4. LAWS OF MOTION

1. Choose the correct answer.

i) Consider following pair of forces of equal magnitude and opposite directions:

(P) Gravitational forces exerted on each other by two point masses separated by a distance.

(Q) Couple of forces used to rotate a water tap.

(R) Gravitational force and normal force experienced by an object kept on a table.
For which of these pair/pairs the two forces do NOT cancel each other's translational effect?

<u>(A) Only P</u>
(B) Only P and Q
(C) Only R
(D) Only Q and R

ii) Consider following forces: (w) Force due to tension along a string, (x) Normal force given by a surface, (y) Force due to air resistance and (z) Buoyant force or upthrust given by a fluid.
Which of these are electromagnetic forces?

(A) Only w, y and z
(B) Only w, x and y
(C) Only x, y and z
<u>(D) All four.</u>

iii) At a given instant three point masses m, 2m and 3m are equidistant from each other. Consider only the gravitational forces between them. Select correct statement/s for this instance only:

(A) Mass m experiences maximum force.
(B) Mass 2m experiences maximum force.
<u>(C) Mass 3m experiences maximum force.</u>
(D) All masses experience force of same magnitude.

iv) The rough surface of a horizontal table offers a definite maximum opposing force to initiate the motion of a block along the table, which is proportional to the resultant normal force given by the table. Forces F_1 and F_2 act at the same angle θ with the horizontal and both are just initiating the sliding motion of the block

along the table. Force F_1 is a pulling force while the force F_2 is a pushing force. $F_2 > F_1$, because

<u>(A) Component of F_2 adds up to weight to increase the normal reaction.</u>
(B) Component of F_1, adds up to weight to increase the normal reaction.
(C) Component of F_2 adds up to the opposing force.
(D) Component of F_1, adds up to the opposing force.

v. A mass 2m moving with some speed is directly approaching another mass m moving with double speed. After some time, they collide with coefficient of restitution 0.5. Ratio of their respective speeds after collision is

(A) 2/3
<u>(B) 3/2</u>
(C) 2
(D) ½

vi. A uniform rod of mass 2m is held horizontal by two sturdy, practically inextensible vertical strings tied at its ends. A boy of mass 3m hangs himself at one third length of the rod. Ratio of the tension in the string close to the boy to that in the other string is

(A) 2
<u>(B) 1.5</u>
(C) 4/3
(D) 5/3

vii. Select WRONG statement about centre of mass:

(A) Centre of mass of a 'C' shaped uniform rod can never be a point on that rod.
(B) If the line of action of a force passes through the centre of mass, the moment of that force is zero.
(C) Centre of mass of our Earth is not at its geometrical centre.
<u>(D) While balancing an object on a pivot, the line of action of the gravitational force of the earth passes through the centre of mass of the object.</u>

viii. For which of the following objects will the centre of mass NOT be at their geometrical centre?

(I) An egg
(II) a cylindrical box full of rice
(III) a cubical box containing assorted sweets

(A) Only (1)

(B) Only (I) and

(II) (C) Only (III)

(D) All, (I), (II) and (III).

2. Answer the following questions.

i) In the following table, every entry on the left column can match with any number of entries on the right side. Pick up all those and write respectively against A, B, C and D.

Name of the force	Type of the force
A Force due to tension in a string	P EM force
B Normal force	Q Reaction force
C Frictional force	R Conservative force
D Resistive force offered by air or water for objects moving through it.	S Non- conservative force

Ans.

A. Force due to tension in a string:
Electromagnetic (EM) force, reaction force, non-conservative force.

B. Normal force:
Electromagnetic (EM) force, non conservative force, Reaction force

C. Frictional force:
Electromagnetic (EM) force, reaction force, non-conservative force.

D. Resistive force offered by air or water for objects moving through it:
Electromagnetic (EM) force, non conservative force.

ii) In real life objects, never travel with uniform velocity, even on a horizontal surface, unless something is done? Why is it so? What is to be done?

Ans. 1. According to Newton's first law, for a body to achieve uniform velocity, the net force acting on it should be zero.

2. In real life, a body in motion is constantly being acted upon by resistive or opposing force like friction, in the direction opposite to that of the motion.

3. To overcome these opposing forces, an additional external force is required. Thus, the net force is not maintained at zero, making it hard to achieve uniform velocity.

4. For an object to travel with uniform velocity, the surface has to be frictionless i.e., the motion has to be free of resistive or opposing forces.

iii) For the study of any kind of motion, we never use Newton's first law of motion directly. Why should it be studied?
Ans. Newton's first law of motion is not redundant for two reasons:

(1) Before Newton synthesized and firmly founded that branch of mechanics called dynamics, Artistotelians had a different view of the role of force. They held that force was the cause of uniform (unaccelerated) motion. Newton's first law of motion implies that a net force is the cause of change of velocity. Thus, the first law was an axiomatic emphasis on the changing point of view in mechanics.

(2) Newton's first law defines inertial frames of reference and limits the scope of the second law to such frames. In deriving the first law from the second law, $\vec{v}$ is assumed to be measured in an inertial frame; i.e., Newton's first law is built into the argument and not truly 'derived' from the second law. The first law is independent and necessary to understand Newton's concept of force. The argument that the first law is redundant, as some sceptics would like to impress upon us, is a deceptive fallacy.
Or
Easy and short answer:

1. Newton's first law shows an equivalence between the 'state of rest' and 'state of uniform motion along a straight line.'

2. Newton's first law of motion defines force as a physical quantity that brings about a change in 'state of rest' or 'state of uniform motion along a straight line' of a body.

3. Newton's first law of motion defines inertia as a fundamental property of every physical object by which the object resists any change in its state of rest or of uniform motion along a straight line.
Due to all these reasons, Newton's first law should be studied.

iv) Are there any situations in which we cannot apply Newton's laws of motion? Is there any alternative for it?

Ans. Limitations of Newton's laws of motion:

(1) Newton's laws of motion are applicable only in inertial frames of reference which they define. When applying the laws in a noninertial frame of reference, the acceleration of the reference frame must be considered by including a pseudo force equal to the mass of the body times the acceleration of the reference frame.

(2) Newton's laws of motion are applicable to particles (point objects) and rigid bodies. The laws may be applied to real bodies if these can be approximated as particles.

(3) At very high speeds comparable to the speed of light, or for very strong gravity, Newton's laws do not give results that match with experiments. In these cases, Einstein's special and general theories of relativity must be used.

(4) Newtonian mechanics also fails to explain the interactions of elementary particles. Quantum mechanics must be used for interactions at the atomic and subatomic scale.

v) You are inside a closed capsule from where you are not able to see anything about the outside world. Suddenly you feel that you are pushed towards your right. Can you explain the possible cause (s)? Is it a feeling or a reality? Give at least one more situation like this.

Ans. The push to the right is due to the acceleration of the observer's frame of reference - the capsule - to the left. This could be due to an acceleration of the capsule in a straight line towards the left or due to the circular motion of the capsule, in which case it is the centripetal acceleration.

The force to the right is a reality in the observer's reference frame.

Similar pseudo force is experienced in a lift which accelerates up or down, and in an accelerating vehicle.

vi) Among the four fundamental forces, only one force governs your daily life almost entirely. Justify the statement by stating that force.

Ans. Common contact forces, such as solid and liquid friction, pushes and pulls, normal reaction, tension in a string, elastic forces and those exerted during collisions are electromagnetic in origin. Thus, the electromagnetic force – the second strongest of the four fundamental forces – almost governs our daily life.

vii) Find the odd man out: (i) Force responsible for a string to become taut on stretching (ii) Weight of an object (iii) The force due to which we can hold an object in hand.

Ans. Weight of an object; it is due to the gravitational force of the Earth. The other two – tension in a string and friction – are electromagnetic in origin.

viii) You are sitting next to your friend on ground. Is there any gravitational force of attraction between you two? If so, why are you not coming together naturally? Is any force other than the gravitational force of the earth coming in picture?

Ans. Gravitational attraction is universal. However, the gravitational forces between ordinary uncharged objects are masked by frictional forces of one kind or another, or by the tremendously larger force of gravitation due to the Earth. Hence, we do not see all ordinary objects colliding due to their mutual pulls.

Here, the friction forces between the friends and the ground prevent them from colliding due to their mutual gravitational pulls.

ix) Distinguish between:

(A) Real and pseudo forces,

(B) Conservative and non-conservative forces,

(C) Contact and non-contact forces,

(D) Inertial and non-inertial frames of reference.

Ans.

(A) Real and pseudo forces

Real force	Pseudo force
1. A real force has its origin in any of the four basic interactions between matter.	1. A pseudo force arises due to the acceleration of the observer's frame of reference and not due to any of the four basic interactions between matter.
2. A real force on one body due to another body is always accompanied by an equal and opposite reaction on the other body.	2. Newton's third law of motion does not apply in that there is no reaction force on some other body.

