03

Graphical Solution of Linear Equation

1. Equation of a straight line : General equation of a straight line in xy plane is given by $ax + by + c = 0$. In various situations its graph is drawn as follows :

Case (i) : When $a \neq 0, b \neq 0, c \neq 0$

This straight line intersects x–axis at $\left(-\frac{c}{a}, 0\right)$ and y–axis at $\left(0, -\frac{c}{b}\right)$

Explanation & Putting $y = 0$ in the equation $ax + by + c = 0$ we get $ax + c = 0$

or, $x = -\frac{c}{a}$ i.e. straight line $ax + by + c = 0$ cuts x–axis at $\left(-\frac{c}{a}, 0\right)$

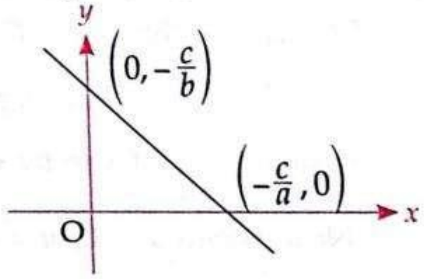
Again, putting $x = 0$ in the equation $ax + by + c = 0$, we get $by + c = 0$ or $y = -\frac{c}{b}$ i.e. y–axis intersects straight line $ax + by + c = 0$ at $\left(0, -\frac{c}{b}\right)$

It can be learned as follows, $ax + by + c = 0$

or, $ax + by = -c$

or, $\frac{ax}{-c} + \frac{by}{-c} = 1$

or, $\frac{x}{\left(-\frac{c}{a}\right)} + \frac{y}{\left(-\frac{c}{b}\right)} = 1$



This is known as intercept forms of a straight line. The terms in denominator are respectively known as x–intercept and y–intercept Clearly,

Length intercepted by the straight line $ax + by + c = 0$ between the axes is $\sqrt{\left(\frac{-c}{a}\right)^2 + \left(-\frac{c}{b}\right)^2}$. Here $\left(-\frac{c}{a}\right)$ and $\left(-\frac{c}{b}\right)$ are intercepts made by the line respectively on the x–axis and y–axis.

Case (ii) : When $a \neq 0, b \neq 0, c = 0$ i.e. equation of the straight line is $ax + by = 0$

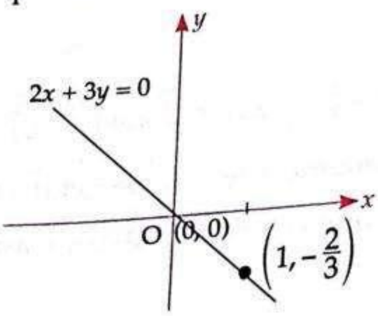
This straight line always passes through origin. If a and b are of opposite sign, it passes through first and third quadrant while when a and b are of same sign, it passes through second and fourth quadrant.

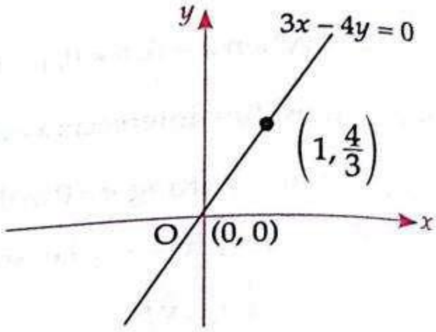
For example, Draw the graph of straight lines

(a) $2x + 3y = 0$ (b) $4x - 3y = 0$

Soln : For the straight line $2x + 3y = 0$

When $x = 0, y = 0$ and when $x = 1, y = -\frac{2}{3}$, thus the line passes through origin (0, 0) and another point $\left(1, -\frac{2}{3}\right)$. Clearly it lies in second and fourth quadrant.





For the straight line $4x - 3y = 0$

When $x = 0, y = 0$ and when $x = 1, y = \frac{4}{3}$ i.e. the straight line goes through origin (0, 0) and another point $\left(1, \frac{4}{3}\right)$. Clearly it lies in the first and third quadrant.

Case (iii) : When $a = 0, b \neq 0, c \neq 0$ i.e. equation of the line is $by + c = 0$.

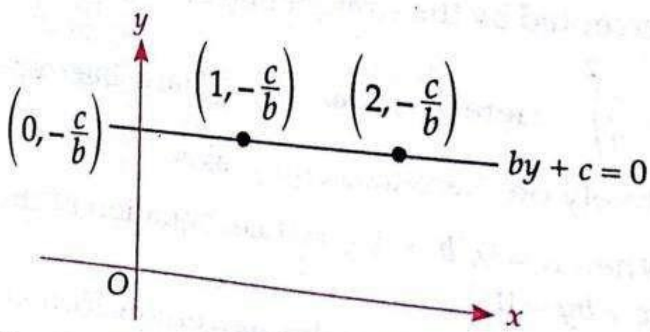
This line is parallel to x–axis and intersects y–axis at $\left(0, -\frac{c}{b}\right)$

Explanation : From $by + c = 0$, we have $y = -\frac{c}{b}$

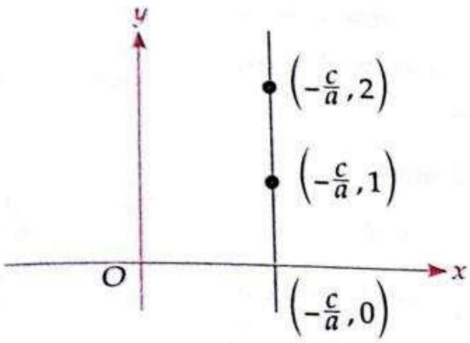
Now, when $x = 0, y = -\frac{c}{b}$, $x = 1, y = -\frac{c}{b}$

$x = 2, y = -\frac{c}{b}$ etcetera.

Thus this line passes through the points $\left(0, -\frac{c}{b}\right), \left(1, -\frac{c}{b}\right), \left(2, -\frac{c}{b}\right)$ etcetera. Clearly it is parallel to x–axis



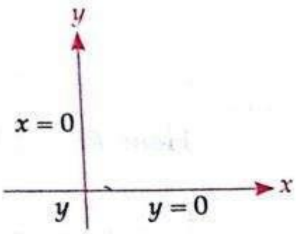
Case (iv) : When $a \neq 0, b = 0$ and $c \neq 0$ i.e. equation of straight line is of the form $ax + c = 0$



This line is parallel to y–axis and cuts x–axis at $\left(-\frac{c}{a},0\right)$.

2. Equation of axes : Equation of x–axis is $y = 0$ because y-coordinates of all the points lie on x-axis are zero.

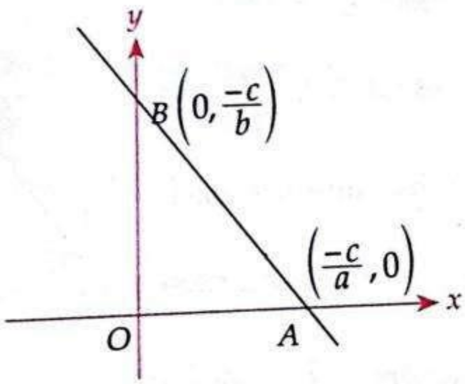
   Equation of y–axis is $x = 0$ because x-coordinates of all the points lie on y-axis are zero.



3. Point of intersection : To find the coordinates of point of intersection of two straight lines, solve their equations.

4. Area of triangle formed by straight lines

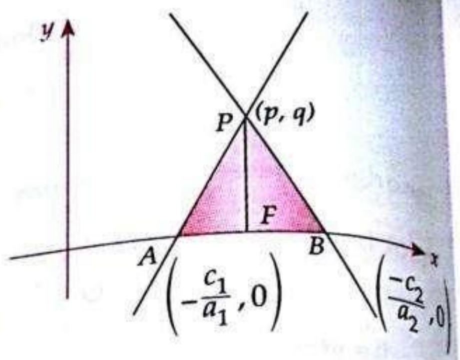
   4.1. Area of triangle formed by straight lines $ax + by + c = 0$, $a \neq 0$, $b \neq 0$, $c \neq 0$ with coordinate axes is $\left|\frac{1}{2}\frac{c^2}{ab}\right|$.



Explanation : Since straight lines $ax + by + c = 0$ cuts x–axis at $A\left(-\frac{c}{a},0\right)$ and cuts y–axis at $B\left(0,-\frac{c}{b}\right)$, then $OA = -\frac{c}{a}$ and $OB = -\frac{c}{b}$.

Hence, area of triangle formed by straight lines $ax + by + c = 0$ with x–axis and y–axis $= \left|\frac{1}{2}(OA)(OB)\right| = \left|\frac{1}{2}\left(\frac{-c}{a}\right)\left(\frac{-c}{b}\right)\right| = \left|\frac{1}{2}\frac{c^2}{ab}\right|$

4.2. Area of triangle formed by two straight lines either with x–axis or with y–axis :

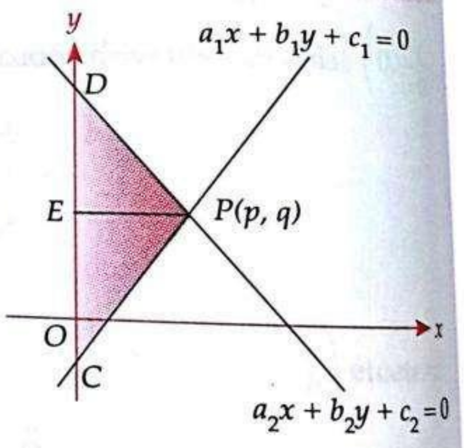


If two straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ intersect at point $P(p, q)$ then area of the triangle formed by these lines with x–axis $= \left|\frac{1}{2}(AB)(PF)\right|$

(see figure)

Here $PF = q$ is the y coordinate of point P and $A\left(\frac{-c_1}{a_1}, 0\right)$ and $B\left(\frac{-c_2}{a_2}, 0\right)$ are respectively point of intersection of given lines on the x–axis

Hence area of triangle formed by two straight lines with x–axis $= \frac{1}{2}$ (difference of x–intercept of the two lines) × (y coordinate of point of intersection of two lines)



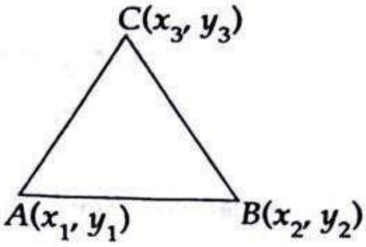
Similarly, straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ intersect y–axis respectively at $C\left(0, \frac{-c_2}{b_2}\right)$ and $D\left(0, \frac{-c_2}{b_2}\right)$ and if they intersect each other at $P(p, q)$ *[see figure]*, then

Area of triangle $CPD = \left|\frac{1}{2}(CD)(PE)\right|$

Here, $PE = p$ is the x–coordinate of point P.

