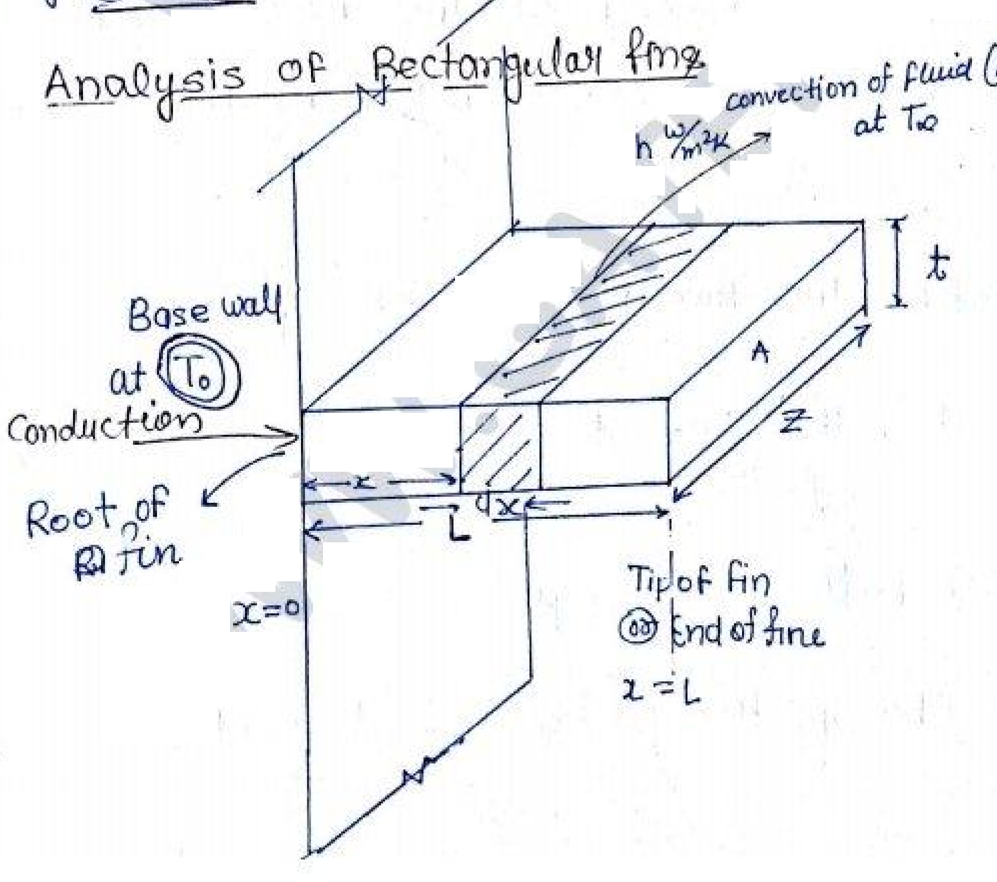
Fins (Extended surface) :-

Fins are the projections protruding (coming out) from a hot surface in ambiant ~~surface~~ fluid and they are meant for increase heat transfer rate by increasing surface Area of H.T.

Ex ① Air cooled I.C. engine
② Reciprocating air compressor
③ Refrigerator (R-134a) & A/C condenser Unit
④ Electric Motor & transformers
⑤ electronic devices
⑥ Automobile Radiator

Fins are always used only when the convective heat transfer coefficient are relatively low ~~tha~~ i.e. with gases or air.

Analysis of Rectangular fins



A = profile Area (Zt)

L = length of fin

Z = width of fin

t = thickness of fin

P = perimeter of fin $= 2(Z+t)$

The actual Mechanisms of H.T. in any fin is that First Get Conducted into fin at its root and then while conduction along the length of fin i.e. in x-direction heat is also simultensously convecting from the surface of fin into ambint fluid at T_∞ with a convecting H.T. Coefficient of h W/m^2K

Objectives:- (1) To get temp distribution within the fin i.e. $T = f(x)$

(2) To get heat transfer rate throughout the fin $q_{fin} = ?$

Assume:- • steady state conditions of fin i.e. $T \neq f(\text{time})$

→ Consider a differential element of the fin of length dx at a distance of x, measured from the base of fin.