(B) Conservative and non-conservative forces

Conservative force	Non-conservative force
1. The force required to move a body from one point to another is called a conservative force if the work done in moving the body between these points is independent of the path taken.	1. A force that is not conservative is called a non-conservative force. It is a dissipative path- dependent force.
2. The work done on a body against a conservative force is stored in the body as its potential energy.	2. The work done against a non-conservative force is lost and cannot be converted into useful work later.
3. The work done on a body by a conservative force decrease the potential energy of the body, but the total mechanical energy of the body is conserved.	3. The work done on a body by a non conservative force changes (decreases) the total mechanical energy of the body, i.e., the total mechanical energy is not conserved.

(C) Contact and non-contact forces,

Contact force	Non-contact force
1. A contact force arises due to direct physical contact between interaction objects..	1. A non-contact force acts without the necessity of physical contact between interacting objects.
2. A contact force has its origin in electro-magnetic interaction between the objects.	2. Each of the four fundamental forces of nature are non-contact action-at-a-distance type force.

(D) Inertial and non-inertial frames of reference.

Inertial frame of reference	Noninertial frame of reference
1. A frame of reference in which every free body moves with a constant velocity (including zero) is called an inertial frame of reference. It is taken to be a frame fixed to the fixed distant stars. A frame of reference which itself has uniform motion in a straight line with respect to an inertial frame of reference is also an inertial frame.	1. An accelerated frame of reference, that is, one which has an acceleration with respect to an inertial frame of reference, is called a noninertial frame of reference.
2. Newton's laws are obeyed in an inertial frame.	2. Newton's laws are not obeyed in a noninertial frame.
3. In this frame, any force acting on a body is a real force.	3. The acceleration of the frame gives rise to a pseudo force.

x) State the formula for calculating work done by a force. Are there any conditions or limitations in using it directly? If so, state those clearly. Is there any mathematical way out for it? Explain.

<u>Solution:</u>

1. Suppose a constant force $\vec{F}$
acting on a body produces a displacement $\vec{s}$
in the body along the positive X-direction. Then the work done by the force is given as,
W = F.s cos θ
Where θ is the angle between the applied force and displacement.

2. If displacement is in the direction of the force applied, $\theta = 0^0$
$W = \vec{F}.\vec{s}$

Conditions/limitations for application of work formula:

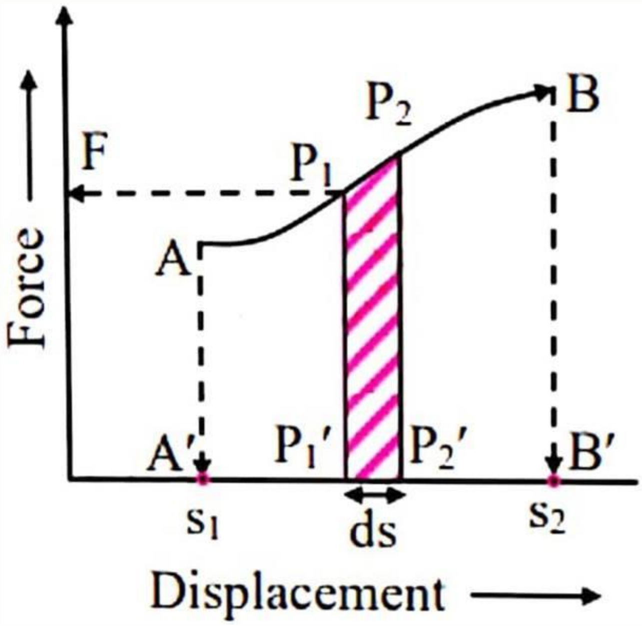
1. The formula for work done is applicable
only if both force $\vec{F}$ and displacement $\vec{s}$

are constant and finite i.e., it cannot be applied when the force is variable.

2. The formula is not applicable in several real-life situations like lifting an object through several thousand kilometers since the gravitational force is not constant. It is not applicable to viscous forces like fluid resistance as they depend upon speed and thus are often. not constant with time.

3. The method of integration has to be applied to find the work done by a variable force.

Integral method to find work done by a variable force:



1. Let the force vary non-linearly in magnitude between points A and B as shown in the above figure.

2. In order to calculate the total work done during the displacement from s_1 to s_2 we need to use integration. For integration, we need to divide the displacement into large numbers of infinitesimal (infinitely small) displacements.

3. Let at P_1, the magnitude of force be $F = P_1P_1$. Due to this force, the body displaces through infinitesimally small displacement ds, in the direction of the force. It moves from P_1 to P_2.

$\therefore\ d\vec{s} = \overrightarrow{P_1P_2}$

4. But direction of force and displacement are same, we have

$d\vec{s} = P_1'P_2'$

5. ds is so small that the force F is practically constant for the displacement. As the force is constant, the

area of the strip $\vec{F}.d\vec{s}$ is the work done dW for this displacement.

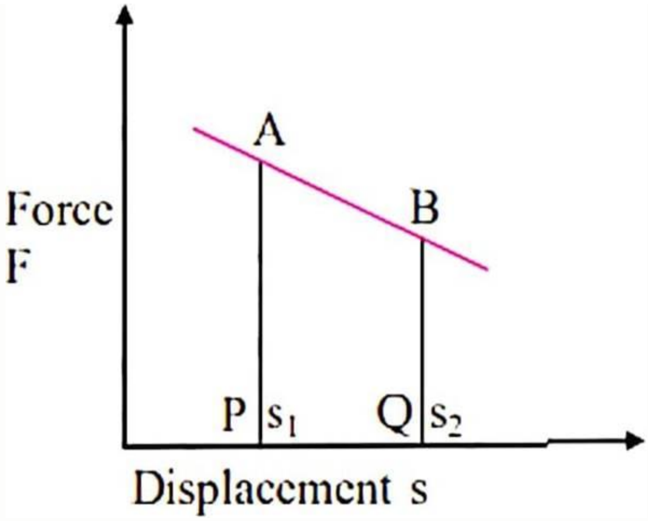
6. Hence, small work done between P_1 to P_2 is dW and is given by
$dW = \vec{F}.d\vec{s} = P_1P_1' \times P_1'P_2'$
$=$ Ares of the strip $P_1P_2P_2P_1$.

7. The total work done can be found out by dividing the portion AB into small strips like $P_1P_2P_2P_1$.and taking sum of all the areas of the strips.
$\therefore W = \int_{s_1}^{s_2} \vec{F}.d\vec{s} = Area\ ABB'A'$

8. Method of integration is applicable if
the exact way of variation in $\vec{F}$ *and* $\vec{s}$
is known and that function is integrable.

9. The work done by the non-linear variable force is represented by the area under the portion of the force displacement graph.

10. Similarly, in the case of a linear variable force, the area under the curve from s_1 to s_2 (trapezium APQB) gives total work done W in the following figure.



xi) Justify the statement, "Work and energy are the two sides of a coin".

Ans. Work is said to be done when a body moves with the application of an external force. External force is the influence of other body or bodies. Thus, if body 2 does work on body 1, the body 2 transfers energy to the body 1 whose change in kinetic energy, by the work-energy theorem, is equal to the work done on it. Hence, it is said work and energy are two sides of the same coin.

xii) From the terrace of a building of height H. you dropped a ball of mass m. It reached the ground with speed v. Is the relation

$$\boldsymbol{mgH = \frac{1}{2}mv^2}$$

applicable exactly? If not, how can you account for the difference? Will the ball bounce to the same height from where it was dropped?

Ans.

1. Let the ball dropped from the terrace of a building of height h have mass m. During free fall, the ball is acted upon by gravity (accelerating conservative force).

2. While coming down, the work that is done is equal to the decrease in potential energy.

3. This work done however is not entirely converted into kinetic energy but some part of it is used in overcoming the air resistance (retarding non-conservative force). This part of energy appears in some other forms such as heat, sound, etc.

4. Thus, in this case of an accelerating conservative force along with a retarding non-conservative force, the work-energy theorem is given as, Decrease in the gravitational P.E. Increase in the kinetic energy + work done against non-conservative forces.

5. Thus, the relation

$$mgH = \frac{1}{2}mv^2$$

is not applicable when non-conservative forces are considered. The part of the energy converted to heat, sound, etc also needs to be added to the equation.
6. The ball will not bounce to the same height from where it was dropped due to the loss in kinetic energy during the collision making it an inelastic collision.

xiii) State the law of conservation of linear momentum. It is a consequence of which law? Given an example from our daily life for conservation of momentum. Does it hold good during burst of a cracker?

Ans.

