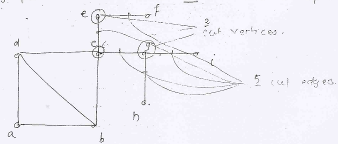
Connectivity →

→ A graph is G is connected if there exists a path betn every pair of vertices.

→ A graph G is connected iff G has a "spanning tree."



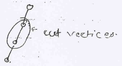
Cut vertex / Articulation point →

Let 'G' be a connected graph. A vertex $v \in G$ is called a "cut vertex of G" if $G-\{v\}$ results in a disconnected graph; then V

For the graph given above, c, e and g are cut vertices.

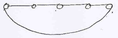
→ A connected graph G with n vertices can have at most '(n-2)' cut vertices.

ex → a tree



→ Cut vertices are not necessary to be there in a graph.

ex →



5) Cut edge / Bridge →

Let 'G' be a connected graph. An edge $E \in G$ is called a "cut edge" if 'G-E' results in a disconnected graph.

Note →

∴ Let 'G' be a connected graph and edge $E \in G$ is a cut-edge iff the edge E is not a part of any cycle in G.

If 'G' is a connected graph with 'n' vertices, then max. no. of cut edges possible is (n-1)

In a connected graph, wherever cut edge exists, cut vertex also exists because at least one vertex of a cut edge is a cut vertex.

In a connected graph, if cut vertex exists, then a cut-edge may or may not exist.

ex →

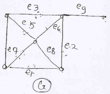
(cut vertex but no cut edge)

*) Cut set →

Let G be a connected graph $G = (V, E)$. A subset E' of E is called a "cut set of G" if deletion of all the edges of

E' from G makes G disconnected and deletion of no ~~global~~ proper subset of E' can disconnect G.

Q.1. ex. → For the graph shown below, which of the foll. are cut sets of G ?



1) $E_1 = \{e_1, e_3, e_5, e_7\}$.

\+ It is a cut-set.

2) $E_2 = \{e_1, \underbrace{e_2, e_3, e_6}_{\text{(proper subset)}}\}$.

So, E_2 is not a cut set.

E_2 is not a cutset because it has proper subset $\{e_2, e_3, e_6\}$ whose deletion can disconnect the graph.

3) $E_3 = \{e_9\}$. ✓ cut set.

4) $E_4 = \{e_3, e_4, e_5\}$. ✓ cut set.

5) $E_5 = \{e_1, e_3, e_5, e_8\}$. ✗ not a cut set

Edege connectivity of graph G $\lambda(G)$ (Imp) →

Let G be a connected graph. The min. no of edges whose removal makes G disconnected is called edge connectivity of G.

If G has a cut edge then $\boxed{\lambda(G)=1}$.

1/10/13

Vertex connectivity → $K(G)$

→ Let G be a connected graph. The minimum no. of vertices whose removal makes G either disconnected or reduces G into a trivial graph is called "Vertex connectivity" of G.

Denoted by $K(G)$ (say).

→ If G has a cut vertex, then $K(G) = 1$.

ote →.

For any connected graph G, vertex connectivity of G

$$\boxed{K(G) \leq \lambda(G)}$$

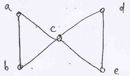
edge connectivity.

and $\boxed{\lambda(G) \leq \delta(G)}$

min. degree of vertices.

$$\therefore \boxed{K(G) \leq \lambda(G) \leq \delta(G).}$$

1. For the graph shown below, what is vertex connectivity and edge connectivity?



→ C is a cut vertex of graph G.

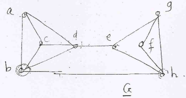
$\therefore$ $K(G)$ (vertex connectivity) $= \boxed{1}$

The graph G has no cut edge.

$\therefore \lambda(G) \geq 2$. and also $\lambda(G) \leq \underset{2}{\underset{\uparrow}{\delta(G)}}$

$\therefore \boxed{\lambda(G) = 2}$

Q.2. For the graph shown below, find $K(G)$ and $\lambda(G)$.



→ The graph has no cut edge.

