Quadrilaterals

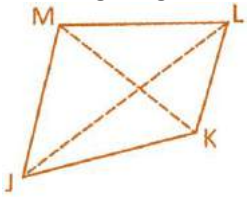
TALENT & OLYMPIAD

Introduction

We know that a plane figure which is bounded by four line segments is called quadrilateral. Previously we have studied about different types of quadrilateral like rectangle, square, parallelogram etc. In this chapter, we will discuss about different properties of quadrilateral with the help of theorems.

Quadrilateral

In the figure given below:



(i) Points J, K, L and M are the vertices of quadrilateral JKLM.
(ii) The line segments J, K, L, LM and JM are the sides of this quadrilateral.
(iii) The two sides of a quadrilateral having a common point is called adjacent side.
(iv) The two sides have no common end point is called opposite side.
(v) Two angles of a quadrilateral having common arm is called adjacent angle.
(vi) Two angles of a quadrilateral having no common arm is called vertically opposite angles.

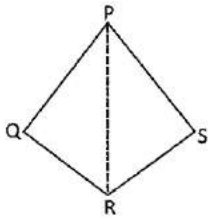
Theorem on a Quadrilateral

Theorem 1:
The sum of all interior angles of a quadrilateral is 360°

given:
A quadrilateral PQSR

To Prove:
$\angle P + \angle Q + \angle R + \angle S = 360°$

Construction:
Join PQ



Proof:
In ΔPQR

$\angle Q + \angle QPR + \angle QRP = 180°$ .....(i)

In ΔPRS

$\angle S + \angle SRP + \angle SPR = 180°$ .....(ii)

Adding (i) and (ii), we get:

$\angle Q + \angle S + (\angle QPR + \angle SPR) + (\angle QRP + \angle SRP) = 360° \Rightarrow \angle Q + \angle S + \angle P + \angle R = 360°$

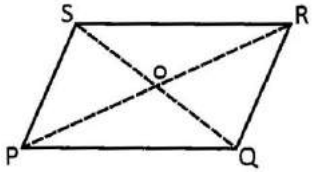
[Because $\angle QPR + \angle SPR = \angle P, \angle QRP + \angle SRP = \angle R$]

Hence, proved

Theorem 2;
In a parallelogram opposite sides and opposite angles are equal and diagonals bisect each other.

Given:
A parallelogram PQSR, PQ II RS and PS II QR

To Prove:
(i) $PQ = RS, \quad PS = QR$
(ii) $\angle P = \angle R$ and $\angle Q = \angle S$
(iii) 0 is the midpoint of PR and SQ



Proof:
In ΔPQR and ΔRSP
$\angle RPQ = \angle PRS,$ (Alternate)
$\angle PRQ = \angle RPS$
and $PR = PR$ (Common)
$\Delta PQR = \Delta PRS$
By C.P.C.T. (Corresponding part of congruence triangle PQ = RS, and PS = QR and $\angle Q = \angle S$.
Similarly
In ΔPQS and ΔQSR
we get:
$\Delta PQS \cong \Delta QSS \quad \Rightarrow \quad \angle P = \angle R$
so,

$\boxed{PQ = RS \text{ and } PS = QR}$ Proved

$\boxed{\angle P = \angle R \text{ and } \angle Q = \angle S}$ Proved

Now
In ΔPOQ and ΔROS
$\angle OPQ = \angle ORS$ (Alternate)
$\angle OQP = \angle OSR$
And $PQ = QR$
(Opposite of Parallelogram)
$\Rightarrow \quad \Delta POQ \cong \Delta SOR$ (By ASA)
$\Rightarrow$ PO = OR and QO = OS (By CPCT)
$\Rightarrow$ O is the midpoint of PR and QS
(Hence, proved)

Important Points

(i) A quadrilateral is a parallelogram if their opposite sides are equal.
(ii) A quadrilateral is a parallelogram if their opposite angles are equal.
(iii) If the diagonal of a quadrilateral bisect each other then it is a parallelogram.
(iv) A quadrilateral is a parallelogram if its one pair of opposite sides are equal and parallel.
(v) The diagonals of rectangle are equal.

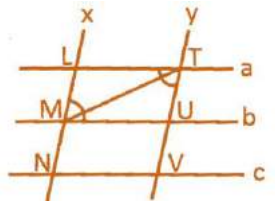
(vi) If the two diagonals of a parallelogram is equal then the parallelogram is a rectangle.
(vii) The diagonal of the rhombus are perpendicular bisector of each other.
(viii) A parallelogram is a square if the diagonals of a parallelogram are equal and bisector at right angle.

