CHAPTER 04

Analysis and Design by Limit State Method (LSM)

4.1 Introduction

In any type of structure, the basic structural elements viz. beams, columns, walls, slabs, footings etc. are subjected to bending, transverse shear, torsion and axial tension or compression. In the subject of Strength of Materials, we talk of pure flexure i.e. shear stresses are zero. But in practical situations, the concept of pure flexure rarely exists; rather flexure is often associated with transverse shear and/or axial forces (tension or compression). Torsion is also sometimes encountered along with flexure.

In this chapter, we will look into the analysis of reinforced concrete beams subjected to pure flexure only. Furthermore, we will discuss the design aspects of reinforced concrete beams by **limit state method.** The additional effects associated with flexure viz. transverse shear, torsion etc. are dealt in separate chapters.

4.2 Analysis of Beams by LSM

In the previous chapters, we studied the WSM of analysis and design wherein the structure was subjected to **working loads**. This chapter deals with the **limit state method** of analysis and design at **ultimate loads**. Two types of Limit States are defined in Limit State Method of design viz. Limit State of Collapse and Limit State of Serviceability which are defined as below:

Limit State of Collapse: In this state, loads are corresponding to impending failure of structure and lead to complete collapse of the structure. It depicts the **imaginary behavior of structure** at the time of failure. It includes limit state of flexure, shear, compression and torsion.

Limit State of Serviceability: In this state, the loads and stresses which are applicable in the day-to-day service of the structure and structure is expected to perform its intended function. It depicts the **actual behavior of structure** at the time of service of the structure. It includes limit state of deflection, cracking, vibration, corrosion etc.

4.3 Assumptions in the Analysis Design by LSM

As per Cl. 38.1 of IS 456: 2000, the following assumptions are made while analyzing the reinforced concrete beam by LSM:

1. Plane sections normal to the beam axis remain plane after bending, i.e., in an initially straight beam, strain varies linearly over the depth of the section. Thus, strain diagram is linear as shown in the **Fig. 4.1**.

2. The maximum compressive strain in concrete at the outermost fiber (ε_{cu}) is taken as 0.0035, regardless of whether the beam is under-reinforced or over-reinforced, because collapse invariably occurs by the crushing of concrete.

3. The design stress-strain curve of concrete in flexural compression as recommended by IS 456: 2000 is as shown in **Fig. 4.2**. The IS 456: 2000 also allows the use of any other possible shape of the stress-strain curve which results in substantial agreement with the results of the tests on reinforced concrete.

4. For design purposes, compressive strength of concrete may be assumed as **0.67** times the characteristic strength of concrete. The partial safety factor of $\gamma_c = 1.5$ shall be applied in addition to this.

5. The tensile strength of concrete is ignored i.e. not taken into account. **Cl. B-1.3(b) of IS 456: 2000** states that all tensile stresses are to be taken up by reinforcement and none by concrete, except as otherwise specifically permitted.

6. The stress in reinforcement is derived from representative stress - strain curve for the type of steel used. The design stress-strain curves for mild steel and cold-worked bars are as shown in **Fig. 4.4** and **Fig. 4.5**.

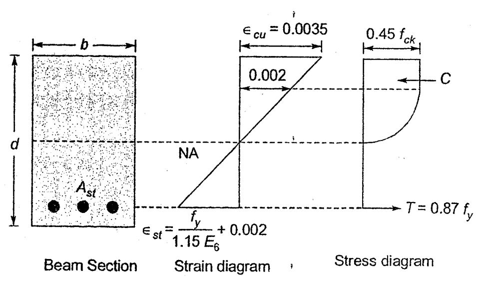


Fig. 4.1 *Singly reinforced beam section*

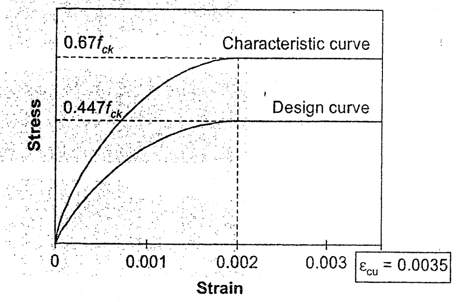


Fig. 4.2 *Stress-strain curve of concrete in flexural compression*

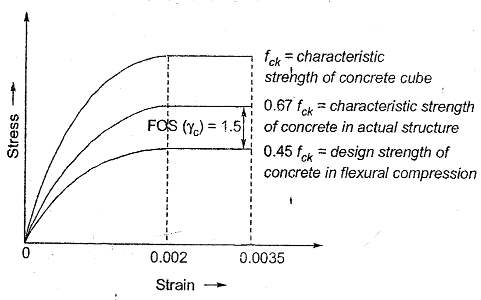


Fig. 4.3 *Characteristic, actual and design stress-strain curve of concrete in flexural compression*

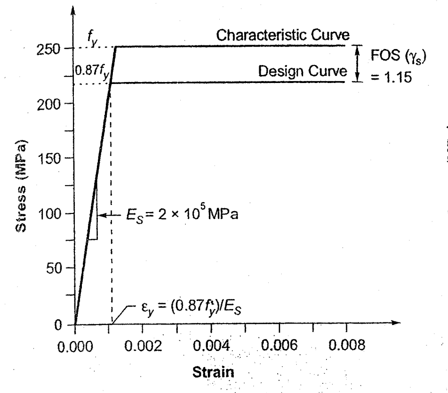


Fig. 4.4 *Characteristic and design stress strain curve for mild steel (Fe 250)*

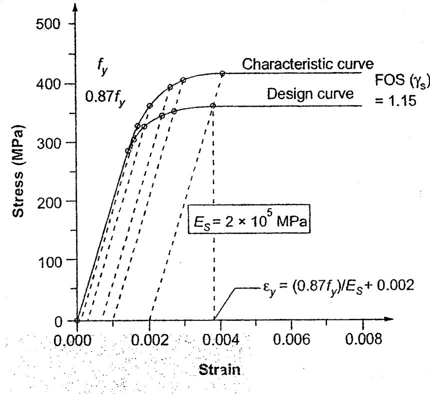


Fig. 4.5 *Characteristic and design stress strain curve for cold worked Fe 415 grade steel*

7. For design purpose, the partial safety factor for steel is taken as $\gamma_s = 1.15$ i.e.

$$\text{design stress of steel} = \frac{f_y}{1.15} = 0.87 f_y$$

8. The maximum strain (ε_{st}) in the tension reinforcement at the level of centroid of reinforcement steel at the ultimate limit state shall not be less than ε_{st} which is defined as:

$$\varepsilon_{st} = \frac{0.87 f_y}{E_s} + 0.002$$

Mild steel has a well defined yield point $\left(\varepsilon_y = \frac{0.87 f_y}{E_s}\right)$ but this is not so for HYSD bars. IS 456: 2000 specify a uniform criterion of yield for all grades of steel. This is done to ensure that the yielding of the tension steel takes place at the ultimate limit state, so that the consequent failure is ductile in nature and thus providing ample warning of the impending collapse by limiting the minimum permissible steel strain to $\left(\frac{0.87 f_y}{E_s} + 0.002\right)$.

A very rare type of RCC failure is "FRACTURE OF REINFORCING STEEL" which happens due to extremely low amounts of reinforcing steel and under the conditions of dynamic loading.

Remember

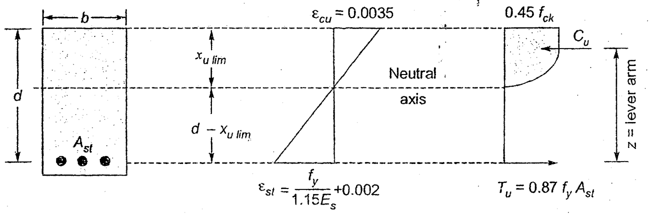
Limitation of the Assumption "Plane Sections Remain Plane before and after the Bending".

This assumption is NOT applicable in case of **deep beams** where warping of cross-section occurs due to shear deformations.

4.4 Analysis of Singly Reinforced Sections

4.4.1 Limiting Depth of Neutral Axis

It corresponds to the loading when concrete compression and steel tension, both reach their ultimate limit state simultaneously.



(a) Beam section (b) Strains (c) Stresses

Fig. 4.6 *Reinforced concrete rectangular section at ultimate limit state of flexure*

Strain diagram is used for calculating the limiting depth of neutral axis. From strain diagram, by similar triangles,

$$\frac{0.0035}{x_{u\,lim}} = \frac{\frac{f_y}{1.15E_s} + 0.002}{d - x_{u\,lim}}$$

$$\frac{d - x_{u\,lim}}{x_{u\,lim}} = \frac{\frac{f_y}{1.15E_s} + 0.002}{0.0035}$$

Thus,

$$\frac{x_{u\,lim}}{d} = k = \frac{700}{0.87f_y + 1100}$$

$$\Rightarrow \quad x_{u\,lim} = kd = \left(\frac{700}{0.87f_y + 1100}\right)d$$

4.4.2 $\frac{x_{u\,lim}}{d}$ For Different Grades of Steel

The limiting depth of neutral axis corresponds to balanced failure at ultimate limit state i.e. concrete and steel will reach to their limiting strain in simultaneously. If $x_u < x_{ulim}$, then the section is under-reinforced leading to yielding in steel first and if $x_u > x_{ulim}$ then the section is over-reinforced leading to crushing of concrete first.

Table 4.1 : *Limiting depth of neutral axis*

Steel grade	Fe 250	Fe 415	Fe 500
$k = \frac{x_{u\,lim}}{d}$	0.53	0.48	0.46

4.4.3 Analysis of Singly Reinforced Rectangular Section

Analyzing a reinforced concrete member at ultimate limit state is equivalent to determining the ultimate moment of resistance (M_{uR}) of the reinforced concrete section. This is computed by,

$$M_{uR} = C_u \cdot z = T_u \cdot z$$

Where, C_u and T_u are the ultimate compression and tension forces in the concrete and steel respectively.

Analysis of Stress Diagram

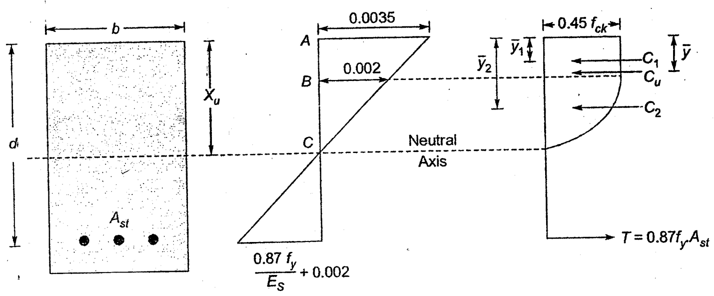


Fig. 4.7 *Analysis of stress diagram*

By similar triangles of strain diagram,

$$\frac{BC}{AC} = \frac{0.002}{0.0035} = \frac{4}{7}$$

$$BC = \frac{4}{7}AC = \frac{4}{7}x_u$$

$$AB = x_u - \frac{4}{7}x_u = \frac{3}{7}x_u$$

Compressive Force

For determining the value of C_u and its line of action, it is essential to analyze the concrete stress block under compression.

$$C_1 = \text{width x area of stress diagram}$$

$$= b \times 0.45\, f_{ck} \times \left(\frac{3}{7}\right)x_u = 0.193\, f_{ck}\, bx_u$$

$$y_1 = \left(\frac{1}{2}\right) \times \left(\frac{3}{7}\right)x_u = \left(\frac{3}{14}\right)x_u$$

$$C_2 = b \times \left(\frac{2}{3} \times 0.45 f_{ck} \times \frac{4}{7}x_u\right) = 0.1714\, f_{ck}\, bx_u$$

$$y_2 = \left(\frac{3}{7}\right)x_u + \left(\frac{3}{8}\right) \times \left(\frac{4}{7}\right)x_u = 0.643\, x_u$$

Total Compressive Force

$$C_u = C_1 + C_2$$

$$= 0.193 f_{ck}\, b x_u + 0.1714 f_{ck}\, bx_u = 0.36\, f_{ck}\, bx_u$$

This total compressive force (C) acts at a distance $\bar{y}$ from top of compression fibre which is given by,

$$\bar{y} = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2} = \frac{(0.193 \times 0.214 + 0.1714 \times 0.643)}{0.3644} x_u$$

$= 0.416x_u$ (As per previous code IS 456: 1978)

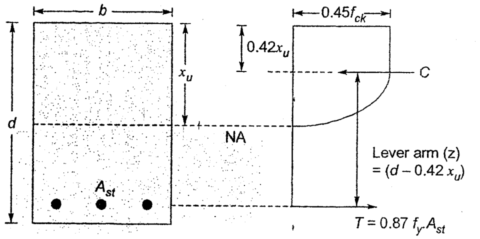
$= 0.42x_u$ (As per recent code IS 456: 2000)

Alternatively it can also be derived as,

The line of action of C_u is determined by the centroid of the concrete stress block in compression. Thus distance of C_u from the fibers subjected to the maximum compressive strain is:

$$0.362 f_{ck} b x_u \cdot \bar{x} = (0.447 f_{ck} b x_u)\left[\left(\frac{3}{7}\right)\cdot\left(\frac{1.5x_u}{7}\right)+\left(\frac{2}{3}\cdot\frac{4}{7}\right)\left(x_u-\frac{5}{8}\cdot\frac{4x_u}{7}\right)\right]$$

$$\Rightarrow \quad \bar{x} = 0.42x_u$$



Tensile Force

$$T_u = f_{st} \cdot A_{st}$$

Where $f_{st} = 0.87 f_y$ for all $x_u \le x_{u\,lim}$

$\therefore \quad T_u = 0.87 f_y A_{st}$

The line of action of T_u is the centroid of tension steel.

Actual Depth of Neutral Axis

For any given section, the location of neutral axis will be such that it satisfies the condition of static equilibrium i.e., $C_u = T_u$. Thus,

Total force of compression = Total force of tension

$$C = T$$

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st} \qquad (f_{st} = 0.87 f_y \text{ if } x_u \le x_{u\,lim})$$

$$\Rightarrow \quad x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \text{ for all } x_u \le x_{u\,lim}$$

NOTE: When $x_u > x_{ulim}$, then section is over-reinforced and $f_{st} \ne 0.87 f_y$ and in this case, a trial and error procedure is followed for determining the value of x_u.

Table 4.2 : *Difference between Tension and Compression failure of a beam section*

TENSION FAILURE	COMPRESSION FAILURE
Tension reinforcement yields before limit state of collapse of beam.	Tension reinforcement does not yield prior to limit state of collapse of beam.
In this, the beam shows large amounts of deflection and excessive cracks before its actual failure. This type of failure gives ample warning and is said to be ductile failure.	In this, the beam fails by crushing of concrete. This type of failure is sudden without giving any warning and thus this type of failure is always to be avoided.