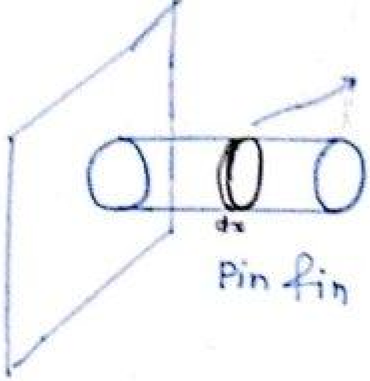
let Heat Conducted into the element $= q_x = -kA\left(\frac{dT}{dx}\right)$

Heat conducted out of the element $= q_{x+dx} = q_x + \frac{\partial (q_x)}{\partial x} dx$

$\frac{\partial}{\partial x}(q_x)$ → Rate of change

total change through dx $= \frac{\partial}{\partial x}(q_x) \cdot dx$

* Pinfin



$q_{conv.} = h \pi d \, (dx) (T - T_\infty)$

$P = \pi d$ (perimeter

writting the enegy balance for steady state condition of fin we get

$$q_x = q_{x+dx} + q_{conv}$$

$$q_x = q_x + \frac{\partial}{\partial x}(q_x)\,dx + hP\,dx\,(T - T_\infty)$$

$$0 = \frac{\partial}{\partial x}\left(-KA\frac{dT}{dx}\right)dx + hP\,dx\,(T - T_\infty)$$

$$\Rightarrow \quad \frac{d^2T}{dx^2} - \frac{hP}{KA}(T - T_\infty) = 0$$

let $\quad T - T_\infty = \theta = f(x)$

$$\frac{dT}{dx} = \frac{d\theta}{dx}$$

$$\frac{d^2T}{dx^2} = \frac{d^2\theta}{dx^2}$$

and put $\quad m^2 = \frac{hP}{KA}$

$$\Rightarrow \quad \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

this is a strandard format of second order differential equation in θ whose soln directly given as

$$\theta = C_1 e^{-mx} + C_2 e^{mx} \quad \text{where } m = \sqrt{\frac{hP}{KA}} \ \ 1/\text{mete}$$

C_1 & C_2 are constant of integration that are to be obtained from boundary conditions

* one Common boundary conditions is.

at $x=0$ (i.e. at the root of fin)

$\Rightarrow \quad T = T_0$ and $\theta = \theta_0 = (T_0 - T_\infty)$

* the second boundary condition depends upon three differance case.

Case(1) Fin is Infinitely long (OR) long fin

Than the temp. at the tip of fin will be essentially that of the ambient temp

At $x = \infty \Rightarrow T = T_\infty$ and then $\theta = 0$

$$T \rightarrow T_\infty$$

$$\theta \rightarrow 0$$

$$\theta = 0 = C_1 e^{-m(\infty)} + C_2 e^{m(\infty)}$$

$$\Rightarrow \quad C_2 = 0$$

@ $x=0 \quad \theta = \theta_0 \Rightarrow C_1 + C_2 = \theta_0$

$$C_1 = \theta_0 = T_0 - T_\infty$$

$$\theta = \theta_0 e^{-mx}$$

$$T - T_\infty = (T_0 - T_\infty)\, e^{-mx}$$

$$\boxed{\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = e^{-mx}}$$

To find q_{find} :- (for any fin Case)

For any fin case H.T. Rate through find is equal to Heat Conducted into fin at it's Root/base

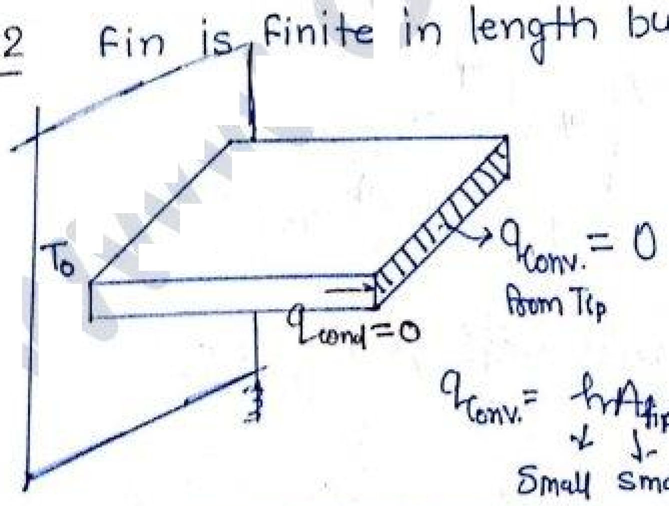
$$q_{fin} = -KA \left.\frac{dT}{dx}\right)_{x=0}$$

