CHAPTER 6

Work, Energy and Power

6.1 Introduction

1. A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}$, where ω is a constant. Which of the following is true?
(a) Velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin.
(b) Velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin.
(c) Velocity and acceleration both are perpendicular to $\vec{r}$.
(d) Velocity and acceleration both are parallel to $\vec{r}$.
(NEET-I 2016)

2. If vectors $\vec{A} = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$ and $\vec{B} = \cos\frac{\omega t}{2}\,\hat{i} + \sin\frac{\omega t}{2}\,\hat{j}$ are functions of time, then the value of t at which they are orthogonal to each other is
(a) $t = \frac{\pi}{\omega}$ (b) $t = 0$
(c) $t = \frac{\pi}{4\omega}$ (d) $t = \frac{\pi}{2\omega}$ *(2015)*

3. If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{j} - 4\hat{i} + \alpha\hat{k}$, then the value of α is
(a) 1/2 (b) –1/2
(c) 1 (d) –1 *(2005)*

4. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces
(a) are equal to each other
(b) are equal to each other in magnitude
(c) are not equal to each other in magnitude
(d) cannot be predicted. *(2003)*

5. The position vector of a particle is $\vec{r} = (a\cos\omega t)\,\hat{i} + (a\sin\omega t)\,\hat{j}$. The velocity of the particle is
(a) directed towards the origin
(b) directed away from the origin
(c) parallel to the position vector
(d) perpendicular to the position vector. *(1995)*

6. The angle between the two vectors $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ will be
(a) 90° (b) 180°
(c) zero (d) 45°. *(1994)*

6.2 Notions of Work and Kinetic Energy : The Work-Energy Theorem

7. Consider a drop of rain water having mass 1 g falling from a height of 1 km. It hits the ground with a speed of 50 m s^{-1}. Take g constant with a value 10 m s^{-2}. The work done by the (i) gravitational force and the (ii) resistive force of air is
(a) (i) 1.25 J (ii) –8.25 J
(b) (i) 100 J (ii) 8.75 J
(c) (i) 10 J (ii) –8.75 J
(d) (i) –10 J (ii) –8.25 J *(NEET 2017)*

8. A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion?
(a) 0.18 m/s^2 (b) 0.2 m/s^2
(c) 0.1 m/s^2 (d) 0.15 m/s^2
(NEET-I 2016)

9. A bullet of mass 10 g leaves a rifle at an initial velocity of 1000 m/s and strikes the earth at the same level with a velocity of 500 m/s. The work done in joule for overcoming the resistance of air will be
(a) 375 (b) 3750 (c) 5000 (d) 500
(1989)

6.3 Work

10. A particle moves from a point $(-2\hat{i} + 5\hat{j})$ to $(4\hat{j} + 3\hat{k})$ when a force of $(4\hat{i} + 3\hat{j})$ N is applied. How much work has been done by the force?
(a) 8 J (b) 11 J (c) 5 J (d) 2 J
(NEET-II 2016)

11. A uniform force of $(3\hat{i}+\hat{j})$ newton acts on a particle of mass 2 kg. Hence the particle is displaced from position $(2\hat{i}+\hat{k})$ metre to position $(4\hat{i}+3\hat{j}-\hat{k})$ metre. The work done by the force on the particle is
(a) 13 J (b) 15 J
(c) 9 J (d) 6 J *(NEET 2013)*

12. A body moves a distance of 10 m along a straight line under the action of a 5 N force. If the work done is 25 J, then angle between the force and direction of motion of the body is
(a) 60° (b) 75° (c) 30° (d) 45° *(1997)*

13. A body, constrained to move in y-direction, is subjected to a force given by $\vec{F}=(-2\hat{i}+15\hat{j}+6\hat{k})$ N. The work done by this force in moving the body through a distance of $10\hat{j}$ m along y-axis, is
(a) 150 J (b) 20 J
(c) 190 J (d) 160 J *(1994)*

14. When a body moves with a constant speed along a circle
(a) no work is done on it
(b) no acceleration is produced in it
(c) its velocity remains constant
(d) no force acts on it. *(1994)*

6.4 Kinetic Energy

15. A particle of mass $5m$ at rest suddenly breaks on its own into three fragments. Two fragments of mass m each move along mutually perpendicular direction with speed v each. The energy released during the process is
(a) $\frac{3}{5}mv^2$ (b) $\frac{5}{3}mv^2$
(c) $\frac{3}{2}mv^2$ (d) $\frac{4}{3}mv^2$
(Odisha NEET 2019)

16. A body of mass $(4m)$ is lying in x-y plane at rest. It suddenly explodes into three pieces. Two pieces, each of mass (m) move perpendicular to each other with equal speeds (v). The total kinetic energy generated due to explosion is
(a) mv^2 (b) $\frac{3}{2}mv^2$
(c) $2mv^2$ (d) $4mv^2$ *(2014)*

17. An engine pumps water continuously through a hose. Water leaves the hose with a velocity v and m is the mass per unit length of the water jet. What is the rate at which kinetic energy is imparted to water?
(a) mv^3 (b) $\frac{1}{2}mv^2$
(c) $\frac{1}{2}m^2v^2$ (d) $\frac{1}{2}mv^3$ *(2009)*

18. A shell of mass 200 gm is ejected from a gun of mass 4 kg by an explosion that generates 1.05 kJ of energy. The initial velocity of the shell is
(a) 40 m s^{-1} (b) 120 m s^{-1}
(c) 100 m s^{-1} (d) 80 m s^{-1} *(2008)*

19. A bomb of mass 30 kg at rest explodes into two pieces of masses 18 kg and 12 kg. The velocity of 18 kg mass is 6 m s^{-1}. The kinetic energy of the other mass is
(a) 324 J (b) 486 J
(c) 256 J (d) 524 J. *(2005)*

20. A particle of mass m_1 is moving with a velocity v_1 and another particle of mass m_2 is moving with a velocity v_2. Both of them have the same momentum but their different kinetic energies are E_1 and E_2 respectively. If $m_1 > m_2$ then
(a) $E_1 < E_2$ (b) $\frac{E_1}{E_2}=\frac{m_1}{m_2}$
(c) $E_1 > E_2$ (d) $E_1 = E_2$ *(2004)*

21. A ball of mass 2 kg and another of mass 4 kg are dropped together from a 60 feet tall building. After a fall of 30 feet each towards earth, their respective kinetic energies will be in the ratio of
(a) $\sqrt{2}:1$ (b) 1 : 4
(c) 1 : 2 (d) $1:\sqrt{2}$ *(2004)*

22. A stationary particle explodes into two particles of masses m_1 and m_2 which move in opposite directions with velocities v_1 and v_2. The ratio of their kinetic energies E_1/E_2 is
(a) m_2/m_1 (b) m_1/m_2
(c) 1 (d) m_1v_2/m_2v_1 *(2003)*

23. If kinetic energy of a body is increased by 300% then percentage change in momentum will be
(a) 100% (b) 150%
(c) 265% (d) 73.2%. *(2002)*

24. A particle is projected making an angle of 45° with horizontal having kinetic energy K. The kinetic energy at highest point will be
(a) $\frac{K}{\sqrt{2}}$ (b) $\frac{K}{2}$ (c) $2K$ (d) K
(2001, 1997)

25. Two bodies with kinetic energies in the ratio of 4 : 1 are moving with equal linear momentum. The ratio of their masses is
(a) 4 : 1 (b) 1 : 1 (c) 1 : 2 (d) 1 : 4. *(1999)*

26. Two bodies of masses m and $4m$ are moving with equal kinetic energies. The ratio of their linear momenta is
(a) 1 : 2 (b) 1 : 4 (c) 4 : 1 (d) 1 : 1.
(1998, 1997, 1989)

27. The kinetic energy acquired by a mass m in travelling distance d, starting from rest, under the action of a constant force is directly proportional to
(a) m (b) m^0
(c) $\sqrt{m}$ (d) $1/\sqrt{m}$ *(1994)*

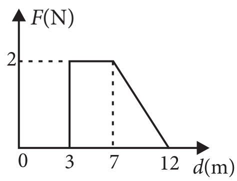
28. Two masses of 1 g and 9 g are moving with equal kinetic energies. The ratio of the magnitudes of their respective linear momenta is
(a) 1 : 9 (b) 9 : 1 (c) 1 : 3 (d) 3 : 1 *(1993)*

29. A particle of mass M is moving in a horizontal circle of radius R with uniform speed v. When it moves from one point to a diametrically opposite point, its
(a) kinetic energy change by $Mv^2/4$
(b) momentum does not change
(c) momentum change by $2Mv$
(d) kinetic energy changes by Mv^2 *(1992)*

