

## Elements, their Atomic Number and Molar Mass

| Element     | Symbol | Atomic Number | Molar mass/<br>(g mol <sup>-1</sup> ) |
|-------------|--------|---------------|---------------------------------------|
| Actinium    | Ac     | 89            | 227.03                                |
| Aluminium   | Al     | 13            | 26.98                                 |
| Americium   | Am     | 95            | (243)                                 |
| Antimony    | Sb     | 51            | 121.75                                |
| Argon       | Ar     | 18            | 39.95                                 |
| Arsenic     | As     | 33            | 74.92                                 |
| Astatine    | At     | 85            | 210                                   |
| Barium      | Ba     | 56            | 137.34                                |
| Berkelium   | Bk     | 97            | (247)                                 |
| Beryllium   | Be     | 4             | 9.01                                  |
| Bismuth     | Bi     | 83            | 208.98                                |
| Bohrium     | Bh     | 107           | (264)                                 |
| Boron       | B      | 5             | 10.81                                 |
| Bromine     | Br     | 35            | 79.91                                 |
| Cadmium     | Cd     | 48            | 112.40                                |
| Caesium     | Cs     | 55            | 132.91                                |
| Calcium     | Ca     | 20            | 40.08                                 |
| Californium | Cf     | 98            | 251.08                                |
| Carbon      | C      | 6             | 12.01                                 |
| Cerium      | Ce     | 58            | 140.12                                |
| Chlorine    | Cl     | 17            | 35.45                                 |
| Chromium    | Cr     | 24            | 52.00                                 |
| Cobalt      | Co     | 27            | 58.93                                 |
| Copper      | Cu     | 29            | 63.54                                 |
| Curium      | Cm     | 96            | 247.07                                |
| Dubnium     | Db     | 105           | (263)                                 |
| Dysprosium  | Dy     | 66            | 162.50                                |
| Einsteinium | Es     | 99            | (252)                                 |
| Erbium      | Er     | 68            | 167.26                                |
| Europium    | Eu     | 63            | 151.96                                |
| Fermium     | Fm     | 100           | (257.10)                              |
| Fluorine    | F      | 9             | 19.00                                 |
| Francium    | Fr     | 87            | (223)                                 |
| Gadolinium  | Gd     | 64            | 157.25                                |
| Gallium     | Ga     | 31            | 69.72                                 |
| Germanium   | Ge     | 32            | 72.61                                 |
| Gold        | Au     | 79            | 196.97                                |
| Hafnium     | Hf     | 72            | 178.49                                |
| Hassium     | Hs     | 108           | (269)                                 |
| Helium      | He     | 2             | 4.00                                  |
| Holmium     | Ho     | 67            | 164.93                                |
| Hydrogen    | H      | 1             | 1.0079                                |
| Indium      | In     | 49            | 114.82                                |
| Iodine      | I      | 53            | 126.90                                |
| Iridium     | Ir     | 77            | 192.2                                 |
| Iron        | Fe     | 26            | 55.85                                 |
| Krypton     | Kr     | 36            | 83.80                                 |
| Lanthanum   | La     | 57            | 138.91                                |
| Lawrencium  | Lr     | 103           | (262.1)                               |
| Lead        | Pb     | 82            | 207.19                                |
| Lithium     | Li     | 3             | 6.94                                  |
| Lutetium    | Lu     | 71            | 174.96                                |
| Magnesium   | Mg     | 12            | 24.31                                 |
| Manganese   | Mn     | 25            | 54.94                                 |
| Meitneium   | Mt     | 109           | (268)                                 |
| Mendelevium | Md     | 101           | 258.10                                |

| Element       | Symbol | Atomic Number | Molar mass/<br>(g mol <sup>-1</sup> ) |
|---------------|--------|---------------|---------------------------------------|
| Mercury       | Hg     | 80            | 200.59                                |
| Molybdenum    | Mo     | 42            | 95.94                                 |
| Neodymium     | Nd     | 60            | 144.24                                |
| Neon          | Ne     | 10            | 20.18                                 |
| Neptunium     | Np     | 93            | (237.05)                              |
| Nickel        | Ni     | 28            | 58.71                                 |
| Niobium       | Nb     | 41            | 92.91                                 |
| Nitrogen      | N      | 7             | 14.0067                               |
| Nobelium      | No     | 102           | (259)                                 |
| Osmium        | Os     | 76            | 190.2                                 |
| Oxygen        | O      | 8             | 16.00                                 |
| Palladium     | Pd     | 46            | 106.4                                 |
| Phosphorus    | P      | 15            | 30.97                                 |
| Platinum      | Pt     | 78            | 195.09                                |
| Plutonium     | Pu     | 94            | (244)                                 |
| Polonium      | Po     | 84            | 210                                   |
| Potassium     | K      | 19            | 39.10                                 |
| Praseodymium  | Pr     | 59            | 140.91                                |
| Promethium    | Pm     | 61            | (145)                                 |
| Protactinium  | Pa     | 91            | 231.04                                |
| Radium        | Ra     | 88            | (226)                                 |
| Radon         | Rn     | 86            | (222)                                 |
| Rhenium       | Re     | 75            | 186.2                                 |
| Rhodium       | Rh     | 45            | 102.91                                |
| Rubidium      | Rb     | 37            | 85.47                                 |
| Ruthenium     | Ru     | 44            | 101.07                                |
| Rutherfordium | Rf     | 104           | (261)                                 |
| Samarium      | Sm     | 62            | 150.35                                |
| Scandium      | Sc     | 21            | 44.96                                 |
| Seaborgium    | Sg     | 106           | (266)                                 |
| Selenium      | Se     | 34            | 78.96                                 |
| Silicon       | Si     | 14            | 28.08                                 |
| Silver        | Ag     | 47            | 107.87                                |
| Sodium        | Na     | 11            | 22.99                                 |
| Strontium     | Sr     | 38            | 87.62                                 |
| Sulphur       | S      | 16            | 32.06                                 |
| Tantalum      | Ta     | 73            | 180.95                                |
| Technetium    | Tc     | 43            | (98.91)                               |
| Tellurium     | Te     | 52            | 127.60                                |
| Terbium       | Tb     | 65            | 158.92                                |
| Thallium      | Tl     | 81            | 204.37                                |
| Thorium       | Th     | 90            | 232.04                                |
| Thulium       | Tm     | 69            | 168.93                                |
| Tin           | Sn     | 50            | 118.69                                |
| Titanium      | Ti     | 22            | 47.88                                 |
| Tungsten      | W      | 74            | 183.85                                |
| Ununbium      | Uub    | 112           | (277)                                 |
| Ununnilium    | Uun    | 110           | (269)                                 |
| Unununium     | Uuu    | 111           | (272)                                 |
| Uranium       | U      | 92            | 238.03                                |
| Vanadium      | V      | 23            | 50.94                                 |
| Xenon         | Xe     | 54            | 131.30                                |
| Ytterbium     | Yb     | 70            | 173.04                                |
| Yttrium       | Y      | 39            | 88.91                                 |
| Zinc          | Zn     | 30            | 65.37                                 |
| Zirconium     | Zr     | 40            | 91.22                                 |

The value given in parenthesis is the molar mass of the isotope of largest known half-life.

## Some Useful Conversion Factors

### Common Unit of Mass and Weight

**1 pound = 453.59 grams**

1 pound = 453.59 grams = 0.45359 kilogram  
 1 kilogram = 1000 grams = 2.205 pounds  
 1 gram = 10 decigrams = 100 centigrams  
           = 1000 milligrams  
 1 gram =  $6.022 \times 10^{23}$  atomic mass units or u  
 1 atomic mass unit =  $1.6606 \times 10^{-24}$  gram  
 1 metric tonne = 1000 kilograms  
                       = 2205 pounds

### Common Unit of Volume

**1 quart = 0.9463 litre**

**1 litre = 1.056 quarts**

1 litre = 1 cubic decimetre = 1000 cubic centimetres = 0.001 cubic metre  
 1 millilitre = 1 cubic centimetre = 0.001 litre  
               =  $1.056 \times 10^{-3}$  quart  
 1 cubic foot = 28.316 litres = 29.902 quarts  
               = 7.475 gallons