Moment of Resistance

For determining the moment of resistance M_R, the lever arm z i.e., the distance between compressive force (C) and tensile force (T) is given by:

$$z = d - 0.42\,x_u$$

Thus moment of resistance w.r.t. compressive force of beam section

$$M_R = 0.362\, f_{ck}\, b x_u (d - 0.42\,x_u)$$

and the moment of resistance w.r.t. tensile force of beam section

$$M_R = f_{st} \cdot A_{st} (d - 0.42\,x_u)$$

$$= 0.87 f_y A_{st} (d - 0.42\,x_u) \quad \text{if } x_u \le x_{u\,lim}$$

Limiting Moment of Resistance

The limiting moment of resistance for a singly reinforced rectangular section is obtained when $x_u = x_{u\,lim}$.

Thus, $$M_{ulim} = 0.362\, f_{ck}\, b x_{u\,lim} (d - 0.42\,x_{u\,lim})$$

$$\Rightarrow \quad \frac{M_{u\,lim}}{f_{ck} b d^2} = 0.362\left(\frac{x_{u\,lim}}{d}\right)\left(1 - 0.42\frac{x_{u\,lim}}{d}\right)$$

The values of the above quantity i.e. $\frac{M_{u\,lim}}{f_{ck} b d^2}$ is obtained as 0.148, 0.138 and 0.133 for Fe 250, Fe 415 and Fe 500 respectively by substituting the values of $\frac{x_{u\,lim}}{d}$ as discussed earlier.

Comparison of x_u and x_{ulim}

(a) When $x_u < x_{ulim}$, then the section is under - reinforced

and, $$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \text{ i.e., steel has been yielded } (f_{st} = 0.87 f_y)$$

(b) When $x_u = x_{u\,lim}$, then section is **balanced** and use $x_{u\,lim}$ for calculation the moment of resistance (M_R)

$$M_{u\,lim} = 0.36 f_{ck}\, b x_{ulim} (d - 0.42 x_{ulim}) \qquad \text{(from compression side)}$$

$$M_{u\,lim} = 0.87 f_y A_{st} (d - 0.42 x_{ulim}) \qquad \text{(from tension side)}$$

(c) When $x_u > x_{u\,lim}$, then section is over-reinforced and M_u is limited to $M_{u\,lim}$ by limiting x_u to $x_{u\,lim}$.

$M_{u\,lim}$ can be found using compression side formula as:

$$M_{ulim} = 0.36 f_{ck}\, b x_{ulim} (d - 0.42 x_{ulim})$$

Remember: Do not use tension side formula because $f_{st} \ne 0.87 f_y$ in this case. Due to more reinforcement, steel has not reached to its yield stress value.

Design Formula

While designing a beam section, x_u is limited to $x_{u\,lim}$ i.e.

$$x_u = x_{ulim}$$

and $$M_u = M_{ulim}$$

From compression side,

$$M_u = 0.362\, f_{ck}\, b x_{ulim} (d - 0.42\,x_{ulim})$$

Put $$x_{ulim} = kd$$

$\therefore$ $$M_u = 0.362\, f_{ck}\, bkd (d - 0.42\,kd)$$

$$= [0.362\, f_{ck}\, k (1 - 0.42\,k)]\, bd^2$$

Thus, $M_u = Qbd^2$

$\Rightarrow$ $$d = \sqrt{\frac{M_u}{Qb}}$$

where, $Q = 0.362\, f_{ck} k(1 - 0.42k)$

For example: For Fe 415, $k = 0.48$ $(\because x_{u\,lim} = 0.48d$ for Fe 415)

$\therefore$ $Q = 0.36\, f_{ck} 0.48\,(1 - 0.42 \times 0.48) = 0.138\, f_{ck}$

Lever Arm: For limiting section, lever arm is given as:

$$\text{Lever arm} = d - 0.42\, x_{ulim}$$
$$= d - 0.42\,kd = (1 - 0.42\,k)\,d = jd$$

where j = Lever arm factor $= 1 - 0.42\,k$

$= 1 - 0.42 \times 0.53 = 0.777$ For Fe 250

$= 1 - 0.42 \times 0.48 = 0.79$ For Fe 415

$= 1 - 0.42 \times 0.46 = 0.806$ For Fe 500

Thus if can be inferred that $j \simeq 0.8$ for all the thre grades of steel.

Reinforcement Steel Requirement

$$M_{u\,lim} = 0.87 f_y A_{st}(d - 0.42x_{ulim})$$

$$A_{st} = \frac{M_{ulim}}{0.87 f_y (d - 0.42x_{ulim})} = \frac{M_{ulim}}{0.87 f_y jd}$$

Alternate procedure for determining the area of steel required

Let $M_u = M_{u\,lim}$ and $x_u \le x_{u\,lim}$ so that $f_{st} = 0.87 f_y$

Now, $C = T$

$\Rightarrow$ $0.36\, f_{ck}\, bx_u = 0.87 f_y A_{st}$

$\Rightarrow$ $$\left(\frac{x_u}{d}\right) = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}$$

$$M_u = 0.36\, f_{ck} bx_u (d - 0.42\, x_u)$$

$\Rightarrow$ $$M_u = 0.36 f_{ck} b\left(\frac{x_u}{d}\right)\left(1 - 0.42\frac{x_u}{d}\right)d^2$$

$\Rightarrow$ $$0.42\left(\frac{x_u}{d}\right)^2 - \left(\frac{x_u}{d}\right) + \frac{M_u}{0.36 f_{ck} bd^2} = 0$$

This is a quadratic equation in $\left(\frac{x_u}{d}\right)$ which on solving gives,

$$\frac{x_u}{d} = 1.202\left[1 - \sqrt{1 - \frac{4.598R}{f_{ck}}}\right]$$

where, the quantity $\frac{M_u}{bd^2}$ is sometimes denoted as 'R'

Now, $M_u = 0.87\, f_y A_{st}(d - 0.42x_u)$

$\Rightarrow$ $$A_{st} = \frac{M_u}{0.87 f_y (d - 0.42x_u)}$$

$$= \frac{M_u}{0.87 f_y d\left(1 - 0.42\frac{x_u}{d}\right)}$$

$\Rightarrow$ $$\frac{A_{st}}{bd} = \frac{M_u}{0.87 f_y bd^2\left(1 - 0.42\frac{x_u}{d}\right)}$$

By substituting the value of $\left(\frac{x_u}{d}\right)$ as computed above, area of steel required (A_{st}) can be determined.

Alternatively: Area of steel can also be determined without actually calculating $\left(\frac{x_u}{d}\right)$.

$$M_u = 0.87\, f_y A_{st}(d - 0.416x_u) \text{ for all } x_u \le x_{u\,lim}$$

Now, $C = T$

$\Rightarrow$ $0.362\, f_{ck}\, bx_u = 0.87 f_y A_{st}$

$\Rightarrow$ $$\frac{A_{st}}{bd} = \frac{p_t}{100} = \frac{0.362 f_{ck} x_u}{0.87 f_y d} \quad \text{where, } p_t = \frac{A_{st}}{bd} \times 100 \text{ and } p_t \le p_{tlim}$$

$\Rightarrow$ $$\left(\frac{x_u}{d}\right) = \frac{0.87 f_y p_t}{100(0.362) f_{ck}}$$

$\therefore$ $M_u = 0.87 f_y A_{st}[d - 0.416x_u]$

$\Rightarrow$ $$\frac{M_u}{bd^2} = \frac{0.87 f_y A_{st}}{bd}\left(1 - 0.416\frac{x_u}{d}\right)$$

$$= 0.87 f_y\left(\frac{p_t}{100}\right)\left[1 - \frac{0.416(0.87 f_y p_t)}{100(0.362 f_{ck})}\right]$$

$$= 0.87 f_y\left(\frac{p_t}{100}\right)\left[1 - \frac{f_y p_t}{100 f_{ck}}\right]$$

$$= 0.87 f_y\left(\frac{p_t}{100}\right)\left[1 - \left(\frac{p_t}{100}\right)\frac{f_y}{f_{ck}}\right]$$

$\Rightarrow$ $$\left(\frac{f_y}{f_{cu}}\right)\left(\frac{p_t}{100}\right)^2 - \left(\frac{p_t}{100}\right) + \frac{R}{0.87 f_y} = 0 \qquad \text{where, } R = \frac{M_u}{bd^2}$$

This is a quadratic equation in $\left(\frac{p_t}{100}\right)$ which on solving gives,

$$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-4.598\frac{R}{f_{ck}}}\right] \qquad \text{where, } R = \frac{M_u}{bd^2}$$

The above expression can be directly used for calculating the area of tension reinforcement required.

Limiting Percentage of Tension Steel

Corresponding to limiting moment of resistance for singly reinforced sections, there is a limiting percentage of tension steel given by:

$$p_{t\,lim} = \frac{100\,A_{st\,lim}}{bd} \qquad \text{when, } x_u = x_{u\,lim}$$

Now,

$$C = T$$

$$\Rightarrow \quad 0.362 f_{ck}\, b\, x_{u\,lim} = 0.87 f_y A_{st\,lim}$$

$$\frac{x_{u\,lim}}{d} = \frac{0.87 f_y}{0.362 f_{ck}} \cdot \frac{p_{t\,lim}}{100} \qquad \text{where, } p_{t\,lim} = \frac{A_{st\,lim}}{bd} \times 100$$

$$\Rightarrow \quad \frac{A_{st\,lim}}{bd}\left(\frac{0.87 f_y}{0.362 f_{ck}}\right) = \left(\frac{x_{u\,lim}}{d}\right)$$

$$\Rightarrow \quad p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u\,lim}}{d}\right)$$

Under reinforced sections undergo ductile failure giving ample warning to the users prior to its complete collapse. For this, $p_t \le p_{t\,lim}$ and corresponding $x_u \le x_{u\,lim}$.

Thus,

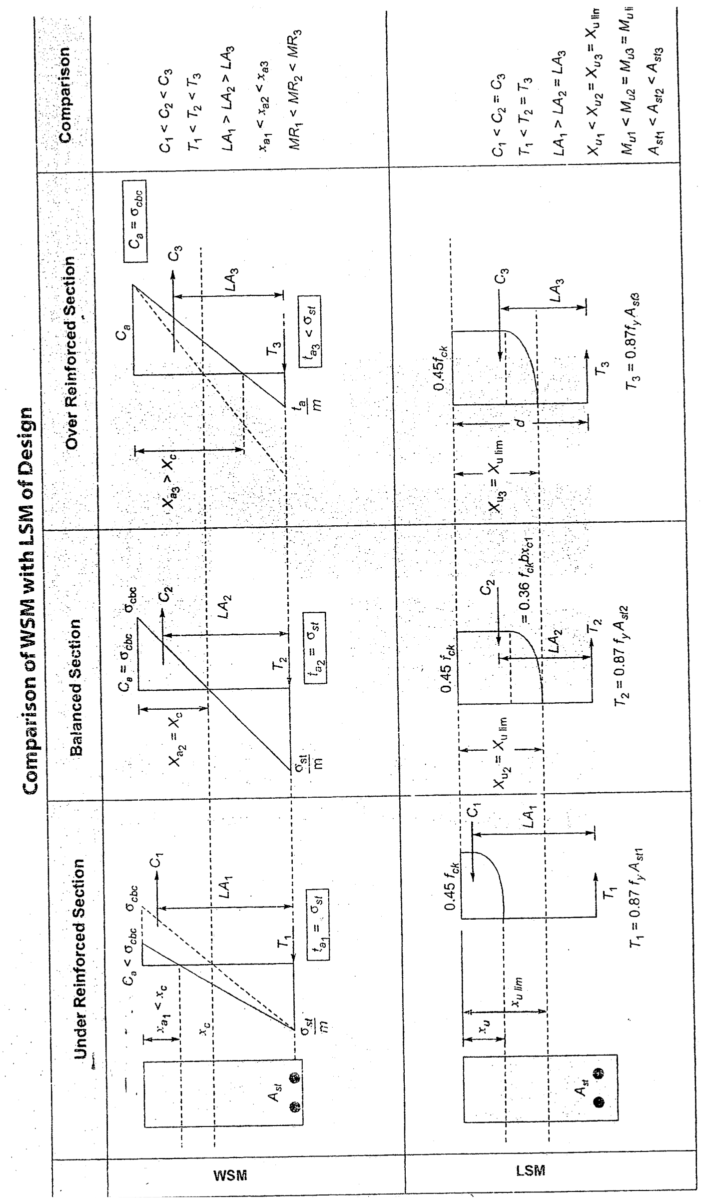
When $p_t = p_{t\,lim}$, then section is **balanced**.

When $p_t < p_{t\,lim}$, then section is **under-reinforced**.

When $p_t > p_{t\,lim}$, then section is **over-reinforced**.

Table 4.3: *$p_{t\,lim}$ (in %) for different grades of concrete and steel*

f_{ck} (N/mm^2)	Fe 250 ($f_y = 250\ N/mm^2$)	Fe 415 ($f_y = 415\ N/mm^2$)	Fe 500 ($f_y = 500\ N/mm^2$)
20	1.77	0.96	0.77
25	2.21	1.2	0.96
30	2.65	1.44	1.15
35	3.09	1.68	1.34
40	3.54	1.92	1.53
45	3.98	2.16	1.72
50	4.42	2.4	1.91



Example 4.1 Determine the depth of neutral axis of a beam section of 250 mm x 400 mm size reinforced with 3-20 mm diameter bars of Fe 250 grade steel. The beam is made up of M 20 concrete and effective cover to reinforcement is 50 mm.

Solution:

Effective depth of the beam (d) = 400 mm − 50 mm = 350 mm

Area of tensile steel (A_{st}) = $3\times\left(\frac{\pi}{4}\right)\times 20^2 = 942.5\ \text{mm}^2$

Limiting depth of neutral axis ($x_{u\,lim}$) for Fe250 steel

= 0.53 d = 0.53 × 350 mm = 185.5 mm

Let depth of neutral axis (x_u) < $x_{u\,lim}$ so that $f_{st} = 0.87 f_y$

From statical equilibrium, $C = T$

$$0.36 f_{ck}\, b x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87\times250\times942.5}{0.36\times20\times250} = 113.26\ \text{mm}$$

$$< x_{u\,lim} (= 185.5\ \text{mm})$$

So, the assumption of $x_u < x_{u\,lim}$ is true and thus stress in steel reaches to yield stress i.e. 0.87 f_y.
Therefore, depth of neutral axis (x_u) = 113.26 mm.

Example 4.2 Determine the lever arm for a rectangular beam of size 300 mm x 450 mm with 50 mm effective cover which is made up of M 25 concrete and is reinforced with 4-20 mm diameter bars of Fe 415 grade of steel.

