Matrices

Short Answer Type Quesrtoins

1. Construct a matrix $A=[a_{ij}]_{2\times 2}$ whose elements aij are given by $a_{ij}=e^{2ix}\sin jx$

Sol. For $i=1,\ j=1,\ a_{11}=e^{2x}\sin x$

For $i=1,\ j=2,\ a_{21}=e^{2x}\sin 2x$

For $i=2,\ j=1,\ a_{21}=e^{4x}\sin x$

For $i=2,\ j=2,\ a_{22}=e^{4x}\sin 2x$

Thus, $A=\begin{bmatrix} e^{2x}\sin x & \mathrm{e}^{2x}\sin 2x \\ e^{4x}\sin x & e^{4x}\sin 2x \end{bmatrix}$

2. If $A=\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 3 & 2 \\ 4 & 3 & 1 \end{bmatrix}$, $C=\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $D=\begin{bmatrix} 4 & 6 & 8 \\ 5 & 7 & 9 \end{bmatrix}$, then which of the sums A + B, B + C, C + D and B + D is defined?

Sol. Only B + D is defined since matrices of the same order can only be added.

3. Show that a matrix which is both symmetric and skew symmetric is a zero matrix.

Sol. Let A = [a_{ij}] be a matrix which is both symmetric and skew symmetric.
Since A is a skew symmetric matrix, so A′= –A.
Thus, for all i *and* j, we have $a_{ij}= - a_{ji}$ (1)
Again, since A is a symmetric matrix, so A′= A.
Thus, for all i *and* j, we have

$a_{ji}=a_{ij}$ (2)

Therefore, from (1) and (2), we get

$a_{ij}=-a_{ij}$ *for all i and j*

Or $2a_{ij}=0,$

i.e., $a_{ij}=0$ for all i and j. Hence A is Zero matrix.

4. If $\begin{bmatrix} 2x & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix}=O$, find the value of x.

Sol. We have $[2x \quad 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix}=0 \Rightarrow [2x-9 \quad 4x]\begin{bmatrix} x \\ 8 \end{bmatrix}=0$

Or $[2x^2-9 \quad +32x]=[0] \Rightarrow 2x^2+23x=0$

Or $x(2x+23)=0 \Rightarrow x=0,\ x=\dfrac{-23}{2}$

5. If A is 3×3 invertible matrix, then show that for any scalar k(non-zero), kA is invertible and $(kA)^{-1} = \frac{1}{k}A^{-1}$

Sol. We have

$$(kA)\left(\frac{1}{k}A^{-1}\right) = \left(k.\frac{1}{k}\right)(A.A^{-1}) = 1(I) = I$$

Hence (kA) is inverse of $\left(\frac{1}{k}A^{-1}\right)$ or $(kA)^{-1} = \frac{1}{k}A^{-1}$

Long Answer Type Questions

6. Express the matrix A as the sum of a symmetric and a skew symmetric matrix, where

$$A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$$

Sol. We have

$$A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}, \text{ then } A' = \begin{bmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{bmatrix}$$

Hence $\frac{A+A'}{2} = \frac{1}{2}\begin{bmatrix} 4 & 11 & -5 \\ 11 & 6 & 3 \\ -5 & 3 & 8 \end{bmatrix} = \begin{bmatrix} 2 & \frac{11}{2} & \frac{5}{2} \\ \frac{11}{2} & 3 & \frac{3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 4 \end{bmatrix}$

And $\frac{A-A'}{2} = \frac{1}{2}\begin{bmatrix} 0 & -3 & -7 \\ 3 & 0 & 7 \\ 7 & -7 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-3}{2} & \frac{-7}{2} \\ \frac{3}{2} & 3 & \frac{7}{2} \\ \frac{7}{2} & \frac{-7}{2} & 4 \end{bmatrix}$

Therefore,

$$\frac{A+A'}{2}+\frac{A-A'}{2}=\begin{bmatrix} 2 & \frac{11}{2} & \frac{-5}{2} \\ \frac{11}{2} & 3 & \frac{3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 4 \end{bmatrix}+\begin{bmatrix} 0 & \frac{-3}{2} & \frac{-7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & \frac{-7}{2} & 0 \end{bmatrix}=\begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 0 & -2 & 4 \end{bmatrix}=A$$

7. **If** $A=\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$**, then show that A satisfies the equation**

$A^3-4A^2-3A+11I=O.$

Sol. $A^2 = A\times A=\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}\times\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$

$$\begin{bmatrix} 1+6+2 & 3+0+4 & 2-3+6 \\ 2+0-1 & 6+0-2 & 4+0-3 \\ 1+4+3 & 3+0+6 & 2-2+9 \end{bmatrix} = \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$$

And $A^3 = A^2\times A=\begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}\times\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$

$$=\begin{bmatrix} 9+14+5 & 27+0+10 & 18-7+15 \\ 1+8+1 & 3+0+2 & 2-4+3 \\ 8+18+9 & 24+0+18 & 16-9+27 \end{bmatrix}=\begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix}$$

Now $A^3-4A^2-3A+11(I)$

$$=\begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix}-4\begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}-3\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}+11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 28-36-3+11 & 37-28-9+0 & 26-20-6+0 \\ 10-4-6+0 & 5-16+0+11 & 1-4+3+0 \\ 35-32-3+0 & 42-36-6+0 & 34-36-9+11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=0$$

8. **Let** $A=\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}$. **Then show that** $A^2-4A+7I=0$. **Using this result calculate** A^5 **also.**

Sol. We have $A^2=\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}=\begin{bmatrix}1 & 12\\ -4 & 1\end{bmatrix}$

$-4A=\begin{bmatrix}-8 & -12\\ 4 & -8\end{bmatrix}$ *and* $7I=\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$

Therefore, $A^2-4A+7I=\begin{bmatrix}1-8+7 & 12-12+0\\ -4+4+0 & 1-8+7\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=O$

$\Rightarrow \mathrm{A}^2=4A-7I$

Thus $A^3=A.A^2=A(4A-7I)=4(4A-7I)-7A$

$=16A-28I-7A=9A-28I$

And so $A^5=A^3A^2$

= (9A – 28I) (4A – 7I)

= $36A^2$– 63A – 112A + 196I

= 36 (4A – 7I) – 175A + 196I

= – 31A – 56I

$=-31\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}-56\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}-118 & -93\\ 31 & -118\end{bmatrix}$

Objective Type Questions

Choose the correct answer from the given four options in s 9 to 12.

9. **If A and B are square matrices of the same order, then (A+B) (A–B) is equal to**

(A) A^2-B^2

(B) $A^2-BA-AB-B^2$

(C) $A^2-B^2+BA-AB$

(D) A^2-BA+B^2+AB

Sol. (C) is correct answer. (A + B) (A – B) = A (A – B) + B (A – B)= A^2– AB + BA – B^2

10. **If A =** $A=\begin{bmatrix}2 & 1 & 3\\ 4 & 5 & 1\end{bmatrix}$ **and** $B=\begin{bmatrix}2 & 3\\ 4 & 2\\ 1 & 5\end{bmatrix}$**, then**

(A) only AB is defined

(B) only BA is defined

(C) AB and BA both are defined

(D) AB and BA both are not defined.

Sol. (C) is correct answer. Let A = $[a_{ij}]_{2\times3}$ B = $[b_{ij}]_{3\times2}$. Both AB and BA are defined.

11. The matrix $A=\begin{bmatrix}0&0&5\\0&5&0\\5&0&0\end{bmatrix}$ **is a**

(A) scalar matrix

(B) diagonal matrix

(C) unit matrix

(D) square matrix

Sol. (D) is correct answer.

12. If A and B are symmetric matrices of the same order, then (AB′–BA′) is a

(A) Skew symmetric matrix

(B) Null matrix

(C) Symmetric matrix

(D) None of these

Sol. (A) is correct answer since

(AB′–BA′)′ = (AB′)′– (BA′)′

= (BA′– AB′)

= – (AB′–BA′)

Fill in the blanks in each of the s 13 to 15:

13. If A and B are two skew symmetric matrices of same order, then AB is symmetric matrix if _______.

Sol. AB = BA.

14. If A and B are matrices of same order, then (3A –2B)′is equal to _______.

Sol. 3A′–2B′.

15. Addition of matrices is defined if order of the matrices is _______

Sol. Same.

State whether the statements in each of the s 16 to 19 is true or false:

16. If two matrices A and B are of the same order, then 2A + B = B + 2A.

Sol. True

17. Matrix subtraction is associative

Sol. False

18. For the non-singular matrix A, $(A')^{-1} = (A^{-1})'$.

Sol. True

19. AB = AC ⇒B = C for any three matrices of same order.

Sol. False

Matrices

Objective Type Questions

Choose the correct answer from the given four options in each of the Exercises 53 to 67.

53. **The matrix** $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ **is a**

(A) square matrix
(B) diagonal matrix
(C) unit matrix
(D) none of these

Sol. We know that, in a square matrix number of rows are equal to the number of columns. So, the matrix $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a square matrix.

54. **Total number of possible matrices of order** 3×3 **with each entry 2 or 0 is**

(A) 9
(B) 27
(C) 81
(D) 512

Sol. (D) Total number of possible matrices of order 3×3 with each entry 2 or 0 is 2^9 i.e., 512.

55. **If** $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+16 \end{bmatrix}$**, then the value of** $x+y$ **is**

(A) x= 3, y= 1
(B) x= 2, y= 3
(C) x= 2, y= 4
(D) x= 3, y= 3

Sol. (B) We have, $4x = x+6 \Rightarrow x = 2$

And $4x = 7y-13 \Rightarrow 8 = 7y-13$

$\Rightarrow 7y=21 \Rightarrow y=3$

$\therefore x+y=2+3=5$

56. **If** $A=\frac{1}{\pi}\begin{bmatrix}\sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right)\\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x)\end{bmatrix}$ **and** $B=\frac{1}{\pi}\begin{bmatrix}-\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right)\\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x)\end{bmatrix}$ **then A – B is equal to**

(A) I

(B) O

(C) 2I

(D) $\frac{1}{2}I$

Sol. (D) We have, $A=\begin{bmatrix}\frac{1}{\pi}\sin^{-1}(x\pi) & \frac{1}{\pi}\tan^{-1}\left(\frac{x}{\pi}\right)\\ \frac{1}{\pi}\sin^{-1}\left(\frac{x}{\pi}\right) & \frac{1}{\pi}\cot^{-1}(\pi x)\end{bmatrix}$

$$B=\begin{bmatrix}-\frac{1}{\pi}\cos^{-1}(x\pi) & \frac{1}{\pi}\tan^{-1}\left(\frac{x}{\pi}\right)\\ \frac{1}{\pi}\sin^{-1}\left(\frac{x}{\pi}\right) & -\frac{1}{\pi}\tan^{-1}(\pi x)\end{bmatrix}$$

$$\therefore\quad A-B=\begin{bmatrix}\frac{1}{\pi}\sin^{-1}x\pi+\cos^{-1}x\pi & \frac{1}{\pi}\left(\tan^{-1}\frac{x}{\pi}\right)-\tan^{-1}\frac{x}{\pi}\\ \frac{1}{\pi}\left(\sin^{-1}\frac{x}{\pi}-\sin^{-1}\frac{x}{\pi}\right) & \frac{1}{\pi}\cot^{-1}\pi x+\tan^{-1}\pi x\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{\pi}.\frac{\pi}{2} & 0\\ 0 & \frac{1}{\pi}.\frac{\pi}{2}\end{bmatrix}\left[\begin{array}{l}\because \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\\ and\ \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\end{array}\right]$$

$$=\frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\frac{1}{2}I$$

57. **If A and B are two matrices of the order 3 × m and 3 × n, respectively and m= n, then order of matrix (5A – 2B) is**

(A) m× 3

(B) 3 × 3

(C) m× n

(D) 3 × n

Sol. (d) $A_{3\times m}$ and $B_{3\times n}$ are two matrices. If m=n, then A and B have same orders as $3\times n$ each, so the order of (5A-2B) should be same as $3\times n$.