Hence area of triangle formed by two straight lines with y–axis $= \frac{1}{2}$ (difference of y–intercept of two lines) × (x coordinate of point of intersection of two lines)

4.3. If three straight lines intersect each other at the points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ then from coordinate Geometry.

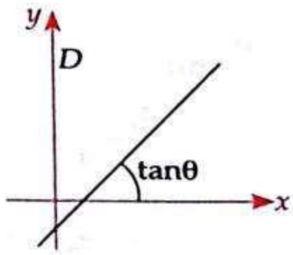


Area of ΔABC,

$$= \frac{1}{2}\left|x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right|$$

4.4. If $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$ then three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear.

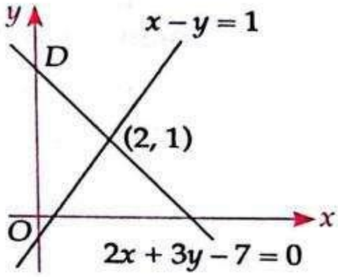
4.5. Slope of the line : If a straight line makes angle θ with x–axis in positive direction (anti-clockwise direction) then slope of the line is $\tan\theta$.



Slope of the straight line $ax + by + c = 0$ is $\frac{-a}{b}$

[For more details see exercise on coordinate Geometry]

5. Solution of corresponding equation of straight lines: If two straight lines intersect at a point then x–coordinate and y–coordinate of the point are called solution of equation of the straight lines.



For example, solving $2x + 3y - 7 = 0$ and $x - y = 1$, we get $x = 2, y = 1$. Hence two straight lines intersect at (2, 1).

6. Consistent and Inconsistent system of equations : A system of equation $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has

(i) a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) Infinitely many solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(iii) no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

to understand it clearly, consider the following examples :

6.1. Consider the system of equations $2x + 3y = 7$ and $3x - y = 5$

here, $a_1 = 2, b_1 = 3, c_1 = -7$

and $a_2 = 3, b_2 = -1, c_2 = -5$

$\because \quad \frac{a_1}{a_2} = \frac{2}{3}$ and $\frac{b_1}{b_2} = -3$

$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

So we can conclude that the system of equation has unique solution. Solving these equations we get $x = 2, y = 1$ (do yourself). Geometrically, the two lines of the system (having unique solution) intersect each other at a unique point.

6.2. Consider the system of equation $2x + y = 10$ and $4x + 2y = 20$

here, $a_1 = 2, b_1 = 1, c_1 = -10$

and, $a_2 = 4, b_2 = 2, c_2 = -20$

$\because \quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-10}{-20} = \frac{1}{2}$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

From this we can conclude that the system of equations has infinitely many solutions.

To solve the equation, multiply first equation by 2 and subtract second equation from it

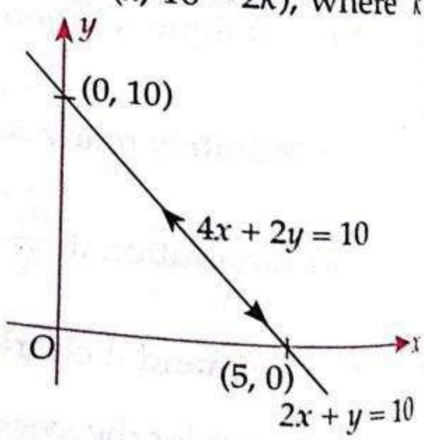
$$[2x + y = 10] \times 2$$
$$\underline{\underset{-}{4x} + \underset{-}{2y} = \underset{-}{20}}$$
$$0 = 0$$

here $0 = 0$ indicates that system of equations has infinitely many solutions. To find its solution, proceed as follows—

From first equation $y = 10 - 2x$ when $x = k, y = 10 - 2k$

Clearly $x = k, y = 10 - 2k$ also satisfy the second equation.

Hence, solution of the system of equation is $(k, 10 - 2k)$, where k is a real number. For each real value of k, the system has a solution. Putting $k = 1, 2, 3, 4,$ .... we get the solution as (1, 8), (2, 6), (3, 4), (4, 2)... etcetera, which are infinitely many is counting



Geometrically, these two lines are coincident. Both lines cut x–axis at (5, 0) and y–axis at (0, 10).

6.3. Consider the system of equations $2x + y = 6$ and $4x + 2y = 16$.

Here, $a_1 = 2, b_1 = 1, c_1 = -6$

$a_2 = 4, b_2 = 2, c_2 = -16$

$\because \quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-6}{-16} = \frac{3}{8}$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

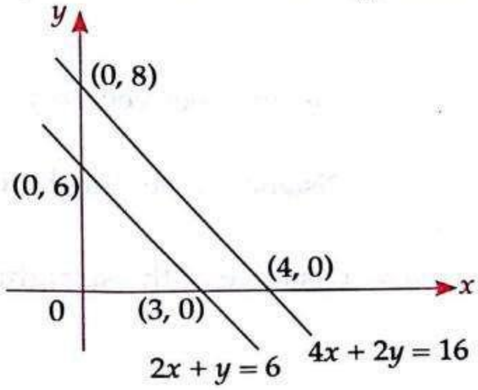
From this, we can conclude the system of equation has no solution.

Solving,

$$\begin{array}{r} 2x + y = 6 \,] \times 2 \\ \underline{\overset{-}{}4x \overset{-}{+} 2y \overset{-}{=} 16} \\ 0 = -2 \end{array}$$

$0 = -2$ indicates that solution is not possible. Geometrically, these two straight lines are parallel

[see figure]



The system of equations having solution is called **consistent** . It is of two types—

(i) Unique solution

(ii) Infinitely many solution

The system of equations having no solution is called **inconsistent**

Conclusion : For the system of equations $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Unique Solution	Consistent (independent)	Intersecting lines
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Infinitely many Solution	Consistent (dependent)	Coincident lines
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	No Solution	Inconsistent	Parallel lines

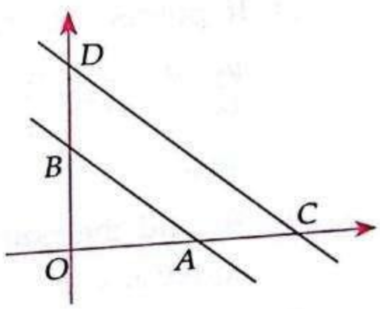
7. Area of trapezium between two parallel lines and axes :

Suppose $ax + by + c = 0$

and $ax + by + d = 0$ are two parallel lines.

First line cuts x–axis at A, y–axis at B while second line cuts x–axis at C and y–axis at D .

[see figure]



Hence, Area of trapezium $ACBD$

= area of ΔOCD – area of ΔOAB

$$= \frac{1}{2}\left|\frac{d^2}{ab}\right| - \frac{1}{2}\left|\frac{c^2}{ab}\right| = \frac{1}{2}\left(\left|\frac{d^2}{ab}\right| - \left|\frac{c^2}{ab}\right|\right)$$

Note : Donot write it as $\frac{1}{2}\left|\frac{d^2-c^2}{ab}\right|$. In the above figure, above fact ca also be used if AB and CD are not parallel.

8. Some important points about coordinate Geometry regarding straig lines :

8.1. Distance between two points (x_1, y_1) and (x_2, y_2)

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

8.2. Distance between origin and $(x, y) = \sqrt{x^2 + y^2}$

8.3. Distance of the straight line $ax + by + c = 0$ from origin $= \left|\frac{c}{\sqrt{a^2+b^2}}\right|$

8.4. Distance of the straight line $ax + by + c = 0$ from the point $(x_1, y_1$

$$= \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

8.5. If point P divides the line segment joining the points $(x_1, y$ and (x_2, y_2) in the ratio $m : n$ internally then coordinates of P a

$$\left(\frac{mx_2 + nx_1}{m-n}, \frac{my_2 + ny_1}{m-n}\right)$$

8.6. If point Q divides the line joining the points (x_1, y_1) and (x_2, y_2) the ratio $m : n$ externally then coordinates of Q are

$$\left(\frac{mx_2 - nx_1}{m+n}, \frac{my_2 - ny_1}{m+n}\right)$$

8.7. If P be the midpoint of line segment joining the points (x_1, y_1) a (x_2, y_2) then coordinates of P are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

8.8. If points $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$ ... are collinear th

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2} = \frac{y_4 - y_3}{x_4 - x_3}$ .... Here each term is slope of the strai line.