$$q_{fin} = -KA\left[\theta_0 e^{-mx}\right](-m)\Big|_{x=0}$$

$$q_{fin} = +KA\,\theta_0\sqrt{\frac{hP}{KA}}$$

$$\boxed{q_{fin} = \sqrt{hPKA}\;\theta_0} \qquad \theta_0 = T_0 - T_\infty$$

Case-2 Fin is finite in length but it's tip is insulated or Adiabatic tip



Heat Conduction into the tip of fin = 0

$$q_{conv.} = \underset{\text{Small}}{h}\,\underset{\text{Small}}{A_{tip}}\,(\underbrace{T_{x=L} - T_\infty}_{\text{Small}}) \rightarrow \text{in praticle}$$

$$\Rightarrow \quad -KA\left(\frac{dT}{dx}\right)_{at\ x=L} = 0$$

$$\left(\frac{dT}{dx}\right)_{at\ x=L} = 0 \quad \Rightarrow \quad \left(\frac{d\theta}{dx}\right)_{at\ x=L} = 0$$

$$\theta = T - T_\infty$$

then the sol^n for the temp distribution within the fin is

$$* \Rightarrow \boxed{\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}}$$

Heat transfer rate in fin $q_{fin} = -KA \left.\frac{dT}{dx}\right)_{x=0}$

$$\boxed{q_{fin} = \sqrt{hPKA}\,(T_0 - T_\infty)\tanh(mL)} \text{ watt}$$

Note if no case is mention in any fin problem by default assume case 2

* In practice we never insulate the tip of the fin rather neglected the convection heat loss from the tip

$$q_{conv.} = \underset{\text{Small}}{h}\ \underset{\text{Small}}{A_{tip}}\ (\underbrace{T_{x=L} - T_\infty}_{\text{Small}})$$

Product will negligible small

Case (3) Fin is finite in length and fin's tip is uninsulated (tip also loses heat by Convection)

The solⁿ for temp. distribution within fin is

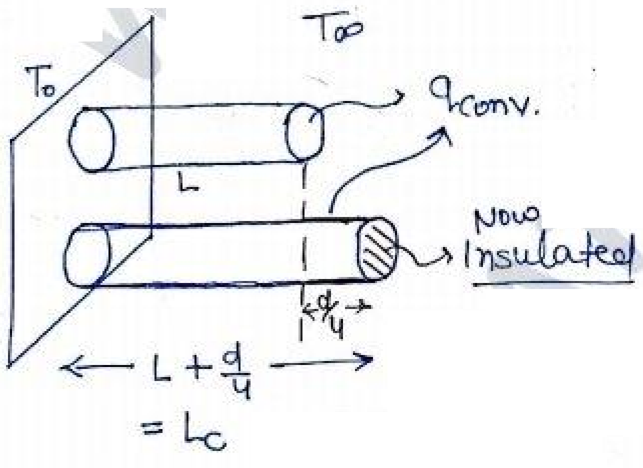
$$\Rightarrow \boxed{\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(L_c - x)}{\cosh mL_c}}$$

where L_c = Corrected length of fin

$L_c = L + \frac{t}{2}$ (for Ractangular fin)

$L_c = L + \frac{d}{4}$ (For ~~Cir~~ Pin fin)

$$\boxed{q_{fin} = \sqrt{hPKA}\,(T_0 - T_\infty)\tanh(mL_c)} \text{ watt}$$



Q 34

$D = 5mm$, $L = 100$ mm

$k = 400$ W/mK , $h = 40$ W/m²K

$T_0 = 130^\circ C$ $T_\infty = 30^\circ C$

$P = \pi d$

$A = \frac{\pi}{4} d^2$ $m = \sqrt{\frac{hP}{KA}} = \cancel{\sqrt{\frac{40 \times \pi D}{400 \times \pi D^2 L}}} = \sqrt{\frac{40 \times \pi D}{400 \times \frac{\pi D^2}{4}}}$

Cond. Area

$\cancel{m = \sqrt{\frac{4 \times 1000}{40 \times 100}}}$ $m = \sqrt{\frac{40 \times 1000}{400 \times 5}}$