58. **If** $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$**, then A^2 is equal to**

(A) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

(B) $\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix}$

(C) $\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix}$

(D) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Sol. (D) $\because\ A^2=A.A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}.\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

59. **If matrix $A = [a_{ij}]_{2\times 2}$, where $a_{ij}= 1$ if $i \neq j= 0$ if $i= j$ then A^2 is equal to**

(A) I

(B) A

(C) 0

(D) None of these

Sol. (a) We have, $A = [a_{ij}]_{2\times2}$, where $a_{ij}= 1$ if $i \neq j= 0$ if $i= j$

$\therefore\ A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

And $A^2=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$

60. **The matrix** $\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 4\end{bmatrix}$ **is a**

(A) identity matrix

(B) symmetric matrix

(C) skew-symmetric matrix

(D) None of these

Sol. (B) Let $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 4\end{bmatrix}$

$$\therefore A' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} = A$$

So, the given matrix is a symmetric matrix.

[Since, in a square matrix A, if A'=A, then A is called symmetric matrix]

61. The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a

(A) diagonal matrix
(B) symmetric matrix
(C) skew symmetric matrix
(D) scalar matrix

Sol. (C) We know that, in a square matrix, if $b_{ij} = 0$, when $i \neq j$, then it is said to be a diagonal matrix. Here, $b_{12}, b_{13}..... \neq 0$, so the given matrix is not a diagonal matrix.

$$Now,\ B = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$$

$$\therefore \text{B}' = \begin{bmatrix} 0 & 5 & -8 \\ -5 & 0 & -12 \\ 8 & 12 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix} = -B$$

So, the given matrix is a skew-symmetric matrix, since we know that in a square matrix B, if B'=-B, then it is called skew-symmetric matrix.

62. If A is matrix of order m× n and B is a matrix such that AB′and B′A are both defined, then order of matrix B is

(A) m × m
(B) n × n
(C) n × m
(D) m × n

Sol. (D) Let $A = \left[a_{ij}\right]_{m\times n}$ $and\ B = \left[b_{ij}\right]_{p\times q}$

$$B' = \left[b_{ji}\right]_{q\times p}$$

Now, AB' is defined, so n=q

and B'A is also defined, so p=m

$\therefore$ Order of $\text{B}' = \left[b_{ji}\right]_{n\times m}$

And order of $\text{B} = \left[b_{ij}\right]_{m\times n}$

63. If A and B are matrices of same order, then (AB′–BA′) is a

(A) skew symmetric matrix

(B) null matrix

(C) symmetric matrix

(D) unit matrix

Sol. (a) We have matrices A and B of same order.

Let $P(AB'-BA')$

Then, $P'=(AB'-BA')'=(AB')'-(BA')'$

$=(B')'(A)'-(A')'B'=BA'-AB'$

$=-(AB'-BA')=-P$

Hence, $(AB'-BA')$ is a skew-symmetric matrix.

64. If A is a square matrix such that A^2= I, then $(A-I)^3+(A+I)^3-7A$ is equal to

(A) A

(B) I – A

(C) I + A

(D) 3A

Sol. (a) We have, $A^2=I$

$\therefore (A-I)^3+(A+I)^3-7A$

$=\left[(A-I)+(A+I)\left\{(A-I)^2+(A+I)^2-(A-I)(A+I)\right\}\right]-7A$

$\left[\because a^3+b^3=(a+b)\left(a^2+b^2-ab\right)\right]$

$=\left[(2A)\left\{A^2+I^2-2AI+A^2+I^2+AI-\left(A^2-I^2\right)\right\}\right]-7A$

$=2A[I+I^2+I+I^2-A^2+I^2]-7A \quad \left[\because A^2=AI\right]$

$=2A[5I-I]-7A$

$=8AI-7AI \quad [\because A=AI]$

= AI = A

65. For any two matrices A and B, we have

(A) AB = BA

(B) AB ≠BA

(C) AB = 0

(D) None of the above

Sol. (D) For any two matrices A and B, we may have AB=BA=I, $AB \neq BA$ and AB=0 but it is not always true.

66. On using elementary column operations $C_2 \to C_2 - 2C_1$ in the following matrix

equation $\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$**, we have:**

(A) $\begin{bmatrix} 1 & -5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -0 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$

Sol. (D) Given that, $\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

On using $C_2 \to C_2 - 2C_1$, $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$

Since, on using elementary column operation on X=AB, we apply these operations simultaneously on X and on the second matrix B of the product AB on RHS.

67. On using elementary row operation $R_1 \to R_1 - 3R_2$ in the following matrix equation $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$, **we have**

(a) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1 & -3 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -7 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 2 \\ -5 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -3 & -3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Sol. We have, $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Using elementary row operation $R_1 \to R_1 - 3R_{2,}$

$$\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

Since, on using elementary row operation on X=AB, we apply these operation simultaneously on X and on the first matrix A of the product AB on RHS.

Fill in the blanks in each of the Exercises 68–81.

68. ________ matrix is both symmetric and skew-symmetric matrix.

Sol. Null matrix is both symmetric and skew-symmetric matrix.

69. Sum of two skew-symmetric matrices is always ________ matrix.

Sol. Let A is a given matrix, then (-A) is a skew-symmetric matrix.
Similarly, for a given matrix-B is a skew-symmetric matrix.
Hence, $-A-B=-(A+B) \Rightarrow$ sum of two skew-symmetric matrices is always skew-symmetric matrix.

70. The negative of a matrix is obtained by multiplying it by ________.

Sol. Let A is a given matrix.
$\therefore$ -A=-1$[A]$
So, the negative of a matrix is obtained by multiplying it by -1.

71. The product of any matrix by the scalar ________ is the null matrix.

Sol. The product of any matrix by the scalar 0 is the null matrix. i.e., 0. A=0
[where, A is any matrix]

72. A matrix which is not a square matrix is called a ________ matrix.

Sol. A matrix which is not a square matrix is called a rectangular matrix. For example, a rectangular matrix is $A=\left[a_{ij}\right]_{m\times n}$, where $m \neq n$

73. Matrix multiplication is ________ over addition.

Sol. Matrix multiplication is distributive over addition.
e.g., For three matrices A, B and C.
(i) A(B+C) = AB+AC
(ii) (A+B)C = AC+BC

74. If A is a symmetric matrix, then A^3 is a ________ matrix.

Sol. If A is a symmetric matrix, then $\mathbf{A^3}$ is a symmetric matrix.
$\because A' = A$
$\therefore (A^3)' = A'^3$
$= A^3 \quad \left[\because (A')^n = (A^n)'\right]$

75. If A is a skew-symmetric matrix, then A^2 is a ________.

Sol. If A is a skew-symmetric matrix, then A^2 is a symmetric matrix.

$\because A' = -A$

$\therefore \left(A^2\right)' = A'^2$

$= (-A)^2 \quad [\because A' = -A]$

$= A^2$

So, A^2 is a symmetric matrix.

76. If A and B are square matrices of the same order, then
(i) (AB)′= ________.
(ii) (kA)′= ________. (Where, k is any scalar)
(iii) [k (A – B)]′= ________.

Sol.

$(\mathbf{i})\ (\mathbf{AB})' = \mathrm{B'A'}$

$(\mathbf{ii})\ (\mathbf{kA})' = \mathrm{kA'}$

$(\mathbf{iii})\ \left[\mathbf{k}(\mathbf{A} - \mathbf{B})\right]' = \mathrm{k}(A' - B')$

77. If A is skew-symmetric, then kA is a ________. (where, k is any scalar)

Sol. If A is a skew-symmetric, then kA is a skew-symmetric matrix (where, k is any scalar).

$\left[\because\ A' = -A \Rightarrow (kA)' = k(A)' = -(kA)\right]$

78. If A and B are symmetric matrices, then
(i) AB – BA is a ________.
(ii) BA – 2AB is a ________.

Sol. (i)AB-BA is a skew-symmetric matrix.

Since, $[AB - BA]' = (AB') - (BA)'$

$= \mathrm{B'A'\text{-}A'B'} \quad \left[\because (AB)' = B'A'\right]$

$= \mathrm{BA\text{-}AB} \quad [\because A'] = A \text{ and } B' = B]$

$= -[AB - BA]$

So, [AB-BA] is a skew-symmetric matrix.

(ii) [BA-2AB] is a neither symmetric nor skew-symmetric matrix.

$\therefore\ (BA - 2AB)' = (BA)' - 2(AB)'$

$= \mathrm{A'B'\text{-}2B'A'}$

$= \mathrm{AB\text{-}2BA}$

$= -(2BA - AB)$

So, [BA-2AB] is neither symmetric nor skew-symmetric matrix.

79. If A is symmetric matrix, then B′AB is ________.

Sol. If A is a symmetric matrix, then B’AB is a symmetric matrix.

$\because [B'AB]' = [B'(AB)]'$

$= (AB)'(B')' [\because (AB)' = B'A']$

$= B'A'B$

$[= B'A'B]$

So, B'AB is a symmetric matrix.

80. If A and B are symmetric matrices of same order, then AB is symmetric if and only if ________.

Sol. If A and B are symmetric matrices of same order, then AB is symmetric if and only if AB=BA.

$\therefore \ (AB)'$

= B'A'=BA $[\because \ AB = BA]$

= AB

81. In applying one or more row operations while finding A^{-1} by elementary row operations, we obtain all zeroes in one or more, then A^{-1} ________.

Sol. In applying one or more row operations while finding A^{-1} by elementary row operations, we obtain all zeroes in one or more, then A^{-1} does not exist.