8.9. To find the point of intersection of two straight lines, solve th equations.

8.10. Equation of x–axis is $y = 0$ and Equation y–axis is $x = 0$.

8.11. Equation of a straight line parallel to x–axis is $y = c$. It cuts y–axis at $(0, c)$

8.12. Equation of a straight line parallel to y–axis is $x = k$. It cuts x–axis at $(k. 0)$.

8.13. Distance between two parallel lines $ax + by + c = 0$ and $ax + by + d = 0$ is equal to $= \left| \frac{d - c}{\sqrt{a^2 + b^2}} \right|$

8.14. If point (α, β) lies on the line $ax + by + c = 0$ then $a\alpha + b\beta + c = 0$

8.15. Equation of a straight line passing through points (x_1, y_1) and (x_2, y_2) is $y - y_1 \; \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

where, $m = \frac{y_2 - y_1}{x_2 - x_1}$ = slope of the line.

9. Definition of Modulus and its graph :

$|x|$ shows the absolute value of x, it is therefore $|4| = 4$ and $|-4| = 4$.

But it is incorrect to write $|x| = \pm x$. $|x|$ is defined as follows

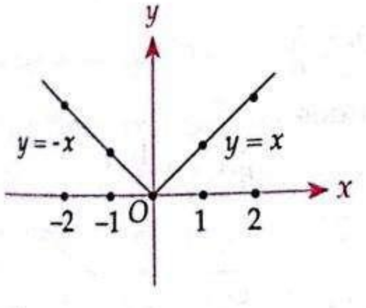
$$|x| = \begin{cases} x \text{ when } x \geq 0 \\ -x \text{ when } x < 0 \end{cases}$$

Hence, $|4| = 4$ and $|-4| = -(-4)$

Similarly $|x - 1| = \begin{cases} x - 1 \text{ when } x - 1 \geq 0 \\ -(x - 1) \text{ when } x - 1 < 0 \end{cases}$

or, $|x - 1| = \begin{cases} x - 1 \text{ when } x \geq 1 \\ 1 - x \text{ when } x < 1 \end{cases}$

Graph of $y = |x|$ is as follows :



x	-2	-1	0	1	2
y	2	1	0	1	2

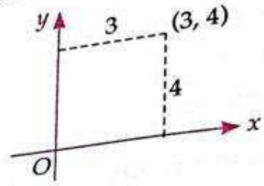
This graph contains two different lines (in fact rays). For $x \geq 0$, it shows $y = x$ and for $x < 0$ it shows $y = -x$

Solved Examples

1. Find the distance of point (3, 4) from (i) x–axis (ii) y–axis (iii) origin.

Solution : Point (3, 4) is at a distance of 4 unit from x–axis and at a distance of 3 unit from y–axis.

Its distance from origin = $\sqrt{3^2+4^2}=5$.



2. What is the distance between points (–2, 5) and (6, –1).

Solution : From coordinate Geometry, distance between

points (x_1, y_1) and $(x_2, y_2) = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ (Note it)

$\because$ Required distance = $\sqrt{(-2-6)^2+(5-(-1))^2} = \sqrt{64+36} = \sqrt{100} = 10$

3. Find the point where the straight line $2x-3y=12$ cuts x–axis and y–axis. Also find the length intercepted by the line between the axes.

Solution : $2x-3y=12$

or, $\frac{2x}{12}-\frac{3y}{12}=1$

or, $\frac{x}{6}+\frac{y}{(-4)}=1$

Thus straight line cuts x–axis at (6, 0) and y–axis at (0, –4)

Length intercepted between axes = $\sqrt{6^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

Second method : In the equation of line $2x-3y=12$

putting $y=0$, $2x=12 \Rightarrow x=6$

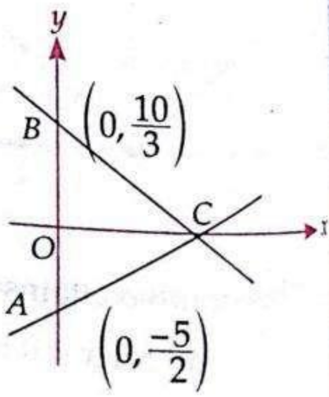
putting $x=0$ $-3y=12 \Rightarrow x=-4$

i.e. line cuts x–axis at (6, 0) and y–axis at (0, –4)

4. Find the area of triangle formed by lines $x-2y=5$ and $2x+3y=10$ with y–axis.

Solution : Solving equation $x-2y=5$ and $2x+3y=10$, $(x, y)=(5, 0)$

Let $C \equiv (5, 0)$



putting $x = 0$ in $x - 2y = 5$ we get $y = \frac{-5}{2}$ i.e.

first line cuts y–axis at $A\left(0, \frac{-5}{2}\right)$

putting $x = 0$ in $2x + 3y = 10$ we get $y = \frac{10}{3}$ i.e.

second line cuts y–axis at $B\left(0, \frac{10}{3}\right)$

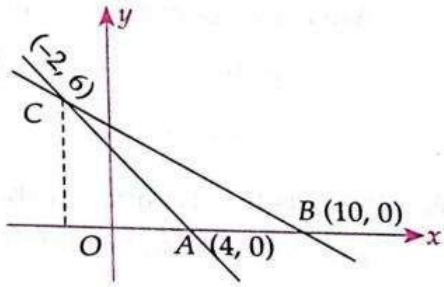
Hence, Area of $\Delta ABC = \frac{1}{2} \cdot AB \cdot OC$

$$= \frac{1}{2}\left(\frac{10}{3} - \left(\frac{-5}{2}\right)\right) \times 5 = \frac{1}{2}\left(\frac{20+15}{6}\right) \times 5 = \frac{175}{12}$$

5. Find the area of triangle formed by straight lines $x + y - 4 = 0, x + 2y - 10 = 0$ and $y = 0$

Solution : $y = 0$ represents x–axis.

Solving $x + y - 4 = 0$ and $x + 2y - 10$ we get $(x, y) = (-2, 6)$ i.e. two lines intersect at $C \equiv (-2, 6)$



putting $y = 0$ in $x + y - 4 = 0$ we get $x = 4$ i.e. first line cuts x–axis at $A(4, 0)$

putting $y = 0$ in $x + 2y - 10 = 0$ we get $x = 10$ i.e. second line cuts x–axis at $B(10, 0)$

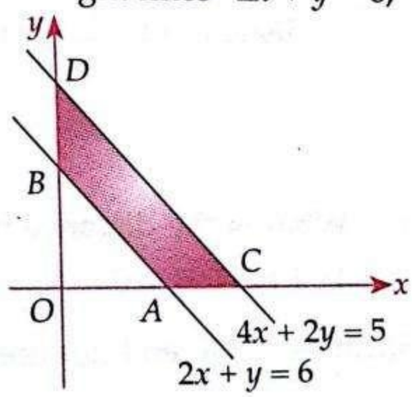
$\therefore$ Required area $= \frac{1}{2}$ × (difference of x–intercept of the two lines) × (y coordinate of point of intersection of two lines)

$$= \frac{1}{2}(10 - 4)\, 6 = \frac{1}{2} \times 6 \times 6 = 18 \text{ (unit)}^2$$

6. Find the area of quadrilateral intercepted by straight lines $2x + y = 6$, $4x + 2y = 25$ between the axes.

Solution : For the given lines $2x + y - 6 = 0$ and $4x + 2y - 25 = 0$, $\frac{2}{4} = \frac{1}{2} \neq \frac{-6}{25}$, thus two lines are parallel.

These two lines along with coordinate axes formed trapezium $ABCD$ (see fig.)



Area of trapezium $ABCD$ = area of ΔOCD – area of ΔOAB

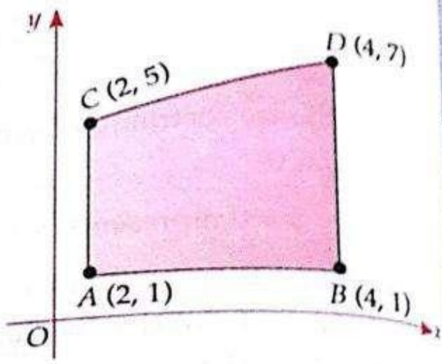
$$= \frac{1}{2}\left|\frac{c_2^2}{a_2 b_2}\right| - \frac{1}{2}\left|\frac{c_1^2}{a_1 b_1}\right| = \left|\frac{1}{2}\left(\frac{-25}{4 \times 2}\right)\right| - \frac{1}{2}\left|\frac{(-6)}{2 \times 1}\right|$$

$= \frac{25}{16} - \frac{6}{4} = \frac{25-24}{16} = \frac{1}{16}$ (unit)^2

(Recall that area of triangle formed by straight line $ax + by + c = 0$, with coordinate axes $= \frac{1}{2}\left|\frac{c^2}{ab}\right|$)

7. Find the area of quadritateral formed by joining points (2, 1), (4, 1), (2, 5) and (4, 7)

Solution : Let $A \equiv (2, 1)$, $B \equiv (4, 1)$, $C \equiv (2, 5)$ and $D \equiv (4, 7)$



Clearly (see fig.) AB is parallel to x-axis while AC and BD are parallel to y-axis.

So $ABDC$ is a trapezium.

Area of trapezium $ABCD$

$= \frac{1}{2}(AC + BD) \times AB = \frac{1}{2}(4+6) \times 2$

= 10 square unit

8. What is the distance of the line $3x - 4y + 15 = 0$ from origin ? What is its distance from point (–5, – 1).

Solution : Distance of the line $ax + by + c = 0$ from origin $= \left|\frac{c}{\sqrt{a^2+b^2}}\right|$

Thus, distance of $3x - 4y + 15 = 0$ from origin $= \left|\frac{15}{\sqrt{3^2+4^2}}\right| = \left|\frac{15}{5}\right| = 3$

distance of point (x_1, y_1) from $ax + by + c = 0$ is $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$

$\therefore$ distance of (–5, –1) from $3x - 4y + 15 = 0$ is $\left|\frac{3(-5) - 4(-1) + 15}{\sqrt{3^2+4^2}}\right|$

$= \frac{4}{5}$ unit.

