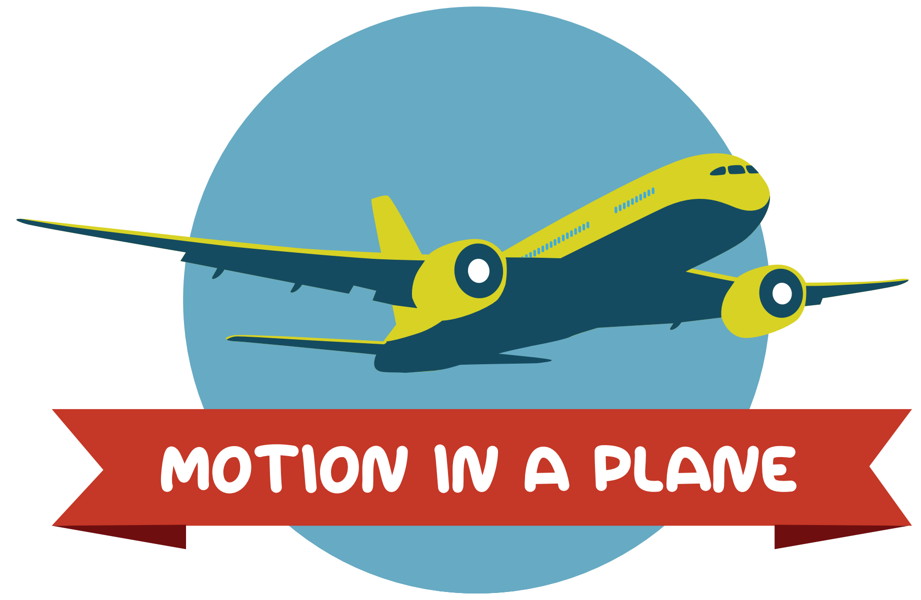
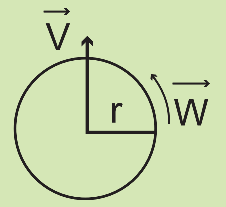


MOTION IN A PLANE

Vector (Magnitude + Direction)

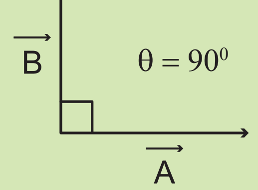
Basic Terminologies

- **Null vector:** $|\vec{A}| = 0$
- **Unit vector:** $\hat{A} = \frac{\vec{A}}{|\vec{A}|} = 1$
- **Equal vector:** $\left.\begin{matrix}\vec{A}\\ \vec{B}\end{matrix}\right\} |\vec{A}| = |\vec{B}|$
- **Axial vector:** used in rotation



$\vec{V} = \vec{r} \times \vec{W}$

- **Orthogonal vector Angle b/w** $\vec{A}$ & $\vec{B}$ $(\theta = 90^0)$



- **Parallel Vector:**

Angel b/w $\vec{A}$ & $\vec{B}$ $(\theta = 0^0)$

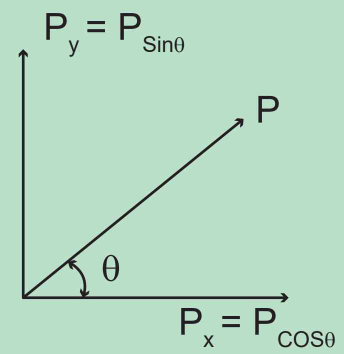
$\Rightarrow |\vec{A}| = n|\vec{B}|$

- **Anti-Parallel Vector:**

Angel b/w $\vec{A}$ & $\vec{B}$ $(\theta = 180^0)$

$\Rightarrow |\vec{A}| = -n|\vec{B}|$

Resolution of vector



Mathematical operations

$(\,9g\ \vec{A} = (a_1\hat{i} + b_1\hat{j} + c_1\hat{K})$ & $\vec{B} = (a_1\hat{i} + b_1\hat{j} + c_1\hat{K})\,)$

Arithmetic operations

Addition

$\vec{A} + \vec{B} = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j} + (c_1 + c_2)\hat{k}$

Subtraction

$\vec{A} + \vec{B} = (a_1 - a_2)\hat{i} + (b_1 - b_2)\hat{j} + (c_1 - c_2)\hat{k}$

Multiplication

Dot Product (Scalar Product)