State Exercises 82 to 101 which of the following statements are True or False

82. A matrix denotes a number.

Sol. False

A matrix is an ordered rectangular array of numbers of functions.

83. Matrices of any order can be added.

Sol. False

Two matrices are added, if they are of the same order.

84. Two matrices are equal if they have same number of rows and same number of columns.

Sol. False

If two matrices have same number of rows and same number of columns, then they are said to be square matrix and if two square matrices have same elements in both the matrices, only then they are called equal.

85. Matrices of different order cannot be subtracted.

Sol. True

Two matrices of same order can be subtracted.

86. Matrix addition is associative as well as commutative.

Sol. True

Matrix addition is associative as well as commutative i.e.,

$(A+B)+C = A+(B+C) \, and \, A+B = B+A,$ where A, B and C are matrices of same order.

87. Matrix multiplication is commutative.

Sol. False

Since, $AB \neq BA$ is possible when AB and BA are both defined.

88. A square matrix where every element is unity is called an identity matrix.

Sol. False

Since, in an identity matrix, the diagonal elements are all one and rest are all zero.

89. If A and B are two square matrices of the same order, then A + B = B + A.

Sol. True

Since, matrix addition is commutative ie, A+B=B+A, where A and B are two square matrices.

90. If A and B are two matrices of the same order, then A – B = B – A.

Sol. False

Since the addition of two matrices of same order are commutative.

$\therefore$ $A+(-B)=A-B=-[B-A]\neq B-A$

91. If matrix AB = 0, then A = 0 or B = 0 or both A and B are null matrices.

Sol. False

Since, for two non-zero matrices A and B of same order, it can be possible that A.B=0= null matrix

92. Transpose of a column matrix is a column matrix.

Sol. False

Transpose of a column matrix is a row matrix.

93. If A and B are two square matrices of the same order, then AB = BA.

Sol. False

For two square matrices of same order it is not always true that AB=BA.

94. If each of the three matrices of the same order are symmetric, then their sum is a symmetric matrix.

Sol. True

Let A, B and C are three matrices of same order

$\therefore A'=A, B'=B \text{ and } C'=C$

$\therefore (A+B+C)'=A'+B'+C'$

$=(A+B+C)$

95. If A and B are any two matrices of the same order, then (AB)′= A′B′.

Sol. False

$\because (AB)'=B'A'$

96. If (AB)′= B′A′, where A and B are not square matrices, then number of rows in A is equal to number of columns in B and number of columns in A is equal to number of rows in B.

Sol. True

Let A is of order $m\times n$ and B is of order $p\times q$

Since, $(AB)' = B'A'$

$\therefore$ $A_{(m\times n)}B_{(p\times p)}$ *is defined* $\Rightarrow n = p$ ...(*i*)

and AB is of order $m\times q$

$\Rightarrow (AB)'$ is of order $q\times m$ ...(*ii*)

Also, B' is of order $q\times p$ and A' is of order $n\times m$

$\therefore$ B'A' is defined $\Rightarrow$ p=n

And $B'A'$ *is of order* $q\times m.$ ...(*iii*)

Also, equality of matrices (AB)' =B'A', we get the given statement as true.

e.g. If A is order (3×1) and B is of order (1×3), we get

Order of (AB)' = Order of $(B'A') = 3\times 3$

97. **If A, B and C are square matrices of same order, then AB = AC always implies that B = C.**

Sol. False

If AB=AC=0, then it can be possible that B and C are two non-zero matrices such that $B \neq C.$

$\therefore A.B = 0 = A.C$

Let $A\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix}$

And $C = \begin{bmatrix} 0 & 0 \\ 3 & 1 \end{bmatrix}$

$\therefore$ AB$=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

And AC$=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.\begin{bmatrix} 0 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow AB = AC \text{ but } B \neq C$

98. **AA′ is always a symmetric matrix for any matrix A.**

Sol. True

$\because [AA']' = (A')'A' = [AA']$

99. **If $A = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{vmatrix}$, then AB and BA are defined and equal.**

Sol. False

Since, AB is defined,

$$\therefore AB = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 20 \\ 22 & 25 \end{bmatrix}$$

Also, BA is defined

$$\therefore \text{ BA} \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 18 & 4 \\ 13 & 32 & 6 \\ 5 & 10 & 0 \end{bmatrix}$$

$\therefore AB \neq BA$

100. If A is skew-symmetric matrix, then A² is a symmetric matrix.

Sol. True

$\because \left[A^2\right]' = \left[A'\right]^2$

$= \left[-A\right]^2 \; \left[\because A' = -A\right]$

$= A^2$

Hence, A^2 *is symmetric matrix.*

101. (AB)⁻¹= A⁻¹. B⁻¹, where A and B are invertible matrices satisfying commutative property with respect to multiplication.

Sol. True

We know that, if A and B are invertible matrices of the same order, then

$(AB)^{-1} = (BA)^{-1} \; [\because \; AB = BA]$

Here, $(AB)^{-1} = (AB)^{-1}$

$\Rightarrow \text{B}^{-1}A^{-1} = A^{-1}B^{-1}$

[Since, A and B are satisfying commutative property with respect to multiplications].

Matrices

Short Answer Type Questions

1. If a matrix has 28 elements, what are the possible orders it can have? What if it has 13 elements?

Sol. We know that, if a matrix is of order $m\times n$, it has mn elements, where m and n are natural numbers.

We have, $m\times$n =28

$\Rightarrow(m,n)=\{(1,28),(2,14),(4,7),(7,4),(14,2),(28,1)\}$

So, the possible orders are $1\times28,\ 2\times14,\ 4\times7,\ 7\times4,\ 14\times2,\ 28\times1$

Also, if it has 13 elements, then $m\times n=13$

$(m,n)=\{(1,13),(13,1)\}$

Hence, the possible orders are $1\times13, 13\times1$.

2. In the matrix $A=\begin{bmatrix} a & 1 & x \\ 2 & \sqrt{3} & x^2-y \\ 0 & 5 & \frac{-2}{5} \end{bmatrix}$, write

(i) The order of the matrix A

(ii) The number of elements

(iii) elements $a_{23},\ a_{31}$ and a_{12}

Sol. We have, $A=\begin{bmatrix} a & 1 & x \\ 2 & \sqrt{3} & x^2-y \\ 0 & 5 & \frac{-2}{5} \end{bmatrix}$

(i) the order of matrix $A=3\times3$

(ii) the number of elements= $3\times3=9$

[Since, the number of elements in an $m\times n$, matrix will be equal to $m\times n=mn$]

(iii) $a_{23}=x^2-y,\ a_{31}=0,\ a_{12}=1$

[since, we know that $a_{ij,}$ is a representation of element lying in the i^{th} row and j^{th} column]

3. Construct $a_{2\times2}$ matrix, where

(i) $a_{ij}=\frac{(i-2j)^2}{2}$

(ii) $a_{ij}=|-2i+3j|$

Sol. We know that, the notation, namely $A\left[a_{ij}\right]_{m\times n}$ indicates that A is a matrix of order $m\times n$, also $1\le i\le m, 1\le j\le n; i, j\in N.$

(i) Here, $A=\left[a_{ij}\right]_{2\times 2}$

$\Rightarrow A=\dfrac{(i-2j)^2}{2}, 1\le i\le 2;\ 1\le j\le 2 \ldots(i)$

$\therefore \mathrm{a}_{11}=\dfrac{(1-2)^2}{2}=\dfrac{1}{2}$

$\mathrm{a}_{12}=\dfrac{(1-2\times 2)^2}{2}=\dfrac{9}{2}$

$\mathrm{a}_{21}=\dfrac{(2-2\times 1)^2}{2}=0$

$\mathrm{a}_{22}=\dfrac{\left(2-2\times 2^2\right)^2}{2}=2$

Thus, $A=\begin{bmatrix}\dfrac{1}{2} & \dfrac{9}{2}\\ 0 & 2\end{bmatrix}_{2\times 2}$

(ii) Here, $A=\left[a_{ij}\right]_{2\times 2}=\left|-2i+3j\right|, 1\le i\le 2; 1\le j\le 2$

$\therefore\ \mathrm{a}_{11}=\left|-2\times 1+3\times 1\right|=1$

$\mathrm{a}_{12}=\left|-2\times 1+3\times 2\right|=4\ \left[\because \left|-1\right|=1\right]$

$a_{21}=\left|-2\times 2+3\times 1\right|=1$

$a_{22}=\left|-2\times 2+3\times 2\right|=2$

$\therefore\ A\begin{vmatrix}1 & 4\\ 1 & 2\end{vmatrix}_{2\times 2}$

4. Construct a 3×2 matrix whose elements are given by $a_{ij}=e^{i.x}=\sin jx.$

Sol. Since, $A=\left[a_{ij}\right]_{m\times n}\ \ 1\le i\le m\ \ and\ \ 1\le j\le n, i, j\in N$

$\therefore\ A=\left[e^{i.x}\sin jx\right]_{3\times 2}; 1\le i\le 3; 1\le j\le 2$

$\Rightarrow\ \mathrm{a}_{11}=e^{1.x}.\sin 1.x=e^{x}\sin x$

$\mathrm{a}_{12}=e^{1.x}.\sin 2.x=e^{x}\sin 2x$

$\mathrm{a}_{21}=e^{2.x}.\sin 1.x=e^{2x}\sin x$

$\mathrm{a}_{22}=e^{2.x}.\sin 2.x=e^{2x}\sin 2x$

$a_{31} = e^{3.x}.\sin 1.x = e^{3x}\sin x$

$a_{32} = e^{3.x}.\sin 2.x = e^{3x}\sin 2x$

$$\therefore\ A=\begin{bmatrix} e^x \sin x & e^x \sin 2x \\ e^{2x}\sin x & e^{2x}\sin 2x \\ e^{3x}\sin x & e^{3x}\sin 2x \end{bmatrix}_{3\times 2}$$

5. Find values of a and b if A = B, where

$$A=\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} and\ B=\begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

Sol. We have,

$$A=\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}_{2\times 2}\ and\ B=\begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}_{2\times 2}$$

Also, A=B

By equality of matrices we know that each element of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ for all i and j.

$a_{11} = b_{11} \Rightarrow a+4 = 2a+2 \Rightarrow a = 2$

$a_{12} = b_{12} \Rightarrow 3b = b^2+2 \Rightarrow b^2 = 3b-2$

And $a_{22} = b_{22} \Rightarrow -6 = b^2 - 5b$

$\Rightarrow$ -6=3b-2-5b $\left[\because b^2 = 3b-2\right]$

$\Rightarrow 2b = 4 \Rightarrow b = 2$

$\therefore a = 2\ and\ b = 2$

6. If possible, find the sum of the matrices A and B, where $A=\begin{bmatrix} \sqrt{3} & 1 \\ 2 & 3 \end{bmatrix}$ **and**

$$B=\begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix}$$

Sol. We have, $A=\begin{bmatrix} \sqrt{3} & 1 \\ 2 & 3 \end{bmatrix}_{2\times 2}$ and $B=\begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix}_{2\times 3}$

Here, A and B are of different Orders. Also, we know that the addition of two matrices A and B is possible only if order of both the matrices A and B should be same. Hence, the sum of matrices A and B is not possible.

