Chapter 9
Algebra

Exercise 9.1

Question 1.

Find the rule which gives the number of matchsticks required to make the following matchsticks patterns. Use a variable to write he rule.

(i) A Pattern of letter T as T

(ii) A Pattern of letter V as V

(iii) A Pattern of letter Z as Z

(iv) A Pattern of letter U as U

(v) A Pattern of letter F as F

(vi) A Pattern of letter S as S

Solution:

(i) Number of matchsticks required = 2n

(ii) Number of matchsticks required = 2n

(iii) Number of matchsticks required = 3n

(iv) Number of matchsticks required = 3n

(v) Number of matchsticks required = An

(vi) Number of matchsticks required = 5n

Question 2.

If there are 24 mangoes in a box, how will you write the number of mangoes in terms of the number of boxes ? (use b for the number of boxes.)

Solution:

Total number of mangoes = 24b

Question 3.

Anuradha is drawing a dot Rangoli (a beautiful pattern of lines joining dots). She has 8 dots in a row. How many dots will her Rangoli have for rows ? How many dots are there if there are 12 rows ?

Solution:

∵ Numbers of dots in 1 row = 8

∵ Number of dots in r rows = $8 \times r = 8$

Number of dots in 12 rows = $12 \times 8 = 96$

Question 4.

Anu and Meenu are sisters. Anu is 5 years younger than Meenu. Can you write Anu's age in terms of Meenu's age ? Take Meenu's age as x years.

Solution:

Yes ! we can write Anu's age in terms of Meenu's age.

Age of Meenu = x

∵ Anu is 5 years younger than Meenu

∴ Age of Anu = (x – 5) years

Question 5.

Oranges are to be transferred from larger boxes to smaller boxes. When a larger box is empited, the oranges from it fill 3 samller boxes and still 7 oranges are lest. If the numbre of oranges in a small box are taken to be x, then what is the number of oranges in the larger box ?

Solution:

Let the number of oranges in a smaller box be x.

∴ Number of oranges in three smaller boxes = 3x

Number of oranges remained outside = 7

∴ Number of oranges in the larger box = 3x + 7

Question 6.

Harsha's score in Mathematics is 15 more than three-fourth of her score in Science. If she scores x marks in Science, find her score in Mathematics ?
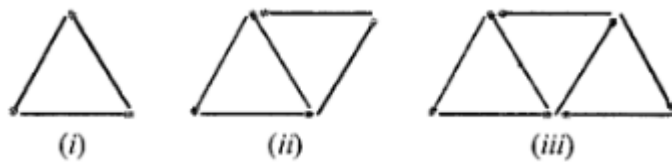
Solution:

Let the score of science be x

Harsha score's in Mathematics = $\frac{3}{4}$th of $x + 15$

∴ Score of Harsha's in Mathematics = $\frac{3}{4}x + 15$

Question 7.

Look at the following matchstick pattern of equilateral triangles. The triangles are not sepaeate. Two neighbouring triangles have a common

matchstick. Observe the pattern and find the rule that gives the number of matchsticks.

(i)   (ii)   (iii)

Solution:

Figure	Number of triangles	Number of matchsticks	Pattern
(i)	1	3	$2 \times 1 + 1$
(ii)	2	5	$2 \times 2 + 1$
(iii)	3	7	$2 \times 3 + 1$

Number of matchsticks required $= 2x + 1,$ where x is the number of triangles.

Question 8.

Look at the following matchsticks pattern of letter A. The A's are not separate. Two neighbouring A's have two common matchsticks. Observe the pattern and find the rule that gives the number of matchsticks.
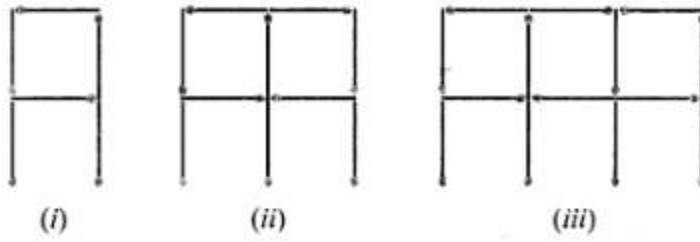
(i)   (ii)   (iii)

Solution:

Figure	Number of letter 'A'	Number of matchsticks	Pattern
(i)	1	6	$4 \times 1 + 2$
(ii)	2	10	$4 \times 2 + 2$
(iii)	3	14	$4 \times 3 + 2$

Number of matchsticks required = $4x + 2$,

where x is the number of letter 'A' formed.

Exercise 9.2

Question 1.

If the side of an equilateral triangle is 1, then express the perimeter of the triangle in terms of 1.

Solution:
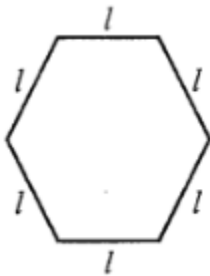
Perimeter (P) of the equilateral triangle with side l = Sum of the lengths of sides of the equilateral triangle = 1 + 1 + 1 = 3l

Question 2.

The side of a regular hexagon is 1. Express its perimeter in terms of 1.

Solution:

Perimeter (P) of the regular hexagon with side 1 = Sum of the lengths of all sides of the regular hexagon.

= 1 + 1 + 1+ 1 + 1+ 1 = 6l

Question 3.

The length of an edge of a cube is l. Find the formula for the sum of lengths of all the edges of the cube.
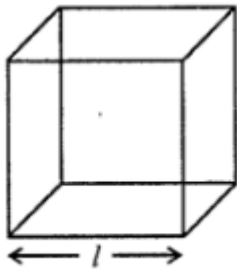
Solution:

Total length (L) of the edges of a cube = Sum of the lengths of all (12) edges of the cube.

$$= l+l+l+l+l+l+l+l+l+l+l+l = 12l$$

Question 4.

If the radius of a circle is r units, then express the legngth of a diameter of the circle in terms of r.

Solution:

Draw a cicle :
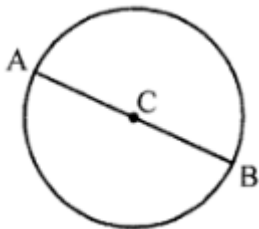
Let AB is a diameter of the circle

C its centre

$AB = AC + CB$

$\Rightarrow d = r + r$

$\Rightarrow d = 2r$

Exercise 9.3

Question 1.