$\cancel{m = 1}$ $m = 8.943$ /meter

$q_{fin} = \sqrt{hPKA} \tanh(mL) (T_0 - T_\infty)$

$q_{fin} = \sqrt{40 \times \pi \times 5 \times 10^{-3} \times 400 \times \pi \frac{(5 \times 10^{-3})^2}{4}} (130 - 30)$

$\tan\left(8.943 \times \frac{1}{10}\right)$

$q_{fin} = 5.0$ watt

D.54

$L = 30\ cm$ $\quad T_0 = 300°C$

$z = 30\ cm$ $\quad k = 204\ W/mK$

$t = 2\ mm$ $\quad T_\infty = 30°C$

$h = 15\ W/m^2K$

~~$A = 2(z+t)$~~

~~$P = 64\ cm$~~

~~$A = zt$~~

~~$A = 60\ cm^2$~~

$P = 2(300+2)$

$P = 604$

$A = zt$

$A = 300 \times 2$

$A = 600\ mm^2$

$$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{15 \times 604 \times 1000}{204 \times 600}}$$

~~$m = 7.84$ /meter~~

$m = 8.6$ /met

$$L_c = L + \frac{t}{2}$$

$$L_c = \left(0.3 + \frac{1}{1000}\right)$$

$$L_c = 0.301\ m$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(L_c - x)}{\cosh m L_c}$$

Put $x = 0.3\ m$

$$\frac{T - 30}{300 - 30} = \frac{\cosh(8.6(0.301 - 0.3))}{\cosh(8.6(0.30))}$$

$$T = 70.3°C$$

$$q_{fin} = \sqrt{hPKA}\ \tan mL_c\ (T_0 - T_\infty)$$

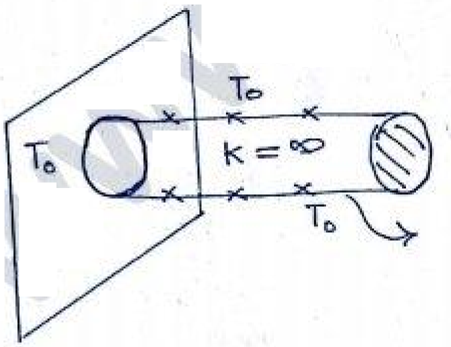
$$q_{fin} = 281.1\ \text{watt}$$

Fin efficiecy :-

It is defined as the ratio between actual H.T. rate taking through fin and max. possible H.T. rate that can occur through the fin that when the entire fin surface ~~at~~ is at it root ~~and~~ or base temp.

$$\boxed{\eta_{fin} = \frac{q_{act.}}{q_{max.possible}}}$$

$$q_{act.} = \sqrt{hPKA}\,(T_0 - T_\infty)\tanh(mL) \text{ watt}$$



$$\left\{ T_{\substack{entire\\ surface}} = T_0 \right\}$$

Note:- The entire fin surface will be at it's root or base temp. only when the material of fin has infinite thermal conductivity ($K = \infty$)

$$q_{max.} = h(P \times L)(T_0 - T_\infty)$$

$$\eta_{fin} = \frac{\sqrt{hPKA}\,\cancel{(T_0 - T_\infty)}\tanh(mL)}{h(P \times L)\cancel{(T_0 - T_\infty)}}$$

Case ② $$\boxed{\eta_{fin} = \frac{\tanh(mL)}{mL}}$$ $\therefore \; m = \frac{hP}{KA}$

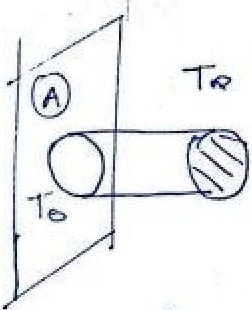
Case ③ $$\boxed{\eta_{fin} = \frac{\tanh(mL_c)}{mL_c}}$$

$$\eta_{fin} \propto \sqrt{K}$$

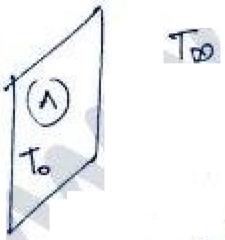
$\therefore$ fin material Must have high k (Al, Copper)

fin effectiveness (ϵ_{fin}) :- It is ~~the~~ defined as the ratio b/w H.T. rate with fin and the H.T. rate without fin