### Common Units of Energy

**1 joule =  $1 \times 10^7$  ergs**

1 thermochemical calorie\*\*  
                                       = 4.184 joules  
                                       =  $4.184 \times 10^7$  ergs  
 =  $4.129 \times 10^{-2}$  litre-atmospheres  
 =  $2.612 \times 10^{19}$  electron volts  
 1 ergs =  $1 \times 10^{-7}$  joule =  $2.3901 \times 10^{-8}$  calorie  
 1 electron volt =  $1.6022 \times 10^{-19}$  joule  
                       =  $1.6022 \times 10^{-12}$  erg  
                       = 96.487 kJ/mol†  
 1 litre-atmosphere = 24.217 calories  
                           = 101.32 joules  
                           =  $1.0132 \times 10^9$  ergs  
 1 British thermal unit = 1055.06 joules  
                               =  $1.05506 \times 10^{10}$  ergs  
                               = 252.2 calories

### Common Units of Length

**1 inch = 2.54 centimetres (exactly)**

1 mile = 5280 feet = 1.609 kilometres  
 1 yard = 36 inches = 0.9144 metre  
 1 metre = 100 centimetres = 39.37 inches  
           = 3.281 feet  
           = 1.094 yards  
 1 kilometre = 1000 metres = 1094 yards  
               = 0.6215 mile  
 1 Angstrom =  $1.0 \times 10^{-8}$  centimetre  
               = 0.10 nanometre  
               =  $1.0 \times 10^{-10}$  metre  
               =  $3.937 \times 10^{-9}$  inch

### Common Units of Force\* and Pressure

1 atmosphere = 760 millimetres of mercury  
               =  $1.013 \times 10^5$  pascals  
               = 14.70 pounds per square inch  
 1 bar =  $10^5$  pascals  
 1 torr = 1 millimetre of mercury  
 1 pascal = 1 kg/ms<sup>2</sup> = 1 N/m<sup>2</sup>

### Temperature

**SI Base Unit: Kelvin (K)**

K = -273.15°C  
 K = °C + 273.15  
 °F = 1.8(°C) + 32  

$$^{\circ}\text{C} = \frac{^{\circ}\text{F} - 32}{1.8}$$

\* Force: 1 newton (N) = 1 kg m/s<sup>2</sup>, i.e., the force that, when applied for 1 second, gives a 1-kilogram mass a velocity of 1 metre per second.

\*\* The amount of heat required to raise the temperature of one gram of water from 14.5°C to 15.5°C.

† Note that the other units are per particle and must be multiplied by  $6.022 \times 10^{23}$  to be strictly comparable.

# Standard potentials at 298 K in electrochemical order

| Reduction half-reaction                                                                                         | $E^\circ/\text{V}$ | Reduction half-reaction                                                                                   | $E^\circ/\text{V}$ |
|-----------------------------------------------------------------------------------------------------------------|--------------------|-----------------------------------------------------------------------------------------------------------|--------------------|
| $\text{H}_4\text{XeO}_6 + 2\text{H}^+ + 2\text{e}^- \longrightarrow \text{XeO}_3 + 3\text{H}_2\text{O}$         | +3.0               | $\text{Cu}^+ + \text{e}^- \longrightarrow \text{Cu}$                                                      | +0.52              |
| $\text{F}_2 + 2\text{e}^- \longrightarrow 2\text{F}^-$                                                          | +2.87              | $\text{NiOOH} + \text{H}_2\text{O} + \text{e}^- \longrightarrow \text{Ni(OH)}_2 + \text{OH}^-$            | +0.49              |
| $\text{O}_3 + 2\text{H}^+ + 2\text{e}^- \longrightarrow \text{O}_2 + \text{H}_2\text{O}$                        | +2.07              | $\text{Ag}_2\text{CrO}_4 + 2\text{e}^- \longrightarrow 2\text{Ag} + \text{CrO}_4^{2-}$                    | +0.45              |
| $\text{S}_2\text{O}_8^{2-} + 2\text{e}^- \longrightarrow 2\text{SO}_4^{2-}$                                     | +2.05              | $\text{O}_2 + 2\text{H}_2\text{O} + 4\text{e}^- \longrightarrow 4\text{OH}^-$                             | +0.40              |
| $\text{Ag}^+ + \text{e}^- \longrightarrow \text{Ag}^+$                                                          | +1.98              | $\text{ClO}_4^- + \text{H}_2\text{O} + 2\text{e}^- \longrightarrow \text{ClO}_3^- + 2\text{OH}^-$         | +0.36              |
| $\text{Co}^{3+} + \text{e}^- \longrightarrow \text{Co}^{2+}$                                                    | +1.81              | $[\text{Fe(CN)}_6]^{3-} + \text{e}^- \longrightarrow [\text{Fe(CN)}_6]^{4-}$                              | +0.36              |
| $\text{H}_2\text{O}_2 + 2\text{H}^+ + 2\text{e}^- \longrightarrow 2\text{H}_2\text{O}$                          | +1.78              | $\text{Cu}^{2+} + 2\text{e}^- \longrightarrow \text{Cu}$                                                  | +0.34              |
| $\text{Au}^+ + \text{e}^- \longrightarrow \text{Au}$                                                            | +1.69              | $\text{Hg}_2\text{Cl}_2 + 2\text{e}^- \longrightarrow 2\text{Hg} + 2\text{Cl}^-$                          | +0.27              |
| $\text{Pb}^{4+} + 2\text{e}^- \longrightarrow \text{Pb}^{2+}$                                                   | +1.67              | $\text{AgCl} + \text{e}^- \longrightarrow \text{Ag} + \text{Cl}^-$                                        | +0.27              |
| $2\text{HClO} + 2\text{H}^+ + 2\text{e}^- \longrightarrow \text{Cl}_2 + 2\text{H}_2\text{O}$                    | +1.63              | $\text{Bi}^{3+} + 3\text{e}^- \longrightarrow \text{Bi}$                                                  | +0.20              |
| $\text{Ce}^{4+} + \text{e}^- \longrightarrow \text{Ce}^{3+}$                                                    | +1.61              | $\text{SO}_4^{2-} + 4\text{H}^+ + 2\text{e}^- \longrightarrow \text{H}_2\text{SO}_3 + \text{H}_2\text{O}$ | +0.17              |
| $2\text{HBrO} + 2\text{H}^+ + 2\text{e}^- \longrightarrow \text{Br}_2 + 2\text{H}_2\text{O}$                    | +1.60              | $\text{Cu}^{2+} + \text{e}^- \longrightarrow \text{Cu}^+$                                                 | +0.16              |
| $\text{MnO}_4^- + 8\text{H}^+ + 5\text{e}^- \longrightarrow \text{Mn}^{2+} + 4\text{H}_2\text{O}$               | +1.51              | $\text{Sn}^{4+} + 2\text{e}^- \longrightarrow \text{Sn}^{2+}$                                             | +0.15              |
| $\text{Mn}^{3+} + \text{e}^- \longrightarrow \text{Mn}^{2+}$                                                    | +1.51              | $\text{AgBr} + \text{e}^- \longrightarrow \text{Ag} + \text{Br}^-$                                        | +0.07              |
| $\text{Au}^{3+} + 3\text{e}^- \longrightarrow \text{Au}$                                                        | +1.40              | $\text{Ti}^{4+} + \text{e}^- \longrightarrow \text{Ti}^{3+}$                                              | 0.00               |
| $\text{Cl}_2 + 2\text{e}^- \longrightarrow 2\text{Cl}^-$                                                        | +1.36              | $2\text{H}^+ + 2\text{e}^- \longrightarrow \text{H}_2$                                                    | 0.0 by definition  |
| $\text{Cr}_2\text{O}_7^{2-} + 14\text{H}^+ + 6\text{e}^- \longrightarrow 2\text{Cr}^{3+} + 7\text{H}_2\text{O}$ | +1.33              | $\text{Fe}^{3+} + 3\text{e}^- \longrightarrow \text{Fe}$                                                  | -0.04              |
| $\text{O}_3 + \text{H}_2\text{O} + 2\text{e}^- \longrightarrow \text{O}_2 + 2\text{OH}^-$                       | +1.24              | $\text{O}_2 + \text{H}_2\text{O} + 2\text{e}^- \longrightarrow \text{HO}_2^- + \text{OH}^-$               | -0.08              |
| $\text{O}_2 + 4\text{H}^+ + 4\text{e}^- \longrightarrow 2\text{H}_2\text{O}$                                    | +1.23              | $\text{Pb}^{2+} + 2\text{e}^- \longrightarrow \text{Pb}$                                                  | -0.13              |
| $\text{ClO}_4^- + 2\text{H}^+ + 2\text{e}^- \longrightarrow \text{ClO}_3^- + 2\text{H}_2\text{O}$               | +1.23              | $\text{In}^+ + \text{e}^- \longrightarrow \text{In}$                                                      | -0.14              |
| $\text{MnO}_2 + 4\text{H}^+ + 2\text{e}^- \longrightarrow \text{Mn}^{2+} + 2\text{H}_2\text{O}$                 | +1.23              | $\text{Sn}^{2+} + 2\text{e}^- \longrightarrow \text{Sn}$                                                  | -0.14              |
| $\text{Pt}^{2+} + 2\text{e}^- \longrightarrow \text{Pt}$                                                        | +1.20              | $\text{AgI} + \text{e}^- \longrightarrow \text{Ag} + \text{I}^-$                                          | -0.15              |
| $\text{Br}_2 + 2\text{e}^- \longrightarrow 2\text{Br}^-$                                                        | +1.09              | $\text{Ni}^{2+} + 2\text{e}^- \longrightarrow \text{Ni}$                                                  | -0.23              |
| $\text{Pu}^{4+} + \text{e}^- \longrightarrow \text{Pu}^{3+}$                                                    | +0.97              | $\text{V}^{3+} + \text{e}^- \longrightarrow \text{V}^{2+}$                                                | -0.26              |
| $\text{NO}_3^- + 4\text{H}^+ + 3\text{e}^- \longrightarrow \text{NO} + 2\text{H}_2\text{O}$                     | +0.96              | $\text{Co}^{2+} + 2\text{e}^- \longrightarrow \text{Co}$                                                  | -0.28              |
| $2\text{Hg}^{2+} + 2\text{e}^- \longrightarrow \text{Hg}_2^{2+}$                                                | +0.92              | $\text{In}^{3+} + 3\text{e}^- \longrightarrow \text{In}$                                                  | -0.34              |
| $\text{ClO}^- + \text{H}_2\text{O} + 2\text{e}^- \longrightarrow \text{Cl}^- + 2\text{OH}^-$                    | +0.89              | $\text{Tl}^+ + \text{e}^- \longrightarrow \text{Tl}$                                                      | -0.34              |
| $\text{Hg}^{2+} + 2\text{e}^- \longrightarrow \text{Hg}$                                                        | +0.86              | $\text{PbSO}_4 + 2\text{e}^- \longrightarrow \text{Pb} + \text{SO}_4^{2-}$                                | -0.36              |
| $\text{NO}_3^- + 2\text{H}^+ + \text{e}^- \longrightarrow \text{NO}_2 + \text{H}_2\text{O}$                     | +0.80              | $\text{Ti}^{3+} + \text{e}^- \longrightarrow \text{Ti}^{2+}$                                              | -0.37              |
| $\text{Ag}^+ + \text{e}^- \longrightarrow \text{Ag}$                                                            | +0.80              | $\text{Cd}^{2+} + 2\text{e}^- \longrightarrow \text{Cd}$                                                  | -0.40              |
| $\text{Hg}_2^{2+} + 2\text{e}^- \longrightarrow 2\text{Hg}$                                                     | +0.79              | $\text{In}^{2+} + \text{e}^- \longrightarrow \text{In}^+$                                                 | -0.40              |
| $\text{Fe}^{3+} + \text{e}^- \longrightarrow \text{Fe}^{2+}$                                                    | +0.77              | $\text{Cr}^{3+} + \text{e}^- \longrightarrow \text{Cr}^{2+}$                                              | -0.41              |
| $\text{BrO}^- + \text{H}_2\text{O} + 2\text{e}^- \longrightarrow \text{Br}^- + 2\text{OH}^-$                    | +0.76              | $\text{Fe}^{2+} + 2\text{e}^- \longrightarrow \text{Fe}$                                                  | -0.44              |
| $\text{Hg}_2\text{SO}_4 + 2\text{e}^- \longrightarrow 2\text{Hg} + \text{SO}_4^{2-}$                            | +0.62              | $\text{In}^{3+} + 2\text{e}^- \longrightarrow \text{In}^+$                                                | -0.44              |
| $\text{MnO}_4^{2-} + 2\text{H}_2\text{O} + 2\text{e}^- \longrightarrow \text{MnO}_2 + 4\text{OH}^-$             | +0.60              | $\text{S} + 2\text{e}^- \longrightarrow \text{S}^{2-}$                                                    | -0.48              |
| $\text{MnO}_4^- + \text{e}^- \longrightarrow \text{MnO}_4^{2-}$                                                 | +0.56              | $\text{In}^{3+} + \text{e}^- \longrightarrow \text{In}^{2+}$                                              | -0.49              |
| $\text{I}_2 + 2\text{e}^- \longrightarrow 2\text{I}^-$                                                          | +0.54              | $\text{U}^{4+} + \text{e}^- \longrightarrow \text{U}^{3+}$                                                | -0.61              |
| $\text{I}_3^- + 2\text{e}^- \longrightarrow 3\text{I}^-$                                                        | +0.53              | $\text{Cr}^{3+} + 3\text{e}^- \longrightarrow \text{Cr}$                                                  | -0.74              |
|                                                                                                                 |                    | $\text{Zn}^{2+} + 2\text{e}^- \longrightarrow \text{Zn}$                                                  | -0.76              |