7. If $X=\begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix} and\ Y=\begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$ **, then find**

(i) X + Y.

(ii) 2x - 3Y

(iii) a matrix Z such that X + Y + Z is a zero matrix.

Sol. We have, $X=\begin{bmatrix}3 & 1 & -1\\ 5 & -2 & -3\end{bmatrix}_{2\times 3}$ *and* $Y=\begin{bmatrix}2 & 1 & -1\\ 7 & 2 & 4\end{bmatrix}_{2\times 3}$

(i) $X+Y=\begin{bmatrix}3+2 & 1+1 & -1-1\\ 5+7 & -2+2 & -3+4\end{bmatrix}=\begin{bmatrix}5 & 2 & -2\\ 12 & 0 & 1\end{bmatrix}$

(ii) $\because 2x=2\begin{bmatrix}3 & 1 & -1\\ 5 & -2 & -3\end{bmatrix}=\begin{bmatrix}6 & 2 & -2\\ 10 & -4 & -6\end{bmatrix}$

$and\ 3y=3\begin{bmatrix}2 & 1 & -1\\ 7 & 2 & 4\end{bmatrix}=\begin{bmatrix}6 & 3 & -3\\ 21 & 6 & 12\end{bmatrix}$

$\therefore\ 2x-3y=\begin{bmatrix}6-6 & 2-3 & -2+3\\ 10-21 & -4-6 & -6-12\end{bmatrix}=\begin{bmatrix}0 & -1 & 1\\ -11 & -10 & -18\end{bmatrix}$

(iii) $x+y=\begin{bmatrix}3+2 & 1+1 & -1-1\\ 5+7 & -2+2 & -3+4\end{bmatrix}=\begin{bmatrix}5 & 2 & -2\\ 12 & 0 & +1\end{bmatrix}$

Also, $X+Y+Z=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

We see that Z is the additive inverse of $(x+y)$ or negative of $(x+y)$

$\therefore Z=\begin{bmatrix}-5 & -2 & 2\\ -12 & 0 & -1\end{bmatrix}[\because Z=-(X+Y)]$

8. Find non-zero values of x satisfying the matrix equation:

$x\begin{bmatrix}2x & 2\\ 3 & x\end{bmatrix}+2\begin{bmatrix}8 & 5x\\ 4 & 4x\end{bmatrix}=2\begin{bmatrix}(x^2+8) & 24\\ (10) & 6x\end{bmatrix}.$

Sol. Given that,

$x\begin{bmatrix}2x & 2\\ 3 & x\end{bmatrix}+2\begin{bmatrix}8 & 5x\\ 4 & 4x\end{bmatrix}=2\begin{bmatrix}(x^2+8) & 24\\ (10) & 6x\end{bmatrix}$

$\Rightarrow\begin{bmatrix}2x^2 & 2x\\ 3x & x^2\end{bmatrix}+\begin{bmatrix}16 & 10x\\ 8 & 8x\end{bmatrix}=\begin{bmatrix}2x^2+16 & 48\\ 20 & 12x\end{bmatrix}$

$\Rightarrow\begin{bmatrix}2x^2+16 & 2x+10x\\ 3x+8 & x^2+8x\end{bmatrix}=\begin{bmatrix}2x^2+16 & 48\\ 20 & 12x\end{bmatrix}$

$\Rightarrow 2x+10x=48$

$\Rightarrow 12x=48$

$\therefore x=\dfrac{48}{12}=4$

9. **If** $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ *and* $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$**, then show that** $(A+B)(A-B) \neq A^2 - B^2$.

Sol. We have, $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ *and* $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\therefore (A+B) = \begin{bmatrix} 0+0 & 1-1 \\ 1+1 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}_{2\times 2}$

and $(A-B) = \begin{bmatrix} 0-0 & 1+1 \\ 1-1 & 1-0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

Since, (A+B). (A-B) is defined, if the number of columns of (A+B) is equal to the number of rows of (A-B), so here multiplication of matrices (A+B). (A-B) is possible.

Now, $(A+B)_{2\times 2}.(A-B)_{2\times 2} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 4+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$ ......(i)

Also, $A^2 = A.A$

$= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} . \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0+1 & 0+1 \\ 0+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

And $B^2 = B.B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} . \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0-1 & 0+0 \\ 0+1 & -1+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$\therefore A^2 - B^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$.....$(ii)$

Thus, we see that

$(A+B).(A-B) \neq A^2 - B^2$ $[\text{using } Eqs.(i)\, and\, (ii)]$

$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ Hence proved.

10. **Find the value of x, if** $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

Sol. We have, $\begin{bmatrix}1 & x & 1\end{bmatrix}_{1\times3}\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}_{3\times3}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}_{3\times1}=0$

$$\Rightarrow\begin{bmatrix}1+2x+15 & 3+5x+3 & 2+x+2\end{bmatrix}_{1\times3}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}_{3\times1}=0$$

$$\Rightarrow\begin{bmatrix}16+2x & 5x+6 & x+4\end{bmatrix}_{1\times3}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}_{3\times1}=0$$

$$\Rightarrow\begin{bmatrix}16+2x+(5x+6).2+(x+4).x\end{bmatrix}_{1\times1}=0$$

$$\Rightarrow\begin{bmatrix}16+2x+10x+12+x^2+4x\end{bmatrix}=0$$

$$\Rightarrow\begin{bmatrix}x^2+16x+28\end{bmatrix}=0$$

$$\Rightarrow\begin{bmatrix}x^2+2x+14x+28\end{bmatrix}=0$$

$$\Rightarrow(x+2)(x+14)=0$$

$$\therefore x=-2,-14$$

11. Show that $A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$ satisfies the equation $A^2-3A-7I=0$ and hence find A^{-1}.

Sol. We have, $A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$

$$\therefore A^2=A.A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}.\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$$

$$=\begin{bmatrix}25-3 & 15-6\\ -5+2 & -3+4\end{bmatrix}=\begin{bmatrix}22 & 9\\ -3 & 1\end{bmatrix}$$

$$3A=3\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}=\begin{bmatrix}15 & 9\\ -3 & -6\end{bmatrix}$$

And $7I=7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$

$$\therefore A^2-3A-7I=\begin{bmatrix}22 & 9\\ -3 & 1\end{bmatrix}-\begin{bmatrix}15 & 9\\ -3 & -6\end{bmatrix}-\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$

$$=\begin{bmatrix}22-15-7 & 9-9-0\\ -3+3-0 & 1+6-7\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

=0 Hence proved.

Since, $A^2-3A-7I=0$

$$\Rightarrow A^{-1}\left[\left(A^2\right)-3A-7I\right]=A^{-1}0$$

$$\Rightarrow A^{-1}A.A-3A^{-1}A-7A^{-1}I=0\ \left[\because A^{-1}0=0\right]$$

$$\Rightarrow IA-3I-7A^{-1}=0\ \left[\because A^{-1}A=I\right]$$

$$\Rightarrow A-3I-7A^{-1}=0\left[\because A^{-1}I=A^{-1}\right]$$

$$\Rightarrow -7A^{-1}=-A+3I$$

$$=\begin{bmatrix}-5 & -3\\1 & 2\end{bmatrix}+\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}=\begin{bmatrix}-2 & -3\\1 & 5\end{bmatrix}$$

$$\therefore\ A^{-1}=\frac{-1}{7}\begin{bmatrix}-2 & -3\\1 & 5\end{bmatrix}$$

12. Find the matrix A satisfying the matrix equation:

$$\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}.$$

Sol. We have, $\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}_{2\times2} A.\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}_{2\times2}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}_{2\times2}$

Let $A\begin{bmatrix}a & b\\c & d\end{bmatrix}_{2\times2}$

$$\therefore\ \begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2a+c & 2b+d\\3a+2c & 3b+2d\end{bmatrix}\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}-6a-3c+10b+5d & 4a+2c-6b-3d\\-9a-6c+15b+10d & 6a+4c-9b-6d\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\Rightarrow -6a-3c+10b+5d=1\quad ......(i)$$

$$\Rightarrow 4a+2c-6b-3d=0\ ....(ii)$$

$$\Rightarrow -9a-6c+15b+10d=0\quad ......(iii)$$

$$\Rightarrow\ 6a+4c\text{-}9c\text{-}6d=1\quad .....(iv)$$

On adding Eqs. (i) and (iv), we get

$$c+b-d=2\Rightarrow d=c+b-2...............(v)$$

On adding Eqs. (ii) and (iii), we get

$-5a-4c+9b+7d=0 \quad ....(vi)$

On adding Eqs. (vi) and (iv), we get

$a+0+0+d=1 \Rightarrow d=1-a \quad .....(vii)$

From Eqs. (v) and (vii)

$\Rightarrow c+b-2=1-a \Rightarrow a+b+c=3 \quad .....(viii)$

$\Rightarrow a=3-b-c$

Now, using the values of a and d in Eq. (iii) , we get

$-9(3-b-c)-6c+15b+10(-2+b+c)=0$

$\Rightarrow -27+9b+9c-6c+15b-20+10b+10c=0$

$\Rightarrow \ 34b+13c=47 \quad .....(ix)$

Now, using the values of a and d in Eq. (ii), we get

$4(3-b-c)+2c-6b-3(b+c-2)=0$

$\Rightarrow 12-4b-4c+2c-6b-3b-3c+6=0$

$\Rightarrow$ -13b+5c=-18 ....(x)

On multiplying Eq. (ix) by 5 and Eq. (x) by 13, then adding, we get

$$\begin{array}{c} -169b-65c=-234 \\ \underline{170b+65c=235} \\ b=1 \end{array}$$

$\Rightarrow$ -13×1-5c=-18 $\left[from\ Eq.(x) \right]$

$\Rightarrow -5c=-18+13=-5 \Rightarrow c=1$

$\therefore \quad a=3-1-1=1 \ \ and \ \ d=1-1=0$

$\therefore \quad A\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

13. **Find A, if** $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A=\begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$.