Form four expressions with numbers 7, 5 and 8 (no variables) using operations of addition, subtraction or multiplication with the condition that every number should be used but not more than once.

Solution:

The possible expressions are:

$5 \times 7 + 8, 5 \times 8 - 7$

$(5 + 8) - 7, 8 \times (5 + 7)$

Question 2.

Which out of following are expressions with numbers only?

(i) $2y + 3$

(ii) $(7 \times 20) - 82$

(iii) $5 \times (21 - 7) + 9 \times 2$

(iv) $5 - 11n$

(v) $(5 \times 4) - 45 + p$

(vi) $3 \times (11 + 7) - 24 + 3$

Solution:

(iii) $5 \times (21 - 7) + 9 \times 2$

(vi) $3 \times (11 + 7) - 24 + 3$ are expressions with numbers only.

Question 3.

Identify the Operations (addition, subtraction, multiplication, division) in forming the following expressions and tell how the expressions have been formed:

(i) $x + 5$

(ii) $y - 7$

(iii) $3z$

(iv) $\frac{p}{5}$

(v) $2x + 17$

(vi) $3y - 5$

(vii) $-7m + \frac{2}{3}$

(viii) $\frac{x}{3} - 15$

Solution:

(i) $x + 5$

Addition on → 5 added to x.

(ii) $y - 7$

Subtraction on → 7 subtracted from y.

(iii) $3z$

Multiplication → z multiplied by 3.

(iv) $\frac{p}{5}$

Division $\rightarrow$ p divided by 5.

(v) $2x + 17$

Multiplication and addition $\rightarrow$ First x multiplies by 2 and then 17 added to the product.

(vi) $3y - 5$

Multiplication and Subtraction $\rightarrow$ First y multiplied by 3 and then 5 Subtracted from the product.

(vii) -7m + 23

Multiplication and addition $\rightarrow$ first m multiplied by -7 and then $\frac{2}{3}$ added to the product.

(viii) $\frac{x}{3} - 15$

Division and Subtraction $\rightarrow$ First x divided by 3 and then 15 subtracted from the quotient.

Question 4.

Write expression for the following :

(i) 7 added to p

(ii) p subtracted from 7

(iii) p multiplies by 7

(iv) p divided by 7

(v) 7 divided by p

(vi) 7 subtracted from -m

(vii) p multiplied by -5

(viii) –p divided by 5

Solution:

(i) p + 7

(ii) 7 – p

(iii) 7p

(iv) $\frac{p}{7}$

(v) $\frac{7}{p}$

(vi) –m -7

(vii) -5p

(viii) $\frac{-p}{5}$

Question 5.

Write expression for the following:

(i) 11 added to 2m

(ii) 11 subtracted from 2m

(iii) 3 added to 5 times y

(iv) 3 subtracted from 5 times y

(v) y is multiplies by -8 and then 5 is added to the result

(vi) y is multiplies by 5 and then the result is subtracted from 16.

Solution :

(i) 2m + 11

(ii) 2m - 11

(iii) 5y +3

(iv) 5y - 3

 (v) -8y + 5

(vi) 16 – 5y

Question 6:

Write the following in mathematical form using signs and symbols :

(i) 6 more than thrice a number x.

(ii) 7 taken away from y.

(iii) 3 less than quotient of x by y.

Solution;

(i) $3x + 6$

(ii) $\frac{x}{y} - 3$

(iii) $y - 7$

Question 7

Form six expressions using t and 4. Use not more than one number operation and every expression must have t in it.

Solution:

$$t + 4, t - 4, 4 - t, 4t, \quad \frac{t}{4}, \frac{4}{t}$$

Question.8

Form expressions using y, 2 and 7. Use only two different number Operations and every expression must have y in it.

Solution:

$$2y + 7, 2y - 1, 7y + 2, 7y - 2, \frac{y}{2} + 7, \frac{y}{2} - 7$$

Question 9.

A student scored x marks in English but the teacher deducted 5 marks for bad handwriting. What was the student's fina; score in English ?

Solution:

Marks in English = x

Deducted = 5

Final score = $x - 5$

Question 10.

Raju's father's age is 2 years more than 3 times Raju's age. If Raju's present age is y years, then what is his father's age?

Solution:

(3y + 2) years

Question 11.

Mohini is x years old. Express the following in algebric form:

(i) three times Mohini's age next year.

(ii) four time Mohini's age 3 years ago.

(iii) the present age of Mohini's uncle, if his uncle is 5 times as old as Mohini will be two years from now.

(iv) the present age of Mohini's cousin, if her cousin is two years less than one-third of Mohini's age five years ago.

Solution:

(i) $3(x + 1) years$

(ii) $4(x - 3) years$

(iii) $5(x + 2)$ years

(iv) $\left[\frac{1}{3}(x - 5) - 2\right]$ years

Question 12.

A cubodial box has height h cm. Its length is 5 times the height and breadth is 10 cm less than the length. Express the length and the breadth of the box in terms of the height.

Solution:

Length of the box = 5h cm

Breadth of the box = (5h – 10) cm

Question 13.

A bus travels at v km per hour. It is going from Delhi to jaipur. After the bus has travelled 5 hours, jaipur is still 20km away. What is the distance from Delhi to Jaipur?

Solution:

Speed of the bus = v km/ hr

Distance travelled in 5 hours = 5v km

∴ Total distane = (5v + 20) km

Question 14.

Change the following statements using expressions into staements in ordinary language :

(i) A notebook cost ₹p. A book cost ₹ 3p.

(ii) The cost of rice per kg is ₹p. The cost of oil per litre is ₹5p.

(iii) The speed of a truck is v km per hour. The speed of a bus is (v + 10) km per hour.

(iv) Tony's box contains 8 times the marbles he puts on the table.

(v) The total number of students in the school is 20 times that of our class.

(vi) Raju is x years old. His uncle is 4x years old and his aunt is (4x-3) years old.

(vii) In arrsngement of dots there are r rows. Each row contains 5 dots.

Solution:

(i) The cost of a book is 3 times the cost of a note boook.

(ii) The cost of oil per litre is 5 time the cost of rice per kg.

(iii) The speed of a bus is 10 km per hour more than the speed of a truck.

