☆ Steady State errors:

⇒ The error is nothing but deviation of the output from the input.

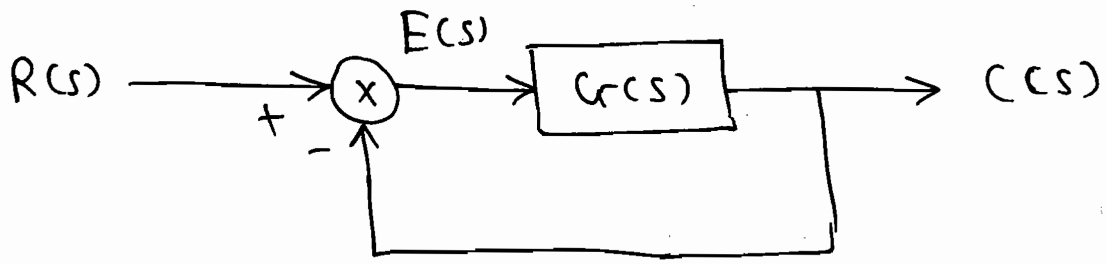
⇒ Steady state error is the error at $t \to \infty$.

$$e_{ss} = \lim_{t \to \infty} e(t).$$

$$\therefore \boxed{e_{ss} = \lim_{s \to 0} s\,E(s).}$$

(∵ Final value ~~theorom~~ theorom).

⇒ Consider the UFB system as shown in figure:



⇒ $E(s) = R(s) - C(s)$

$C(s) = G(s) \cdot E(s).$

$\therefore E(s) = R(s) - G(s) \cdot E(s).$

⇒ $$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)}.$$

$$\therefore \boxed{e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1 + G(s)}} \leftarrow \text{H.B}$$

⇒ The steady state errors depends on two factors ① Type of Input.
② Type of System.

⇒ The steady state error are calculate to only CL stable system.

⇒ The steady state errors are valid only for UFB system.

⇒ If Non Unity FB system is given it should be converted into UFB.

* Type of Input:

	Step	Ramp	Parabolic
I/P $r(t)$	$A\,u(t)$	$At\,u(t)$	$A\,t^2/2\;u(t)$
e_{ss}	$\frac{A}{1+K_p}$	$\frac{A}{K_v}$	$\frac{A}{K_a}$
Error Constant	K_p: Possion con^t $\lim_{s \to 0} G(s)$	K_v: velocity cons. $\lim_{s \to 0} s\,G(s)$	K_a: Acceleration cons. $\lim_{s \to 0} s^2 G(s)$

* Type of System:

$\Rightarrow$ The standard form of the system is

$$G(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{s^{n}(1+sT_a)(1+sT_b)\cdots}$$

$H(s)=1$

$s^n \longrightarrow$ Type $= n$ system.

$\Rightarrow$ Consider the step ~~inpd~~ input and the different type of the system.

⊙ $\underline{0.\ \text{Step}} \Rightarrow (\underline{t^0 . u(t)})$.

$\Rightarrow$

$$e_{ss} = \frac{A}{1+K_p}$$

$$K_p = \lim_{s \to 0} G(s).$$

① Type - 0:

$$K_p = \lim_{s \to 0} \frac{K(1+sT_1)(1+sT_2)\cdots}{s^0(1+sT_a)(1+sT_b)\cdots}$$

$$\boxed{K_p = K}$$

$$\Rightarrow \quad e_{ss} = \frac{A}{1+K_p} = \frac{A}{1+K}.$$

$$\boxed{e_{ss} = \frac{A}{1+K} = \text{constant}}$$

② Type - 1:

$$\Rightarrow \quad K_p = \lim_{s \to 0} \frac{K(1+s\tau_1)(1+s\tau_2)\cdots}{s^1 (1+s\tau_a)(1+s\tau_b)\cdots}$$

$$\boxed{K_p = \infty} \Rightarrow e_s = \frac{A}{1+\infty} = 0$$

$$\Rightarrow \boxed{e_{ss} = 0}$$

③ Type - 2:

$$\Rightarrow \quad K_p = \lim_{s \to 0} \frac{K(1+s\tau_1)(1+s\tau_2)\cdots}{s^2 (1+s\tau_a)(1+s\tau_b)\cdots}$$

$$\boxed{K_p = \infty} \Rightarrow \boxed{e_{ss} = 0}$$

⇒ The S.S. errors are required to calculate only in 3 - cases:

i.e ① Type - 0 & step input (t^0).

② Type - 1 & ramp input (t^1).

③ Type - 2 & Parabolic input (t^2).