Sol. We have, $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3\times 1} A=\begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times 3}$

Let $A=\begin{bmatrix} x & y & z \end{bmatrix}$

$$\therefore \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3\times 1} \begin{bmatrix} x & y & z \end{bmatrix}_{1\times 3}=\begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times 3}$$

$$\Rightarrow \begin{bmatrix} 4x & 4y & 4z \\ x & y & z \\ 3x & 3y & 3z \end{bmatrix} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

$\Rightarrow 4x = -4 \Rightarrow x = -1, 4y = 8$

$\Rightarrow y = 2 \text{ and } 4z = 4$

$\Rightarrow z = 1$

$\therefore A = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

14. If A $\begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$ and B=$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, then verify $(BA)^2 \neq B^2A^2$.

Sol. We have, A $\begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}_{3\times2}$ and B=$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times3}$

$$\therefore BA = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times3} \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}_{3\times2}$$

$$= \begin{bmatrix} 6+1+4 & -8+1+0 \\ 3+2+8 & -4+2+0 \end{bmatrix} = \begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix}$$

And $(BA).(BA) = \begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix} \begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix}$

$$\Rightarrow (BA)^2 = \begin{bmatrix} 121-91 & -77+14 \\ 143-26 & -91+4 \end{bmatrix} = \begin{bmatrix} 30 & -63 \\ 117 & -87 \end{bmatrix} \quad ....(i)$$

Also, $B^2 = B.B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times3} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times3}$

So B^2 is not possible, since the B is not a square matrix.

Hence, $(BA)^2 \neq B^2A^2$.

15. If possible, find the value of BA and AB, Where

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Sol. We have, $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times3}$ $B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3\times2}$

So, AB and BA both are possible.
[Since, in both A.B and B. A, the number of columns of first is equal to the number of rows of second.]

$$\therefore AB = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times3} \quad B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3\times2}$$

$$= \begin{bmatrix} 8+2+2 & 2+3+4 \\ 4+4+4 & 1+6+8 \end{bmatrix} = \begin{bmatrix} 12 & 9 \\ 12 & 15 \end{bmatrix}$$

And $BA = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3\times2} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times3}$

$$= \begin{bmatrix} 4\times2+1 & 4+2 & 8+4 \\ 4+3 & 2+6 & 4+12 \\ 2+2 & 1+4 & 2+8 \end{bmatrix} = \begin{bmatrix} 9 & 6 & 12 \\ 7 & 8 & 16 \\ 4 & 5 & 10 \end{bmatrix}$$

16. Show by an example that for $A \neq 0,\ B \neq 0, AB = 0.$

Sol. Let $A = \begin{bmatrix} 0 & -4 \\ 0 & 2 \end{bmatrix} \neq 0 \ and\ B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$

$$\therefore AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0\ Hence\ \ proved.$$

17. Given $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$**. Is (AB)′= B′A′?**

Sol. We have, $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}_{2\times3}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}_{3\times2}$

$$\therefore AB = \begin{bmatrix} 2+8+0 & 8+32+0 \\ 3+18+6 & 12+72+18 \end{bmatrix} = \begin{bmatrix} 10 & 40 \\ 27 & 102 \end{bmatrix}$$

And $(AB)' = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix} \quad ....(i)$

Also, $B' = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 8 & 3 \end{bmatrix}_{2\times3}$ and $A' = \begin{bmatrix} 2 & 3 \\ 4 & 9 \\ 0 & 6 \end{bmatrix}_{3\times2}$

$$\therefore B'A' = \begin{bmatrix} 2+8+0 & 3+18+6 \\ 8+32+0 & 12+72+18 \end{bmatrix} = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix} \quad (ii)$$

Thus, we see that, (AB)'=B'A' [Using Eqs. (i) and (ii)]

18. **Solve for** x *and* y, $x\begin{bmatrix} 2 \\ 1 \end{bmatrix} + y\begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$

Sol. We have, $x\begin{bmatrix} 2 \\ 1 \end{bmatrix} + y\begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$

$$\Rightarrow \begin{bmatrix} 2x \\ x \end{bmatrix} + \begin{bmatrix} 3.y \\ 5.y \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x & 3y & -8 \\ x & 5y & -11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore\ 2x+3y-8=0$

$\Rightarrow 4x+6y=16 \quad ....(i)$

and $x+5y-11=0$

$\Rightarrow 4x+20y=44 \quad ......(ii)$

On subtracting Eq. (i) and from (ii), we get

$14y=28 \Rightarrow y=2$

$\therefore\ 2x+3\times 2-8=0$

$\Rightarrow 2x=2 \Rightarrow x=1$

$\therefore\ x=1$ *and* $y=2$

19. **If X and Y are 2×2 matrices, then solve the following matrix equations for X and Y**

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}, 3X+2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$$

Sol. We have,

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \quad ....(i)$$

And $3X+2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \quad ....(ii)$

On subtracting Eq. (i) from Eq. (ii), we get

$$\therefore\ (3X+2Y)-(2X+3Y) = \begin{bmatrix} -2-2 & 2-3 \\ 1-4 & -5-0 \end{bmatrix}$$

$$(x-y) = \begin{bmatrix} -4 & -1 \\ -3 & -5 \end{bmatrix} \quad ....(iii)$$

On adding Eqs. (i) and (ii), we get

$$(5X+5Y)=\begin{bmatrix}0 & 5\\ 5 & -5\end{bmatrix}$$

$$\Rightarrow (X+Y)=\frac{1}{5}\begin{bmatrix}0 & 5\\ 5 & -5\end{bmatrix}=\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}$$

On adding Eqs. (iii) and (iv), we get

$$\Rightarrow (X-Y)+(X+Y)=\begin{bmatrix}-4 & 0\\ -2 & -6\end{bmatrix}$$

$$\Rightarrow \ 2X=2\begin{bmatrix}-2 & 0\\ -1 & 3\end{bmatrix}$$

$$\therefore X=\begin{bmatrix}-2 & 0\\ -1 & 3\end{bmatrix}$$

From Eq. (iv),

$$\begin{bmatrix}-2 & 0\\ -1 & -3\end{bmatrix}+Y=\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}$$

$$\therefore Y=\begin{bmatrix}2 & 1\\ 2 & 2\end{bmatrix} \ and \ X=\begin{bmatrix}-2 & 0\\ -1 & -3\end{bmatrix}$$

20. If $A=[3 \quad 5]$ and $B=[7 \quad 3]$, then find a non-zero matrix C such that AC = BC.

Sol. We have, $A=[3 \quad 5]_{1\times 2}$ and $B=[7 \quad 3]_{1\times 2}$

Let $C=\begin{bmatrix}x\\ y\end{bmatrix}_{2\times 1}$ is a non-zero matrix of order 2×1.

$$\therefore AC=[3 \quad 5]\begin{bmatrix}x\\ y\end{bmatrix}=[3x+5y]$$

And $BC=[7 \quad 3]\begin{bmatrix}x\\ y\end{bmatrix}=[7x+3y]$

For AC=BC,

$$[3x+5y]=[7x+3y]$$

On using equality of matrix, we get

$$\Rightarrow \quad 3x+5y=7x+3y$$

$$\Rightarrow 4x=2y$$

$$\Rightarrow x=\frac{1}{2}y$$

$$\Rightarrow y=2x$$

$$\therefore C=\begin{bmatrix} x \\ 2x \end{bmatrix}$$

We see that on taking C of order $2\times1,\ 2\times2,\ 2\times3,\ \ldots\ldots$ we get

$$C=\begin{bmatrix} x \\ 2x \end{bmatrix},\begin{bmatrix} x & x \\ 2x & 2x \end{bmatrix},\begin{bmatrix} x & x & x \\ 2x & 2x & 2x \end{bmatrix}\ldots..$$

In general,

$$C=\begin{bmatrix} k \\ 2k \end{bmatrix},\begin{bmatrix} k & k \\ 2k & 2k \end{bmatrix}\ \ etc.....$$

Where, k is any real number.

21. Give an example of matrices A, B and C such that AB = AC, where A is non-zero matrix, but B ≠C.

Sol. Let $A=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},B=\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} and\ C=\begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix}[\because B\neq C]$

$$\therefore AB=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}=\begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}\ \ \ldots\ldots(i)$$

$$\text{And } \therefore AC=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}=\begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}\ \ \ldots\ldots(ii)$$

Thus, we see that AB=AC [using Eqs. (i) and (ii)]

Where, A is non-zero matrix but $B\neq C$.

22. If $A=\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix},\ B=\begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$ **and** $C=\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$**, verify**

(i) $(AB)C\ =\ A(BC)$

(ii) $A(B+C)\ =\ AB+AC.$

Sol. We have, $A=\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix},\ B=\begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$ and $C=\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

$$\text{(i) (AB) }=\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}=\begin{bmatrix} 2+6 & 3-8 \\ -4+3 & -6-4 \end{bmatrix}=\begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}$$

$$\text{And }\ (AB)C=\begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} 8+5 & 0 \\ -1+10 & 0 \end{bmatrix}=\begin{bmatrix} 13 & 0 \\ 9 & 0 \end{bmatrix}\ \ \ldots.(i)$$

$$\text{Again, } (BC)=\begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} 2-3 & 0 \\ 3+4 & 0 \end{bmatrix}=\begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix}$$

And $A(BC)=\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix}$

$$=\begin{bmatrix} -1+14 & 0 \\ +2+7 & 0 \end{bmatrix}=\begin{bmatrix} 13 & 0 \\ 9 & 0 \end{bmatrix} \quad ...(ii)$$

$\therefore$ $(AB)C = A(BC)$ [using Eqs. (i) and (ii)]

(ii) $(B+C)=\begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}+\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}=\begin{bmatrix} 3 & 3 \\ 2 & -4 \end{bmatrix}$

And $A.(B+C)=\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 2 & -4 \end{bmatrix}$

$$=\begin{bmatrix} 3+4 & 3-8 \\ -6+2 & -6-4 \end{bmatrix}=\begin{bmatrix} 7 & -5 \\ -4 & -10 \end{bmatrix} \quad ...(iii)$$

Also, $AB=\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}.\begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$

$$=\begin{bmatrix} 2+6 & 3-8 \\ -4+3 & -6-4 \end{bmatrix}=\begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}$$

And $AC=\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}=\begin{bmatrix} 1-2 & 0 \\ -2-1 & 0 \end{bmatrix}=\begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix}$

$$\therefore AB+AC=\begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}+\begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix}$$

$$\Rightarrow AB+AC=\begin{bmatrix} 7 & -5 \\ -4 & -10 \end{bmatrix} \quad ....(iv)$$

From Eqs. (iii) and (iv),

$A(B+C)=AB+AC$

23. If $P=\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ and $Q=\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, then prove that

$$PQ=\begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix}=QP.$$

Sol. $$PQ=\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}=\begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} \quad ....(i)$$

$$and\ QP=\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}=\begin{bmatrix} ax & 0 & 0 \\ 0 & by & 0 \\ 0 & 0 & zc \end{bmatrix} \quad ...(ii)$$

Thus, we see that, PQ=QP [using Eqs. (i) and (ii)]
Hence proved.

