

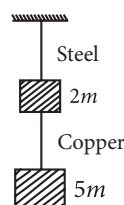
# Mechanical Properties of Solids

## 9.5 Stress-Strain Curve

1. The stress-strain curves are drawn for two different materials  $X$  and  $Y$ . It is observed that the ultimate strength point and the fracture point are close to each other for material  $X$  but are far apart for material  $Y$ . We can say that materials  $X$  and  $Y$  are likely to be (respectively)
- (a) ductile and brittle (b) brittle and ductile  
(c) brittle and plastic (d) plastic and ductile  
(Odisha NEET 2019)

## 9.6 Elastic Moduli

2. A wire of length  $L$ , area of cross section  $A$  is hanging from a fixed support. The length of the wire changes to  $L_1$  when mass  $M$  is suspended from its free end. The expression for Young's modulus is
- (a)  $\frac{MgL_1}{AL}$  (b)  $\frac{Mg(L_1 - L)}{AL}$   
(c)  $\frac{MgL}{AL_1}$  (d)  $\frac{MgL}{A(L_1 - L)}$  (NEET 2020)
3. When a block of mass  $M$  is suspended by a long wire of length  $L$ , the length of the wire becomes  $(L + l)$ . The elastic potential energy stored in the extended wire is
- (a)  $\frac{1}{2}MgL$  (b)  $Mgl$   
(c)  $MgL$  (d)  $\frac{1}{2}Mgl$  (NEET 2019)
4. Two wires are made of the same material and have the same volume. The first wire has cross-sectional area  $A$  and the second wire has cross-sectional area  $3A$ . If the length of the first wire is increased by  $\Delta l$  on applying a force  $F$ , how much force is needed to stretch the second wire by the same amount?
- (a)  $9F$  (b)  $6F$   
(c)  $4F$  (d)  $F$  (NEET 2018)
5. The bulk modulus of a spherical object is ' $B$ '. If it is subjected to uniform pressure ' $p$ ', the fractional decrease in radius is
- (a)  $\frac{B}{3p}$  (b)  $\frac{3p}{B}$  (c)  $\frac{p}{3B}$  (d)  $\frac{p}{B}$  (NEET 2017)
6. The Young's modulus of steel is twice that of brass. Two wires of same length and of same area of cross section, one of steel and another of brass are suspended from the same roof. If we want the lower ends of the wires to be at the same level, then the weights added to the steel and brass wires must be in the ratio of
- (a) 4 : 1 (b) 1 : 1 (c) 1 : 2 (d) 2 : 1 (2015)
7. The approximate depth of an ocean is 2700 m. The compressibility of water is  $45.4 \times 10^{-11} \text{ Pa}^{-1}$  and density of water is  $10^3 \text{ kg/m}^3$ . What fractional compression of water will be obtained at the bottom of the ocean?
- (a)  $1.2 \times 10^{-2}$  (b)  $1.4 \times 10^{-2}$   
(c)  $0.8 \times 10^{-2}$  (d)  $1.0 \times 10^{-2}$   
(2015 Cancelled)
8. Copper of fixed volume  $V$  is drawn into wire of length  $l$ . When this wire is subjected to a constant force  $F$ , the extension produced in the wire is  $\Delta l$ . Which of the following graphs is a straight line?
- (a)  $\Delta l$  versus  $1/l$  (b)  $\Delta l$  versus  $l^2$   
(c)  $\Delta l$  versus  $1/l^2$  (d)  $\Delta l$  versus  $l$  (2014)
9. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied?
- (a) length = 200 cm, diameter = 2 mm  
(b) length = 300 cm, diameter = 3 mm  
(c) length = 50 cm, diameter = 0.5 mm  
(d) length = 100 cm, diameter = 1 mm (NEET 2013)
10. If the ratio of diameters, lengths and Young's modulus of steel and copper wires shown in the figure are  $p$ ,  $q$  and  $s$  respectively, then the corresponding ratio of increase in their lengths would be
- (a)  $\frac{5q}{(7sp^2)}$  (b)  $\frac{7q}{(5sp^2)}$   
(c)  $\frac{2q}{(5sp)}$  (d)  $\frac{7q}{(5sp)}$



