CAT 2022 Question Paper Slot 2

Quant

45. Mr. Pinto invests one-fifth of his capital at 6%, one-third at 10% and the remaining at 1%, each rate being simple interest per annum. Then, the minimum number of years required for the cumulative interest income from these investments to equal or exceed his initial capital is

46. The average of a non-decreasing sequence of N numbers $a_1, a_2, ..., a_N$ is 300. If a_1, is replaced by $6a_1$, the new average becomes 400. Then, the number of possible values of a_1, is

47. The number of integer solutions of the equation $\left(x^2-10\right)^{\left(x^2-3x-10\right)}=1$ is

48. Manu earns ₹4000 per month and wants to save an average of ₹550 per month in a year. In the first nine months, his monthly expense was ₹3500, and he foresees that, tenth month onward, his monthly expense will increase to ₹3700. In order to meet his yearly savings target, his monthly earnings, in rupees, from the tenth month onward should be

A 4400

B 4200

C 4300

D 4350

49. In triangle ABC, altitudes AD and BE are drawn to the corresponding bases. If $\angle BAC = 45^\circ$ and $\angle ABC = \theta$, then $\frac{AD}{BE}$ equals

A $\sqrt{2}\cos\theta$

B $\frac{(\sin\theta+\cos\theta)}{\sqrt{2}}$

C 1

D $\sqrt{2}\sin\theta$

50. Let $f(x)$ be a quadratic polynomial in x such that $f(x) \geq 0$ for all real numbers x. If f(2) = 0 and f(4) = 6, then f(-2) is equal to

A 12

B 24

C 6

D 36

51. Let r and c be real numbers. If r and -r are roots of $5x^3 + cx^2 - 10x + 9 = 0$, then c equals

A $-\frac{9}{2}$

B $\frac{9}{2}$

C -4

D 4

52. Two ships meet mid-ocean, and then, one ship goes south and the other ship goes west, both travelling at constant speeds. Two hours later, they are 60 km apart. If the speed of one of the ships is 6 km per hour more than the other one, then the speed, in km per hour, of the slower ship is

A 20

B 12

C 18

D 24

54. In an examination, there were 75 questions. 3 marks were awarded for each correct answer, 1 mark was deducted for each wrong answer and 1 mark was awarded for each unattempted question. Rayan scored a total of 97 marks in the examination. If the number of unattempted questions was higher than the number of attempted questions, then the maximum number of correct answers that Rayan could have given in the examination is

55. Regular polygons A and B have number of sides in the ratio 1 : 2 and interior angles in the ratio 3 : 4. Then the number of sides of B equals

56. In an election, there were four candidates and 80% of the registered voters casted their votes. One of the candidates received 30% of the casted votes while the other three candidates received the remaining casted votes in the proportion 1 : 2 : 3. If the winner of the election received 2512 votes more than the candidate with the second highest votes, then the number of registered voters was

A 50240

B 40192

C 60288

D 62800

57. On day one, there are 100 particles in a laboratory experiment. On day n, where $n \geq 2$, one out of every n articles produces another particle. If the total number of particles in the laboratory experiment increases to 1000 on day m, then m equals

A 19

B 16

C 18

D 17

58. The number of integers greater than 2000 that can be formed with the digits 0, 1, 2, 3, 4, 5, using each digit at most once, is

A 1440

B 1200

C 1480

D 1420

59. For some natural number n, assume that (15,000)! is divisible by (n!)!. The largest possible value of n is

A 4

B 7

C 6

D 5

60. Working alone, the times taken by Anu, Tanu and Manu to complete any job are in the ratio 5 : 8 : 10. They accept a job which they can finish in 4 days if they all work together for 8 hours per day. However, Anu and Tanu work together for the first 6 days, working 6 hours 40 minutes per day. Then, the number of hours that Manu will take to complete the remaining job working alone is

61. There are two containers of the same volume, first container half-filled with sugar syrup and the second container half-filled with milk. Half the content of the first container is transferred to the second container, and then the half of this mixture is transferred back to the first container. Next, half the content of the first container is transferred back to the second container. Then the ratio of sugar syrup and milk in the second container is

A 4 : 5

B 6 : 5

C 5 : 4

D 5 : 6

62. Consider the arithmetic progression 3, 7, 11, ... and let A_n denote the sum of the first n terms of this progression. Then the value of $\frac{1}{25}\sum_{n=1}^{25} A_n$ is

A 455

B 442

C 415

D 404

63. The number of distinct integer values of n satisfying $\frac{4-\log_2 n}{3-\log_4 n} < 0$, is

64. If a and b are non-negative real numbers such that $a+ 2b = 6$, then the average of the maximum and minimum possible values of $(a+ b)$ is