24. If $\begin{bmatrix} 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ = A, then find the value of A.

Sol. We have, $\begin{bmatrix} 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}=A$

$$\therefore \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}=\begin{bmatrix} -2-1+0 & 0+1+3 & -2+0+3 \end{bmatrix}$$

$=[-3 \quad 4 \quad 1]$

Now, $\begin{bmatrix} -3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}=A$

$$\therefore A=\begin{bmatrix} -3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$=\begin{bmatrix} -3+0-1 \end{bmatrix}=\begin{bmatrix} -4 \end{bmatrix}$

25. If $A=\begin{bmatrix} 2 & 1 \end{bmatrix}, B=\begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C=\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then verify that $A(B+C)=(AB+AC)$.

Sol. We have to verify that, $A(B+C)=(AB+AC)$

We have, $A=\begin{bmatrix} 2 & 1 \end{bmatrix}, B=\begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C=\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

$$\therefore A(B+C)=\begin{bmatrix}2 & 1\end{bmatrix}\begin{bmatrix}5-1 & 3+2 & 4+1\\ 8+1 & 7+0 & 6+2\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1\end{bmatrix}\begin{bmatrix}4 & 5 & 5\\ 9 & 7 & 8\end{bmatrix}$$

$$=\begin{bmatrix}8+9 & 10+7 & 10+8\end{bmatrix}$$

$$=\begin{bmatrix}17 & 17 & 18\end{bmatrix} \ldots(i)$$

Also, $AB=\begin{bmatrix}2 & 1\end{bmatrix}\begin{bmatrix}5 & 3 & 4\\ 8 & 7 & 6\end{bmatrix} \ldots..(i)$

$$=[10+8 \quad 6+7 \quad 8+6]=[18 \quad 13 \quad 14]$$

And $AC=[2 \quad 1]\begin{bmatrix}-1 & 2 & 1\\ 1 & 0 & 2\end{bmatrix}$

$$=\begin{bmatrix}-2+1 & 4+0 & 2+2\end{bmatrix}=\begin{bmatrix}-1 & 4 & 4\end{bmatrix}$$

$$\therefore AB+AC=\begin{bmatrix}18 & 13 & 14\end{bmatrix}+\begin{bmatrix}-1 & 4 & 4\end{bmatrix}$$

$$=\begin{bmatrix}17 & 17 & 18\end{bmatrix} \ldots(ii)$$

$\therefore A(B+C)=(AB+AC)$ [using Eqs. (i) and (ii)]

Hence proved.

26. **If** $A=\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}$**, then verify that** $A^2+A=A(A+I)$, **where I is** 3×3 **unit matrix.**

Sol. We have, $A=\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}$

$$\therefore A^2=A.A$$

$$=\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & -1 & -2\\ 4 & 4 & 4\\ 2 & 2 & 4\end{bmatrix}$$

$$\therefore A^2+A=\begin{bmatrix}1 & -1 & -2\\ 4 & 4 & 4\\ 2 & 2 & 4\end{bmatrix}+\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}2 & -1 & -3\\ 6 & 5 & 7\\ 2 & 3 & 5\end{bmatrix} \ldots(i)$$

Now, $A+I = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

And $A(A+I) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} . \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix} \quad ...(ii)$

Thus, we see that $A^2 + A = A(A+I)$ [using Eqs. (i) and (ii)]

27. **If** $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ **and** $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$ **then verify that:**

(i) $(A')' = A$

(ii) $(AB)' = B'A'$

(iii) $(kA)' = (kA')$ **.**

Sol. We have, $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$

(i) We have to verify that, A'=A

$\therefore A' = \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix}$

And $A' = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix} = A$ Hence Proved.

(ii) We have to verify that, AB'=B'A'

$\therefore AB = \begin{bmatrix} 3 & 9 \\ 11 & -15 \end{bmatrix}$

$\Rightarrow AB = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix}$

And $B'A' = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix}$

=(AB)' Hence proved.

(iii) We have to verify that, (kA)'=(kA')

Now, $(kA)=\begin{bmatrix}0 & -k & 2k\\ 4k & 3k & -4k\end{bmatrix}$

And $(kA)'=\begin{bmatrix}0 & 4k\\ -k & 3k\\ 2k & -4k\end{bmatrix}$

Also, $kA'=\begin{bmatrix}0 & 4k\\ -k & 3k\\ 2k & -4k\end{bmatrix}=(kA)'$ Hence proved.

28. **If** $A=\begin{bmatrix}1 & 2\\ 4 & 1\\ 5 & 6\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2\\ 6 & 4\\ 7 & 3\end{bmatrix}$**, then verify that:**

(i) $(2A+B)'=2A'+B'$

(ii) $(A-B)'=A'-B'$

Sol. We have, $A=\begin{bmatrix}1 & 2\\ 4 & 1\\ 5 & 6\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2\\ 6 & 4\\ 7 & 3\end{bmatrix}$

(i) $\therefore (2A+B)=\begin{bmatrix}2 & 4\\ 8 & 2\\ 10 & 12\end{bmatrix}+\begin{bmatrix}1 & 2\\ 6 & 4\\ 7 & 3\end{bmatrix}=\begin{bmatrix}3 & 6\\ 14 & 6\\ 17 & 15\end{bmatrix}$

And $(2A+B)'=\begin{bmatrix}3 & 14 & 17\\ 6 & 6 & 15\end{bmatrix}$

Also, $2A'+B'=2\begin{bmatrix}1 & 4 & 5\\ 2 & 1 & 6\end{bmatrix}+\begin{bmatrix}1 & 6 & 7\\ 2 & 4 & 3\end{bmatrix}$

$=\begin{bmatrix}3 & 14 & 17\\ 6 & 6 & 15\end{bmatrix}=(2A+B)'$ Hence proved.

(ii) $(A-B)=\begin{bmatrix}1 & 2\\ 4 & 1\\ 5 & 6\end{bmatrix}-\begin{bmatrix}1 & 2\\ 6 & 4\\ 7 & 3\end{bmatrix}=\begin{bmatrix}0 & 0\\ -2 & -3\\ -2 & 3\end{bmatrix}$

And $(A-B)'=\begin{bmatrix}0 & -2 & -2\\ 0 & -3 & 3\end{bmatrix}$

Also, $A'-B'=\begin{bmatrix}1 & 4 & 5\\ 2 & 1 & 6\end{bmatrix}-\begin{bmatrix}1 & 6 & 7\\ 2 & 4 & 3\end{bmatrix}$

$$=\begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix}$$

= (A-B)’ Hence Proved.

29. Show that A′A and AA′ are both symmetric matrices for any matrix A.

Sol. Let P=A’A

$\therefore$ $P'=(AA')'$

$= A'(A')' \left[\because (AB')' \right] = B'A']$

$= A'A = P$

So, A’A is symmetric matrix for any matrix A.

Similarly, let Q=AA’

$\therefore Q'=(AA')'=(A')'(A)'$

$= A(A')' = Q$

So, AA’ is symmetric matrix for any matrix A.

30. Let A and B be square matrices of the order 3×3. Is $(AB)^2 = A^2B^2$? Give reasons.

Sol. Since, A and B are square matrices of order 3×3 .

$\therefore AB^2 = AB.AB$

=ABAB

=AABB $[\because\ AB = BA]$

$= A^2B^2$

So, $AB^2 = A^2B^2$ is true when AB=BA

31. Show that if A and B are square matrices such that AB = BA, then $(A + B)^2 = A^2 + 2AB + B^2$

Sol. Since, A and B are square matrices such that AB=BA

$\therefore (A+B)^2 = (A+B).(A+B)$

$= A^2 + AB + BA + B^2$

$= A^2 + AB + AB + B^2$ $[\because AB = BA]$

$= A^2 + 2AB + B^2$ Hence proved.

32. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$, a= 4, b= –2, then show that:

(a) A + (B + C) = (A + B) + C

(b) A (BC) = (AB) C

(c) (a+ b)B = aB + bB

(d) a(C–A) = aC –aA

(e) $(A^T)^T$= A

(f) $(bA)^T$= bA^T

(g) $(AB)^T = B^TA^T$
(h) $(A - B)C = AC - BC$
(i) $(A - B)^T = A^T - B^T$

Sol. We have, $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix},$

$C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$ and a= 4, b= −2.

(i) $A + (B + C) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$

And $(A + B) + C = \begin{bmatrix} 5 & 2 \\ 0 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix} = A + (B + C)$ Hence proved.

(ii) $(BC) = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$

And $A(BC) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$

$= \begin{bmatrix} 8+14 & 0-20 \\ -8+21 & 0-30 \end{bmatrix} = \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix}$

Also, $(AB) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} . \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}$

$(AB)C = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix} = A(BC)$ Hence Proved.

(iii) $(a + b)B = (4 - 2)\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$ $[\because a = 4, \mathbf{b} = -2]$

$= \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$

and $aB + bB = 4B - 2B$

$= \begin{bmatrix} 16 & 0 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$

$$=\begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$$

=(a+b)B Hence Proved.

(iv) $(C-A)=\begin{bmatrix} 2-1 & 0-2 \\ 1+1 & -2-3 \end{bmatrix}=\begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix}$

and $a(C-A)=\begin{bmatrix} 4 & -8 \\ 8 & -20 \end{bmatrix}$

Also, $aC-aA=\begin{bmatrix} 8 & 0 \\ 4 & -8 \end{bmatrix}-\begin{bmatrix} 4 & 8 \\ -4 & 12 \end{bmatrix}=\begin{bmatrix} 4 & -8 \\ 8 & -20 \end{bmatrix}$

=a(C-A) Hence Proved.

(v) $A^T=\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}^T=\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Now, $\left(A^T\right)^T=\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}^T$

= A. Hence Proved.

(vi) $(bA)^T=\begin{bmatrix} -2 & -4 \\ 2 & -6 \end{bmatrix}^T$ $[\because\ b=-2]$

$$=\begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix}$$

and $A^T=\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$\therefore$ b$A^T=\begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix}=(bA)^T$ Hence proved.

(vii) $AB=\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}=\begin{bmatrix} 4+2 & 0+10 \\ -4+3 & 0+15 \end{bmatrix}=\begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}$

$\therefore (AB)^T=\begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix}$

Now, $B^TA^T=\begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}=\begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix}$

$=(AB)^T$ Hence proved.