Solution:

Area of steel (A_{st}):

$$A_{st} = 4\times\frac{\pi}{4}\times20^2 = 1256.64\ \text{mm}^2$$

Compressive force of concrete (C) = $0.36 f_{ck}\, b x_u$

Tensile force of steel (T) = $0.87 f_y A_{st}$ if $x_u \le x_{u\,lim}$

From statical equilibrium, $C = T$

$$0.36 f_{ck}\, b x_u = 0.87 f_y A_{st}$$

$$0.36\times25\times300 x_u = 0.87\times415\times1256.64$$

$$x_u = 167.11\ \text{mm}$$

Also limiting depth of neutral axis ($x_{u\,lim}$) = 0.48d (for Fe 415)

= 0.48 × (450 − 50) = 192 mm > x_u (= 167.11 mm)

Thus,

steel will reach its yield point first i.e. $f_{st} = 0.87 f_y$

Thus, lever arm (z) = $d - 0.42 x_u$

= (400 − 0.42 × 167.11) mm = 330.48 mm

Example 4.3 A rectangular beam of size 300 mm x 500 mm is required to resist a moment of 30 kNm. Find the reinforcement required for the beam if grade of concrete is M25 and steel is Fe500. Assume effective cover is 50 mm.

Solution:

Working moment to be resisted = 30 kNm

Factored moment to be resisted (M_u) = 1.5 x 30 kNm = 45 kNm

Let effective cover of the beam section = 50 mm

Thus, effective depth of beam section (d) = 500 mm − 50 mm = 450 mm

$$R = \frac{M_u}{bd^2} = \frac{45\times10^6}{(300\times450^2)} = 0.74\ \text{N/mm}^2$$

Area of steel required:

$$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-4.598\frac{R}{f_{ck}}}\right]$$

$$\Rightarrow \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{25}{2\times500}\left[1-\sqrt{1-4.598\frac{0.74}{25}}\right] = 0.0017653$$

$$\Rightarrow \quad p_t = 0.176\%$$

$$\therefore \quad A_{st} = \frac{0.176}{100}\times300\times450 = 238.3\ \text{mm}^2$$

Depth of Neutral Axis:

From statical equilibrium, $C = T$

$$0.36 f_{ck}\, b x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87\times500\times238.3}{0.36\times25\times300} = 38.18\ \text{mm}$$

Check:

Limiting depth of neutral axis ($x_{u\,lim}$) for Fe500 steel = 0.46d = 0.46 × 450 mm = 207 mm > x_u (OK)

Thus, the steel reaches its yield point first i.e. $f_{st} = 0.87 f_y$

Also limiting percentage of tension steel (p_{tlim}),

$$p_{tlim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{ulim}}{d}\right)$$

$$p_{tlim} = 41.61\left(\frac{25}{500}\right)(0.46) = 0.957\% > 0.176\% \qquad \text{(OK)}$$

Example 4.4 Determine the ultimate moment of resistance of a rectangular beam of width 350 mm with 550 mm effective depth which is casted with M30 grade of concrete and reinforced with 4 numbers of 25 mm diameter bars of Fe250 steel grade.

Solution:

Area of steel (A_{st}):

$$A_{st} = 4\times\frac{\pi}{4}\times d^2 = 4\times\frac{\pi}{4}\times25^2 = 1963.5\ \text{mm}^2$$

Limiting depth of neutral axis (x_{ulim}) for Fe250 steel = 0.53d = 0.53 × 550 = 291.5 mm

Actual depth of neutral axis (x_u):

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87\times250\times1963.5}{0.36\times30\times350} = 112.36\,\text{mm} < x_{u\,lim}\ (= 291.5\text{ mm})$$

Thus, steel stress $f_{st} = 0.87\, f_y$ is correct.

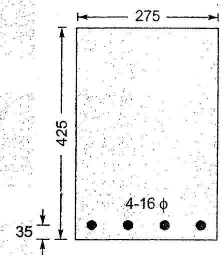
Ultimate moment of resistance of beam:

Since, the section is under-reinforced, the ultimate moment of resistance of the beam is given by

$$M_{uR} = 0.87 f_y A_{st}(d - 0.42\,x_u)$$
$$= 0.87\,(250)\,(1963.5)\,(550 - 0.42 \times 112.36)\text{ Nmm}$$
$$= 214.92\text{ kNm} \simeq 215\text{ kNm}$$

Example 4.5 Determine the lever arm and moment of resistance of the beam section as shown. Use M 20 concrete and Fe 250 steel.

Solution:



Given $b = 275$ mm

$$A_{st} = 4\times\frac{\pi}{4}\times16^2 = 804.25\text{ mm}^2$$

Effective cover = 35 mm

∴ Effective depth (d) = 425 − 35 = 390 mm

Limiting depth of neutral axis for Fe 250 steel ($x_{u\,lim}$) = 0.53 d

= 0.53 (390) = 207.09 mm

Let depth of neutral axis $(x_u) < x_{u\,lim}$

∴ $f_{st} = 0.87\, f_y$

From statical equilibrium $C = T$

⇒ $0.36\, f_{ck}\, b x_u = 0.87\, f_y A_{st}$

$$\Rightarrow \quad x_u = \frac{0.87(250)804.25}{0.36(20)275}$$

$= 87.86\text{ mm} < x_{u\,lim}$

Thus assumption of $x_u < x_{u\,lim}$ is correct.

⇒ Steel will yield first i.e. $f_{st} = 0.87\, f_y$ is correct

∴ Lever arm (z) = $d - 0.42\,x_u = 390 - 0.42\,(87.86)$

= 353.45 mm

∴ Moment of resistance = 0.87(250) (804.25) (353.45) Nmm = 61.83 kNm

Ultimate moment of resistance corresponding to balanced section

$M_{u\,lim} = 0.148\, f_{ck}\, bd^2$ (for Fe 250)

$= 0.148(20)\,275\,(390)^2$

$= 123.81\text{ kNm} > 61.83\text{ kNm}$

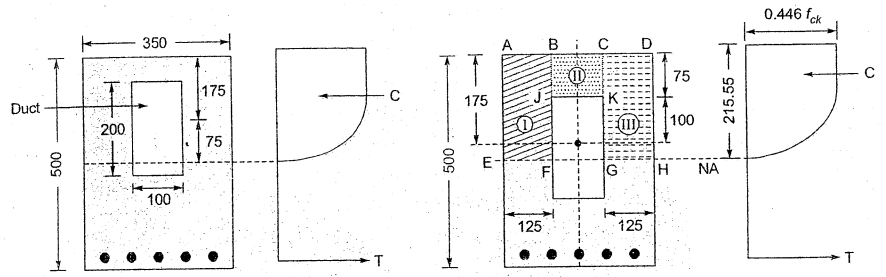
Thus the given beam section is under-reinforced ($x_u < x_{u\,lim}$)

with lever arm (z) = 353.45 mm

and moment of resistance of 61.83 kNm

Example 4.6 A beam of size 350 × 500 mm is having a duct of rectangular section of size 100 × 200 mm which is symmetric about the vertical axis of beam section and the centre of duct is at a distance of 175 mm from the top of the beam section. Compute the moment of resistance of the balanced section and thus the area of steel required. Assume M 20 concrete and Fe 415 steel. Assume effective cover is 50 mm.

Solution:



Given, M 20 concrete ⇒ f_{ck} = 20 N/mm²

Fe 415 steel ⇒ f_y = 415 N/mm²

b = 350 mm

D = 500 mm

Effective cover = 50 mm

∴ Effective depth (d) = 500 mm − 50 mm = 450 mm

Distance of top edge of duct from top of beam = CK = 75 mm

Limiting depth of neutral axis ($x_{u\,lim}$) = 0.48 d = 0.48 (450) = 215.55 mm

Depth of rectangular portion of compressive stress = $\frac{3}{7}x_{u\,lim} = \frac{3}{7}(215.55)$ mm = 92.38 mm

Divide the area above the balanced neutral axis into 3 parts with

portion ABFE → Part I

portion BCKJ → Part II

and portion CDHG → Part III

Part I and III are each of width 125 mm and are under the influence of complete compressive stress block over a depth of $x_{u\,lim}$ (= 215.55 mm). Part II is of width 100 mm which is under the influence of compressive stress block for a depth of h,= 75 mm. This part II is within the rectangular stress block because h = 75 mm depth $< \frac{3}{7}x_{u\,lim}$ (= 92.38 mm)

Let, C_1 = Compressive force due to Part I

∴ $C_1 = C_3 = 0.36\, f_{ck}\, b\, x_{u\,lim}$

= 0.36 (20)(125) 215.55 N = 193.995 kN

C_2 = Compressive force due to Part II

$C_2 = 0.45\, f_{ck}\, bh = 0.45\,(20)\,(100)\,75\text{ N} = 67.5\text{ kN}$

C_3 = Compressive force due to Part III

From statical equilibrium,

$$C_1 + C_2 + C_3 = T$$

$\Rightarrow \quad 195.995 + 67.5 + 193.99 = T$

$\Rightarrow \quad T = 455.49\text{ kN}$

At balanced stage, $\quad f_{st} = 0.87\, f_y$

$\therefore \quad T = 0.87\, f_y A_{st} = 455.49 \times 10^3\text{ N}$

$$\Rightarrow \quad A_{st} = \frac{457.046 \times 10^3}{0.87(415)} = 1261.57\text{ mm}^2$$

Moment of resistance at balanced stage (MOR)

$= (C_1 + C_3)\,(d - 0.42\, x_{u\,lim}) + C_2\,(d - 75/2)$

$= 2 \times 193.995 \times 10^3(450 - 0.42 \times 215.55) + 67.5 \times 10^3(450 - 37.5)\text{Nmm}$

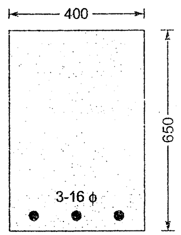
$= 167.314\text{ kNm}$

Example 4.7 Calculate MR of a rectangular beam of size 400 × 650 mm. Which is reinforced with: (a) 3-16 mmϕ (b) 6-25 mmϕ (c) 8-25 mmϕ

Calculate area of steel required for limiting section also. Use M 20 concrete and Fe 415 steel. Assume effective cover is 50 mm.

Solution:

Case (a) 3-16 mm ϕ bars



$D = 650\text{ mm}$

$d = 650 - 50 = 600\text{ mm}$

Area of steel $= 3 \times \frac{\pi}{4} \times 16^2 = 603.2\text{ mm}^2$

Limiting depth of NA

$x_{u\,lim} = 0.48 \times d \quad$ (for Fe 415)

$= 0.48 \times 600 = 288\text{ mm}$

Actual depth of NA

$C = T$

$0.36 f_{ck}\, b x_u = 0.87 f_y\, A_{st}$

(Assuming under reinforced section so that $f_{st} = 0.87 f_y$)

$0.36 \times 30 \times 400 \times x_u = 0.87 \times 415 \times 603.2$

$x_u = 50.41\text{ mm} < x_{u\,lim} (= 288\text{ mm})$

Thus assumption of under-reinforced section is correct.

i.e., $\quad x_u < x_{u\,lim}$

Moment of resistance

$M_R = 0.36 f_{ck} b x_u (d - 0.42 x_u)$

$= 0.36 \times 30 \times 400 \times 50.41\,(600 - 0.42 \times 50.41)$

$= 126.05\text{ kNm}$

$\Rightarrow$ Unfactored moment/working moment $= \frac{126.05}{1.5} = 84.03\text{ kNm}$

From tension side

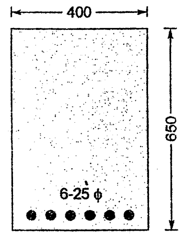
$M_R = 0.87 f_y A_{st}(d - 0.42 x_u)$

$= 0.87 \times 415 \times 603.2\,(600 - 0.42 \times 50.41)$

$= 126.06\text{ kNm}$

Unfactored $M_R = \frac{126.05}{1.5} = 84.04\text{ kNm}$

Case (b) 6-25 mm ϕ bars



Given : $\quad A_{st} = 6 \times \frac{\pi}{4} \times 25^2 = 2945.24\text{ mm}^2$

Limiting depth of NA $= 0.48d$

$\Rightarrow \quad x_{u\,lim} = 0.48(650 - 50)\text{ mm} = 288\text{ mm}$

Actual depth of NA

$C = T$

$0.36 f_{ck} b x_u = 0.87 f_y A_{st} \quad$ (assuming under reinforced section)

$0.36 \times 30 \times 400 \times x_u = 0.87 \times 415 \times 2945.24$

$x_u = 246.15\text{ mm} < x_{u\,lim} (= 288\text{ mm})$

So section is under reinforced section

Moment of resistance

$M_R = 0.36 f_{ck} b x_u (d - 0.42 x_u)$

$= 0.36 \times 30 \times 400 \times 246.15(600 - 0.42 \times 246.15)$

$= 528.09\text{ kNm}$

Unfactored moment/working moment $= \frac{528.09}{1.5} = 352.06\text{ kNm}$

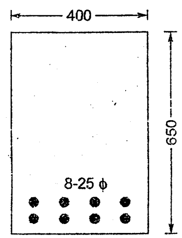
Moment of resistance from tension side

$M_R = 0.87 f_y A_{st}(d - 0.42 x_u) = 528.09\text{ kNm}$

$0.87 \times 415 \times 2945.24\,[600 - 0.42 \times 246.15]$

Unfactored moment $= \frac{528.09}{1.5} = 352.06\text{ kNm}$

Case (c) 8-25 mm ϕ bars



Given: $\quad$ Area of steel $= 8 \times \frac{\pi}{4} \times 25^2 = 3927\text{ mm}^2$

Limiting depth of NA $\quad x_{u\,lim} = 0.48d = 0.48 \times 600 = 288\text{ mm}$

Actual depth of NA

$x_u = 0.36 f_{ck}\, b x_u = 0.87\, f_y\, A_{st}$

$x_u = 328\text{ mm} > x_{u\,lim} (= 288\text{ mm})$

Thus section is over reinforced and $f_{st} \neq 0.87 f_y$. So limit x_u to $x_{u\,lim}$

$\therefore \quad M_u = 0.36 f_{ck}\, b x_{u\,lim}\,(d - 0.42\, x_{u\,lim})$

$= 0.36 \times 30 \times 400 \times 288\,(d - 0.42 \times 288) = 596\text{ kNm}$

Alternatively $\quad M_u = 0.138 f_{ck}\, bd^2 = 596\text{ kNm}$ (for Fe 415)

$$\therefore \quad \text{Unfactored } M_R = \frac{596}{1.5} = 397 \text{ kNm}$$

Area of steel for limiting section:

$$A_{st\,lim} = \frac{M_{u\,lim}}{0.87 f_y \left(d - 0.42 x_{u\,lim}\right)}$$

$$= \frac{596 \times 10^6}{0.87 \times 415(600 - 0.42 \times 288)} = 3445.9 \text{ mm}^2$$

Alternatively

$$p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u\,lim}}{d}\right) = 41.61\left(\frac{30}{415}\right)(0.48) = 1.44\%$$

$$\therefore \quad A_{st\,lim} = \frac{1.44}{100} \times 400 \times 600 = 3465.6 \text{ mm}^2$$

4.5 Requirements of Flexural Design

1. **Concrete Cover to Reinforcement:** Concrete cover is provided to protect the reinforcing bars from corrosion, fire and also provides sufficient bond between steel and concrete to prevent slipping out of reinforcing bars from concrete. **Clear cover** is defined as the distance of exposed surface of concrete to the nearest surface of reinforcing steel. **Cl. 26.4.1 of IS 456: 2000** defines **nominal cover** as **"The design depth of concrete cover to all steel reinforcement, including links."** **IS 456: 2000** specifies the deviation in concrete cover from 0 mm to +10 mm. The tolerance '0 mm' indicates that no reduction in the prescribed nominal cover is allowed.