6.5 Work done by a Variable Force

30. A force $F = 20 + 10y$ acts on a particle in y-direction where F is in newton and y in meter. Work done by this force to move the particle from $y = 0$ to $y = 1$ m is
(a) 20 J (b) 30 J (c) 5 J (d) 25 J
(NEET 2019)

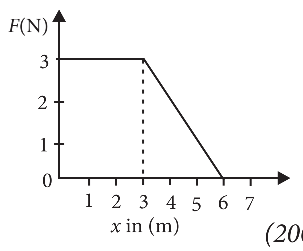
31. Force F on a particle moving in a straight line varies with distance d as shown in figure. The work done on the particle during its displacement of 12 m is



(a) 18 J (b) 21 J (c) 26 J (d) 13 J *(2011)*

32. A body of mass 3 kg is under a constant force which causes a displacement s in metres in it, given by the relation $s = \frac{1}{3}t^2$, where t is in seconds. Work done by the force in 2 seconds is
(a) $\frac{19}{5}$ J (b) $\frac{5}{19}$ J (c) $\frac{3}{8}$ J (d) $\frac{8}{3}$ J *(2006)*

33. A force F acting on an object varies with distance x as shown here. The force is in N and x in m. The work done by the force in moving the object from $x = 0$ to $x = 6$ m is



(a) 18.0 J
(b) 13.5 J
(c) 9.0 J
(d) 4.5 J. *(2005)*

34. A force acts on a 3 g particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metres and t is in seconds. The work done during the first 4 second is
(a) 490 mJ (b) 450 mJ
(c) 240 mJ (d) 530 mJ *(1998)*

35. A position dependent force,
$F = (7 - 2x + 3x^2)$ N acts on a small body of mass 2 kg and displaces it from $x = 0$ to $x = 5$ m. The work done in joule is
(a) 135 (b) 270
(c) 35 (d) 70 *(1994, 1992)*

6.7 The Concept of Potential Energy

36. The potential energy of a particle in a force field is $U = \frac{A}{r^2} - \frac{B}{r}$ where A and B are positive constants and r is the distance of the particle from the centre of the field. For stable equilibrium, the distance of the particle is
(a) $\frac{B}{2A}$ (b) $\frac{2A}{B}$ (c) $\frac{A}{B}$ (d) $\frac{B}{A}$ *(2012)*

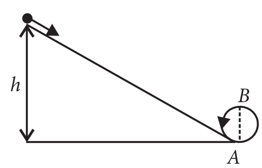
37. The potential energy of a system increases if work is done
(a) upon the system by a nonconservative force
(b) by the system against a conservative force
(c) by the system against a nonconservative force
(d) upon the system by a conservative force. *(2011)*

38. The potential energy between two atoms, in a molecule, is given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$ where a and b are positive constants and x is the distance between the atoms. The atom is in stable equilibrium, when
(a) $x = \left(\frac{2a}{b}\right)^{1/6}$ (b) $x = \left(\frac{11a}{5b}\right)^{1/6}$
(c) $x = 0$ (d) $x = \left(\frac{a}{2b}\right)^{1/6}$ *(1995)*

6.8 The Conservation of Mechanical Energy

39. A mass m is attached to a thin wire and whirled in a vertical circle. The wire is most likely to break when
(a) inclined at an angle of 60° from vertical
(b) the mass is at the highest point
(c) the wire is horizontal
(d) the mass is at the lowest point *(NEET 2019)*

40. A body initially at rest and sliding along a frictionless track from a height h (as shown in the figure) just completes a vertical circle of diameter $AB = D$. The height h is equal to



(a) $\frac{3}{2}D$ (b) D (c) $\frac{7}{5}D$ (d) $\frac{5}{4}D$
(NEET 2018)

41. What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop?

(a) $\sqrt{3gR}$ (b) $\sqrt{5gR}$

(c) $\sqrt{gR}$ (d) $\sqrt{2gR}$ *(NEET-I 2016)*

42. A particle with total energy E is moving in a potential energy region $U(x)$. Motion of the particle is restricted to the region when

(a) $U(x) < E$ (b) $U(x) = 0$

(c) $U(x) \leq E$ (d) $U(x) > E$

(Karnataka NEET 2013)

43. A stone is tied to a string of length l and is whirled in a vertical circle with the other end of the string as the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in velocity as it reaches a position where the string is horizontal (g being acceleration due to gravity) is

(a) $\sqrt{2(u^2-gl)}$ (b) $\sqrt{u^2-gl}$

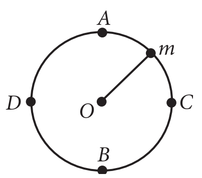
(c) $u-\sqrt{u^2-2gl}$ (d) $\sqrt{2gl}$ *(2003)*

44. A child is sitting on a swing. Its minimum and maximum heights from the ground 0.75 m and 2 m respectively, its maximum speed will be

(a) 10 m/s (b) 5 m/s

(c) 8 m/s (d) 15 m/s *(2001)*

45. As shown in the figure, a mass is performing vertical circular motion. The average velocity of the particle is increased, then at which point will the string break?



(a) A (b) B (c) C (d) D *(2000)*

6.9 The Potential Energy of a Spring

46. Two similar springs P and Q have spring constants K_P and K_Q, such that $K_P > K_Q$. They are stretched first by the same amount (case a), then by the same force (case b). The work done by the springs W_P and W_Q are related as, in case (a) and case (b) respectively

(a) $W_P > W_Q$; $W_Q > W_P$

(b) $W_P < W_Q$; $W_Q < W_P$

(c) $W_P = W_Q$; $W_P > W_Q$

(d) $W_P = W_Q$; $W_P = W_Q$ *(2015 Cancelled)*

47. A block of mass M is attached to the lower end of a vertical spring. The spring is hung from a ceiling and has force constant value k. The mass is released from rest with the spring initially unstretched. The maximum extension produced in the length of the spring will be

(a) $2Mg/k$ (b) $4Mg/k$

(c) $Mg/2k$ (d) Mg/k *(2009)*

48. A vertical spring with force constant k is fixed on a table. A ball of mass m at a height h above the free upper end of the spring falls vertically on the spring so that the spring is compressed by a distance d. The net work done in the process is

(a) $mg(h+d)-\frac{1}{2}kd^2$

(b) $mg(h-d)-\frac{1}{2}kd^2$

(c) $mg(h-d)+\frac{1}{2}kd^2$

(d) $mg(h+d)+\frac{1}{2}kd^2$ *(2007)*

49. The potential energy of a long spring when stretched by 2 cm is U. If the spring is stretched by 8 cm the potential energy stored in it is

(a) $U/4$ (b) $4U$

(c) $8U$ (d) $16U$ *(2006)*

50. A mass of 0.5 kg moving with a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant k = 50 N/m. The maximum compression of the spring would be

(a) 0.15 m (b) 0.12 m

(c) 1.5 m (d) 0.5 m *(2004)*

51. When a long spring is stretched by 2 cm, its potential energy is U. If the spring is stretched by 10 cm, the potential energy stored in it will be

(a) $U/5$ (b) $5U$

(c) $10U$ (d) $25U$ *(2003)*

52. Two springs A and B having spring constant K_A and K_B ($K_A = 2K_B$) are stretched by applying force of equal magnitude. If energy stored in spring A is E_A then energy stored in B will be

(a) $2E_A$ (b) $E_A/4$

(c) $E_A/2$ (d) $4E_A$ *(2001)*

6.10 Various Forms of Energy : The Law of Conservation of Energy

53. A body of mass 1 kg is thrown upwards with a velocity 20 m/s. It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction? (g = 10 m/s^2)

(a) 30 J (b) 40 J (c) 10 J (d) 20 J

(2009)

54. 300 J of work is done in sliding a 2 kg block up an inclined plane of height 10 m. Work done against friction is (Take g = 10 m/s^2)

(a) 1000 J (b) 200 J

(c) 100 J (d) zero. *(2006)*

6.11 Power

55. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = (2t\,\hat{i} + 3t^2\,\hat{j})$ N, where $\hat{i}$ and $\hat{j}$ are unit vectors along x and y axis. What power will be developed by the force at the time t?
(a) $(2t^3 + 3t^4)$ W (b) $(2t^3 + 3t^5)$ W
(c) $(2t^2 + 3t^3)$ W (d) $(2t^2 + 4t^4)$ W
(NEET-I 2016)