1. Statement: The total momentum of an isolated system is conserved during any interaction.

2. The law of conservation of linear momentum is a consequence of Newton's second law of motion. (in combination with Newton's third law)

3. Example: When a nail is driven into a wall by striking it with a hammer, the hammer is seen to rebound after striking the nail. This is because the hammer imparts a certain amount of momentum to the nail and the nail imparts an equal and opposite amount of momentum to the hammer.

Linear momentumconservation during the burst of a cracker:

a. The law of conservation of linear momentum holds good during the bursting of a cracker.

b. When a cracker is at rest before the explosion, the linear momentum of the cracker is zero.

c. When the cracker explodes into number of pieces, scattered in different directions, the vector sum of the linear momentum of these pieces is also zero. This is as per the law of conservation of linear momentum.

xiv) Define coefficient of restitution and obtain its value for an elastic collision and a perfectly inelastic collision.

Ans. Definition: When two bodies collide with each other, the negative ratio of their relative velocity after collision to their relative velocity before collision, is called the coefficient of restitution (e).

Consider the head-on collision of two particles of masses m_1 and m_2, moving before the collision with
constant velocities $\overrightarrow{u_1}$ *and* $\overrightarrow{u_2}$
in the laboratory frame of reference After the collision they have
velocities $\overrightarrow{v_1}$ *and* $\overrightarrow{v_2}$ again
considered to be constant. Since the collision is head-on, i.e., the motion is confined to one dimension, the velocity vectors all lie in the same line and hence we can write the equations in scalar form with the usual sign convention.
Therefore, by the principle of conservation of linear momentum,
$m_1 u_1 + m_2u_2 = m_1v_1 + m_2v_2$

... (1)
or $m_1(u_1 - v_1) = m_2 (v_2 - u_2)$

... (2)
Also, since the collision is elastic, total kinetic energy before collision = total kinetic energy after collision :
$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$
$\therefore\ m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$
or $m_1 (u_1 + v_1) (u_1 - v_1) = m_2 (v_2 + u_2) (v_2 - u_2)$

... (3)
Dividing Eq. (3) by Eq. (2), we get,
$u_1 + v_1 = v_2 + u_2$

... (4)
Rearranging the terms in Eq. (4),
$v_2 - v_1 = u_1 - u_2$
...(5)
Here, $(u_1 - u_2)$ is the velocity with which particle 1 approaches particle 2 before collision, and $(v_2 - v_1)$ is the velocity with which particle 2 separates away from particle 1 after collision. Hence, in a head-on elastic collision, velocity of separation = velocity of approach.
From Eq. (5), the coefficient of restitution,
$$e = \frac{(v_2 - v_1)}{(u_1 - u_2)} = 1$$
3. For a perfectly inelastic collision, the colliding bodies move jointly after the collision, i.e.,
$V_1 = V_2$
$\therefore V_1 - V_2 = 0$
$e = 0$

xv) Discuss the following as special cases of elastic collisions and obtain their exact or approximate final velocities in terms of their initial velocities.

(i) Colliding bodies are identical.

(ii) A veru heavy object collides on a lighter object, initially at rest.

(iii) A very light object collides on a comparatively much massive object, initially at rest.

Ans. Two particles of masses m_1 and m_2 move with
initial velocities $\overrightarrow{u_1}$ *and* $\overrightarrow{u_2}$
such that particle 1 collides hea elastically with particle 2. Their respective velocities after the collision are
$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$
$$and\ v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$
(1) If the particles have equal masses, substituting $m_1 = m_2$ in the above equations,
$$v_1 = (0)u_1 + \left(\frac{2m_2}{2m_2}\right)u_2 = u_2$$
$$and\ v_2 = \left(\frac{2m_1}{2m_1}\right)u_1 + (0)u_2 = u_1$$
This shows that in a head-on elastic collision of two particles of equal masses, the particles exchange their velocities.
Further, if the second particle is at rest before the collision, $u_2 = 0$, so that after the collision,
$v_1 = 0$ and $v_2 = u_1$

that is, the first particle initially moving stops and the second one takes off with the initial velocity of the first.

(2) If particle 1 is very massive and initially moving while the lighter particle 2 is at rest, $m_1 >> m_2$ and $u_2 = 0$. Then,

$$v_1 \cong \left(\frac{m_1}{m_1}\right)u_1 \cong u_1$$

and $v_2 \cong \left(\frac{2m_1}{m_1}\right)u_1 \cong 2u_1$

that is, if a massive particle makes an elastic head-on collision with a light one at rest, the massive one continues its motion at almost the same speed, and the light one takes off at nearly twice this speed.

(3) If the second particle is very massive and initially at rest, $m_2 >> m_1$ and $u_2 = 0$. Then, ignoring m_1 in comparison to m_2, the final velocities are

$$v_1 \cong \left(-\frac{m_2}{m_2}\right)u_1 \cong -u_1 \; and \; v_2 \cong (0)u_1 \cong 0$$

Thus, if a light particle makes an elastic collision with a massive one at rest, it rebounds with almost its initial speed; the massive one is almost unaffected.

xvi) A bullet of mass m, travelling with a velocity u strikes a stationary wooden block of mass m_2, and gets embedded into it. Determine the expression for loss in the kinetic energy of the system. Is this violating the principle of conservation of energy? If not, how can you account for this loss?

Ans. The loss in KE of the bullet is partly utilised in doing work against the retarding force of the block, partly in deforming the wood, and the rest is converted into heat and vibrational energy. Total energy is conserved, and the principle of energy conservation is not violated.
For the expression for the loss in KE, refer
Consider a one-dimensional (head-on) collision of two particles, of masses m_1 and m_2, moving with constant initial velocities u_1 and u_2. If the collision is perfectly inelastic, the particles stick together and move with a common velocity v after the collision along the same straight line.
Then, by the principle of conservation of linear momentum,
$m_1u_1 + m_2u_2 = (m_1 + m_2)\,v$

$$\therefore v = \frac{m_1u_1 + m_2u_2}{m_1 + u_2}$$

The change in kinetic energy,
ΔKE = final KE - initial KE

$$= \frac{1}{2}(m_1 + m_2)v^2 - \left(\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2\right)$$

$$= \frac{1}{2}(m_1 + m_2)\left(\frac{m_1u_1 + m_2u_2}{m_1 + m_2}\right)^2 - \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right)$$

$$= \frac{1}{2}\left[\frac{(m_1u_1 + m_2u_2)^2}{(m_1 + m_2)} - (m_1u_1^2 + m_2u_2^2)\right]$$

$$= \frac{1}{2}\left[\frac{m_1^2u_1^2 + 2m_1m_2u_1u_2 + m_2^2u_2^2 - m_1^2u_1^2 - m_2^2u_2^2 - m_1m_2u_1^2 - m_1m_2u_2^2}{m_1 + m_2}\right]$$

$$= \frac{1}{2}\left[-\frac{m_1m_2(u_1^2 + u_2^2 - 2u_1u_2)}{m_1 + m_2}\right] = -\frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$$

Since the bracketed terms are always positive, the minus sign in Eq. (2) shows that the final kinetic energy is less than the initial kinetic energy. That is, there is a loss in kinetic energy.

∴Loss in KE = initial KE - final KE

$$= \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$$

xvii) One of the effects of a force is to change the momentum. Define the quantity related to this and explain it for a variable force. Usually when do we define it instead of using the force?

Ans.

For a force $\vec{F}$

which is constant over the time interval Δt during which it acts, the impulse of the force is defined as the product

$\vec{F}\,\Delta t$:

$\vec{J} = \vec{F}\,\Delta t$

Newton's second law of motion, which duplicates the first law with quantitative precision, states that the change in momentum of a body equals the impulse of the applied force and is made in the direction of that force. Then,

$\vec{J} = \overrightarrow{dp} = \overrightarrow{p_f} - \overrightarrow{p_i} = m(\vec{v} - \vec{u})$

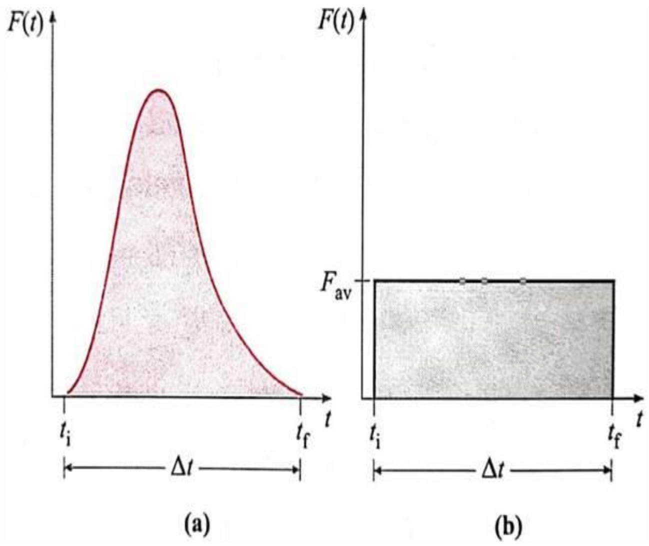
where m is the mass of the body;

$\overrightarrow{p_i}$ and $\overrightarrow{p_f}$

are the initial and final momenta of the body over the time interval of the impulse.