(iv) Tony puts q marbles on the table. He has 8q marbles in his box.

(v) Our class has n students. The school has 20n students.

(vi) Raju's uncle is 4 times oldere than Raju and his aunt is 3 years.

(vii) The total number of dots is 5 times the number of rows.

Exercise 9.4

Question 1.

Find the value of the following:

(i) 4^3

(ii) $(-6)^4$

(iii) $\left(\dfrac{2}{3}\right)^4$

(iv) $(-2)^3 \times 5^2$

Solution:

(i) $4^3 = 4 \times 4 \times 4 = 64$

(ii) $(-6)^4 = (-6) \times (-6) \times (-6) \times (-6)$

(iii) $\left(\dfrac{2}{3}\right)^4 = \dfrac{2}{3} \times \dfrac{2}{3} \times \dfrac{2}{3} \times \dfrac{2}{3}$

$= \dfrac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3} = \dfrac{16}{81}$

(iv) $(-2)^3 \times 5^2$

$= (\text{-}2) \times (\text{-}2) \times (\text{-}2) \times (5) \times (5)$

$= (\text{-}8) \times 25 = \text{-}200$

Question 2.

Find the value of:

(i) 3x + 2y when $x = 3$ and $y - 2$

(ii) $5x - 3y$ when $x = 2$ and $y = -5$

(iii) $a + 2b - 5c$ when $a = 2, b = -3 \ and \ c = 1$

(iv) $2p + 3q + 4r + pqr$ when p = -1, q = 2 and r =3

(v) $3ab + 4bc - 5ca \ when \ a = 4, 6 = 5and \ c = -2.$

Solution:

(i) $3x + 2y, x = 3, y = 2$

$(3 \times 3) + (2 \times 2) = 9 + 4 = 13$

(ii) $5x - 3y, \quad x = 2, \quad y = -5$

$(5 \times 2) - (3 \times -5) = 10 + 15 = 25$

(iii) $a + 2b - 5c, \quad a = 2, b = -3, c = 1$

$2 + (2 \times -3) - 5 \times (1)$

$= 2 \times -6 - 5 = -9$

(iv) $2p + 3q + 4r + pqr, \quad p = -1, \quad q = 2, \quad r = 3$

$= (2 \times -1) + (3 \times 2) + (4 \times 3) + (-1) \times 2 \times 3$

$= -2 + 6 + 12 - 6 = 10$

(v) $3ab + 4bc - 5ca, a = 4, b = 5, c = -2 \ (3 \times 4 \times 5) + (4 \times 5 \times -2) - 5 \times -2 \times 4$

$= 60 - 40 + 40 = 60$

Question 3.

Find the value of:

(i) $2x^2 - 3x + 4$ when $x = 2$

(ii) $4x^3 - 5x^2 - 6x + 7$ when $x = 3$

(iii) $3x^3 + 9x^2 - x + 8$ when $x = -2$

(iv) $2x^4 - 5x^3 + 7x - 3$ when $x = -3$

Solution:

(i) $2x^2 - 3x + 4$, $x = 2$

$= 2 \times (2)^2 - 3x^2 + 4$

$= 8 - 6 + 4 = 6$

(ii) $4x^3 - 5x^2 - 6x + 7$, $x = 3$

$= 4(3)^3 - 5(3)^2 - 6(3) + 7$

$= 108 - 45 - 18 + 7 = 52$

(iii) $3x^3 + 9x^2 - x + 8$, x = -2

$= 3(-2)^3 + 9(-2)^2 - (-2) + 8$

$=$ -24 $+ 36 + 2 + 8 = 22$

(iv) $2x^4 - 5x^3 + 7x - 3$, $x = -3$

$= 2(-3)^4 - 5(-3)^3 + 7(-3) - 3$

$= 162 + 135 - 21 - 3 = 273$

Question 4.

If $x = 5$, find the value of:

(i) $6 - 7x^2$

(ii) $3x^2 + 8x - 10$

(iii) $2x^3 - 4x^2 - 6x + 25$

Solution :

(i) $6 - 7x^2$

$= 6 - -7(5)^2 = 6 - 7(25)$

$= 6 - 175 = -169$

(ii) $3(5)^2 + 8(5) - 10$

$= 3(25) + 40 - 10$

$= 75 + 40 - 10$

$= 75 + 30$

$= 105$

(iii) $2(5)^3 - 4(5)^2 - 6(5) + 25$

$= 2(125) - 4(25) - 30 + 25$

$= 250 - 100 - 30 + 25$

$= 145$

Question 5.

If x =2, y = 3 and z = -1, find the values of:

(i) x + y

(ii) $\frac{xy}{z}$

(iii) $\frac{2x+3y-4z}{3x-z}$

Solution:

(i) $x + y$

x = 2, y =3

$2 \div 3 = \frac{2}{3}$

(ii) $\frac{xy}{z} = \frac{2 \times 3}{-1} = -6$

(iii) $\frac{2x+3y-4z}{3x-z}$

$= \frac{2 \times 2 + 3 \times 3 - 4 \times (-1)}{3 \times 2 - (-1)}$

$= \frac{4+9+4}{6+1}$

$= \frac{17}{7}$

$= 2\frac{3}{7}$

Question 6.

If $a = 2, b = 3$ and $c = -2$, find the value of $a^2 + b^2 + c^2 - 2ab - 2bc - -2ca + 3abc$.

Solution:

$a = 2, b = 3$ and $c = -2$

$= a^2 + b^2 + c^2 - 2ab - 2bc - 2ca + 3abc$

$= (2)^2 + (3)^2 + (-2)^2 - 2 \times 23 - 2 \times 3 \times -2 - 2 \times -2 \times 2 + 3 \times 2 \times 3 \times -2$

$= 4 + 9 + 4 - 12 + 12 + 8 - 36$

$= 25 - 36 = $ -11

Question 7.

If $p = 4, q = -3$ and $r = 2$ find the value of : $P^3 + q^3 - r^3 - 3pqr$.

Solution:

P = 4, q = -3, r = 2

$= P^3 + q^3 - r^3 - 3pqr$

$= (4)^3 + (-3)^3 - (2)^3 - 3 \times 4 \times -3 \times 2$

$= 64 - 27 - 8 + 72$

$= 136 - 35 = 101$

Question 8.