56. The heart of a man pumps 5 litres of blood through the arteries per minute at a pressure of 150 mm of mercury. If the density of mercury be 13.6×10^3 kg/m^3 and $g = 10$ m/s^2 then the power (in watt) is
(a) 3.0 (b) 1.50 (c) 1.70 (d) 2.35 *(2015)*

57. A particle of mass m is driven by a machine that delivers a constant power k watts. If the particle starts from rest, the force on the particle at time t is
(a) $\sqrt{2mk}\,t^{-1/2}$ (b) $\frac{1}{2}\sqrt{mk}\,t^{-1/2}$
(c) $\sqrt{\frac{mk}{2}}\,t^{-1/2}$ (d) $\sqrt{mk}\,t^{-1/2}$
(2015 Cancelled)

58. One coolie takes 1 minute to raise a suitcase through a height of 2 m but the second coolie takes 30 s to raise the same suitcase to the same height. The powers of two coolies are in the ratio
(a) 1 : 3 (b) 2 : 1 (c) 3 : 1 (d) 1 : 2
(Karnataka NEET 2013)

59. A car of mass m starts from rest and accelerates so that the instantaneous power delivered to the car has a constant magnitude P_0. The instantaneous velocity of this car is proportional to
(a) t^2P_0 (b) $t^{1/2}$ (c) $t^{-1/2}$ (d) $\frac{t}{\sqrt{m}}$
(Mains 2012)

60. A body projected vertically from the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest
(a) at the highest position of the body
(b) at the instant just before the body hits the earth
(c) it remains constant all through
(d) at the instant just after the body is projected
(2011)

61. An engine pumps water through a hose pipe. Water passes through the pipe and leaves it with a velocity of 2 m/s. The mass per unit length of water in the pipe is 100 kg/m. What is the power of the engine?
(a) 400 W (b) 200 W
(c) 100 W (d) 800 W *(2010)*

62. A particle of mass M, starting from rest, undergoes uniform acceleration. If the speed acquired in time T is V, the power delivered to the particle is
(a) $\frac{MV^2}{T}$ (b) $\frac{1}{2}\frac{MV^2}{T^2}$
(c) $\frac{MV^2}{T^2}$ (d) $\frac{1}{2}\frac{MV^2}{T}$ *(Mains 2010)*

63. Water falls from a height of 60 m at the rate of 15 kg/s to operate a turbine. The losses due to frictional forces are 10% of energy. How much power is generated by the turbine ? ($g = 10$ m/s^2)
(a) 12.3 kW (b) 7.0 kW
(c) 8.1 kW (d) 10.2 kW *(2008)*

64. If $\vec{F} = (60\hat{i} + 15\hat{j} - 3\hat{k})$ N and $\vec{v} = (2\hat{i} - 4\hat{j} + 5\hat{k})$ m/s, then instantaneous power is
(a) 195 watt (b) 45 watt
(c) 75 watt (d) 100 watt *(2000)*

65. How much water a pump of 2 kW can raise in one minute to a height of 10 m ? (take $g = 10$ m/s^2)
(a) 1000 litres (b) 1200 litres
(c) 100 litres (d) 2000 litres *(1990)*

6.12 Collisions

66. Body A of mass $4m$ moving with speed u collides with another body B of mass $2m$, at rest. The collision is head on and elastic in nature. After the collision the fraction of energy lost by the colliding body A is
(a) 5/9 (b) 1/9 (c) 8/9 (d) 4/9
(NEET 2019)

67. A moving block having mass m, collides with another stationary block having mass $4m$. The lighter block comes to rest after collision. When the initial velocity of the lighter block is v, then the value of coefficient of restitution (e) will be
(a) 0.5 (b) 0.25 (c) 0.8 (d) 0.4
(NEET 2018)

68. A bullet of mass 10 g moving horizontally with a velocity of 400 m s^{-1} strikes a wooden block of mass 2 kg which is suspended by light inextensible string of length 5 m. As a result, the centre of gravity of the block is found to rise a vertical distance of 10 cm. The speed of the bullet after it emerges out horizontally from the block will be
(a) 100 m s^{-1} (b) 80 m s^{-1}
(c) 120 m s^{-1} (d) 160 m s^{-1}
(NEET-II 2016)

69. Two identical balls A and B having velocities of 0.5 m s^{-1} and –0.3 m s^{-1} respectively collide elastically in one dimension. The velocities of B and A after the collision respectively will be
(a) –0.5 m s^{-1} and 0.3 m s^{-1}

(b) 0.5 m s^{-1} and –0.3 m s^{-1}
(c) –0.3 m s^{-1} and 0.5 m s^{-1}
(d) 0.3 m s^{-1} and 0.5 m s^{-1}

(NEET-II 2016, 1994, 1991)

70. Two particles A and B, move with constant velocities $\vec{v}_1$ and $\vec{v}_2$. At the initial moment their position vectors are $\vec{r}_1$ and $\vec{r}_2$ respectively. The condition for particles A and B for their collision is

(a) $\vec{r}_1 \times \vec{v}_1 = \vec{r}_2 \times \vec{v}_2$ (b) $\vec{r}_1 - \vec{r}_2 = \vec{v}_1 - \vec{v}_2$

(c) $\dfrac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \dfrac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$ (d) $\vec{r}_1 \cdot \vec{v}_1 = \vec{r}_2 \cdot \vec{v}_2$

(2015)

71. A ball is thrown vertically downwards from a height of 20 m with an initial velocity v_0. It collides with the ground, loses 50 percent of its energy in collision and rebounds to the same height. The initial velocity v_0 is (Take g = 10 m s^{-2})

(a) 28 m s^{-1} (b) 10 m s^{-1}
(c) 14 m s^{-1} (d) 20 m s^{-1} *(2015)*

72. On a frictionless surface, a block of mass M moving at speed v collides elastically with another block of same mass M which is initially at rest. After collision the first block moves at an angle θ to its initial direction and has a speed $v/3$. The second block's speed after the collision is

(a) $\dfrac{3}{\sqrt{2}}v$ (b) $\dfrac{\sqrt{3}}{2}v$ (c) $\dfrac{2\sqrt{2}}{3}v$ (d) $\dfrac{3}{4}v$

(2015)

73. Two particles of masses m_1, m_2 move with initial velocities u_1 and u_2. On collision, one of the particles get excited to higher level, after absorbing energy ε. If final velocities of particles be v_1 and v_2 then we must have

(a) $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \varepsilon = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

(b) $\frac{1}{2}m_1^2u_1^2 + \frac{1}{2}m_2^2u_2^2 + \varepsilon = \frac{1}{2}m_1^2v_1^2 + \frac{1}{2}m_2^2v_2^2$

(c) $m_1^2u_1 + m_2^2u_2 - \varepsilon = m_1^2v_1 + m_2^2v_2$

(d) $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \varepsilon$

(2015 Cancelled)

74. Two spheres A and B of masses m_1 and m_2 respectively collide. A is at rest initially and B is moving with velocity v along x-axis. After collision B has a velocity $\dfrac{v}{2}$ in a direction perpendicular to the original direction. The mass A moves after collision in the direction

(a) same as that of B
(b) opposite to that of B
(c) $\theta = \tan^{-1}\left(\dfrac{1}{2}\right)$ to the x-axis
(d) $\theta = \tan^{-1}\left(-\dfrac{1}{2}\right)$ to the x-axis *(2012)*

75. A mass m moving horizontally (along the x-axis) with velocity v collides and sticks to a mass of $3m$ moving vertically upward (along the y-axis) with velocity $2v$. The final velocity of the combination is

(a) $\frac{3}{2}v\hat{i} + \frac{1}{4}v\hat{j}$ (b) $\frac{1}{4}v\hat{i} + \frac{3}{2}v\hat{j}$
(c) $\frac{1}{3}v\hat{i} + \frac{2}{3}v\hat{j}$ (d) $\frac{2}{3}v\hat{i} + \frac{1}{3}v\hat{j}$

(Mains 2011)

76. A ball moving with velocity 2 m/s collides head on with another stationary ball of double the mass. If the coefficient of restitution is 0.5, then their velocities (in m/s) after collision will be

(a) 0, 1 (b) 1, 1 (c) 1, 0.5 (d) 0, 2 *(2010)*

77. Two equal masses m_1 and m_2 moving along the same straight line with velocities + 3 m/s and –5 m/s respectively collide elastically. Their velocities after the collision will be respectively

(a) – 4 m/s and +4 m/s
(b) +4 m/s for both
(c) – 3 m/s and +5 m/s
(d) – 5 m/s and + 3 m/s. *(1998)*

78. A rubber ball is dropped from a height of 5 m on a plane. On bouncing it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of

(a) $\dfrac{3}{5}$ (b) $\dfrac{2}{5}$
(c) $\dfrac{16}{25}$ (d) $\dfrac{9}{25}$ *(1998)*

79. A metal ball of mass 2 kg moving with speed of 36 km/h has a head on collision with a stationary ball of mass 3 kg. If after collision, both the balls move as a single mass, then the loss in K.E. due to collision is

(a) 100 J (b) 140 J (c) 40 J (d) 60 J. *(1997)*

80. A moving body of mass m and velocity 3 km/hour collides with a body at rest of mass $2m$ and sticks to it. Now the combined mass starts to move. What will be the combined velocity?