$\therefore \lambda(G) \geq 2$. $\therefore \boxed{\lambda(G) = 2}$.

(by deleting edges d-e and bh).

The graph has no cut vertex.

$\therefore K(G) \geq 2$. $\therefore \boxed{K(G) = 2}$

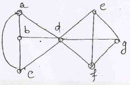
(by deleting vertices d and b (or) e and h.)

edge conn = 2

(As there are two cut edges,)

vertex conn. should be ≥ 2.

Q.3: For the graph show below, find $K(G)$ and $\lambda(G)$



G has a cut vertex d

$\therefore \boxed{K(G) = 1}$

Min. deg. $\delta(G) = 3$.

$\lambda(G) \leq \delta(G) = 3$. $\therefore \lambda(G) \leq 3$. — (1)

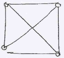
and G has no cut edge

$\therefore \boxed{\lambda(G) = 3.}$

Q. 4. For complete graph K_n, $K(G)$ and $\lambda(G)$?

→ We can reduce the complete graph into trivial graph (as a complete graph cannot be reduced to disconnected graph.) by deleting $(n-1)$ vertices.

K_4.



$\lambda(G) = 3 \cdot (n-1)$ for K_n.

$\therefore \boxed{K(G) = n-1}$

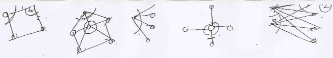
Also, $\boxed{\lambda(G) = n-1.}$

For a complete graph.
$(K(G) \neq \delta(G) = \lambda(G))$.

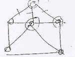
Q. 5 For a cycle graph, $K(G)$ and $\lambda(G)$?

→ $\lambda(G) = 2$ $K(G) = 2$ $\delta(G) = 2$.

For a cycle graph, $\lambda(G) = K(G) = \delta(G) = 2$.



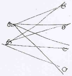
Q.6. For a wheel graph $\lambda(G)$ and $K(G)$?



→ $\lambda(G) = K(G) = \delta(G) = 3.$

Q.7. For a complete bipartite graph, $\lambda(G)$ and $K(G)$?

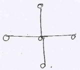
→ $\lambda(G) = K(G) = \min(m, n)$



$K_{2,4}$.

Q.8. For a star graph with n vertices, $(n \geq 2)$, $\lambda(G)$ and $K(G)$?

→ ∴ $\lambda(G) = K(G) = 1.$



Every star graph is a tree.
For any tree with n vertices,
$\lambda(G) = 1$, $K(G) = 1.$

$S_5 = K_{1,4}$.

Note →

1) A simple graph with 'n' vertices is necessarily disconnected if $\boxed{|E| < (n-1)}$ because a simple connected graph with min. no. of edges is a tree and no. of edges in a tree with n vertices is $(n-1)$.

2) A simple graph with 'n' vertices is necessarily connected if $\boxed{|E| > \frac{(n-1)(n-2)}{2}}$

no. of edges in $K_{n-1} = \frac{(n-1)(n-2)}{2}$

and suppose we have n vertices, to make a graph connected we need one more edge.

Q.9. Min. no. of edges necessary in a simple connected graph to ensure connectedy is ?

→. $|E| > \frac{(n-1)(n-2)}{2}$

$> 36.$ $\therefore \boxed{|E| = 37}$

Q.10. Which of the foll. graphs is necessarily connected?

a) simple graph with 7 vertices and 14 edges.

$\left(\frac{6 \times 5}{2} = \frac{30}{2} = 15 \right)$ $(14 < 15)$

$\therefore$ not necessarily connected.

b) A simple graph with 8 vertices and 21 edges

$\frac{7 \times 6}{2} = 21.$ $21 = 21$ $\therefore$ not necessarily connected.

✓c) A simple graph with 9 vertices and 29 edges.

$\frac{8 \times 7}{2} = \frac{56}{2} = 28$ $\therefore 29 > 28.$

$\therefore$ necessarily connected.

10-3.

Note ⇒

1) A simple graph with 'n' vertices and 'k' edges has at least $(n-k)$ components.