A 3

B 4

C 3.5

D 4.5

65. Five students, including Amit, appear for an examination in which possible marks are integers between 0 and 50, both inclusive. The average marks for all the students is 38 and exactly three students got more than 32. If no two students got the same marks and Amit got the least marks among the five students, then the difference between the highest and lowest possible marks of Amit is

A 22

B 21

C 24

D 20

66. The length of each side of an equilateral triangle ABC is 3 cm. Let D be a point on BC such that the area of triangle ADC is half the area of triangle ABD. Then the length of AD, in cm, is

A $\sqrt{8}$

B $\sqrt{6}$

C $\sqrt{7}$

D $\sqrt{5}$

Answers

Quant

45.**20**	46.**14**	47.**4**	48.**A**	49.**D**	50.**B**	51.**A**	52.**C**
53.**12**	54.**24**	55.**10**	56.**D**	57.**A**	58.**A**	59.**B**	60.**6**
61.**D**	62.**A**	63.**47**	64.**D**	65.**D**	66.**C**		

Explanations

Quant

45. **20**

Let the total investment me 15x and the no. of years required be T years

$\frac{(3x\times6\times T)}{100}+\frac{(5x\times10\times T)}{100}+\frac{(7x\times1\times T)}{100}\geq 15x$

or, $\frac{75xT}{100}\geq 15x$

or, $T\geq 20$

So minimum value of T is 20 years

46. **14**

$a_1+a_2+.....+a_N=300N$

$6a_1+a_2+.....+a_N=400N$

$5a_1=100N$

$a_1=20N$

As the given sequence of numbers is non-decreasing sequence, N can take values from 2 to 15.

N is not equal to 1, if N = 1, then average of N numbers is 300 wouldn't satisfy.

Therefore, N can take values from 2 to 15, i.e. 14 values.

47. **4**

Case 1: When $x^2-3x-10=0$ and $x^2-10\neq\ 0$

$x^2-3x-10=0$ or, $(x-5)(x+2)=0$

or, x= 5 or -2

Case 2: $x^2-10=1$

$x^2-11=0$

No integer solutions

Case 3: $x^2-10=-1\ and\ x^2-3x-10\ is\ even$

$x^2-9=0$

or, (x+3)(x-3)=0

or, x= -3 and 3

for x= -3 and +3 $x^2-3x-10$ is even

In total 4 values of x satisfy the equations

48. **A**

Savings target in a year = 550*12 = Rs 6600

Saving in first 9 months = 9(4000-3500) = Rs 4500

Saving for remaining 3 months should be 6600-4500, i.e. Rs 2100

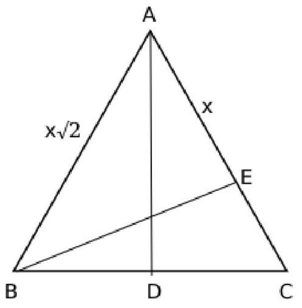
Savings for each month in last 3 months = $\frac{2100}{3}$ = Rs 700

It is given, monthly expenses in last 3 months = Rs 3700

This implies, his monthly earnings from 10th month should be 3700+700, i.e. Rs 4400

The answer is option A.

49. **D**



It is given, Angle BAE = 45 degrees

This implies AE = BE

Let AE = BE = x

In right-angled triangle ABD, it is given $\angle ABC = \theta$

$\sin\theta = \frac{AD}{AB}$

$\sin\theta = \frac{AD}{x\sqrt{2}}$

$\sqrt{2}\sin\theta = \frac{AD}{BE}$

The answer is option D.

50. **B**

$f(x) \geq 0$for all real numbers x, so D<=0

Since f(2)=0 therefore x=2 is a root of f(x)

Since the discriminant of f(x) is less than equal to 0 and 2 is a root so we can conclude that D=0

Therefore f(x) = $a\left(x-2\right)^2$

f(4)=6

or, 6 = $a\left(x-2\right)^2$

a= 3/2

$f\left(-2\right) = -\frac{3}{2}\left(-4\right)^2 = 24$

51. **A**

Let the roots of the given equation $5x^3 + cx^2 - 10x + 9 = 0$ be r, -r and p

r - r + p = $-\frac{c}{5}$

p = $-\frac{c}{5}$ ...... (1)

$-r^2 - pr + pr = -2$

$r^2 = 2$ ...... (2)

$-r^2p = -\frac{9}{5}$

$p = \frac{9}{10}$ ...... (3)

Substituting p in (1), we get

$\frac{9}{10} = -\frac{c}{5}$

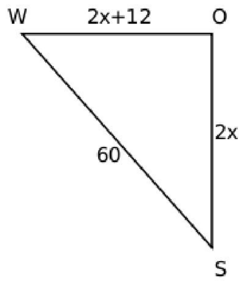
$-\frac{9}{2} = c$

The answer is option A.