⇒ Remain all the cases the steady state error either become zero (or) infinity.

=>

Type = i/p	e_{ss} = Constant
Type > i/p	e_{ss} = 0.
Type < i/p	$e_{ss} = \infty$.

$K = \dfrac{\text{Nr. constant}}{\text{Dr. constant}}$

A = Amplitude of i/p.

$\boxed{Q}$ Find e_{ss} to the given unity FB system $G(s) = \dfrac{10(s+1)}{s^2(s+2)(s+10)}$, $H(s) = 1$.

to the following input

$$r(t) = (10 + 2t + 1 \cdot t^2/2) \cdot u(t).$$

Soln:

$$R(s) = \frac{10}{s} + \frac{2}{s^2} + \frac{1}{s^3}.$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{1 + G(s)}.$$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{s \cdot \left(\frac{10}{s} + \frac{2}{s^2} + \frac{1}{s^3}\right)}{1 + \frac{10(s+1)}{s^2(s+2)(s+10)}}.$$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{(10s^2 + 2s + 1)(s+2)(s+10)}{s^2(s+2)(s+10) + 10(s+1)}.$$

$$= \frac{(0+2)(0+10)}{0 + 10(0+1)} = \frac{20}{10} = 2.$$

Method-2: Compare type and input.

→ Type - 2:

Type		i/p		e_{ss}
2	>	0	→	0.
				+
2	>	1	→	0
				+
2	=	2	→	$\frac{A}{K}$

$$= \frac{1.}{\frac{10\times 1}{10\times 2}}$$

$$= 2$$

$$\boxed{e_{ss} = 2}$$

Q Find the e_{ss} to the following i/ps to the given UFB system.

$$G(s) = \frac{10}{s(s+5)} \;;\; H(s) = 1.$$

① $10u(t)$ ③ $10t^2u(t)$ ⑤ $(1+t+t^2)u(t)$.

② $10t\,u(t)$ ④ $(1+t)u(t)$

Sol^n: $G(s) = \frac{10}{s(s+5)}$.

Type- 1:

① Type > (i/p = 0)

So, $\boxed{e_{ss} = 0}$

② Type = (i/p = 1)

$$\therefore \quad e_{ss} = \frac{A}{K} = \frac{\cancel{10}\,5}{2\,\cancel{10}/\cancel{5}}$$

$\boxed{e_{ss} = 5}$

③ Type < (i/p = 2)

$\therefore \quad \boxed{e_{ss} = \infty}$

④ $(1+t)u(t) = u(t) + tu(t)$

$u(t) \downarrow e_{ss} = 0$

$tu(t) \downarrow e_{ss} = \frac{A}{K} = \frac{1}{10/5}$

$\boxed{e_{ss} = 0.5}$

⑤ $(1+t+t^2)u(t)$.

$= u(t) + tu(t) + t^2u(t)$

$\boxed{e_{ss} = 0} + \boxed{e_{ss} = 0.5} + \boxed{e_{ss} = \infty}$

$\Downarrow$

$\boxed{e_{ss} = \infty}$

Q Repeat the above Problem for

$$G(s) = \frac{(s+1)}{s^2(s+5)(s+10)}.$$

Soln: Type - 2:

① Type > (i/p = 0)

$\boxed{e_{ss} = 0}$

(2) Type > (i/p = 1)

$\Rightarrow \boxed{e_{ss} = 0}$

(3) Type = (i/p = 2)

$$\therefore \quad e_{ss} = A/K = \frac{2 \times 10}{1/5 \times 10} = 1000$$

($\because$ given is $t^2 u(t)$).

(4) Type > i/p

$\Rightarrow \boxed{e_{ss} = 0}$

(5) $(1 + t + t^2) u(t)$

$$= \underbrace{u(t)}_{0} + \underbrace{t \cdot u(t)}_{0} + \underbrace{t^2 u(t)}_{e_{ss} = 100}$$