Intercept Theorem

If a transversal intersect three and more than three parallel lines in such a way that all the intercepts are equal then the intercept on any other transversal is also equal.

Given:

Three parallel lines a, b, c intersected by a transversal x at L, M, N respectively so that LM = MN. Another transversal y cutting a, b, c at T, U, V respectively



Proof:

Since $LT \parallel MU$ and $LM \parallel TU$

$\therefore$ LMUT is a parallelogram

$\therefore$ TU = LM .....(i)

Similarly

MNVU is a parallelogram

$\therefore$ MN=UV .....(ii)

But

$\therefore$ TU = UV

Now

In ΔMLT and ΔTUM (Alternate)

$\angle MTL = \angle MTU$

(Opposite angle of Parallelogram)

LT = UM

(Opposite sides of Parallelogram)

MT=MT (Common)

$\therefore$ By SOS

$\Delta MTL \cong \Delta TUM$

By CPCT,

LM=TU

Similarly

MN=UV

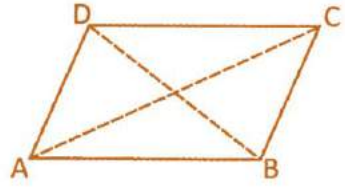
But

LM=MN

$\therefore$ TU = UV

Results on Area of Quadrilateral

(i) Any diagonal of a parallelogram divides it into two triangles of equal area.



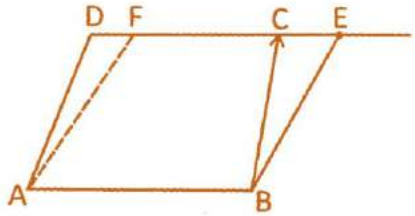
In the above given figure,

$\text{ar}(\Delta ABC) = \text{ar}(\Delta ACD)$

Similarly

$\text{ar}(\Delta ABD) = \text{ar}(\Delta BCD)$

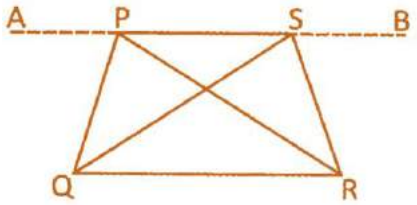
(ii) Parallelograms which are on the same base and between the same parallel lines are equal in area.



In the above given figure,

$ar(ll^{gm} ABCD) = ar(ll^{gm} ABEF)$

(v) Triangles which are on the same base and between the same parallel lines are equal in area.



In the above given figure $QR \,||\, AB$

Then $\text{ar}(\Delta PQR) = \text{ar}(\Delta QRS)$

(iv) Area of trapezium= $\frac{1}{2}$ (sum of parallel sides) $\times$ (distance between them)

You Must KNOW

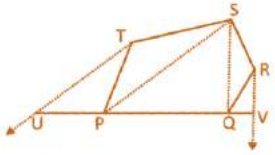
- A quadrilateral is known as a concave quadrilateral if one interior angle is reflex.
- A self-intersecting quadrilateral is called a cross-quadrilateral, butterfly quadrilateral or bow-tie quadrilateral.
- A non-planar quadrilateral is called a skew quadrilateral.

SUMMARY

- The sum of all interior angles of a quadrilateral is 360^o .
- In a parallelogram opposite sides and opposite angles are equal and diagonals bisect each other.
- If a transversal intersect three and more than three parallel lines in such a way that all the intercepts are equal, then the intercept on any other transversal is also equal.
- Parallelograms which one on the same base and between the same parallel lines are" equal in area.
- Triangles which are on the same base and between same parallel lines are equal in area.

Commonly Asked QUESTIONS

In the figure given below shows a pentagon in which TU drawn parallel to SP meet at QP produced at U and RV parallel to SQ, meet PQ produced at V then:



(a) ar(pentagon PQRST) $= \text{ar}(\Delta STU) + \text{ar}(\Delta QRV)$

(b) ar(pentagon PQRST) $= \text{ar}(\Delta SUV)$

(c) ar(pentagon PQRST) $= \text{ar}(\Delta SUV) + \text{ar}(\Delta TUP)$

(d)ar(pentagon PQRST) $= \text{ar}(\Delta SUV) + \text{ar}(\Delta QRV)$

(e) None of these

Answer: (b)

Explanation

Since $RV \parallel SQ \Rightarrow \text{ar}(\Delta SPT) = \text{ar}(\Delta TUP)$

$\text{ar}(\Delta SQR) + \text{ar}(\Delta SPT) + \text{ar}(\Delta SPQ) = \text{ar}(\Delta SRV) + \text{ar}(\Delta SUP) + \text{ar}(\Delta BPQ)$

$\Rightarrow \text{ar}(\text{pentagon PQRST}) = \text{ar}(\Delta SUV)$

In a trapezium non-parallel sides are equal. When we join the midpoint of diagonals and parallel sides a quadrilateral is formed, the quadrilateral formed is.....