(viii) $(A-B)=\begin{bmatrix} 1-4 & 2-0 \\ -1-1 & 3-5 \end{bmatrix}=\begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}$

$$(A-B)C=\begin{bmatrix}-3 & 2\\ -2 & -2\end{bmatrix}\begin{bmatrix}2 & 0\\ 1 & -2\end{bmatrix}=\begin{bmatrix}-4 & -4\\ -6 & 4\end{bmatrix}\quad ...(i)$$

Now, $AC=\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\ 1 & -2\end{bmatrix}=\begin{bmatrix}4 & -4\\ 1 & -6\end{bmatrix}\quad ....(ii)$

and $BC=\begin{bmatrix}4 & 0\\ 1 & 5\end{bmatrix}\begin{bmatrix}2 & 0\\ 1 & -2\end{bmatrix}=\begin{bmatrix}8 & 0\\ 7 & -10\end{bmatrix}\quad ....(iii)$

$\therefore$ AC-BC$=\begin{bmatrix}4-8 & -4-0\\ 1-7 & -6+10\end{bmatrix}$ [using Eq. (ii) and (iii)]

$=\begin{bmatrix}-4 & -4\\ -6 & 4\end{bmatrix}$

=(A-B)C [using Eq. (i)] Hence proved.

(ix) $(A-B)^T=\begin{bmatrix}1-4 & 2-0\\ -1-1 & 3-5\end{bmatrix}^T$

$=\begin{bmatrix}-3 & 2\\ -2 & -2\end{bmatrix}^T=\begin{bmatrix}-3 & -2\\ 2 & -2\end{bmatrix}$

$A^T-B^T=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}-\begin{bmatrix}4 & 1\\ 0 & 5\end{bmatrix}$

$=\begin{bmatrix}-3 & -2\\ 2 & -2\end{bmatrix}=(A+B)^T$ Hence proved.

33. If $A=\begin{bmatrix}\cos q & \sin q\\ -\sin q & \cos q\end{bmatrix}$ **then show that** $A^2=\begin{bmatrix}\cos 2q & \sin 2q\\ -\sin 2q & \cos 2q\end{bmatrix}$

Sol. We have, $A=\begin{bmatrix}\cos q & \sin q\\ -\sin q & \cos q\end{bmatrix}$

$\therefore A^2=A.A=\begin{bmatrix}\cos q & \sin q\\ -\sin q & \cos q\end{bmatrix}.\begin{bmatrix}\cos q & \sin q\\ -\sin q & \cos q\end{bmatrix}$

$=\begin{bmatrix}\cos^2 q-\sin^2 q & \cos q.\sin q+\sin q\cos q\\ -\sin q\cos q-\cos q\sin q & -\sin^2+\cos^2 q\end{bmatrix}$

$=\begin{bmatrix}\cos 2q & 2\sin q\cos q\\ -\sin q\cos q & \cos 2q\end{bmatrix}\quad [\because \cos^2\theta-\sin^2\theta=\cos 2\theta]$

$=\begin{bmatrix}\cos 2q & \sin 2q\\ -\sin 2q & \cos 2q\end{bmatrix}\quad [\because \sin 2\theta=2\sin\theta.\cos\theta]$ Hence proved.

34. **If** $A=\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}, B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ **and** $x^2=-1$**, then show that** $(A+B)^2=A^2+B^2$

Sol. We have, $A=\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}, B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ and $x^2=-1$

$\therefore\ (A+B)=\begin{bmatrix}0 & -x+1\\ x+1 & 0\end{bmatrix}$

And $(A+B)^2=\begin{bmatrix}0 & -x+1\\ x+1 & 0\end{bmatrix}\begin{bmatrix}0 & -x+1\\ x+1 & 0\end{bmatrix}$

$=\begin{bmatrix}1-x^2 & 0\\ 0 & 1-x^2\end{bmatrix}\ ...(i)$

Also, $A^2=A.A=\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}=\begin{bmatrix}-x^2 & 0\\ 0 & -x^2\end{bmatrix}$

And $B^2=B.B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Now, $A^2+B^2=\begin{bmatrix}-x^2+1 & 0\\ 0 & -x^2+1\end{bmatrix}=\begin{bmatrix}1-x^2 & 0\\ 0 & 1-x^2\end{bmatrix}$ [using Eq. (i)]

$=(A+B)^2$ Hence Proved.

35. **Verify that A²= I, when** $A=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$**.**

Sol. We have, $A=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$

$\therefore A^2=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}.\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}\ [\because A^2=A.A]$

$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$ Hence proved.

36. **Prove by Mathematical Induction that** $(A')^n=(A^n)'$ **where n**$\in$ **N for any square matrix A.**

Sol. Let $P(n):(A')^n=(A^n)'$

$\therefore\ \mathrm{P}(1):(A)^1=(A)'$

$\Rightarrow$ A'=A' $\Rightarrow \mathrm{P}(1)$ is true.

Now, $P(k):(A')^k=(A^k)'$,

Where $k\in\mathrm{N}$

And $P(k+1):(A')^{k+1}=(A^{k+1})'$

where P(k+1) is true whenever P(k) is true.

$\therefore\ \mathrm{P}(k+1):(A')^k.(A')^1=\left[A^{k+1}\right]'$

$(A^k)'.(A)'=\left[A^{k+1}\right]'$

$(A.A^k)'=\left[A^{k+1}\right]'\ \left[\because (A')^k=(A^k)'\, and\, (AB)=B'A'\right]$

$(A^{k+1})'=\left[A^{k+1}\right]'$ Hence proved.

37. Find inverse, by elementary row operations (if possible), of the following matrices

(i) $\begin{bmatrix}1 & 3\\-5 & 7\end{bmatrix}$

(ii) $\begin{bmatrix}1 & -3\\-2 & 6\end{bmatrix}$

Sol. (i) Let $A=\begin{bmatrix}1 & 3\\-5 & 7\end{bmatrix}$

In order to use elementary row operations we may write A=IA.

$$\therefore\ \begin{bmatrix}1 & 3\\-5 & 7\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A$$

$$\Rightarrow\ \begin{bmatrix}1 & 3\\0 & 22\end{bmatrix}=\begin{bmatrix}1 & 0\\5 & 1\end{bmatrix}A\ \left[\because\ R_2\to R_2+5R_1\right]$$

$$\Rightarrow\ \begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\5/22 & 1/22\end{bmatrix}A\ \left[\because\ R_2\to\frac{1}{22}R_2\right]$$

$$\Rightarrow\ \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}7/22 & -3/22\\5/22 & 1/22\end{bmatrix}A\ \left[\because\ R_1\to R_1-3R_2\right]$$

$$\Rightarrow\ \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\frac{1}{22}\begin{bmatrix}7 & -3\\5 & 1\end{bmatrix}A$$

$\Rightarrow I=BA,$ where B is the inverse of A.

$\therefore$ B=$\frac{1}{22}\begin{bmatrix} 7 & -3 \\ 5 & -1 \end{bmatrix}$

(ii) Let $A=\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$

In order to use elementary row operations, we write A=IA

$\Rightarrow \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}A$

$\Rightarrow \begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}A\ [\because R_2 \to R_2+2R_1]$

Since, we obtain all zeroes in a row of the matrix A on LHS, so A^{-1} does not exist.

38. If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix}=\begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, then find values of x, y, z and w.

Sol. We have, $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix}=\begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$

By equality of matrix, $x+y=6$ *and* $xy=8$

$\Rightarrow$ x=6-y and $(6-y)$.y=8

$\Rightarrow y^2-6y+8=0$

$\Rightarrow y^2-4y-2y+8=0$

$\Rightarrow (y-2)(y-4)=0$

$\Rightarrow y=2$ *or* $y=4$

$\therefore x=6-4=2$

or $x=6-4=2\ [\because x=6-y]$

Also, z + 6=0

$\Rightarrow z=-6$ *and* $w=4$

$\therefore$ x=2, y=4 or x=4, y=2, z=-6 and w=4

39. If $A=\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B=\begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, find a matrix C such that 3A + 5B + 2C is a null matrix.

Sol. We have, $A=\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B=\begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$

Let $C=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore 3A+5B+2C=0$

$$\Rightarrow \begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 48+2a & 20+2b \\ 56+2c & 76+2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow 2a+48=0 \Rightarrow a=-24$

Also, $20+2b=0 \Rightarrow b=-10$

$56+2c=0 \Rightarrow c=-28$

And $76+2d=0 \Rightarrow d=-38$

$$\therefore C\begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$$

40. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2-5A-14I$. Hence, obtain A^3.

Sol. We have, $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \quad ...(i)$

$$\therefore A^2 = \text{A.A} = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} \quad ...(ii)$$

$$\therefore \text{A}^2-5A-14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now, $A^2-5A-14I=0$

$\Rightarrow \text{A.A}^2-5A.A-14AI=0$

$\Rightarrow A^3-5A^2-14A=0 \; [\because \; AI=A]$

$\Rightarrow \; \text{A}^3 = 5A^2 = 14A$

$$= 5\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \text{ [Using Eqs. (i) and (ii)]}$$

$$= \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$$

$$= \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

41. Find the values of a, b, c and d, if

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix} z.$$

Sol. We have, $3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix} = \begin{bmatrix} a+4 & 6+a+b \\ c+d-1 & 3+2d \end{bmatrix}$

$\Rightarrow 3a = a+4 \Rightarrow a = 2;$

$3b = 6+a+b$

$\Rightarrow 3b-b = 8 \Rightarrow b = 4;$

$3d = 3+2d \Rightarrow d = 3$

and 3c=c+d-1

$\Rightarrow$ 2c=3-1 $\Rightarrow$ c=1

$\therefore$ a=2, b=4, c=1 and d=3

42. Find the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$**.**

Sol. We have, $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}_{3\times 2} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}_{3\times 3}$

From the given equation, it is clear that order of A should be 2×3

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$\therefore \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a-d & 2b-e & 2c-f \\ a+0d & b+0.e & c+0.f \\ -3a+4d & -3b+4e & -3c+4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a-d & 2b-e & 2c-f \\ a & b & c \\ -3a+4d & -3b+4e & -3c+4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

By equality of matrices, we get

$a = 1, b = -2 \;\; c = -5$

And $2a-d=-1 \Rightarrow d=2a+1=3;$

$\Rightarrow 2b-e=-8 \Rightarrow e=2(-2)+8=4$

$2c-f=-10 \Rightarrow f=2c+10=0$

$$\therefore A=\begin{bmatrix}1 & -2 & -5\\ 3 & 4 & 0\end{bmatrix}$$

43. **If** $A=\begin{bmatrix}1 & 2\\ 4 & 1\end{bmatrix}$, *then find* $A^2+2A+7I.$

Sol. We have, $A=\begin{bmatrix}1 & 2\\ 4 & 1\end{bmatrix},$

$$\therefore\ \mathrm{A}^2-\begin{bmatrix}1 & 2\\ 4 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 4 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1-8 & 2+2\\ 4+4 & 8+1\end{bmatrix}=\begin{bmatrix}9 & 4\\ 8 & 9\end{bmatrix}$$

$$\therefore A^2+2A+7=\begin{bmatrix}9 & 4\\ 8 & 9\end{bmatrix}+\begin{bmatrix}2 & 4\\ 8 & 2\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}=\begin{bmatrix}18 & 8\\ 16 & 18\end{bmatrix}$$

44. **If** $A=\begin{bmatrix}cos\alpha & sin\alpha\\ -sin\alpha & cos\alpha\end{bmatrix}$ *and* $A^{-1}=A'$, **then find value of α.**

Sol. We have, $A=\begin{bmatrix}cos\alpha & sin\alpha\\ -sin\alpha & cos\alpha\end{bmatrix}$ and $A'=\begin{bmatrix}\cos a & -\sin a\\ \sin a & \cos a\end{bmatrix}$

Also, $A^{-1}=A'$

$\Rightarrow\ \mathrm{AA}^{-1}=AA'$

$$\Rightarrow I=\begin{bmatrix}cos\alpha & sin\alpha\\ -sin\alpha & cos\alpha\end{bmatrix}\begin{bmatrix}cos\alpha & -sin\alpha\\ sin\alpha & cos\alpha\end{bmatrix}$$

$$\Rightarrow\ \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & 0\\ 0 & \sin^2\alpha+\cos^2\alpha\end{bmatrix}$$

By Using equality of matrices, we get

$\cos^2\alpha+\sin^2\alpha=1$

Which is true for all real values of α .