$\therefore \boxed{e_{ss} = 100}$

Q Repeat the above Problem for

$$G(s) = \frac{1}{s^2 (s+5)(s+10)}, \quad H(s) = 1.$$

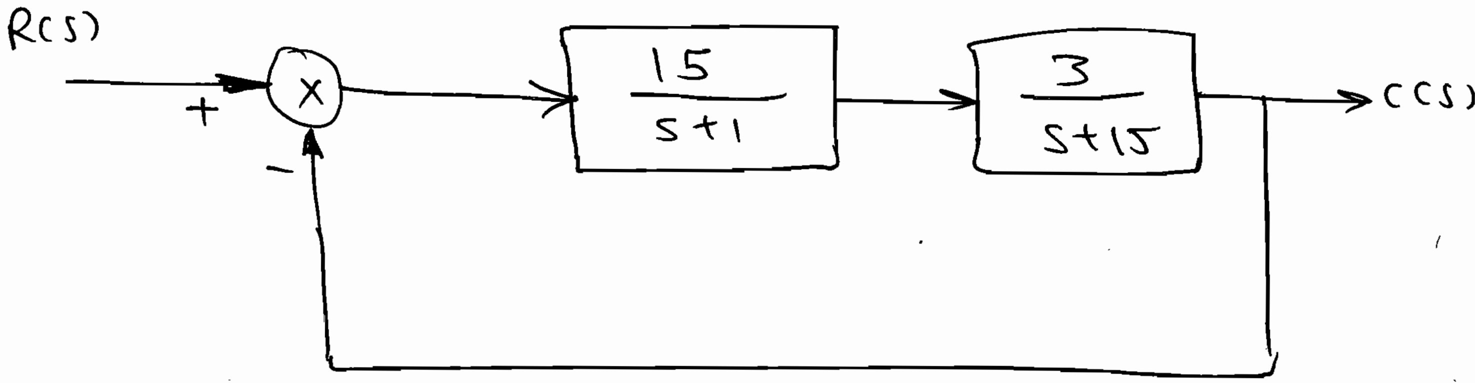
Soln: The above system is unstable,

$$CLTF = \frac{G}{1+G} = \frac{1}{s^4 + 15s^3 + \underbrace{50s^2 + 1}_{s^1 \text{ missing} \to US}}$$

$\Rightarrow$ The e_{ss} are calculated to only CLTF Stable system.

Note: → Before calculating SS error observe the options. If any one of option is 'none' then verify the CLTF system stability by using the RH criteria.

$\boxed{a}$ Calculate the e_{ss} to the given UFB system to the unit step input.



Sol^n:

$$G(s) = \frac{45}{(s+1)(s+15)}$$

$\Rightarrow$ (Type = 0) = (I/p = 0)

$$\Rightarrow \text{SSE} \quad e_{ss} = \frac{A}{1+K} = \frac{1}{1+\frac{\cancel{45}\,3}{\cancel{15}}}$$

$$\Rightarrow \boxed{e_{ss} = 0.25}$$

Note: e_{ss} are calculated to only UFB system by using OLTF i.e. $G(s)$, $H(s) = 1$.

Method : 2 :

⇒ If any BO (or) SFG is given (or) Non UFB is given, then

$$e_{ss} = \lim_{t \to \infty} [r(t) - c(t)]$$

$$\Rightarrow e_{ss} = \lim_{s \to 0} s[R(s) - C(s)].$$

$$= \lim_{s \to 0} s \cdot R(s)\left[1 - \frac{C(s)}{R(s)}\right].$$

$$\boxed{e_{ss} = \lim_{s \to 0} s \cdot R(s)[1 - CLTF].}$$ Imp.

So, $CLTF = \dfrac{45}{s^2 + 16s + 60}$.

$$\therefore e_{ss} = \lim_{s \to 0} \cancel{s} \cdot \left(\frac{1}{\cancel{s}}\right)\left(1 - \frac{45}{s^2+16s+60}\right).$$

$$= 1 - \frac{45}{60}$$

$$= 1 - 3/4.$$

$$= 1/4$$

$$\Rightarrow \boxed{e_{ss} = 0.25}$$

Q) The OLTF of UFBS

$$G(s) = \frac{K}{s(s+1)(s+2)}$$. The value of

k to get the s.s error 0.1 to the unit ramp i/p is — ?

Soln:

Type = i/p

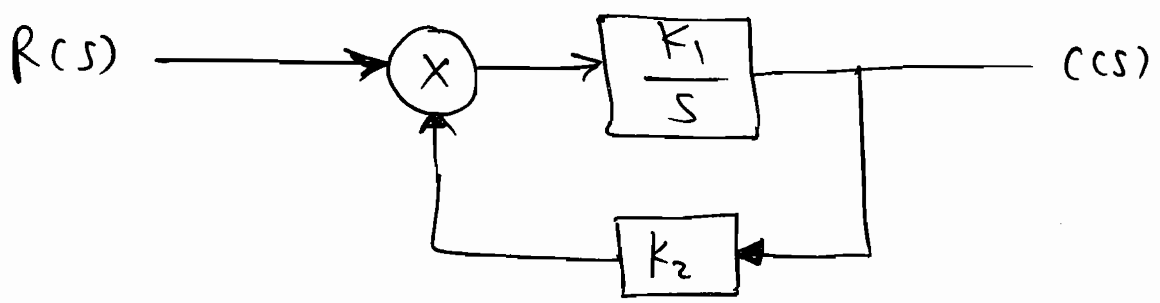
$$\Rightarrow e_{ss} = \frac{A}{K}$$

$$\therefore 0.1 = \frac{1}{K/(1\times 2)}$$

$$\therefore K = \frac{2}{0.1}.$$

$$\boxed{K = 20}$$

Q) For the system shown in fig. the s.s. o/p is 2. for unit step input and system time constant is 0.4 sec the values of K_1 & K_2 are ?



Soln:

$$CLTF = \frac{C(s)}{R(s)} = \frac{K_1/s}{1 + \frac{K_1 . K_2}{s}} = \frac{K_1}{s + K_1 . K_2}.$$

$\therefore \quad \tau = \dfrac{1}{K_1 \cdot K_2} = 0.4.$

Now, $\quad C_{ss} = C(t) = C(\infty) = 2.$

$$\therefore \quad C(\infty) = \lim_{t \to \infty} c(t)$$

$$= \lim_{s \to 0} s\, C(s).$$

$$C(\infty) = \lim_{s \to 0} s \cdot R(s) \cdot \frac{K_1}{s + K_1 \cdot K_2}$$

$$\therefore \quad 2 = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot \frac{K_1}{s + K_1 \cdot K_2}.$$

$$2 = \frac{\cancel{K_1}}{\cancel{K_1} \cdot K_2}$$

$$\boxed{K_2 = 0.5}$$

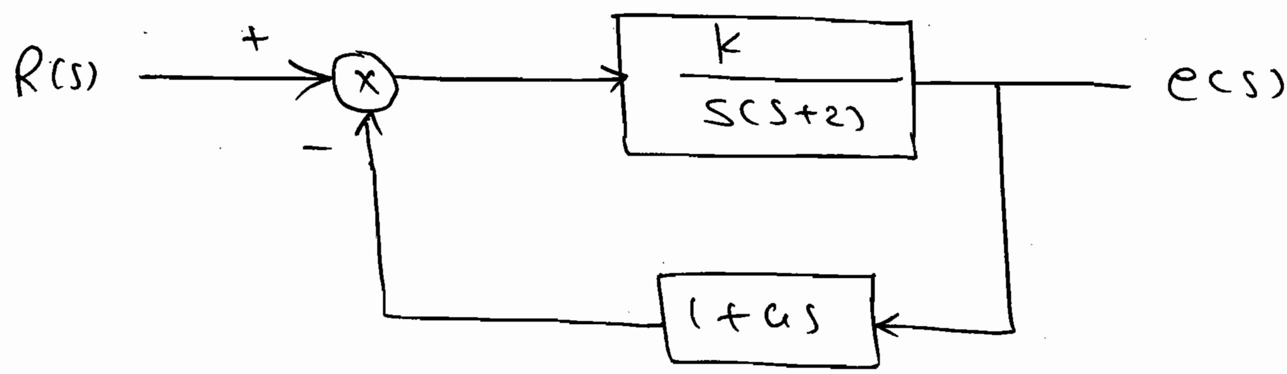
Now, $\quad K_1 \cdot K_2 = \dfrac{1}{0.4}$

$$K_1 = \frac{1}{0.4 \times 0.5}$$

$$\boxed{K_1 = 5}$$

$\boxed{a}$ For the system shown in figure the natural freq. of osc. is 4 rad/s and damping ratio is 0.7. The values of K & A are?

Soln: $\quad \omega_n = 4$ rad/sec, $\quad \zeta = 0.7$



$$\Rightarrow \quad CLTF = \frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+2)}}{1 + \frac{K(1+as)}{s(s+2)}}$$

$$= \frac{K}{s^2+2s+K+Kas}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{K}{s^2+(2+Ka)s+K}$$

$$\Rightarrow \quad \omega_n^2 = K$$

$$\boxed{K=16}$$

$$2\xi\omega_n = 2+Ka$$

$$\therefore 2\times 0.7\times 4 = 2+Ka$$

$$3.6 = Ka$$

$$\Rightarrow \quad a = \frac{3.6}{16}$$

$$\boxed{a = 0.225}$$

$\boxed{e}$ A Control system describe by following differential eqn.

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 5y = 10(1-e^{-2t}).$$

the response at $t \to \infty$.