For a time − varying force $\vec{F}(t)$

whose magnitude over the interval Δt is given by the curve in Fig. (a), the impulse of the force is given by the area under the curve:



Impulse of a time-varying force as the area under the curve
For a time-varying force, the magnitude of the impulse can also be written as
$J = F_{av}\,\Delta t$
where F_{av} is the average
is the average force that would impart the same impulse as the variable force over the time interval, i.e.,
the area $F_{av}\,\Delta t$
of the rectangle in Fig. (b) is equal to the area under the curve in Fig.(a).

(2) Force is an abstract concept while what we actually observe is the change in momentum. The concept of impulse becomes more useful when the force varies with time, as in a two body collision.

xviii) While rotating an object or while opening a door or a water tap we apply a force or forces. Under which conditions is this process easy for us? Why? Define the vector quantity concerned. How does it differ for a single force and for two opposite forces with different lines of action?

Ans. 1. Opening a door can be done with ease if the force applied is:

a. proportional to the mass of the object

b. far away from the axis of rotation and the direction of force is perpendicular to the line joining the axis of rotation with the point of application of force.

2. This is because, the rotational ability of a force depends not only upon the magnitude and direction of the force but also on the point where the force acts with respect to the axis of rotation.

3. Rotating an object like a water tap can be done with ease if the two forces are equal in magnitude but opposite in direction are applied along different lines of

action.

4. The ability of a force to produce rotational motion is measured by its turning effect called 'moment of force' or 'torque'.

5. However, a moment of couple or rotational effect of a couple is also called torque.

6. For differences in the two vector quantities.
Moment of a force:
i. Moment of a force is given as,
$\vec{\tau} = \vec{r} \times \vec{F}$
ii. It depends upon the axis of rotation and the point of application of the force.

iii. It can produce translational acceleration also, if the axis of rotation is not fixed or if
friction is not enough.

iv. Its rotational effect can be balanced by a proper single force or by a proper couple.
Moment of a couple
i. Moment of a couple is given as,
$\vec{\tau} = \overrightarrow{r_{12}} \times \overrightarrow{F_1} = \overrightarrow{r_{21}} \times \overrightarrow{F_2}$
ii. It depends only upon the two forces, i.e., it is independent of the axis of rotation or the points of application of forces.

iii. Does not produce any translational acceleration, but produces only rotational or angular acceleration.

iv. Its rotational effect can be balanced only by another couple of equal and opposite torque.

xix) Why is the moment of a couple independent of the axis of rotation even if the axis is fixed?
Ans.

1. Consider a rectangular sheet free to rotate only about a fixed axis of rotation, perpendicular to the plane.

2. A couple of forces $\vec{F}$ $and - \vec{F}$
is acting on the sheet at two different locations.

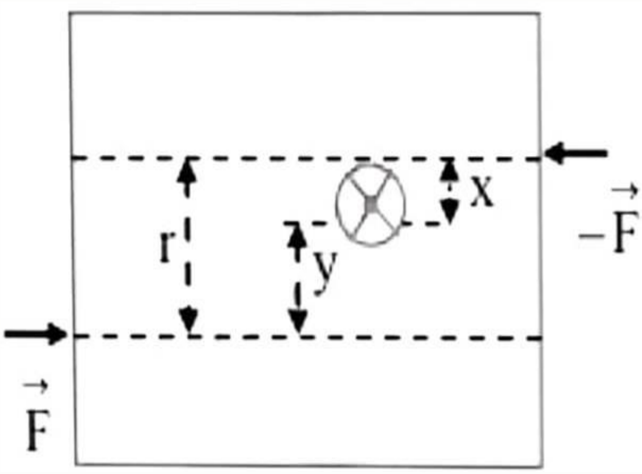
3. Consider the torque of the couple as two torques due to individual forces causing rotation about the axis of rotation.

4. **Case 1:** The axis of rotation is between the lines of action of the two forces constituting the couple. Let x and y be the perpendicular distances of the axis of rotation from the
forces $\vec{F}$ and $-\vec{F}$ respectively.

In this case, the pair of forces cause anticlockwise rotation. As a result, the direction of individual torques due to the two forces is the same.

$\therefore\ \tau = \tau_{+} + \tau = xF + yF = (x+y)F = rF$

...(1)



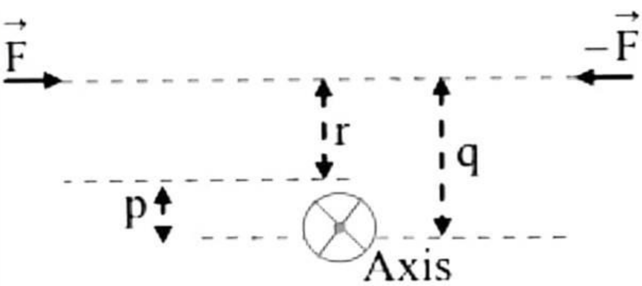
5. **Case 2:** Lines of action of both the forces are on the same side of the axis of rotation. Let q and p be the perpendicular distances of the axis of rotation from the
forces $\vec{F}$ and $-\vec{F}$ respectively.

In this case, the rotation of $+\vec{F}$
is anticlockwise, while that of $-\vec{F}$
is clockwise (from the top view). As a result, their individual torques are oppositely directed.

$\therefore \tau = \tau_{+} - \tau = qF - pF$

= (q - p) F = rF ......(2)

From equation (1) and (2), it is clear that that torque of a couple is independent of the axis of rotation.



xx) Explain balancing or mechanical equilibrium. Linear velocity of a rotating fan as a whole is generally zero. Is it in mechanical equilibrium? Justify your answer.

Ans.
A rigid body is said to be in equilibrium if there is no change in its state of motion. If there is no change in its velocity, the body is said to be in translational equilibrium. If there is no change in the rotational state of the body, it is said to be in rotational equilibrium.

(1) Condition for translational equilibrium: A rigid body is in translational equilibrium when the resultant force acting on it is zero. By Newton's first law of motion, a body remains in its natural state of constant velocity (including zero) if the net force on the body is zero. For translational equilibrium, the forces acting on the body must be concurrent. Thus, if F_1, F_2, F_3, ... are concurrent forces acting on a rigid body, the first condition of equilibrium is

$$\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \cdots = 0$$

Or

$$\sum \overrightarrow{F_i} = 0$$

(2) Condition for rotational equilibrium: A rigid body is in rotational equilibrium when the resultant torque acting on it is zero. If the forces acting on a rigid body are not concurrent, then their resultant may form a couple. Since the sum of the forces constituting a couple is zero, the first condition for equilibrium is satisfied. But the action of the couple is to change the state of rotation of the body. A body at rest will start rotating, or if it was initially rotating, it will speed up or slow down.
Thus, if $\overrightarrow{F_1}, \overrightarrow{F_2}, \overrightarrow{F_3}, \ldots$
are nonconcurrent forces acting on a rigid body and
$\vec{\tau}_1, \vec{\tau}_2, \vec{\tau}_3 \ldots$
are the respective torques produced by them about any point, the second condition for equilibrium is

$$\vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \cdots = 0$$

$$\sum \vec{\tau}_i = 0$$

[Is a fan rotating with constant angular velocity in rotational equilibrium? Justify.]

A ceiling/table fan is generally stationary. Hence it satisfies the condition for translational equilibrium. When the blades of the fan rotate with constant angular velocity, their angular acceleration is zero. Hence, it also satisfies the condition for rotational equilibrium. Therefore, a fan rotating with constant angular velocity is in mechanical equilibrium.

xxi) Why do we need to know the centre of mass of an object? For which objects, its position may differ from that of the centre of gravity?
Use g = 10 m s^{-2}, unless, otherwise stated.

Ans. Definition: The centre of mass of a system of particles or a rigid body is the point which moves in agreement with Newton's laws of motion as though all the mass of the system is concentrated at this point, and all the external forces act at this point.

The centre of gravity of a body does not coincide with its centre of mass when the body is in a nonuniform gravitational field or, equivalently, when the extent of the body is very large.

3. Solve the following problems.

i) A truck of mass 5 ton is travelling on a horizontal road with 36 km hr $^{-1}$ stops on traveling 1 km after its engine fails suddenly. What fraction of its weight is the frictional force exerted by the road? If we assume that the story repeats for a car of mass 1 ton i.e., can moving with same speed stops in similar distance same how much will the fraction be?