(continued)

### APPENDIX III CONTINUED

| Reduction half-reaction                                                         | $E^\ominus/\text{V}$ | Reduction half-reaction                                  | $E^\ominus/\text{V}$ |
|---------------------------------------------------------------------------------|----------------------|----------------------------------------------------------|----------------------|
| $\text{Cd}(\text{OH})_2 + 2\text{e}^- \longrightarrow \text{Cd} + 2\text{OH}^-$ | -0.81                | $\text{La}^{3+} + 3\text{e}^- \longrightarrow \text{La}$ | -2.52                |
| $2\text{H}_2\text{O} + 2\text{e}^- \longrightarrow \text{H}_2 + 2\text{OH}^-$   | -0.83                | $\text{Na}^+ + \text{e}^- \longrightarrow \text{Na}$     | -2.71                |
| $\text{Cr}^{2+} + 2\text{e}^- \longrightarrow \text{Cr}$                        | -0.91                | $\text{Ca}^{2+} + 2\text{e}^- \longrightarrow \text{Ca}$ | -2.87                |
| $\text{Mn}^{2+} + 2\text{e}^- \longrightarrow \text{Mn}$                        | -1.18                | $\text{Sr}^{2+} + 2\text{e}^- \longrightarrow \text{Sr}$ | -2.89                |
| $\text{V}^{2+} + 2\text{e}^- \longrightarrow \text{V}$                          | -1.19                | $\text{Ba}^{2+} + 2\text{e}^- \longrightarrow \text{Ba}$ | -2.91                |
| $\text{Ti}^{2+} + 2\text{e}^- \longrightarrow \text{Ti}$                        | -1.63                | $\text{Ra}^{2+} + 2\text{e}^- \longrightarrow \text{Ra}$ | -2.92                |
| $\text{Al}^{3+} + 3\text{e}^- \longrightarrow \text{Al}$                        | -1.66                | $\text{Cs}^+ + \text{e}^- \longrightarrow \text{Cs}$     | -2.92                |
| $\text{U}^{3+} + 3\text{e}^- \longrightarrow \text{U}$                          | -1.79                | $\text{Rb}^+ + \text{e}^- \longrightarrow \text{Rb}$     | -2.93                |
| $\text{Sc}^{3+} + 3\text{e}^- \longrightarrow \text{Sc}$                        | -2.09                | $\text{K}^+ + \text{e}^- \longrightarrow \text{K}$       | -2.93                |
| $\text{Mg}^{2+} + 2\text{e}^- \longrightarrow \text{Mg}$                        | -2.36                | $\text{Li}^+ + \text{e}^- \longrightarrow \text{Li}$     | -3.05                |
| $\text{Ce}^{3+} + 3\text{e}^- \longrightarrow \text{Ce}$                        | -2.48                |                                                          |                      |

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## Logarithms

Sometimes, a numerical expression may involve multiplication, division or rational powers of large numbers. For such calculations, logarithms are very useful. They help us in making difficult calculations easy. In Chemistry, logarithm values are required in solving problems of chemical kinetics, thermodynamics, electrochemistry, etc. We shall first introduce this concept, and discuss the laws, which will have to be followed in working with logarithms, and then apply this technique to a number of problems to show how it makes difficult calculations simple.

We know that

$$2^3 = 8, 3^2 = 9, 5^3 = 125, 7^0 = 1$$

In general, for a positive real number  $a$ , and a rational number  $m$ , let  $a^m = b$ , where  $b$  is a real number. In other words the  $m^{\text{th}}$  power of base  $a$  is  $b$ .