52. **C**

Let the speeds of two ships be 'x' and 'x+6' km per hour

Distance covered in 2 hours will be 2x and 2x+12



It is given,

$$(2x)^2 + (2x+12)^2 = 60^2$$

$$(x)^2 + (x+6)^2 = 30^2$$

$$2x^2 + 12x + 36 = 900$$

$$x^2 + 6x + 18 = 450$$

$$x^2 + 6x - 432 = 0$$

Solving, we get x = 18

The speed of slower ship is 18 kmph

The answer is option C.

53. **12**

Given,

$f(x) + f(x-1) = 1$ ...... (1)

$f(x^2 - x) = 5$ ...... (2)

$g(x) = x^2$

Substituting x = 1 in (1) and (2), we get

f(0) = 5

f(1) + f(0) = 1

f(1) = 1 - 5 = -4

f(2) + f(1) = 1

f(2) = 1 + 4 = 5

f(n) = 5 if n is even and f(n) = -4 if n is odd

f(g(5)) + g(f(5)) = f(25) + g(-4) = -4 + 16 = 12

54. **24**

Let the number of questions attempted be x+y out of which x are correct and y are incorrect and the number of questions unattempted be z.

It is given,

x + y + z = 75 ...... (1)

3x - y + z = 97 ...... (2)

(2)-(1) -> x - y = 11

(1)+(2) -> 2x + z = 86

z > x + y

z > 75 - z

z > 37.5

Minimum possible value of z is 38

2x + 38 = 86

2x = 48

x = 24

The maximum number of correct answers is 24.

55. **10**

Let the number of sides of polygons A and B be n and 2n, respectively.

$$\frac{\frac{(n-2)\,180}{n}}{\frac{(2n-2)\,180}{2n}} = \frac{3}{4}$$

$$\frac{n-2}{n-1} = \frac{3}{4}$$

$$4n - 8 = 3n - 3$$

$$n = 5$$

The number of sides of polygon B is 2*5, i.e. 10.

56. **D**

Let the number of registered votes be 100x

The number of votes casted = 80x

Votes received by one of the candidates = $\frac{30}{100} \times 80x$ = 24x

Remaining votes = 80x - 24x = 56x

Votes received by other three candidates is $\frac{56x}{6}$, $\frac{2\times56x}{6}$, $\frac{3\times56x}{6}$

It is given,

28x - 24x = 2512

4x = 2512

x = 628

The number of registered votes = 100x = 62800

The answer is option D.

57. **A**

Given, the number of particles on day 1 = 100

On day 2, one out of every 2 articles produces another particle.

The number of particles on day 2 will be $\frac{100}{2}$, i.e. 50 particles

On day 3, one out of every 3 articles produces another particle.

The number of particles on day 3 will be $\frac{100+50}{3}$, i.e. 50 particles

On day 4, one out of every 4 articles produces another particle.

The number of particles on day 4 will be $\frac{100+50+50}{4}$, i.e. 50 particles

Series will be 100, 50, 50, 50,....

It is given,

100 + (m-1)50 = 1000

m = 19

The answer is option A.

58. **A**

Case 1: 4-digit numbers

Given digits - 0, 1, 2, 3, 4, 5

_, _, _, _

As the numbers should be greater than 2000, first digit can be 2, 3, 4 and 5.

For remaining digits, we need to arrange 3 digits from the remaining 5 digits, i.e. 5*4*3 = 60 ways

Total number of possible 4-digit numbers = 4*60 = 240

Case 2: 5-digit numbers

_, _, _, _, _

First digit cannot be zero.

Therefore, total number of cases = 5*5*4*3*2 = 600

Case 3: 6-digit numbers

_, _, _, _, _, _

First digit cannot be zero.

Therefore, total number of cases = 5*5*4*3*2*1 = 600

Total number of integers possible = 600 + 600 + 240 = 1440

The answer is option A.

59. **B**

To find the largest possible value of n, we need to find the value of n such that n! is less than 15000.

7! = 5040

8! = 40320 > 15000

This implies 15000! is not divisible by 40320!

Therefore, maximum value n can take is 7.

The answer is option B.

60. **6**

Let the time taken by Anu, Tanu and Manu be 5x, 8x and 10x hours.

