Variation

Practice set 7.1

Q. 1. Write the following statements using the symbol of variation.

(1) Circumference (c) of a circle is directly proportional to its radius (r).
(2) Consumption of petrol (I) in a car and distance traveled by that car (D) are in direct variation.

Answer :

(1) Circumference $= c$ and radius $= r$

Therefore, $c \propto r$ or $c = kr$, where $k =$ constant

(2) Consumption of petrol in a car $= I$

Distance traveled by that car $= D$

$I \propto D$ OR $I = K \times D$

Q. 2. Complete the following table considering that the cost of apples and their number are in direct variation.

Number of apples (x)	1	4	...	12	...
Cost of apples (y)	8	32	56	...	160

Answer : Since, cost of apples and their number are in direct variation it means that as the number of apple increases, the cost also increases and as the number of apple decreases, the cost also decreases.

Number of apples (x)	1	4	7	12	20
Cost of apples (y)	8	32	56	96	160

$X \propto Y$

EXPLANATION:

• When no. of apples is 1 [$X = 1$]

Cost of apple is 8 [$Y = 8$]

Now, when no. of apples become 4 times then the cost of apples will also become 4 times because they are inversely proportion.

$X = 1 \times 4 = 4$

$Y = 8 \times 4 = 32$

• When Cost of apple is 8 [$Y = 32$]

No. of apples is 4 [$X = 4$]

Now, the cost of apples becomes $\frac{7}{4}$ times.

$Y = 32 \times \frac{7}{4}$

$Y = 56$

∴ No. of apples will also become $\frac{7}{4}$ times.

$$X = 4 \times \frac{7}{4}$$

$$X = 7$$

• When no. of apples is 7 [$X = 7$]

Cost of apple is 56[$Y = 56$]

Now, no. of apples becomes $\frac{12}{7}$ times.

$$X = 7 \times \frac{12}{7}$$

$$X = 12$$

∴ Cost of apples will also become $\frac{12}{7}$ times.

$$Y = 56 \times \frac{12}{7}$$

$$Y = 96$$

• When Cost of apple is 96 [$Y = 96$]

No. of apples is 12 [$X = 12$]

Now, the cost of apples becomes $\frac{5}{3}$ times.

$$Y = 96 \times \frac{5}{3}$$

$$= 160$$

∴ No. of apples will also become $\frac{5}{3}$ times.

$X = 12 \times \frac{5}{3}$

$X = 20$

Q. 3. If $m \propto n$ and when m=154, n=7. Find the value of m, when n=14.

Answer : GIVEN:

M ∝ n

M = 154

N = 7

TO FIND: Value of m, when n=14

PROOF:

M ∝ n

That is, 154∝7

It means that as the value of m increases, the value of n also increases.

When n=14, n becomes 2 times of its original value.

∵ M ∝ n

∴ M will also get double.

M = 154×2

M = 308

M ∝ n

308 ∝ 14

∴ Value of m is 308.

Q. 4. If n varies directly as m, complete the following table.

m	3	5	6.5	. . .	1.25
n	12	20	. . .	28	. . .

Answer : It is given that $m \propto n$. It means that as the value of n increases, the value of m also increases and if the value of n decreases, the value of n also decreases.

m	3	5	6.5	7	1.25
n	12	20	26	28	5

• When m =3

n =12

Now, when m becomes $\frac{5}{3}$ times. ∴ n will also become $\frac{5}{3}$ times.

$m = 3 \times \frac{5}{3}$

$n = 12 \times \frac{5}{3}$

$n = 20$

• When $m = 5$

$n = 20$

Now, when m becomes 1.3 times.

$m = 5 \times 1.3$

$m = 6.5$

∴ n will also become 1.3 times.

$n = 20 \times 1.3$

$n = 26$

• When $m = 6.5$

$n = 26$

Now, when m becomes $\frac{14}{13}$ times.

$m = 5 \times \frac{14}{13} = 28$

∴ m will also become $\frac{14}{13}$ times.