Another way of stating the same fact is logarithm of  $b$  to base  $a$  is  $m$ .

If for a positive real number  $a$ ,  $a \neq 1$

$$a^m = b,$$

we say that  $m$  is the logarithm of  $b$  to the base  $a$ .

We write this as  $\log_a^b = m$ ,

“log” being the abbreviation of the word “logarithm”.

Thus, we have

$$\log_2 8 = 3, \quad \text{Since } 2^3 = 8$$

$$\log_3 9 = 2, \quad \text{Since } 3^2 = 9$$

$$\log_5 125 = 3, \quad \text{Since } 5^3 = 125$$

$$\log_7 1 = 0, \quad \text{Since } 7^0 = 1$$

### Laws of Logarithms

In the following discussion, we shall take logarithms to any base  $a$ , ( $a > 0$  and  $a \neq 1$ )

**First Law:**  $\log_a (mn) = \log_a m + \log_a n$

**Proof:** Suppose that  $\log_a m = x$  and  $\log_a n = y$

Then  $a^x = m$ ,  $a^y = n$

$$\text{Hence } mn = a^x \cdot a^y = a^{x+y}$$

It now follows from the definition of logarithms that

$$\log_a (mn) = x + y = \log_a m + \log_a n$$

**Second Law:**  $\log_a \left( \frac{m}{n} \right) = \log_a m - \log_a n$

**Proof:** Let  $\log_a m = x$ ,  $\log_a n = y$

Then  $a^x = m$ ,  $a^y = n$

$$\text{Hence } \frac{m}{n} = \frac{a^x}{a^y} = a^{x-y}$$

Therefore

$$\log_a \left( \frac{m}{n} \right) = x - y = \log_a m - \log_a n$$

**Third Law :**  $\log_a(m^n) = n \log_a m$

**Proof :** As before, if  $\log_a m = x$ , then  $a^x = m$

$$\text{Then } m^n = (a^x)^n = a^{nx}$$

giving  $\log_a(m^n) = nx = n \log_a m$

Thus according to First Law: “the log of the product of two numbers is equal to the sum of their logs. Similarly, the Second Law says: the log of the ratio of two numbers is the difference of their logs. Thus, the use of these laws converts a problem of multiplication/division into a problem of addition/subtraction, which are far easier to perform than multiplication/division. That is why logarithms are so useful in all numerical computations.

### Logarithms to Base 10

Because number 10 is the base of writing numbers, it is very convenient to use logarithms to the base 10. Some examples are:

$$\begin{array}{ll} \log_{10} 10 = 1, & \text{since } 10^1 = 10 \\ \log_{10} 100 = 2, & \text{since } 10^2 = 100 \\ \log_{10} 10000 = 4, & \text{since } 10^4 = 10000 \\ \log_{10} 0.01 = -2, & \text{since } 10^{-2} = 0.01 \\ \log_{10} 0.001 = -3, & \text{since } 10^{-3} = 0.001 \\ \text{and } \log_{10} 1 = 0 & \text{since } 10^0 = 1 \end{array}$$

The above results indicate that if  $n$  is an integral power of 10, i.e., 1 followed by several zeros or 1 preceded by several zeros immediately to the right of the decimal point, then  $\log n$  can be easily found.

If  $n$  is not an integral power of 10, then it is not easy to calculate  $\log n$ . But mathematicians have made tables from which we can read off approximate value of the logarithm of any positive number between 1 and 10. And these are sufficient for us to calculate the logarithm of any number expressed in decimal form. For this purpose, we always express the given decimal as the product of an integral power of 10 and a number between 1 and 10.

### Standard Form of Decimal

We can express any number in decimal form, as the product of (i) an integral power of 10, and (ii) a number between 1 and 10. Here are some examples:

(i) 25.2 lies between 10 and 100

$$25.2 = \frac{25.2}{10} \times 10 = 2.52 \times 10^1$$

(ii) 1038.4 lies between 1000 and 10000.

$$\therefore 1038.4 = \frac{1038.4}{1000} \times 10^3 = 1.0384 \times 10^3$$

(iii) 0.005 lies between 0.001 and 0.01

$$\therefore 0.005 = (0.005 \times 1000) \times 10^{-3} = 5.0 \times 10^{-3}$$

(iv) 0.00025 lies between 0.0001 and 0.001

$$\therefore 0.00025 = (0.00025 \times 10000) \times 10^{-4} = 2.5 \times 10^{-4}$$

In each case, we divide or multiply the decimal by a power of 10, to bring one non-zero digit to the left of the decimal point, and do the reverse operation by the same power of 10, indicated separately.

Thus, any positive decimal can be written in the form

$$n = m \times 10^p$$

where  $p$  is an integer (positive, zero or negative) and  $1 \leq m < 10$ . This is called the “standard form of  $n$ .”

### Working Rule

1. Move the decimal point to the left, or to the right, as may be necessary, to bring one non-zero digit to the left of decimal point.
2. (i) If you move  $p$  places to the left, multiply by  $10^p$ .  
(ii) If you move  $p$  places to the right, multiply by  $10^{-p}$ .  
(iii) If you do not move the decimal point at all, multiply by  $10^0$ .  
(iv) Write the new decimal obtained by the power of 10 (of step 2) to obtain the standard form of the given decimal.

### Characteristic and Mantissa

Consider the standard form of  $n$

$$n = m \times 10^p, \text{ where } 1 \leq m < 10$$

Taking logarithms to the base 10 and using the laws of logarithms

$$\begin{aligned} \log n &= \log m + \log 10^p \\ &= \log m + p \log 10 \\ &= p + \log m \end{aligned}$$

Here  $p$  is an integer and as  $1 \leq m < 10$ , so  $0 \leq \log m < 1$ , i.e.,  $m$  lies between 0 and 1. When  $\log n$  has been expressed as  $p + \log m$ , where  $p$  is an integer and  $0 \leq \log m < 1$ , we say that  $p$  is the “characteristic” of  $\log n$  and that  $\log m$  is the “mantissa of  $\log n$ .” Note that characteristic is always an integer – positive, negative or zero, and mantissa is never negative and is always less than 1. If we can find the characteristics and the mantissa of  $\log n$ , we have to just add them to get  $\log n$ .

Thus to find  $\log n$ , all we have to do is as follows:

1. Put  $n$  in the standard form, say  
 $n = m \times 10^p, 1 \leq m < 10$
2. Read off the characteristic  $p$  of  $\log n$  from this expression (exponent of 10).
3. Look up  $\log m$  from tables, which is being explained below.
4. Write  $\log n = p + \log m$

If the characteristic  $p$  of a number  $n$  is say, 2 and the mantissa is .4133, then we have  $\log n = 2 + .4133$  which we can write as 2.4133. If, however, the characteristic  $p$  of a number  $m$  is say -2 and the mantissa is .4123, then we have  $\log m = -2 + .4123$ . We cannot write this as -2.4123. (Why?) In order to avoid this confusion we write  $\bar{2}$  for -2 and thus we write  $\log m = \bar{2}.4123$ .

Now let us explain how to use the table of logarithms to find mantissas. A table is appended at the end of this Appendix.

Observe that in the table, every row starts with a two digit number, 10, 11, 12,... 97, 98, 99. Every column is headed by a one-digit number, 0, 1, 2, ...9. On the right, we have the section called “Mean differences” which has 9 columns headed by 1, 2...9.

|    | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |    | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|----|------|------|------|------|------|------|------|------|------|------|----|----|----|----|----|----|----|----|----|----|
| .. | ..   | ..   | ..   | ..   | ..   | ..   | ..   | ..   | ..   | ..   | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. |
| 61 | 7853 | 7860 | 7868 | 7875 | 7882 | 7889 | 7896 | 7803 | 7810 | 7817 | .. | 1  | 1  | 2  | 3  | 4  | 4  | 5  | 6  | 6  |
| 62 | 7924 | 7931 | 7935 | 7945 | 7954 | 7959 | 7966 | 7973 | 7980 | 7987 | .. | 1  | 1  | 2  | 3  | 3  | 4  | 5  | 6  | 6  |
| 63 | 7993 | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 | .. | 1  | 1  | 2  | 3  | 3  | 4  | 5  | 6  | 6  |
| .. | ..   | ..   | ..   | ..   | ..   | ..   | ..   | ..   | ..   | ..   | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. |

Now suppose we wish to find  $\log(6.234)$ . Then look into the row starting with 62. In this row, look at the number in the column headed by 3. The number is 7945. This means that

$$\log(6.230) = 0.7945^*$$

But we want  $\log(6.234)$ . So our answer will be a little more than 0.7945. How much more? We look this up in the section on Mean differences. Since our fourth digit is 4, look under the column headed by 4 in the Mean difference section (in the row 62). We see the number 3 there. So add 3 to 7945. We get 7948. So we finally have

$$\log(6.234) = 0.7948.$$

Take another example. To find  $\log(8.127)$ , we look in the row 81 under column 2, and we find 9096. We continue in the same row and see that the mean difference under 7 is 4. Adding this to 9096, and we get 9100. So,  $\log(8.127) = 0.9100$ .