(a) 3 km/hour (b) 4 km/hour
(c) 1 km/hour (d) 2 km/hour *(1996)*

81. The coefficient of restitution e for a perfectly elastic collision is

(a) 1 (b) 0 (c) ∞ (d) –1 *(1988)*

ANSWER KEY

1. (a)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (d)	**6.** (a)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (c)
11. (c)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (d)	**16.** (b)	**17.** (d)	**18.** (c)	**19.** (b)	**20.** (a)
21. (c)	**22.** (a)	**23.** (a)	**24.** (b)	**25.** (d)	**26.** (a)	**27.** (b)	**28.** (c)	**29.** (c)	**30.** (d)
31. (d)	**32.** (d)	**33.** (b)	**34.** (c)	**35.** (a)	**36.** (b)	**37.** (b)	**38.** (a)	**39.** (d)	**40.** (d)
41. (b)	**42.** (c)	**43.** (a)	**44.** (b)	**45.** (b)	**46.** (a)	**47.** (a)	**48.** (a)	**49.** (d)	**50.** (a)
51. (d)	**52.** (a)	**53.** (d)	**54.** (c)	**55.** (b)	**56.** (c)	**57.** (c)	**58.** (d)	**59.** (b)	**60.** (b)
61. (d)	**62.** (d)	**63.** (c)	**64.** (b)	**65.** (b)	**66.** (c)	**67.** (b)	**68.** (c)	**69.** (b)	**70.** (c)
71. (d)	**72.** (c)	**73.** (a)	**74.** (d)	**75.** (b)	**76.** (a)	**77.** (d)	**78.** (a)	**79.** (d)	**80.** (c)
81. (a)									

Hints & Explanations

1. (a) : Given, $\vec{r} = \cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}$

$\therefore\quad \vec{v} = \dfrac{d\vec{r}}{dt} = -\omega\sin\omega t\,\hat{x} + \omega\cos\omega t\,\hat{y}$

$\vec{a} = \dfrac{d\vec{v}}{dt} = -\omega^2\cos\omega t\,\hat{x} - \omega^2\sin\omega t\,\hat{y} = -\omega^2\vec{r}$

Since position vector $(\vec{r})$ is directed away from the origin, so, acceleration $(-\omega^2\vec{r})$ is directed towards the origin.

Also,

$\vec{r}\cdot\vec{v} = (\cos\omega t\,\hat{x} + \sin\omega t\,\hat{y})\cdot(-\omega\sin\omega t\,\hat{x} + \omega\cos\omega t\,\hat{y})$

$= -\omega \sin\omega t \cos\omega t + \omega \sin\omega t \cos\omega t = 0$

$\Rightarrow\quad \vec{r} \perp \vec{v}$

2. (a) : Two vectors $\vec{A}$ and $\vec{B}$ are orthogonal to each other, if their scalar product is zero *i.e.* $\vec{A}\cdot\vec{B} = 0$

Here, $\vec{A} = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$

and $\vec{B} = \cos\dfrac{\omega t}{2}\,\hat{i} + \sin\dfrac{\omega t}{2}\,\hat{j}$

$\therefore\quad \vec{A}\cdot\vec{B} = (\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})\cdot\left(\cos\dfrac{\omega t}{2}\,\hat{i} + \sin\dfrac{\omega t}{2}\,\hat{j}\right)$

$= \cos\omega t\cos\dfrac{\omega t}{2} + \sin\omega t\sin\dfrac{\omega t}{2} = \cos\left(\omega t - \dfrac{\omega t}{2}\right)$

But $\vec{A}\cdot\vec{B} = 0$ (as $\vec{A}$ and $\vec{B}$ are orthogonal to each other)

$\therefore\quad \cos\left(\omega t - \dfrac{\omega t}{2}\right) = 0$

$\cos\left(\omega t - \dfrac{\omega t}{2}\right) = \cos\dfrac{\pi}{2}$ or $\omega t - \dfrac{\omega t}{2} = \dfrac{\pi}{2}$

$\dfrac{\omega t}{2} = \dfrac{\pi}{2}$ or $t = \dfrac{\pi}{\omega}$

3. (b) : $\vec{a} = 2\hat{i} + 3\hat{j} + 8\hat{k},\ \vec{b} = 4\hat{j} - 4\hat{i} + \alpha\hat{k}$

$\vec{a}\cdot\vec{b} = 0$ if $\vec{a} \perp \vec{b}$

$(2\hat{i} + 3\hat{j} + 8\hat{k})\cdot(-4\hat{i} + 4\hat{j} + \alpha\hat{k}) = 0$

or, $-8 + 12 + 8\alpha = 0 \ \Rightarrow\ 4 + 8\alpha = 0$

$\Rightarrow\ \alpha = -1/2.$

4. (b) : Given: $(\vec{F}_1 + \vec{F}_2) \perp (\vec{F}_1 - \vec{F}_2)$

$\therefore\quad (\vec{F}_1 + \vec{F}_2)\cdot(\vec{F}_1 - \vec{F}_2) = 0$

$F_1^2 - F_2^2 - \vec{F}_1\cdot\vec{F}_2 + \vec{F}_2\cdot\vec{F}_1 = 0 \Rightarrow F_1^2 = F_2^2$

i.e. F_1, F_2 are equal to each other in magnitude.

5. (d) : Position vector of the particle

$\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$

velocity vector

$\vec{v} = \dfrac{d\vec{r}}{dt} = (-a\omega\sin\omega t)\hat{i} + (a\omega\cos\omega t)\hat{j}$

$= \omega[(-a\sin\omega t)\hat{i} + (a\cos\omega t)\hat{j}]$

$\vec{v}\cdot\vec{r} = \omega a[-\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j})]\cdot[a\cos\omega t\,\hat{i} + a\sin\omega t\,\hat{j}]$

$= \omega[-a^2\sin\omega t\cos\omega t + a^2\cos\omega t\sin\omega t] = 0$

Therefore velocity vector is perpendicular to the position vector.

6. (a) : $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \dfrac{(3\hat{i} + 4\hat{j} + 5\hat{k})\cdot(3\hat{i} + 4\hat{j} - 5\hat{k})}{[\sqrt{(3)^2 + (4)^2 + (5)^2}]\times[\sqrt{(3)^2 + (4)^2 + (5)^2}]}$

$= \dfrac{9 + 16 - 25}{50} = 0$ or $\theta = 90°.$

7. (c) : Here, $m = 1$ g $= 10^{-3}$ kg,

$h = 1$ km $= 1000$ m, $v = 50$ m s^{-1}, $g = 10$ m s^{-2}.

(i) The work done by the gravitational force

$= mgh = 10^{-3} \times 10 \times 1000 = 10$ J

(ii) The total work done by gravitational force and the resistive force of air is equal to change in kinetic energy of rain drop.