Solution:

Data: m = 5 ton = 5000 kg, u = 36 km/h
= 36000 m/3600 s = 10 m/s, v = 0, s = 1 km
= 1000 m, g = 10 m/s^2
$v^2 - u^2 = 2as$
$\therefore 0 - u^2 = 2$

$$\therefore a = -\frac{u^2}{2s} = -\frac{(10)^2}{2 \times 1000} = -\frac{100}{2000}$$

$$= -\frac{1}{20} m/s^2$$

The frictional force exerted by the road on the truck = |ma|

$$\frac{|ma|}{|mg|} = \left|\frac{a}{g}\right| = \frac{1/20}{10} = \mathbf{\frac{1}{200}}$$

This gives the required fraction. It has the same value for the truck and the car.

ii) A lighter object A and a heavier object B are initially at rest. Both are imparted the same linear momentum. Which will start with greater kinetic energy: A or B or both will start with the same energy?

Solution:

1. Let m_1 and m_2 be the masses of light object A and heavy objects B and v_1 and v_2 be their respective velocities.
2. Since both are imparted with the same linear momentum,
   $m_1v_1 = m_2v_2$
3. The kinetic energy of the lighter object A
   $= K.E._A = \frac{1}{2}m_2v_1^2$
   The kinetic energy of the heavier object B
   $= K.E._B = \frac{1}{2}m_2v_2^2$
   $$\therefore \frac{K.E._A}{K.E._B} = \frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2} = \frac{(m_1v_1)^2/m_1}{(m_2v_2)^2/m_2}$$
   $$\therefore \frac{K.E._A}{K.E._B} = \frac{m_2}{m_1} \quad ....[\because m_1v_1 = m_2v_2]$$
4. As $m_1 < m_2$, therefore $K.E._A > K.E._B$ i.e, the lighter body A has more kinetic energy.

iii) As I was standing on a weighing machine inside a lift it recorded 50 kg wt. Suddenly for few seconds it recorded 45 kg wt. What must have happened during that time? Explain with complete numerical analysis.

Solution:

Data: mg = 50 kg wt, mg - ma = 45 kg wt as the reading recorded by the weighing machine is less than mg, g = 10 m/s^2

$$\therefore \frac{mg - ma}{mg} = \frac{45}{50}$$

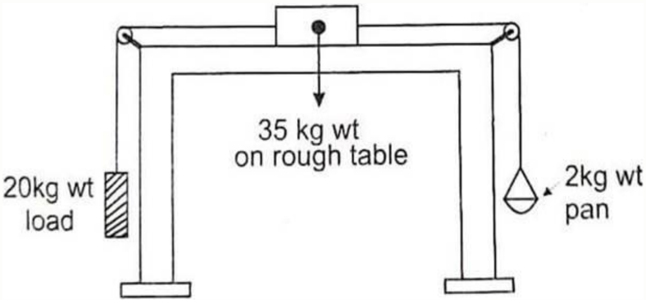
$$\therefore 1 - \frac{a}{g} = \frac{9}{10}$$

$$\therefore \frac{a}{g} = 1 - \frac{9}{10} = \frac{1}{10}$$

$\therefore a = \frac{g}{10} = \frac{10 m/s^2}{10} = 1\, m/s^2$

The lift must be coming down with this acceleration.

iv) Figure below shows a block of mass 35 kg resting on a table. The table is so rough that it offers a self adjusting resistive force 10% of the weight of the block for its sliding motion along the table. A 20 kg wt load is attached to the block and is passed over a pulley to hang freely on the left side. On the right side there is a 2 kg wt pan attached to the block and hung freely. Weights of 1 kg wt each, can be added to the pan. Minimum how many and maximum how many such weights can be added into the pan so that the block does not slide along the table?



Solution:

Let $\boldsymbol{m_B}$ and m_L

be the masses of the block and the hanging load, respectively. Let m_0 be the mass of the pan and m_1 the mass added to the pan.

Let f_s be the friction

(self-adjusting resistive) force on the block.

Data:

$m_B = 35kg,$

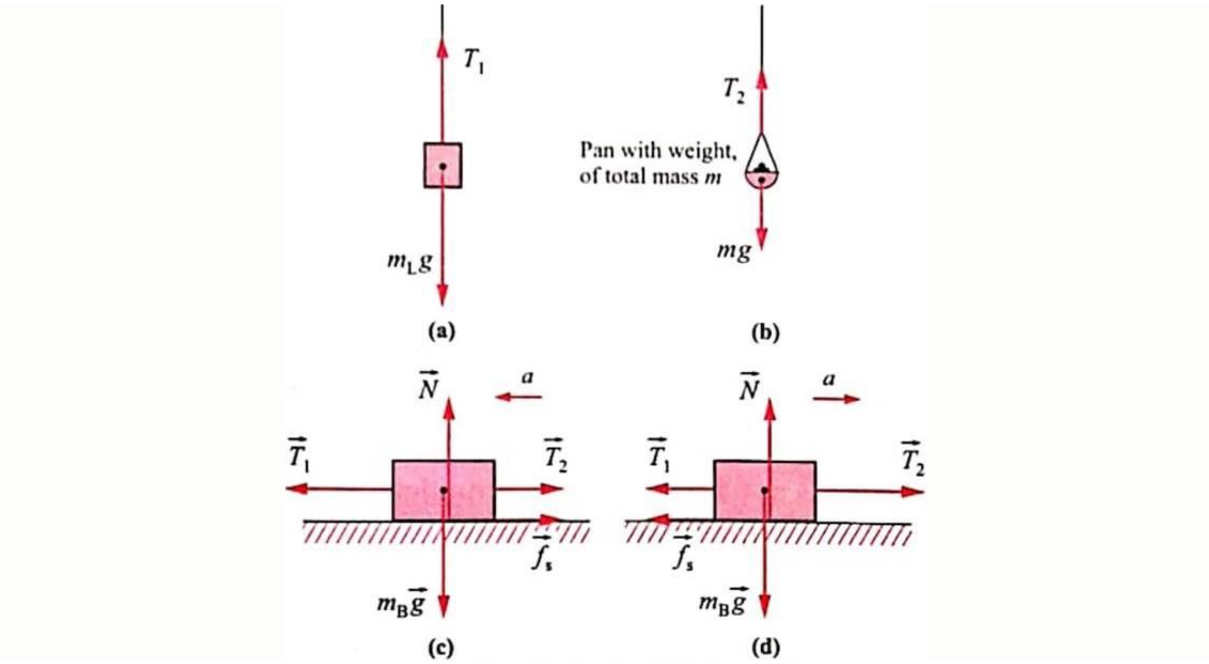
$m_L = 20kg,$

$m_0 = 2\ kg,$

$f = 10\%\ of\ m_B g = 0.1 m_B g$

The total mass on the right hand side is

$m = m_0 + m_1.$



From the free-body diagrams for the hanging load and the pan, shown in Figs. (a) and (b), for the system to be at rest,

$T_1 - m_L g = 0$

$\therefore T_1 = m_L g$

...(1)

$and\ T_2 - mg = 0$

$\therefore T_2 = mg$

...(2)

If the total mass on the right hand side, m, is less than a certain minimum, the block will tend to slide to the left,

so that f_s will be to the right

Fig.(c). Then,

$T_1 - T_2 - f_s = m_B a\ (a\ is\ to\ the\ laft)$

$\therefore m_L g - mg - f_s = m_B a$

...(3)

If m is more than a certain maximum, the block will tend to slide to the right,

so that f_s will be to the left,

Fig. (d). Then,

$T_2 - T_1 - f_s = m_B a\ (a\ is\ to\ the\ laft)$

$\therefore mg - m_L g - f_s = m_B a$

...(4)

For the block to be at rest, i.e., a = 0, Eqs. (3) and (4) respectively sets the minimum and maximum values of m.

From Eq. (3),

$(m_L - m - 0.1m_B)g = 0$

$\therefore m = m_L - 0.1m_B = 20kg - 0.1\ (35kg)$

$\therefore m_0 + m_1 = 20 - 3.5 = 16.5kg$

$\therefore m_1 = 16.5kg - m_0 = 16.5kg - 2kg = 14.5kg$

Hence, minimum 15 weights, each of 1 kg wt, can be added to the pan.

From Eq. (4),

$(m - m_L - 0.1m_B)g = 0$

$\therefore m = m_L + 0.1m_B = 20kg + 0.1\ (35kg)$

$\therefore m_0 + m_1 = 20 + 3.5 = 23.5kg$

$\therefore m_1 = 23.5kg - m_0 = 23.5kg - 2kg = 21.5kg$

Hence, maximum 21 weights, each of 1 kg wt, can be added to the pan.

v) Power is rate of doing work or the rate at which energy is supplied to the system. A constant force F is applied to a body of mass m. Power delivered by the force at time t from the start is proportional to

(a) t

(b) t^2

$(c)\sqrt{t}$

(d)t^0

Derive the expression for power in terms of F, m and t.