45. **If the matrix** $\begin{bmatrix}0 & a & 3\\ 2 & b & -1\\ c & 1 & 0\end{bmatrix}$ **is a skew symmetric matrix, then find the values of a, b and c.**

Sol. Let $A=\begin{bmatrix}0 & a & 3\\ 2 & b & -1\\ c & 1 & 0\end{bmatrix}$

Since, A is skew-symmetric matrix.

$\therefore A'=-A$

$$\Rightarrow \begin{bmatrix}0 & 2 & c\\ a & b & 1\\ 3 & -1 & 0\end{bmatrix}=-\begin{bmatrix}0 & a & 3\\ 2 & b & -1\\ c & 1 & 0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}0 & 2 & c\\ a & b & 1\\ 3 & -1 & 0\end{bmatrix}=\begin{bmatrix}0 & -a & -3\\ -2 & -b & +1\\ -c & -1 & 0\end{bmatrix}$$

By equality of matrices, we get

$a=-2, c=-3 \; and \;\; b=-b \Rightarrow b=0$

$\therefore$ a=-2, b=0 and c=-3

46. If $P(x)=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$, then show that $P(x).P(y)=P(x+y)=P(y).P(x)$.

Sol. We have,

$$P(x)=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix},$$

$$\therefore P(y)=\begin{bmatrix}\cos y & \sin y\\ -\sin y & \cos y\end{bmatrix},$$

Now, $P(x).P(y)=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}\begin{bmatrix}\cos y & \sin y\\ -\sin y & \cos y\end{bmatrix}$

$$=\begin{bmatrix}\cos x.\cos y-\sin x.\sin y & \cos x.\sin y+\sin x.\cos y\\ -\sin x.\cos y-\cos x.\sin y & -\sin x.\sin y+\cos x.\cos y\end{bmatrix}$$

$$=\begin{bmatrix}\cos(x+y) & \sin(x+y)\\ -\sin(x+y) & \cos(x+y)\end{bmatrix} \quad ....(i)$$

$$\begin{bmatrix}\therefore \; \cos(x+y)=\cos x.\cos y-\sin x.\sin y\\ and \; \sin(x+y)=\sin x.\cos y+\cos x.\sin y\end{bmatrix}$$

And $P(x+y)=\begin{bmatrix}\cos(x+y) & \sin(x+y)\\ -\sin(x+y) & \cos(x+y)\end{bmatrix} \; .....(ii)$

Also, $P(y).P(x)=\begin{bmatrix}\cos y & \sin y\\ -\sin y & \cos y\end{bmatrix}\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$

$$= \begin{bmatrix} \cos y.\cos x - \sin y.\sin x & \cos y.\sin x + \sin y.\cos x \\ -\sin y.\cos x - \sin x - \cos y & -\sin y.\sin x + \cos y.\cos x \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix}$$

Thus, we see from the Eqs. (i), (ii) and (iii) that,

$P(x).P(y) = P(x+y) = P(y).P(x)$ Hence Proved.

47. If A is square matrix such that A^2= A, show that $(I + A)^3$= 7A + I.

Sol. Since, $A^2 = A$ and $(I+A).(I+A) = I^2 + IA + AI + A^2$

$= I^2 + 2AI + A^2$

$= I + 2A + A = I + 3A$

And $(I+A).(I+A)(I+A) = (I+A)(I+3A)$

$= I^2 + 3AI + AI + 3A^2$

$= I + 4AI + 3A$

$= I + 7A = 7A + I$ Hence proved.

48. If A, B are square matrices of same order and B is a skew-symmetric matrix, show that A′BA is skew symmetric.

Sol. Since, A and B are square matrices of same order and B is a skew-symmetric matrix i.e. B'=-B.

Now, we have to prove that A'BA is a skew-symmetric matrix.

$\therefore A'BA' = A'BA' = BA'A'$ $[\because AB' = B'A']$

$= A'BA' = A' - BA = -A'BA$

Hence, A'BA is a skew-symmetric matrix.

Matrices

Long Answer Type Questions

49. If AB = BA for any two square matrices, then prove by mathematical induction that $(AB)^n = A^n B^n$

Sol. Let $P(n):(AB)^n = A^n B^n$

$\therefore \text{P}(1):(AB)^1 = A^1 B^1 \Rightarrow AB = AB$

So, P(1) is true.

Now, $P(k):(AB)^k = A^k B^k . k \in N$

So, P(K) is true, whenever P(k+1) is true.

$\therefore P(K+1: AB)^{k+1} = A^{k+1} B^{k+1}$

$\Rightarrow AB^k . AB^1 [\because AB = BA]$

$\Rightarrow A^k B^k . BA \Rightarrow A^k B^{k+1} A$

$\Rightarrow A^k A . B^{k+1} \Rightarrow A^{k+1} B^{k+1}$

$\Rightarrow (A.B)^{k+1} = A^{k+1} B^{k+1}$

So, P(k+1) is true for all $n \in N$, whenever P(k) is true.

By mathematical induction (AB)=$A^n B^n$ is true for all $n \in N$.

50. Find x, y, z if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies A′= A^{-1}.

Sol. We have, $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ and $A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

Also, $A' = A^{-1}$

$\Rightarrow AA' = AA^{-1} \left[\because AA^{-1} = I \right]$

$\Rightarrow AA' = I$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4y^2 + z^2 & 2y^2 - z^2 & -2y^2 + z^2 \\ 2y^2 - z^2 & x^2 + y^2 + z^2 & x^2 - y^2 - z^2 \\ -2y^2 + z^2 & x^2 - y^2 - z^2 & x^2 + y^2 - z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \ 2y^2 - z^2 = 0 \Rightarrow 2y^2 = z^2$

$\Rightarrow 4y^2 + z^2 = 1$

$\Rightarrow 2.z^2 + z^2 = 1$

$z = \pm\dfrac{1}{\sqrt{3}}$

$\therefore y^2 = \dfrac{z^2}{2} \Rightarrow y = \pm\dfrac{1}{\sqrt{6}}$

Also, $x^2 + y^2 + z^2 = 1$

$\Rightarrow x^2 = 1 - y^2 - z^2 = 1 - \dfrac{1}{6} - \dfrac{1}{3}$

$= 1 - \dfrac{3}{6} = \dfrac{1}{2}$

$\Rightarrow$ $x = \pm\dfrac{1}{\sqrt{2}}$

$\therefore x = \pm, \dfrac{1}{\sqrt{2}}, y = \pm\dfrac{1}{\sqrt{6}}$

and $z = \pm\dfrac{1}{\sqrt{3}}$

51. If possible, using elementary row transformations, find the inverse of the following matrices

(i) $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Sol. For getting the inverse of the given matrix A by row elementary operations we may write the given matrix as A=IA

(i) $\because \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 4 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \left[\because\ R_2 \to R_2 - R_1\right]$$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \left[\because\ R_3 \to R_3 - R_2\right]$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 7 \\ -3 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \left[\because\ R_1 \to R_1 - R_2\right]$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 7 \\ 0 & -1 & -17 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -5 & -2 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \left[\because\ R_2 \to R_2 - 3R_1\right]$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & -10 \\ 0 & -1 & -17 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & -1 & 0 \\ -5 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix} A \begin{bmatrix} \because\ R_1 \to R_1 + R_2 \\ and\ R_3 \to -1.R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ 1 & 1 & -1 \end{bmatrix} A \begin{bmatrix} \because\ R_1 \to R_1 + 10R_3 \\ and\ R_2 \to R_2 + 17R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix} A \begin{bmatrix} \because\ R_1 \to -1R_1 \\ and\ R_2 \to -1R_2 \end{bmatrix}$$

So, the inverse of A is $\begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$

(ii) $\therefore \begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$$\Rightarrow \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} \because\ R_2 \to R_2 + R_3 \\ and\ R_1 \to R_1 - 2R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A \ [\because\ R_2 \to R_2 + R_1]$$

Since, second row of the matrix A on LHS is containing all zeroes, so we can say that inverse of matrix A does not exist.

(iii) $\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \ [\because\ R_2 \to R_2 - R_1]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} \because\ R_2 \to R_2 - R_1 \\ and\ R_3 \to R_3 + R_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & \frac{5}{2} \\ 4 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \begin{bmatrix} \because\ R_3 \to R_3 + R_1 \\ and\ R_2 \to R_2 - \frac{1}{2} R_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \ [\because\ R_3 \to R_3 - 2R_1]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A \ [\because\ R_3 \to R_3 - R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A \begin{bmatrix} \because\ R_1 \to \frac{1}{2} R_1 \\ and\ R_3 \to 2R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A \left[\begin{array}{l} \because\ R_1 \to R_1 + \frac{1}{2} R_3 \\ and\ R_2 \to R_2 - \frac{5}{2} R_3 \end{array} \right]$$

Hence, $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ is the inverse of given matrix A.

52. Express the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Sol. We have, $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$

$$\therefore\ A' = \begin{bmatrix} 2 & 1 & 4 \\ 3 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Now, $\dfrac{A+A'}{2} = \dfrac{1}{2}\begin{bmatrix} 4 & 4 & 5 \\ 4 & -2 & 3 \\ 5 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ 5 & \frac{3}{2} & 2 \end{bmatrix}$

And $\dfrac{A-A'}{2} = \dfrac{1}{2}\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$

$$\therefore \frac{A+A'}{2} = \frac{A-A'}{2} = \begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

Which is the required expression.