$n = 6.5 \times \frac{14}{13} = 7$

• When m =7

n = 28

Now, when m becomes $\frac{5}{28}$ times.

$m = 7 \times \frac{5}{28} = 1.25$

∴ n will also become $\frac{5}{28}$ times.

$n = 28 \times \frac{5}{28} = 5$

Q. 5. y varies directly as the square root of x. When x = 16, y = 24. Find the constant of variation and equation of variation.

Answer : It is given that y varies directly as the square root of x.

$x = 16$

$y = 24$

$y = k \times \sqrt{x}$ (k is a constant)

$24 = k \times \sqrt{16}$

$24 = 4K = \frac{24}{4}$

$K = 6$

∴ Required constant $= 6$

Equation: $y = 6\sqrt{x}$

Q. 6. The total remuneration paid to laborers, employed to harvest soybeans is indirect variation with the number of laborers. If remuneration of 4 laborers is Rs1000, find the remuneration of 17 laborers.

Answer : It is given that the total remuneration paid to laborers, employed to harvest soybeans is direct variation with the no. of laborers.

$\text{Total remuneration} \propto \text{No. of laborers}$

Remuneration of 4 labourers $= \text{Rs } 1000$

Remuneration of 1 labour $= \frac{1000}{4} = \text{Rs } 250$

Remuneration of 17 labourers $= \text{Remuneration of 7 labourers} \times 17$

$= 250 \times 17$

$= \text{Rs } 4250$

ALTERNATIVE METHOD

Total remuneration of 4 laborers is Rs 1000.

$1000 \propto 4$

Remuneration of 17 laborers

To get the number of laborers to be 17, the present number of laborers will be multiplied by $\frac{17}{4}$

Since total remuneration is in direct variation with the number of laborers total remuneration will also get $\frac{17}{4}$ times.

$$1000 \times \frac{17}{4} \propto 4 \times \frac{17}{4}$$

$$= 4250 \propto 17$$

∴ Remuneration of 17 labourers = Rs 4250

Practice set 7.2

Q. 1. The information about numbers of workers and the number of days to complete work is given in the following table. Complete the table.

Number of workers	30	20		10	
Days	6	9	12		36

Answer : Number of workers and the number of days to complete a work will be inversely proportional because if the number of workers increases the number of days to complete the work will reduce.

Number of workers $\alpha \frac{1}{\text{number of days}}$

Number of workers	30	20	15	10	5
Days	6	9	12	18	36

When the number of workers = 30

Number of days = 6

$30 \ \alpha \ \frac{1}{6}$

$30 = \frac{k}{6}$ [k = constant]

$k = 180$ - (1)

The value of k will remain the same in all the cases

When the number of days = 12

Number of workers $\alpha \ \frac{1}{12}$

Number of workers $= \frac{k}{12}$ (k=constant)

Number of workers $= \frac{180}{12}$ (from 1)

Number of workers = 15

∴ Number of workers = 15 when the number of days is 12

When the number of workers = 10

$10 \ \alpha \ \frac{1}{\text{number of days}}$

$10 \ = \ \frac{k}{\text{number of days}}$

Number of days $= \frac{180}{10}$ (from 1)

Number of days = 18

∴ When the number of workers=10, number of days is 18

When the number of days = 36

Number of workers $\alpha \ \frac{1}{36}$

Number of workers $= \frac{k}{36}$

Number of workers $= \frac{180}{36}$ (from1)

Number of workers = 5

∴ When the number of days is 36, the number of workers will be 5

Q. 2. Find constant of variation and write equation of variation for every example given below.

(1) $p \alpha \frac{1}{q}$ **; if** $p = 15$ **then** $q = 4$

(2) $z \alpha \frac{1}{w}$**; when** $z = 2.5$ **then** $w = 24$

(3) $s \alpha \frac{1}{t^2}$ **; if** $s = 4$ **then** $t = 5$

(4) $x \alpha$**; if** $x = 15$ **then** $y = 9$

Answer :

(1) $P \propto \frac{1}{q}$ $(p = 15, q = 4)$

$15 \propto \frac{1}{4}$

$15 = \frac{k}{4}$ (k=constant)

