase crossover frequency (ω_{pc}):-

→ The frequency at which phase of OLTF will be $-180°$ is referred as phase cross over frequency.

$$\angle GH(j\omega_{pc}) = -180°$$
$$\angle GH(j\omega)|_{\omega=\omega_{pc}} = -180°$$

OR

→ The frequency at which Imaginary part of OLTF will be zero is referred as phase crossover frequency.

$$Im\,[GH(j\omega)]|_{\omega=\omega_{pc}} = 0$$

Eg(1) $GH(s) = \frac{1}{s(s+1)(s+2)}$

Method 1:

$$\angle GH(j\omega) = -90° - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

$$-180°+90° = -\tan^{-1}\omega_{pc} - \tan^{-1}\frac{\omega_{pc}}{2}$$

$$90° = \tan^{-1}\omega_{pc} + \tan^{-1}\frac{\omega_{pc}}{2}$$

Method 2 : $GH(s) = \frac{1}{s^3+3s^2+2s}$

$GH(j\omega_{pc}) = \frac{1}{-j\omega_{pc}^3 - 3\omega_{pc}^2 + 2j\omega_{pc}}$

$Im\,[GH(j\omega_{pc})] = 0$

$-\omega_{pc}^3 + 2\omega_{pc} = 0$ (qki $\omega = 0$ imaginary me hai according to polar plot)

$\omega_{pc} = 0, \sqrt{2}, -\sqrt{2}$ (ω cannot be -ve).

**

Method 2 tabhi valid hai jab Polar Plot 180° ko cross karegi.

Special Note : For second order system (w/o zeroes) $\omega_{pc} = \infty$

Kabhi bhi $\omega_{pc} = 0$ aya to tabular table me check karna hai ki $\omega = 0$ pe kitna angle hai.

aur yad rakhna $\tan 180° = -0$.

Special Note : For type 2 system (w/o zeroes) $\omega_{pc} = 0$

2. Gain crossover frequency (ω_{gc}) :-

The frequency at which gain or magnitude of OLTF will be unity or 0 dB.

$|GH(j\omega_{gc})| = 1.0$

$|GH(j\omega)|_{\omega=\omega_{gc}} = 1.0$

→ ω_{gc} and ω_{pc} are positive real numbers.

Concept for directly finding whether ω_{gc} will exist or not :

$GH(s) = \frac{10}{(s+1)(s+2)}$

$|GH(j0)| = 5$

$|GH(j\infty)| = 0$

5 se 0 jane me 1 aega bich me isliye ω_{gc} exist karega

→ Isi tarah $\omega = 0$ pe angle aur $\omega = \infty$ pe angle ke bich 180° ya -180° aya to ω_{pc} bhi exist karega.

3. Gain Margin (GM) : It is defined as the reciprocal of magnitude of OLTF at ω_{pc}

$$GM = \frac{1}{|GH(j\omega_{pc})|}$$ **

→ So of ∞ esa no. hai jiska log bhi ∞ ajata hai.

$\log 0 = -\infty$

$\log \infty = \infty$

4. Phase Margin :-

$$PM = 180° + \angle GH(j\omega_{gc})$$

If ω_{gc} does not exist, that means P.M also does not exist.

- ω_{pc} and GM are related whereas ω_{gc} and PM are related.

Concept :

$\angle GH(j\omega_{gc}) = ?? \rightarrow PM = 180° + \angle GH(j\omega_{gc})$

$\angle GH(j\omega_{pc}) = -180°$ — fixed.

$|GH(j\omega_{pc})| = ? \rightarrow GM = \frac{1}{|GH(j\omega_{pc})|}$

$|GH(j\omega_{gc})| = 1.0 \rightarrow$ fixed.

$\angle GH(j\omega)$ ka range me dekh lena ki uske interval -180° ara ki hai, agar ara hoga to exist karega ω_{pc} varna nai (For w/o zeroes OLTF).

$$\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = 90°$$ *** ye tab use karege jab ω_{pc} nikalte rahenge aur niche sirf 2 pole rahega (except pole at origin).

Case : $GH(s) = \frac{K}{s(s+a)(s+b)}$ (esi koi bhi GH(s) lelie hai)

(1) For GH(s), let ω_{pc} be ω_{pc} and G.M be GM

then for $GH(s) = \frac{K'}{s(s+a)(s+b)}$, ω_{pc} will be ω_{pc} but $GM \neq GM$.

** → matlab OLTF ke N^r me jo constant rehta hai usko badal bhi die to ω_{pc} nai badlega but GM badal jaega.

Concept 1:

$$GH(s) = \frac{K}{s(s+1)(s+2)}$$

If K = change

(1) ω_{pc} = No change

(2) ω_{gc} = changed

(3) GM = changed

(4) PM = changed

Concept 2:

GM	PM	Stability
GM > 1 or GM > 0 dB	PM > 0°	Stable
GM = 1 or GM = 0 dB	PM = 0°	Marginal stable
GM < 1 or GM < 0 dB	PM < 0°	Unstable

Keypoints: (1) If ω_{gc} occurs earlier than ω_{pc} then CLTF will be stable. [i.e. $\boxed{\omega_{gc} < \omega_{pc}}$]

because as we move on ω value ↑, matlab jo phle aya uska value kum hoga aur jo bad me aya wo jada value hoga.

(2) If ω_{pc} occurs earlier than ω_{gc} then CLTF will be unstable. [$\omega_{pc} < \omega_{gc}$]

(3) If ω_{pc} and ω_{gc} occurs simultaneously then CLTF will be marginal stable. [$\omega_{pc} = \omega_{gc}$].

Hum GM, PM vagera se CLTF ki stability batate hai.

(i) GM > 0 dB and PM < 0° (ii) GM < 0 dB and PM > 0°

Not possible.

Calculation of gain margin using Root locus:-

Note: ye concept tabhi applicable hai jab Kmarginal exist karra ho given OLTF ke RoL se.

$$\boxed{GM = \frac{K_{marginal}}{K_{desired}}}$$

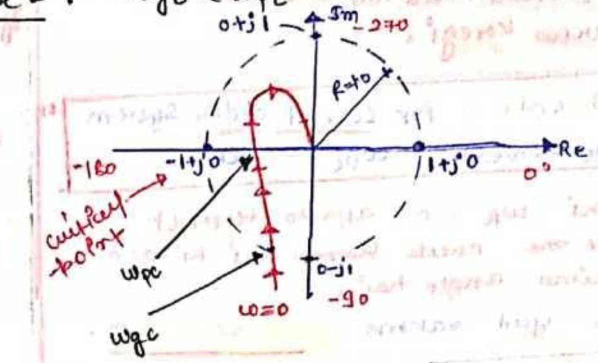
Kdesired Kmax nai kerdung hai.

yaha se jo GM aega wo dB me nai aega.

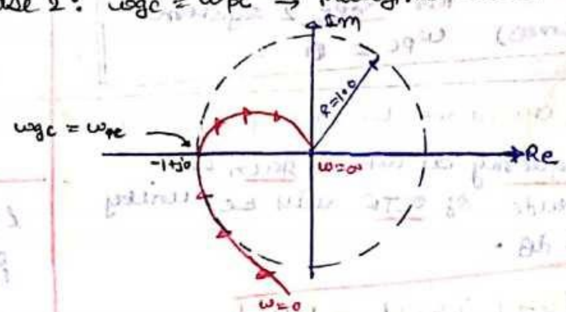
For eg: $GH(s) = \frac{1}{s(s+1)(s+2)}$ ← ye Kdesired hai.

ab $GH(s) = \frac{K}{s(s+1)(s+2)}$ likhenge aur phir Kmax nikalenge, 3rd order hai to Routh se asani se nikal lenge agar exist kia to aur phir $\left(GM = \frac{K_{max}}{1}\right)$ ∵ Kdesired = 1).

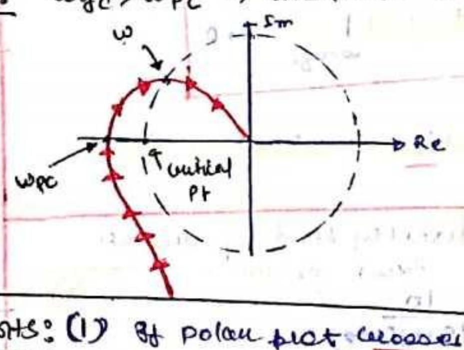
Case 1: $\omega_{gc} < \omega_{pc}$ → stable case



Case 2: $\omega_{gc} = \omega_{pc}$ → marginal stable case.



Case 3: $\omega_{gc} > \omega_{pc}$ → unstable case:



Keypoints: (1) If polar plot crosses −180° line before critical point (−1+j0), then CLTF will be stable.

(2) If polar plot crosses 180° line at critical point (−1+j0) then CLTF will be marginal stable.

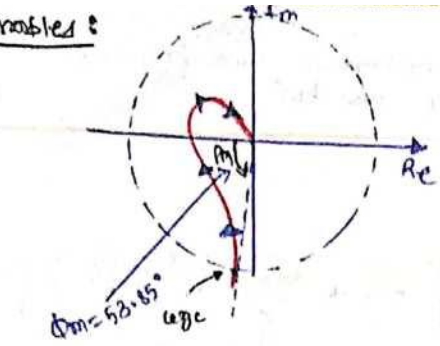
(3) If polar plot crosses −180° line after critical point (−1+j0), then CLTF will be unstable.

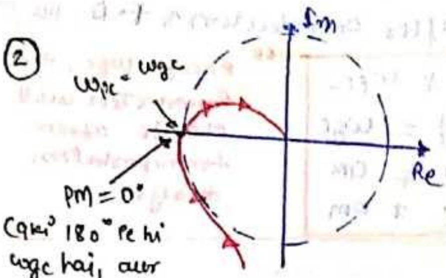
Graphical calculation of Phase Margin:-

(1) PM measure karne ke lie hamesa −180° line se measure karonge. Agar PM −180° ke niche ara hoga to use +ve lena hai aur agar −180° line ke upar ara hoga to PM ko −ve lena hai.

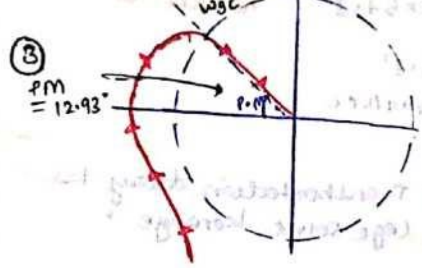
(2) ∠GH(jωgc) hamesa 0° line se measure karenge, agar clockwise lie to negative aega aur anticlockwise lie to positive aega.

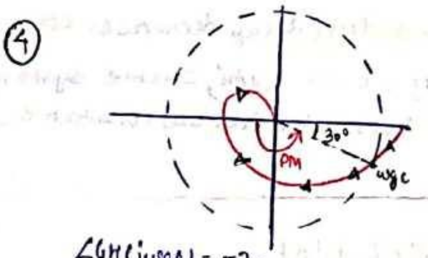
Examples :

① 

② 

(qki 180° pe hi ωgc hai, aur 180 line se hi hum PM naple hai)

③ 

④ 

$\angle GH(j\omega_{gc}) = -30$

$\therefore PM = 180 - 30$
$= 150$

Niche jane me magnitude ghat ra . matlab 'ω' badhra aur abhi table se ω_{gc} phle mila aur ω_{pc} bad me mila i.e $\omega_{pc} > \omega_{gc}$ stable condition .

$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
1.3	$-130°$
1.2	-140
1.0	-150
ω_{gc} 0.8	$-160°$
0.5 ω_{pc}	$-180°$
0.3	-200

Note : order 1 me ω_{pc} exist nai karega qki order me angle 0 to $-90°$ hota hai i.e $-180°$ uske interval me nai ata. aur ω_{pc} exist nai karega to GM bhi exist nai karega.

+ stable system me GM does not exist ka matlab GM is ∞ or ∞ dB .

approach : agar question me OLTF dedia aur 4 option dedia GM and PM ko ++, +-, -+, -- karke to snap RH table se OLTF ka stability nikal ke option se answer dedunga .

Note : agar ω_{gc} does not exist aya for stable system, we can say $PM = \infty$.

(2) Positive frequency line plotting of Nyquist plot is referred as polar plot.

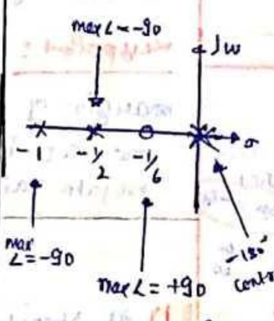
agar kabhi bole for $\omega \to 0^+$ ke lie Nyquist plot banao to hum polar plot draw karenge .

→ kabhi bhi humko ω ke kisi bhi value pe coordinates x intercept and y intercept nikalna hai to sidha sidha OLTF ka rationalise karo aur Re part or Im part alag karke ω ki kisi bhi value pe required chiz nikallo .