### Finding $N$ when $\log N$ is given

We have so far discussed the procedure for finding  $\log n$  when a positive number  $n$  is given. We now turn to its converse i.e., to find  $n$  when  $\log n$  is given and give a method for this purpose. If  $\log n = t$ , we sometimes say  $n = \text{antilog } t$ . Therefore our task is given  $t$ , find its antilog. For this, we use the ready-made antilog tables.

Suppose  $\log n = 2.5372$ .

To find  $n$ , first take just the mantissa of  $\log n$ . In this case it is .5372. (Make sure it is positive.) Now take up antilog of this number in the antilog table which is to be used exactly like the log table. In the antilog table, the entry under column 7 in the row .53 is 3443 and the mean difference for the last digit 2 in that row is 2, so the table gives 3445. Hence,

$$\text{antilog}(.5372) = 3.445$$

Now since  $\log n = 2.5372$ , the characteristic of  $\log n$  is 2. So the standard form of  $n$  is given by

$$n = 3.445 \times 10^2$$

$$\text{or } n = 344.5$$

#### Illustration 1:

If  $\log x = 1.0712$ , find  $x$ .

**Solution:** We find that the number corresponding to 0712 is 1179. Since characteristic of  $\log x$  is 1, we have

$$\begin{aligned} x &= 1.179 \times 10^1 \\ &= 11.79 \end{aligned}$$

#### Illustration 2:

If  $\log_{10} x = \bar{2}.1352$ , find  $x$ .

**Solution:** From antilog tables, we find that the number corresponding to 1352 is 1366. Since the characteristic is  $\bar{2}$  i.e., -2, so

$$x = 1.366 \times 10^{-2} = 0.01366$$

### Use of Logarithms in Numerical Calculations

#### Illustration 1:

Find  $6.3 \times 1.29$

**Solution:** Let  $x = 6.3 \times 1.29$

$$\text{Then } \log_{10} x = \log(6.3 \times 1.29) = \log 6.3 + \log 1.29$$

Now,

$$\log 6.3 = 0.7993$$

$$\log 1.29 = 0.1106$$

$$\therefore \log_{10} x = 0.9099,$$

---

\* It should, however, be noted that the values given in the table are not exact. They are only approximate values, although we use the sign of equality which may give the impression that they are exact values. The same convention will be followed in respect of antilogarithm of a number.

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Taking antilog

$$x = 8.127$$

**Illustration 2:**

Find  $\frac{(1.23)^{1.5}}{11.2 \times 23.5}$

**Solution:** Let  $x = \frac{(1.23)^{\frac{3}{2}}}{11.2 \times 23.5}$

Then  $\log x = \log \frac{(1.23)^{\frac{3}{2}}}{11.2 \times 23.5}$

$$= \frac{3}{2} \log 1.23 - \log (11.2 \times 23.5)$$

$$= \frac{3}{2} \log 1.23 - \log 11.2 - \log 23.5$$

Now,

$$\log 1.23 = 0.0899$$

$$\frac{3}{2} \log 1.23 = 0.13485$$

$$\log 11.2 = 1.0492$$

$$\log 23.5 = 1.3711$$

$$\log x = 0.13485 - 1.0492 - 1.3711$$

$$= -3.71455$$

$$\therefore x = 0.005183$$

**Illustration 3:**

Find  $\sqrt{\frac{(71.24)^5 \times \sqrt{56}}{(2.3)^7 \times \sqrt{21}}}$

**Solution:** Let  $x = \sqrt{\frac{(71.24)^5 \times \sqrt{56}}{(2.3)^7 \times \sqrt{21}}}$

$$\begin{aligned} \text{Then } \log x &= \frac{1}{2} \log \left[ \frac{(71.24)^5 \times \sqrt{56}}{(2.3)^7 \times \sqrt{21}} \right] \\ &= \frac{1}{2} [\log (71.24)^5 + \log \sqrt{56} - \log (2.3)^7 - \log \sqrt{21}] \\ &= \frac{5}{2} \log 71.24 + \frac{1}{4} \log 56 - \frac{7}{2} \log 2.3 - \frac{1}{4} \log 21 \end{aligned}$$

Now, using log tables

$$\log 71.24 = 1.8527$$

$$\log 56 = 1.7482$$

$$\log 2.3 = 0.3617$$

$$\log 21 = 1.3222$$

$$\begin{aligned} \therefore \log x &= \frac{5}{2} \log (1.8527) + \frac{1}{4} (1.7482) - \frac{7}{2} (0.3617) - \frac{1}{4} (1.3222) \\ &= 3.4723 \end{aligned}$$

$$\therefore x = 2967$$

# LOGARITHMS

TABLE I

| N  | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|----|------|------|------|------|------|------|------|------|------|------|---|---|----|----|----|----|----|----|----|
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 | 5 | 9 | 13 | 17 | 21 | 26 | 30 | 34 | 38 |
|    |      |      |      |      |      |      |      |      |      |      | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 | 4 | 8 | 12 | 16 | 20 | 23 | 27 | 31 | 35 |
|    |      |      |      |      |      |      |      |      |      |      | 4 | 7 | 11 | 15 | 18 | 22 | 26 | 29 | 33 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 | 3 | 7 | 11 | 14 | 18 | 21 | 25 | 28 | 32 |
|    |      |      |      |      |      |      |      |      |      |      | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 3 | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
|    |      |      |      |      |      |      |      |      |      |      | 3 | 7 | 10 | 13 | 16 | 19 | 22 | 25 | 29 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 | 3 | 6 | 9  | 12 | 15 | 19 | 22 | 25 | 28 |
|    |      |      |      |      |      |      |      |      |      |      | 3 | 6 | 9  | 12 | 14 | 17 | 20 | 23 | 26 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 | 3 | 6 | 9  | 11 | 14 | 17 | 20 | 23 | 26 |
|    |      |      |      |      |      |      |      |      |      |      | 3 | 6 | 8  | 11 | 14 | 17 | 19 | 22 | 25 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 | 3 | 6 | 8  | 11 | 14 | 16 | 19 | 22 | 24 |
|    |      |      |      |      |      |      |      |      |      |      | 3 | 5 | 8  | 10 | 13 | 16 | 18 | 21 | 23 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 | 3 | 5 | 8  | 10 | 13 | 15 | 18 | 20 | 23 |
|    |      |      |      |      |      |      |      |      |      |      | 3 | 5 | 8  | 10 | 12 | 15 | 17 | 20 | 22 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 | 2 | 5 | 7  | 9  | 12 | 14 | 17 | 19 | 21 |
|    |      |      |      |      |      |      |      |      |      |      | 2 | 4 | 7  | 9  | 11 | 14 | 16 | 18 | 21 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 | 2 | 4 | 7  | 9  | 11 | 13 | 16 | 18 | 20 |
|    |      |      |      |      |      |      |      |      |      |      | 2 | 4 | 6  | 8  | 11 | 13 | 15 | 17 | 19 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 2 | 4 | 6  | 8  | 11 | 13 | 15 | 17 | 19 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 | 2 | 4 | 6  | 8  | 10 | 12 | 14 | 16 | 18 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 | 2 | 4 | 6  | 8  | 10 | 12 | 14 | 15 | 17 |
| 23 | 3617 | 3636 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 | 2 | 4 | 6  | 7  | 9  | 11 | 13 | 15 | 17 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 | 2 | 4 | 5  | 7  | 9  | 11 | 12 | 14 | 16 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 | 2 | 3 | 5  | 7  | 9  | 10 | 12 | 14 | 15 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 | 2 | 3 | 5  | 7  | 8  | 10 | 11 | 13 | 15 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 | 2 | 3 | 5  | 6  | 8  | 9  | 11 | 13 | 14 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 | 2 | 3 | 5  | 6  | 8  | 9  | 11 | 12 | 14 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 | 1 | 3 | 4  | 6  | 7  | 9  | 10 | 12 | 13 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 | 1 | 3 | 4  | 6  | 7  | 9  | 10 | 11 | 13 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 | 1 | 3 | 4  | 6  | 7  | 8  | 10 | 11 | 12 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 | 1 | 3 | 4  | 5  | 7  | 8  | 9  | 11 | 12 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 | 1 | 3 | 4  | 5  | 6  | 8  | 9  | 10 | 12 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 | 1 | 3 | 4  | 5  | 6  | 8  | 9  | 10 | 11 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 | 1 | 2 | 4  | 5  | 6  | 7  | 9  | 10 | 11 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 | 1 | 2 | 4  | 5  | 6  | 7  | 8  | 10 | 11 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 | 1 | 2 | 3  | 5  | 6  | 7  | 8  | 9  | 10 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 | 1 | 2 | 3  | 5  | 6  | 7  | 8  | 9  | 10 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 | 1 | 2 | 3  | 4  | 5  | 7  | 8  | 9  | 10 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 | 1 | 2 | 3  | 4  | 5  | 6  | 8  | 9  | 10 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 45 | 6532 | 6542 | 6551 | 6561 | 6471 | 6580 | 6590 | 6599 | 6609 | 6618 | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 7  | 8  |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 | 1 | 2 | 3  | 4  | 5  | 5  | 6  | 7  | 8  |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 | 1 | 2 | 3  | 4  | 4  | 5  | 6  | 7  | 8  |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 | 1 | 2 | 3  | 4  | 4  | 5  | 6  | 7  | 8  |