Ans.

(a) t

Derive the expression for power in terms of F, m and t.

Solution:

1. A constant force F is applied to a body of mass (m) initially at rest (u = 0).
2. We have, v = u + at

   $\therefore$ v = 0 + at

   $\therefore$ v = at ....(1)
3. Now, power is the rate of doing work,

   $\therefore P = \frac{dW}{dt}$

   $\therefore P = F \cdot \frac{ds}{dt}$ .... $[\because dW = F \cdot ds]$
4. But $\frac{ds}{dt} = v$, the instantaneous velocity of the particle.

   $\therefore$ P = F·v ....(2)
5. According to Newton's second law,

   F = ma .....(3)
6. Substituting equations (1) and (3) in equation (2)

   P = (ma)(at)

   $\therefore P = ma^2t$

   $\therefore P = \frac{m^2a^2}{m} \times t$

   $\therefore P = \frac{F^2}{m}t$
7. As F and m are constant, therefore, P $\propto$ t.

vi) 40000 litre of oil of density 0.9 g cc is pumped from an oil tanker ship into a storage tank at 10 m higher level than the ship in half an hour. What should be the power of the pump?

Solution:

Data : $V = 40000 \quad L = 40\ m^3$,

$\rho = 0.9\ g/cm^3 = 900\ kg/m^3$, $h = 10$ m,

$g = 10\ m/s^2$, $t = 30$ min $= 1800$ s

The power of the pump, $P = \frac{\text{work done}}{\text{time}}$

$$\therefore P = \frac{mgh}{t} = \frac{(V\rho)gh}{t}$$

$$= \frac{(40\ m^3)(900\ kg/m^3)(10\ m/s^2)(10\ m)}{(1800\ s)}$$

$$= 20 \times 100 = \mathbf{2000\ W = 2\ kW}$$

vii) Ten identical masses (m each) are connected one below the other with 10 strings. Holding the topmost string, the system is accelerated upwards with acceleration g/2. What is the tension in the 6th string from the top (Topmost string being the first string)?

Solution: For the mass at the top (Fig.),

$T_1 - (T_2 + mg) = ma$ (upward) ...(1)

For the subsequent masses,

$T_2 - (T_3 + mg) = ma$ ...(2)

$T_3 - (T_4 + mg) = ma$ ...(3)

$T_4 - (T_5 + mg) = ma$ ...(4)

$T_5 - (T_6 + mg) = ma$ ...(5)

Adding all these equations, we get,

$T_1 - T_6 - 5mg = 5\ ma$
$\therefore T_6 = T_1 - 5mg - 5ma$
Now, $T_1 - 10\ mg = 10\ ma$
$\therefore T_1 = 10\ mg + 10\ ma$
$\therefore T_6 = 10\ mg + 10\ ma - 5\ mg - 5ma$
$\therefore T_6 = 5mg + 5\ ma$
$= 5m\ (g + a)$
Now, $a = \frac{g}{2}$
$\therefore$The tension in the 6th string,
$T_6 = 5m\left(g + \frac{g}{2}\right) = 5m\left(\frac{3g}{2}\right)$
$= \frac{15}{2} mg$

=7.5mg
[Important Note: If we take $a = 0.2g$, then $T_6 = 5mg + 5\ m(0.2\ g) = 5mg + mg = \mathbf{6\ mg}$ which is the answer given in the textbook.]

viii) Two galaxies of masses 9 billion solar mass and 4 billion solar mass are 5 million light years apart. If, the Sun has to cross the line joining them, without being attracted by either of them, through what point it should pass?

<u>Solution:</u>

Data:
$r = r_A + r_B = 5 \times 10^6 ly$
$m_A = 9 \times 10^9 M_S, m_B = 4 \times 10^9 M_S,$
$M_S = solar\ mass$
For the net gravitational force on the Sun due to the two galaxies to be zero, the path of the Sun must be perpendicular to the line joining the two galaxies.
Let $r_A\ and\ r_B$
be the distances of the galaxies A and B from the Sun as it crosses the line joining the galaxies. At this point, the magnitudes of the gravitational force on the Sun due to the galaxies A and B are, respectively,

$$F_{SA} = \frac{Gm_A M_S}{r_A^2} \quad \text{and} \quad F_{SB} = \frac{Gm_B M_S}{r_B^2}$$

Since, $F_{SA} = F_{SB}$

$$\frac{m_A}{r_A^2} = \frac{m_B}{r_B^2}$$

$$\therefore \frac{r_A^2}{r_B^2} = \frac{m_A}{m_B} = \frac{9 \times 10^9 M_S}{4 \times 10^9 M_S} = \frac{9}{4}$$

$$\therefore \frac{r_A}{r_B} = \frac{3}{2} \quad \therefore r_B = \frac{2}{3} r_A$$

$$r_A + r_B = 5 \times 10^6 \text{ ly}$$

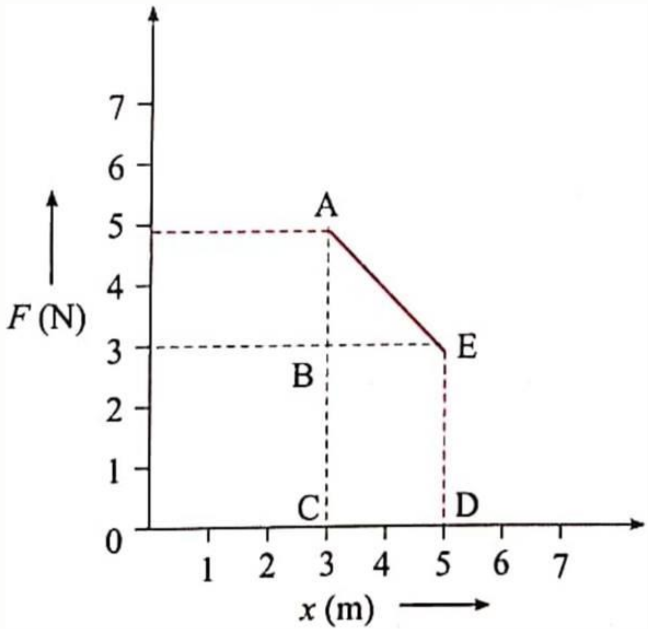
$$\therefore r_A + \frac{2}{3} r_A = \frac{5}{3} r_A = 5 \times 10^6 \text{ ly}$$

$$\therefore r_A = 3 \times 10^6 \text{ ly} \quad \text{and} \quad r_B = 2 \times 10^6 \text{ ly}$$

The neutral point on the line joining the two galaxies, where the sun experiences no net force due to the galaxies, is 3 million light years from the more massive galaxy.

ix) While decreasing linearly from 5 N to 3 N, a force displaces an object from 3 m to 5 m. Calculate the work done by this force during this displacement.

Solution:



From the graph (Fig.), the work done by the linearly varying force
= area of the trapezium ACDE
$= \frac{1}{2}(AC + DE) \times CD$

$$= \frac{1}{2}(5\,N + 3\,N) \times (5\,m - 3\,m)$$
$$= \frac{1}{2} \times 8 \times 2 = \mathbf{8J}$$

x) Variation of a force in a certain region is given by F = $6x^2$ - 4x - 8. It displaces an object from x= 1 m to x=2 m in this region. Calculate the amount of work done.

Solution:

$$W = \int_{x=1}^{x=2} (6x^2 - 4x - 8)$$

$\therefore$ W =

$$\int_{x=1}^{x=2} 6x^2dx - \int_{x=1}^{x=2} 4x^2dx - \int_{x=1}^{x=2} 8dx$$

$$= \left[\frac{6x^3}{3}\right]_1^2 - \left[\frac{4x^2}{2}\right]_1^2 - [8x]_1^2$$

= (16 - 2) - (8 - 2) - (16 - 8) = 0

The work done is **zero**.

xi) A ball of mass 100 g dropped on the ground from 5 m bounces repeatedly. During every bounce 64% of the potential energy is converted into kinetic energy. Calculate the following:

(a) Coefficient of restitution.

(b) Speed with which the ball comes up from the ground after third bounce.

(c) Impulse given by the ball to the ground during this bounce.

(d) Average force exerted by the ground if this impact lasts for 250 ms.

(e) Average pressure exerted by the ball on the ground during this impact if contact area of the ball is 0.5 cm^2.