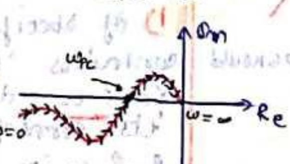
Introduction of Zeroes :

Jab zeroes ajate hai OLTF me to polar plot kesa banega wo table se batana thoda muskil hojata hai. To hum s plane me plot karlenge sare poles aur zeroes aur simultaneously ka pole / zero aur last ke poles / zeroes ka full contribution lenge .

For eg : $G(s) = \dfrac{1+6s}{s^2(1+s)(1+2s)}$



matlab $-180°$ se chalu hoga fir +ve angle add hoga to $-90°$ ki taraf bhagega but turant pole ke karan -ve angle add hoga to wapas $-180°$ ki taraf bhagega but last me pole add hoga to wo full contribution dega -90 toh $-270°$ pe jake rukega .



+ Jitne bar x-axis ko cut karega utna ω_{pc} aega .

Tip : Rationalise karte waqt phle chote poles ya zeroes ko multiply kare .

eg : $G(s) = \dfrac{1+6s}{s^2(1+s)(1+2s)} = \dfrac{(1+6s)(1-s)(1-2s)}{s^2(1+s^2)(1-4s^2)} = \dfrac{(1+6s)(1-3s+2s^2)}{s^2(1-s^2)(1-4s^2)}$.

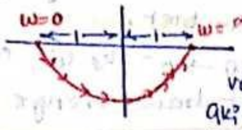
Note : agar idea nai lagra ki fig kesa hoga to Rationalise karke Im axis aur Real axis me ω nikal lo agar exist kie to matlab x axis aur y axis ko kat karega .

Concept : Late me zero add karne se Im ko cut karta hai polar plot . Late matlab pole se 10-12 unit bad and jaldi matlab 1-2 unit bad .

+ yad rakhna hai ki s plane wala sioftabhi bana ke check karenge jab stable OLTF hoga .

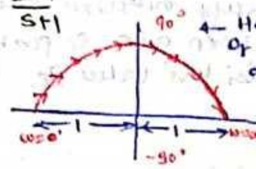
Note: (1) agar Improper Transfer function raha type 2 wala ie No. of zeroes = No. of poles excluding zeroes or poles at origin to Polar plot hamesa semicircle Banega.

Eg: (1) $GH(s) = \frac{s+1}{s-1}$



← same har w ki value loge hogi qki har w pe magnitude 1 hai

(2) $GH(s) = \frac{s-1}{s+1}$



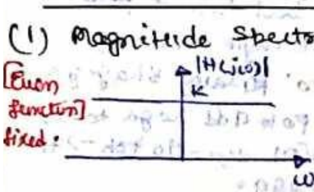
← Here all value of w are logic; at every value of w, magnitude is 1.

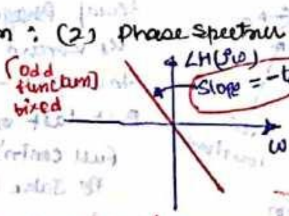
(2) Au agar proper transfer function hua with zeroes < poles and no poles and zeroes at origin to bhi semi-circle Banega.

Eg: $GH(s) = \frac{(s+2)}{(s+1)(s-1)}$

Transportation Delay :-

(1) Magnitude Spectrum : (2) Phase Spectrum





Keypoint: For distortionless Transmission o/p should be exact replica of i/p system.

Condition of distortionless transmission :

(1) magnitude spectrum should be constant i.e $|H(j\omega)| =$ constant.

(2) Phase spectrum should be linear functions of frequency.

Eg. $K = 1.0$, $t_d = 1$ sec, $H(s) = e^{-s}$.

$\Rightarrow H(j\omega) = e^{-j\omega}$

$|H(j\omega)| = 1.0$

$\angle H(j\omega) = -\omega\,(rad) = -\omega \times 57.3°$ degree. convert karna hai.

$H(s) = e^{-s}$

$H(s) \simeq 1-s$ for RH [Linear Approx]

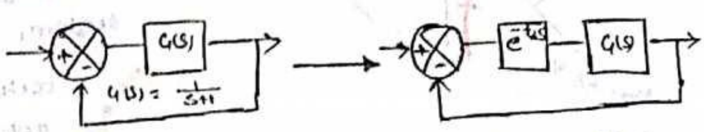
$H(s) \simeq 1-s$ for RL [Linear Approx]

$H(s) = e^{-s}$ for Polar and Nyquist Plot

(No approximation in case of Polar and Nyquist Plot as they use Both Phase and magnitude hence e^{-s} represent both magnitude and Phase)

→ agar optical type se plot dikha to samajh jana chahiye ki delay introduce hua i.e Transportation delay introduce hua hai.

Concept :-



Before and after introduction of T.D, we have

$\omega_{pc}' \neq \omega_{pc}$	
$\omega_{gc}' = \omega_{gc}$	
$GM' \neq GM$	
$PM' \neq PM$	

*** except ω_{gc}, all Parameters will change after transportation delay.

Eg: $GH(s) = e^{-s}$, Find $\omega_{pc} = ?$

$\angle GH(j\omega) = -\omega \times 57.3°$ — Transportation delay ka angle itna hi hota hai.

$\angle GH(j\omega_{pc}) = -\omega \times 57.3$

$-180° = -\omega \times 57.3°$

$\omega = 3.14$ rad/sec

Tip: agar kabhi Bhi Transportation delay ka sawal aya to phle ω_{gc} solve karenge.

keypoint: Transportation delay decreases the margin of stability. ~~kabhi~~ Kabhi, stable system me T.D introduce karne ke Bad wo unstable hojata hai.

Nyquist plot

(1) If specified region [S plane of G(s) or 1+G(s)] contains 'P' no. of poles and we are moving in C.W direction across that region then its corresponding frequency plane will Encircle its origin 'P' times in CCW/ACW direction (opp direction of C.W) and vice-versa.

(2) If specified region [S plane or G(s) or 1+G(s)] contain 'Z' no. of zeroes and we are moving in C.W direction across that region then its corresponding frequency plane will Encircle its origin Z times in C.W (same direction) and vice versa.

(3) If specified region contain 'P' no. of poles and 'Z' no. of zeroes and we are moving in C.W direction then its corresponding frequency plane will Encircle its origin (P-Z) times in CCW direction provided P>Z and (Z-P) times in CW direction provides Z>P and vice versa.

compensator aur controller aur Bode plot me bhi approximation nai chalega.

Concept 1 :

$N = P - Z$ [Assume s-plane movement = CW]

(i) P = No. of poles of $G(s)$

(ii) Z = no. of poles of $G(s)$

(iii) N = no. of encirclement about origin in frequency plane of $G(s)$ [CCW]

Note: agar $N = -ve$ aaya, to $N = +ve$ karenge par direction CCW ki jagah CW hojaega.

Concept 2 : $N = P - Z$ [Assume s-plane movement = C.W]

(i) P = No. of poles of RHS $1+G(s)$

(ii) Z = No. of Zeroes of RHS $1+G(s)$

(iii) N = no. of encirclement about origin in frequency plane of $1+G(s)$ [CCW]

Concept 3 : isi ko use karna hai

$N = P - Z$ [Assume s-plane movement = C.W]

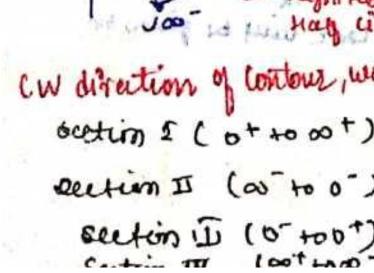
(1) P = No. of RH Pole of $1+G(s)$ = No. of RH Pole of $G(s)$.

(2) Z = No. of RH Zeroes of $1+G(s)$ = No. of RH Poles of CLTF

(3) N = no. of encirclement about critical point $(-1+j0)$ in frequency plane of $G(s)$ [CCW]

Note: Agar $1+G(s)$ or $G(s)$ ke frequency plane pe jaye to at the same time jitne baar $1+G(s)$ ka frequency plane origin ko encircle karra hoga utne baar $G(s)$ ka frequency plane critical point ko encircle karra hoga.

Nyquist contour :-



Section I : Positive frequency line.

Section II : Negative frequency line.

Section III : Zero frequency line.

Section IV : ∞ frequency line.

CW direction of contour, we have assumed it.

Section I (0^+ to ∞^+) → Polar plot

Section II (∞^- to 0^-) → mirror image of Polar Plot

Section III (0^- to 0^+)

Section IV (∞^+ to ∞^-)

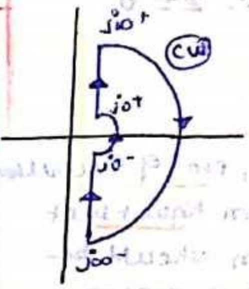
- No. of ∞ magnitude half circle $= m - n$ ($m > n$) * rare case or special case.

m = no. of zeroes

n = no. of poles

- No. of ∞ magnitude half circle i.e. of frequency line circle = no. of poles at origin. [CW direction].

Note: If there is no information of direction of Nyquist contour then by default we will consider CW direction of contour.



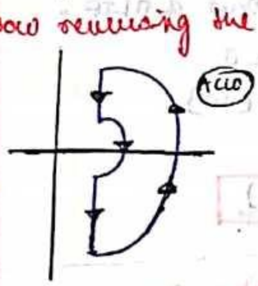
$N = P - Z$

P = No. of RH Pole of OLTF.

Z = No. of RH Pole of CLTF

$N \to -1 + j0$ (CCW / ACW)

Now reversing the direction of contour i.e. ACW.



$N = P - Z$

$N = -1 + j0$ [CW]

→ Contour ke 4eg dimag me yad rakhna hai.

→ Mirror image of P.P ke analysis me bus P.P ke data me angle ka sign badal dena hai aur ∞^- to 0^- karna hai.

Eg : $GH(s) = \frac{10}{s(s+1)}$

Analysis of P.P

ω	$\lvert GH(j\omega)\rvert$	$\angle GH(j\omega)$
0^+	∞	$-90°$
∞^+	0	$-180°$

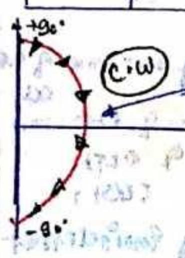
Analysis of Mirror of P.P :

ω	$\lvert GH(j\omega)\rvert$	$\angle GH(j\omega)$
∞^-	0	$+180$
0^-	∞	$+90$

→ Analysis of section III :

ω	$\lvert GH(j\omega)\rvert$	$\angle GH(j\omega)$
0^-	∞	$+90°$
0^+	∞	-90

upper table ke info utha ke is table me dal diye aur exam me bhi esi karna hai.



No. of Infinite magnitude Half circle = 1 = No. of poles at origin.

Jab tak Proper transfer function rahega, tab tak Section 4 me bus ek • banega $j0$ pe ∞ % case me hoga.

Concept :-

$GH(s) = \frac{10}{s(s+1)}$

$P = 0$ ($\because$ No RHS poles of GH)

$1 + GH(s) = 0$

$s^2 + s + 10 = 0$

s^2	1	10
s^1	1	0
s^0	10	0

$\boxed{\text{No of sign change} = \text{No. of RHS poles of CLTF} = Z}$

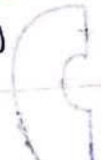
Here no. of sign change = 0. $\therefore Z = 0$

$N = P - Z$
$= 0 - 0$
$N = 0$ [CCW].

Note: For stability of CLTF, No. of encirclement about critical point from Nyquist plot of OLTF G(s) in CCW direction should be equal to No. of RHS pole of OLTF.

$\therefore$ For stability ; $Z = 0$

[CCW] $N = P - Z$ [CCW]



If $Z = 0$

$\boxed{N = P \text{ [CCW]}}$

Special note: Jab Jab Improper transfer function aega, ∞ frequency line ka importance badh jaega kyuki 0 wala circle to nai banega kuch finite ya infinite wala circle banega. Aur jab origin pe pole nai aega to zero frequency line ka bhi importance zero hojaega.

Concept: $GH(s) = \frac{s-1}{s+1}$

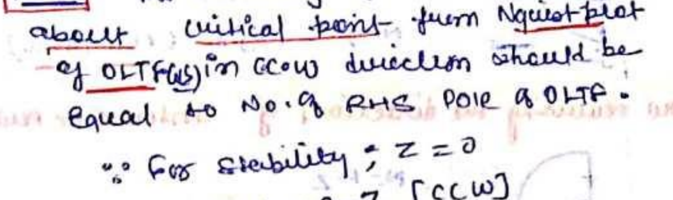


fig (1) s plane of 1+GH(s)

(1) P = No. of RHS pole of OLTF

(2) Z = no. of RHS pole of CLTF

(3) $N = -1 + j0$ encirclement about c.p. [CCW]

P = No. of RHS pole of OLTF G(s)

Z = no. of RHS zero of OLTF (G(s)),

N = No. of encirclement about origin (in CCW)

→ Agar question critical point ke around aya to concept se jaye aur agar aya to origin ya kisi aur pt. pe around aya to Nyquist plot bana le. (yabhi abhi jo concept likhe hai left me bottom of page me usko command me lao but kahin kahin pe ye kam nai karta, wo example is chapter ke last me dalunga).