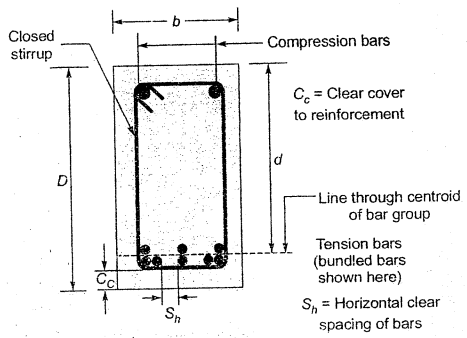
Table 4.4: *Nominal cover requirements based on exposure conditions*

Exposure condition	Minimum Grade of concrete	Nominal Cover (mm)	Permitted allowance
Mild	M 20	20	Can be reduced by 5 mm for bars less than 12 mm dia
Moderate	M 25	30	
Severe	M 30	45	Can by reduced by 5 mm if concrete grade is M 35 or higher
Very Severe	M 35	50	
Extreme	M 40	75	

NOTE

The earlier version of code i.e. **IS 456: 1978** specified clear cover requirements based on the type of structural elements (like 15 mm in slabs, 25 mm in beams, 40 mm in columns etc.), but in the latest version i.e. IS **456: 2000**, clear cover requirements are now made applicable for all types of structural elements. Cl. 26.4.2.1 of **IS 456: 2000** specifies certain minimum clear cover requirements for columns (generally 40 mm) and Cl. 26.4.2.2 of IS **456: 2000** specifies a certain minimum clear cover requirements for footings (generally 50 mm).

Apart from the above stated clear cover requirements, **Cl. 26.4.3 of IS 456: 2000** specifies clear cover requirements based on **fire resistance (expressed in terms of hours).** For 1 hour fire occurrence, a nominal cover of 20 mm for beams and 40 mm for columns has been specified in the code. Larger cover is required if the member has to be specifically designed for fire resistance.



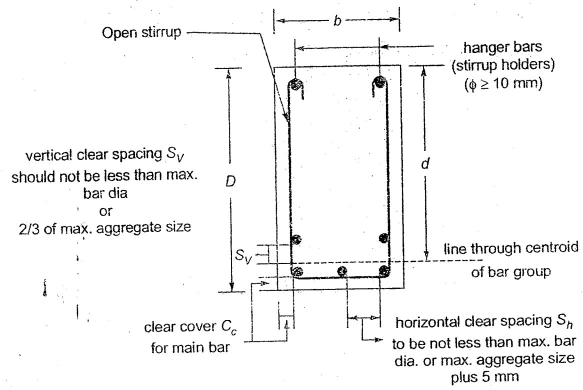
(c) Doubly reinforced beam

Fig. 4.8 *IS 456: 2000 requirements for placement of reinforcing bars*

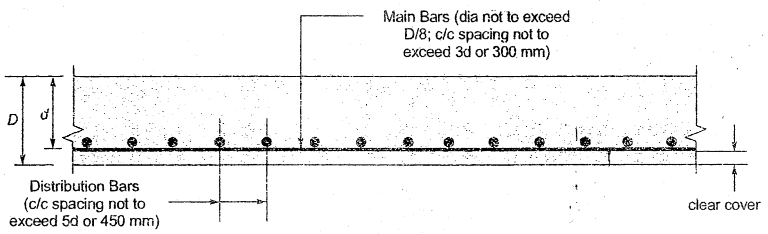
2. **Spacing of Reinforcing Bars:** Minimum spacing of parallel reinforcing bars are specified in order to ensure that the concrete can be easily placed and compacted during casting. Maximum limit on spacing of parallel reinforcing bars are specified for controlling the cracks in concrete i.e. crack widths.

In beams, where the width of the member (i.e. beam) is fixed, often it is the minimum spacing requirement that dictates the selection of bar diameter. When all the reinforcing bars cannot be accommodated in one layer conforming to the requirements of minimum spacing of reinforcing bars and the clear cover, then either we have to increase the width of the beam, or place bars in more than one layer or bundle the group of parallel bars. This problem does not occur in slabs where the width of slab is large enough to accommodate the bars and also the percentage of reinforcement in slabs is low as compared to beams in general.

Cl. 26.3 of IS 456: 2000 specifies the requirements of placement of reinforcing bars in concrete members (i.e. beams, slabs) which is explained in Fig. 4.9.



(a) Singly reinforced beam



(b) One-way slab

***Fig. 4.9** Placement of reinforcing bars in beam and slab*

For a given area of tension reinforcement, it is **beneficial to use large number of small diameter bars (distributed uniformly in one or more layers)** rather than small number of large diameter bars to have efficient control on cracks.

In case of slab, Cl. 26.5.2.2 and **Cl. 26.3.3b of IS 456: 2000** limits the maximum diameter of reinforcing bars to **1/8th of the total slab thickness** with maximum spacing of these bars being limited to 3d or 300 mm (whichever is less).

In very deep structural members (like *deep beams*), a considerable portion of the web is under tension. Wider cracks may develop in the higher regions of web. Also, in regions of large exposed unreinforced concrete surfaces, cracks appear more frequently due to shrinkage (because of temperature variations). **Cl. 26.5.1.3 of IS 456: 2000** states that side face reinforcement has to be provided on the two faces of the beam if overall depth of beam exceeds 750 mm (or 450 mm for beams subjected to torsion) such that:

"The total area of web reinforcement is NOT less than 0.1% of web area and shall be distributed equally on the two faces at a spacing not exceeding 300 mm or web thickness, whichever is small."

3. **Maximum and Minimum Areas of Reinforcement:** A minimum area of tension steel is required in flexural members (like beams) in order to resist the effect of loads and also control the cracking in concrete due to shrinkage and temperature variations.

Minimum flexural steel reinforcement in beams: Cl. 26.5.1.1 of IS 456: 2000 specifies minimum area of tension reinforcing steel as:

$$\frac{A_{st\,min}}{bd} \geq \frac{0.85}{f_y}$$

or

$$p_{t\,min} \geq \frac{85}{f_y}$$

$= 0.34\%$ for Fe 250

$= 0.205\%$ for Fe 415

$= 0.17\%$ for Fe 500

For flanged beams, replace 'b' by width of the web 'b_w'.

Minimum flexural steel reinforcement in slabs: Minimum steel requirements in slab **are based** on ***shrinkage and temperature considerations*** **and not on strength consideration** because in slabs, there occurs a better distribution of load effects unlike in beams, where minimum steel requirement is based on *strength consideration*. **Cl. 26.5.2 of IS 456: 2000** limits minimum steel reinforcement requirement in slabs in the either direction as:

$A_{st\,min} = 0.15\%$ of gross area for Fe 250

$= 0.12\%$ of gross area for Fe 415

where, gross area (A_g) is $b \times D$. For one way slabs also, this minimum area of reinforcement has to be provided in the transverse direction as *secondary or distributor reinforcement.* The **spacing** of distribution bars shall NOT exceed **5d or 450 mm**, whichever is less.

Maximum flexural steel reinforcement in beams: Excessive reinforcement leads to congestion of bars which adversely affects the placement and compaction of concrete. **Cl. 26.5.1 of IS 456: 2000** limits the maximum area of steel reinforcement in tension ($A_{st\,max}$) and compression ($A_{sc\,max}$) to 4% of gross area i.e. $0.04bD$.

4. **Control of Deflection:** Excessive deflection in slabs or beams is not desirable as it leads to discomfort to the occupants and also adversely affects the flooring and finishes. Cross sectional sizes of the flexural elements (particularly depth of the section) governs the criterion of deflection control. It is the ratio of span/depth that needs to be controlled in order to control the deflection. **Cl. 23.2a of IS 456: 2000** limits this span/depth ratio to **250** as final deflection due to all types of loads (including the long term effects of creep and shrinkage).

4.6 Deflection Control by Limiting the Span/Depth Ratio

Let a uniform rectangular simply supported beam of size $b \times D$ and made up of a linear elastic material is subjected to a uniformly distributed load 'w' per unit length over the entire span 'l'. The central deflection (Δ) in the beam is given by

$$\Delta = \frac{5}{384} \cdot \frac{wl^4}{EI} \quad \text{...(i)}$$

Also maximum mid-span moment, $M_{max} = \dfrac{wl^2}{8}$

$$\Rightarrow \quad w = \frac{8M_{max}}{l^2} = (\sigma Z)\frac{8}{l^2} = \left(\sigma \frac{bD^2}{6}\right)\frac{8}{l^2} \quad \text{...(ii)}$$

Substituting eq. (ii) in (i) and writing $I = \dfrac{bD^3}{12}$, it comes out that:

$$\frac{\Delta}{l} = \text{constant} \times \frac{l}{D}$$

Where, the constant is $\dfrac{5\sigma}{24E}$. Thus by limiting the $\left(\dfrac{l}{D}\right)$ ratio, the deflection in the member can be controlled for any type of loading and support conditions. The above result does not apply to reinforced concrete as such because reinforced concrete is a non-homogeneous and composite material. However **IS 456: 2000** adopts this concept and prescribes limiting values of $\dfrac{l}{D}$ ratios.

Cl. 23.2.1 of IS 456: 2000 prescribes limits for limiting l/D ratios for prismatic rectangular beams of span upto 10 m as:

$$\left(\frac{l}{D}\right)_{max} = \left(\frac{l}{D}\right)_{basic} . k_t . k_c \quad \text{here } D \text{ is effective depth}$$

where, $\left(\frac{l}{D}\right)_{basic}$ = 7 for cantilever spans

= 20 for simply supported spans

= 26 for continuous spans

For spans above 10 m, the $(l/D)_{basic}$ values may be multiplied by [10/span (m)], except for cantilever.

Here the modification factor k_t varies with percentage of tension reinforcement (p_t) and f_{st} as given in **Fig. 4 of IS 456: 2000** and the modification factor k_c varies with percentage of compression reinforcement (p_c) as given in **Fig. 5 of IS 456: 2000.**

The factor 'f_{st}' is defined as:

$$f_{st} = 0.58 f_y \frac{\text{Area of steel required}}{\text{Area of steel provided}}$$

For flanged beams, **Cl. 23.2.1e of IS 456: 2000** recommends the values of p_t and p_c to be based on the area equal to '$b_f d$' and the calculated $(l/D)_{max}$ should be multiplied by a suitable reduction factor, the value of which depends on the ratio b_w/b_f. Alternatively, for deflection control in flanged beams, the overhang portions are ignored and the flanged beam is considered as a rectangular beam of effective depth 'd' and width 'b_w' (width of the web).

4.7 Selection of Member Sizes

The ratio of overall depth (D) to width (b) of rectangular beam section should in general lie in the range of **1.5 to 2.** This ratio can go up to **3 for beams carrying heavy loads.** The size of the beam is also governed by shear force at the section of the beam. Many a times, architectural requirements also dictate the sizes of beams and columns. If these architectural requirements are too stringent then required strength of the beam is achieved by making the beam doubly reinforced and using higher grade of concrete and steel.

For building frames, the width of beams should, in general, be less than or equal to the lateral dimension of the columns into which they frame in. Generally adopted beam widths are of 200 mm, 250 mm and 300 mm.

Often the beam is required to support a masonry wall and in such cases, the width of the beam is often made such that the beam sides are flushed with the finished surfaces of the wall and therefore beam widths of 230 mm are quite common in India.

In designs, the beam depth is often taken as 1/10th to 1/15th of span for simply supported and continuous beams. For cantilever beams, lower ratios are adopted and depth of cantilever beam is usually tapered.

1. **Preliminary proportioning of slab thickness:** For *slabs*, the thicknesses are very small as compared to depths of beams and thus limiting the span /depth ratios will generally govern the proportioning.

   For determining the thickness of the slab, the clear cover (based on exposure condition) plus half the diameter of the main reinforcement (usually along the shorter span) have to be added to the effective depth. The calculated value of the thickness should be rounded off to the nearest multiple of 5 mm or 10 mm.

2. **Deep beams and slender beams:**

   Deep Beam: When span to depth (l/d) ratio is less than 2 **for simply supported** and 2.5 for **continuous beams,** then such beams are called as deep beams.

   Slender Beam: When span of the beam is very large as compared to cross sectional dimension of the beam, then such beams are called as slender beams. There is always possibility of instability in slender beams and thus **Cl. 23.3 of IS 456: 2000** specify 'slenderness limits' to ensure lateral stability. The clear distance between the lateral restraints must be minimum of **60*b*** or **250*b*²/*d*** for simply supported and continuous beams and minimum of **25*b*** or **100*b*²/*d*** for cantilever beams.

4.8 Design of Reinforced Concrete Rectangular Beams

The design of reinforced concrete rectangular beams consists of determination of the following:

1. Cross sectional dimensions of the beam i.e. width (b), effective depth (d) and thus the overall depth (D).
2. Area of tension reinforcement required to resist the factored bending moment (M_u).

Apart from the above, the material properties viz. grade of concrete (f_{ck}) and the grade of steel (f_y) are fixed at the prior stage itself.

Do You Know? In under-reinforced sections, the effect of concrete grade on the ultimate moment of resistance of the beam section is NOT significant and thus it is not so much economical to use higher grades of concrete but at the same time higher grades of concrete is beneficial from the durability point of view.