$k = 60$

$P \propto \frac{1}{q}$

$P = \frac{k}{q}$

$P = \frac{60}{q}$

$p \times q = 60$

∴constant of variation is 60 and equation is $p \times q = 60$

(2) $z \propto \frac{1}{w}$ $(z = 2.5, w = 24)$

$2.5 \propto \frac{1}{24}$

$2.5 = \frac{k}{24}$ (k=constant)

$K = 60$ (1)

$z \, \alpha \, \frac{1}{w}$

$z = \frac{k}{w}$

$z \times w = 60$ (from 1)

∴constant of variation is 60 and equation is $z \times w = 60$

(3) $s \, \alpha \, \frac{1}{t^2}$ $(s = 4, t = 5)$

$4 \, \alpha \, \frac{1}{5^2}$

$4 = \frac{k}{25}$ (k=constant)

$K = 100$ (1)

$s \, \alpha \, \frac{1}{t^2}$

$s = \frac{k}{t^2}$

$st^2 = 100$ (from 1)

∴constant of variation is 100 and the equation is $st^2 = 100$

(4) $x \, \alpha \, \frac{1}{\sqrt{y}}$ ($x = 15, y = 9$)

$15 \, \alpha \, \frac{1}{\sqrt{9}}$

$15 = \frac{K}{3}$ (k=constant)

$K = 45$ (1)

$$x \, \alpha \, \frac{1}{\sqrt{y}}$$

$$x = \frac{k}{\sqrt{y}}$$

$x\sqrt{y} = 45$ (from 1)

∴constant of variation is 45 and equation is

$$x\sqrt{y} = 45$$

Q. 3. The boxes are to be filled with apples in a heap. If 24 apples are put in a box then 27 boxes are needed. If 36 apples are filled in a box how many boxes will be needed?

Answer : The number of apples and number of boxes will be inversely proportional as if the number of apples will be filled in a box then fewer boxes will be needed.

$$\text{Number of apples } \alpha \; \frac{1}{\text{number of boxes}}$$

Number of apples in a box = 24

Number of boxes needed = 27

$$24 \, \alpha \, \frac{1}{27}$$

$24 = \frac{k}{27}$ (k = constant)

$K = 648$ (1)

In the case, if 36 apples are filled in a box

Let the number of boxes be x

$36\ \alpha\ \frac{1}{x}$

$36 = \frac{k}{x}$ (k = constant)

$36 = \frac{648}{x}$

$X = 18$

Therefore, 18 boxes are needed.

Q. 4. Write the following statements using the symbol of variation.

(1) The wavelength of sound (l) and its frequency (f) are in inverse variation.
(2) The intensity (I) of light varies inversely with the square of the distance (d) of a screen from the lamp.

Answer : (1) Wavelength of sound (l) and frequency (f) are in inverse proportion.

$l\ \alpha\ \frac{1}{f}$

(2) Intensity (I) of light varies inversely

With the square of the distance (d)

$I\ \alpha\ \frac{1}{d^2}$

Q. 5. $x\ \alpha \frac{1}{\sqrt{y}}$ **and when x = 40 then y = 16. If x = 10, find y.**

Answer : We are given that $x\ \alpha\ \frac{1}{\sqrt{y}}$

When $x = 40$ then $y = 16$

$40 \; \alpha \; \frac{1}{\sqrt{16}}$

$40 \; \alpha \; \frac{1}{4}$

$40 = \frac{k}{4}$ (k=constant)

$k = 160$ (1)

If $x = 10$

$x \; \alpha \; \frac{1}{\sqrt{y}}$

$10 \; \alpha \; \frac{1}{\sqrt{y}}$

$10 = \frac{k}{\sqrt{y}}$

$10 = \frac{160}{\sqrt{y}}$

$\sqrt{y} = \frac{160}{10}$

$\sqrt{y} \; = 16$

$y = 16^2$

$y = 256$

Q. 6. X varies inversely as y, when x = 15 then y = 10, if x = 20 then y =?

Answer : We are given that x varies inversely as y

i.e. $x \; \alpha \; \frac{1}{\sqrt{y}}$

When $x = 15$

$Y = 10$

$15 \alpha \frac{1}{10}$

$15 = \frac{k}{10}$ (k=constant)