→ Agar stable OLTF bol dia, to $P = 0$ hojaega formulae me.

→ Agar koi information nai hai OLTF ki stability se related to by default we will consider stable OLTF i.e $P = 0$.

→ Nyquist plot is symmetrical about real axis. i.e mirror real axis pe rakhenge symmetry lete time P.P ka.

→ polar plot unique nai hota hai par jo figure dia rehta hai uska hum minimum phase system lete hai i.e w/o coordinate change karne se jo banega.

→ In MPS system, all poles and zeroes are on LHS of s-plane.

→ Loop state wagk dhyan dena hai loop me ek bhi node repeat nai hona chahi except jisse start kie the uske alawa.

Ex - A B C A ✓ loop
C C ✓ loop
A B C A C B ✗ Not loop
A C C A B ✗ Not loop
C A B C A ✗ Not loop.

→ Proper transfer function ke case me ∞ frequency line bus ek dot dega. i.e. direct zero frequency line ke figure ke bad Nyquist plot draw karenge.

How to Handle 'K' wale questions :

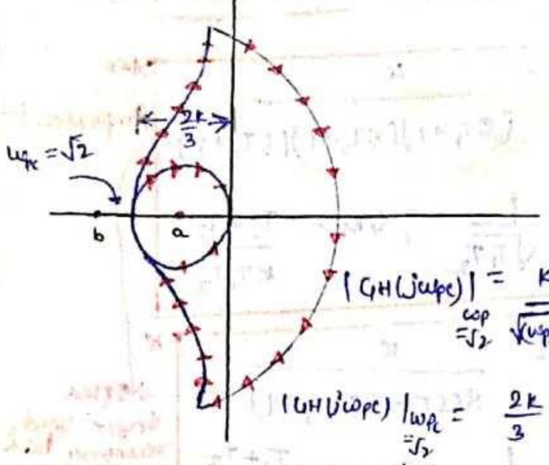
Q. Draw Nyquist plot for given OLTF,

$GH(s) = \frac{Ks^3}{(s+1)(s+2)}$. Determine range of 'K' for stability of OLTF.

⇒ No. of zeroes = 3

No. of poles = 2

→ Improper transfer function, No importance of 0 frequency line. care will be given to ∞ frequency line.



$$|GH(j\omega_{pc})|_{\omega_{pc}=\sqrt{2}} = \frac{K\omega_{pc}^3}{\sqrt{(\omega_{pc}^2+1)(\omega_{pc}^2+4)}}$$

$$|GH(j\omega_{pc})|_{\omega_{pc}=\sqrt{2}} = \frac{2K}{3}$$

Case 1: Assume C.P at 'a'.
i.e. Distance from origin to a is 1

and $\frac{2K}{3} > 1$

$K > \frac{3}{2}$

$N = P - Z$ (CCW)

$P = 0$

$N = 2$ (CCW)

$\boxed{Z = 2}$ i.e for $K > \frac{3}{2}$, there will be 2 poles of CLTF in RHS side, Hence system will be Unstable.

Case 2: Assume C.P at b.
Distance b/n origin and b is 1

and $\frac{2K}{3} < b$

$\therefore \frac{2K}{3} < 1$

$K > \frac{3}{2}$

$N = P - Z$ [CCW]

$P = 0$

$N = 0$

$\boxed{Z = 0}$

This mean for $K < \frac{3}{2}$, the no. of RHS poles of CLTF are there, Hence system will be stable.

Hence range of CLTF stability will be $\boxed{0 < K < \frac{3}{2}}$

[By RH table se bhi ja sakte hai.]

- Agar OLTF deke conditional stability puchha to, RH table se jada clean hoga. But indirectly puchha to phir to Nyquist se hi jana padega.

- Agar question me OLTF ke bare me mention nai hai to hum hamesa Proper transfer system lenge aur stable manenge aur ek PTF hamesa origin pe end hota hai for $\omega = \infty$ pe magnitude zero hoga.

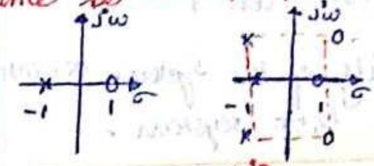
Concept: $N = 1$ [CCW] $+ 1$ [CW]
$= 1$ [CCW] $- 1$ [CCW]
$= 0$.

Q.28
2005
GATE
revise
from copy.

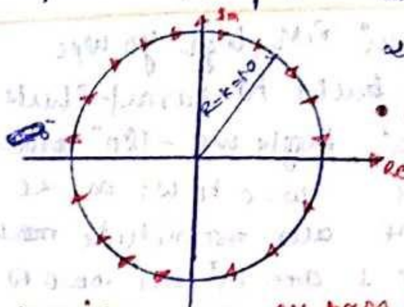
All pass filter Nyquist plot :-

Distribution of roots/poles and zeroes about origin symmetrically is called all pass filter. matlab har ek pole ka same distance pe zero ke form me image hoga.

Eg:



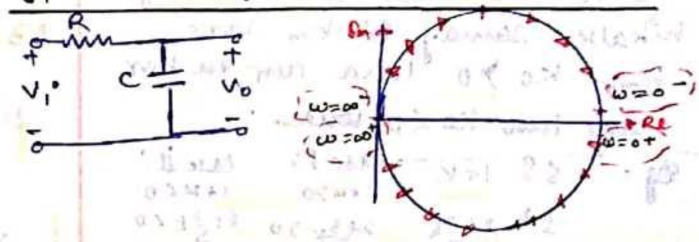
- In case of All pass filter, RHand zeros and left-hand poles are distributed symmetrically about origin.
- In case of all pass filter, magnitude spectrum will be constant.



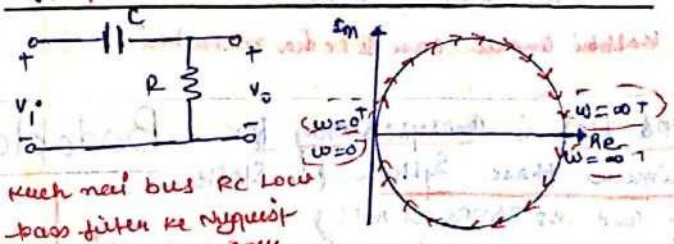
Nyquist plot of All pass filter.

- We know that Nyquist plot is always symmetrical about Real axis. But in case all pass filter, it will also be symmetrical about origin [locus of circle with a centre (0,0)].

Nyquist plot of RC Low pass filter :



Nyquist plot for RC High pass filter :



Kuch nai bus RC Low pass filter ke Nyquist plot se frequency opp. kardie hai

- Revise Q 44 and 46 from copy, also Q. 58 and 40.
- ** Minimum phase system (all poles and zeroes on left hand side) me jisme no. of zeroes = no. of poles rahenge uska Nyquist plot circle hi banega wo bhi right side me.

Relative Stability Parameter for Positive feedback System :

$|GH(s)| = 1.0$ (Magnitude condition)

$\angle GH(s) = 0°$ or $360°$

$\angle GH(\omega_{pc}) = 0°$

$|GH(j\omega_{gc})| = 1$ $\quad PM = \angle GH(j\omega_{gc})$.

$GM = \frac{1}{|GH(j\omega_{pc})|}$

Effect of time delay :-

(1) $\omega_{pc} \downarrow$ (3) ω_{gc} unchanged

(2) $GM \downarrow$ (4) $PM \downarrow$

(5) $MPO \uparrow$ (6) $t_{sett} \uparrow$

(7) Relative stability of the system decrease in the closed loop system.

Note: Kabhi bhi P.M, ω_{gc} ya ω_{pc} ya GM puche to turant check karna ki Angle me $-180°$ ara ki hai $\omega=0$ to $\omega=\infty$ ke range me aur magnitude ke range me 1 ara ki hai $\omega=0$ to $\omega=\infty$ me.

agar nai aya to turant ∞ answer dedenge accordingly.

→ Kabhi R-H Table me solve karte waqt Ist coloumn me sare term me K hai to K ka range nikalte samay ek bar sare term ko >0 lena aur ek bar sare term ko <0 lena.

Eg:

		Case I:	Case II:
s^3	$1+K$	$1+K>0$	$1+K<0$
s^2	$2+\frac{3}{7}K$	$2+\frac{3}{7}K>0$	$2+\frac{3}{7}K<0$
s^1	$1+3K$	$1+3K>0$	$1+3K<0$
s^0	$K+2$	$K+2$	$K+2<0$

Qki kabhi answer case II se dia rehta hai!!

Phase crossover frequency and gain margin for standard third order transfer functions :- ★★★ Important

(1) $$G(s)H(s) = \frac{K}{(sT_1+1)(sT_2+1)(sT_3+1)}$$

$$\omega_{pc} = \frac{1}{\sqrt{T_1T_2}} \; ; \; GM = \frac{T_1+T_2}{KT_1T_2}$$

Iska style yad rakhna hai

(2) ★ $$GH(s) = \frac{K}{s(sT_1+1)(sT_2+1)}$$

$$\omega_{pc} = \frac{1}{\sqrt{T_1T_2}} \quad GM = \frac{T_1+T_2}{KT_1T_2}$$

above formulal is not valid for $GH(s) = \frac{K}{(sT_1+1)^3}$

Is type ke liye $GM = \frac{K_{marg}}{K_{desire}}$ use karo Best hoga.

(3) $$GH(s) = \frac{K}{s(sT_1+1)^2}$$

$$\omega_{pc} = \frac{1}{\sqrt{T_1T_2}} \quad GM = \frac{T_1+T_2}{KT_1T_2}$$

$T_1 = T_2$

$$\omega_{pc} = \frac{1}{T_1} \; ; \; GM = \frac{2}{KT_1}$$

Bodeplot

$$\text{Slope} = \frac{Y_A - Y_B}{\log\omega_B - \log\omega_A}$$ dB me rehne chahiye.

→ Bode plot is analysed only for Minimum phase system (i.e stable OLTF and no zeroes on RHS)

$|G(j\omega)|$ — ω axis me jo bhi value hogi wo 'ω' ki hogi na ki $\log\omega$ ki.

→ But ye axis logarthmic hota hai, slope nikalte samay $\log\omega_2 - \log\omega_1$ karte hai.

→ $\frac{\omega_2}{\omega_1} = 2$ → octave frequency

$\frac{\omega_2}{\omega_1} = 10$ → Decade frequency.

→ ★★★

$6\,dB/oct = 20\,dB/dec$

$12\,dB/oct = 40\,dB/dec$

$-24\,dB/oct = -80\,dB/dec$

Conclusion:

(1) Pole ke karand hamesa slope negative aega.

(2) Zeroes ke karand slope hamesa positive aega.

(3) • agar $K<1$ hai Tab bode plot niche shift hoga $20\log K$ se wrt Bode plot at $K=1$

• agar $K>1$ hai Tab bode plot upar shift hoga $20\log K$ se wrt Bode plot at $K=1$.

GH(s)	ω'	Slope
$\frac{K}{S}$	$\omega' = K$	−20dB/dec
$\frac{K}{S^2}$	$\omega' = K^{1/2}$	−40dB/dec
⋮		
$\frac{K}{S^n}$	$\omega' = K^{1/n}$	−20×n dB/dec

$\omega' = \omega'$ is defined as the value of ω at which Gain of Bode magnitude plot will be zero (0) dB.

- 'n' pole ka slope −20×n dB/dec hoga.

GH(s)	ω'	Slope
KS	$\omega' = \frac{1}{K}$	+20 dB/dec
KS²	$\omega' = \frac{1}{\sqrt{K}}$	+40 dB/dec
⋮		
KS^n	$\omega' = K^{-1/n}$	+20×n dB/dec

Definition of corner frequency:

The frequency at which slope of Bode magnitude plot and phase of Bode phase plot will change is referred as corner frequency.

Bode plot of 1st order system:

$$GH(s) = \frac{1}{1+S\tau}$$

Representation of Ist order system in Bode plot is

$$\boxed{GH(s) = \frac{K}{1+\frac{s}{\omega_c}}}$$

K — d.c gain of given function.

ω_c = corner frequency.

- Error in Bode magnitude plot is even symmetrical about corner frequency ω_c.
- Error will be maximum at corner frequency.
- For 1st order system maximum error will be −3 dB.
- Error in Bode phase plot is odd symmetrical about corner frequency.
- For 1st order system maximum error will be −45° at corner frequency.