Sol^n: L.T.

$$\Rightarrow \quad (s^2 + 5s + 5)Y(s) = 10\left(\frac{1}{s} - \frac{1}{s+2}\right) X(s).$$

$$\therefore \quad \text{CLTF} \quad \frac{Y(s)}{X(s)} = \frac{\frac{20}{s(s+2)}}{(s^2+5s+5)}$$

$$\frac{Y(s)}{X(s)} = \frac{20}{s(s+2)(s^2+5s+5)}.$$

$$\therefore \quad y(\infty) = \lim_{t \to \infty} y(t) \qquad \boxed{X(s) = 1}$$

$$\therefore \quad Y(s) = \lim_{s \to 0} s.Y(s).$$

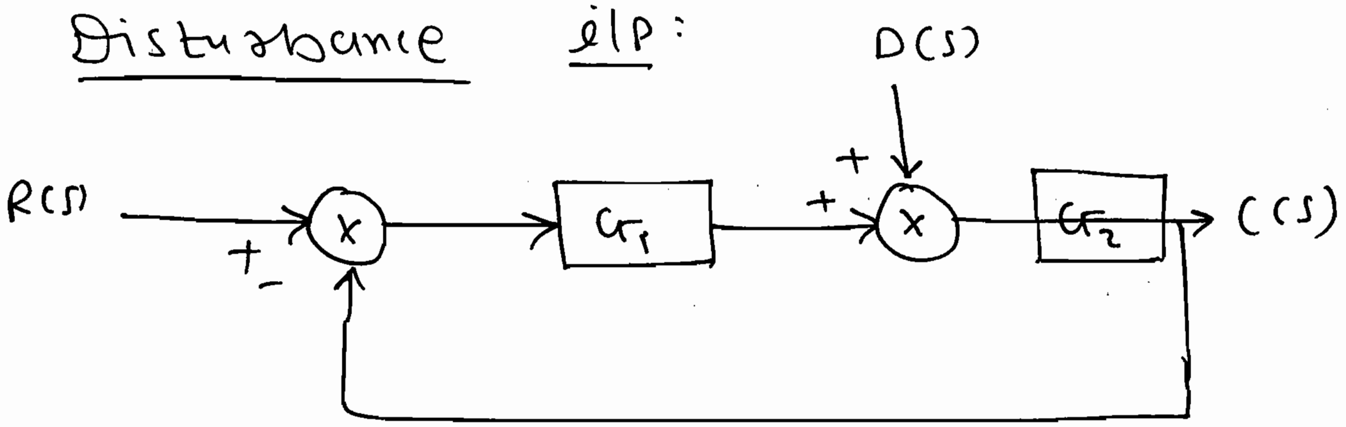
$$= \lim_{s \to 0} \cancel{s} . X(s) \times \frac{20}{\cancel{s}(s+2).(s^2+5s+5)}.$$

$$= \frac{20}{2 \times 5}$$

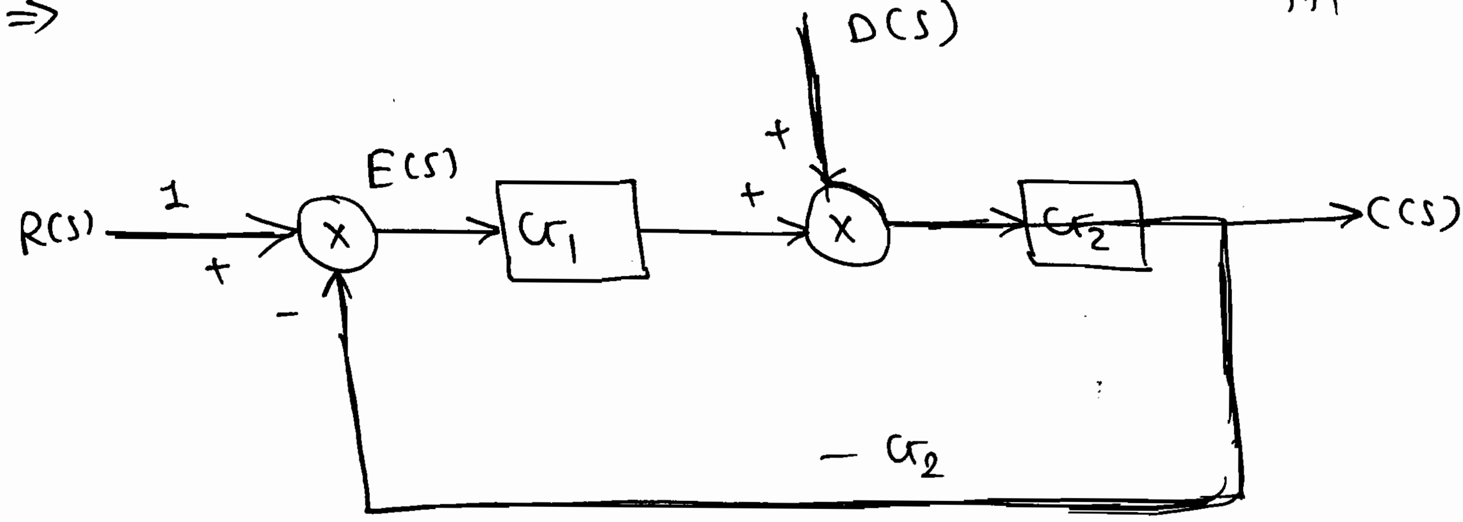
$$\therefore \quad \boxed{y(\infty) = 2}$$