$\therefore\quad W_g + W_r = \dfrac{1}{2}mv^2 - 0$

$10 + W_r = \dfrac{1}{2}\times 10^{-3}\times 50\times 50$

or $\quad W_r = -8.75$ J

8. (c) : Here, $m = 10\text{ g} = 10^{-2}\text{ kg}$, $R = 6.4\text{ cm} = 6.4 \times 10^{-2}\text{ m}$, $K_f = 8 \times 10^{-4}\text{ J}$, $K_i = 0$, $a_t = ?$

Using work energy theorem,

Work done by all the forces = Change in KE

$W_{\text{tangential force}} + W_{\text{centripetal force}} = K_f - K_i$

$$\Rightarrow a_t = \frac{K_f}{4\pi Rm} = \frac{8\times10^{-4}}{4\times\frac{22}{7}\times6.4\times10^{-2}\times10^{-2}}$$

$= 0.099 \approx 0.1\text{ m s}^{-2}$

9. (b) : Work done = change in kinetic energy of the body

$$W = \frac{1}{2}\times0.01[(1000)^2 - (500)^2] = 3750\text{ joule}$$

10. (c) : Here $\vec{r}_1 = (-2\hat{i} + 5\hat{j})\text{ m}$, $\vec{r}_2 = (4\hat{j} + 3\hat{k})\text{ m}$

$\vec{F} = (4\hat{i} + 3\hat{j})\text{ N}$, $W = ?$

Work done by force F in moving from $\vec{r}_1$ to $\vec{r}_2$,

$W = \vec{F}\cdot(\vec{r}_2 - \vec{r}_1) \Rightarrow W = (4\hat{i}+3\hat{j})\cdot(4\hat{j}+3\hat{k}+2\hat{i}-5\hat{j})$

$= (4\hat{i}+3\hat{j})\cdot(2\hat{i}-\hat{j}+3\hat{k}) = 8 + (-3) = 5\text{ J}$

11. (c) : Here, $\vec{F} = (3\hat{i} + \hat{j})\text{ N}$

Initial position, $\vec{r}_1 = (2\hat{i} + \hat{k})\text{ m}$

Final position, $\vec{r}_2 = (4\hat{i} + 3\hat{j} - \hat{k})\text{ m}$

Displacement, $\vec{r} = \vec{r}_2 - \vec{r}_1$

$\vec{r} = (4\hat{i}+3\hat{j}-\hat{k})\text{ m} - (2\hat{i}+\hat{k})\text{ m} = 2\hat{i}+3\hat{j}-2\hat{k}\text{ m}$

Work done, $W = \vec{F}\cdot\vec{r} = (3\hat{i}+\hat{j})\cdot(2\hat{i}+3\hat{j}-2\hat{k})$

$= 6 + 3 = 9\text{ J}$

12. (a) : Distance (s) = 10 m; Force (F) = 5 N and work done (W) = 25 J

Work done (W) = $Fs\cos\theta$

$\therefore$ $25 = 5 \times 10\cos\theta = 50\cos\theta$

or $\cos\theta = 25/50 = 0.5$ or $\theta = 60°$

13. (a) : Force $\vec{F} = (-2\hat{i}+15\hat{j}+6\hat{k})$ N, and distance, $d = 10\hat{j}\text{ m}$

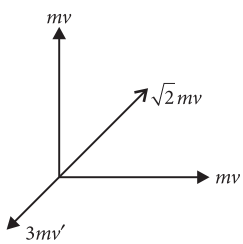
Work done, $W = \vec{F}\cdot\vec{d} = (-2\hat{i}+15\hat{j}+6\hat{k})\cdot(10\hat{j})$

$= 150\text{ Nm} = 150\text{ J}$

14. (a)

15. (d) : Let the speed of the third fragment of mass $3m$ be v'.

From law of conservation of linear momentum,



$$3mv' = \sqrt{2}\,mv \Rightarrow v' = \frac{\sqrt{2}\,v}{3} \quad \text{...(i)}$$

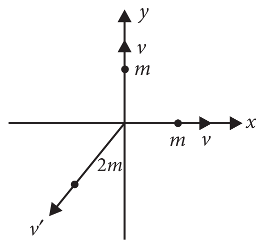
$\therefore$ Energy released during the process is,

$$\text{K.E.} = 2\left(\frac{1}{2}mv^2\right) + \frac{1}{2}(3m)v'^2 = mv^2 + \frac{1}{2}(3m)\frac{2v^2}{9}$$

(Using eqn. (i))

$$= mv^2 + \frac{mv^2}{3} = \frac{4}{3}mv^2$$

16. (b) :



Let $\vec{v}'$ be velocity of third piece of mass $2m$.

Initial momentum, $\vec{p}_i = 0$ (As the body is at rest)

Final momentum, $\vec{p}_f = mv\hat{i} + mv\hat{j} + 2m\vec{v}'$

According to law of conservation of momentum

$$\vec{p}_i = \vec{p}_f \text{ or, } 0 = mv\hat{i} + mv\hat{j} + 2m\vec{v}' \text{ or, } \vec{v}' = -\frac{v}{2}\hat{i} - \frac{v}{2}\hat{j}$$

The magnitude of v' is

$$v' = \sqrt{\left(-\frac{v}{2}\right)^2 + \left(-\frac{v}{2}\right)^2} = \frac{v}{\sqrt{2}}$$

Total kinetic energy generated due to explosion

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)v'^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)\left(\frac{v}{\sqrt{2}}\right)^2$$

$$= mv^2 + \frac{mv^2}{2} = \frac{3}{2}mv^2$$

17. (d) : Velocity of water is v, mass flowing per unit length is m.

$\therefore$ Mass flowing per second = mv

$\therefore$ Rate of kinetic energy or K.E. per second

$$= \frac{1}{2}(mv)v^2 = \frac{1}{2}mv^3$$

18. (c) : $mv = Mv' \Rightarrow v' = \left(\frac{m}{M}\right)v$

Total K.E. of the bullet and gun $= \frac{1}{2}mv^2 + \frac{1}{2}Mv'^2$

Total K.E. $= \frac{1}{2}mv^2 + \frac{1}{2}M\cdot\frac{m^2}{M^2}v^2$

Total K.E. $= \frac{1}{2}mv^2\left\{1 + \frac{m}{M}\right\}$

or $\left\{\frac{1}{2}\times0.2\right\}\left\{1 + \frac{0.2}{4}\right\}v^2 = 1.05\times1000\text{ J}$

$\Rightarrow v^2 = \frac{4\times1.05\times1000}{0.1\times4.2} = 100^2 \quad \therefore v = 100\text{ m s}^{-1}$

19. (b) : According to law of conservation of linear momentum,

$30 \times 0 = 18 \times 6 + 12 \times v$

$\Rightarrow -108 = 12v \Rightarrow v = -9\text{ m/s}.$

Negative sign indicates that both fragments move in opposite directions.

K.E. of 12 kg $= \frac{1}{2}mv^2 = \frac{1}{2}\times 12\times 81 = 486$ J

20. (a) : Kinetic energy $= \frac{p^2}{2m}$

$\therefore \quad \frac{E_1}{E_2} = \frac{p_1^2/2m_1}{p_2^2/2m_2} \Rightarrow \frac{E_1}{E_2} = \frac{m_2}{m_1}$

or $E_1 < E_2$ [as $m_1 > m_2$]

21. (c) : Ratio of their kinetic energy is given as

$$\frac{\mathrm{KE}_1}{\mathrm{KE}_2} = \frac{(1/2)\, m_1 v_1^2}{(1/2)\, m_2 v_2^2}$$

$v^2 = 2gs$ (zero initial velocity)

which is same for both

$\therefore \quad \frac{\mathrm{KE}_1}{\mathrm{KE}_2} = \frac{m_1}{m_2} = \frac{2}{4} = \frac{1}{2}$

22. (a) : $m_1v_1 = m_2v_2$

(conservation of linear momentum)

$$\frac{E_1}{E_2} = \frac{(1/2)m_1v_1^2}{(1/2)m_2v_2^2} = \frac{m_1^2v_1^2}{m_2^2v_2^2}\cdot\frac{m_2}{m_1} = \frac{m_2}{m_1}.$$

23. (a) : Let m be the mass of the body and v_1 and v_2 be the initial and final velocities of the body respectively.

$\therefore \quad$ Initial kinetic energy $= \frac{1}{2}mv_1^2$

Final kinetic energy $= \frac{1}{2}mv_2^2$

Initial kinetic energy is increased 300% to get the final kinetic energy.

$\therefore \quad \frac{1}{2}mv_2^2 = \frac{1}{2}\left(1+\frac{300}{100}\right)mv_1^2$

$\Rightarrow \quad v_2 = 2v_1 \quad$ or $\quad v_2/v_1 = 2$

Initial momentum $= p_1 = mv_1$

Final momentum $= p_2 = mv_2$

$\therefore \quad \frac{p_2}{p_1} = \frac{mv_2}{mv_1} = \frac{v_2}{v_1} = 2 \quad$ or, $\quad p_2 = 2p_1 = \left(1+\frac{100}{100}\right)p_1$

So momentum has increased 100%.

24. (b) : Kinetic energy of the ball $= K$ and angle of projection $(\theta) = 45°$.

Velocity of the ball at the highest point $= v\cos\theta$

$= v\cos 45° = \frac{v}{\sqrt{2}}$

Therefore kinetic energy of the ball

$$= \frac{1}{2}m\times\left(\frac{v}{\sqrt{2}}\right)^2 = \frac{1}{4}mv^2 = \frac{K}{2}$$

25. (d) : $\mathrm{K.E.} = \frac{p^2}{2m} \Rightarrow \frac{\mathrm{K.E.}_1}{\mathrm{K.E.}_2} = \frac{m_2}{m_1} = \frac{4}{1}$

or $\frac{m_1}{m_2} = \frac{1}{4}$

26. (a) : Mass of first body $= m$;

Mass of second body $= 4m$ and $\mathrm{KE}_1 = \mathrm{KE}_2$.