2) A simple graph with 'n' vertices and 'k' components at least $(n-k)$ edges.
i.e. no. of edges $\geq (n-k)$.

Q.11. Minimum no. of edges necessary in a simple graph with 10 vertices and 3 components is ?

→ $|E| = n - k = 10 - 3 = \boxed{7}$.

Q.12. ~~A simple graph with~~

Note ⇒

3) A simple graph with 'n' vertices and k components has at most $\frac{(n-k).(n-k+1)}{2}$ edges.

$$\therefore \boxed{|E| \leq \frac{(n-k).(n-k+1)}{2}}$$

Q.12. Max no. of edges possible in a simple ~~connected~~ graph with 10 vertices and 3 components is ?

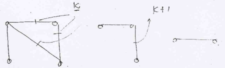
→ $|E| = \frac{(n-k).(n-k+1)}{2} = \frac{7 \times 8}{2} = \boxed{28}$ (max no. of edges).

300

3. Let G be a simple graph with 'n' vertices and 'k' components and 'm' edges. If we delete an edge in G then the no. of components in G is ?

√a) (K) or (K+1) b) (K-1) or (n-K)

c) (K+1) or (m-K). d) (n-K) or (m-K)

→ 

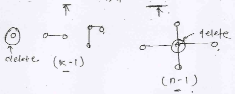
* If the edge we are deleting is not a cut edge for any component then the no. of components remain same as 'K'.

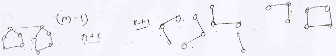
* On the other hand, if the edge we are deleting is a cut edge for any component, then the no. of components become (K+1).

1.14. Let G be a simple graph with n vertices, m edges and k components. If we delete a vertex in G, then the no. of components in G should be between.

a) K and n-1 b) K-1 and n-K.

c) K and n-K. √d) K-1 and n-1.



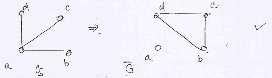


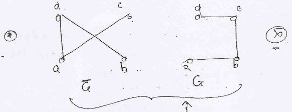
* If the vertex we are deleting from the graph is an isolated vertex, which is component itself then the no. of components become (k-1).

* If the given graph is a star graph with n vertices, then by deleting the cut vertex of star graph, we get (n-1) components.

Q-15 Which of the foll. statements is/are true?

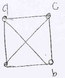
S1) If A simple graph G is connected then, then its complement $\bar{G}$ is not connected. (FALSE)



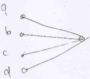


we have a counter example

S2) If a simple graph is not connected, then $\bar{G}$ is connected.



$e_{\bar{G}}$ $\Rightarrow$



G (disconnected graph)

$\bar{G}$

So, always, there will exist a path betⁿ the vertices and no vertex will be left isolated.

16. which of the foll: is /are true ?

(S1) A simple graph with n vertices is connected if $\delta(G) = \frac{n-1}{2}$

→ Suppose G is not connected.

Let G1 and G2 are two connected components of G.

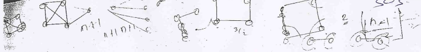
Let $v \in G_1$, $\deg(v) \geq \frac{n-1}{2}$ $\left(\because \delta(G) = \left(\frac{n-1}{2}\right) \right)$

$\Rightarrow$ No. of vertices in $G_1 \geq \left(\frac{n-1}{2}\right) + 1$

$= \left(\frac{n+1}{2}\right)$

$\therefore |V(G_1)| \geq \left(\frac{n+1}{2}\right)$

Similarly, no. of $|V(G_2)| \geq \left(\frac{n+1}{2}\right)$



Now,

$|V(G)| \geq |V(G_1)| + |V(G_2)|$

$|V(G)| \geq \left(\frac{n+1}{2}\right) + \left(\frac{n+1}{2}\right)$

$|V(G)| \ngeq (n+1)$ (which is a contradiction to the our hypothesis.)

$\therefore$ the given statement is true.

52) If a simple graph 'G' has exactly two vertices of odd degree then there exists a path betn the two vertices of odd degree.