# LOGARITHMS

TABLE 1 (Continued)

| N  | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|------|------|------|------|------|------|------|------|------|------|---|---|---|---|---|---|---|---|---|
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 7 |
| 53 | 7243 | 7251 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| 55 | 7404 | 7412 | 7419 | 7427 | 7435 | 7443 | 7451 | 7459 | 7466 | 7474 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 56 | 7482 | 7490 | 7497 | 7505 | 7513 | 7520 | 7528 | 7536 | 7543 | 7551 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 57 | 7559 | 7566 | 7574 | 7582 | 7589 | 7597 | 7604 | 7612 | 7619 | 7627 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 58 | 7634 | 7642 | 7649 | 7657 | 7664 | 7672 | 7679 | 7686 | 7694 | 7701 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 59 | 7709 | 7716 | 7723 | 7731 | 7738 | 7745 | 7752 | 7760 | 7767 | 7774 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 60 | 7782 | 7789 | 7796 | 7803 | 7810 | 7818 | 7825 | 7832 | 7839 | 7846 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 61 | 7853 | 7860 | 7768 | 7875 | 7882 | 7889 | 7896 | 7903 | 7910 | 7917 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 62 | 7924 | 7931 | 7938 | 7945 | 7952 | 7959 | 7966 | 7973 | 7980 | 7987 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 63 | 7993 | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 64 | 8062 | 8069 | 8075 | 8082 | 8089 | 8096 | 8102 | 8109 | 8116 | 8122 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 65 | 8129 | 8136 | 8142 | 8149 | 8156 | 8162 | 8169 | 8176 | 8182 | 8189 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 66 | 8195 | 8202 | 8209 | 8215 | 8222 | 8228 | 8235 | 8241 | 8248 | 8254 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 67 | 8261 | 8267 | 8274 | 8280 | 8287 | 8293 | 8299 | 8306 | 8312 | 8319 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 68 | 8325 | 8331 | 8338 | 8344 | 8351 | 8357 | 8363 | 8370 | 8376 | 8382 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| 69 | 8388 | 8395 | 8401 | 8407 | 8414 | 8420 | 8426 | 8432 | 8439 | 8445 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 70 | 8451 | 8457 | 8463 | 8470 | 8476 | 8482 | 8488 | 8494 | 8500 | 8506 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 71 | 8513 | 8519 | 8525 | 8531 | 8537 | 8543 | 8549 | 8555 | 8561 | 8567 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 72 | 8573 | 8579 | 8585 | 8591 | 8597 | 8603 | 8609 | 8615 | 8621 | 8627 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 73 | 8633 | 8639 | 8645 | 8651 | 8657 | 8663 | 8669 | 8675 | 8681 | 8686 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 74 | 8692 | 8698 | 8704 | 8710 | 8716 | 8722 | 8727 | 8733 | 8739 | 8745 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 75 | 8751 | 8756 | 8762 | 8768 | 8774 | 8779 | 8785 | 8791 | 8797 | 8802 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 76 | 8808 | 8814 | 8820 | 8825 | 8831 | 8837 | 8842 | 8848 | 8854 | 8859 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 77 | 8865 | 8871 | 8876 | 8882 | 8887 | 8893 | 8899 | 8904 | 8910 | 8915 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 78 | 8921 | 8927 | 8932 | 8938 | 8943 | 8949 | 8954 | 8960 | 8965 | 8971 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 79 | 8976 | 8982 | 8987 | 8993 | 8998 | 9004 | 9009 | 9015 | 9020 | 9025 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 80 | 9031 | 9036 | 9042 | 9047 | 9053 | 9058 | 9063 | 9069 | 9074 | 9079 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 81 | 9085 | 9090 | 9096 | 9101 | 9106 | 9112 | 9117 | 9122 | 9128 | 9133 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 82 | 9138 | 9143 | 9149 | 9154 | 9159 | 9165 | 9170 | 9175 | 9180 | 9186 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 83 | 9191 | 9196 | 9201 | 9206 | 9212 | 9217 | 9222 | 9227 | 9232 | 9238 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 84 | 9243 | 9248 | 9253 | 9258 | 9263 | 9269 | 9274 | 9279 | 9284 | 9289 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 85 | 9294 | 9299 | 9304 | 9309 | 9315 | 9320 | 9325 | 9330 | 9335 | 9340 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 86 | 9345 | 9350 | 9355 | 9360 | 9365 | 9370 | 9375 | 9380 | 9385 | 9390 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 87 | 9395 | 9400 | 9405 | 9410 | 9415 | 9420 | 9425 | 9430 | 9435 | 9440 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 88 | 9445 | 9450 | 9455 | 9460 | 9465 | 9469 | 9474 | 9479 | 9484 | 9489 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 89 | 9494 | 9499 | 9504 | 9509 | 9513 | 9518 | 9523 | 9528 | 9533 | 9538 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 90 | 9542 | 9547 | 9552 | 9557 | 9562 | 9566 | 9571 | 9576 | 9581 | 9586 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 91 | 9590 | 9595 | 9600 | 9605 | 9609 | 9614 | 9619 | 9624 | 9628 | 9633 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 92 | 9638 | 9643 | 9647 | 9652 | 9657 | 9661 | 9666 | 9671 | 9675 | 9680 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 93 | 9685 | 9689 | 9694 | 9699 | 9703 | 9708 | 9713 | 9717 | 9722 | 9727 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 94 | 9731 | 9736 | 9741 | 9745 | 9750 | 9754 | 9759 | 9763 | 9768 | 9773 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 95 | 9777 | 9782 | 9786 | 9791 | 9795 | 9800 | 9805 | 9809 | 9814 | 9818 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 96 | 9823 | 9827 | 9832 | 9836 | 9841 | 9845 | 9850 | 9854 | 9859 | 9863 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 97 | 9868 | 9872 | 9877 | 9881 | 9886 | 9890 | 9894 | 9899 | 9903 | 9908 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 98 | 9912 | 9917 | 9921 | 9926 | 9930 | 9934 | 9939 | 9943 | 9948 | 9952 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 99 | 9956 | 9961 | 9965 | 9969 | 9974 | 9978 | 9983 | 9987 | 9997 | 9996 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 |