(Karnataka NEET 2013)

## ANSWER KEY

1. (b) 2. (d) 3. (d) 4. (a) 5. (c) 6. (d) 7. (a) 8. (b) 9. (c) 10. (b)

## Hints &amp; Explanations

1. (b)

2. (d) : Given : initial length =  $L$ , area of cross section =  $A$   
New length after mass  $M$  is suspended on the wire =  $L$   
 $\therefore$  Change in length,  $\Delta L = L_1 - L$ .

Now Young's modulus,  $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A} \times \frac{L}{\Delta L}$

$$= \frac{mg}{A} \frac{L}{\Delta L} \text{ or } \frac{MgL}{A(L_1 - L)}$$

3. (d) : Stress =  $\frac{F}{A} = \frac{Mg}{A}$ ,

$$\text{Strain} = \frac{\Delta L}{L} = \frac{L + l - L}{L} = \frac{l}{L}$$

Energy stored in the wire is,

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

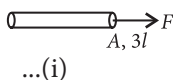
$$= \frac{1}{2} \times \frac{Mg}{A} \times \frac{l}{L} \times A \times L = \frac{1}{2} Mgl$$

4. (a) : Young's modulus,  $Y = \frac{Fl}{A\Delta l}$

Since initial volume of wires are same and their areas of cross sections are  $A$  and  $3A$  so lengths are  $3l$  and  $l$  respectively.

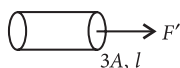
For wire 1,

$$\Delta l = \left( \frac{F}{AY} \right) 3l$$



For wire 2, let  $F'$  force is applied

$$\frac{F'}{3A} = Y \frac{\Delta l}{l}$$



$$\Rightarrow \Delta l = \left( \frac{F'}{3AY} \right) l$$

From eqns (i) and (ii),

$$\left( \frac{F}{AY} \right) 3l = \left( \frac{F'}{3AY} \right) l \Rightarrow F' = 9F$$

5. (c) : Bulk modulus  $B$  is given as

$$B = \frac{-pV}{\Delta V} \quad \dots (i)$$

The volume of a spherical object of radius  $r$  is given as

$$V = \frac{4}{3} \pi r^3, \Delta V = \frac{4}{3} \pi (3r^2) \Delta r$$

$$\therefore -\frac{V}{\Delta V} = \frac{\frac{4}{3} \pi r^3}{\frac{4}{3} \pi 3r^2 \Delta r} \text{ or } -\frac{V}{\Delta V} = -\frac{r}{3\Delta r}$$

Put this value in eqn. (i), we get  $B = -\frac{pr}{3\Delta r}$

Fractional decrease in radius is  $-\frac{\Delta r}{r} = \frac{p}{3B}$

6. (d) : Let  $L$  and  $A$  be length and area of cross section of each wire. In order to have the lower ends of the wires to be at the same level (*i.e.* same elongation is produced in both wires), let weights  $W_s$  and  $W_b$  are added to steel and brass wires respectively. Then, by definition of Young's modulus, the elongation produced in the steel wire is

$$\Delta L_s = \frac{W_s L}{Y_s A} \quad \left( \text{as } Y = \frac{W/A}{\Delta L/L} \right)$$

and that in the brass wire is  $\Delta L_b = \frac{W_b L}{Y_b A}$

But  $\Delta L_s = \Delta L_b$  (given)

$$\therefore \frac{W_s L}{Y_s A} = \frac{W_b L}{Y_b A} \text{ or } \frac{W_s}{W_b} = \frac{Y_s}{Y_b}$$