Bode plot me stability ki bat bahot kum korte hai islie GH(s) exam me bahot kum likha rahega, koi bhi random T(s) dia rahega aur uska Bode plot banana rahega.

GH(s)	DC gain	ω_c	Max. Angle	E_{max}(dB) at ω_c	E_{max}(degree) at ω_c	Slope at high frequency
$\frac{1}{S\tau+1}$	K=1	$\frac{1}{\tau}$	−90°	−3dB	−45°	−20dB/dec
$\frac{1}{(S\tau+1)^2}$	K=1	$\frac{1}{\tau}$	−180	−6dB	−90°	−40 dB/dec
$\frac{1}{(S\tau+1)^n}$	K=1	$\frac{1}{\tau}$	−90×n	−3×n dB	−45°×n	−20×n dB/dec

GH(s)	DC gain	ω_c	max. Angle	E_{max}(dB) at ω_c	E_{max}(degree) at ω_c	Slope at high frequency
$1+S\tau$	K=1.0	$\frac{1}{\tau}$	+90	+3dB	+45°	+20dB/dec
$(1+S\tau)^2$	K=1.0	$\frac{1}{\tau}$	+180	+6dB	+90	+40dB/dec
$(1+S\tau)^n$	K=1.0	$\frac{1}{\tau}$	+n×90	+3×n dB	+45×n	+20×n dB/dec

Concept of DC gain in Bode plot:

$$GH(s) = \frac{K}{S\left(1+\frac{S}{\omega_{c_1}}\right)\left(1+\frac{S}{\omega_{c_2}}\right)}$$

written in Bode plot style.

DC gain = K

- Kisi bhi T(s) ya GH(s) ko Bode plot ki style me likhenge to Nr me jo constant hoga wo DC gain hoga.

eg: Find DC gain of $GH(s) = \frac{10s}{s+5}$.

$K_p = 0$
$K_v = 0$
$K_a = 0$

agar upar zero agya or Kp, Kv, Ka sab zero aya to Bode plot style me jaenge.

$$GH(s) = \frac{2s}{1+s/5}$$

Bode plot style

→ DC gain cannot be 0 and ∞.

$\therefore K = 2$ dc gain.

→ Kp, Kv, Ka wale proper transfer system ke lie applicable hai.

Recovery of Transfer function from Bode magnitude plot :-

Case 1 : If Low frequency Region or Initial Region is constant (flat) then

$$\boxed{20\log K = \text{magnitude of constant region.}}$$

Case 2 : If Low frequency Region or Initial region consist slope then,

$$\boxed{20\log K = \text{magnitude at } \omega = 1\text{ rad/sec}} \text{ (Conditional)}$$

Case 2.1 : Initial region = Negative slope

-20 dB/dec = 1 Pole at origin

-40 dB/dec = 2 poles at origin

Negative slope at Initial region represents no. of poles at origin.

Case 2.2 :- Initial region = positive slope

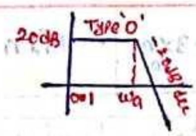
$+20$ dB/dec = 1 zero at origin

$+40$ dB/dec = 2 zero at origin

positive slopes at Initial region represent no. of zeroes at origin.

Calculation of Error coefficient from Bode plot :-

(1) In case of type '0' system, position error coefficient will be dc gain of Bode plot ($K_p = K$).

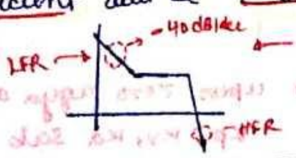


← Type 0 system.

(2) In case of Type '1' system velocity Error coefficient will be dc gain of Bodeplot ($K_v = K$)

← Type 1 System

(3) In case of type '2' system acceleration Error coefficient will be dc gain of Bode plot ($K_a = K$)



← Type 2 System

⇒ $\boxed{\omega' = \omega_{gc}}$ qki dono ka definition same hai i.e at this point gain of Bode magnitude plot will be 0 dB.

Error at corner frequency :-

$$\boxed{E(u=1) = -20\log 2\xi}$$

Concept :- To find K :

(1) $40 = 20\log K - 60\log 0.1$

$\omega = 0.1$ pe 40 dB magnitude hai aur us line ka slope -60 dB/dec hai.

(2) $-20 = 20\log K - 60\log 1$

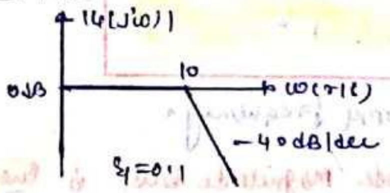
$\omega = 1$ pe -20 dB magnitude hai aur us line ka slope -60 dB hai.

Strict note : ye uparwala method se K bus low frequency region ko use karke nikalenge.

$$\boxed{\text{Time constant} = \frac{1}{\text{Corner frequency}}}$$

Questions Involving 'ξ' in Bodeplots:

Q) Find the Transfer function :-



$20\log K = 0$

$K = 1$.

$$G(s) = \frac{1}{1 + 2\times\xi\times\frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2}$$

$$= \frac{1}{1 + 2\times 0.1 + \frac{s}{10} + \left(\frac{s}{10}\right)^2}$$

$$= \frac{1}{1 + 0.02s + \left(\frac{s}{10}\right)^2}$$

→ ξ based system me -40 dB/dec, -80 dB/dec, -120 dB/dec slope hoga qki ξ based system 2nd order k lie define hoga aur uske integral multiple ke lie.

Q. Find the corner frequency of given Transfer function ;

$$G(s) = \frac{5(s+4)}{s(s+0.25)(s^2+4s+25)}$$

$$\Rightarrow \frac{20\left(\frac{s}{4}+1\right)}{0.25\,s\left(\frac{s}{0.25}+1\right)25\left(1+\frac{4s}{25}+\left(\frac{s}{5}\right)^2\right)}$$

$$= \frac{3.2\left(1+s/4\right)}{s\left(1+\frac{s}{0.25}\right)\left[1+\frac{4s}{25}+\left(\frac{s}{5}\right)^2\right]}$$

D.C gain ↑ ; Perfect style of Bode plot.

$\omega_c = 0.25, 4, 5$

Critical Damped System ($\xi=1$):-

Is case me Normally
Jesa Transfer function
milta hai vese hi milega
from Bode-plot.

Overdamped system ($\xi>1$):-

In Bode plot Dominant pole concept is not applicable.

$$T(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n+\omega_n^2} \qquad D.C = 1.0$$

$$T(s) = \frac{1}{\left(1+\frac{s}{\xi\omega_n-\omega_n\sqrt{\xi^2-1}}\right)\left(1+\frac{s}{\xi\omega_n+\omega_n\sqrt{\xi^2-1}}\right)}$$

Undamped Oscillation ($\xi=0$):-

$$T(s) = \frac{\omega_n^2}{s^2+\omega_n^2} \qquad D.C\ gain = 1.0$$

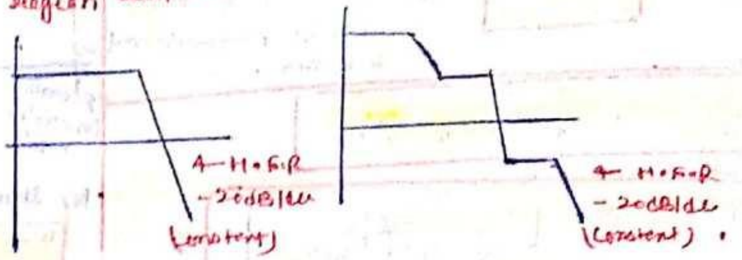
$$T(s) = \frac{1}{1+\left(\frac{s}{\omega_n}\right)^2}$$

$\boxed{\text{Corner frequency } (\omega_c) \neq \omega_n}$

Key point:

→ Bode plot is not valid for Undamped Oscillation.

→ In Bode magnitude plot, slope of high frequency region will be constant.



→ Bode plot is only valid for Absolutely stable system i.e minimum phase system (MPS).

→ Bode plot is an asymptotic plot of a constant slope at high frequency.

→ dB me Reciprocal lene se bus sign badalte hai.

i.e $|GH(j\omega_{pc})| = 10\,dB$

$$GM = \frac{1}{|GH(j\omega_{pc})|} = -10\,dB$$ ← Signed changed after taking reciprocal.

Error:

$\boxed{\text{Error} = \text{Exact} - \text{Approx}}$ **
(By correction) (By Bode plot)

Frequency Responce of Standard IInd Order System

$$T(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$T(j\omega) = \frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2+2j\xi\frac{\omega}{\omega_n}}$$

$\boxed{\mu = \frac{\omega}{\omega_n} = \text{Normalised frequency}}$

*** $\boxed{\mu_r = \frac{\omega_r}{\omega_n} = \sqrt{1-2\xi^2}}$ M_r: Normalised Resonant frequency.

ye wo Normalised frequency jisme maxm magnitude aega $|T(j\omega)|$ ka.

$\Rightarrow \boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}}$

ω_r: Resonant Frequency.

[The frequency at which magnitude of frequency response of standard 2nd order system will be maximum is referred as ω_r.]

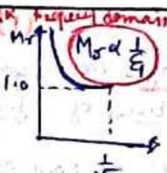
→ Range of ξ : $0 \le \xi \le \frac{1}{\sqrt{2}}$ in frequency domain of 2nd order system.

→ Range of ξ : $0 \le \xi \le 1$ in time domain analysis ∵ $\omega_d = \omega_n\sqrt{1-\xi^2}$

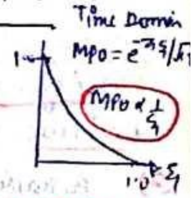
→ $\boxed{\omega_r \propto \frac{1}{\xi}}$ ***

$\boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}$ *** Frequency domain

Resonant peak. V.Imp formula.



Time Domain
$M_{po} = e^{-\pi\xi/\sqrt{1-\xi^2}}$



→ Resonant Peak describes the maximum magnitude of frequency Response of second order system at a Resonance frequency.

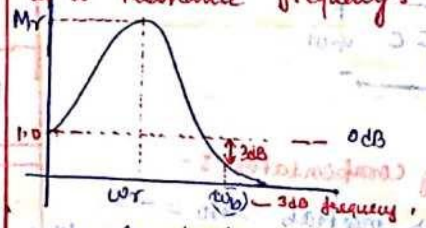


Fig: Second order LPF frequency Response

● 3dB lete hai same maximum value se 3dB niche ate hai na ki Peak value se i.e 1.0 se 3dB niche aenge na ki M_r se.

$T(s)$	ω_r	M_r
$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$\omega_n\sqrt{1-2\xi^2}$	$\frac{1}{2\xi\sqrt{1-\xi^2}}$
$\frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$\omega_n\sqrt{1-2\xi^2}$	$\frac{K}{2\xi\sqrt{1-\xi^2}}$ **

→ $\boxed{\mu_b = \frac{\omega_b}{\omega_n} = \text{Normalised 3-dB frequency}}$ **

$$\boxed{\omega_b = \omega_n \sqrt{1-2\xi^2 + \sqrt{4\xi^4 - 4\xi^2 + 2}}}$$

Note : $\boxed{\text{for } \xi = \frac{1}{\sqrt{2}},\ \omega_b = \omega_n}$

$\boxed{\omega_b \propto \frac{1}{\xi}}$ $\omega_r, \omega_d, \omega_b$ teeno hi inversely proportional to ξ hai.

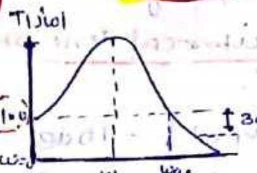
$$\boxed{\omega_{gc} = \omega_n \sqrt{-2\xi^2 + \sqrt{4\xi^4+1}}}$$

$$\boxed{P.M = \tan^{-1}\left[\frac{2\xi}{\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}\right]}$$

$$\boxed{\xi = 0.5\sqrt{\sin P.M \cdot \tan P.M}}$$

$\omega_{gc} \propto \frac{1}{\xi}$ and $P.M \propto \xi$

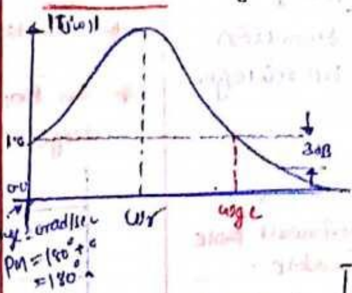
Special Note :



$\omega = 0$, pe jo value y-axis me hogi wo value K = DC gain ki hogi qki at $\omega = 0 \Rightarrow s = 0$ pe DC gain hi nikalta hai. Ya question pe dedi thi magnitude is 5 at $\omega = 0$, then $K = 5$.