The basic requirements for the safe design of beam in flexure are:

1. Factored moment (M_u) should be less than or at most equal to the limiting moment of resistance of the beam section i.e.

$$M_u \le M_{u\,lim}$$

2. The failure at the ultimate limit state should be **ductile** and not brittle.

   Thus

$$M_u \le M_{u\,lim} \quad \text{such that } x_u \le x_{u\,lim} \text{ so that } f_{st} = 0.87 f_y$$

But for under-reinforced sections, moment of resistance is given from tension side as steel yields earlier than the attainment of ultimate strain (i.e. 0.0035) by concrete.

Therefore,

$$M_u = 0.87 f_y A_{st}(d - 0.416 x_u) \text{ for all } x_u \le x_{u\,lim}$$

From statical equilibrium,

$$C = T$$

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

Substituting this value of x_u in M_u,

$$M_u = 0.87 f_y A_{st}\left(d - 0.42\frac{0.87 f_y A_{st}}{0.36 f_{ck} b}\right) = 0.87 f_y A_{st}\left(d - \frac{f_y A_{st}}{f_{ck} b}\right)$$

$$\Rightarrow \quad \frac{M_u}{bd^2} = \frac{0.87 f_y A_{st}}{bd^2}\left(d - \frac{f_y A_{st}}{f_{ck} b}\right)$$

$$\Rightarrow \quad \frac{M_u}{bd^2} = \frac{0.87 f_y p_t}{100}\left(1 - \frac{f_y}{f_{ck}}\frac{p_t}{100}\right) \quad \text{Provided } p_t \le p_{t\,lim}$$

where

$$p_t = \frac{A_{st}}{bd} \times 100$$

The expression for $p_{t\,lim}$ as derived in previous sections of this chapter is:

$$p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u\,lim}}{d}\right)$$

	$= 0.0882f_{ck}$	for Fe 250
	$= 0.048f_{ck}$	for Fe 415
	$= 0.038f_{ck}$	for Fe 500
Thus	$\frac{M_{u\,lim}}{f_{ck}bd^2} = 0.148$	for Fe 250
	$= 0.138$	for Fe 415
	$= 0.133$	for Fe 500

Arriving at the Dimensions of Rectangular Beam

Once we know the factored moment (M_u) from the principles of *Structural Analysis,* this factored moment is equated to the moment of resistance of the beam section depending on the grade of steel (viz. $0.148f_{ck}bd^2$, $0.138f_{ck}bd^2$ or $0.133f_{ck}bd^2$ for Fe 250, Fe 415 and Fe 500 respectively). From there, effective depth of the beam section can be computed as:

$$M_u = Rbd^2$$

$$\Rightarrow \quad d = \sqrt{\frac{M_u}{Rb}}$$

Where	$R = 0.148f_{ck}$	for Fe 250
	$= 0.138f_{ck}$	for Fe 415
	$= 0.133f_{ck}$	for Fe500

The overall depth of the beam (D) is then computed as:

$$D = d + \text{Clear cover} + (1/2) \times \text{Dia. of reinforcing bar} + \text{Dia. of stirrup}$$

The value of D so calculated must be **rounded up (and NOT rounded down)** to the nearest multiple of 10 or 5.

Determining the Area of Tension Steel Required

As stated earlier, for design purpose

$$M_u = M_{u\,lim} \qquad \text{so that } x_u \le x_{u\,lim} \text{ and } f_{st} = 0.87f_y$$

Now

$$A_{st} = \frac{M_u}{0.87f_y(d-0.42x_u)} = \frac{M_u}{0.87f_yd\left(1-0.42\frac{x_u}{d}\right)}$$

Here the question arises:

"How to compute $\frac{x_u}{d}$ for determination of area of steel required A_{st}?"

Design the beam as balanced section by equating factored moment (M_u) with the ultimate moment of resistance of the beam section ($M_{u\,lim}$) and in that case, $M_{u\,lim}$ can also be computed from the compression side of concrete as:

$$M_u = 0.36f_{ck}bx_u(d-0.42x_u)$$

$$\Rightarrow \quad 0.42\left(\frac{x_u}{d}\right)^2 - \left(\frac{x_u}{d}\right) + \frac{M_u}{0.36f_{ck}bd^2} = 0$$

This is a quadratic equation in $\left(\frac{x_u}{d}\right)$ the solution of which comes out to be:

$$\frac{x_u}{d} = 1.202\left[1-\sqrt{1-4.598\frac{R}{f_{ck}}}\right] \qquad \text{where, } R = \frac{M_u}{bd^2}$$

Alternate Solution

Earlier we arrived at the following expression:

$$\frac{M_u}{bd^2} = \frac{0.87f_yp_t}{100}\left(1-\frac{f_y}{f_{ck}}\frac{p_t}{100}\right)$$

The above equation can be rearranged as:

$$\left(\frac{f_y}{f_{ck}}\right)\left(\frac{p_t}{100}\right)^2 - \left(\frac{p_t}{100}\right) + \frac{M_u}{0.87f_ybd^2} = 0$$

Solution of the above quadratic equation gives,

$$\frac{p_t}{100}\left(=\frac{A_{st\,required}}{bd}\right) = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-4.598\frac{R}{f_{ck}}}\right] \qquad \text{where, } R = \frac{M_u}{bd^2}$$

(As derived in **section 4.4.3**)

From the above expression, we can directly compute the percentage of steel required and hence the area of steel required to resist the factored moment M_u. **Alternatively,** we can use ***Design Aid SP: 16*** wherein different charts are given for different values of f_{ck}, f_y and $\frac{M_u}{bd^2}$.

Computing the Number of Bars Required in Beams

Once we have $A_{st\,required}$, number of bars required of particular dia (say ϕ) is calculated as:

$$\text{No. of bars} = \frac{A_{st\,required}}{\frac{\pi}{4}\phi^2}$$

The computed number of bars must be **rounded up** to the nearest whole number.

Computing the Spacing of Bars Required in Slabs

In case of slabs (where b is taken as 1000 mm), instead of computing the number of bars required, we compute the **spacing of bars required (s)** of particular dia (say ϕ) which is given as:

$$\text{Spacing of bars of dia } \phi = \frac{1000A_\phi}{A_{st\,required}}$$

where,

$$A_\phi = \frac{\pi}{4}\phi^2 = \text{Area of one bar}$$

4.9 Reinforcement Arrangement in Different Types of Beams

The figures given below depict the arrangement / placement of reinforcing bars in various types of beams commonly encountered in day to day life.

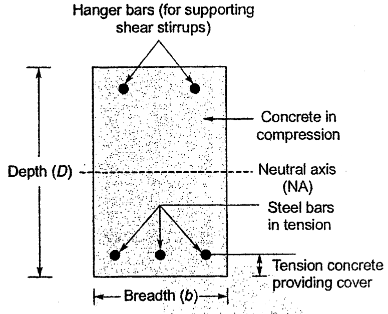


Fig. 4.10 *Singly reinforced rectangular beam under positive bending moment (near midspan)*

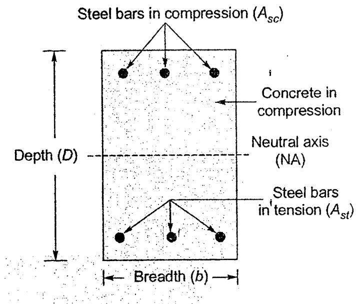


Fig. 4.11 *Doubly reinforced rectangular beam under positive bending moment (near midspan)*

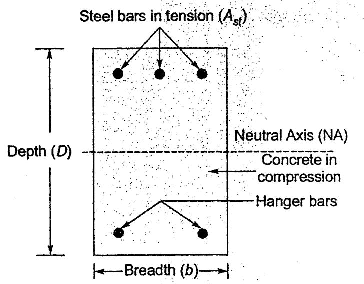


Fig. 4.12 *Singly reinforced rectangular beam under negative bending moment (over the support).*

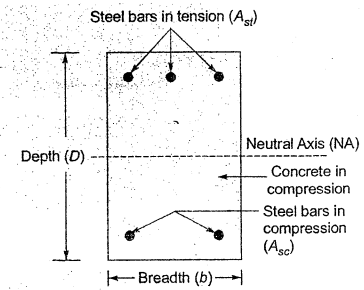


Fig. 4.13 *Doubly reinforced rectangular beam under negative bending moment (over the support).*

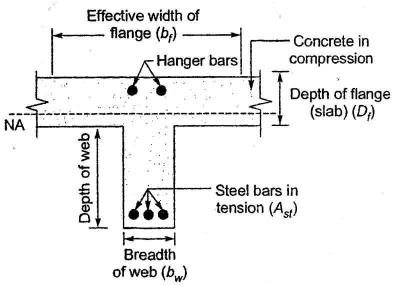


Fig. 4.14 *Singly reinforced T-beam under positive bending moment (near midspan)*

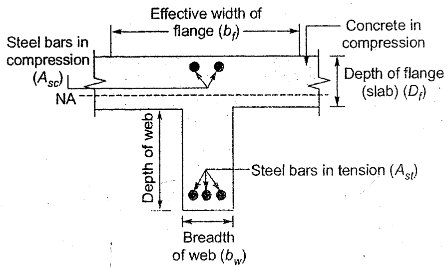
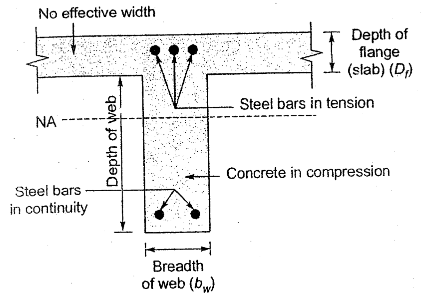
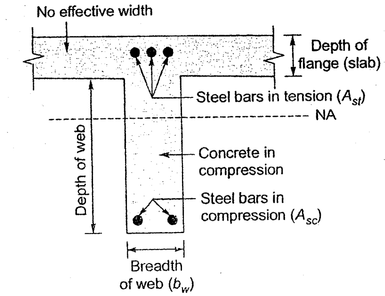


Fig. 4.15 *Doubly reinforced T-beam under positive bending moment (near midspan).*



T-beam ≡ Rectangular beam (no flange action as concrete above NA is in tension)

Fig. 4.16 *Singly reinforced T-beam under negative bending moment (over the support). (concrete above neutral axis is in tension)*



T-beam ≡ Rectangular beam (no flange action as concrete above NA is in tension)

Fig. 4.17 *Doubly reinforced T-beam under negative bending moment (over the support).*

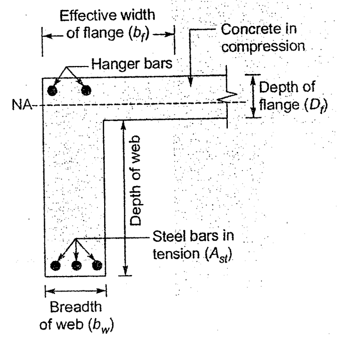


Fig. 4.18 *Singly reinforced L-beam under positive bending moment (near midspan).*

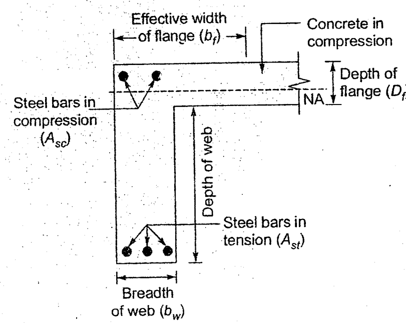
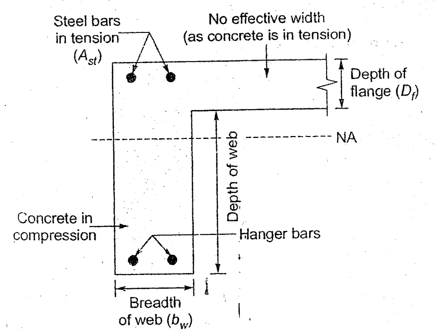
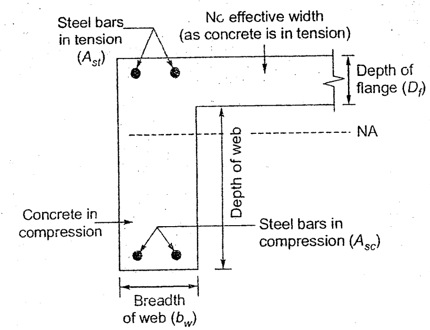


Fig. 4.19 *Doubly reinforced L-beam under positive bending moment (near midspan).*



L-beam ≡ Rectangular beam (no flange action as concrete above NA is in tension)

Fig. 4.20 *Singly reinforced L-beam under negative bending moment (over the support). (Concrete above neutral axis is in tension)*



L-beam ≡ Rectangular beam (no flange action as concrete above NA is in tension)

Fig. 4.21 *Doubly reinforced L-beam under negative bending moment (over the support). (Concrete above neutral axis in in tension)*

Do You Know?

Why is it advantageous to use Design Aid SP: 16?

Use of Design Aids like SP: 16 helps a designer in the following ways:

1. The numerous sets of values of width (b), depth (d) and reinforcing steel (A_{st}) for particular design problems are obtained very quickly.
2. The values given in SP: 16 already take into account the values of steel reinforcement where it is not admissible.

4.10 Comparison between WSM and LSM of Design

Table 4.5: *WSM versus LSM of design*

	WSM	LSM
Concrete	Stress factor of safety $= \frac{\text{Characteristic Stress}}{\text{Allowable stress}} = 3$ For M15, 15/5 = 3 For M20, 20/7 = 2.86 For M25, 25/8.5 = 2.94 For M30, 30/10 = 3.0	Stress factor of safety $= \frac{\text{Characteristic Stress}}{\text{Allowable stress}} = \frac{f_{ck}}{0.45 f_{ck}} = 2.22$
Loads	Load factor of safety = 1.0	Load factor of safety = γ_f = 1.5
	Total FOS = Stress FOS × Load FOS = 3.0 × 1.0 = 3.0	Total FOS = Stress FOS × Load FOS = 2.22 × 1.5 = 3.33

Example 4.8 Design a rectangular beam of effective span 4.3 m subjected to a live load of 12.5 kN/m use M 25 concrete and Fe 415 steel.