If $m = 1, n = 2$ and $p = -3$, find the value of $2mn^4 - 15m^2n + p$.

Solution:

m = 1, n = 2, p = -3

$= 2mn^4 - 15m^2n + p$

$= 2 \times 1 \times (2)^4 - 15 \times (1)^2 \times (2) + (-3)$

$= 32 - 30 - 3 = $ -1

Question 9.

State true or false :

(i) The value of $3x - 2$ is 1 when x =0.

(ii) The value of $2x^2 - x - 3$ is 0 when $x = -1$.

(iii) $p^2 + q^2 - r^2$ when p =5, q = 12 and r = 13.

(iv) $16 - 3x = 5x$ when $x = 2$.

Solution:

(i) The value of $3x - 2$ is 1 when $x = 0. False$

Correct :

$\because$ $3 \times 0 - 2 = -2$

(ii) The value of $2x^2 - x - 3$ is 0 when $x = -1$. True

$2(-1)^2 - (-1) - 3$

$= 2 + 1 - 3 = 0$

(iii) $p^2 + q^2 = r^2$ when p = 5, q = 12 and r = 13. **True**

$(5)^2 + (12)^2 = (13)^2$

$= 25 + 144 = 169$

$= 169 = 169$

(iv) 16-3x = 5x when x = 2 . True

$16 - 3x^2 = 5x^2$

$16 - 6 = 10$

$\Rightarrow 10 = 10$

Question 10.

For $x = 2$ and $y = -3$, verify the following:

(i) $(x + y)^2 = x^2 + 2xy + y^2$

(ii) $(x - y)^2 = x^2 - 2xy + y^2$

(iii) $x^2 - y^2 = (x + y)(x - y)$

(iv) $(x + y)^2 = (x - y)^2 + 4xy$

(v) $(x + y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$

Solution:

$x = 2$ and $y = -3$

(i) $(x + y)^2 = x^2 + 2xy + y^2$

L.H.S. $= (x + y)^2 = (2 - 3)^2 = (-1)^2 = 1$

R.H.S. $= x^2 + 2xy + y^2$

$= (2)^2 + 2 \times 2(-3) + (-3)^2$

$= 4 - 12 + 9 = 13 - 12 = 1$

L.H.S. $=$ R.H.S.

Exercise 9.5

Question 1.

State which of the following are equations with a variable. In case of an equation with a variable, identity the variable.

(i) $17 + x = 5$

(ii) $2b - 3 = 7$

(iii) $(y-7) > 5$

(iv) $\frac{9}{3} = 3$

(v) $7 \times 3 - 19 = 2$

(vi) $5 \times 4 - 8 = 31$

(vii) $2p < 15$

(viii) $7 = 11 \times 5 - 12 \times 4$

(ix) $\frac{3}{2}q = 5$

Solution:

(i) $17 + x = 5$ is an equation → L.H.S. = R.H.S. → Related variable x.

(ii) $2b - 3 = 7$ *is an equation* → L.H.S. = R.H.S. → Related variable b.

(iii) $(y -7) > 5$
is not an equation → L.H.S. ≠ R.H.S.
It has no sign of equality (=).

(iv) $\frac{9}{3} = 3$

Is an equation = L.H.S. = R.H.S.

It has no variable.

(v) $7 \times 3 - 19 = 2$

Is an numerical equation = L.H.S. = R.H.S. it has no variable.

(vi) $5 \times 4 - 8 = 31$

Is an equation = L.H.S. = R.H.S. It has no variable.

(vii) $2p < 15$

Is not an equation = L.H.S. $\neq$ R.H.S.

It has no sign of equality.

(viii) $7 = 11 \times 5 - 12 \times 4$

Is an numerical equation = L.H.S. = R.H.S. It has no variable.

(ix) $\frac{3}{2}q = 5$

Is an equation $\rightarrow$ L.H.S. = R.H.S.$\rightarrow$ Related variable q.

Question 2.

Solve each of the following equation:

(i) x = 6 = 8

(ii) $2 - x = 5$

(iii) $4x = -6$

(iv) $\frac{x}{2} = 5$

(v) $2y - 3 = 2$

(vi) $4 - 5y = 2$

Solution:

(i) x = 6 = 8

$= x = 8 - 6 \Rightarrow x = 2$

(ii) $2 - x = 5$

$= -x = 5 - 2 \Rightarrow -x = 3 \Rightarrow x = -3$

(iii) $4x = -6$

$= x = \frac{-6}{4} = \frac{-3}{2}$

(iv) $\frac{x}{2} = 5$

$= x = 5 \times 2$

$x = 10$

(v) $2y - 3 = 2$

$= 2y = 2 + 3$

$\Rightarrow 2y = 5$

$\Rightarrow y = \dfrac{5}{2}$

(vi) $4 - 5y = 2$

$= 4 - 2 = 5y \Rightarrow 5y = 2$

$\Rightarrow y = \dfrac{2}{5}$

Question 3.

Solve the following linear equations:

(i) $5 (x + 1) = 25$

(ii) $2(3x - 1) = 25$

(iii) $3x - 14 = 11$

Solution:

(i) Given:

$5 (x + 1) = 25$

$\Rightarrow \dfrac{5(x+1)}{5} = \dfrac{25}{5}$ (dividing both sides by 5)

$\Rightarrow x + 1 = 5$

$\Rightarrow x + 1 - 1 = 5 - 1$ (Subtracting 1 from both sides)

$\Rightarrow x = 4$

(ii) $2(3x - 1) = 10$

$= \left(\frac{2(3x-1)}{2}\right) = \frac{10}{2}$ (dividing both sides by 2)

$\Rightarrow 3x - 1 = 5$

$\Rightarrow 3x - 1 + 1 = 5 + 1$ (*adding 1 to both sides*)

$\Rightarrow 3x = 6$

$\Rightarrow \frac{3x}{6} = \frac{6}{3}$ (dividing both sides by 3)

$\Rightarrow x = 2$

(iii) Given $\frac{3x-1}{4} = 11$

$\Rightarrow 4 \times \frac{3x-1}{4} = 4 \times 11$ (multiplying both sides by 4)

$\Rightarrow 3x - 1 = 44$

$\Rightarrow 3x - 1 + 1 = 44 + 1$ (adding 1 to both sides)

$\Rightarrow 3x = 45$

$\Rightarrow \frac{3x}{3} = \frac{45}{3}$ (dividing both sides by 3)

$\Rightarrow x = 15$

Question 4.