$\rightarrow$

By sum of degrees thm, if a graph has exactly two vertices with odd degree, because a component with only one odd degree as not possible.

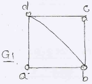
Traversible Graphs →

A connected graph 'G' is said to be traversible if there exists a path which contains each ~~vertex~~ edge of G exactly once. and each vertex of G atleast once.

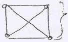
Such a path is called "Euler path".

ex → ~~a~~b - c - d - b - a - d

b - c - d - a - b - d



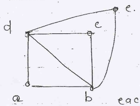
So given graph is traversible.



Not traversible.

Euler circuit →

In a Euler path, if the starting vertex is same as ending vertex, then it is called "Euler circuit."



e - d - a - b - d - c - b - e.

So, euler circuit.

each edge is used exactly once.

Theorem. →

A connected graph 'G' is traversible iff no of vertices with odd degree in G is exactly 2 (or) 0.

case 1) → In a connected graph G, if no. of vertices with odd degree is exactly 2, then Euler path exists but Euler circuit does not exist.

(*) The Euler path begins with a vertex of odd degree and ends with the other vertex of odd degree.

case 2) → In a connected graph G, if no. of vertices with odd degree is 0, then Euler circuit exists. But Euler circuit is also a Euler path. So, both Euler circuit and Euler path exist.

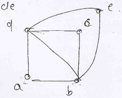
Hamiltonian Graph →

* A connected graph 'G' is said to be Hamiltonian if there exists a cycle which contains each vertex of G exactly once.

Note →

* Every cycle is a circuit but not every circuit is a cycle

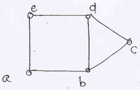
ex



a - b - d - c - b - e - d - a. it is not a cycle.

* The cycle in hamiltonian graph is called "Hamiltonian cycle".

* A connected graph G is said to be "Semi-Hamiltonian" if there exists a path which contains each vertex of G exactly once. Such a path is called "Hamiltonian path".



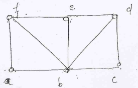
a - b - c - d - e - a.
Hamiltonian cycle.

a - b - c - d - e
Hamiltonian path.

* Note :-

Euler circuit contains each edge of the graph exactly once, whereas, in hamiltonian circuit some edges can be skipped.

Q-1. For the graph given below, denoted by G, which of the foll. statements are/is true?



S1) Euler path exists (traversible)

S2) Euler circuit exists.

S3) Hamiltonian cycle exists.

S4) Hamiltonian path exists.

→

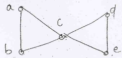
G has 4 vertices with odd degree. ∴ it is not traversible.

∴ S1 and S2 are false.

By skipping internal edges, the graph has hamiltonian cycle passing th' all vertices, (a-b-c-d-e-f-a).

a-b-c-d-e-f ← Hamiltonian path.

Q.2. For the graph shown below, which of the foll. is/are true? (same options).



→ G has no vertices with odd degree.

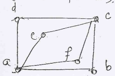
∴ S1 and S2 are true.

Note →

If G has a cut vertex, then Hamiltonian cycle is not possible (Hamiltonian path may exist)

So, by deleting 2 edges at C, hamiltonian path exists but hamiltonian cycle is not possible

Q.3. " — " (same options)



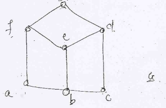
→ G has no vertices with odd degree.

∴ S1 and S2 are true.

In hamiltonian cycle, degree of each vertex should be 2. ∴ we have to delete two edges at vertices a and c. If we delete any vertex of a, hamiltonian cycle is not possible. Also, hamiltonian path is not possible.

If we delete two edges each at a and c, we are left with 6 vertices and 4 edges. ∴ Hamiltonian path is also not possible.

Q.4. " - " (same options).



G has 4 vertices with odd degree.
∴ G is not traversible. ∴ S_1 and S_2 are false.

To construct a Hamiltonian cycle, we have to delete one edge each at b, d and f. Then we are left with 7 vertices and 6 edges.

However, with 6 edges and 7 vertices, path exists

∴ S_3 false. S_4 true.