Solution:

$$\text{Effective span } (l) = 4.3 \text{ m} = 4300 \text{ mm}$$

For preliminary proportioning of beam section, take

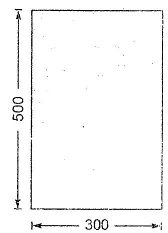
$$D = \frac{l}{10} = \frac{4300}{10} \text{ mm} = 430 \text{ mm}$$

Adopt overall depth, D = 500 mm

So that effective depth, d = 500 – 35 = 465 mm (Assuming 35 mm effective cover)

b = 300 mm

Loads



$$\text{Self weight of beam} = 0.3 \times 0.5 \times 25 = 3.75 \text{ kN/m}$$

$$\text{Live load} = 12.5 \text{ kN/m}$$

$$\text{Total load} = 3.75 + 12.5 = 16.25 \text{ kN/m}$$

$$\text{Factored load } (w_u) = 1.5 \times 16.25 = 24.375 \text{ kN/m}$$

$$\text{Factored moment } (M_u) = \frac{w_u l^2}{8} = 24.375 \times \frac{4.3^2}{8}$$

$$= 56.34 \text{ kNm}$$

Calculation of depth required

Ultimate moment of resistance for balanced section for Fe 415 = $0.138 f_{ck} bd^2$

$$\Rightarrow \quad 56.34 \times 10^6 = 0.138(25)(300)d^2$$

$$\Rightarrow \quad d = 233.31 \text{ mm}$$

$$\therefore \quad D = 233.31 + 35 \quad \text{(Assuming 35 mm effective cover)}$$

$$= 268.31 \text{ mm} \simeq 300 \text{ mm (say)}$$

Revise cross-section size to 250 mm × 300 mm

$$\therefore \quad \text{Self weight of beam} = 0.25 \times 0.3 \times 25 = 1.875 \text{ kN/m}$$

$$\text{Live load} = 12.5 \text{ kN/m}$$

$$\therefore \quad \text{Total load} = 14.375 \text{ kN/m}$$

$$\therefore \quad \text{Factored load} = 1.5 \times 14.375 = 21.56 \text{ kN/m}$$

$$\text{Factored moment } (M_u) = 21.56 \times \frac{4.3^2}{8} = 49.83 \text{ kNm}$$

Now,

$$49.83 \times 10^6 = 0.138 f_{ck} bd^2$$

$$= 0.138(25)\,250\, d^2$$

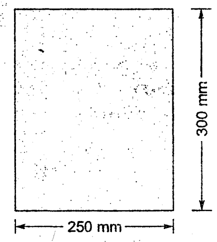
$$\therefore \quad d = 240.36 \text{ mm}$$

$$\therefore \quad D = 240.36 + 35$$

$$= 275.36 \text{ mm} < 300 \text{ mm (assumed } D) \quad \text{(OK)}$$

$$\therefore \quad d = 300 - 35 = 265 \text{ mm}$$

Thus adopt beam size as 250 × 300



Reinforcement requirement

$$R = \frac{M_u}{bd^2} = \frac{49.83 \times 10^6}{250 \times 265^2} = 2.838$$

$$\therefore \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1 - \sqrt{1 - 4.598\frac{R}{f_{ck}}}\right]$$

$$= \frac{25}{2(415)}\left[1 - \sqrt{1 - 4.598 \times \frac{2.838}{25}}\right] = 9.295 \times 10^{-3}$$

$$\therefore \quad p_t = 0.93\%$$

Limiting percentage of tensile reinforcement

$$p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u\,lim}}{d}\right) = 41.61\left(\frac{25}{415}\right)(0.48)$$

$$= 1.203\% > p_t (= 0.93\%) \quad \text{(OK)}$$

$$\Rightarrow \quad A_{st} = \frac{0.9295}{100} \times 250 \times 265 = 615.79 \text{ mm}^2$$

$$\therefore \quad \text{No. of } 20\,\phi \text{ bars required} = \frac{615.79}{\frac{\pi}{4}(20)^2} = 1.96 \simeq 3 \text{ (say)}$$

∴ As per IS : 456, minimum two number of bars is to be provided so provide 3 – 20 ϕ bars.

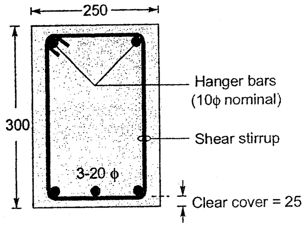
Minimum reinforcement

$$A_{st\,min} = 0.85\frac{bd}{f_y}$$

$$= \frac{0.85\times250\times265}{415}$$

$$= 135.7\ \text{mm}^2$$

$$< 615.79\ \text{mm}^2 \quad \text{(OK)}$$



Example 4.9 An RCC beam of size 380 × 700 mm overall depth is designed for an effective span 6 m and is subjected to a live load of 20 kN/m. Design the beam using LSM. Use M30 concrete and Fe500 steel.

Solution:

Given:

Width of beam (b) = 380 mm

Overall depth of the beam (D) = 700 mm

Span (L) = 6 m

Assuming an effective cover of 50 mm,

Effective depth of the beam (d) = 700 – 50 mm = 650 mm

Loads and bending moment

Live load = 20 kN/m

Self weight of the beam = 0.38 × 0.7 × 25 kN/m = 6.65 kN/m

Total load = Dead load + Live load

= Self weight of the beam + Live load

= 6.65 + 20 kN/m = 26.65 kN/m

Factored load (w_u) = 1.5 × 26.65 kN/m = 39.975 kN/m

$$\text{Factored bending moment } (M_u) = \frac{w_uL^2}{8} = \frac{39.975\times6^2}{8}\ \text{kNm} = 179.89\ \text{kNm}$$

Limiting depth of neutral axis

$$x_{u\,lim} = 0.46d \qquad \text{(For Fe 500 steel)}$$

$$= 0.46\times650\ \text{mm} = 299\ \text{mm}$$

Actual depth of neutral axis

Compression force = Tension force

$$0.36f_{ck}bx_u = 0.87\,f_yA_{st}$$

and

$$M_u = 0.87\,f_yA_{st}(d-0.42x_u)$$

$$M_{u\,lim} = 0.36f_{ck}\,bx_{u\,lim}(d-0.42\,x_{u\,lim})$$

$$= 0.36\times30\times380\times299\,(650-0.42\times299) = 643.5\ \text{kNm}$$

Alternatively

$$M_{u\,lim} = 0.133\,f_{ck}\,bd^2 \qquad \text{(for Fe 500)}$$

= 640.6 kNm (which is very close to 643.5 kNm)

∴ $M_u < M_{u\,lim}$, and thus we require an under reinforced section

Calculation of x_u for M_u

$$M_u = 0.87\,f_yA_{st}(d-0.42x_u)$$

$$179.89\times10^6 = 0.36\times30\times x_u\times380(650-0.42\,x_u)$$

$$43832.846 = 650\,x_u - 0.42\,x_u^2$$

$$x_u = 70.7\ \text{mm}$$

Area of tension steel required

$$A_{st} = \frac{M_u}{0.87f_y(d-0.42x_u)} = \frac{179.89\times10^6}{0.87\times500(650-0.42\times70.7)}$$

$$= 666.7\ \text{mm}^2$$

Alternatively

$$R = \frac{M_u}{bd^2} = \frac{179.89\times10^6}{380\times650^2} = 1.120461$$

$$\therefore \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{30}{2(500)}\left[1-\sqrt{1-4.598\times\frac{1.120461}{30}}\right]$$

$$\Rightarrow \quad p_t = 0.2679\%$$

$$\Rightarrow \quad A_{st} = \frac{0.2697}{100}\times380\times650$$

$$= 666.16\ \text{mm}^2\ \text{(same as above)}$$

Example 4.10 Design a RCC beam for an effective span of 8.5 m subjected to a live load of 45 kN/m. Width of the beam is 400 mm. Use M 30 concrete and Fe 500 steel. Design by both LSM and WSM and compare the results.

Solution:

By LSM

Let overall depth of the beam is 850 mm

Assuming an effective cover of 50 mm

Live load = 45 kN/m

Self weight of the beam = 0.85 × 0.4 × 25 kN/m = 8.5 kN/m

Total load = 45 + 8.5 kN/m = 53.5 kN/m

Factored load (w_u) = 1.5 × 53.5 kN/m = 80.25 kN/m

$$\text{Factored bending moment } (M_u) = \frac{w_uL^2}{8} = \frac{80.25\times8.5^2}{8}\ \text{kNm} = 724.76\ \text{kNm}$$

Design of beam as balanced section

$$M_u = M_{u\,lim} = Qbd^2$$

Where

$$Q = 0.36\,f_{ck}\,k(1-0.42\,k)$$

$$= 0.36\times30\times0.46\,(1-0.42\times0.46)\ \text{(for Fe 500, } k = 0.46)$$

$$= 4.008$$

$$\therefore \quad 724.76\times10^6 = Qbd^2$$

$$724.76\times10^6 = 4.008\times400\times d^2$$

$$d = 672.36\ \text{mm}$$

Alternatively

$$M_{u\,lim} = 0.133\,f_{ck}\,bd^2 \qquad \text{(for Fe 500)}$$

$$\Rightarrow \quad 724.76\times10^6 = 0.133(30)(400)\,d^2$$

$$d = 673.87\ \text{mm}$$

$$\therefore \quad D = 673.87 + 50\ \text{mm} = 723.87\ \text{mm}$$

Provide $D = 725$ mm, $d = 675$ mm

$\therefore$ self weight of the beam = $0.725 \times 0.4 \times 25 = 7.25$ kN/m

Total load = $45 + 7.25 = 52.25$ kN/m

Factored load = $1.5 \times 52.25 = 78.375$ kN/m

$$\text{Factored bending moment } (M_u) = \frac{78.375 \times 8.5^2}{8} = 707.82 \text{kNm}$$

Area of steel required

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left(1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}\right) bd$$

$$= \frac{0.5 \times 30}{500}\left(1 - \sqrt{1 - \frac{4.6 \times 707.82 \times 10^6}{30 \times 400 \times 675^2}}\right) \times 400 \times 675$$

$$= 2948.485 \text{ mm}^2$$

$$\text{Using 25 mm dia. bars, no. of bars required} = \frac{2948.485}{\frac{\pi}{4} \times 25^2} = 6 \text{ bars}$$

By WSM

Total load = 52.25 kN/m

$$\text{Bending moment } (M) = \frac{52.25 \times 8.5^2}{8} \text{kNm} = 471.88 \text{ kNm}$$

Beam depth required

$$d = \sqrt{\frac{M}{Qb}}$$

$$Q = \frac{1}{2} \times cjk$$

$$k = \frac{mc}{mc + t} = \frac{9 \times 10}{9 \times 10 + 275} = 0.246$$

$$j = 0.918$$

$$Q = \frac{1}{2} \times 10 \times 0.246 \times 0.918 = 1.13$$

$$d = \sqrt{\frac{483.17 \times 10^6}{1.13 \times 400}} = 1021.76 \approx 1022 \text{ mm}$$

$D = 1022 + 50$ mm = 1072 mm = 1100 mm

$\therefore$ $d = 1100 - 50$ mm = 1050 mm

Self weight of the beam = $0.4 \times 1.1 \times 25$ kN/m = 11 kN/m

Total load = $11 + 45$ kN/m = 56 kN/m

$$\text{Bending moment } (M) = \frac{wL^2}{8} = \frac{56 \times 8.5^2}{8} \text{ kNm} = 505.75 \text{ kNm}$$

$$d = \sqrt{\frac{505.75 \times 10^6}{1.13 \times 400}} \text{ mm} = 1058 \text{ mm} > 1050 \text{ mm} \quad \text{(Not OK)}$$

Assume $D = 1110$ mm

$\therefore$ $d = 1110 - 50$ mm = 1060 mm

Self weight of the beam = $1.11 \times 0.4 \times 25$ kN/m = 11.10 kN/m

Total load = $11.1 + 45$ kN/m = 56.1 kN/m

$$\text{Bending moment } (M) = \frac{wL^2}{8} = \frac{56.1 \times 8.5^2}{8} \text{ kNm} = 506.65 \text{ kNm}$$

$$d = \sqrt{\frac{506.65 \times 10^6}{1.129 \times 400}} \text{ mm} = 1059 \text{ mm} < 1060 \text{ mm} \quad \text{(OK)}$$

Area of steel required

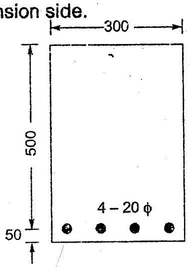
$$A_{st} = \frac{M}{\sigma_{st} jd} = \frac{506.65 \times 10^6}{275 \times 0.918 \times 1060} \text{ mm} = 1893 \text{ mm}^2$$

Comparison of results

LSM	WSM
Width (b) = 400 mm Depth (D) = 725 mm d = 675 mm A_{st} = 2948.485 mm^2	Width (b) = 400 mm Depth (D) = 1110 mm d = 1060 mm A_{st} = 1893 mm^2

Remember: Thus with LSM, section comes out to be smaller than that of WSM but in LSM, steel comes out to be larger than that coming out from WSM.

Example 4.11 **Determine the permissible value of service load that can be imposed on a beam of effective span 7 m. The beam size is 300 mm × 550 mm with an effective cover of 50 mm. Use M 25 concrete and Fe 500 steel. The beam is reinforced with 4–20 φ bars on tension side.**



Solution:

Given: $f_{ck} = 25$ N/mm^2, $f_y = 500$ N/mm^2, $b = 300$ mm

$$A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.64 \text{ mm}^2$$

D = 550 mm

$\therefore$ $d = 550 - 50$ (effective cover)

Let $x_u < x_{u\,lim}$

Where, $x_{u\,lim} = 0.46\, d = 0.46 (500) = 230$ mm

$$\therefore \quad x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck}\, b} = \frac{0.87(500)(1256.64)}{0.36(25)(300)} = 201.34 \text{ mm} < x_{u\,lim}$$

$\therefore$ Assumption is true and $f_{st} = 0.87 f_y$ is correct

$\therefore$ Ultimate moment of resistance of the beam (MOR)

$$= 0.87 f_y A_{st}(d - 0.42 x_u)$$

$$= 0.87(500)(1256.64)(500 - 0.42 \times 201.34) = 227.094 \text{ kNm}$$

$$\text{Now} \quad \text{MOR} = \frac{w_u l^2}{8} \text{ (for simply supported beam)}$$

$$\Rightarrow \quad w_u = \frac{8(227.094)}{7^2} = 37.077 \text{ kN/m} = \text{Factored load}$$

$$\therefore \quad \text{Service load} = \frac{\text{Factored load}}{1.5} = \frac{37.077}{1.5} = 24.718 \text{ kN/m}$$

Self weight of beam = $0.3 \times 0.55 \times 25$ kN/m = 4.125 kN/m

∴ Maximum service imposed loads that the beam is able to take

$$w_L = (24.718 - 4.125) \text{ kN/m}$$
$$= 20.593 \text{ kN/m} \simeq 20 \text{ kN/m}$$

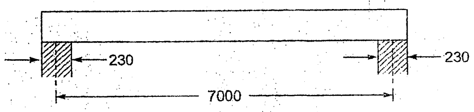
Example 4.12 **Design a simply supported rectangular beam whose centre to centre distance between the supports is 7 m and is supported on 230 mm thick walls. Assume live load 7 kN/m length of beam. Use M25 concrete and Fe415 steel.**

Solution:

Step: 1 Unknown Parameters : b, d (and thus D), A_{st}

Step: 2 Effective Span : As per Cl. 22.2 (a) of IS 456: 2000, effective span is taken as :

(i) Clear span + effective depth
(ii) Centre to centre distance between the two supports } whichever is less



∴ Clear span $= 7000 - \frac{230}{2} - \frac{230}{2} = 6770$ mm

Now Cl. 23.2.1 of IS : 456 – 2000 gives basic ratios of span to effective depth which is 20 for simply supported beam for beam upto 10 m span.