Solve the following linear equations:

(i) $5x - 6 = 12 - x$

(ii) $\frac{n}{3} + 1 = 4 - n$

(iii) $5p - 6 = 12 - x$

(iv) $2x + \dfrac{5}{2} = \dfrac{2}{3} - x$

(v) $\dfrac{x}{2} - 5 = \dfrac{x}{3} - 4$

(vi) $18 - \dfrac{3y}{4} = 11 + y$

Solution:

(i) $5x - 6 = 12 - x$

$\Rightarrow 5x + x = 12 + 6$

$6x = 18 \Rightarrow x = \dfrac{18}{6} = 3$

Verification

$5x - 6 = 12 - x$

$\Rightarrow 5(3) - 6 = 12 - 3$

$\Rightarrow 15 - 6 = 9$

$\Rightarrow 9 = 9$

(ii) $\dfrac{n}{3} + 1 = 4 - n$

$\dfrac{n+3}{3} = \dfrac{4-n}{1}$

$\Rightarrow 3(4 - n) = 1(n + 3)$

$\Rightarrow 12 - 3n = n + 3$

$\Rightarrow -3n - n = 3 - 12$

$\Rightarrow$ -4n = -9

$\Rightarrow n = \dfrac{-9}{-4} = \dfrac{9}{4}$

Verification

$5p + 7 = 19 - 2p \Rightarrow 5\left(\dfrac{12}{7}\right) + 7 = 19 - 2\left(\dfrac{12}{7}\right)$

$\Rightarrow \dfrac{60}{7} + 7 = 19 - \dfrac{24}{7} \Rightarrow \dfrac{60+49}{7} = \dfrac{133-24}{7}$

$\Rightarrow \dfrac{109}{7} = \dfrac{109}{7}$

(iv) $2x + \dfrac{5}{2} = \dfrac{2}{3} - x$

$\dfrac{4x+5}{2} = \dfrac{2-3x}{3}$

$\Rightarrow 3(4x + 5) = 2(2 - 3x)$

$\Rightarrow 12x + 15 = 4 - 6x$

$\Rightarrow 12x + 6x = 4 - 15$

$\Rightarrow 18x = -11$

$\Rightarrow x = \dfrac{-11}{18}$

Verification :

$\Rightarrow 2x + \dfrac{5}{2} = \dfrac{2}{3} - x$

$\Rightarrow 2\left(\dfrac{-11}{18}\right) + \dfrac{5}{2} = \dfrac{2}{3} - \dfrac{-11}{18}$

$\Rightarrow \dfrac{-22}{18} + \dfrac{5}{2} = \dfrac{2}{3} + \dfrac{11}{18}$

$$\Rightarrow \frac{-22+45}{18} = \frac{12+11}{18}$$

$$\Rightarrow \frac{23}{18} = \frac{23}{18}$$

(v) $\frac{x}{2} - 5 = \frac{x}{3} - 4$

$$\Rightarrow \frac{x-10}{2} = \frac{x-12}{3}$$

$$\Rightarrow 3(x - 10) = 2(x - 12)$$

$$\Rightarrow 3x - 30 = 2x - 24$$

$$\Rightarrow 3x - 2x = -24 + 30$$

$$\Rightarrow x = 6$$

verification

$$\Rightarrow \frac{x}{2} - 5 = \frac{x}{3} - 4$$

$$\Rightarrow \frac{6}{2} - 5 = \frac{6}{3} - 4$$

$$\Rightarrow 3 - 5 = 2 - 4$$

$$\Rightarrow -2 = -2$$

(vi) $18 - \frac{3y}{4} = 11 + y$

$$\Rightarrow \frac{72 - 3y}{4} = 11 + y$$

$$\Rightarrow 72 - 3y = 44 + 4y \text{ (cross multiplication)}$$

$$\Rightarrow \mathbf{-3y - 4y = 44 - 72}$$

$\Rightarrow$ -7y = - 28

$\Rightarrow$ y = $\frac{-28}{-7}$ = 4

Verification

18 - $\frac{3y}{4}$ = 11 + y

$\Rightarrow$ 18 - $\frac{3 \times 4}{4}$ = 11 + 4

$\Rightarrow$ 18 – 3 = 15

$\Rightarrow$ 15 = 15

Question : 5

Solve the following equations and verify your answers :

(i) $3(+ 7) = 18$

(ii) $2(- 1) = + 2$

(iii) $3x - \frac{1}{3} = 2\left(x - \frac{1}{2}\right) + 5$

(iv) $4(2x - 1) - 2(x - 5) = 5(x + 1) + 3$

Solution:

(i) $3(x + 7) = 18$

$\Rightarrow 3x + 21 = 18$

$\Rightarrow 3x = 18 - 21$

$\Rightarrow 3x = -3$

$\Rightarrow x = \frac{-3}{3}$

$\Rightarrow x = -1$

Verification

$\Rightarrow 3(x + 7) = 18$

$\Rightarrow 3(-1 + 7) = 18$

$\Rightarrow 3(6) = 18$

$\Rightarrow 18 = 18$

(ii) $2(x - 1) = x + 2$

$\Rightarrow 2x - 2 = x + 2$

$\Rightarrow 2x - x = 2 + 2$

$\Rightarrow x = 4$

Verification

$\Rightarrow 2(x - 1) = x + 2$

$\Rightarrow 2(4-1) = 4+2$

$\Rightarrow 2(3) = 6$

$\Rightarrow 6 = 6$

(iii) $3x - \frac{1}{3} = 2\left(x - \frac{1}{2}\right) + 5$

$\frac{9x-1}{3} = 2\left(\frac{2x-1}{2}\right) + 5$

$\Rightarrow \frac{9x-1}{3} = \frac{4x-2}{2} + 5$

$\Rightarrow \frac{9x-1}{3} = \frac{4x-2+10}{2}$

$\Rightarrow 2(9x-1) = 3(4x+8)$

$\Rightarrow 18x - 2 = 12x + 24$

$\Rightarrow 18x - 12x = 24 + 2$

$\Rightarrow 6x = 26$

$\Rightarrow x = \frac{26}{6} = \frac{13}{3} = 4\frac{1}{3}$

Verification

$\Rightarrow 3x - \frac{1}{3} = 2\left(x - \frac{1}{2}\right) + 5$

$\Rightarrow 3\left(\frac{13}{3}\right) - \frac{1}{3} = 2\left(\frac{13}{3} - \frac{1}{2}\right) + 5$