Linear momentum of a body

$p = \sqrt{2mE} \propto \sqrt{m}$

Therefore $\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m}{4m}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

or $\quad p_1 : p_2 = 1 : 2$.

27. (b) : $v^2 = u^2 + 2as \quad$ or $\quad v^2 - u^2 = 2as$

or $\quad v^2 - (0)^2 = 2\times\frac{F}{m}\times s \quad$ or $\quad v^2 = \frac{2Fs}{m}$

and $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\times\frac{2Fs}{m} = Fs.$

Thus K.E. is independent of m or directly proportional to m^0.

28. (c) : $\frac{K_1}{K_2} = \frac{p_1^2}{p_2^2}\times\frac{M_2^2}{M_1^2}$

Here $K_1 = K_2$

$\therefore \quad \frac{p_1}{p_2} = \sqrt{\frac{M_1}{M_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3} \quad$ or, $\quad p_1 : p_2 = 1 : 3$

29. (c) : On the diametrically opposite points, the velocities have same magnitude but opposite directions. Therefore change in momentum is

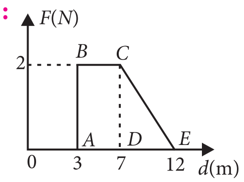
$Mv - (-Mv) = 2Mv$

30. (d) : Given : $F = 20 + 10y$

work done, $W = \int F.dy$

$$= \int_0^1 (20+10y)\,dy = \left[20y + \frac{10}{2}y^2\right]_0^1 = 20 + \frac{10}{2} = 25\text{ J}$$

31. (d) :



Work done = Area under (F-d) graph

= Area of rectangle $ABCD$ + Area of triangle DCE

$= 2\times(7-3) + \frac{1}{2}\times 2\times(12-7) = 8 + 5 = 13$ J

32. (d) : $s = \frac{t^2}{3}; \ \frac{ds}{dt} = \frac{2t}{3}; \ \frac{d^2s}{dt^2} = \frac{2}{3}$ m/s^2

Work done, $W = \int F ds = \int m\frac{d^2s}{dt^2}ds$

$$= \int m\frac{d^2s}{dt^2}\frac{ds}{dt}dt = \int_0^2 3\times\frac{2}{3}\times\frac{2t}{3}dt = \frac{4}{3}\int_0^2 t\,dt$$

$$= \frac{4}{3}\left|\frac{t^2}{2}\right|_0^2 = \frac{4}{3}\times 2 = \frac{8}{3}\text{ J}$$

33. (b) : Work done = area under F-x curve

= area of trapezium

$$= \frac{1}{2}\times(6+3)\times 3 = \frac{9\times 3}{2} = 13.5\text{ J}$$

34. (c) : $x = 3t - 4t^2 + t^3$

$$V = \frac{dx}{dt} = 3 - 8t + 3t^2$$

$V_0 = 3 - 8 \times 0 + 3 \times 0^2 = 3$ m/s

$V_4 = 3 - 8 \times 4 + 3 \times 16 = 13$ m/s

$$W = \frac{1}{2}m\left(V_4^2 - V_0^2\right) = \frac{1}{2} \times 3 \times 10^3 (13^2 - 3^2)$$

$= 240 \times 10^3$ J $= 240$ mJ

35. (a) : Force $(F) = 7 - 2x + 3x^2$;
Mass $(m) = 2$ kg and displacement $(d) = 5$ m. Therefore work done

$$(W) = \int F dx = \int_0^5 (7 - 2x + 3x^2) dx = [7x - x^2 + x^3]_0^5$$

$= (7 \times 5) - (5)^2 + (5)^3 = 35 - 25 + 125 = 135$ J

36. (b) : Here, $U = \frac{A}{r^2} - \frac{B}{r}$

For equilibrium, $\frac{dU}{dr} = 0$

$\therefore \quad -\frac{2A}{r^3} + \frac{B}{r^2} = 0$ or $\frac{2A}{r^3} = \frac{B}{r^2}$ or $r = \frac{2A}{B}$

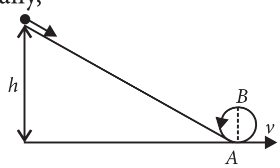
37. (b)

38. (a) : $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$ or $-\frac{12a}{x^{13}} - \frac{-6b}{x^7} = 0$

or $x^6 = \frac{2a}{b}$. Therefore $x = \left(\frac{2a}{b}\right)^{1/6}$

39. (d): In vertical circular motion, tension in the wire is maximum at the lowermost point, so the wire is most likely to break when the mass is at the lowermost point.

40. (d) : As body is at rest initially, *i.e.*, speed = 0.
At point A, speed = v.
As track is frictionless, so total mechanical energy will remain constant.



$\therefore \quad (\text{T.M.E})_i = (\text{T.M.E})_f$

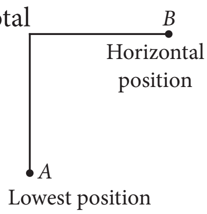
$$0 + mgh = \frac{1}{2}mv^2 + 0 \quad \text{or} \quad h = \frac{v^2}{2g}$$

For completing the vertical circle, $v \geq \sqrt{5gR}$

$$\therefore \quad h = \frac{5gR}{2g} = \frac{5}{2}R = \frac{5}{4}D$$

41. (b) **42. (c)**

43. (a) : The total energyat A = the total energy at B



$$\Rightarrow \quad \frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgl$$

$$\Rightarrow \quad v = \sqrt{u^2 - 2gl}$$

The change in magnitude of velocity $= \sqrt{u^2 + v^2}$

$= \sqrt{2(u^2 - gl)}$

44. (b) : Maximum drop in P.E. = maximum gain in K.E.

$$mg(2 - 0.75) = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2g(1.25)} = 5 \text{ m/s}$$

45. (b) : When a mass is moving in a vertical circle, it string experiences the maximum force when it is at the lowest point B.
Therefore, tension at B is maximum

$$= \text{Weight} + \frac{mv^2}{R}$$

So, the string breaks at point B.

46. (a) : Here, $K_P > K_Q$
Case (a) : Elongation (x) in each spring is same.

$$W_P = \frac{1}{2}K_P x^2, \; W_Q = \frac{1}{2}K_Q x^2 \quad \therefore \quad W_P > W_Q$$

Case (b) : Force of elongation is same.

So, $x_1 = \frac{F}{K_P}$ and $x_2 = \frac{F}{K_Q}$

$$W_P = \frac{1}{2}K_P x_1^2 = \frac{1}{2}\frac{F^2}{K_P}$$

$$W_Q = \frac{1}{2}K_Q x_2^2 = \frac{1}{2}\frac{F^2}{K_Q} \quad \therefore W_P < W_Q$$

47. (a) : When the mass attached to a spring fixed at the other end is allowed to fall suddenly, it extends the spring by x. Potential energy lost by the mass is gained by the spring.

$$Mgx = \frac{1}{2}kx^2 \Rightarrow x = \frac{2Mg}{k}$$

48. (a) : Net work done = $W_{mg} + W_{spring}$

$$= mg(h + d) - \frac{1}{2}kd^2$$

49. (d) : Potential energy of a spring

$= \frac{1}{2} \times$ force constant $\times$ (extension)^2

$\therefore$ Potential energy $\propto$ (extension)^2

or, $\frac{U_1}{U_2} = \left(\frac{x_1}{x_2}\right)^2$ or, $\frac{U_1}{U_2} = \left(\frac{2}{8}\right)^2$

or, $\frac{U_1}{U_2} = \frac{1}{16}$ or, $U_2 = 16U_1 = 16U \qquad (\because U_1 = U)$

50. (a) : The kinetic energy of mass is converted into energy required to compress a spring which is given by

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad x = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{0.5 \times (1.5)^2}{50}} = 0.15 \text{ m}$$

51. (d) : $U = -kx^2$, k = Spring constant

$$\frac{U_1}{U_2} = \frac{x_1^2}{x_2^2} = \frac{4}{100} \quad \Rightarrow \quad U_2 = 25\,U_1$$

52. (a) : Energy $=\frac{1}{2}Kx^2=\frac{1}{2}\frac{F^2}{K}$

$\because \frac{K_A}{K_B}=2;\ \therefore \frac{E_A}{E_B}=\frac{1}{2}$ or $E_B=2E_A$