Relative stability parameter for standard 2nd order system :-

$$T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$



$$|T(j\omega)| = \frac{\omega_n^2}{\sqrt{(\omega_n^2-\omega^2)^2 + 4\xi^2\omega^2\omega_n^2}}$$

$$\angle T(j\omega) = -\tan^{-1}\left[\frac{2\xi\omega\omega_n}{\omega_n^2 - \omega^2}\right]$$

$\phi = 0$ rad/sec

$PM = 180° + 0$
$= 180°$

For standard 2nd order ;

$\boxed{\begin{array}{l}\omega_{pc} = \infty \\ GM = \infty / \infty\ dB\end{array}}$

$\boxed{\omega_{gc} = 0}$

$PM = 180° + 0°$

$\boxed{PM = 180°}$

ω	$\lvert T(j\omega)\rvert$	$\angle T(j\omega)$
0 (ω_{gc})	1	0°
∞ (ω_{pc})	0	180°

Note : (1) Bode plot me bhi transportation delay ko approximate nai karna hai.

(2) Magnitude of the error at the corner frequency is
$= \boxed{\pm n(20\log 2\xi)}$ dB

COMPENSATOR AND CONTROLLER

* Compensator is a device which is used for improving the transient and steady state performance of the system.

Special Note : all compensators are minimum phase system.

Lagging compensator : (1) Lag compensator provides lagging angle or negative angle for all positive values of frequency.

(2) Behaviour of Lag compensator is same as Low pass filter (LPF).

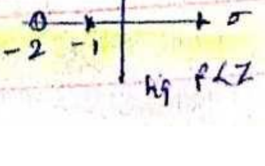
LPF

Basic concept of Lag compensator :-

$\boxed{\text{Zero} > \text{Pole}}$ → matlab zero = −2 to pole −2 se chota hona chahi magnitude i.e pole = −1.

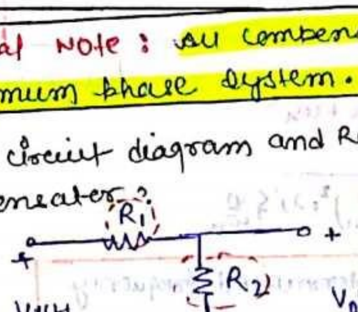
* Lag compensator is a "pole dominating system" i.e Pole exist near origin.

Eg $T(s) = \frac{s+2}{s+1}$ $\quad P = -1, \ Z = -2$ Lag compensator



$P < Z$

Basic circuit diagram and Results of Lag compensator :



(i) $T(s) = \dfrac{\tau s + 1}{\beta\tau s + 1}$

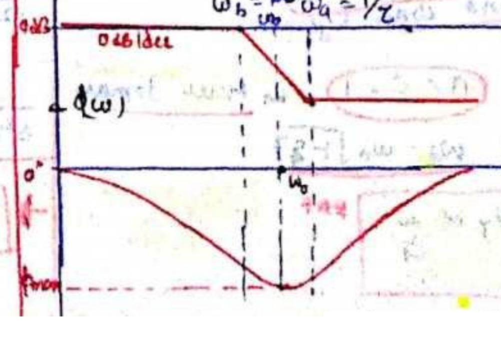
(ii) $\omega_0 = \dfrac{1}{\tau\sqrt{\beta}}$ wo frequency jab jab $\angle T(j\omega)$ maximum hoga.

(iii) $\phi_{max} = \sin^{-1}\left(\dfrac{1-\beta}{1+\beta}\right)$

$\beta = \dfrac{R_1 + R_2}{R_2}$ $(\beta > 1)$

$\tau = R_2 C$

Bode plot of Lag compensator:

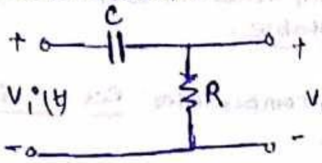


Lead Compensator :

→ Lead compensator provide leading angle or positive angle for all +ve values of frequency.

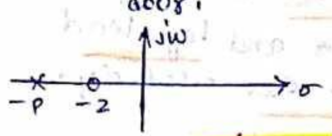
→ Behaviour of lead compensator is same as high pass filter.

→ Basic circuit :



Basic concept of lead compensator :-

$\boxed{Z < P}$ → matlab zero origin ke pass hoga aur pole origin se door.



Lead compensator is zero dominating compesator.

→ $T(s) = \dfrac{s+a}{s+b}$

$a > b$ — Lag comp.

$b > a$ — Lead compo.

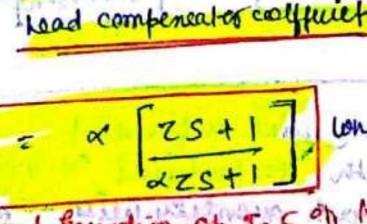
→ $\boxed{\omega_0 = \sqrt{ab}}$

ω_0 to hai jispe angle of $\angle T(j\omega)$ maximum hoga.

→ $\boxed{\tan\phi_{max} = \dfrac{b-a}{2\sqrt{ab}}}$

→ $\boxed{\sin\phi_{max} = \dfrac{b-a}{b+a}}$

Standard circuit for lead compensator :-

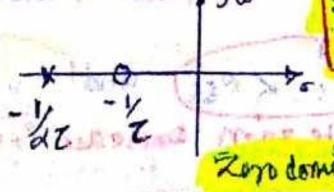


$\boxed{\tau = R_1 C}$ — time constant

Lead compensator coefficient, $\boxed{\alpha = \dfrac{R_2}{R_1+R_2}}$ $(\alpha < 1)$

$$\boxed{T(s) = \alpha\left[\frac{\tau s+1}{\alpha\tau s+1}\right]}$$ where $\alpha < 1$, $\tau = R_1 C$

Standard Equation of T.F of lead compensator.

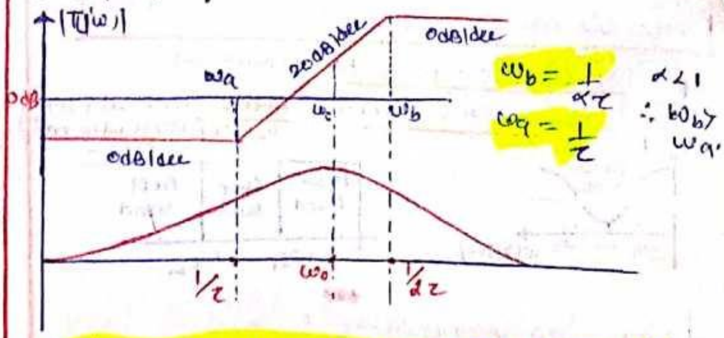


$\boxed{\sin\phi_{max} = \dfrac{1-\alpha}{1+\alpha}}$

$\alpha = \dfrac{R_2}{R_1+R_2}$

Zero dominating System.

Bode plot of lead compensator :



$\omega_b = \dfrac{1}{\alpha\tau}$, $\omega_a = \dfrac{1}{\tau}$, $\alpha < 1$ $\therefore \omega_b > \omega_a$

α ki value less than 1 isliye $20\log\alpha$ = −ve dB me hoga isliye 0dB line se niche se start kia hai.

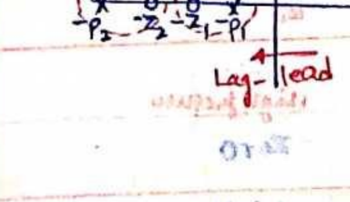
Important Difference b/w Lag compensator and lead.

Lag Compensator	Lead Compensator
(1) $\xi\downarrow$, $\omega_n\downarrow$	(1) $\xi\uparrow$ $\omega_n\uparrow$
(2) Actual damping $\xi\omega_n\downarrow$	(2) Actual damping $\xi\omega_n\uparrow$
(3) $\omega_{gc}\downarrow$	(3) $\omega_{gc}\uparrow$
(4) Bandwidth ↓	(4) Bandwidth ↑
(5) Settling time ↑	(5) Settling time ↓
(6) Rise time is high, MPO↑	(6) Rise time is low, MPO↓
(7) Stability ↓	(7) Stability ↑
(8) For type 1 system velocity error constant ↑.	(8) For type 1 system velocity error constant decreases ↓
(9) For type 1 system ess decreases for ramp input	(9) For type 1 system, ess ↑ for ramp i/p
(10) Lag compensator improve the steady state performance of system (↓ the ess)	(10) Lead compensator improve the transient performance of system (↓ the time domain parameter).

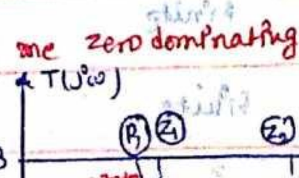
Lag-Lead compensator :

→ In lag-lead compensator both phase lag and phase lead occurs but in different frequency region (phase lag at low frequency and phase lead at high frequency).

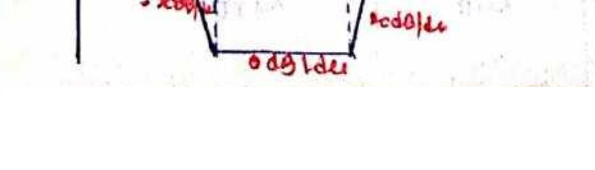
Standard circuit diagram :-



Lag-lead compesater — o/p me lag, i/p me lead



→ lag-lead me lag wala origin ke pass aega aur lag me pole dominate karta hai islie pole of lag sabse phle aega. phir lead aega aur lead me phle zero aega qki lead me zero dominating nature hota hai.

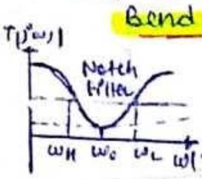


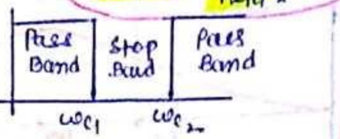
aage dekhenge → lag lead ka Bode Plot.

Note: All passfilter ek NMPS hota hai. MPS me all pass filter nai ban sakta.

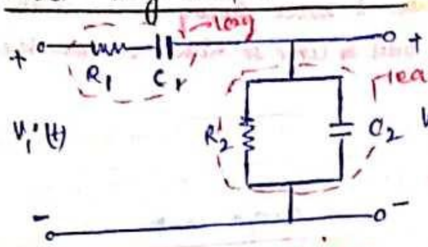
→ Lag-lead compensator behave as Band stop filter

Note: Band stop filter ka koi bandwidth nai hota.



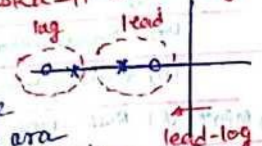


Lead - Lag compensator :



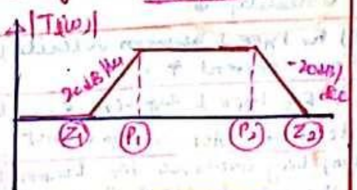
Lead-Lag: o/p me lead, i/p me lead.

Jo phle likha rahega uska a/t o/p me banega, aur jo bad me likha rahega uska I/P me banega.

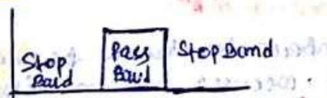


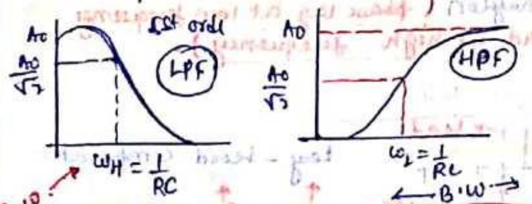
Lead-lag bolne me lead phle ara matlab origin ke karib lead aega aur lead me zero dominate karte hai so, 0 origin ke karib dega.

Bode plot:

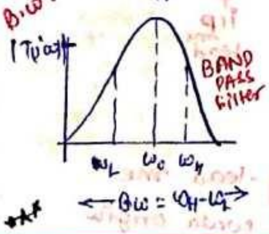


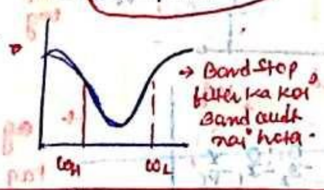
→ "Lead lag compensator behaves as band pass filter"





∴ B.W of HPF = ∞





Filter	o/p at $\omega=0$ (low frequency)	o/p at $\omega=\infty$ (High frequency)
(1) LPF	finite	Zero
(2) HPF	Zero	finite
(3) Band stop filter	finite	finite
(4) Band pass filter	Zero	Zero

Note: likha hai kahi amplification of 20dB matlab +20dB aur attenuation of 20dB matlab −20dB.

Note (1) Lead compensator does not affect steady state error (qki ye transient part ko improve karta hai na ki S.S part)

(2) Lag compensator approximately acts as a proportional plus integral controller and thus tends to make system less stable.

(3) kabhi bhi compensator e_{ss} ko 0 nai karega.

Special point:

(1) Lead compensator and lead-lag compensator ↑ e_{ss} relatively.

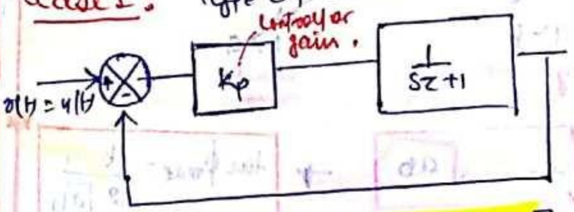
(2) Lag compensator and lag-lead compensator ↓ e_{ss} relatively.