$\Rightarrow \frac{39-1}{3} = 2\left(\frac{23}{6}\right) + 5$

$\Rightarrow \frac{38}{3} = \frac{23}{3} + 5$

$\Rightarrow \frac{38}{3} = \frac{23+15}{3}$

$$\Rightarrow \frac{38}{3} = \frac{38}{3}$$

(iv) $4(2x - 1) - 2(x - 5) = 5(x + 1) + 3$

$\Rightarrow 8x - 4 - 2x + 10 = 5x + 5 + 3$

$\Rightarrow 8x - 2x - 4 + 10 = 5x + 5 + 3$

$\Rightarrow 6x + 6 = 5x + 8$

$\Rightarrow 6x - 5x = 8 - 6$

$\Rightarrow x = 2$

Verification :

$4(2x - 1) - 2(x - 5) = 5(x + 1) + 3$

$\Rightarrow 4 (2 \times 2\text{-}1) - 2(2\text{-}5) = 5(2+1)+3$

$\Rightarrow 4 (4 - 1) - 2 (\text{-}3) = 5 (3) + 3$

$\Rightarrow 4 (3) - 2 (\text{-}3) = 15 + 3$

$\Rightarrow 12 + 6 = 18$

$\Rightarrow 18 = 18$

Objective Types Questions

Mental Maths

Question 1.

Fill in the blanks:

(i) In algebra, we use ………to present variables (generalized numbers).

(ii) A symbol or letter which can be given various numerical values is called a………….

(iii) If jaggu'ss present age is x years, then his age 7 years from now is ……….

(iv) If one pen costs ₹X x, then the cost of 9 pens is……….

(v) An equation is a statements that the two expressions are…………..

(vi) Trial an error is one of methods to obtain ………….of an equation.

(vii) 7 less than thrice a number y is………..

(viii) If 3x +4 = 19, then the value of x is………

(ix) The number of pencils bought for

(viii) If 3x + 4 = 19, then the value of x is ……………

(ix) The number of pencils bought for ₹x at the rate of ₹2 per pencil is……….

(x) In the expression $(-7)^5$, base = ……… and exponent = ………..

(xi) If base = 6 and exponent – 5, then the exponetial from = ………….

Solution:

(i) In algebra, we use **letters** to present variables (generalized numbers).

(ii) A symbol or letter which can be given various numerical values is
called a **variable.**

(iii) If jaggu'ss present age is x years, then his age 7 years from now is
(x+7) years

(iv) If one pen costs ₹X x, then the cost of 9 pens is ₹9x.

(v) An equation is a statements that the two expressions are **equal.**

(vi) Trial an error is one of methods to obtain **The solution** of an
equation.

(vii) 7 less than thrice a number y is $3y - 7$.

(viii) If 3x +4 = 19, then the value of x is **5.**

(ix) The number of pencils bought for ₹x at the rate of ₹2 per pencil
is $\frac{x}{2}$.

(x) In the expression $(-7)^5$, base = -7 and exponent = 5.

(xi) If base = 6 and exponent – 5, then the exponetial from = 6^5.

Question 2.

State whether the following statements are true (T) or false (F):

(i) If x is variable then 5x is also variable.

(ii) If y is variable then y – 5 is also variable.

(iii) The number of angles in a triangle is a variable.

(iv) The value of an algebraic expression changes with the change in the value of the variable.

(v) If the length of a rectangle is twice its breadth, then its area is a constant.

(vi) An equation is satisfied only for a definite value of the variable.

(vii) if x toffees are distributed equally among 5 children, then each child gets 5x toffees.

(viii) t minutes are equal to 60 t seconds.

(ix) If x is a negative integer, then –x is a positive integer.

(x) $x = 5$ is a solution of the equation $3x + 2 = 13$

(xi) $2y - 7 > 13$ is an equation.

(xii) 'One third of a number x added to itself gives 8' can be expressed as $\frac{x}{3} + 8 = x$.

(xiii) The difference between the ages of two sisters Lata and Asha is a variable.

Solution:

(i) If x is variable then 5x is also variable. **True**

(ii) If y is variable then y – 5 is also variable. **True**

(iii) The number of angles in a triangle is a variable. **False**

(iv) The value of an algebraic expression changes with the change in the value of the variable. **True**

(v) If the length of a rectangle is twice its breadth, then its area is a constant. **False**

(vi) An equation is satisfied only for a definite value of the variable. **True**

(vii) if x toffees are distributed equally among 5 children, then each child gets 5x toffees. **False**

(viii) t minutes are equal to 60 t seconds. **True**

(ix) If x is a negative integer, then –x is a positive integer. **True**

(x) $x = 5$ is a solution of the equation $3x + 2 = 13$. **False**

(xi) $2y - 7 > 13$ is an equation. **False**

(xii) 'One third of a number x added to itself gives 8' can be expressed as $\frac{x}{3} + 8 = x$. **False**

(xiii) The difference between the ages of two sisters Lata and Asha is a variable. **False**

Multiple Choice Questions
Choose the correct answer from the given four options (3 to 19):

Question 3.
I think of a number x, add 5 to it. The result is then multiplies by 2 and the final result is 24. The correct algebraic statement is
(a) $x + 5 \times 2 = 24$
(b) $(x + 5) \times 2 = 24$
(c) $2 \times x + 5 = 24$
(d) $x + 5 = 2 \times 24$
Solution:
Let number = x
Add 5 to the number
$\Rightarrow$ i.e. $x + 5$
Now multiply result with 2
i.e. $(x + 5) \times 2 = 24$ (b)
Now final result is 24
i.e. $(x + 5) \times 2 = 24$ (b)

Question 4.

Which of the following is an equation ?

(a) $x + 5$

(b) $7x$

(c) $2y + 3 = 11$

(d) $2p < 1$

Solution:

(c) $2y + 3 = 11$

Question 5.