# ANTILOGARITHMS

TABLE II

| N   | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|------|------|------|------|------|------|------|------|------|------|---|---|---|---|---|---|---|---|---|
| .00 | 1000 | 1002 | 1005 | 1007 | 1009 | 1012 | 1014 | 1016 | 1019 | 1021 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| .01 | 1023 | 1026 | 1028 | 1030 | 1033 | 1035 | 1038 | 1040 | 1042 | 1045 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| .02 | 1047 | 1050 | 1052 | 1054 | 1057 | 1059 | 1062 | 1064 | 1067 | 1069 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| .03 | 1072 | 1074 | 1076 | 1079 | 1081 | 1084 | 1086 | 1089 | 1091 | 1094 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| .04 | 1096 | 1099 | 1102 | 1104 | 1107 | 1109 | 1112 | 1114 | 1117 | 1119 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| .05 | 1122 | 1125 | 1127 | 1130 | 1132 | 1135 | 1138 | 1140 | 1143 | 1146 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| .06 | 1148 | 1151 | 1153 | 1156 | 1159 | 1161 | 1164 | 1167 | 1169 | 1172 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| .07 | 1175 | 1178 | 1180 | 1183 | 1186 | 1189 | 1191 | 1194 | 1197 | 1199 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| .08 | 1202 | 1205 | 1208 | 1211 | 1213 | 1216 | 1219 | 1222 | 1225 | 1227 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |
| .09 | 1230 | 1233 | 1236 | 1239 | 1242 | 1245 | 1247 | 1250 | 1253 | 1256 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |
| .10 | 1259 | 1262 | 1265 | 1268 | 1271 | 1274 | 1276 | 1279 | 1282 | 1285 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |
| .11 | 1288 | 1291 | 1294 | 1297 | 1300 | 1303 | 1306 | 1309 | 1312 | 1315 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .12 | 1318 | 1321 | 1324 | 1327 | 1330 | 1334 | 1337 | 1340 | 1343 | 1346 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .13 | 1349 | 1352 | 1355 | 1358 | 1361 | 1365 | 1368 | 1371 | 1374 | 1377 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| .14 | 1380 | 1384 | 1387 | 1390 | 1393 | 1396 | 1400 | 1403 | 1406 | 1409 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| .15 | 1413 | 1416 | 1419 | 1422 | 1426 | 1429 | 1432 | 1435 | 1439 | 1442 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| .16 | 1445 | 1449 | 1452 | 1455 | 1459 | 1462 | 1466 | 1469 | 1472 | 1476 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| .17 | 1479 | 1483 | 1486 | 1489 | 1493 | 1496 | 1500 | 1503 | 1507 | 1510 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| .18 | 1514 | 1517 | 1521 | 1524 | 1528 | 1531 | 1535 | 1538 | 1542 | 1545 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| .19 | 1549 | 1552 | 1556 | 1560 | 1563 | 1567 | 1570 | 1574 | 1578 | 1581 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 |
| .20 | 1585 | 1589 | 1592 | 1596 | 1600 | 1603 | 1607 | 1611 | 1614 | 1618 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 |
| .21 | 1622 | 1626 | 1629 | 1633 | 1637 | 1641 | 1644 | 1648 | 1652 | 1656 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 |
| .22 | 1660 | 1663 | 1667 | 1671 | 1675 | 1679 | 1683 | 1687 | 1690 | 1694 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 |
| .23 | 1698 | 1702 | 1706 | 1710 | 1714 | 1718 | 1722 | 1726 | 1730 | 1734 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 |
| .24 | 1738 | 1742 | 1746 | 1750 | 1754 | 1758 | 1762 | 1766 | 1770 | 1774 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 |
| .25 | 1778 | 1782 | 1786 | 1791 | 1795 | 1799 | 1803 | 1807 | 1811 | 1816 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 |
| .26 | 1820 | 1824 | 1828 | 1832 | 1837 | 1841 | 1845 | 1849 | 1854 | 1858 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 |
| .27 | 1862 | 1866 | 1871 | 1875 | 1879 | 1884 | 1888 | 1892 | 1897 | 1901 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 |
| .28 | 1905 | 1910 | 1914 | 1919 | 1923 | 1928 | 1932 | 1936 | 1941 | 1945 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 |
| .29 | 1950 | 1954 | 1959 | 1963 | 1968 | 1972 | 1977 | 1982 | 1986 | 1991 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 |
| .30 | 1995 | 2000 | 2004 | 2009 | 2014 | 2018 | 2023 | 2028 | 2032 | 2037 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 |
| .31 | 2042 | 2046 | 2051 | 2056 | 2061 | 2065 | 2070 | 2075 | 2080 | 2084 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 |
| .32 | 2089 | 2094 | 2099 | 2104 | 2109 | 2113 | 2118 | 2123 | 2128 | 2133 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 |
| .33 | 2138 | 2143 | 2148 | 2153 | 2158 | 2163 | 2168 | 2173 | 2178 | 2183 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 |
| .34 | 2188 | 2193 | 2198 | 2203 | 2208 | 2213 | 2218 | 2223 | 2228 | 2234 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| .35 | 2239 | 2244 | 2249 | 2254 | 2259 | 2265 | 2270 | 2275 | 2280 | 2286 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| .36 | 2291 | 2296 | 2301 | 2307 | 2312 | 2317 | 2323 | 2328 | 2333 | 2339 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| .37 | 2344 | 2350 | 2355 | 2360 | 2366 | 2371 | 2377 | 2382 | 2388 | 2393 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| .38 | 2399 | 2404 | 2410 | 2415 | 2421 | 2427 | 2432 | 2438 | 2443 | 2449 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| .39 | 2455 | 2460 | 2466 | 2472 | 2477 | 2483 | 2489 | 2495 | 2500 | 2506 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| .40 | 2512 | 2518 | 2523 | 2529 | 2535 | 2541 | 2547 | 2553 | 2559 | 2564 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| .41 | 2570 | 2576 | 2582 | 2588 | 2594 | 2600 | 2606 | 2612 | 2618 | 2624 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| .42 | 2630 | 2636 | 2642 | 2649 | 2655 | 2661 | 2667 | 2673 | 2679 | 2685 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| .43 | 2692 | 2698 | 2704 | 2710 | 2716 | 2723 | 2729 | 2735 | 2742 | 2748 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| .44 | 2754 | 2761 | 2767 | 2773 | 2780 | 2786 | 2793 | 2799 | 2805 | 2812 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| .45 | 2818 | 2825 | 2831 | 2838 | 2844 | 2851 | 2858 | 2864 | 2871 | 2877 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| .46 | 2884 | 2891 | 2897 | 2904 | 2911 | 2917 | 2924 | 2931 | 2938 | 2944 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| .47 | 2951 | 2958 | 2965 | 2972 | 2979 | 2985 | 2992 | 2999 | 3006 | 3013 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| .48 | 3020 | 3027 | 3034 | 3041 | 3048 | 3055 | 3062 | 3069 | 3076 | 3083 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| .49 | 3090 | 3097 | 3105 | 3112 | 3119 | 3126 | 3133 | 3141 | 3148 | 3155 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |

# ANTILOGARITHMS

TABLE II (Continued)