9. What is the distance between two parallel lines $2x + 3y + 13 = 0$ and $4x + 6y - 91 = 0$

Solution : Given lines are

$2x + 3y + 13 = 0$ and $4x + 6y - 91 = 0$

or, $4x + 6y + 26 = 0$ and $4x + 6y - 91 = 0$

(*Make the coefficients of x and y same*

Since, distance between $ax + by + c = 0$ and $ax + by + d = 0$ is $\left|\frac{d-c}{\sqrt{a^2+b^2}}\right|$

$\therefore$ Required distance $= \left|\frac{-91-26}{\sqrt{4^2+6^2}}\right| = \left|\frac{-117}{\sqrt{52}}\right| = \frac{13\times 9}{2\sqrt{13}} = \frac{9\sqrt{13}}{2}$ unit

10. For what values of a and b points $(1, 1)$, $(2, 3)$, $(3, a)$ and $(b, 7)$ are collinear.

Solution : Points (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) are collinear i.e. lie on a straight line if $\frac{y_2-y_1}{x_2-x_1} = \frac{y_3-y_2}{x_3-x_2} = \frac{y_4-y_3}{x_4-x_3}$

$\therefore$ points $(1, 1)$, $(2, 3)$, $(3, a)$ and $(b, 7)$ are on a straight line if

$$\frac{3-1}{2-1} = \frac{a-3}{3-2} = \frac{7-a}{b-3}$$

or, $2 = a - 3 = \frac{7-a}{b-3}$

from first two relations $a - 3 = 2 \Rightarrow a = 5$

from first and third relations $\frac{7-a}{b-3} = 2$

or, $\frac{7-5}{b-3} = 2$ $\quad (\because a = 5)$

or, $2 = 2(b-3)$

or, $1 = b - 3$

or, $b = 4$

11. For what value of k given system of equations has infinitely many solution.

or, For what value of k following equations show coincident lines ?

or, For what value of k given system of equation is dependent ?

$kx + 3y = k - 3$, $12x + ky = k$

Solution : Given equations are

$$kx + 3y - (k-3) = 0 \text{ and } 12x + ky - k = 0$$

here, $a_1 = k$, $b_1 = 3$, $c_1 = -(k-3)$ and $a_2 = 12$, $b_2 = k$, $c_2 = -k$

$\therefore$ from, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{k}{12} = \frac{3}{k} = \frac{-(k-3)}{-k}$$

or, $\frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$

from first two ratios, $\frac{k}{12} = \frac{3}{k}$

or, $k^2 = 36$ $\quad \ldots$ (i)

or, $k = \pm 6$

from last two ratios, $\frac{3}{k} = \frac{k-3}{k}$

or, $3k = k^2 - 3k$

or, $k^2 - 6k = 0$

or, $k = 0, 6$

from (i) and (ii) we get that common value of k is 6.

Thus for $k = 6$, given system of equation has infinitely many solution

12. Find the value of a and b so that following system of equation has infinitely many solutions

$$2x - (a-4)y = 2b + 1$$

$$4x - (a-1)y = 5b - 1$$

Solution : System of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has infinitely many solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Given equations are $2x - (a-4)y - (2b+1) = 0$

and $4x - (a-1)y - (5b-1) = 0$

here $a_1 = 2, b_1 = -(a-4), c_1 = -(2b+1)$

and $a_2 = 4, b_2 = -(a-1), c_2 = -(5b-1)$

Hence from (i), $\frac{2}{4} = \frac{-(a-4)}{-(a-1)} = \frac{-(2b+1)}{-(5b-1)}$

or, $\frac{1}{2} = \frac{a-4}{a-1} = \frac{2b+1}{5b-1}$

from first two ratio, $\frac{1}{2} = \frac{a-4}{a-1}$

or, $a - 1 = 2a - 8$

or, $8 - 1 = 2a - a$

or, $7 = a$

or, $a = 7$

from first and third ratio, $\frac{1}{2} = \frac{2b+1}{5b-1}$

or, $5b - 1 = 4b + 2$

or, $5b - 4b = 1 + 2$

or, $b = 3$

Hence, $a = 7,\ b = 3$

13. For what value of k given pair of equation has no solution ?

or, For what value of k given lines are parallel ?

or, For what value of k given system of equation is inconsistent ?

$kx + 3y = 3,\ 12x + ky = 6$

Solution : Pair of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ does not have a solution if

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. In this situation lines are parallel

Given equations are $kx + 3y - 3 = 0$ and $12x + ky - 6 = 0$

here, $a_1 = k, b_1 = 3, c_1 = -3$ and $a_2 = 12, b_2 = k, c_2 = -6$

$\therefore$ from, $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ we get $\frac{k}{12} = \frac{3}{k}$

or, $k^2 = 12 \times 3 = 36$

or, $k = \pm 6$

taking $k = 6$, $\frac{a_1}{a_2} = \frac{6}{12} = \frac{1}{2} = \frac{b_1}{b_2}$

taking $k = -6$, $\frac{a_1}{a_2} = \frac{-6}{12} = \frac{-1}{2} = \frac{b_1}{b_2}$

also $\frac{c_1}{c_2} = \frac{-3}{-6} = \frac{1}{2}$

clearly at $k = 6$, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore$ at $k = 6$ pair of equation has infinitely many solution.

Again at $k = -6$, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ at $k = -6$ pair of equation has no solution

$\therefore$ at $k = -6$ will be the required solution.

14. For what value of k given system of equation has unique solution.

or, For what value of k given lines are intersecting.

or, For what value of k pair of equation has independent solution ?

(i) $kx + 2y = 5,\ 3x + y = 1$ (ii) $kx + 3y = 6,\ 2x - ky = 10$

(iii) $2x + y = 5,\ 6x + 3y = 2k$

Solution : System of equation $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has a unique solution

if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

In this situation corresponding straight lines intersect each other and solution is consistant (independent)

(i) Given equations are

$kx + 2y - 5 = 0$ and $3x + y - 1 = 0$

here $a_1 = k, b_1 = 2, c_1 = -5$ and $a_2 = 3, b_1 = 1, c_2 = -1$

$\therefore$ from $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ from $\frac{k}{3} \neq \frac{2}{1}$

or, $k \neq 6$

(ii) Given equations are

$kx + 3y - 6 = 0$ and $2x - ky - 10 = 0$

here, $a_1 = k, b_1 = 3, c_1 = -6$ and $a_2 = 2, b_2 = -k, c_2 = -10$

$\therefore$ from $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}, \frac{k}{2} = \frac{3}{-k}$

or, $k^2 \neq -6$ which is true for all values of k.

Hence for any real value of k system of equation has unique solution

(iii) Given equations are

$2x + y - 5 = 0$ and $6x + 3y - 2k = 0$

here, $\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$ and $\frac{b_1}{b_2} = \frac{1}{3}$

$\because \frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\therefore$ there is no real value of k for which system of equation has unique solution

15. Find the area of region bounded by straight lines $|x| + |y| = k$

Solution : Given equation is $|x| + |y| = k$

when $x \geq 0, y \geq 0, x + y = k$ ...(i)

$x \leq 0, y \geq 0, -x + y = k$ ...(ii)

$x \leq 0, y \leq 0, -x - y = k$ ...(iii)

$x \geq 0, y \leq 0, x - y = k$ ...(iv)

line (i) lies in the first quadrant and cuts axes respectively at (k, o) and (o, k)

line (ii) line in the second quadrant and cuts axes respectively at $(-k, o)$ and (o, k)

line (iii) l...
and $(o, -$...
line (iv) ...
and $(o, -$...
Hence, ...

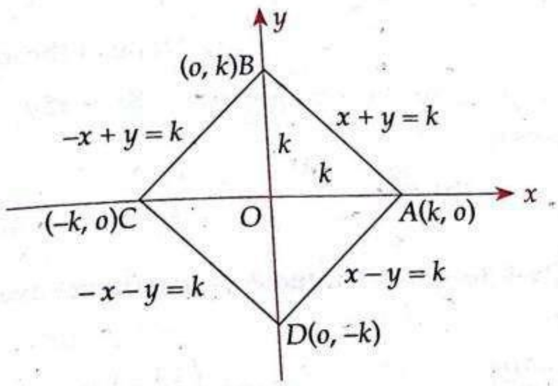
Clearly...
Hence...
Trick :

1. What...
(a) 3
2. If poi...
from...
(a)
3. Wha...
and...
(a)
4. Wha...
and...
(a)
5. Area...
is...
(a)

line (iii) line in the third quadrant and cuts axes respectively at $(-k, o)$ and $(o, -k)$

line (iv) line in the fourth quadrant and cuts axes respectively at (k, o) and $(o, -k)$

Hence, graph of area enclosed by these lines are as follows



Clearly it is a square with each side $= \sqrt{k^2 + k^2} = \sqrt{2}\,k$

Hence, Required Area $= \left(\sqrt{2}\,k\right)^2 = 2k^2$

Trick : Area enclosed by $|x| + |y| = k$ is $2k^2$.