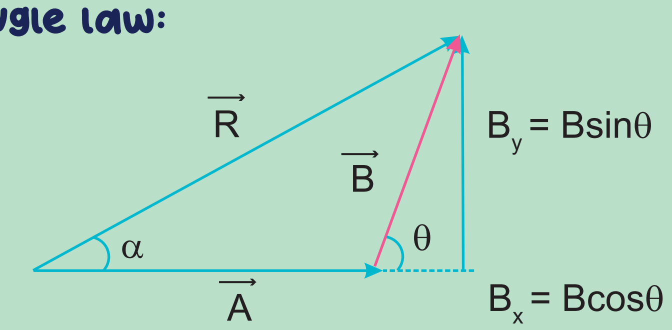
1) $\vec{A} + \vec{B} = |\vec{A}||\vec{B}|\,\mathrm{Cos}\theta$
2) $\vec{A}\cdot\vec{B} = a_1a_2 + b_1b_2 + c_1c_2$
3) $\hat{i}\cdot\hat{i} = 1$
   $\hat{i}\cdot\hat{j} = 0$
   $\hat{i}\cdot\hat{k} = 0$ etc

Cross Product (Vector Product)

1) $\vec{A} + \vec{B} = |\vec{A}||\vec{B}|\sin\theta$
2) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$
   $\hat{i}\,(b_1c_2 - c_1b_2) + \hat{j}\,(a_1c_2 - c_1a_2) + \hat{k}\,(a_1b_2 - b_1a_2)$
3) $\hat{i} \times \hat{i} = 0$
   $\hat{i} \times \hat{j} = 1$
   $\hat{i} \times \hat{k} = -1$

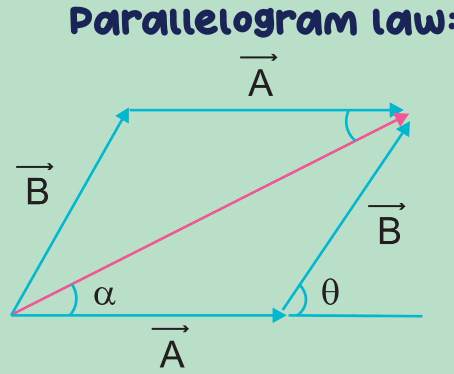
Vector law's

Triangle law:



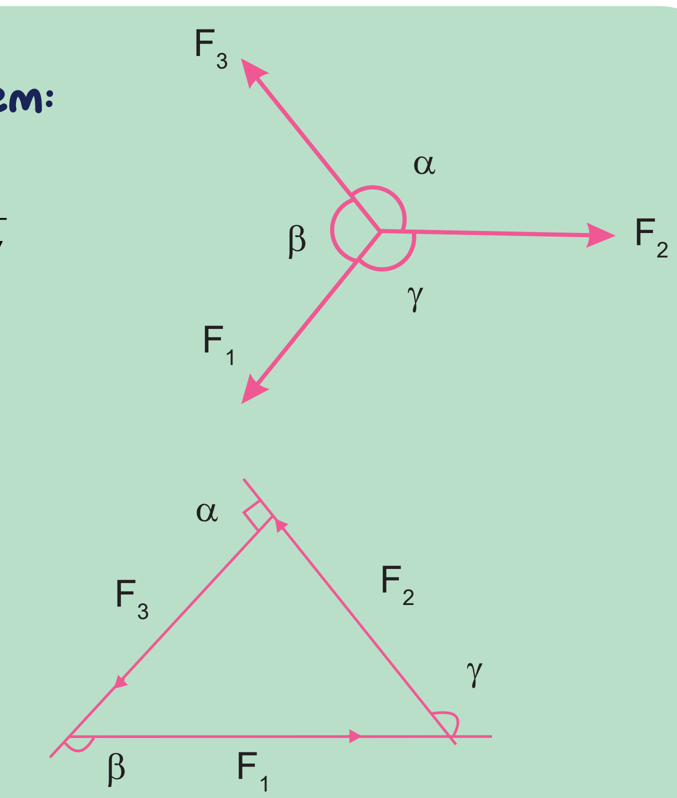
- $\vec{R} = (A + B\cos\theta)\,\hat{i} + B\sin\theta\,\hat{j}$
- $|\vec{R}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\,\mathrm{Cos}\,\theta}$
- $\tan\alpha = \dfrac{|\vec{B}|\sin\theta}{|\vec{A}| + |\vec{B}|\cos\theta}$

Parallelogram law:



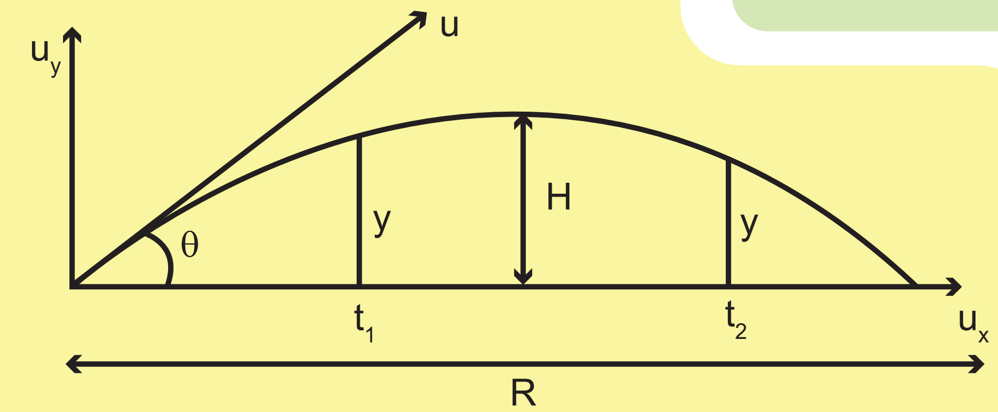
Lammis Theorem:

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$



Projectile motion

Oblique projectile



X – component

- $u_x = u_{\mathrm{Cos}}\theta$
- $a_x = 0$

Y – component

- $u_g = u_{\sin}\theta$
- $a_g = -g$

Equation of Trajectory (parabolic track)

$y = x\tan\theta - \dfrac{1}{2}\dfrac{gx^2}{u^2\mathrm{Cos}^2\theta} = x(1 - \dfrac{x}{R})\tan\theta$

Time of f hight (T). T = 2usinθ/g

Range (R) $= u_xT, = \dfrac{\mu^2\sin^2\theta}{g}$

Height (H) $= \dfrac{\mu^2\sin^2\theta}{2g}$

Projectile passing same height at two different times t_1 and t_2 respectively

1) $y = \dfrac{1}{2}gt_1t_2$
2) $t_1 = \dfrac{\mu\sin\theta}{g}\left[1 - \sqrt{1 - \left(\dfrac{2\,gy}{\mu\sin\theta}\right)^2}\right]$
3) $t_2 = \dfrac{\mu\sin\theta}{g}\left[1 + \sqrt{1 - \left(\dfrac{2\,gy}{\mu\sin\theta}\right)^2}\right]$

Projective with complimentary angles. If θ1 = θ then θ_2 = 90 – θ

1) R = Hcosθ
2) $\dfrac{T\theta}{T_{90-\theta}} = \tan\theta$

Relative motion on 2 D – plane

Motion of one body w.r.t. other: $\vec{V}_{P/Q} = \vec{V}_P - \vec{V}_Q$

$V_{P/Q}$ = velocity of P w.e.t.Q

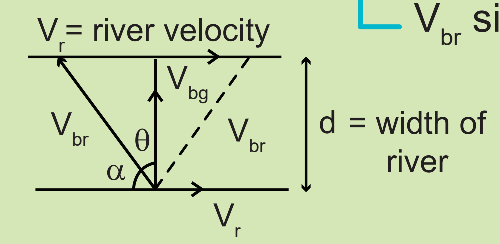
Umbrella problem: $V_{mG} = (V_m - V_G) = V_m$

1) V_{rm} = velocity of rain w.r.t man
2) $V_{rm} = V_r - V_m$
3) $\tan\theta = \dfrac{V_m}{V_r}$

River Boat Problem

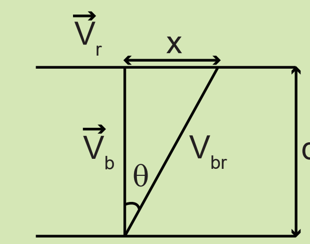
Shortest distance

- $V_r = V_{br}\,\mathrm{Cos}\alpha$ & $V_b = V_{br}\sin\alpha$
- $V_{br}\sin\alpha = \dfrac{d}{t} = d_{min} = (V_{br}\sin\alpha)t$

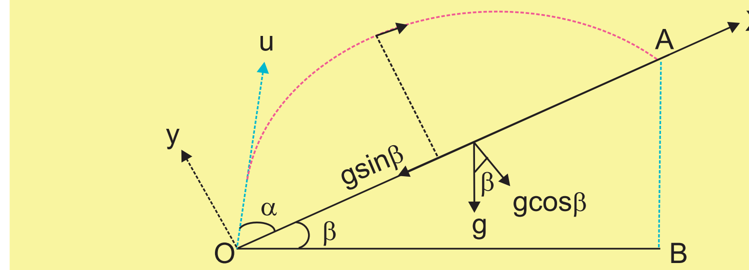


Shortest time

- $V_{br} = \sqrt{V_b^2 + V_r^2}$
- $t_{min} = \dfrac{d}{V_b} = \dfrac{\sqrt{X^2 + d^2}}{\sqrt{V_b^2 + V_r^2}}$
- Drift (x) = $\vec{V}_r\,t_{min}$
- $\tan\theta = \dfrac{V_r}{V_m} = \dfrac{x}{d}$