* Steady State errors to the Disturbance i/p:

$\Rightarrow$



$\Rightarrow$



① e_{ss} due to $R(s)$:

$$\frac{E(s)}{R(s)} = \frac{1}{1+G_1 \cdot G_2}.$$

$$\therefore \quad e_{ss_1} = \lim_{s \to 0} \frac{s \cdot R(s)}{1+G_1 \cdot G_2}.$$

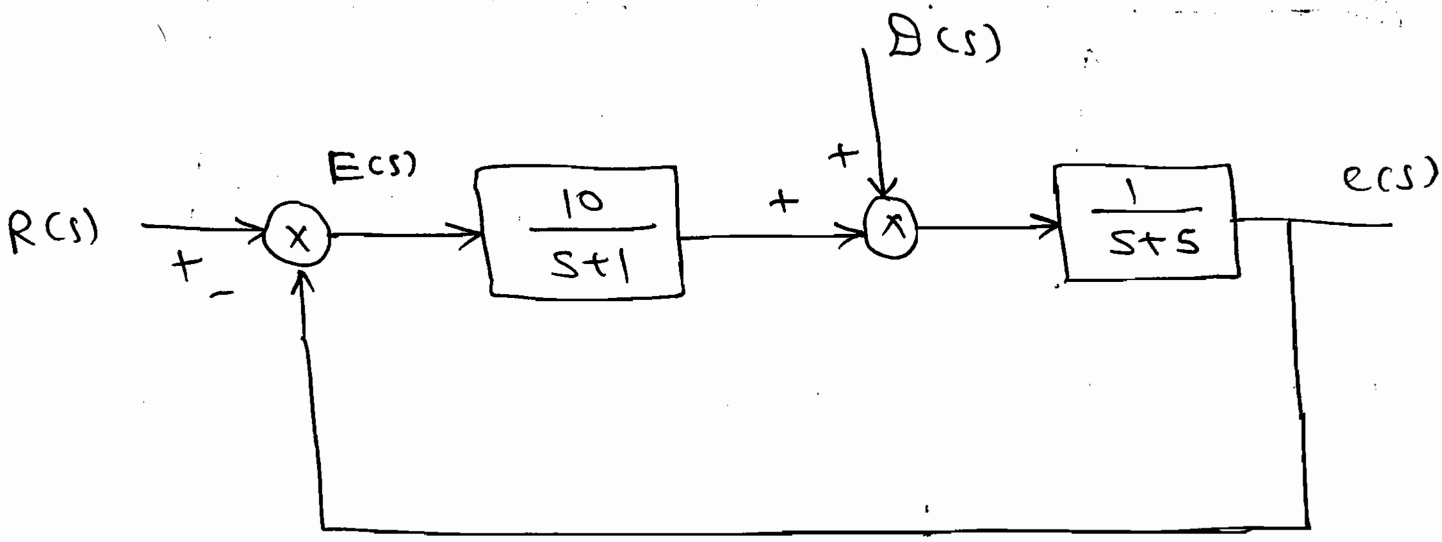
② e_{ss} due to $D(s)$:

$$\therefore \quad \frac{E(s)}{D(s)} = \frac{-G_2}{1+G_1 \cdot G_2}:$$

$$\therefore \quad e_{ss_2} = \lim_{s \to 0} \frac{s.\left[-G_2 \cdot D(s)\right]}{1+G_1 \cdot G_2}$$

$$\therefore \quad \boxed{e_{ss} = \lim_{s \to 0} s\left[\frac{R(s) - G_2 \cdot D(s)}{1+G_1 \cdot G_2}\right].}$$