53. (d) : Initial velocity u = 20 m/s; m = 1 kg
Kinetic energy = maximum potential energy

Initial kinetic energy = $\frac{1}{2}\times1\times20^2=200$ J

Mgh (max) = 200 J
$\therefore$ h = 20 m.
The height travelled by the body, h' = 18 m
$\therefore$ Loss of energy due to air friction
= $mgh - mgh'$
$\Rightarrow$ Energy lost = 200 J – 1 × 10 × 18 J = 20 J

54. (c) : Gain in potential energy = mgh
= 2 × 10 × 10 = 200 J
Gain in potential energy + work done against friction
= work done = 300 J
$\therefore$ Work done against friction = 300 – 200 = 100 J

55. (b) : Here, $\vec{F}=(2t\hat{i}+3t^2\hat{j})$ N, m = 1 kg

Acceleration of the body, $\vec{a}=\frac{\vec{F}}{m}=\frac{(2t\hat{i}+3t^2\hat{j})\text{ N}}{1\text{ kg}}$

Velocity of the body at time t,

$$\vec{v}=\int\vec{a}dt=\int(2t\hat{i}+3t^2\hat{j})\,dt=t^2\hat{i}+t^3\hat{j}\text{ m s}^{-1}$$

$\therefore$ Power developed by the force at time t,

$$P=\vec{F}\cdot\vec{v}=(2t\hat{i}+3t^2\hat{j})\cdot(t^2\hat{i}+t^3\hat{j})\text{ W}=(2t^3+3t^5)\text{ W}$$

56. (c) : Here, Volume of blood pumped by man's heart,
V = 5 litres = 5 × 10^{-3} m^3 ($\because$ 1 litre = 10^{-3} m^3)
Time in which this volume of blood pumps,
t = 1 min = 60 s
Pressure at which the blood pumps,
P = 150 mm of Hg = 0.15 m of Hg
= (0.15 m)(13.6 × 10^3 kg/m^3)(10 m/s^2)
= 20.4 × 10^3 N/m^2

$\therefore$ Power of the heart $=\frac{PV}{t}$

$$=\frac{(20.4\times10^3\text{ N/m}^2)(5\times10^{-3}\text{ m}^3)}{60\text{ s}}=1.70\text{ W}$$

57. (c) : Constant power acting on the particle of mass m is k watt.

or $P=k;\ \frac{dW}{dt}=k;\ \ dW=kdt$

Integrating both sides, $\int_0^W dW=\int_0^t k\,dt$

$\Rightarrow W=kt$ ...(i)

Using work energy theorem,

$$W=\frac{1}{2}mv^2-\frac{1}{2}m(0)^2\Rightarrow kt=\frac{1}{2}mv^2\quad\text{[Using (i)]}$$

$$v=\sqrt{\frac{2kt}{m}}$$

Acceleration of the particle, $a=\frac{dv}{dt}$

$$a=\frac{1}{2}\sqrt{\frac{2k}{m}}\frac{1}{\sqrt{t}}=\sqrt{\frac{k}{2mt}}$$

Force on the particle, $F=ma=\sqrt{\frac{mk}{2t}}=\sqrt{\frac{mk}{2}}\,t^{-1/2}$

58. (d) : Power, $P=\frac{\text{Work done}}{\text{Time taken}}$

Here work done (= mgh) is same in both cases.

$\therefore \frac{P_1}{P_2}=\frac{t_2}{t_1}=\frac{30\text{ s}}{1\text{ min}}=\frac{30\text{ s}}{60\text{ s}}=\frac{1}{2}$

59. (b) : $P_0=Fv$

$\because F=ma=m\frac{dv}{dt}$

$\therefore P_0=mv\frac{dv}{dt}$ or $P_0dt=mvdv$

Integrating both sides, we get $\int_0^t P_0dt=m\int_0^v vdv$

$$P_0t=\frac{mv^2}{2}$$

$$v=\left(\frac{2P_0t}{m}\right)^{1/2}\text{ or } v\propto\sqrt{t}$$

60. (b) : Power, $\vec{F}\cdot\vec{v}=Fv\cos\theta$
Just before hitting the earth θ = 0°. Hence, the power exerted by the gravitational force is greatest at the instant just before the body hits the earth.

61. (d) : Here,
Mass per unit length of water, μ = 100 kg/m
Velocity of water, v = 2 m/s
Power of the engine, $P=\mu v^3$
= (100 kg/m) (2 m/s)^3 = 800 W

62. (d) : Power delivered in time T is
$P=F\cdot V=MaV$

or $P=MV\frac{dV}{dT}\Rightarrow PdT=MVdV$

$\Rightarrow PT=\frac{MV^2}{2}$ or $P=\frac{1}{2}\frac{MV^2}{T}$

63. (c) : Mass of water falling/second = 15 kg/s
h = 60 m, g = 10 m/s^2, loss = 10% *i.e.*, 90% is used.
Power generated = 15 × 10 × 60 × 0.9 = 8100 W = 8.1 kW

64. (b) : $P=\vec{F}\cdot\vec{v}=(60\hat{i}+15\hat{j}-3\hat{k})\cdot(2\hat{i}-4\hat{j}+5\hat{k})$
= 120 – 60 – 15 = 45 watts

65. (b) : Power $=\frac{\text{work done}}{\text{time taken}}=\frac{W}{t}$

$\therefore P=\frac{M\times g\times h}{t}$

$\Rightarrow \quad M = \dfrac{P \times t}{g \times h} = \dfrac{2000 \times 60}{10 \times 10} = 1200 \text{ kg}$

i.e., 1200 litres as one litre has a mass of 1 kg.

66. (c) : According to conservation of mome-ntum,

$4mu_1 = 4mv_1 + 2mv_2 \Rightarrow 2(u_1 - v_1) = v_2$ ...(i)

From conservation of energy,

$$\frac{1}{2}(4m)u_1^2 = \frac{1}{2}(4m)v_1^2 + \frac{1}{2}(2m)v_2^2$$

$\Rightarrow \quad 2(u_1^2 - v_1^2) = v_2^2$ ...(ii)

From (i) and (ii), $2(u_1^2 - v_1^2) = 4(u_1 - v_1)^2$

$3v_1 = u_1$ ...(iii)

Now, fraction of loss in kinetic energy for mass $4m$,

$$\frac{\Delta K}{K_i} = \frac{K_i - K_f}{K_i} = \frac{\frac{1}{2}(4m)u_1^2 - \frac{1}{2}(4m)v_1^2}{\frac{1}{2}(4m)u_1^2} \quad \text{...(iv)}$$

Substituting (iii) in (iv), we get $\dfrac{\Delta K}{K_i} = \dfrac{8}{9}$

67. (b) : Let final velocity of the block of mass, $4\,m = v'$

Initial velocity of block of mass $4\,m = 0$

Final velocity of block of mass $m = 0$

According to law of conservation of linear momentum,

$mv + 4m \times 0 = 4mv' + 0 \Rightarrow v' = v/4$

Coefficient of restitution,

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} = \frac{v/4}{v} = 0.25$$

68. (c) : Mass of bullet, $m = 10 \text{ g} = 0.01 \text{ kg}$

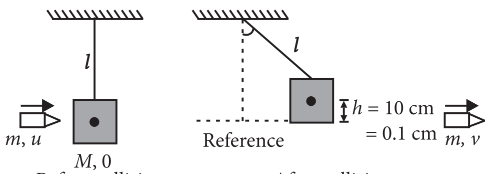
Initial speed of bullet, $u = 400 \text{ m s}^{-1}$

Mass of block, $M = 2$ kg

Length of string, $l = 5$ m

Speed of the block after collision $= v_1$

Speed of the bullet on emerging from block, $v = ?$



Before collision After collision

Using energy conservation principle for the block,

$(\text{KE} + \text{PE})_{\text{Reference}} = (\text{KE} + \text{PE})_h$

$\Rightarrow \quad \dfrac{1}{2}Mv_1^2 = Mgh \quad \text{or,} \quad v_1 = \sqrt{2gh}$

$v_1 = \sqrt{2 \times 10 \times 0.1} = \sqrt{2} \text{ m s}^{-1}$

Using momentum conservation principle for block and bullet system,

$(M \times 0 + mu)_{\text{Before collision}} = (M \times v_1 + mv)_{\text{After collision}}$

$\Rightarrow \quad 0.01 \times 400 = 2\sqrt{2} + 0.01 \times v$

$\Rightarrow \quad v = \dfrac{4 - 2\sqrt{2}}{0.01} = 117.15 \text{ m s}^{-1} \approx 120 \text{ m s}^{-1}$

69. (b) : Masses of the balls are same and collision is elastic, so their velocity will be interchanged after collision.