| N   | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 1 | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |
|-----|------|------|------|------|------|------|------|------|------|------|---|---|---|---|----|----|----|----|----|
| .50 | 3162 | 3170 | 3177 | 3184 | 3192 | 3199 | 3206 | 3214 | 3221 | 3228 | 1 | 1 | 2 | 3 | 4  | 4  | 5  | 6  | 7  |
| .51 | 3236 | 3243 | 3251 | 3258 | 3266 | 3273 | 3281 | 3289 | 3296 | 3304 | 1 | 2 | 2 | 3 | 4  | 5  | 5  | 6  | 7  |
| .52 | 3311 | 3319 | 3327 | 3334 | 3342 | 3350 | 3357 | 3365 | 3373 | 3381 | 1 | 2 | 2 | 3 | 4  | 5  | 5  | 6  | 7  |
| .53 | 3388 | 3396 | 3404 | 3412 | 3420 | 3428 | 3436 | 3443 | 3451 | 3459 | 1 | 2 | 2 | 3 | 4  | 5  | 6  | 6  | 7  |
| .54 | 3467 | 3475 | 3483 | 3491 | 3499 | 3508 | 3516 | 3524 | 3532 | 3540 | 1 | 2 | 2 | 3 | 4  | 5  | 6  | 6  | 7  |
| .55 | 3548 | 3556 | 3565 | 3573 | 3581 | 3589 | 3597 | 3606 | 3614 | 3622 | 1 | 2 | 2 | 3 | 4  | 5  | 6  | 7  | 7  |
| .56 | 3631 | 3639 | 3648 | 3656 | 3664 | 3673 | 3681 | 3690 | 3698 | 3707 | 1 | 2 | 3 | 3 | 4  | 5  | 6  | 7  | 8  |
| .57 | 3715 | 3724 | 3733 | 3741 | 3750 | 3758 | 3767 | 3776 | 3784 | 3793 | 1 | 2 | 3 | 3 | 4  | 5  | 6  | 7  | 8  |
| .58 | 3802 | 3811 | 3819 | 3828 | 3837 | 3846 | 3855 | 3864 | 3873 | 3882 | 1 | 2 | 3 | 4 | 4  | 5  | 6  | 7  | 8  |
| .59 | 3890 | 3899 | 3908 | 3917 | 3926 | 3936 | 3945 | 3954 | 3963 | 3972 | 1 | 2 | 3 | 4 | 5  | 5  | 6  | 7  | 8  |
| .60 | 3981 | 3990 | 3999 | 4009 | 4018 | 4027 | 4036 | 4046 | 4055 | 4064 | 1 | 2 | 3 | 4 | 5  | 6  | 6  | 7  | 8  |
| .61 | 4074 | 4083 | 4093 | 4102 | 4111 | 4121 | 4130 | 4140 | 4150 | 4159 | 1 | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |
| .62 | 4169 | 4178 | 4188 | 4198 | 4207 | 4217 | 4227 | 4236 | 4246 | 4256 | 1 | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |
| .63 | 4266 | 4276 | 4285 | 4295 | 4305 | 4315 | 4325 | 4335 | 4345 | 4355 | 1 | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |
| .64 | 4365 | 4375 | 4385 | 4395 | 4406 | 4416 | 4426 | 4436 | 4446 | 4457 | 1 | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |
| .65 | 4467 | 4477 | 4487 | 4498 | 4508 | 4519 | 4529 | 4539 | 4550 | 4560 | 1 | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |
| .66 | 4571 | 4581 | 4592 | 4603 | 4613 | 4624 | 4634 | 4645 | 4656 | 4667 | 1 | 2 | 3 | 4 | 5  | 6  | 7  | 9  | 10 |
| .67 | 4677 | 4688 | 4699 | 4710 | 4721 | 4732 | 4742 | 4753 | 4764 | 4775 | 1 | 2 | 3 | 4 | 5  | 7  | 8  | 9  | 10 |
| .68 | 4786 | 4797 | 4808 | 4819 | 4831 | 4842 | 4853 | 4864 | 4875 | 4887 | 1 | 2 | 3 | 4 | 6  | 7  | 8  | 9  | 10 |
| .69 | 4898 | 4909 | 4920 | 4932 | 4943 | 4955 | 4966 | 4977 | 4989 | 5000 | 1 | 2 | 3 | 5 | 6  | 7  | 8  | 9  | 10 |
| .70 | 5012 | 5023 | 5035 | 5047 | 5058 | 5070 | 5082 | 5093 | 5105 | 5117 | 1 | 2 | 4 | 5 | 6  | 7  | 8  | 9  | 11 |
| .71 | 5129 | 5140 | 5152 | 5164 | 5176 | 5188 | 5200 | 5212 | 5224 | 5236 | 1 | 2 | 4 | 5 | 6  | 7  | 8  | 10 | 11 |
| .72 | 5248 | 5260 | 5272 | 5284 | 5297 | 5309 | 5321 | 5333 | 5346 | 5358 | 1 | 2 | 4 | 5 | 6  | 7  | 9  | 10 | 11 |
| .73 | 5370 | 5383 | 5395 | 5408 | 5420 | 5433 | 5445 | 5458 | 5470 | 5483 | 1 | 3 | 4 | 5 | 6  | 8  | 9  | 10 | 11 |
| .74 | 5495 | 5508 | 5521 | 5534 | 5546 | 5559 | 5572 | 5585 | 5598 | 5610 | 1 | 3 | 4 | 5 | 6  | 8  | 9  | 10 | 12 |
| .75 | 5623 | 5636 | 5649 | 5662 | 5675 | 5689 | 5702 | 5715 | 5728 | 5741 | 1 | 3 | 4 | 5 | 7  | 8  | 9  | 10 | 12 |
| .76 | 5754 | 5768 | 5781 | 5794 | 5808 | 5821 | 5834 | 5848 | 5861 | 5875 | 1 | 3 | 4 | 5 | 7  | 8  | 9  | 11 | 12 |
| .77 | 5888 | 5902 | 5916 | 5929 | 5943 | 5957 | 5970 | 5984 | 5998 | 6012 | 1 | 3 | 4 | 5 | 7  | 8  | 10 | 11 | 12 |
| .78 | 6026 | 6039 | 6053 | 6067 | 6081 | 6095 | 6109 | 6124 | 6138 | 6152 | 1 | 3 | 4 | 6 | 7  | 8  | 10 | 11 | 13 |
| .79 | 6166 | 6180 | 6194 | 6209 | 6223 | 6237 | 6252 | 6266 | 6281 | 6295 | 1 | 3 | 4 | 6 | 7  | 9  | 10 | 11 | 13 |
| .80 | 6310 | 6324 | 6339 | 6353 | 6368 | 6383 | 6397 | 6412 | 6427 | 6442 | 1 | 3 | 4 | 6 | 7  | 9  | 10 | 12 | 13 |
| .81 | 6457 | 6471 | 6486 | 6501 | 6516 | 6531 | 6546 | 6561 | 6577 | 6592 | 2 | 3 | 5 | 6 | 8  | 9  | 11 | 12 | 14 |
| .82 | 6607 | 6622 | 6637 | 6653 | 6668 | 6683 | 6699 | 6714 | 6730 | 6745 | 2 | 3 | 5 | 6 | 8  | 9  | 11 | 12 | 14 |
| .83 | 6761 | 6776 | 6792 | 6808 | 6823 | 6839 | 6855 | 6871 | 6887 | 6902 | 2 | 3 | 5 | 6 | 8  | 9  | 11 | 13 | 14 |
| .84 | 6918 | 6934 | 6950 | 6966 | 6982 | 6998 | 7015 | 7031 | 7047 | 7063 | 2 | 3 | 5 | 6 | 8  | 10 | 11 | 13 | 15 |
| .85 | 7079 | 7096 | 7112 | 7129 | 7145 | 7161 | 7178 | 7194 | 7211 | 7228 | 2 | 3 | 5 | 7 | 8  | 10 | 12 | 13 | 15 |
| .86 | 7244 | 7261 | 7278 | 7295 | 7311 | 7328 | 7345 | 7362 | 7379 | 7396 | 2 | 3 | 5 | 7 | 8  | 10 | 12 | 13 | 15 |
| .87 | 7413 | 7430 | 7447 | 7464 | 7482 | 7499 | 7516 | 7534 | 7551 | 7568 | 2 | 3 | 5 | 7 | 9  | 10 | 12 | 14 | 16 |
| .88 | 7586 | 7603 | 7621 | 7638 | 7656 | 7674 | 7691 | 7709 | 7727 | 7745 | 2 | 4 | 5 | 7 | 9  | 11 | 12 | 14 | 16 |
| .89 | 7762 | 7780 | 7798 | 7816 | 7834 | 7852 | 7870 | 7889 | 7907 | 7925 | 2 | 4 | 5 | 7 | 9  | 11 | 13 | 14 | 16 |
| .90 | 7943 | 7962 | 7980 | 7998 | 8017 | 8035 | 8054 | 8072 | 8091 | 8110 | 2 | 4 | 6 | 7 | 9  | 11 | 13 | 15 | 17 |
| .91 | 8128 | 8147 | 8166 | 8185 | 8204 | 8222 | 8241 | 8260 | 8279 | 8299 | 2 | 4 | 6 | 8 | 9  | 11 | 13 | 15 | 17 |
| .92 | 8318 | 8337 | 8356 | 8375 | 8395 | 8414 | 8433 | 8453 | 8472 | 8492 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 15 | 17 |
| .93 | 8511 | 8531 | 8551 | 8570 | 8590 | 8610 | 8630 | 8650 | 8670 | 8690 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| .94 | 8710 | 8730 | 8750 | 8770 | 8790 | 8810 | 8831 | 8851 | 8872 | 8892 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| .95 | 8913 | 8933 | 8954 | 8974 | 8995 | 9016 | 9036 | 9057 | 9078 | 9099 | 2 | 4 | 6 | 8 | 10 | 12 | 15 | 17 | 19 |
| .96 | 9120 | 9141 | 9162 | 9183 | 9204 | 9226 | 9247 | 9268 | 9290 | 9311 | 2 | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |
| .97 | 9333 | 9354 | 9376 | 9397 | 9419 | 9441 | 9462 | 9484 | 9506 | 9528 | 2 | 4 | 7 | 9 | 11 | 13 | 15 | 17 | 20 |
| .98 | 9550 | 9572 | 9594 | 9616 | 9638 | 9661 | 9683 | 9705 | 9727 | 9750 | 2 | 4 | 7 | 9 | 11 | 13 | 16 | 18 | 20 |
| .99 | 9772 | 9795 | 9817 | 9840 | 9863 | 9886 | 9908 | 9931 | 9954 | 9977 | 2 | 5 | 7 | 9 | 11 | 14 | 16 | 18 | 20 |