(a) Rhombus (b) Rectangle

(c) Trapezium (d) Square

(e) None of these

Answer: (b)

For a trapezium which one of the following statements is correct?

(a) Adjacent sides are parallel

(b) Vertically opposite sides are parallel

(c) The line joining the mid points of the diagonal is parallel to the parallel side and equal to half their differences

(d) The line joining the mid points of the diagonal is parallel to the parallel side and equal to their differences

(e) None of these

Answer: (b)

If the length of the diagonals of rhombus are 30 cm and 16 cm respectively then the perimeter of rhombus is_____.

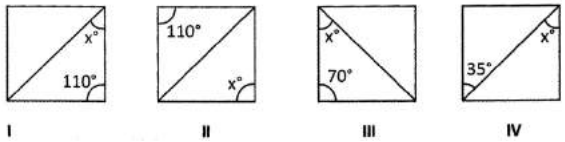
(a) 17cm (b) 69cm

(c) 63cm (d) 68cm

(e) None of these

Answer: (d)

In the adjoining figures which have equal value of angle x ? [Every quadrilateral is rhombus]



I II III IV

(a) I and II
(b) II and III
(c) III and IV
(d) I and IV
(e) None of these

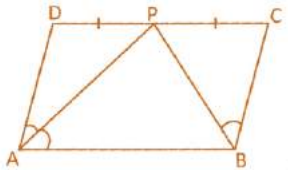
Answer: (d)

Self Evaluation TEST

Duration 10 Minutes

---

1. **In the figure given below:**



ABCD is a parallelogram, in which p is the midpoint of CD and PA bisect $\angle OAB$

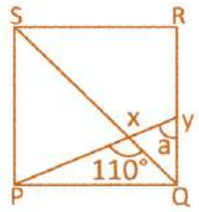
(a) PC = AB (b) PC = BC

(c) $PC^2 = PB^2 + BC^2$ (d) $PA^2 = PB^2 + AB^2$

(e) None of these

---

2. **In the adjoining figure, PQRS is a square. A line segment PY cuts the side QR at QR and diagonal RS at x so that $\angle PXQ = 110°$ then the value of a is_____.**



(a) $65°$ (b) $75°$

(c) $55°$ (d) $45°$

(e) None of these

---

3. **The measurement of angles of parallelogram if one of its angle is $60°$ less than the twice the smallest angle.....**

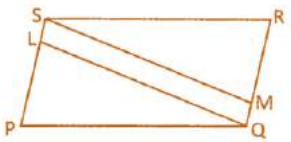
(a) $80°, 100°, 80°$ and $100°$ (b) $70°, 110°, 70°$ and $110°$

(c) $75°, 105°, 75°$ and $105°$ (d) $85°, 95°, 85°$ and $95°$

(e) None of these

---

4. **In the figure given below PQRS is a parallelogram, L and M are the points on PS and QR respectively so that $SL = \frac{1}{3}PS$ and $QM = \frac{1}{3}QRS$ then:**

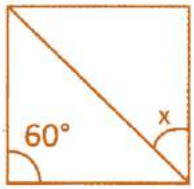


(a) QLSM is rhombus (b) QLSM is a parallelogram

(c) QLSM is a quadrilateral with all angles are equal (d) QLSM is a square

(e) None of these

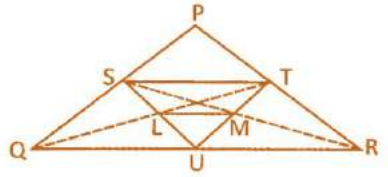
5. **In the adjoining figure, find the value of x ?**



(a) $73°$ (b) $70°$
(c) $60°$ (d) $75°$
(e) None of these

---

6. **In the adjoining figure, S, T and U are the mid points of PQ, PR and QR respectively. If the line joining QT and RS intersect SU and TU at L and M respectively then:**