Exercise—3A

1. What is the distance of point $(3, 4)$ from the x–axis ?

   (a) 3 (b) 4 (c) 5 (d) $\sqrt{5}$

2. If point $(a, a + 2)$ lies on the line $y = 3x + 5$, then distance of the point from y–axis is

   (a) $\frac{3}{2}$ (b) 3 (c) $\frac{1}{2}$ (d) $\frac{7}{2}$

3. What is the distance of point of intersection of straight lines $2x + 3y = 6$ and $y = x + 7$ from origin ?

   (a) 7 (b) 3 (c) 4 (d) 5

4. What is the difference between distances of mid point of points $(-3, 5)$ and $(7, -6)$ from x–axis and y–axis ?

   (a) $\frac{5}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{4}$ (d) $\frac{3}{4}$

5. Area of triangle formed by coordinate axes and straight line $y = 3x - 14$ is

   (a) $\frac{196}{3}$ (b) $\frac{49}{3}$ (c) $\frac{98}{3}$ (d) $\frac{3}{98}$

6. The length intercepted by the straight line $12x - 9y = 108$ between the coordinate axes is
(a) 12 unit (b) 18 unit (c) 15 unit (d) 9 unit

7. The length intercepted by the straight line $y = mx + c$ between the coordinate axes is
(a) $\frac{c}{m}\sqrt{1+m^2}$ (b) $\frac{c}{m}$
(c) $\sqrt{c^2+m^2}$ (d) None of these

8. Length intercepted by the straight line $8x - 15y = 60$ between the coordinate axes is
(a) $\frac{23}{2}$ (b) $\frac{23}{4}$ (c) $\frac{17}{2}$ (d) $\frac{17}{4}$
[SSC Tier-I 2014]

9. Straight line $2x + 3y + 10 = 0$ intersects coordinate axes respectively on the points
(a) $(-5,0)\left(0,\frac{-10}{3}\right)$ (b) $\left(\frac{10}{3},0\right)$ (0, 5)
(c) (–5, 0), $\left(0,\frac{10}{3}\right)$ (d) $\left(\frac{-10}{3},0\right),\left(0,\frac{-5}{3}\right)$

10. Equation of the straight lines passing through points (4, 3) and respectively parallel to x–axis and y–axis are
(a) $x = 4, y = 3$ (b) $x = 3, y = 4$
(c) $x + 4 = 0, y + 3 = 0$ (d) $x + 3 = 0, y + 4 = 0$

11. Equation of straight line passing through the points (2, 0) and (0, –3) is
(a) $\frac{x}{2}-\frac{y}{3}=1$ (b) $\frac{y}{3}-\frac{x}{2}=1$ (c) $\frac{x}{3}-\frac{y}{2}=1$ (d) $\frac{y}{2}-\frac{x}{3}=1$

12. Area of triangle between the straight line $3x + 2y - 6 = 0$ and coordinate axes is ... square unit.
(a) 3 (b) $\frac{3}{2}$ (c) 6 (d) $\frac{9}{2}$

13. Area of triangle formed by the straight line $8x - 3y + 24 = 0$ and coordinate axes is
(a) 24 sq. unit (b) 12 sq. unit (c) 6 sq. unit (d) 18 sq. unit

14. Area of triangle formed by straight line $y = mx + c$ with coordinate axes is
(a) $\frac{c^2}{m}$ (b) $\frac{m}{c^2}$ (c) $\frac{c^2}{2m}$ (d) $\frac{2m}{c^2}$

15. Area of triangle formed by straight lines $2x + 3y = 5$ and $y = 3x - 13$ with x–axis is
(a) $\frac{11}{3}$ sq. unit (b) $\frac{22}{3}$ sq. unit (c) $\frac{11}{6}$ sq. unit (d) $\frac{11}{12}$ sq. unit

6. Area of triangle formed by straight lines $4x - 3y + 4 = 0$, $4x + 3y - 20 = 0$ and x–axis is

(a) 3 sq. unit (b) 6 sq. unit (c) 12 sq. unit (d) 24 sq. unit

7. Area of triangle formed by straight lines $3x - y = 3$, $x - 2y + 4 = 0$ and $y = 0$ is

(a) $\frac{15}{4}$ sq. unit (b) $\frac{15}{2}$ sq. unit

(c) 15 sq. unit (d) Cannot be determined

8. Area of triangle formed by straight lines $4x - y = 4$, $3x + 2y = 14$ and y–axis is

(a) $\frac{11}{2}$ sq. unit (b) $\frac{11}{4}$ sq. unit (c) 22 sq. unit (d) 11 sq. unit

9. Ratio of area of triangle formed by straight lines $2x + 3y = 4$ and $3x - y + 5 = 0$ with x–axis and y–axis is

(a) 1 : 2 (b) 2 : 1

(c) 4 : 1 (d) None of these

0. What is the height of triangle formed by straight lines $3x + y = 10$, $2x - 3y = 6$ and x–axis when x–axis is the base of the triangle ?

(a) 3 (b) 1 (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

1. Area of triangle formed by straight lines $2x - 3y + 6 = 0$, $2x + 3y - 18 = 0$ and $y - 1 = 0$ is

(a) 27 sq. unit (b) $\frac{27}{2}$ sq. unit

(c) 9 sq. unit (d) None of these

2. Area of triangle formed by straight lines $x + y = 4$, $2x - y = 2$ and $x - 2 = 0$ is

(a) 4 sq. unit (b) 9 sq. unit

(c) $\frac{7}{2}$ sq. unit (d) None of these

3. Area of quadrilateral formed by straight lines $x + y = 2$, $3x + 4y = 24$ and coordinate axes is

(a) 22 sq. unit (b) 26 sq. unit (c) 44 sq. unit (d) 11 sq. unit

4. A linear equation $3x + 4y = 24$, intersects x–axis and y–axis respectively at points A and B. If $P(2, 0)$ and $Q\left(0, \frac{3}{2}\right)$ respectively lies on the straight line OA and OB, then area of the quadrilateral $PABQ$ is

(a) $\frac{5}{2}$ sq. unit (b) $\frac{15}{2}$ sq. unit (c) $\frac{35}{2}$ sq. unit (d) $\frac{45}{2}$ sq. unit

25. Area of triangle formed by straight lines $3x - 4y = 0$, $x = 4$ and x-axis
(a) 6 sq. unit (b) 8 sq. unit (c) 12 sq. unit (d) 16 sq. unit

26. If lines are drawn from the point (–5, 3) to the coordinate axes then area of quadrilateral formed by these lines with coordinate axes is
(a) $\frac{15}{2}$ (b) 15 (c) 30 (d) $\frac{25}{2}$

27. If $b > a$, $d > c$ then area of quadrilateral formed by straight lines $x = a$, $x = b$, $y = c$ and $y = d$ is
(a) $(b - a)(d - c)$
(b) $\frac{1}{2}(b - a)(d - c)$
(c) $(b + a)(d + c)$
(d) $\frac{1}{2}(b + a)(d + c)$

28. Area of quadrilateral formed by straight lines $2x = -5$, $2y = 3$, $x + 1 = 0$ and $y + 2 = 0$ is
(a) $\frac{21}{2}$ sq. unit (b) $\frac{21}{4}$ sq. unit (c) $\frac{21}{8}$ sq. unit (d) $\frac{21}{16}$ sq. unit

29. Area of triangle formed by straight lines $3x + 4y = 24$, $x = 8$ and $y = 6$
(a) 12 sq. unit
(b) 24 sq. unit
(c) 48 sq. unit
(d) None of these

30. Area of quadrilateral formed by straight lines $x = 1$, $x = 3$, $y = 2$ and $y = x + 3$ is
(a) 6 sq. unit
(b) 12 sq. unit
(c) 3 sq. unit
(d) None of these

31. Area of triangle formed by straight lines $x - y = 0$, $x + 2y = 0$ and $y = 3$
(a) 27 sq. unit (b) 54 sq. unit (c) 9 sq. unit (d) 13.5 sq. unit

32. Area of triangle formed by straight lines $x - y = 0$, $x + y = 0$ and $2x = 5$
(a) 25
(b) $\frac{25}{2}$
(c) $\frac{25}{4}$
(d) None of these

33. Area enclosed by equation $y = |x| - 5$ with x–axis is
(a) 25 sq. unit
(b) 12.5 sq. unit
(c) 50 sq. unit
(d) None of these

34. Area enclosed by the equation $|x| + |y| = 4$ is
(a) 16 (b) 32 (c) 24 (d) 48

35. Area enclosed by equation $y = |x| - 1$ and $y = 1 - |x|$ is
(a) 2 (b) 4 (c) 8 (d) 16

36. Which of the following system of equations has unique solutions?
(a) $3x + 4y = 11$, $6x + 8y = 15$
(b) $x + 2y = 3$, $2x + 4y = 7$
(c) ...
(d) $4x + 3y = 5$, $4x - 3y = 5$

37. Which of the following system of equations has infinitely many solutions?
(a) $2x + 3y = 12, 4x + 6y = 12$
(b) $x - 3y = 10, 2x - 6y = 20$
(c) $x = 4, y = 3$
(d) $3x - 4y = 0, 3x + 4y = 0$

38. Which of the following system of equations doesnot have a solution ?
(a) $2x = 3y, 4x = 5y$
(b) $2x + y = 7, 4x + 2y = 14$
(c) $3x - 4y = 8, 3x - 4y = 12$
(d) $4x + y = 7, 4y + x = 7$

39. For what value of k system of equations $3x + 4y = 19, y - x = 3$ and $2x + 3y = k$ has a solution ?
(a) 11
(b) −11
(c) 14
(d) −14

40. Which of the following pair represent equation of parallel straight lines.
(a) $2x + 3y = 4, 4x + 6y = 9$
(b) $x + 2y = 4, 2x + y = 4$
(c) $y = 3x + 5, x = 3y + 5$
(d) None of these

41. Which of the following pair of straight lines donot represent intersecting lines ?
(a) $y = \frac{x}{3} + \frac{5}{4}, y = \frac{x}{2} + \frac{7}{3}$
(b) $3x - 4y = 0, x = 0$
(c) $4x + 3y = 1, y = 0$
(d) $2x + 3y = 7, 4x + 6y = 15$

42. The value of K for which system of equation $5x + 2y = K$ and $10x + 4y - 3 = 0$ has infinitely many solution is
(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) 6
(d) $\frac{1}{6}$

43. For what value of K system of equation $x + 3y = K$ and $2x + 6y = 2K$ has infinitely many solution ?
(a) $K = 1$
(b) $K = 2$
(c) for all real values of K
(d) for no real value of K