Q) Find the e_{ss} due to the step input and step disturbance to the following system:



Sol^n:

$$e_{ss} = \lim_{s\to 0} s\left[\frac{R(s) - G_2(s).D(s)}{1 + G_1(s).G_2(s)}\right].$$

$$= \lim_{s\to 0} s\left[\frac{\frac{1}{s} - \frac{1}{s(s+5)}}{1 + \frac{10}{(s+1)(s+5)}}\right].$$

$$= \lim_{s\to 0}\left[\frac{1 - \frac{1}{s+5}}{1 + \frac{10}{(s+1)(s+5)}}\right].$$

$$= \frac{1 - 1/5}{1 + 10/1\times 5}$$

$$\boxed{e_{ss} = \frac{4}{15}}$$

* Steady State error ~~due~~ to Non UFB System :-

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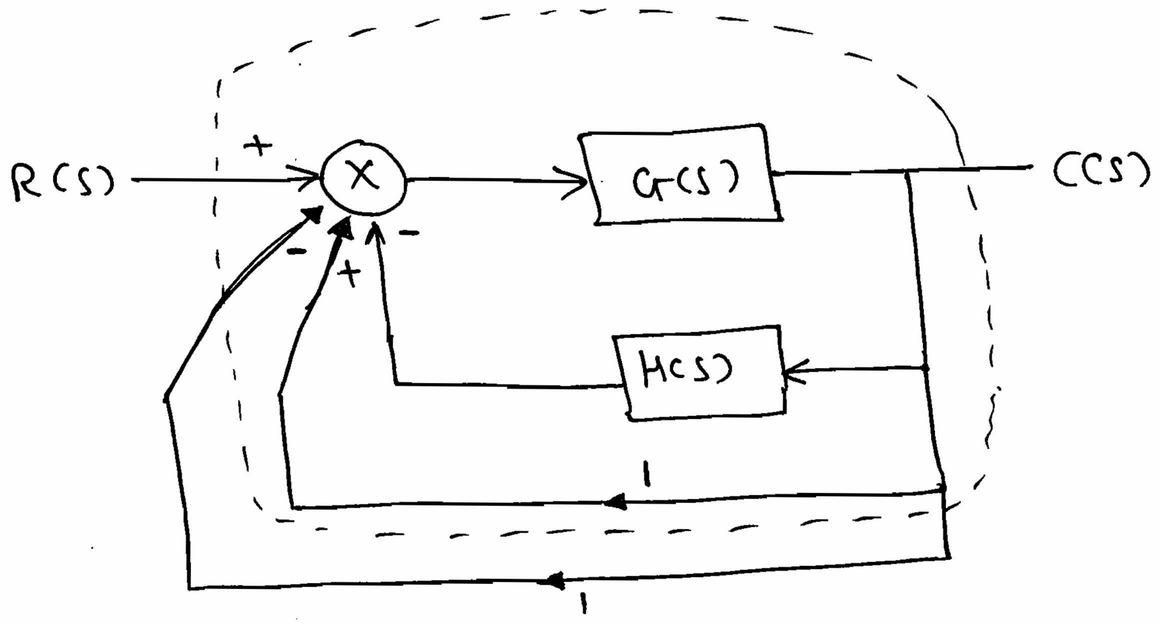
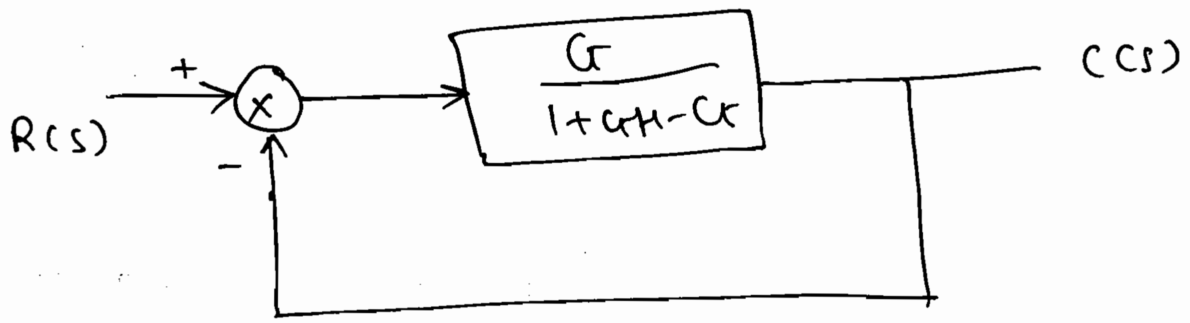
⇒ The SS errors are calculate to only a closed Loop Stable UFB system.