(a) $LM = \frac{1}{16}QR$ (b) $LM = \frac{1}{4}QR$
(c) $LM = \frac{1}{2}QR$ (d) $LM = \frac{1}{8}QR$
(e) None of these

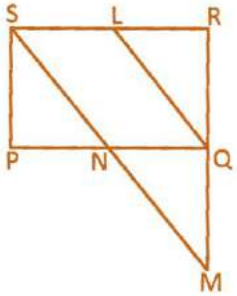
---

7. **Which one of the following statements is incorrect?**

(a) The quadrilateral formed by joining the mid points of adjacent sides of a rectangle is rhombus
(b) The quadrilateral formed by joining the mid points of the pair of adjacent sides of a rhombus is rectangle
(c) The line segment joining the mid points of the sides of a triangle divide it into three congruent triangle
(d) The quadrilateral formed by joining the mid points of the pairs of adjacent sides of a square is square
(e) None of these

---

8. **In the adjoining figure, PQRS is a parallelogram. L is the midpoint of RS and through S a line segment drawn parallel to LQ to meet RQ produced at M and intersect PQ at N**



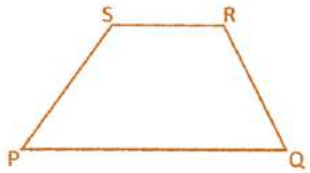
(a) $PS = SL$ (b) $PS = RM - RQ$
(c) $SR = PN$ (d) $SR = NQ$
(e) None of these

9. **In a trapezium, from the midpoint of one non-parallel side a parallel line segment is drawn that cuts the other non-parallel side then:**

(a) Length of line segment is equal to the half of the sum of parallel sides
(b) Length of line segment is equal to the sum of parallel sides
(c) Length of line segment is equal to the one-third of the sum of parallel sides
(d) Length of line segment is equal to the one-fourth of the sum of parallel sides
(e) None of these

---

10. **In the adjoining figure:**



PQ is the longest side and RS is the shortest side then:
(a) $\angle R$ is less than $\angle P$
(b) $\angle R$ is equal to $\angle P$
(c) $\angle R$ is greater than $\angle P$
(d) $\angle D$ is less than $\angle B$
(e) None of these

Answers – Self Evaluation Test																			
1.	B	**2.**	A	**3.**	A	**4.**	B	**5.**	C	**6.**	B	**7.**	C	**8.**	B	**9.**	A	**10.**	C

Self Evaluation Test
SOLUTIONS

---

1. $\angle PAB = \angle APD$ (Alternate Interior)
$\angle APD = \angle PAD \Rightarrow AD = PD$
$\Rightarrow PC = BC$

---

2. In the above given figures
$\angle PXQ + \angle QXY = 180°$
$\Rightarrow \angle QXY = 70° \Rightarrow \angle XQY = 45° \Rightarrow a = 65°$

---

3. According to condition
Let the smallest angle be x
The $x + (2x - 50) = 180°$
$\Rightarrow 3x = 240 \Rightarrow x = \frac{240}{3} \Rightarrow x = 80°$
Another angle $= 100°$

---

4. Since PQRS is a parallelogram
$\Rightarrow PS = QR \Rightarrow \frac{1}{3}PS = \frac{1}{3}QR \Rightarrow SL = QM$ ....(i)
And also $SL \| QM$ .....(ii)
From (i) and (ii), we conclude that QLSM is a parallelogram.

---

6. In ΔPQR , S and T are the mid points of PQ and PR respectively
$\therefore ST \| QR$ and $ST = \frac{1}{2}QR = QU$
Since $ST \| QU$ and $ST = QU$
$\Rightarrow$ STQ.U is a parallelogram whose diagonal QT and SU intersect at L therefore L is the midpoint of SU
Similarly M is the midpoint of TU
$\Rightarrow LM = \frac{1}{4}QR$.

---

7. There are four congruent circles.

---

9. ABCD is a trapezium in which E is the midpoint of AD.
EF drawn parallel to AB then prove, $EF = \frac{1}{2} - (AB + DC)$

10. Join PR and QS

In ΔPQR

$PQ > QR$

$\Rightarrow \angle PRQ > \angle QPR$ .....(i)

In ΔPRS

$PS > SR$

$\Rightarrow \angle PRS > \angle SPR$ .....(ii)

Adding (i) and (ii), we get:

$\angle R$ is greater then $\angle P$