Here, Span = 7 m < 10 m

∴ $\frac{l}{d} \le 20$

⇒ $d \ge \frac{l}{20} = \frac{7000}{20} = 350$ mm

∴ Adopt d = 350 mm

∴ Effective span is

(i) Clear span + d = 6770 + 350 = 7120 mm
(ii) c/c distance between supports = 7000 mm } whichever is less

Hence, L_{eff} = 7000 mm = Effective span of beam

Step: 3 Assume percentage of steel reinforcement for calculation of actual effective depth

For Fe 415, $\frac{x_{u\,lim}}{d} = 0.48$

∴ $p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{ulim}}{d}\right) = 41.61\left(\frac{25}{415}\right)0.48 = 1.203\ \%$

= Maximum percentage of tension steel reinforcement

Also, as per Cl. 26.5 . 1. 1(a) of IS 456: 2000, minimum percentage of steel reinforcement is given by,

$$\frac{A_{st\,lim}}{bd} = \frac{0.85}{f_y}$$

Let $p_t = 0.9\ \% < p_{t\,lim}$ (OK)

Step: 4 Effective Depth (d) : The basic value of span/ depth ratio is multiplied by modification factor (k_t) to account for tension reinforcement.

$$f_s = 0.58 f_y \left(\frac{\text{Area of steel required}}{\text{Area of steel provided}}\right) = 0.58\,(415)$$

Assuming area of steel required = Area of steel provided = 240.7 N/mm²

From Fig. 4 of IS 456: 2000,

$k_t \simeq 1$ for $f_s = 240.7$ N/mm² and $p_t = 0.9$ % (assumed)

∴ Actual span/depth ratio $= \left(\frac{l}{d}\right)_{basic} k_t \times k_c$ Where, $k_c = 1$ for $p_c = 0$ %

= 20 (1) = 20

∴ $d = \frac{\text{span}}{20} = \frac{7000}{20} = 350$ mm

Which is same as assumed earlier

Let Effective cover = 50 mm

∴ Overall depth (D) = 350 + 50 = 400 mm

Width of Beam (b): The ratio $\frac{b}{D}$ should preferably lie between 0.5 and 0.7

Let b = 250 mm

∴ $\frac{b}{D} = \frac{250}{400} = 0.625$ (between 0.5 and 0.7) (OK)

Step: 5 Design loads and design bending moment

Cl. 19.2.1 of IS 456: 2000 specifies unit weight of reinforced concrete as 25 kN/m³

∴ Self weight of beam = 0.25 × 0.4 × 25 kN/m = 2.5 kN/m

Live load = 7 kN/m

∴ Total load = 2.5 + 7 = 9.5 kN/m

∴ Factored load (w_u) = 1.5 × 9.5 kN/m = 14.25 kN/m

(Partial safety factor for dead and live loads is 1.5 for limit state of collapse)

∴ Factored bending moment (M_u) $= \frac{w_u l^2_{eff.}}{8} = 14.25 \times \frac{7^2}{8} = 87.28$ kNm

Step: 6 Effective depth of beam required from design moment consideration

For Fe 415, $M_{u\,lim} = 0.138\, f_{ck}\, bd^2$

⇒ $87.28 \times 10^6 = (0.138)(25)(250)d^2$

⇒ d = 318.11 mm < 350 mm as calculated above (OK)

Step: 7 Area of steel required

Since d = 350 mm and effective depth required from design moment consideration is 318.11 mm which is less than adopted effective depth (= 350 mm).

∴ Design moment is on conservative side and need not to be revised.

$$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1 - \sqrt{1 - 4.598\frac{R}{f_{ck}}}\right]$$

Where $R = \frac{M_u}{bd^2} = \frac{87.28 \times 10^6}{250 \times 350^2} = 2.85$ N/mm²

$$\therefore \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{25}{2(415)}\left[1-\sqrt{1-4.598\times\frac{2.85}{25}}\right]$$

$$= 9.343 \times 10^{-3}$$

$$\therefore \quad p_t = 0.9343\ \%$$

$$\simeq p_t \text{ assumed } (=0.9\ \%)$$

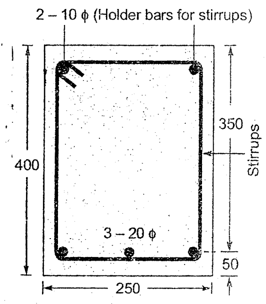
$$\therefore \quad A_{st} = \frac{0.9343}{100}\times 250\times 350$$

$$= 817.51\ \text{mm}^2$$

Using 20 mm ϕ bars, number of bars reqd. $= \dfrac{817.51}{\frac{\pi}{4}(20)^2}$

$$= 2.6 \simeq 3 \text{ nos.}$$

Provide 3-20 ϕ bars at bottom and 2-10 ϕ bars at top for holding the stirrups as hanger bars.



Reinforcement Detailing

Example 4.13 Design a cantilever beam of clear span 2.7 m carrying a superimposed live load of 17.5 kN/m. Use M 25 concrete and Fe 415 steel.

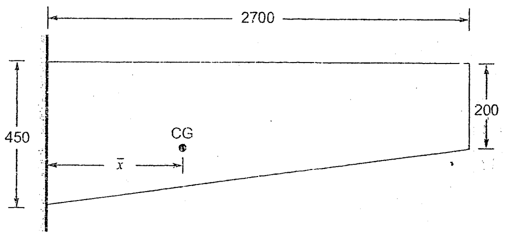
Solution:

Given Clear span = 2700 mm

$$d = \frac{\text{Clear span}}{7} = \frac{2700}{7} = 385.7 \text{ mm}$$

Adopt overall depth as 450 mm at the fixed end with constant width $b = 0.5(D) = 225$ mm. Depth of the beam is reduced to 200 mm at the free end with linear variation of depth along the span.

$$\text{Self weight of beam} = \frac{(0.45+0.2)}{2}\times 2.7\times 0.225\times 25 = 4.94 \text{ kN} \simeq 5 \text{ kN}$$



Distance of centre of gravity (CG) of beam from fixed end

$$= \bar{x}$$

$$= \left(\frac{2\times 200+450}{200+450}\right)\frac{2700}{3} = 1176.92 \text{ mm}$$

$$\therefore \quad \text{Factored } BM(M_u) = 1.5\left[5\times 1.17692+17.5\times\frac{2.7^2}{2}\right] = 104.51 \text{ kNm}$$

$$\therefore \quad M_u = 0.138\, f_{ck}\, bd^2$$

$$\Rightarrow \quad 104.51\times 10^6 = 0.138\,(25)\,(225)\,d^2$$

$$\Rightarrow \quad d = 366.93 \text{ mm at fixed end.}$$

$$\therefore \quad \text{Overall depth, } D = 366.91 + 25 + \frac{20}{2} + 8(\text{for shear stirrups}) = 409.91 \text{ mm}$$

Assuming clear cover 25 mm, dia. of main bar 20 mm and stirrups 8 mm.

Thus adopt D = 450 mm at fixed end

$$\therefore \quad d = 450 - 25 - \frac{20}{2} - 8 = 407 \text{ mm}$$

Limiting depth of neutral axis $(x_{u\,lim}) = 0.48\,d = 0.48(407) = 195.36$ mm

Limiting reinforcement

$$A_{st\,lim} = \frac{0.362\, f_{ck}\, b\, x_u}{0.87 f_y} = \frac{0.362(25)(225)195.36}{0.87(415)}$$

$$= 1101.79 \text{ mm}^2 \text{ (corresponding to balanced section)}$$

Minimum reinforcement

$$A_{st\,min} = 0.85\frac{bd}{f_y} = \frac{0.85(225)(407)}{415}$$

$$= 187.56 \text{ mm}^2 < 1101.79 \text{ mm}^2$$

Provide 3 – 20 ϕ bars at top $\left(A_{st} = 3\times\frac{\pi}{4}\times 20^2 = 942.5\text{mm}^2\right)$

Providing $A_{st} >$ 1101.79 mm² will make the section over-reinforced

Shear Check (is covered in more detail in later chapters)

Factored *SF* at fixed end $(V_u) = 1.5[5 + 17.5 \times 2.7] = 78.375$ kN

$$\text{Nominal shear stress} \quad (\tau_v) = \frac{V_u - \frac{M_u}{d}\tan\beta}{bd}$$

$$\tan\beta = \frac{450-200}{2700} = 0.09259$$

$$\therefore \quad \tau_v = \frac{78.375\times 10^3 - \frac{104.51\times 10^6\,(0.09259)}{407}}{225(407)} = 1.115 \text{ N/m}^2$$

$$p_t = \frac{3\times\frac{\pi}{4}\times 20^2}{225\times 407}\times 100 = 1.029\,\%$$

Shear strength of M 25 concrete for p_t = 1.029 % as per table 19 of IS : 456 $(\tau_c) = 0.65$ N/m²

$$< \tau_v (= 1.115 \text{ N/mm}^2)$$

So, shear reinforcement is required for a design shear stress of

$$\tau_{vs} = \tau_v - \tau_c$$

$$= 1.115 - 0.65 = 0.465 \text{ N/mm}^2$$

Using 2 legged 8 ϕ stirrups, $A_{sv} = 2\times\frac{\pi}{4}\times 8^2 = 100.53$ mm²

Spacing of shear stirrups $(S_v) \le \dfrac{0.87 f_y A_{sv}}{\tau_{us} b} \le \dfrac{0.87(415)100.53}{0.465(225)} \le 346.9$ mm c/c

$\not> 0.75\,d\,(= 0.75 \times 407 \text{ mm} = 305.25 \text{ mm})$

But spacing of shear stirrups $(S_v) < \begin{bmatrix} 300\text{mm} \\ 0.75\text{ d} \end{bmatrix}$ (whichever is less)

∴Provide 2 legged 8 ϕ stirrups @ 250 c/c

$$A_{sv\,min} \ge \frac{0.4 b S_v}{0.87 f_y} = \frac{0.4(225)250}{0.87(415)}$$

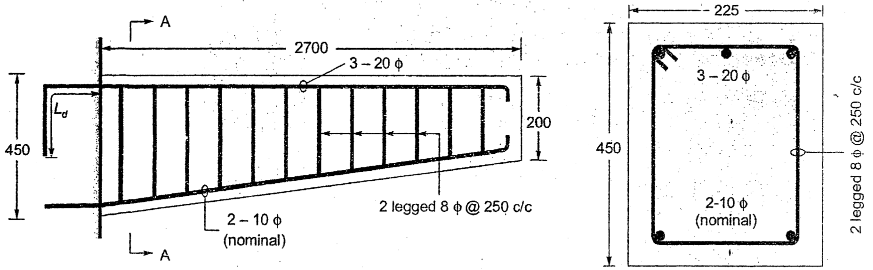
$= 62.32 \text{ mm}^2 < 100.53 \text{ mm}^2$ (OK)

Development Length (is covered in more detail in chapters)

$$L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}$$

where $\tau_{bd} = 1.6 \times 1.4 = 2.24 \text{ N/mm}^2$

∴ $L_d = \dfrac{0.87(415)20}{4(2.24)} = 805.92 \text{ mm} = 810 \text{ mm (say)}$

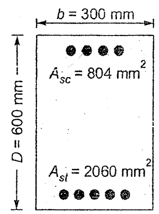


Reinforcement Detailing

Example 4.14 Determine the moment of resistance of a beam of size 300 × 600 mm (overall depth)? It is reinforced with 804 sq. mm compression reinforcement and 2060 sq.mm tension reinforcement. Take effective cover as 50 mm. Use M20 concrete and Fe 415 steel.

Solution:

MOR of over reinforcement section



Effective cover = 50 mm

f_{ck} = 20 N/mm² (M20 concrete)

f_y = 415 N/mm² (Fe 415 steel)

Effective depth (d) = 600 – 50 = 550 mm

$x_{u\,lim}$ = 0.479 d = 0.479 × 550 = 263.45 mm

Let $x_u < x_{u\,lim}$

∴ $0.362 f_{ck} b x_u = 0.87 f_y A_{st}$

⇒ $x_u = \dfrac{0.87 \times 415 \times 2060}{0.362(20)300}$

$= 342.43 \text{ mm} > x_{u\,lim}$

∴ $f_{st} \ne 0.87 f_y$

I[st] iteration

Let $x_u = \dfrac{342.43 + 263.45}{2} = 302.94$ mm

$\epsilon_{sc} = 0.0035\left(\dfrac{550}{302.94} - 1\right) = 0.0028544$

∴ $f_{st} = 351.817 \text{ N/mm}^2$

∴ $x_u = \dfrac{f_{st} A_{st}}{0.362 f_{ck} b} = \dfrac{351.817 \times 2060}{0.362(20)300} = 333.675$ mm

II[nd] iteration

Let $x_u = \dfrac{302.94 + 333.675}{2} = 318.3075$ mm

∴ $= 0.0035\left(\dfrac{550}{318.3075} - 1\right) = 0.0025476$

∴ $f_{st} = 345.538 \text{ N/mm}^2$

∴ $x_u = \dfrac{f_{st} A_{st}}{0.362 f_{ck} b} = \dfrac{345.538 \times 2060}{0.362(20)300} = 327.72$ mm

III[rd] iteration

Let $x_u = \dfrac{318.3075 + 327.72}{2} = 323.01375$ mm

∴ $= 0.0035\left(\dfrac{500}{323.01375} - 1\right) = 0.0024595$

∴ $f_{st} = 343.273 \text{ N/mm}^2$

∴ $x_u = \dfrac{343.273 \times 2060}{0.362(20)300} = 325.572$ mm

IV[th] iteration

Let $x_u = \dfrac{323.01375 + 325.572}{2} = 324.293$ mm

∴ $= 0.0035\left(\dfrac{550}{324.293} - 1\right) = 0.00243599$

∴ $f_{st} = 342.6683 \text{ N/mm}^2$

$$\therefore \quad x_u = \frac{342.6683 \times 2060}{0.362(20)300}$$

$$= 324.99 \text{ mm (converged)} \simeq 325 \text{ mm}$$

Thus $x_u > x_{u\,lim}$

$\therefore$ Moment of resistance (MOR) $= 0.362\, f_{ck} b x_u (d - 0.42\, x_u)$

$= 0.362\,(20)\,300\,(325)\,(550 - 0.42 \times 325)$

$= 291.89$ kNm

Alternatively

MOR $= f_{st} A_{st} (d - 0.42\, x_u)$

$= 342.6683\,(2060)\,(550 - 0.42 \times 325)$

$= 291.89$ kNm

4.11 Slabs as Rectangular Beams

Slabs under uniaxial flexure behave in a same way as that of beams. A slab of constant thickness which is subjected to a bending moment uniformly distributed over its entire width can be treated as wide shallow beam. In such type of slabs, the reinforcing bars are placed uniformly over the width of the slab. As stated above, computations are generally done by considering 1m (=1000 mm) wide strip of the slab. The spacing of bars is given by:

$$s = \frac{1000 \cdot A_\phi}{A_{st}}$$

Here, A_ϕ is the area of one reinforcing bar and A_{st} is the area of tensile reinforcement required for 1 m wide strip. Reinforced concrete slabs are generally singly reinforced.