$$\text{As } \frac{Y_s}{Y_b} = 2; \therefore \frac{W_s}{W_b} = \frac{2}{1}$$

7. (a) : Depth of ocean,  $d = 2700$  m

Density of water,  $\rho = 10^3 \text{ kg m}^{-3}$

Compressibility of water,  $K = 45.4 \times 10^{-11} \text{ Pa}^{-1}$

$$\frac{\Delta V}{V} = ?$$

Excess pressure at the bottom,  $\Delta P = \rho g d$

$$= 10^3 \times 10 \times 2700 = 27 \times 10^6 \text{ Pa}$$

We know,  $B = \frac{\Delta P}{(\Delta V/V)}$

$$\left( \frac{\Delta V}{V} \right) = \frac{\Delta P}{B} = K \cdot \Delta P \quad \left( \because K = \frac{1}{B} \right)$$

$$= 45.4 \times 10^{-11} \times 27 \times 10^6 = 1.2 \times 10^{-2}$$

8. (b) : As  $V = Al$  ... (i)

where  $A$  is the area of cross-section of the wire.

$$\text{Young's modulus, } Y = \frac{(F/A)}{(\Delta l/l)} = \frac{Fl}{A\Delta l}$$

$$\Delta l = \frac{Fl}{YA} = \frac{Fl^2}{YV} \quad (\text{Using (i)})$$

$$\Delta l \propto l^2$$

Hence, the graph between  $\Delta l$  and  $l^2$  is a straight line.

9. (c) : Young's modulus,

$$Y = \frac{FL}{A\Delta L} = \frac{4FL}{\pi D^2 \Delta L} \quad \text{or} \quad \Delta L = \frac{4FL}{\pi D^2 Y}$$

where  $F$  is the force applied,  $L$  is the length,  $D$  is the diameter and  $\Delta L$  is the extension of the wire respectively. As each wire is made up of same material therefore their Young's modulus is same for each wire.

For all the four wires,  $Y$ ,  $F$  (= tension) are the same.

$$\therefore \Delta L \propto \frac{L}{D^2}$$

$$\text{In (a)} \quad \frac{L}{D^2} = \frac{200 \text{ cm}}{(0.2 \text{ cm})^2} = 5 \times 10^3 \text{ cm}^{-1}$$

$$\text{In (b)} \quad \frac{L}{D^2} = \frac{300 \text{ cm}}{(0.3 \text{ cm})^2} = 3.3 \times 10^3 \text{ cm}^{-1}$$

$$\text{In (c)} \quad \frac{L}{D^2} = \frac{50 \text{ cm}}{(0.05 \text{ cm})^2} = 20 \times 10^3 \text{ cm}^{-1}$$

$$\text{In (d)} \quad \frac{L}{D^2} = \frac{100 \text{ cm}}{(0.1 \text{ cm})^2} = 10 \times 10^3 \text{ cm}^{-1}$$

Hence,  $\Delta L$  is maximum in (c).

$$10. \text{ (b) : As } Y = \frac{FL}{A\Delta L} = \frac{4FL}{\pi D^2 \Delta L}$$

$$\Delta L = \frac{4FL}{\pi D^2 Y}$$

$$\therefore \frac{\Delta L_S}{\Delta L_C} = \frac{F_S}{F_C} \frac{L_S}{L_C} \frac{D_C^2}{D_S^2} \frac{Y_C}{Y_S}$$

where subscripts  $S$  and  $C$  refer to copper and steel respectively.

$$\text{Here, } F_S = (5m + 2m)g = 7mg$$

$$F_C = 5mg$$

$$\frac{L_S}{L_C} = q, \quad \frac{D_S}{D_C} = p, \quad \frac{Y_S}{Y_C} = s$$

$$\therefore \frac{\Delta L_S}{\Delta L_C} = \left( \frac{7mg}{5mg} \right) (q) \left( \frac{1}{p} \right)^2 \left( \frac{1}{s} \right) = \frac{7q}{5p^2 s}$$