70. (c) : Let the particles A and B collide at time t. For their collision, the position vectors of both particles should be same at time t, *i.e.*,

$\vec{r}_1 + \vec{v}_1 t = \vec{r}_2 + \vec{v}_2 t\,;\ \vec{r}_1 - \vec{r}_2 = \vec{v}_2 t - \vec{v}_1 t = (\vec{v}_2 - \vec{v}_1)t$ ...(i)

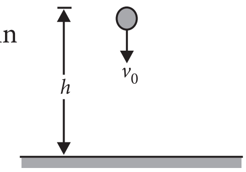
Also, $|\vec{r}_1 - \vec{r}_2| = |\vec{v}_2 - \vec{v}_1|\,t$ or $t = \dfrac{|\vec{r}_1 - \vec{r}_2|}{|\vec{v}_2 - \vec{v}_1|}$

Substituting this value of t in eqn. (i), we get

$$\vec{r}_1 - \vec{r}_2 = (\vec{v}_2 - \vec{v}_1)\frac{|\vec{r}_1 - \vec{r}_2|}{|\vec{v}_2 - \vec{v}_1|}$$

or $\dfrac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \dfrac{(\vec{v}_2 - \vec{v}_1)}{|\vec{v}_2 - \vec{v}_1|}$

71. (d) : The situation is shown in the figure.



Let v be the velocity of the ball with which it collides with ground. Then according to the law of conservation of energy,

Gain in kinetic energy = loss in potential energy

i.e. $\dfrac{1}{2}mv^2 - \dfrac{1}{2}mv_0^2 = mgh$

(where m is the mass of the ball)

or $v^2 - v_0^2 = 2gh$ ...(i)

Now, when the ball collides with the ground, 50% of its energy is lost and it rebounds to the same height h.

$\therefore \quad \dfrac{50}{100}\left(\dfrac{1}{2}mv^2\right) = mgh$

$\dfrac{1}{4}v^2 = gh \quad \text{or} \quad v^2 = 4gh$

Substituting this value of v^2 in eqn. (i), we get

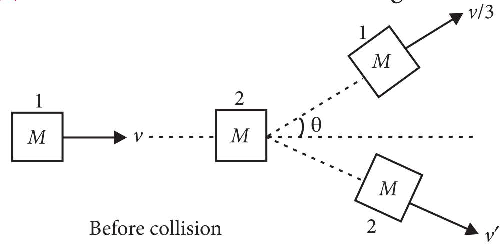
$4gh - v_0^2 = 2gh$

or $v_0^2 = 4gh - 2gh = 2gh$ or $v_0 = \sqrt{2gh}$

Here, $g = 10 \text{ m s}^{-2}$ and $h = 20$ m

$\therefore \quad v_0 = \sqrt{2(10 \text{ m s}^{-2})(20 \text{ m})} = 20 \text{ m s}^{-1}$

72. (c) : The situation is shown in the figure.



Before collision

After collision

Let v' be speed of second block after the collision.

As the collision is elastic, so kinetic energy is conserved.

According to conservation of kinetic energy,

$$\frac{1}{2}Mv^2 + 0 = \frac{1}{2}M\left(\frac{v}{3}\right)^2 + \frac{1}{2}Mv'^2$$

$v^2 = \dfrac{v^2}{9} + v'^2 \quad \text{or} \quad v'^2 = v^2 - \dfrac{v^2}{9} = \dfrac{9v^2 - v^2}{9} = \dfrac{8}{9}v^2$

$$v' = \sqrt{\frac{8}{9}v^2} = \frac{\sqrt{8}}{3}v = \frac{2\sqrt{2}}{3}v$$

73. (a) : Total initial energy of two particles

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

Total final energy of two particles

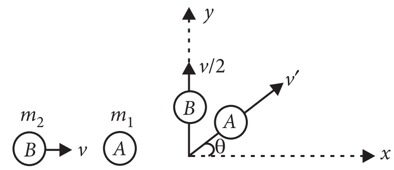
$$= \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_1v_1^2 + \varepsilon$$

Using energy conservation principle,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \varepsilon$$

$$\therefore \quad \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \varepsilon = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

74. (d) :



Before collision After collision

According to law of conservation of linear momentum along x-axis, we get

$$m_1 \times 0 + m_2 \times v = m_1v'\cos\theta$$

$$m_2v = m_1v'\cos\theta$$

$$\text{or} \quad \cos\theta = \frac{m_2v}{m_1v'} \qquad \text{...(i)}$$

According to law of conservation of linear momentum along y-axis, we get

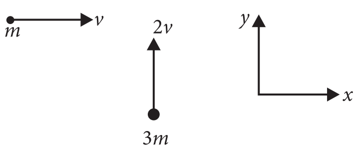
$$m_1 \times 0 + m_2 \times 0 = m_1v'\sin\theta + m_2\frac{v}{2} \Rightarrow -m_2\frac{v}{2} = m_1v'\sin\theta$$

$$\sin\theta = -\frac{m_2v}{2m_1v'} \qquad \text{...(ii)}$$

Divide (ii) by (i), we get

$$\tan\theta = -\frac{1}{2} \quad \text{or} \quad \theta = \tan^{-1}\left(-\frac{1}{2}\right) \text{to the } x\text{-axis}$$

75. (b) :



According to conservation of momentum, we get

$$mv\,\hat{i} + (3m)2v\,\hat{j} = (m + 3m)\vec{v'}$$

where $\vec{v'}$ is the final velocity after collision

$$\vec{v'} = \frac{1}{4}v\,\hat{i} + \frac{6}{4}v\,\hat{j} = \frac{1}{4}v\,\hat{i} + \frac{3}{2}v\,\hat{j}$$

76. (a) : Here, $m_1 = m$, $m_2 = 2m$

$u_1 = 2$ m/s, $u_2 = 0$

Coefficient of restitution, $e = 0.5$

Let v_1 and v_2 be their respective velocities after collision.

Applying the law of conservation of linear momentum, we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\therefore \quad m \times 2 + 2m \times 0 = m \times v_1 + 2m \times v_2$$

$$\text{or} \quad 2m = mv_1 + 2mv_2 \quad \text{or} \quad 2 = (v_1 + 2v_2) \qquad \text{...(i)}$$

By definition of coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$\text{or} \quad e(u_1 - u_2) = v_2 - v_1$$

$$\Rightarrow \quad 0.5(2 - 0) = v_2 - v_1 \qquad \text{...(ii)}$$

$$1 = v_2 - v_1$$

Solving equations (i) and (ii), we get

$v_1 = 0$ m/s, $v_2 = 1$ m/s

77. (d) : Equal masses after elastic collision interchange their velocities.

–5 m/s and +3 m/s.

78. (a) : Initial energy equation

$$mgh = \frac{1}{2}mv^2 \quad i.e.\ 10 \times 5 = \frac{1}{2}v_1^2 \Rightarrow v_1 = 10$$

After one bounce,

$$10 \times 1.8 = \frac{1}{2}v_2^2 \Rightarrow v_2 = 6$$

Loss in velocity on bouncing $\frac{6}{10} = \frac{3}{5}$ a factor.

79. (d) : Mass of metal ball = 2 kg;

Speed of metal ball (v_1) = 36 km/h = 10 m/s and mass of stationary ball = 3 kg

Applying law of conservation of momentum,

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v$$

$$\text{or,} \quad v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(2 \times 10) + (3 \times 0)}{2 + 3} = \frac{20}{5}$$

$$= 4 \text{ m/s}$$

Therefore loss of energy

$$= \left[\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right] - \frac{1}{2} \times (m_1 + m_2)v^2$$

$$= \left[\frac{1}{2} \times 2 \times (10)^2 + \frac{1}{2} \times 3(0)^2\right] - \frac{1}{2} \times (2 + 3) \times (4)^2$$

= 100 – 40 = 60 J

80. (c) : Mass of body (m_1) = m; Velocity of first body (u_1) = 3 km/hour; Mass of second body at rest (m_2) = $2m$ and velocity of second body (u_2) = 0.

After combination, mass of the body

$$M = m + 2m = 3m$$

From the law of conservation of momentum, we get

$$Mv = m_1u_1 + m_2u_2$$

$$\text{or} \quad 3mv = (m \times 3) + (2m \times 0) = 3m \quad \text{or} \quad v = 1 \text{ km/hour.}$$

81. (a) : For a perfectly elastic collision, $e = 1$ and for a perfectly inelastic collision, $e = 0$.

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