44. Values of a and b so that system of equations $2x + 3y = 7$ and $2ax + (a + b)\,y = 28$ has infinitely many solutions are
(a) $a = 4, \; b = 8$
(b) $a = 8, \; b = 4$
(c) $a = -4, b = -8$
(d) $a = -8, b = -4$

45. For what values of k straight lines $2x - ky + 3 = 0$ and $3x + 2y - 1 = 0$ are parallel ?
(a) $\frac{4}{3}$
(b) $\frac{-4}{3}$
(c) $\frac{2}{3}$
(d) $\frac{-2}{3}$

46. Value of k for which system of equations $kx + 2y = 5, 3x + y = 1$ has unique solution is
(a) $k = 1$
(b) $k = 2$
(c) $k = 3$
(d) all are true

Answers—3A

1. (b)	2. (a)	3. (d)	4. (b)	5. (c)	6. (c)	7. (a)	8. (c)
9. (a)	10. (b)	11. (a)	12. (a)	13. (b)	14. (c)	15. (d)	16. (c)
17. (b)	18. (d)	19. (b)	20. (b)	21. (b)	22. (d)	23. (a)	24. (d)
25. (a)	26. (b)	27. (a)	28. (c)	29. (c)	30. (a)	31. (d)	32. (c)
33. (a)	34. (b)	35. (a)	36. (d)	37. (b)	38. (c)	39. (c)	40. (a)
41. (d)	42. (c)	43. (c)	44. (a)	45. (b)	46. (d)		

Explanation

2. (a) $\because$ $(a, a+2)$, passes through line $y = 3x + 5$

$\therefore$ $a + 2 = 3a + 5$

or, $a = \frac{-3}{2}$

$\therefore$ distance from y–axis $= |a| = \left|\frac{-3}{2}\right| = \frac{3}{2}$

3. (d) solving $2x + 3y = 6$

and $y = x + 7$ we get $(x, y) = (-3, 4)$

$\therefore$ distance from origin $= \sqrt{(-3)^2 + 4^2} = 5$

4. (b) Mid point of $(-3, 5)$ and $(7, -6) = \left(\frac{-3+7}{2}, \frac{5-6}{2}\right) = \left(2, \frac{-1}{2}\right)$

distance of point $\left(2, \frac{-1}{2}\right)$ from x–axis $= \left|-\frac{1}{2}\right| = \frac{1}{2}$

distance of point $\left(2, \frac{-1}{2}\right)$ from y–axis $= 2$

Required difference $= \left|\frac{1}{2} - 2\right| = \frac{3}{2}$

5. (c) Equation of line is $3x - y - 14 = 0$

$\therefore$ Required area $= \frac{1}{2}\left|\frac{c^2}{ab}\right| = \frac{1}{2}\left|\frac{(-14)^2}{3(-1)}\right| = \frac{98}{3}$ square unit.

6. (c) Required intercept $= \sqrt{\left(\frac{c}{a}\right)^2 + \left(\frac{c}{b}\right)^2} = \sqrt{\left(\frac{108}{12}\right)^2 + \left(\frac{108}{9}\right)^2}$

$= \sqrt{81 + 144} = \sqrt{225} = 15$

7. (a) Required length $= \sqrt{\left(\frac{c}{m}\right)^2 + \left(\frac{c}{1}\right)^2} = \frac{c}{m}\sqrt{1 + m^2}$

9. (c) Length of intercept made by line $ax + by + c = 0$

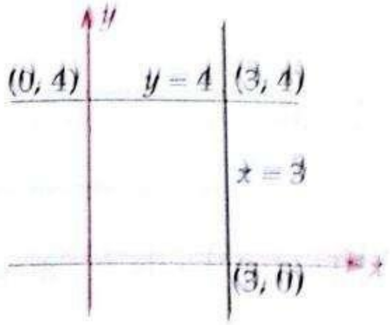
between axes $= \sqrt{\frac{c^2}{a^2} + \frac{c^2}{b^2}}$

$\therefore$ Required length $= \sqrt{\left(\frac{60}{8}\right)^2 + \left(\frac{60}{15}\right)^2}$

$$= 60\sqrt{\left(\frac{1}{8^2} + \frac{1}{15^2}\right)}$$

$$= 60\sqrt{\left(\frac{15^2 + 8^2}{8^2 \times 15^2}\right)} = 60 \times \frac{17}{8 \times 15} = \frac{17}{2} \text{ sq. unit}$$

10. (b) See the figure, solution is obvious



11. (a) If a line cuts x-axis at $(a, 0)$ and cuts y-axis at $(0, b)$ then its equation is $\frac{x}{a} + \frac{y}{b} = 1$

here $a = 2$ and $b = -3$

12. (a) Required Area $= \frac{1}{2}\left|\frac{c^2}{ab}\right| = \frac{1}{2}\left|\frac{6^2}{3 \times 2}\right| = \frac{36}{2 \times 3 \times 2} = 3$ square unit

13. (b) Required Area $= \frac{1}{2}\left|\frac{c^2}{ab}\right| = \frac{1}{2}\left|\frac{24 \times 24}{8(-3)}\right| = 12$ square unit

14. (c) Required Area $= \frac{1}{2}\left|\frac{c^2}{1 \times m}\right| = \frac{c^2}{2m}$

15. (d) Intercept made by $2x + 3y = 5$ on x-axis $= \frac{5}{2}$ (put $y = 0$)

Intercept made by $3x - y = 13$ on x-axis $= \frac{13}{3}$ (put $y = 0$)

(Solving both equations, point of intersection is $(4, -1)$)

$\therefore$ Required Area $= \frac{1}{2}$ (difference between x-intercept)

$\times$ (y-coordinate of point of intersection)

$$= \frac{1}{2}\left|\left(\frac{13}{3} - \frac{5}{2}\right)(-1)\right| = \frac{11}{12}$$

15

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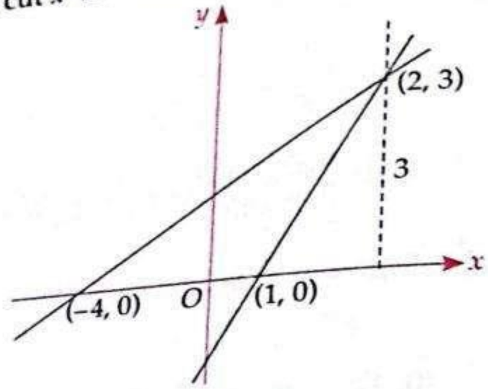
16. (c) Required Area $= \frac{1}{2}\,|5-(-1)| \times 4 = 12$ square unit

17. (b) Solving two given lines $x = 2, y = 3$

Both lines cut x–intercept respectively at $(-4, 0)$ and $(1, 0)$



$\therefore$ Required Area $= \frac{1}{2}$ (difference between x–intercept)

$\times$ (y–coordinate of point of intersection)

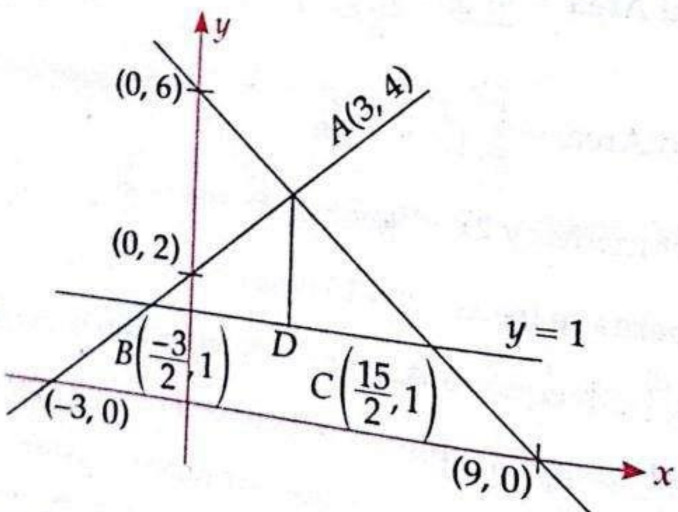
$$= \frac{1}{2}\left|\left(1-(-4)\right)\times 3\right| = \frac{15}{2} \text{ square unit}$$

18. (d) Required Area $= \frac{1}{2}\,|-4-7| \times 2 = 11$ square unit

(b) Required ratio $= \dfrac{\frac{1}{2}\left|\left(2-\left(\frac{-5}{3}\right)\right)2\right|}{\frac{1}{2}\left|\left(5-\frac{4}{3}\right)(-1)\right|} = \dfrac{\frac{22}{3}}{\frac{11}{3}} = \frac{22}{11} = \frac{2}{1}$

21. (b) Lines $2x - 3y + 6 = 0$ and $2x + 3y - 18 = 0$

intersect at $A(3, 4)$



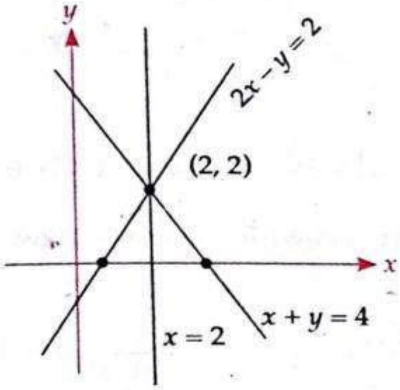
Lines $y - 1 = 0$ and $2x - 3y + 6 = 0$ cuts at $B\left(-\frac{3}{2}, 1\right)$

Lines $y-1=0$ and $2x+3y=18$ cuts at C $\left(\frac{15}{2}, 1\right)$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

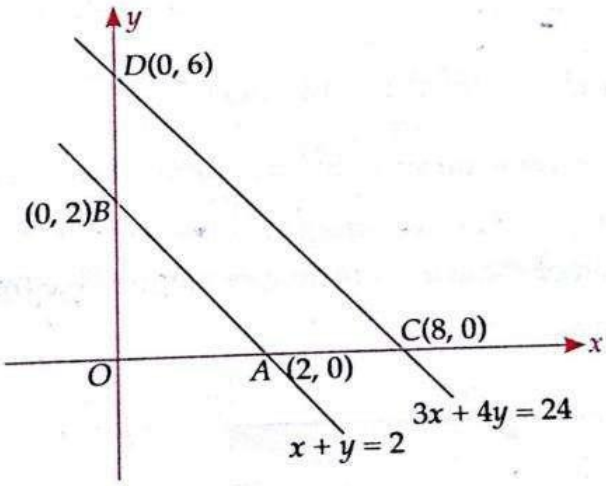
$$= \frac{1}{2}\left|\frac{15}{2}-\left(\frac{-3}{2}\right)\right| \times |4-1| = \frac{1}{2}\times 9 \times 3 = \frac{27}{2} \text{ square unit}$$

22. (d) Solve the equations taking two at a time.