Solution:

Data: m = 100 g = 0.1 kg, g = 10 m/s^2, h=5 m, conversion of 64% of PE into KE

during every bounce, t = 250 ms = 0.25 s,
A = 0.5 cm^2 = 0.5 × 10^{-4} m^2
(i) Just before the ball strikes the ground for the
first time, $\frac{1}{2}mv_1^2$ = mgh
Just after the first bounce,
$\frac{1}{2}mv_2^2 = 0.64\text{mgh}$
$\therefore \frac{{v_2}^2}{{v_1}^2} = 0.64$
$\therefore \frac{v_2}{v_1} = 0.8$
The coefficient of restitution
$e = \frac{|\text{velocity of separation}|}{|\text{velocity of approach}|}$
$= \frac{v_2}{v_1} = \mathbf{0.8}$

(ii) Initial PE = mgh = 0.1 × 10 × 5 = 5J
After the first bounce,
KE= 0.64 x 5J
After the second bounce,
KE=0.64 x 0.64 × 5J
After the third bounce,
KE = 0.64 x 0.64 × 0.64 × 5J
= $(0.64)^3$ × 5J
If v is the corresponding speed of the ball,
$\frac{1}{2}mv^2 = (0.64)^3 \times 5$
∴mv^2 = $(0.64)^3$ × 10
Now, m = 0.1 kg
∴0.1 v^2 = $(0.64)^3$ × 10
∴v^2= $(0.64)^3$ × 100
$\therefore \text{v} = 10 \times (0.64)^{\frac{3}{2}}$
= 10 × [(0.64)½]^3
$= 10 \times (0.8)^3 = 10 \times \frac{512}{1000} = \mathbf{5.12\ m/s}$

(iii) After the second bounce,
$\frac{1}{2}mv^2 = 0.64 \times 0.64 \times 5J$
$\therefore \frac{1}{2} \times 0.1v^2 = (0.64)^2 \times 5$
∴v^2 = $(0.64)^2$ × 10 × 10
∴v=0.64 x 10 = 6.4 m/S
Impulse = change in momentum
= 0.1 × 6.4 kg.m/s - (- 0.1 × 5.12 kg.m/s)
(considering the direction of the velocity)

$= 0.1 \times (6.4 + 5.12) = 0.1 \times 11.52$
$= 1.152$ kg.m/s
$= 1.152$ N.s

(iv)Impulse $\bar{F}t$ 1.152 N. s, t $= 0.25$ s

$\therefore Average\ force, \bar{F} = \frac{1.152}{0.25} = 4 \times 1.152$

$\therefore \boldsymbol{\bar{F} = 4.608\ N}$

(v) Average pressure $= \frac{\bar{F}}{A} = \frac{4.608}{0.5 \times 10^{-4}}$

$= 2 \times 4.608 \times 104$
$= 9.216 \times 10^4$ N/m^2
$= 9.216 \times 10^4$ Pa

xii) A spring ball of mass 0.5 kg is dropped from some height. On falling freely for 10 s, it explodes into two fragments of mass ratio 1:2. The lighter fragment continues to travel downwards with speed of 60 m/s. Calculate the kinetic energy supplied during explosion.

<u>Solution:</u>

Data : $m = 0.5$ kg, $g = 10$ m/s^2, free fall for 10 s,

$\frac{m_1}{m_2} = \frac{1}{2}$, $m_1 + m_2 = 0.5$ kg, v of $m_1 = 60$ m/s

$\therefore m_1 = \frac{m_2}{2}$

Also, $m_1 + m_2 = 0.5$ kg

$\therefore \frac{m_2}{2} + m_2 = 0.5$

$\therefore \frac{3}{2} m_2 = 0.5 = \frac{1}{2} \quad \therefore m_2 = \frac{1}{3}$ kg

$\therefore m_1 = \frac{m_2}{2} = \frac{1}{6}$ kg

Free fall of m : $v = gt = 10 \times 10 = 100$ m/s

The corresponding KE of $m = \frac{1}{2} mv^2$

$= \frac{1}{2} \times 0.5 \times (100)^2 = \frac{0.5 \times 100 \times 100}{2}$

$= 2500$ J

KE of $m_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} \times \frac{1}{6} \times (60)^2$

$= \frac{60 \times 60}{2 \times 6} = 300$ J

By momentum conservation,

$mv = m_1 v_1 + m_2 v_2$

$\therefore 0.5 \times 100 = \frac{1}{6} \times 60 + \frac{1}{3} v_2 = 10 + \frac{v_2}{3}$

$\therefore \frac{v_2}{3} = 50 - 10 = 40 \quad \therefore v_2 = 120$ m/s

$\therefore$ KE of $m_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \times \frac{1}{3} \times (120)^2$

$= \frac{120 \times 120}{6} = 20 \times 120 = 2400$ J

Total KE of the fragments $= 300 + 2400$

$= 2700$ J

KE of the ball just before the explosion $= 2500$ J

Hence, the KE supplied during the explosion

$= 2700$ J $- 2500$ J $=$ **200 J**

xiii) A marble of mass 2m travelling at 6 cm/s is directly followed by another marble of mass m with double speed. After collision, the heavier one travels with the average initial speed of the two. Calculate the coefficient of restitution.

Data : m $\xrightarrow[12\text{ cm/s}]{u_1}$ $2m$ $\xrightarrow[6\text{ cm/s}]{u_2}$

$$v_2 = \frac{12+6}{2} = \frac{18}{2} = 9 \text{ cm/s}$$

By momentum conservation,

$$mu_1 + 2mu_2 = mv_1 + 2mv_2$$

$$\therefore\ 12m + 12m = mv_1 + 18\ m$$

$$\therefore\ v_1 = 24 - 18 = 6 \text{ cm/s}$$

$$\therefore\ v_2 - v_1 = 9 - 6 = 3 \text{ cm/s}$$

$$u_1 - u_2 = 12 - 6 = 6 \text{ cm/s}$$

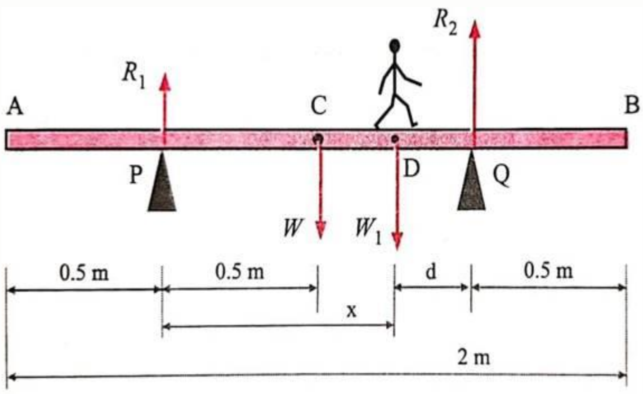
$\therefore$ The coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{3}{6} = \mathbf{0.5}$$

xiv) A, 2 m long wooden plank of mass 20 kg is pivoted (supported from below) at 0.5 m from either end. A person of mass 40 kg starts walking from one of these pivots to the farther end. How far can the person walk before the plank topples?

Solution:

Data: In Fig, AB = 2 m,
AP = PC = CQ = QB = 0.5 m, W = 20 kg wt,
W_1 = 40 kg wt



Suppose the person starts walking from pivot P towards Q. Also, suppose the plank begins to topple when the person is at point D, a distance x from P. At that instant, the upward reaction R_1 at P becomes zero.
Taking moments about pivot Q,

$$W \times QC + W_1 \times QD + (-R_2) \times 0 = 0$$

$$\therefore W_1 \times QD = -W \times QC$$

$$\therefore QD = d = \frac{W \times QC}{W_1}$$

$= \frac{(20kg\ wt)(0.5m)}{40kg\ wt}$

=-0.25m

|d| = 0.25m

PD = x= PQ-|d| = 1 m -0.25 m = 0.75 m

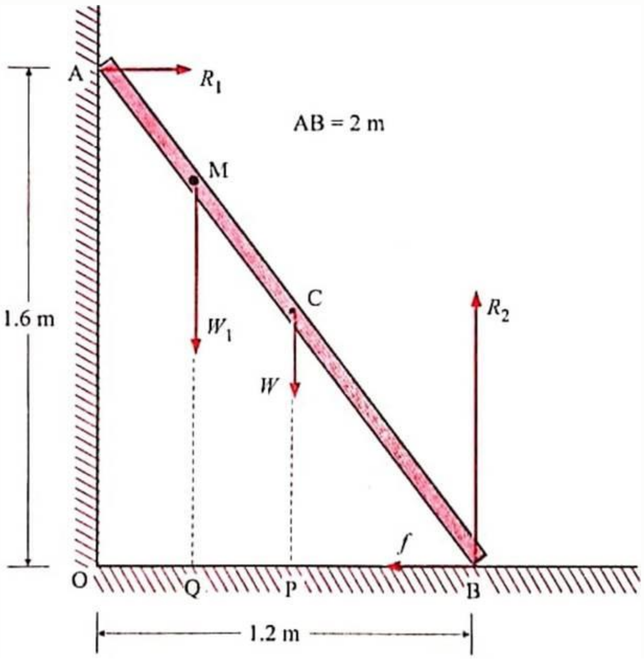
∴The person can walk up to 0.75 m from pivot P, i.e., 1.25 m from end A.

xv) A 2 m long ladder of mass 10 kg is kept against a wall such that its base is 1.2 m away from the wall. The wall is smooth but the ground is rough. Roughness of the ground is such that it offers a maximum horizontal resistive force (for sliding motion) half that of normal reaction at the point of contact. A monkey of mass 20 kg starts climbing the ladder. How far can it climb along the ladder? How much is the horizontal reaction at the wall?