If each matchbox contains 48 matchsticks, then the number of matchsticks required to fill n such boxes is

(i) $48 + n$

(ii) $48 - n$

(iii) $48 \div n$

(iv) 48n

Solution:

Matchstick required to fill 1 matchbox

$= 48 \times 1 = 48$

Matchstick required to fill 2 matchbox

$= 48 \times 2 = 96$

Matchstick required to fill 3 matchbox

$= 48 \times 3 = 144$

∴ Matchsticks required to fill n matchbox

$= 48$ n (d)

Question 6.

If the perimeter of a regular hexagon is x metres, then the length of each of its sides is

(a) $(x + 6) metres$

(b) $(x - 6) metres$

(c) $(x \div 6) metres$

(d) $(6 \div x)$ metres

Solution:

Perimeter of hexagon $= x$ metres

$6(side) = x\ metres$

$side = (x \div 6)$ metres

$\therefore$ Side $= (x \div 6)$ metres (c)

Question 7.

$x = 3$ is the solution of the equation

(a) $x + 7 = 4$

(b) $x + 10 = 7$

(c) $x + 7 = 10$

(d) $x + 3 = 7$

Solution:

When put the value of $x = 3$

$3 + 7 = 10(c)$

Question 8.

The solution of the equation $3x - 2 = 10\ is$

(a) $x = 1$

(b) $x = 2$

(c) $x = 3$

(d) $x = 4$

Solution:

$3x - 2 = 10$

$3x = 10 + 2$

$x = \dfrac{12}{3} = x = 4(d)$

Question 9.

The Operation not involved in forming the expression $5x + \dfrac{5}{x}$ from the variable x and number 5 is

(a) addition

(b) Subtraction

(c) multiplication

(d) divison

Solution:
(b) Subtraction

Question 10.

The quotient of x by 3 added to 7 is written as

(a) $\dfrac{x}{3} + 7$

(b) $\dfrac{3}{x} + 7$

(c) $\dfrac{x+3}{7}$

(d) $\dfrac{x}{3+7}$

Solution:

(a) $\frac{x}{3} + 7$

Question 11.

If there are x chairs in a row, then the number of persons that can be seated in 8 rows are

(a) 64

(b) $x + 8$

(c) 8x

(d) none of these

Solution:

Let the no. of chairs in a row = x

$\Rightarrow$ Number of persons that can be seated in a row = x

Hence, number of persons that can be seoted in 8 row = $8x(c)$

Question 12.

If Arshad earns ₹x per day and spends ₹y per day, then his saving for the month of March is

(a) ₹$(31x - y)$

(b) ₹$31(x - y)$

(c) ₹$31(x + y)$

(d) ₹$31(y - x)$

Solution:

Earning of Arshad for 1 day = ₹x

Spending of Arshad for 1 day = ₹y

Saving for 1 day = ₹$(x - y)$

Saving for 1 day = ₹$31(x - y)$ (b)

Question 13.

If the length of a rectangle is 3 times its breadth and the breadth is x units, then its perimeter is

(a) $4x$ units

(b) $6x \ units$

(c) $8x$ units

(d) $10x$ units

Solution:

Breadth of rectangle = x units

Length of rectangle = 3 (Breadth) = 3x

Perimeter of rectanle = $2(l + b)$

= $2(3x + x)$

= $2(4x) = 8x \ units \ (c)$

Question 14.

Rashmi has a sum of ₹x. She spend ₹800 on grocery, ₹600 on cloths and ₹500 on education and received as ₹200 as a gift. How much money (in ₹) is left with her ?

(a) $x - 1700$

(b) $x - 1900$

(c) $x + 200$

(d) $x - 2100$

Solution:

Total money = ₹x

Money spent = ₹800

Money spent = ₹600

Money spent = ₹500 on Education

Money left with Rashmi

$= x - ₹800 + ₹600 + ₹500$

$= x - 1900$

She received a gift of = ₹200

∴ Money lefy $= x - 1900 + 200$

$= x - 1700 (a)$

Question 15.

For any two integers a and b, which of the following suggests that the operation of addition is commutative ?

(a) $a \times b = b \times a$

(b) $a + b = b + a$

(c) $a - b = b - a$

(d) $a + b > a$

Solution:

(b) $a + b = b + a$

Question 16.

In $(34)^5$, the base is

(a) 3

(b) 4

(c) 5

(d) 34

Solution:

(d) 34

Question 17.

$a \times a \times b \times b \times b$ can be written as

(a) $a^2 b^3$

(b) $a^3 b^2$

(c) a^3b^3

(d) a^5b^5

Solution:

$a \times a \times b \times b \times b$

$= a^2 \times b^3 = a^2b^3$ (a)

Question 18.

$(-5)^2 \times (-1)^3$ is equal to

(a) 25

(b) -25

(c) 10

(d) -10

Solution:

$(-5)^2 \times (-1)^3$
$\Rightarrow (-5) \times (-5) \times (-1) \times (-1) \times (-1)$
$\Rightarrow 25 \times (-1) = -25(b)$

Question 19.

$(-2)^3 \times (-3)^2$ is equal to

(a) 6^5

(b) $(-6)^5$

(c) 72

(d) -72

Solution:

$(-2)^3 \times (-3)^2$

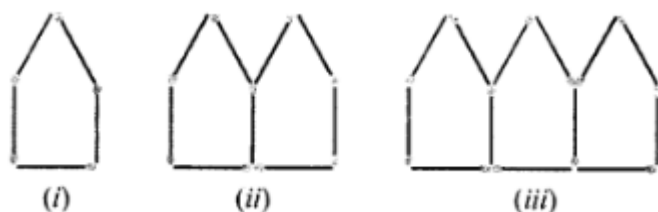$\Rightarrow (-2) \times (-2) \times (-2) \times (-3) \times (-3)$

$\Rightarrow -8 \times 9 = -72 (d)$

Check your Progress

Question 1.

Look at the following matchstick pattern of polygons. Complete the table. Also write the general rule that gives the number of matchsticks.

Solution:

(i)   (ii)   (iii)

Number of polygon	1	2	3	4	5	…..	n
Number of matchsticks	…..	…..	…..	…..	…..	…..	…..

Solution:

Number of polygon	1	2	3	4	5	…..	n
Number of matchsticks	5	9	13	17	21	…..	$4n +$

Question 2.