-: Controllers :-

(1) Proportional controller:

K_p is controller gain here.

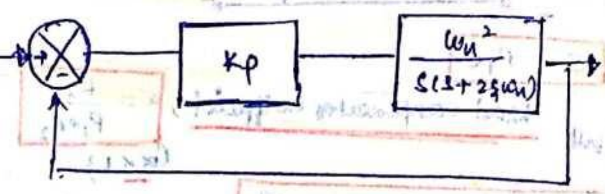
Case 1: Type 0, order 1 system



→ "proportional controller [P.C] decrease the e_{ss} in the same domain"

Time constant [P.C] < T.C [w/o P.C]
Bandwidth [P.C] > Bandwidth [w/o P.C]

Case 2: Type 1, order 2.



• dash quantity will represent system with proportional controller and w/o dash quantity will represent system w/o proportional controller.

Note: $e_{ss}' < e_{ss}$ and $K_v' > K_v$

(1) Steady state gain increase in case of P.C.

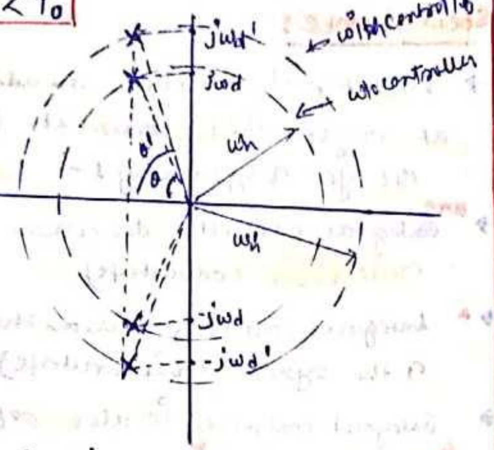
(2) Steady state error decrease in case of P.C.

- $\boxed{\omega_n' > \omega_n}$ • $\boxed{\xi_1' < \xi_1}$
- $\boxed{\alpha' = \alpha}$ * • $\boxed{\tau' = \tau}$ *
- $\boxed{t'_{sett} = t_{sett}}$ * • $\boxed{MPO' > MPO}$
- $\boxed{\theta' > \theta}$ • $\boxed{\omega_d' > \omega_d}$
- $t_p = \frac{\pi}{\omega_d}$ • $t_r = \frac{\pi - \theta}{\omega_d}$

$\boxed{t_p' < t_p}$ $\boxed{t_r' < t_r}$

- $T_0 = \frac{2\pi}{\omega_d}$ $MPO \propto \theta \propto \frac{1}{\xi_1}$ Imp.

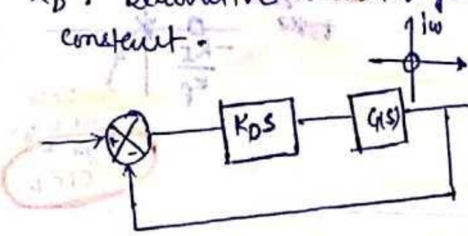
$\boxed{T_0' < T_0}$



(2) Derivative controller :-

$G_C(s) = K_D s$ $[K_D >> 1]$ (isse yad rakhege K_i $G_C(s) = K_D s$ ghone hai) $(u(t) \propto \frac{d}{dt})$

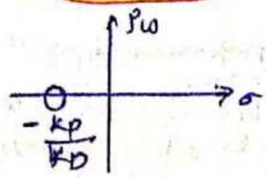
K_D : Derivative controller gain or derivative constant.

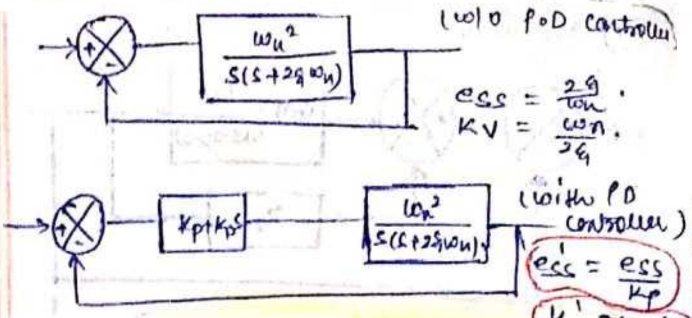


- In Derivative controller we add one zero at origin.
- Derivative controller increases the stability of system (advantage)
- Derivative controller increases the steady state error (e_{ss}) (disadvantage).
- Improves the transient performance

(3) Proportional Derivative Controller (P.D Controller) :-

$G_C(s) = K_P + K_D s$ $[K_P > 1, K_D > 1]$





$e_{ss} = \frac{2\xi_1}{\omega_n}$. $K_V = \frac{\omega_n}{2\xi_1}$.

$e_{ss}' = \frac{e_{ss}}{K_P}$

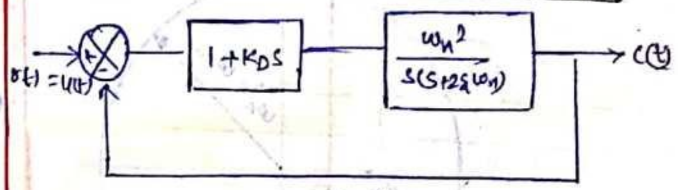
$K_V' = K_P K_V$

Note: (1) PD controller decreases the steady state error (e_{ss}) by the factor of K_P like proportional controller.

(2) In case of PD controller we add one left hand zero that mean P.D controller increases the stability of system.

[addition of zero in OLTF increases the stability of CLTF].

Time domain Analysis of PD controller :-



yahan $K_P = 1$ lelie hai.

- $\boxed{\omega_n' = \omega_n}$
- $\boxed{\xi_1' = \xi_1 + \frac{K_D \omega_n}{2}}$ $\Rightarrow$ $\xi_1' > \xi_1$

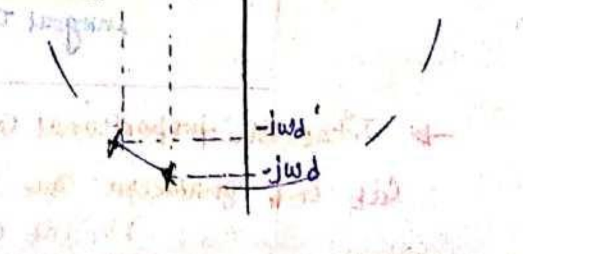
Effective damping ratio of PD controller.

- $\boxed{\alpha' > \alpha}$ $(\because \alpha = \xi_1 \omega_n)$
- $\boxed{\theta' < \theta}$
- $\boxed{\omega_d' < \omega_d}$
- $\boxed{MPO' < MPO}$ $(\because MPO \propto \frac{1}{\xi_1})$
- $\boxed{\tau' < \tau}$ $(\because \tau = \frac{1}{\alpha})$
- $\boxed{t_{sett}' < t_{sett}}$

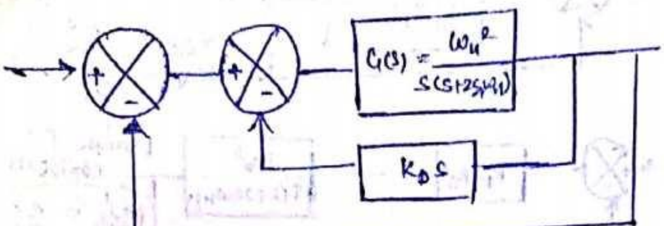
Isko alag se yad rakhna padega.

- $t_p' < t_p$
- $t_r' < t_r$
- $t_d' < t_d$

Isko relate hai karege ki $t_p = \frac{\pi}{\omega_d}$, to ω_d' < $\Rightarrow t_p' > t_p$ galat hojaega.



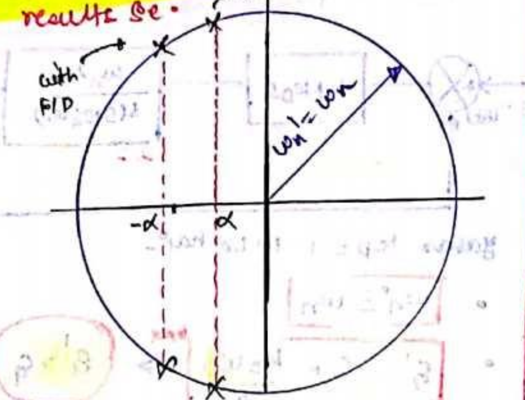
(4) Feedback Derivative controller :-



e_{ss}' [with FD] > e_{ss} [w/o FD]

- $\omega_n' = \omega_n$
- $\xi' = \xi + \frac{\omega_n K_D}{2}$ (same as P.D controller.)

PD controller me aur FP controller me transfer function ka antar hai jiske wajah se but t_p, t_r, t_d wale result badlenge baki sab same rahenge PD controller ke results se.

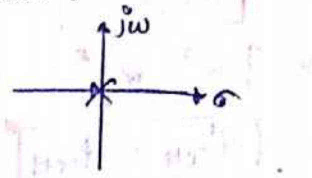


(i) $\omega_n' = \omega_n$
(ii) $\xi' > \xi$
(iii) $\theta' < \theta$
(iv) $MPO' < MPO$
(v) $\omega_d' < \omega_d$
(vi) $\alpha' > \alpha$
(vii) $t_{sett}' < t_{sett}$
(viii) $t_p' > t_p$
(ix) $t_r' > t_r$
(x) $t_d' > t_d$

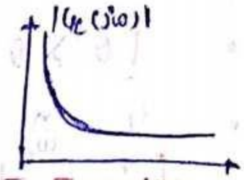
5. Integral controller :-

$G_c(t) \propto \int$

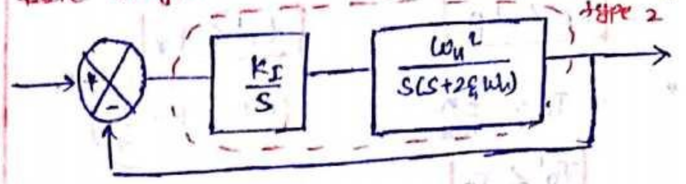
$$G_c(s) = \frac{K_I}{s}$$



$$|G_c(j\omega)| = \frac{K_I}{\omega}$$



- Integral controller me ek pole origin pe add hoga.

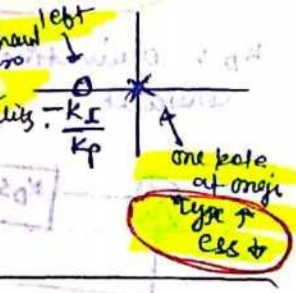


Special Note:

- In Integral controller, we add 1 pole at origin that means it increases the type of system by 1.
- Integral controller decreases the steady state error (advantage)
- Integral controller decrease the stability of the system (Disadvantage)
- Integral controller is also referred as "Reset Controller".

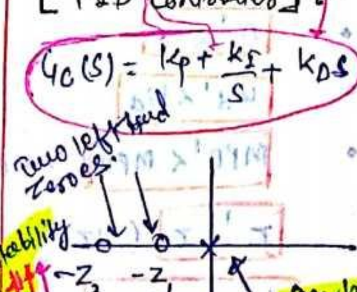
6. Proportional Integral controller (PI controller)

$$G_c(s) = K_p + \frac{K_I}{s}$$



(7). Proportional Integral Derivative [PID controller] :-

$$G_c(s) = K_p + \frac{K_I}{s} + K_D s$$



one pole at origin Type ↑
ess ↓ (Advantage)

→ (PID controller error ko zero kar deta hai.)

Keypoint:
(1) Proportion Band = $\frac{100}{K_p}$
(2) Reset Time or Integral Time = $\frac{K_p}{K_I}$

→ Jitne bhi proportional controller honge sab ess ↓ ghatayenge aur Integral controller bhi ess ↓ baki sab controller ess ↑↑.