Total work = LCM(5x, 8x, 10x) = 40x

Anu can complete 8 units in one hour

Tanu can complete 5 units in one hour

Manu can complete 4 units in one hour

It is given, three of them together can complete in 32 hours.

32(8 + 5 + 4) = 40x

x = $\frac{68}{5}$

It is given,

Anu and Tanu work together for the first 6 days, working 6 hours 40 minutes per day, i.e. 36 + 4 = 40 hours

40(8 + 5) + y(4) = 40x

4y = 24

y = 6

Manu alone will complete the remaining work in 6 hours.

61. **D**

Initially	1st-Sugar syrup-100L	2nd - Milk - 100L
After step 1	Sugar Syrup - 50L	SS- 50L, Milk - 100L
After step 2	SS - 50+25 = 75L Milk - 50L	SS - 25L, Milk - 50L
After step 3	Milk - 25L SS - 37.5L	Milk - 50 + 25 = 75L SS - 25 + 37.5 = 62.5L

Step 1: Half the content of the first container is transferred to the second container

Step 2: Half of the mixture of second container is transferred back to the first container

Step 3: Half the content of the first container is transferred back to the second container

Sugar syrup : Milk in second container = 62.5 : 75 = 5 : 6

The answer is option D.

62. **A**

Sum of n terms in an A.P = $\frac{n}{2}\left(2a+(n-1)\,d\right)$

$$A_n = \frac{n}{2}\left(6+(n-1)\,4\right) = n\,(2n+1)$$

$$\Sigma\, A_n = \Sigma\, n\,(2n+1) = 2\Sigma\, n^2 + \Sigma\, n = \frac{2n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

Substituting n = 25, we get

$$\frac{1}{25}\sum_{n=1}^{25} A_n = \frac{1}{25}\left(\frac{2(25)(25+1)(50+1)}{6} + \frac{25(25+1)}{2}\right)$$

$$\frac{1}{25}\sum_{n=1}^{25} A_n = 26\,(17)+13 = 455$$

The answer is option A.

63. **47**

Let $\log_2 n = y$

$$\frac{4-y}{3-\frac{y}{2}} < 0$$

$$(4-y)\left(3-\frac{y}{2}\right) < 0$$

$$(4-y)\,(6-y) < 0$$

$$(y-4)\,(y-6) < 0$$

$$4 < y < 6$$

$$4 < \log_2 n < 6$$

$$2^4 < n < 2^6$$

$16 < n < 64$

n can take values from 17 to 63(inclusive).

The number of n values possible = 47

64. **D**

a + 2b = 6

From the above equation, we can say that maximum value b can take is 3 and minimum value b can take is 0.

a + b + b = 6

a + b = 6 - b

a + b is maximum when b is minimum, i.e. b = 0

Maximum value of a + b = 6 - 0 = 6

a + b is minimum when b is maximum, i.e. b = 3

Minimum value of a + b = 6 - 3 = 3

Average = $\frac{6+3}{2}$ = 4.5

The answer is option D.

65. **D**

The average marks for all the students is 38.

Sum = 5*38 = 190

To find the minimum marks scored by Amit, we need to maximise the score of remaining students.

Maximum scores sum of remaining students = 50 + 49 + 48 + 32 = 179

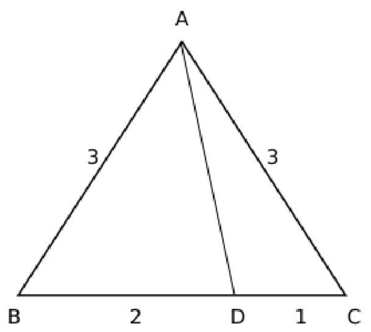
Minimum possible score of Amit = 190 - 179 = 11

It is given, Amit scored least. This implies maximum possible score of Amit is 31.

Difference = 31 - 11 = 20

The answer is option D.

66. **C**



Area of triangle ABD is twice the area of triangle ACD

$\angle ADB = \theta$

$\frac{1}{2}(AD)(BD)\sin\theta = 2\left(\frac{1}{2}(AD(CD)\sin(180 - \theta))\right)$

$BD = 2CD$

Therefore, BD = 2 and CD = 1

$\angle ABC = \angle ACB = 60^\circ$

Applying cosine rule in triangle ADC, we get

$\cos \angle ACD = \frac{AC^2 + CD^2 - AD^2}{2(AC)(CD)}$

$\frac{1}{2} = \frac{9+1-AD^2}{6}$

$AD^2 = 7$

$AD = \sqrt{7}$

The answer is option C.

VIDEO SOLUTION