$$\epsilon_{fin} = \frac{q_{with}}{q_{without}}$$



$$q_{with\ fin} = \sqrt{hPKA}\,(T_0 - T_\infty)\tanh(mL)$$



$$q_{without\ fin} = hA\,(T_0 - T_\infty)$$

Case② $$\epsilon_{fin} = \frac{\sqrt{hPKA}\,(T_0 - T_\infty)\tanh(mL)}{hA\,(T_0 - T_\infty)}$$

$$\boxed{\epsilon_{fin} = \frac{\tanh(mL)}{\sqrt{hA/KP}}}$$

Note:-

(1) effectiveness of fin tells about how much % increase in heat transfer rate we are able to gain by keeping the fins as compared to the case where there is no fin. If effectiveness of fin is low it means that fins are not worth keeping since they do not help us much increasing H.T. Rate.

(2)

$$\epsilon_{fin} \propto \frac{1}{\sqrt{h}}$$

In Cases h less

* use find at Cases side

$$\epsilon_{fin} \propto \sqrt{\frac{P}{A}}$$

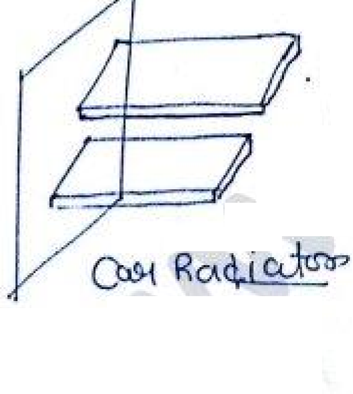
This is the reason why fins are never used with water or Condensation H.T. where the Convective H.T. coefficient high

(3)

$$\epsilon_{fin} \propto \sqrt{\frac{P}{A}}$$

Hence to have high effectiveness of fin. Fin must be thin and closly spaced fins.

⇒



Car Radiator

$$\text{No. of fins required } (n) = \frac{\text{Total H.T. rate i.e. required be transfered}}{\text{H.T. Rate through each fin}}$$

$$\epsilon_{fin} \propto \sqrt{K}$$

⇒ fin material Must have high K.

Q.33 d

1. fins are used on Gas side because h less

Q.55 Long fin

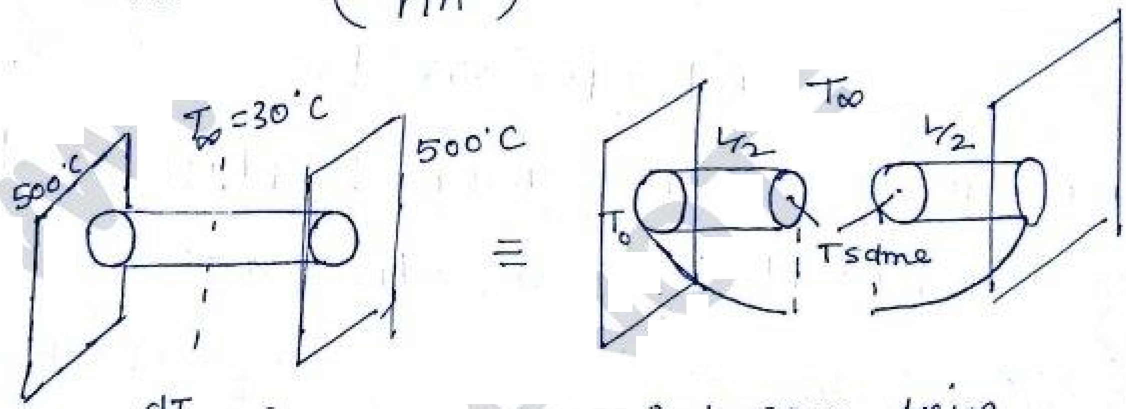
$$q_{fin} = \sqrt{hPKA}\,(T_0 - T_\infty)$$

$$q_{\substack{without\\fin}} = hA\,(T_0 - T_\infty)$$

$$\epsilon_{fin} = \frac{\sqrt{hPKA}\,\cancel{(T_0 - T_\infty)}}{hA\,\cancel{(T_0 - T_\infty)}}$$

$$\epsilon_{fin} = \left(\frac{PK}{hA}\right)^{1/2}$$

Q.21



$\frac{dT}{dx} = ?$

when Rods are join

No heat Cond^n between the ends

i.e. $-KA\frac{dT}{dx} = 0$

$$\Rightarrow \frac{dT}{dx} = 0$$