In each case $x=2$, $y=2$ i.e. lines are concurrent, so donot make a triangle.

23. (a) Required Area = Area of = $\Delta OCD \sim$ area of ΔOAB (see figure)



$$= \frac{1}{2}\times 8 \times 6 - \frac{1}{2}\times 2\times 2 = 24-2 = 22 \text{ square unit}$$

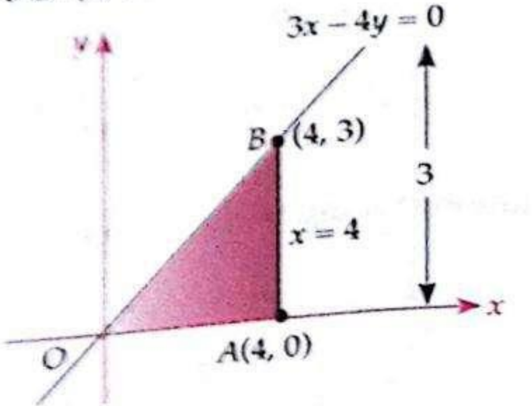
24. (d) Area of quadrilateral $PABQ$ = Area of ΔOAB – area of ΔOPQ

$$= \frac{1}{2}\cdot\left|\frac{c^2}{ab}\right| - \frac{1}{2}\cdot OP\cdot OQ$$

$$= \frac{1}{2}\cdot\left|\frac{24^2}{3\times 4}\right| - \frac{1}{2}\cdot 2\cdot\frac{3}{2}$$

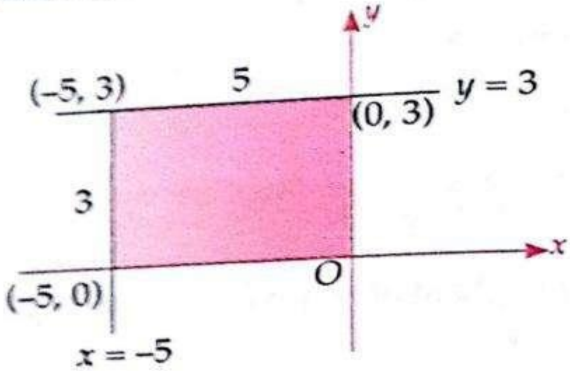
$$= 24 - \frac{3}{2} = \frac{45}{2} \text{ square unit}$$

25. (a) Required Area is show in figure. Point of intersection of line $x = 4$ and $3x - 4y = 0$ is (4, 3)



Required Area $= \frac{1}{2} \times OA \times AB = \frac{1}{2} \times 4 \times 3 = 6$ square unit

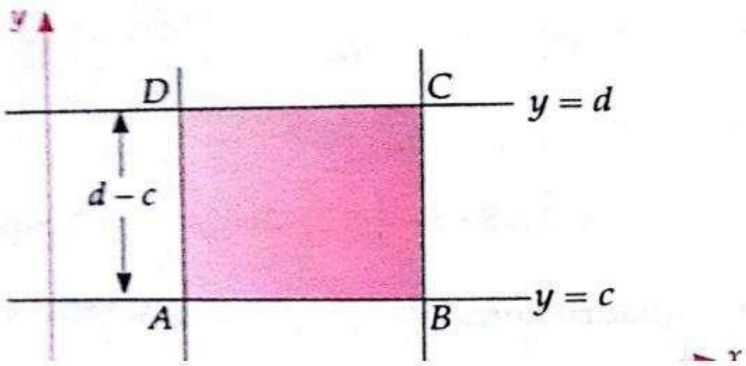
26. (b) See the figure, Area enclosed is a rectangle



Required Area $= |(-5)3| = 15$ square unit

27. (a) $x = a$ and $x = b$, y are straight lines parallel to y–axis.

$y = c$ and $y = d$, x are straight lines parallel to x–axis. Point of intersection of these lines form a rectangle (see figure)

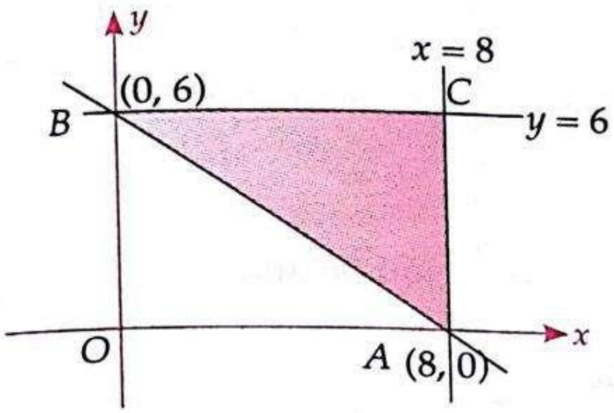


28. (c) Given lines are $x = \frac{-5}{2}$, $y = \frac{3}{2}$, $x = -1$ and $y = -2$

According to above question Required Area $= \frac{1}{2}(b-a)(c-d)$

$$= \frac{1}{2}\left|\left(-1+\frac{5}{2}\right)\left(\frac{3}{2}+2\right)\right| = \frac{1}{2}\times\frac{3}{2}\times\frac{7}{2} = \frac{21}{8} \text{ square unit}$$

29. (c) Line is $3x + 4y = 24$



or, $\frac{3x}{24}+\frac{4y}{24} = 1$

or, $\frac{x}{8}+\frac{y}{6} = 1$

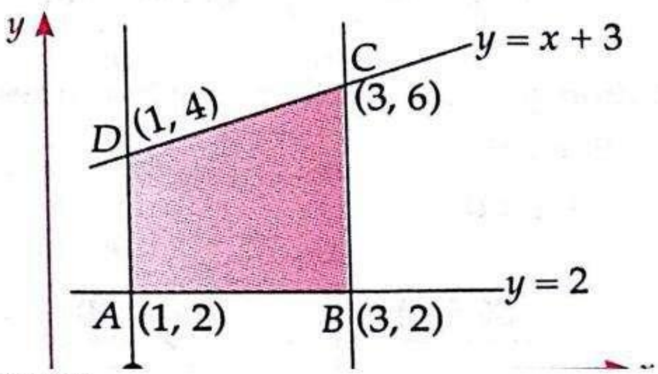
cuts x–axis at (8, 0) and y–axis at (0, 6)

line $x = 8$ is parallel to y–axis while line $y = 6$ is parallal to x–axis. These three lines formed a right angled ΔABC *(see figure)*

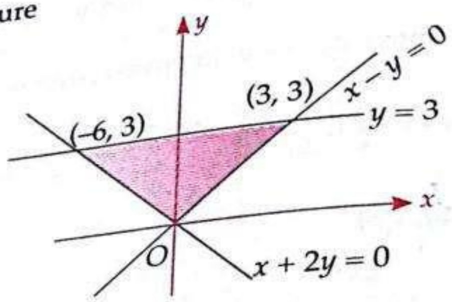
Its area $= \frac{1}{2}\times 8\times 6 = 24$ square unit

30. (a) Point of intersection of given lines are $A(1,2)$, $B(3, 2)$, $C(3, 6)$ and $D(1, 4)$ *(see figure)*

It is a trapezium.

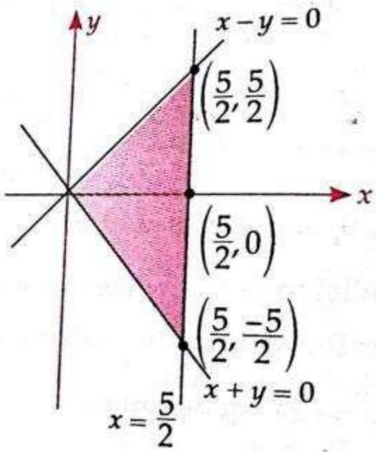


31. (d) See the figure



$$\text{Required Area} = \frac{1}{2}\,|3-(-6)| \times 3$$
$$= \frac{1}{2}\times 9\times 3$$
$$= \frac{27}{2} \text{ square unit}$$

32. (c) See the figure,

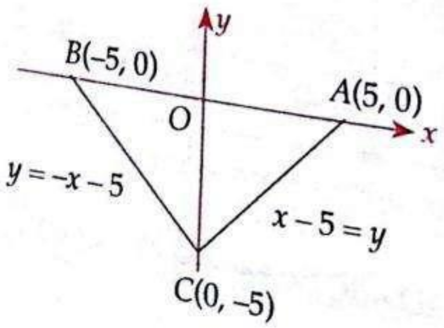


$$\text{Required Area} = \frac{1}{2}\times\left(\frac{5}{2}-\left(-\frac{5}{2}\right)\right)\times\frac{5}{2}$$
$$= \frac{1}{2}\times 5\times\frac{5}{2} = \frac{25}{4} \text{ square unit.}$$

33. (a) In the equation $y = |x| - 5$, there are two lines

$y = x - 5,\ x > 0$

and $y = -x - 5,\ x < 0$



these lines intersect each other at (0, –5) and respectively cut x–axis at (5, 0) and (–5, 0)

$\therefore$ Required Area $= \frac{1}{2} \times AB \times OC$

$= \frac{1}{2} \times 10 \times 5$

$= 25$ square unit.