⇒ If Non - UFB system is given it should be converted into UFB as follows:

⇒



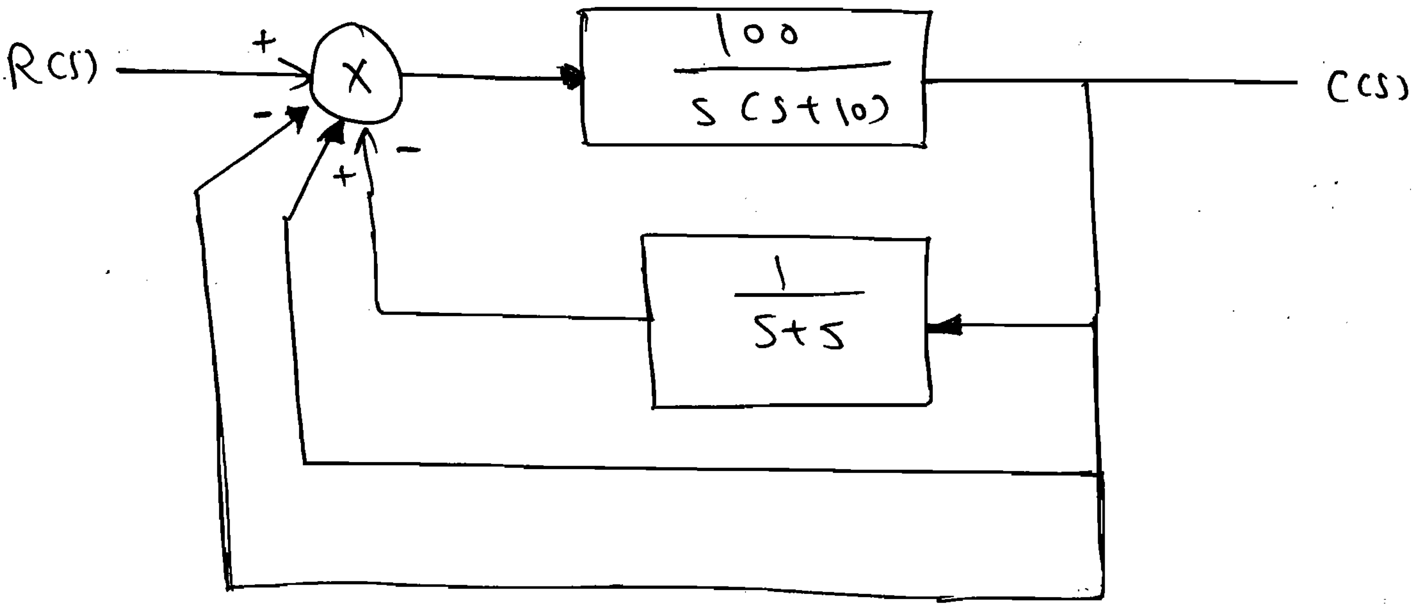
⇓



⇒ $\boxed{G(s) = \frac{G}{1+GH-G}}$

eqn OLTF of NUFB System.

Q Find the e_{ss} to the given NUFB system. to unit step input.



Sol^n:

$$G_{NUF}(s) = \frac{G}{1 + GH - G}$$

$$= \frac{\frac{100}{s(s+10)}}{1 + \frac{100}{s(s+5)(s+100)} - \frac{100}{s(s+10)}}$$

$$G_{NUF}(s) = \frac{100(s+5)}{s(s+5)(s+100) + 100 - 100(s+5)}$$

$$= \frac{100s + 500}{s^3 + 105s^2 + 500s + 100 - 100s + 500}$$

$$G_{NUF}(s) = \frac{100s + 500}{s^3 + 105s^2 + 400s - 400}$$

(Type = 0) = (I/P = 0).

$$\therefore \quad e_{ss} = \frac{A}{1+K}$$

$$= \frac{1}{1 + \frac{500}{-400}} = \frac{1}{1 - 5/4} = -4$$

$$\therefore \quad \boxed{e_{ss} = -4}$$

Method - 2:

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{1 + G(s)}$$

$$= \lim_{s \to 0} \frac{\cancel{s} \times \frac{1}{\cancel{s}}}{1 + \frac{100(s+5)}{s^3 + 105 s^2 + 400 s - 400}}$$

$$= \frac{1}{1 - \frac{500}{400}}$$

$$e_{ss} = \frac{+4}{4-5}$$

$$\Rightarrow \quad \boxed{e_{ss} = -4}$$