Solution:

Data: From Fig., AB = L = 2 m,

W = 10 kg wt, OB = 1.2 m, W_1 = 20 kg wt, f = $0.5R_2$



Since ΔOAB is a right triangle, right-angled at O,

$OA = \sqrt{AB^2 - OB^2}$

$= \sqrt{(2m)^2 - (1.2m)^2}$

$= \sqrt{4 - 1.44}$

$= \sqrt{2.56} = 1.6m$

According to the first condition for equilibrium,

$\sum F_y = 0 \Rightarrow R_2 - W - W_1 = 0$

...(1)

and $\sum F_x = 0 \Rightarrow R_1 - f = 0$

...(2)

From Eq. (1)

$R_2 = W + W_1 = (10 \text{ kg wt}) + (20 \text{ kg wt})$

$= 30 \text{ kg wt}$

From Eq. (2) and data,

$R_1 = f = 0.5\ R_2 = 0.5\ (30 \text{ kg wt}) = 15 \text{ kg wt}$

∴ The horizontal reaction at the wall,

$R_1 = = 15 \text{ kg wt} = (15 \text{ kg})\ (10 \text{ m/s}^2) = 150 \text{ N}$

From the second condition for equilibrium, taking moments about B,

$- R_1 \times OA + W_1 \times BQ + W \times BP + (R_2) \times 0 + (- f) \times 0 = 0$

$\therefore W_1 \times BQ = R_1 \times OA - W \times BP$

$BP = \frac{1}{2}\ BE = \frac{1}{2}\ (1.2m) = 0.6m$

$\therefore BQ = \dfrac{(15kg\ wt)(1.6m) - (10kg\ wt)(0.6m)}{20kg\ wt}$

$= \dfrac{24 - 6}{20} = 0.9m$

$\dfrac{BM}{BA} = \dfrac{BQ}{BE}$

$\therefore BM = (2m)\dfrac{0.9m}{1.2m} = \mathbf{1.5m}$

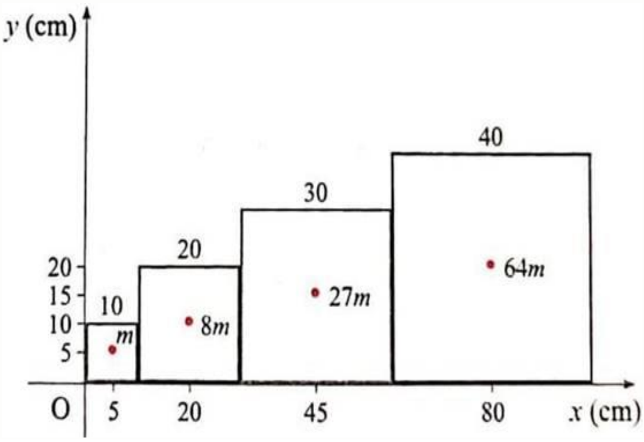
∴ The monkey can climb up 1.5 m along the ladder.

xvi) Four uniform solid cubes of edges 10 cm, 20 cm, 30 cm and 40 cm are kept on the ground, touching each other in order. Locate centre of mass of their

system.

<u>Solution :</u>

Date: $L_1 = 10$ cm, $L_2 = 20$ cm, $L_3 = 30$ cm, $L_4 = 40$ cm.



$\frac{L_2}{L_1} = 2 \qquad \therefore \left(\frac{L_2}{L_1}\right)^3 = 8$

$\frac{L_3}{L_1} = 3 \qquad \therefore \left(\frac{L_3}{L_1}\right)^3 = 27,$

$\frac{L_4}{L_1} = 4 \qquad \therefore \left(\frac{L_4}{L_1}\right)^3 = 64$

Mass = volume × density; volume = L^3 Let $m_1 = m$ = mass of cube A

∴ Mass of cube B, $m_2 = 8m$; mass of cube C,

$m_3 = 27m$; mass of cube D, $m_4 = 64m$

∴ $m_1 + m_2 + M_3 + m_4 = m + 8m + 27m + 64m$

$=100m$

From the figure, $x_1 = 5$ cm, $x_2 = 20$ cm,

$x_3 = 45$ cm, $x_4 = 80$ cm

$\therefore X_{cm} \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$

$= \frac{m \times 5 + m \times 20 + 5 \times 45 + m \times 80}{100m}$

$= \frac{5 + 160 + 1215 + 5120}{100} = \frac{6500}{100}$

= 65 cm

Also $y_1 = 5$ m, $y_2 = 10$ cm, $y_3 = 15$ cm,

$y_4 = 20$ cm

$\therefore Y_{CM} = \frac{\sum_i miyi}{\sum_i mi} = \frac{5 + 8 \times 10 + 27 \times 15 + 64 \times 20}{100}$

$= \frac{5 + 80 + 405 + 1280}{100}$

$= \frac{1770}{100}$

=17.7cm

Similarly, $z_1 = 5$ cm, $z_2 = 10$ cm, $z_3 = 15$ cm,

$2_4 = 20$ cm

$\therefore \boldsymbol{Z_{CM} = 17.7\ cm}$

xvii) A uniform solid sphere of radius R has a hole of radius R/2 drilled inside it. One end of the hole is at the centre of the sphere while the other is at the boundary. Locate centre of mass of the remaining sphere.

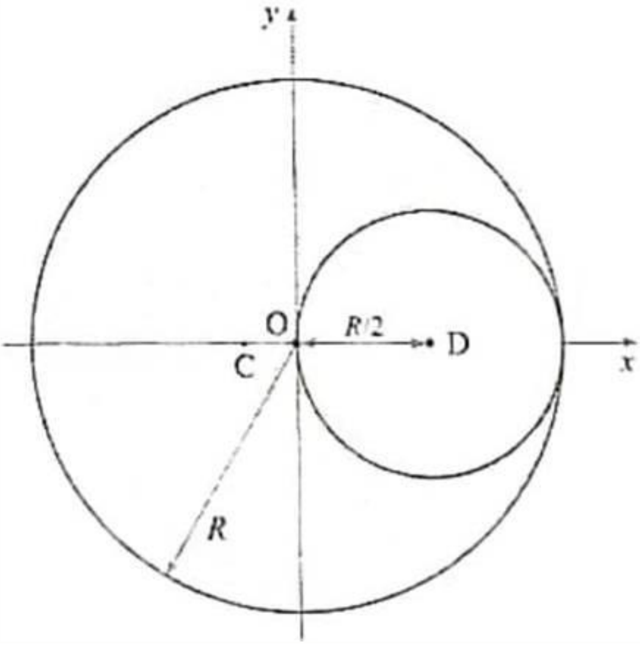
<u>Solution:</u>

Let M = mass of the sphere of radius R,

ρ = density of the material of the sphere

$\rho = \frac{mass}{volume}$

$\therefore M = \left(\frac{4}{3}\pi R^3\right)\rho$



If we imagine the hole (spherical cavity) to be filled with the same material as that of the sphere, the mass of this (smaller) sphere

$$= \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho = \frac{M}{8}, M - \frac{M}{8} = \frac{7M}{8}$$

Treating the hole as a sphere with mass

$$-\frac{M}{8},$$

the X coordinate of the centre of mass (C) of the remaining sphere will be

$$X_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{0 - \left(\frac{M}{8}\right)\left(\frac{R}{2}\right)}{7\frac{M}{8}}$$

$$= -\frac{R}{16} \times \frac{8}{7} = -\frac{\boldsymbol{R}}{\boldsymbol{14}}$$

xviii) In the following table, every item on the left side can match with any number of items on the right hand side. Select all those.

Types of collision	Illustrations
(a) Elastic collision (b) Inelastic collision (c) Perfectly inelastic collision (d) Head on collision	(i) A ball hit by a bat. (ii) Molecular collisions responsible for pressure exerted by a gas. (iii) A stationary marble A is hit by marble B and the marble B comes to rest. (iv) A blob of clay dropped on the ground sticks to the ground. (v) Out of anger, giving a kick to a wall. (vi) A striker hits the boundary of a carrom board in a direction perpendicular to the boundary and rebounds.

Ans.

(a)-(ii);

(b)-(i), (iii), (iv), (v), (vi);

(c)-(iii), (iv);

(d)-(iii), (iv).