Projectile on inclined plane



X – components

$u_x = u\cos\theta$

$a_x = g\sin\theta$

Y – components

ug = usinθ

ay = g cosθ

Time of flight (T) $= \dfrac{2u\sin\theta}{g\cos\propto}$

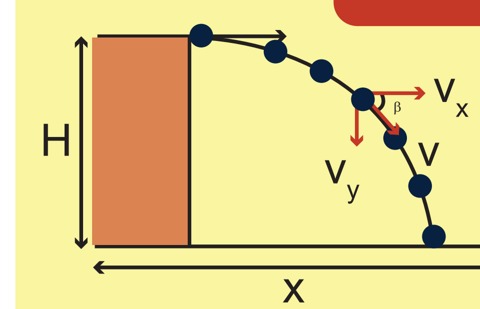
Height (H) $= \dfrac{u^2\sin^2\theta}{2g\cos\propto}$

Range (R) $= \dfrac{2u^2}{g}\,\dfrac{\sin\theta\cos(\propto + \theta)}{\cos^2\propto}$

for $R_{max} = \theta = \pi/4 + \propto/2$

for $H_{max} = \theta = 90^o$ or α = 0°

Horizontal projectile



$u_y = 0, u_x = u$

$x = u_xt = ut$, $t = x/u$

Equation of Trajectory

$Y = \frac{1}{2}gt^2 = \frac{1}{2}g\frac{X^2}{\mu^2}$

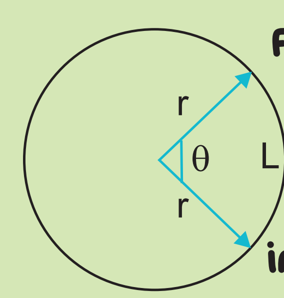
Range (R) $= u_xt = u\sqrt{\dfrac{2H}{g}}$

Time of flight (T) $= \sqrt{\dfrac{2H}{g}}$

$V_{ins} = \sqrt{v_x^2 + v_y^2}$
$= \sqrt{u^2 + g^2t^2}$
$= \sqrt{u^2 + 2gy}$

$\tan\phi = \dfrac{v_y}{v_x} = \dfrac{gt}{u}$

Circular motion



Angular velocity (ω):

$W = \dfrac{d\theta}{dt} = \dfrac{2\pi}{T} = 2\pi f\,(\mathrm{rads}^{-1})$

T= Time period f = frequency

$\boxed{V = RW}$ — linear velocity (ms⁻¹)

Angular displacement (θ):

$\boxed{\ell = R\theta}$ — Angular displacement (rad.) — Radius (m) — Arc length (m)

Angular Acceleration (α):

$\propto = \dfrac{dw}{dt}\,(\mathrm{rads}^{-2})$

$\boxed{a = R\propto}$ — linear acceleration (ms⁻¹)

Equation of motion on Circular track:

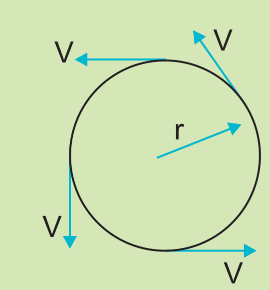
$\omega_f = \omega_i + \alpha t$

$\theta = \omega_{it} + \frac{1}{2}\,N\alpha t^2$

$\omega_f^2 - \omega_i^2 = 2\propto\theta$

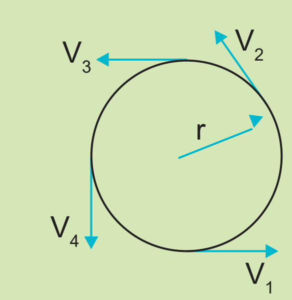
Types of Circular motion:

Uniform circular motion



1) a_τ (Tangential acceleration) = $0ms^{-2}$
2) a_r (Radial acceleration) $= \dfrac{V^2}{R}$
3) $a_{net} = \sqrt{a_T^2 + a_0^2} = a_r$

Non – uniform Circular motion



1) $V_1 \neq V_2 \neq V_3 \neq V_4$
2) $a_T \neq 0ms^{-2}$
3) $a^r = \dfrac{V_{ins}^2}{R}$
4) $a_{net} = \sqrt{a_T^2 + a_r^2}$