34. (b) From solved example 15

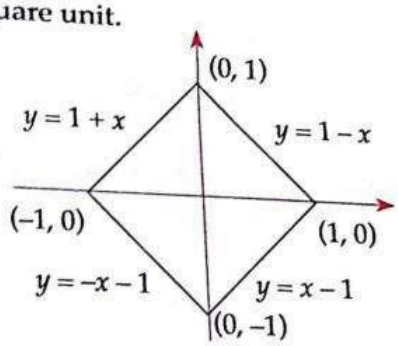
Required Area $= 2k^2 = 2 \times 4^2 = 32$ square unit.

35. (a) Here four lines are $y = x - 1, x \geq 0$

$y = -x - 1, x \leq 0$

$y = 1 - x, x \geq 0$

$y = 1 + x, x \leq 0$



These lines cut axes respectively at (1, 0), (0, 1), (–1, 0) and (0, –1) It is a square with each side $= \sqrt{1^2 + 1^2} = \sqrt{2}$

$\therefore$ Required Area $= \left(\sqrt{2}\right)^2 = 2$ square unit

36. (d) Checking each option one by one, in options (d)

$\frac{a_1}{a_2} = 1$ and $\frac{b_1}{b_2} = -1$ i.e. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So system of equation given in option (d) has unique solution.

37. (b) Option (b) follows (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$; rest are not.

38. (c) In option (c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = 1$ but $\frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

$\therefore$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ i.e. this system of equation has no solution.

39. (c) Solving $3x + 4y = 19$ and $y - x = 3$

we get $x = 1, y = 4$

putting $(x, y) = (1, 4)$ in $2x + 3y = k$

we have $2 \times 1 + 3 \times 4 = k \Rightarrow k = 14$

40. (a) In option (a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. Hence lines given in alternative (a) shows parallel lines.

41. **(d)** In option (d) $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$, which is condition for parallel lines.

43. **(c)** Here $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ is always true. It doesnot depend upon K.

44. **(a)** Required condition is $\frac{2a}{2}+\frac{a+b}{3}=\frac{28}{7}$

or, $a=\frac{a+b}{3}=4$

$\therefore \quad a=4, a+b=12$

or, $a=4, \quad b=8$

46. **(d)** System has unique solution if $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

$$\Rightarrow \frac{k}{3}\neq\frac{2}{1}$$

$$\Rightarrow k\neq 6$$

so options (a), (b), (c) are correct.

Exercise—3B

In the *xy*–coordinate system, if (a, b) and $(a+3, b+k)$ are two points on he line defined by the equation $x=3y-7$, then $k=$

a) 9 (b) 3 (c) $\frac{7}{3}$ (d) 1

[SSC Tier-I 2012]

2. The *x*–intercept of the graph of $5x-4y=20$ is

(a) 4 units (b) 5 units (c) 9 units (d) 1 units

[SSC Tier-I 2012]

3. A triangle is formed by the *x*–axis and the lines $2x+y=4$ and $x-y+1=0$ as three sides. Taking the side along *x*–axis as its base, the corresponding altitude of the triangle is

(a) 2 unit (b) 3 unit (c) $\sqrt{5}$ unit (d) 1 unit

[SSC Tier-I 2012]

4. The length of the portion of the straight line $8x+15y=120$ intercepted between the axes is

(a) 14 units (b) 15 units (c) 16 units (d) 17 units

[SSC Tier-I 2012]

5. The area of the triangle formed by the lines $5x+7y=35$ and $4x+3y=12$ and *x*–axis is

(a) $\frac{160}{13}$ sq. units (b) $\frac{150}{13}$ sq. units (c) $\frac{140}{13}$ sq. units (d) 10 sq. units

[SSC Tier-I 2012]

6. Area of the triangle formed by the graph of the line $2x - 3y + 6 = 0$ along with the coordinate axes is
(a) $\frac{3}{2}$ sq. units (b) 3 sq. units (c) 6 sq. units (d) $\frac{1}{2}$ sq. units
[SSC Tier-I 2012]

7. Area of the trapezium formed by x–axis, y–axis and the lines $3x + 4y = 12$ and $6x + 8y = 60$ is
(a) 31.5 sq. units (b) 48 sq. units (c) 36.5 sq. units (d) 37.5 sq. units
[SSC Tier-I 2012]

8. For what value of k system of equation $x + 2y = 5, 3x + ky + 15 = 0$ doesnot have any solution ?
(a) 2 (b) -2 (c) 6 (d) -6
[SSC CPO 2012]

Answers—3B

1. (d) 2. (a) 3. (a) 4. (d) 5. (a) 6. (b) 7. (a) 8. (c)

Explanation

1. (d) $\because$ (a, b) lies on straight line $x = 3y - 7$

$\therefore$ $a = 3b - 7$ ... (i)

$\therefore$ $(a + 3, b + k)$ lies on straight line $x = 3y - 7$

$\therefore$ $a + 3 = 3(b + k) - 7$ ... (ii)

Subtracting (ii) from, (i) $a + 3 - a = 3(b + k) - 7 - (3b - 7)$

or, $3 = 3k$ $\therefore$ $k = 1$

Second Method

Slope of line joining the points (a, b) and $(a + 3, b + k) = \frac{b+k-b}{a+3-a} = \frac{k}{3}$

For the line $x = 3y - 7$

or, $y = \frac{x}{3} + \frac{7}{3}$

slope $= \frac{1}{3}$

$\therefore$ $\frac{k}{3} = \frac{1}{3}$

$\Rightarrow k = 1$

2. (a) Putting $y = 0$ in $5x - 4y = 20$

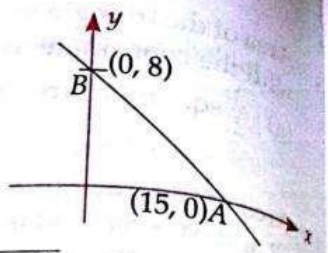
$5x = 20$ $\therefore$ $x = 4$

3. (a) Solving $2x + y = 4$ and $x - y + 1 = 0$, we get

$x = 1, y = 2$ i.e. Vertex of triangle is (1, 2)

It is at a height of 2 unit from the x–axis

4. (d) In the equation $8x + 15y = 120$



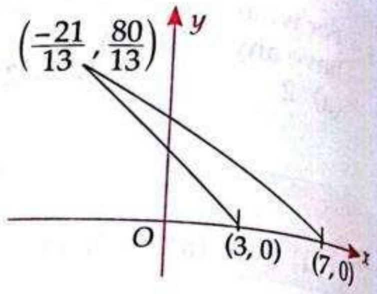
Putting $x = 0$, $y = 8$

Putting $y = 0$, $x = 15$

Thus line cuts x–axis at $A\ (15, 0)$ and y–axis at $B(0, 8)$

$\therefore$ length intercepted between $= AB = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$

5. (a) In $5x + 7y = 35$ putting $y = 0$, $x = 7$



This line cuts x–axis at $(7, 0)$

Similarly $4x + 3y = 12$, cuts x– axis at $(3, 0)$

Solving $5x + 7y = 35$ and $4x + 3y = 12$

$$(x, y) = \left(\frac{-21}{13}, \frac{80}{13}\right)$$

From figure it is clear that base of triangle $= 7 - 3 = 4$

and height $= \frac{80}{13}$

$\therefore$ Area $= \frac{1}{2} \times 4 \times \frac{80}{13} = \frac{160}{13}$ square unit

(b) Required Area $= \frac{1}{2}\left|\frac{c^2}{ab}\right| = \frac{1}{2}\left|\frac{6^2}{2(-3)}\right| = \frac{36}{12} = 3$ square unit

$x + 4y = 12 \Rightarrow \frac{3x + 4y}{12} \Rightarrow \frac{x}{4} + \frac{y}{3} = 1$

$6x + 8y = 60 \Rightarrow \frac{6x + 8y}{60} = 1$

$$\Rightarrow \frac{x}{10} + \frac{y}{\frac{15}{2}} = 1$$

see figure, Area of trapezium $ABCD$ = area of ΔOCD − area of ΔOAB

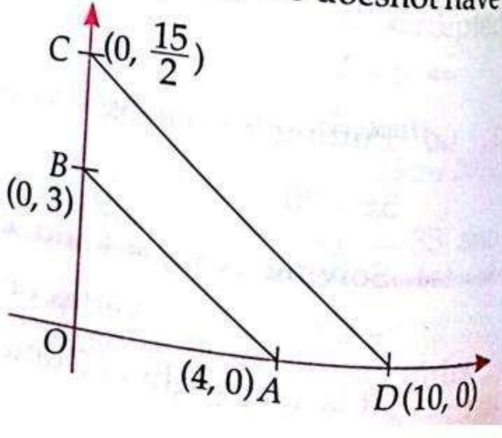
$= \frac{1}{2} \times 10 \times \frac{15}{2} - \frac{1}{2} \times 3 \times 4 = \frac{150}{4} - 6 = 37{\cdot}5 - 6 = 31{\cdot}5$ square unit

8. (c) System of equation $x + 2y - 5 = 0$ and $3x + ky + 15 = 0$ doesnot have Solution if $\frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15}$

$\Rightarrow \frac{1}{3} = \frac{2}{k}$ and $\frac{2}{k} \neq \frac{-5}{11}$

$\therefore$ $k = 6$ and $k \neq -6$

Hence, $k = 6$



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