Controller	ess
(1) Proportion controller.	ess ↓
(2) Proportion Derivative controller.	ess ↓
(3) Integral controller	ess ↓
(4) Proportional Integral controller	ess ↓
(5) Proportional Integral Derivative	ess ↓ or 0
(6) Derivative controller	ess ↑
(7) Feedback Derivative Controller	ess ↑

Controller	State Improve
(1) Derivative Controller	Improves transient Performance
(2) Integral controller	Improves steady state performance
(3) Proportional Integral Controller.	Improve Both state.
(4) Proportional Integral Derivative controller (PID)	Improve Both state

Controller acting as Low pass filter	controller acting as High Pass filter
(1) Integral Controller	(1) Derivative controller
(2) Proportional Integral controller	(2) Proportional Derivative Controller

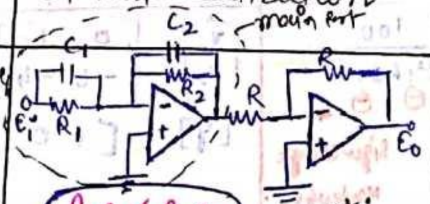
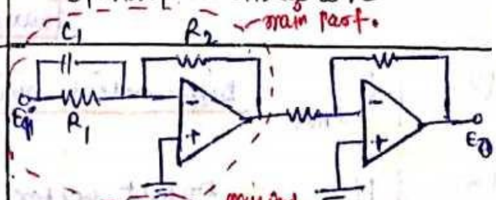
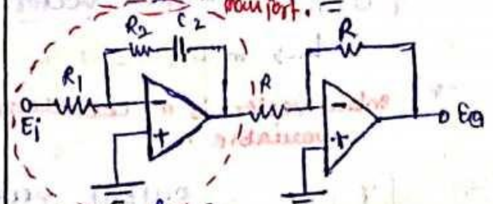
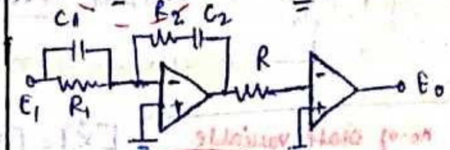
Jisme Derivative term aya (D.C or P.D.C) wo HPF honga aur jisme Integral term (I.C, P.I.C) wo LPF hoga

→ Oscillatory se stable Banane ke liye addition of zero at origin i.e derivative Action.

→ Oscillatory se Unstable Banane ke liye addition of pole at origin karo i.e Integral Action.

Note : (1) Agar ek T(s) deke bola ki esa compensator lagao ki ζ Badh jaye to hum option me lead compensator dhundhenge aur bole ki esa compensator lagao ki ζ Ghat jaye to hum option me Lag compensator dhundhenge.

(2) Dhyaan Dena hai question me ki o/p ka steady state value puchha ki amplitude at steady state.

Compensator	OP-Amp Realization	Controller	OP-Amp Realization
(1) Lag compensator	 $R_1C_1 < R_2C_2$	(1) PD controller	
(2) Lead compensator	 $R_1C_1 > R_2C_2$	(2) PI controller	
		(3) PID controller	 $R_2C_2 > R_1C_1$

Question to be done from Chapter 9 ;
Gate ; 8, 13, 23, 35, 36, 50, 3, 7, 12, 21, 22, 25, 29, 30, 39, 41, 43, 44, 45, 49, 46.
IES ; 12, 4, 25, 43, 47, 57, 26, 52, 76,
Questions to be be done from chapter 10 :-
Gate ; 79, 78, 74,

-: State Space Analysis :-

→ state space variable method is valid for Linear, Non Linear, Time Invariant and time variant system.

→ It is valid for both SISO and MIMO system.

→** State space variable method describes the internal state of the system (initial value of the system).

→ This method gives both ZSR (Zero I/P Response) and ZSR (Zero State Response)

Representation of State Variable Method :

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

→ Basic and very imp equations.

(1) $[X]_{n\times1}$ = State vector (It is collection of state variables)

n represents no. of state variables.

No. of state variable = No. of effective storing element
= No. of Time constant
= No. of initial value
= order of Transfer function

(2) $[\dot{X}]_{n\times1}$ = Differential state vector.

(3) $[U]_{m\times1}$ = Input vector

→ m represent No. of I/P variable

→ Input vector is a collection of I/P variable.

(4) $[Y]_{p\times1}$ = output vector

→ p represents no. of O/P variable

n = no. of state variable
m = no. of I/P variable
P = no. of o/p variable

$$[\dot{X}]_{n\times1} = [A]_{n\times n}[X]_{n\times1} + [B]_{n\times m}[U]_{m\times1}$$
$$[Y]_{p\times1} = [C]_{p\times n}[X]_{n\times1} + [D]_{p\times m}[U]_{m\times1}$$

(5)*** System matrix = $[A]_{n\times n}$ no. of state variable

Ye matrix hamesa square matrix hoga, Baki conditionally square matrix banenge.

(6). Input matrix (Not Input vector)

$[B]_{n\times m}$ n → no. of state variable, m → no. of I/P.

(7) O/P matrix (Not O/P vector)

$[C]_{p\times n}$ n → no. of state variable, p → no. of O/P.

(8) Data transmission matrix :-

$[D]_{p\times m}$ m → no. of I/P, p → no. of O/P

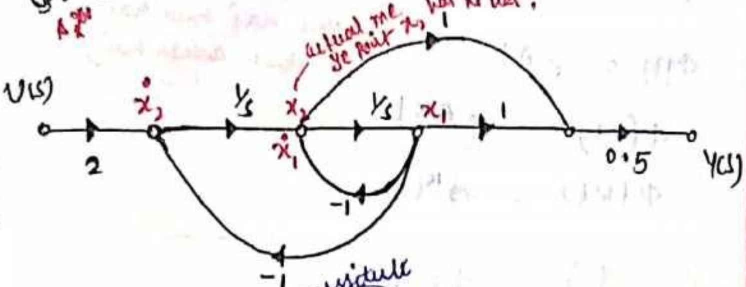
Note: Kisi SFG me jitna no. of $\frac{1}{S}$ terms honge utna no. of state variable hoga.

Method for writing Direct S.V.R from Transfer Function when T.F contains Nr as constant or T.F contains No Zero.

T(S)	S.V.R
(1) $\frac{10}{S+4}$ Sign change kardenge	$[\dot{x}] = [-4]_{1\times1}[x] + [10]_{1\times1}[U]$ Constant term of Nr. $[y] = [1]_{1\times1}[x]_{1\times1}$
(2) $\frac{100}{S^2+4S+6}$ Sign change kardenge. or jidhar se arrow hai udhor ka no. phle likhenge [A] me.	Iska pattern yad rakhna hai $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -6 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 100\end{bmatrix}[u]$ Constant Term of Nr $[y] = [1\ \ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$
(3) $\frac{10}{S^3-4S^2+10S-6}$	Pattern dhyan dena hai $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ +6 & -10 & +4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 10\end{bmatrix}$ Constant term of Nr $[y]_{3\times1} = [1\ \ 0\ \ 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$
(4) $\frac{100}{S^4+20S^3-10S^2+30S+50}$	$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -50 & -30 & 10 & -20\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ 100\end{bmatrix}$ $[y] = [1\ \ 0\ \ 0\ \ 0]\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$

Note : question me $\dot{x}_1 - x_1$, $\dot{x}_2 - x_2$, $\dot{x}_3 - x_3$ kas location hai dia rehta hai hum options ke hisab se locate karenge. yaad rahe ki different location of state variables ke liye different Matrix ...

Q. write the S.V.R for given SFG.



⇒ **Keypoint:** substitute

$$y = 0.5\dot{x}_1 + 0.5x_1 \quad -①$$

$\dot{x}_1 = x_2 - x_1 \Leftarrow \dot{x}_1 = \frac{\dot{x}_2}{s} - x_1 \quad -②$

$(\because \frac{\dot{x}_2}{s} = x_2)$ $\qquad \dot{x}_2 = -x_1 + 2u \quad -③$

doubt: $y = 0.5x_2 + 0.5x_1$ q nai hora?

Reason: qki jo feedback laga hai (-1) ka x_1 se x_2 tak wo easa problem karra. actually me wo point x_2 hai hi nai, wo $\dot{x}_1$ hai qki $\frac{\dot{x}_1}{s} = x_1$ hai aur x_1 pe koi feedback bhi nai hai.

Note: kabhi bhi state space variable $(\dot{x}_1, \dot{x}_2)$ pe feedback path ajaye to ekdam savdhan hojana hai aur equation likh ke dekh lena hai.

Solution:
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-1 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 2\end{bmatrix}[u]$$

$$[y]_{1\times1} = [0 \cdot 0.5]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Recovery of Transfer function from state variable Representation:

$$\boxed{\begin{aligned}\dot{X} &= AX + BU\\ Y &= CX + DU\end{aligned}}$$

for continuous time system transfer function

$$\boxed{T(s) = C\left[(sI - A)^{-1} \cdot B\right] + D}$$ very important

→ multiplication ke order me dhyan dena hai.

→ For discrete time system,

$$T(z) = C\left[(zI - A)^{-1} \cdot B\right] + D$$

Note: kabhi kabhi question me O/P / I/P ka laplace nai puchta but kisi aur ka puchta hai i.e. disturbance / I/P to yad rakhe

$$\frac{Y(s)}{U(s)}\frac{O/P}{I/P} = C\left[(sI-A)^{-1} \cdot B\right] + D$$ ye wala bus O/P / I/P ke lie use karenge.

Note: Jab $|sI - A|$ karenge to eigen values aega, aur 3rd order ke lie two roots aenge to usko dekh ke hum damping of system ke base me bata sakte hai.

Special point: This is not [A] matrix

$$[\dot{x}] = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0.5 & 1 & 2\end{bmatrix}x + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u$$

$$[u] = [-0.5 \; -3 \; -5]\,x + v$$

uppar wala [A] matrix isliye nai hai qki is bar [u] I/P vector matrix nai hai, [u] depend karra [x] ki value me.

$$\Rightarrow [\dot{x}] = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0.5 & 1 & 2\end{bmatrix}[x] + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}_{3\times1}\left[\begin{matrix}[-0.5 \; -3 \; -5]x\\ +v\end{matrix}\right]$$

$$[\dot{x}] = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0.5 & 1 & 2\end{bmatrix}[x] + \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ -0.5 & -3 & -5\end{bmatrix}[x] + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}[v]$$

$$[\dot{x}] = \underbrace{\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -2 & -3\end{bmatrix}}_{A}[x]_{3\times1} + \underbrace{\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}}_{B}[v]_{1\times1}$$

where v is input.

Method to write SVR from T(s) [Provided 's' term in Nr.]

(1) $T(s) = \frac{2s+7}{s^2+3s+2}$
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$
$$[y] = [7 \;\; 2]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

(2) $T(s) = \frac{s+0}{s^2+5s+1}$
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$
$$[y] = [0 \;\; 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

(3) $T(s) = \frac{s^2+s+1}{s^3+2s^2+2s+1}$
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & -2 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u$$
$$[y] = [1 \;\; 1 \;\; 1]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

→ state variable representation is not unique.

→ Gate me, S.V.R deke, T(s) puchta hai.

Solution of State Space Equation :-

$\dot{X} = AX + BU$

$Y = CX + DU$

Case 1: Input = 0 [i.e U = 0]

$\Rightarrow \dot{X} = AX$ (solution nikalte samay initial condition consider karenge)

and $Y = CX$

$$\boxed{[\phi(t)]_{n\times n} = \mathcal{L}^{-1}\left[(SI-A)^{-1}\right]_{n\times n}}$$

$[\phi(t)]_{n\times n}$ = State transition matrix.

$\phi(t) = \mathcal{L}^{-1}\left[\frac{1}{SI-A}\right]$

$\phi(t) = e^{At}$, where $A \to$ Square matrix.

$$\boxed{[x(t)]_{n\times 1} = [\phi(t)]_{n\times n}[x(0)]_{n\times 1}}$$

State Equation (ZIR) / Natural response / State transition equation

* Initial value ka order hamesa column matrix hoga i.e $n\times 1$ (no. of state variable = no. of initial value).

$$\boxed{[y(t)]_{p\times 1} = [C]_{p\times n}[x(t)]_{n\times 1}}$$

Solution

Properties of State transition Matrix (STM):-

$\phi(t) = e^{At}$

1. $\phi(0) = e^{0} = I$

$[\phi(0)]_{n\times 1} = [I]_{n\times n}$

At $t=0$, STM ki value 'I' matrix ke barabar hoti hai.

2. $\phi'(t) = Ae^{At}$

$\phi'(0) = A$

$[\phi'(0)]_{n\times n} = [A]_{n\times n}$

At $t=0$, derivative of STM is system matrix i.e 'A' matrix.

(3.) $\phi^{k}(t) = \phi(kt)$. (exam me abhi tak use nai hua hai but aa skta hai)

$\phi(t) = e^{At}$

$\phi(kt) = e^{Akt}$

$\phi(kt) = \phi^{k}(t)$

(4) $\phi^{-1}(t) = \phi(-t)$

Inverse of Inverse.

(5) $\phi(t_1-t_0)\phi(t_0-t_2) = \phi(t_1-t_2)$

Proof: $\phi(t) = e^{At}$

$\phi(t_1-t_0) = e^{A(t_1-t_0)}$

$\phi(t_0-t_2) = e^{A(t_0-t_2)}$

$\phi(t_1-t_0)\phi(t_0-t_2) = e^{A(t_1-t_0+t_0-t_2)} = e^{A(t_1-t_2)}$

$\phi(t_1-t_0)\phi(t_0-t_2) = \phi(t_1-t_2)$.