Write an algebraic expression for each of the following:

(i) If 1 metre cloth costs ₹x, then what is cost of 6 metre cloth ?

(ii) If the cost of a notebook is ₹x and the cost of a book is ₹y, then what is the cost of 5 notebooks and 2 books ?

(iii) The score of Ragni in Mathematics is 23 more than two-third of her score in English. If she scores x marks in English, what is her score in Mathematics ?

(iv) If the length of a side of a regular pentagon is x cm, then what is the perimeter of the pentagon ?

Solution:

(i) Cost of 1 meter cloth = ₹x

Cost of 6 metre cloth = ₹x × 6 = ₹6x

(ii) Cost of 1 notebook = ₹x

Cost of 1 book = ₹y

Cost of 5 notebooks = 5 × (₹x) = ₹5x

Cost of 2 books = (₹y) × 2 = ₹y

Total cost of 5 notebooks and 2 books = ₹(5x + 2y)

(iii) Score of Ragni in English = x marks

Score of Ragni in Mathematics = 23 more than two-third of her score in English

i.e. $= 23 + \frac{2}{3}(x)$

$= 23 + \frac{2}{3}(x)$

(iv) Side of regular pentagon = x cm

Perimeter of pentagon $= 5 \times Side = 5x \; cm$

Question 3.

When $x = 4$ and $y = 2$, find the value of :

(i) $x + y$

(ii) $x - y$

(iii) $x^2 + 2$

(iv) $x^2 - 2xy + y^2$

Solution:

(i) $x + y$

Put $x = 4$ and $y = 2$

We get,

$\Rightarrow 4 + 2 = 6$

(ii) $x - y$

Put $x = 4, y = 2$

We get,

$\Rightarrow 4 - 2 = 2$

(iii) $x^2 + 2$

Put $x = 4$ We get,

$\Rightarrow (4)^2 + 2$

$= 16 + 2 = 18$

(iv) $x^2 - 2xy + y^2$

Put $x = 4$ and $y = 2$

We get,

$\Rightarrow (4)^2 - 2 \times 4 \times 2 + (2)^2$

$\Rightarrow 16 - 16 + 4 = 4$

Question 4.
When a = 3 and b = -1, find the value of $2a^3 - b^4 + 3a^2b^3$.
Solution:
$2a^3 - b^4 + 3a^2b^3$.
Put the value of $a = 3, b = -1$

$= 2(3)^3 - (-1)^4 + 3(3)^2(-1)^3$.
$= 2 \times 3 \times 3 \times 3 \times (-1) \times (-1) \times (-1) + 3 \times 3 \times 3 \times (-1) \times (-1) \times (-1)$

$= 54 - 1 - 27 = 26$

Question 5.

When $a = 3, b = 0, c = -2,$ find the values of:
(i) $ab + 2bc + 3ca + 4abc$
(ii) $a^3 + b^3 + c^3 - 3abc$

Solution:

(i) $ab + 2bc + 3ca + 4abc$
Put the values of $a = 3, b = 0, c = -2$

$3 \times 0 + 2 \times 0 \times -2 + 3 \times -2 \times 3 + 4 \times 3 \times 0 \times -2$

$= 0 + 0 - 18 + 0 = -18$

(ii) $a^3 + b^3 + c^3 - 3abc$
Put the values of $a = 3, b = 0, c$
$= -2(3)^3 + (0)^3 + (-2)^3 - 3 \times 3 \times 0 \times -2$
$= 27 + 0 - 8 - 0 = 19$
$= 27 + 0 - 8 - 0 = 19$

Question 6.
Solve the following linear equations :
(i) $2x - 1\frac{1}{2} = 4\frac{1}{2}$
(ii) $3(y - 1) = 2(y + 1)$
(iii) $n - 3 = 5n + 21$
(iv) $\frac{1}{3}(7x - 1) = 14$
Solution:

(i) $2x - 1\frac{1}{2} = 4\frac{1}{2}$

$2x - \frac{3}{2} = \frac{9}{2}$

$\Rightarrow 2x = \frac{9}{2} + \frac{3}{2}$

$\Rightarrow 2x = \frac{9+3}{2}$

$\Rightarrow 2x = \frac{12}{2}$

$\Rightarrow 2x = 6$

$\Rightarrow x = \frac{6}{2}$

$\Rightarrow x = 3$

Verification

$\Rightarrow 2x - \frac{3}{2} = \frac{9}{2}$

$\Rightarrow 2x = \frac{9}{2} + \frac{3}{2}$

$\Rightarrow 2x = \frac{9+3}{2}$

$\Rightarrow 2x = \frac{12}{2}$

$\Rightarrow 2 \times 3 = 6$

$\Rightarrow 6 = 6$

(ii) $3(y - 1) = 2(y + 1)$

$3y - 3 = 2y + 2$

$3y - 2y = 2 + 3 \Rightarrow y = 5$

Verification

$\Rightarrow 3(5 - 1) = 2(5 + 1)$

$\Rightarrow 3(4) = 2(6)$

$\Rightarrow 12 = 12$

(iii) $n - 3 = 5n + 21$

$n - 5n = 21 + 3$

$\Rightarrow -4n = 24$

$\Rightarrow n = 24 - 4 = -6$

Verification

$\Rightarrow n - 3 = 5n + 21$

$\Rightarrow$ -6 -3 = 5 (-6) + 21

$\Rightarrow$ -9 = -30 + 21

$\Rightarrow$ -9 = -9

(iv) $\frac{1}{3}(7x - 1) = \frac{1}{4}$

$\Rightarrow \frac{7x-1}{3} = \frac{1}{4}$

By cross multiplication,

$4(7x - 1) = 3 \times 1 \Rightarrow 28x - 4 = 3$

$\Rightarrow 28x = 3 + 4$

$\Rightarrow 28x = 7$

$\Rightarrow x = \frac{7}{28} = \frac{1}{4}$

Hence $x = \frac{1}{4}$

Check :

L.H.S. $= \frac{1}{3}(7x - 1)$

$= \frac{1}{3}\left(7 \times \frac{1}{4} - 1\right) = \frac{1}{3}\left(\frac{7}{4} - 1\right)$

$= \frac{1}{3}\left(\frac{7-4}{4}\right) = \frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$

$=$ R.H.S.