$k = 150$ (1)

If $x = 20$

$x \alpha \frac{1}{y}$

$20 \alpha \frac{1}{y}$

$20 = \frac{k}{y}$

$20 = \frac{150}{y}$ (from 1)

$Y = 7.5$

Practice set 7.3

Q. 1. Which of the following statements is of inverse variation?

(1) The number of workers on a job and time taken by them to complete the job.
(2) The number of pipes of the same size to fill a tank and the time taken by them to fill the tank.
(3) Petrol filled in the tank of a vehicle and its cost.
(4) Area of the circle and its radius.

Answer : (1) Yes, it is of inverse variation because more the number of workers will be lesser time will be taken.

(2) Yes, it is of inverse variation because more the number of pipes will be lesser time will be taken.

(3) No, it is not of inverse variation because the cost of petrol will increase with respect to its quantity.

(4) No, it is not of inverse variation because a larger circle has a longer radius.

Q. 2. If 15 workers can build a wall in 48 hours, how many workers will be required to do the same work in 30 hours?

Answer : The number of workers building a wall and time taken by them is inversely proportional.

Let x be the number of workers and y be the time taken.

$$\text{Number of workers } \alpha \frac{1}{\text{time taken}}$$

$$x \,\alpha\, \frac{1}{y}$$

$$x = \frac{k}{y} \text{ (k is constant) (1)}$$

Number of workers given=15

Time took = 48 hrs

(Put in 1)

$$15 = \frac{k}{48}$$

$$k = 720$$

Wall has to be built in 30 hours

So, $y = 30$

(Put in 1)

$$x = \frac{k}{30}$$

$x = \frac{720}{30}$ (Since $k = 720$)

$x = 24$

So, 24 workers are needed to build the wall in 30 hours.

Q. 3. 120 bags of half liter milk can be filled by a machine within 3 minutes find the time to fill such 1800 bags?

Answer : Number of bags will be directly proportional to the time taken because as the number of bags increases, time to fill them also increases.

Number of bags = 120

Time is taken to fill = 3 min

$\text{Number of bags } \alpha \text{ Time taken}$

$120 \ \alpha \ 3$

$120 = 3k$ (k=constant)

$\frac{120}{3} = k$

$K = 40$ (1)

Number of bags = 1800

Let time taken to fill them be x

$\text{Number of bags } \alpha \text{ Time taken}$

$1800 \ \alpha \ x$

$1800 = k \times x$

$1800 = 40 \times x$ (From 1)

$X = 45$

So, 45 minutes are needed to fill 1800 bags.

Q. 4. A car with a speed of 60 km/hr takes 8 hours to travel some distance. What should be the increase in the speed if the same distance is to be covered in $7\frac{1}{2}$ hours?

Answer : Speed of car and time will be inversely proportional because as the speed increases, time for the journey decreases.

Speed of car $= 60\ \text{km/hr}$

Time $= 8\ \text{hrs}$

$$\text{Speed}\ \alpha\ \frac{1}{\text{time}}$$

$$60\ \alpha\ \frac{1}{8}$$

$60\ = \frac{k}{8}$ (k=constant)

$k = 480$ (1)

Time for which distance is to be covered $= 7\frac{1}{2}\ \text{hrs}$

$$= \frac{15}{2}\ \text{hrs}$$

Let increase in speed $= x$

$$\text{Speed}\ \alpha\ \frac{1}{\text{time}}$$

$$\text{Speed}\ = \frac{k}{\text{time}}$$

$$60 + x = \frac{k}{\frac{15}{2}}$$

From 1

$$60 + x = 480 \times \frac{2}{15}$$

$$60 + x = 64$$

$$X = 4$$

So, speed should be increased by 4 km/hr.