Case 2: Input $\neq 0$, [i.e $U \neq 0$]

$\dot{X} = AX + BU$

$Y = CX + DU$

$$\boxed{[X(s)]_{n\times 1} = \underbrace{[\phi(s)][x(0)]_{n\times 1}}_{ZIR} + \underbrace{[\phi(s)][BU(s)]_{n\times 1}}_{ZSR}}$$

ZIR = f(x(0))   ZSR = f(I/P).

I/P	x(0)	x(t)
0	≠ 0	ZIR
≠0	0	ZSR
≠0	≠0	ZIR+ZSR
0	0	X

$$\boxed{[x(t)]_{n\times 1} = [\phi(t)]_{n\times n}[x(0)]_{n\times 1} + \underbrace{\phi(t)}_{STM} * \underbrace{Bu(t)}_{Input}}$$

$$\boxed{x(t) = e^{AT}\left[x(0) + \int_0^t e^{-A\tau}u(\tau)\,d\tau\right]}$$ yaha τ variable hai.

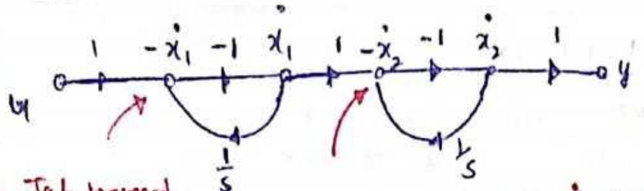
Special note:

(1) kabhi bhi ZSR matrix me zero put karenge to ZSR ko zero hona padega. (qki isme initial condition already 0 rehta hai)

(2) Kabhi bhi ZIR matrix me zero put karenge to ZIR matrix initial condition matrix x(0) ke barabar aega.

(3) kabhi bhi ZIR+ZSR matrix me zero put karenge to usko initial condition x(0) ke barabar aega.

Note:



Jab forward Path me $\frac{1}{s}$ ka term rehta tha to $\dot{x}_2$ ko $\frac{1}{s}$ se pass kara ke direct x_2 likh dete the qki us x_2 pe aur koi path approch nai karra hota tha. But yahan pe us node pe approach hora isliye dhyan denge equation likhte time.

Controllability: (1) controllability defines that all state variable are input dependent.

(2) If SFG is given to check the controllability we observe the path from input to all state variable. If path exist then system is controllable (conditional)*

Kalman's test for controllability:

$$[Q_c]_{n\times m} = [B : AB : A^2B \dots A^{(n-1)}B]_{n\times m}$$

Case 1: $|Q_c| \neq 0$

Q_c = Non singular matrix

$\boxed{\text{Rank of } Q_c = n}$

Then system is referred as controllable.

Case 2: $|Q_c| = 0$

Q_c = Singular matrix

$\boxed{\text{Rank of } Q_c < n}$

Then system is referred as uncontrollable.

Keypoints: (1) In case of controllable, all 'n' state variables are input dependent [conditional]*

(2) In case of uncontrollable, some of the state variables are independent from the inputs.

$Q_c = f(B, A) \Rightarrow f$ (I/P matrix, system matrix)

$Q_c = f$ (I/P variable, state variable)

$Q_c \neq f(C)$

$Q_c \neq f$ (o/p matrix)

$Q_c \neq f$ (o/p variable)

→ Controllability matrix describe the relationship b/n input and state variable.

Observability: (1) Observability defines that o/p depends on 'n' no. of state variables, if there is any change in any state variable then change will reflect directly at o/p.

(2) System is said to observable if it is possible to determine the initial state of the system from desired state in finite time interval.

(3) If the SFG is given to check possibility of system then observe the path from all state variable to output, if path exist then system is observable (conditional).

Kalman's test for observability is given by:

$$[Q_0] = [C^T : A^TC^T : A^{2T}C^T \dots A^{(n-1)T}C^T]_{n\times}$$

Case 1: If $|Q_0| \neq 0$; Q_0 = Non singular matrix.

$\boxed{\text{Rank of } Q_0 = n}$

Then system is referred as Observable.

Case 2: If $|Q_0| = 0$, (singular matrix)

$\therefore \boxed{\text{Rank of } Q_0 < n}$

Then system is referred as Non observable.

** $Q_0 = f(C, A)$; $Q_0 = f$ (o/p matrix, system matrix)

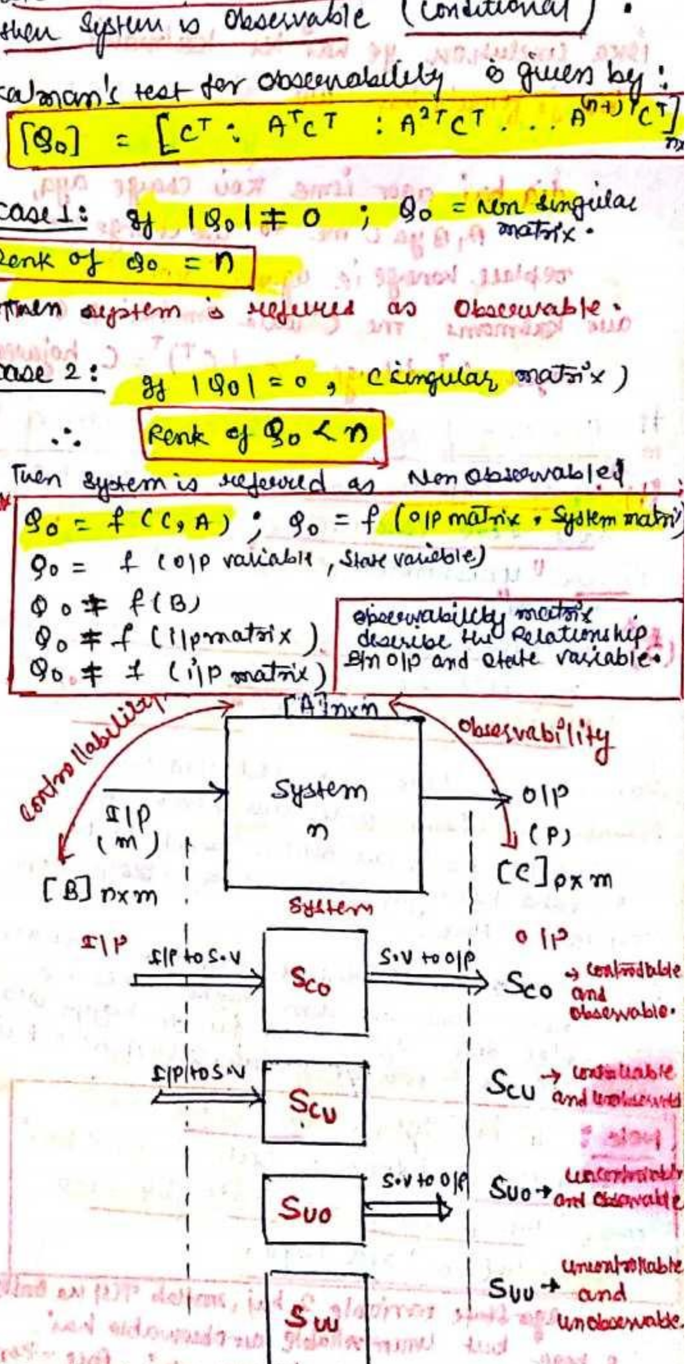
$Q_0 = f$ (o/p variable, state variable)

$Q_0 \neq f(B)$

$Q_0 \neq f$ (I/p matrix)

$Q_0 \neq f$ (I/p matrix)

observability matrix describe the relationship b/n o/p and state variable.



→ Yad rakho
Jab A^2B matrix nikalna hai tab $[A][AB]$ karke nikalenge jaldi hoga.
Similar for $A^{2T}C^T \rightarrow [A^T][A^TC^T]$.

Note: yad rakhe agar Equation is type of A^T ka dia hai

$$\dot{x} = A^T x + B^T U$$
$$y = Cx$$

to $[Q_c] = [B^T : A^T B^T : A^{2T} B^T \cdots]$

$[Q_o] = [C^T : (A^T)^T C^T : \cdots]$

$= [C^T : A C^T : \cdots]$

agar ese de dia : $\dot{x} = A^T x + BU$

$$y = C^T x$$

to $[Q_c] = [B : A^T B : A^{2T} B \cdots]$

$[Q_o] = [(C^T)^T : (A^T)^T (C^T)^T : \cdots]$

$= [C : AC : \cdots]$

iska conclusion ye hai ki Kalman's ka jo fomula hai wo $\dot{x} = Ax + BU$, $y = Cx$ ke hi dia hai agar isme koi change aya, A, B ya C me to use change ko replace karenge ie agar C^T dedia Eqn me aur Kalman me C^T wala term hai to C ki jagah C^T dalenge i.e $(C^T)^T = C$ hojaega.

Concept of pole-zero cancellation :

(1) If the transfer function matrix involve pole-zero cancellation, then system either "uncontrollable" or "unobservable" or both".

(2) If transfer function matrix does not involve any pole zero cancellation, then system "controllable" and "observable" i.e. "both controllable" and "observable".

How to check pole zero cancellation :

Method 1 :- Given poles aur zeroes ko factorise karlo aur dekho exact factor kat raha hai kya, agar kat ra to pole zero cancellation Hora.

Method 2 : Jiska bhi degree kam hai zeros aur poles me wo apne higher wale ko divide dede, agar pura divide hoyga 0/0 remainder to pole-zero concellation hora hai.

Note : Jo bhi system "Observable aur Controllable (dono ek sath) nai hai unke transfer system me 100% Pole zero cancellation hora hoga.

Note : agar state variable 2 hai, matlab T(s) ka order 2 hoga but uncontrollable aur observable hai iska matlab usme order 1 hoga qki pole-zero cancellation hoga i.e. upar niche same function se multiply kardu tab order 2 bana, varna original order 1 hoga.

Transfer function Decomposition :-

(1) Direct decomposition :

Ye apna trick wala method hai. Jo do method sikhe hai wahi wala.

→ Jis transfer function me N^r me constant term hoga to wo 100% controllable and observable hoga (qki pole-zero cancellation hoga hi nai).

→ Form of Direct Decomposition T.F :

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & -c \end{bmatrix}$$

$$T(s) = \frac{K}{s^3 + cs^2 + bs + a}$$

In direct decomposition of Transfer function, the system matrix 'A' has its unity element above its diagonal and negative coefficient in last row and all other elements are zero.

→ System matrix A is known as phase variable cannonical form or Bush form.

→ It is also referred as controllable cannonical form.

→ "System is observable or not depend upon 'c' matrix only. So changing N^r we can change observability but controllability is fixed)".

→ "In direct decomposition method, Pole-Zero cancellation is not possible".

→ Direct decomposition form me N^r me 's' ka term agya to sirf observability check karna hai.

→ "Direct decomposition form is always controllable".

(2) Parallel decomposition :

Iske T.F me D^r me hamesa factor ka form rahega.

→ "Only parallel decomposition gives correct answer for controllability and observability from SFG".

- Incase of parallel decomposition, system matrix 'A' always be diagonal matrix.
- Diagonal elements are pole or eigen value of transfer function.

- There is no pole zero cancellation in parallel decomposition, so parallel decomposition is always "controllable and observable.

(3) Cascade decomposition :

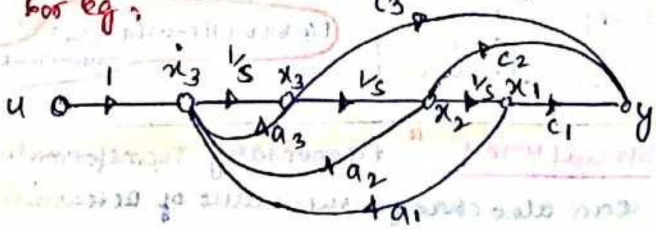
- In case of cascade decomposition diagonal elements represents poles or Eigen values of T.F
- system matrix always forms an upper Triangular or lower triangular matrix in cascade decomposition.

Special Note :(1) Parallel Decomposition me $\dot{x}_n$, x_n pe depend karega, $\dot{x}_{n-1}$, x_{n-1} pe depend karega nai ki $\dot{x}_n$, x_{n-1} pe karega aur na ki $\dot{x}_{n-1}$, x_n pe karega.

(2) For all multiple input multiple o/p system, system matrix [A] will be same.

Kabhi bhi SFG me $\dot{x}_3$ ke turant bad x_2 uske turant bad x_1 mil gya matlab wo direct Decomposition hai. or matlab i/s wale tem line se ek hi line me aaye.

For eg.



Parallel Decomposition of Transfer function having Repeated roots :

- In this case, system matrix A consists poles/roots in diagonal element and also unity element below repeated roots.
- This form is also known as Jordan cannonical form.
- This form is always controllable and observable because there is no pole zero cancellation.

→ Note : Dhyan dena hai Question me kabhi kabhi o/p y(t) ko kisi aur variable se bhi badal sakta hai. For Eg; Bol dena hai ki the output is c(t).