4.12 Transverse Moments in One Way Slabs

In the bending of a normal beam under the influence of gravity loads like dead and live loads, as the beam bends about the neutral axis, the top portion comes under compression and bottom portion under tension. In the top portion under compression, due to Poisson's effect, a lateral expansion/elongation takes place. Similarly due to Poisson's effect, lateral contraction also takes place in the bottom tensile portion of the beam. Thus the cross section of the beam acquires a trapezoidal shape rather than a rectangular shape.

However in the case of one way slab, such lateral displacements are invariably prevented due to the remaining part of the slab on either side of the 1000 mm wide strip of the beam considered except at the two extreme edges. The remaining part of the slab restraints the transverse displacement of the 1000 mm strip by inducing lateral stresses. This is called as plain strain condition. These restraints induce secondary moments in the slab in the transverse direction.

Thus one way slab requires secondary transverse reinforcements in the transverse direction to resist these secondary moments. Shrinkage and temperature also induce secondary moments in slab which is to be resisted by secondary transverse reinforcement.

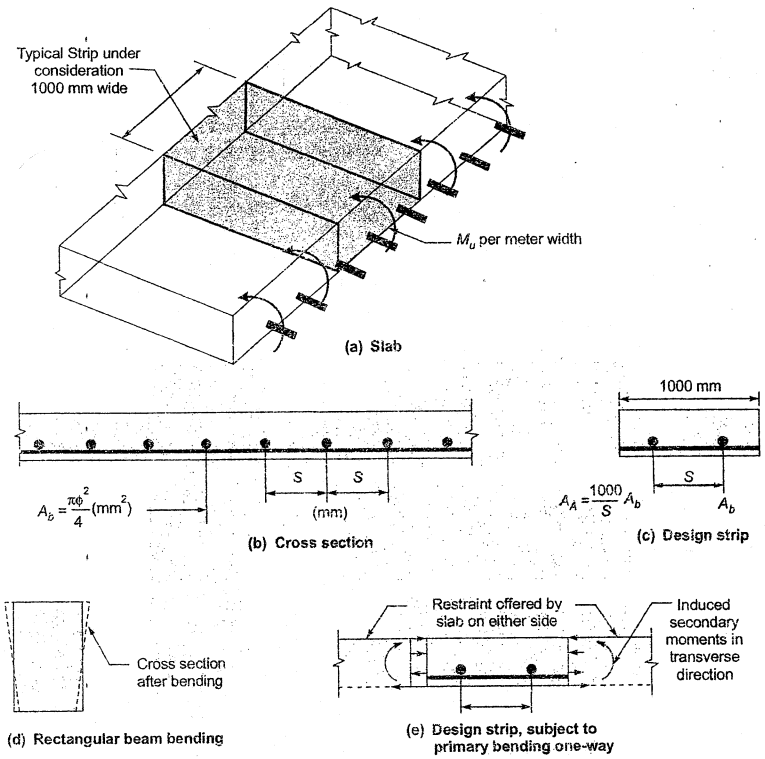
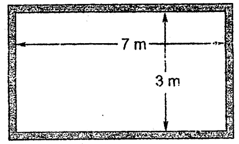


Fig. 4.22 *Transverse moments in one way slabs*

Example 4.15 A simply supported roof slab of clear size 7 m x 3 m subjected to a live load of 4 kN/m². Use M25 concrete and Fe415 steel. The slab rest on 230 mm thick (one brick wall) masonry walls all around.

Solution:



$$\frac{l_y}{l_x} = \frac{7}{3} > 2 \Rightarrow \text{One way slab}$$

Let effective depth of slab $= \frac{3000}{20} = 150$ mm

Assuming clear cover 15 mm and dia. of main reinforcement is 20 mm.

$\therefore$ Overall depth of slab (D) $= 150 + 15 + \frac{20}{2} = 175\text{mm} \simeq 190\text{mm (say)}$

$$d = 190 - 15 - \frac{20}{2} = 165 \text{ mm}$$

$$\text{Effective span } (l_e) = \begin{bmatrix} \text{Clear span} + d \\ \text{c/c distance between the supports} \end{bmatrix} \text{whichever is less}$$

$$= \begin{bmatrix} 3000 + 165 = 3165 \text{ mm} \\ 3000 + \frac{230}{2} + \frac{230}{2} = 3230 \text{ mm} \end{bmatrix} \text{whichever is less}$$

$$= 3165 \text{ mm}$$

Loads

Assuming 1 m strip of slab

Self weight of slab = 0.19 × 25 = 4.75 kN/m

Live load = 4 kN/m

Assume miscellaneous load = 1 kN/m

Total load = 4.75 + 4 + 1 kN/m = 9.75 kN/m

Factored load (w_u) = 1.5 × 9.75 kN/m = 14.625 kN/m

$$\text{Factored moment } (M_u) = \frac{w_u l^2}{8} = \frac{14.625}{8}(3.165)^2 = 18.31 \text{ kNm}$$

Reinforcement requirement in slab

$$R = \frac{M_u}{bd^2} = \frac{18.31 \times 10^6}{1000 \times 165^2} = 0.6725$$

$$\therefore \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1 - \sqrt{1 - \frac{4.598R}{f_{ck}}}\right] = 1.92 \times 10^{-3}$$

$$\Rightarrow \quad p_t = 0.192\%$$

$$\Rightarrow \quad A_{st} = \frac{0.192}{100} \times 1000 \times 165 = 317.493 \text{ mm}^2/\text{m}$$

$$\text{Using 20 mm diameter bars, spacing} = \frac{1000 \times \frac{\pi}{4} \times 20^2}{317.493} = 989.5 \text{mm} > 300 \text{ mm} \quad \text{(Not OK)}$$

$$\text{Using 12 mm diameter bars, spacing} = \frac{1000 \times \frac{\pi}{4} \times 12^2}{317.493} = 356.2 \text{mm} > 300 \text{ mm} \quad \text{(Not OK)}$$

$$\text{Using 10 mm diameter bars, spacing} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{317.493} = 247.4 \text{mm} < 300 \text{ mm} \quad \text{(OK)}$$

$$\text{Maximum bar spacing} \not> \begin{bmatrix} 3d = 3 \times 165 = 495 \text{mm} \\ 300 \text{mm} \end{bmatrix} \text{(whichever is small)}$$

Provide 10ϕ bars @ 230 mm c/c.

Distribution reinforcement

$$\text{Distribution reinforcement @ 0.12\%} = \frac{0.12}{100} \times 1000 \times 190 = 228 \text{mm}^2/\text{m}$$

$$\text{Using 10}\phi \text{ bars, spacing} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{228} = 344.5 \text{mm c/c}$$

$$\text{Maximum bar spacing} \not> \begin{bmatrix} 5d = 5 \times 165 = 825 \text{mm} \\ 450 \text{mm} \end{bmatrix} \text{(whichever is less)} = 450 \text{ mm}$$

Provide 10ϕ bars @ 300 mm c/c.

Check for deflection

$$A_{st\,reqd.} = 317.493 \text{ mm}^2/\text{m}$$

$$A_{st\,provided} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{230} = 341.48 \text{mm}^2/\text{m}$$

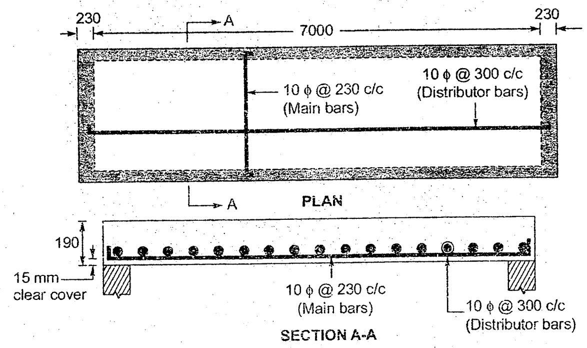
$$f_{st} = 0.58 \times \frac{317.493}{341.48} \times 415 = 223.79 \text{ N/mm}^2$$

$$p_t = \frac{341.48}{1000 \times 165} = 0.207\%$$

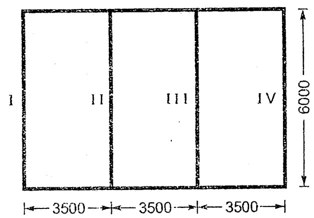
$$k_t \simeq 1.8 \quad \text{(from fig. 4 of IS 456: 2000)}$$

$$\left(\frac{l}{d}\right)_{max} = k_t \cdot k_c \left(\frac{l}{d}\right)_{basic} = 1.8\,(1)\,(20) = 36 \text{ where } k_c = 1 \text{ for } p_c = 0$$

$$\left(\frac{l}{d}\right)_{provided} = \frac{3165}{165} = 19.18 < \left(\frac{l}{d}\right)_{max} \quad \text{(OK)}$$



Example 4.16 Design a one-way slab as shown in figure below which is subjected to imposed load of 5 kN/m². Use M 20 and Fe 415. Take dead load of finish as 0.5 kN/m². Width of beams at support is 300 mm.



Solution:

Step 1: Preliminary depth of slab

∵ The slab is simply supported on edges I and IV and continuous on edges II and III.

$$\therefore \quad \text{Span/depth ratio} = \frac{20 + 26}{2}$$

Average of simply supported and continuous limiting span/depth values.

= 23 (Cl. 23.2.1 of IS 456: 2000)

Now for p_t = 0.5% and f_y = 240 N/mm², k_t = 1.2 (Fig. 4 of IS 456: 2000)

Effective depth required from deflection criterion = $d = \frac{3500}{23 \times 1.2} = 126.8$ mm

Let effective cover = 25 mm

∴ Overall depth of slab, D = 126.8 + 25 = 151.8 mm ≈ 155 mm (say)

∴ d = 155 − 25 = 130 mm

Step 2: Design loads, bending moment and shear force

DL of slab per meter width = 0.155 × 25 = 3.875 kN/m

DL of floor finish given as 0.5 kN/m² = 0.5 kN/m per meter width of slab.

∴ Total *DL* = 3.875 + 0.5 = 4.375 kN/m

LL = 5 kN/m² = 5 kN/m per meter width of slab.

∴ Total load = 4.375 + 5 = 9.375 kN/m

∴ Factored load = 1.5 × 9.375 = 14.06 kN/m

$$\text{Maximum positive B.M.} = (14.06)\frac{3.5^2}{12} = 14.4 \text{ kNm/m}$$

Maximum shear force (V_u) = (14.06) 3.5 (0.4) = 19.7 kN

Step 3: Determining the effective depth required

For Fe 415, $M_{u\,min} = 0.138 f_{ck} bd^2$

⇒ $17.2 \times 10^6 = 0.138(20)(1000)d^2$

⇒ $d = 78.9 \text{ mm} < 130 \text{ mm} (= d_{provided})$ (OK)

Step 4: Check for slab depth w.r.t. shear

Design shear strength of M 20 concrete for

$p_t \le 0.15\,\%$, τ_c = 0.28 N/mm² (Table 19 of IS 456: 2000)

k = 1.29 (As per Cl. 40.2.1 of IS 456: 2000)

∴ $k\tau_c$ = 1.29 (0.28) = 0.36 N/mm²

$\tau_{c\,max}$ (M 20 concrete) = 2.8 N/mm² (Table 20 of IS 456: 2000)

$$\tau_v = \frac{V_u}{bd} = \frac{19.7 \times 1000}{1000 \times 130} = 0.15 \text{ N/mm}^2$$

Thus $\tau_v < \tau_c < \tau_{c\,max}$

∴ Depth adopted is OK and no shear reinforcement is required.

Step 5: Flexural reinforcement requirement

Negative reinforcement

$M_u^- = 17.2$ kNm/m

∴ $R = \frac{17.2 \times 10^6}{10^3 \times 130^2} = 1.018$

∴ $$\frac{p_t^-}{100} = \frac{A_{st}^-}{bd} = \frac{20}{2 \times 415}\left[1 - \sqrt{1 - \frac{4.598 \times 1.018}{20}}\right] = 0.003$$

∴ p_t^- = 0.3 %

and A_{st}^- = 390 mm²/m

Using 10 mm φ bars, $$\text{spacing} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{390} = 201.4 \text{ mm} = 200 \text{ mm c/c (say)}$$

Provide 10 φ bars @ 200 c/c

M_u^+ = 14.4 kNm/m

∴ $R = \frac{14.4 \times 10^6}{10^3 \times 130^2} = 0.852$

∴ $$\frac{p_t^+}{100} = \frac{A_{st}^+}{bd} = \frac{20}{2(415)}\left[1 - \sqrt{1 - \frac{4.598 \times 0.852}{20}}\right] = 0.0025$$

∴ p_t^+ = 0.25 %

A_{st}^+ = 325 mm²/m

Using 10 mm φ bars, $$\text{spacing} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{325} = 241.7 \text{ mm c/c} = 240 \text{ mm c/c (say)}$$

Provide 10φ bars @ 200 c/c

Distribution reinforcement @ 0.12 % along long span

$$= \frac{0.12}{100} \times 1000 \times 130 = 156 \text{ mm}^2/\text{m}$$

Also, minimum reinforcement ($A_{st\,min}$) = 0.12% of bd

$$= \frac{0.12}{100} \times 1000 \times 130 = 156 \text{ mm}^2/\text{m}$$

Using 8 mm φ bars for distribution reinforcement,

$$\text{Spacing} = \frac{1000 \times \frac{\pi}{4}(8)^2}{156} = 322.2 \text{ mm c/c} = 280 \text{ mm c/c (say)}$$

Maximum spacing for main bars ≯ 3d or 300 mm which ever is less.

≯ 3 × 130 or 300 mm

≯ 390 or 300 mm

= 300 mm

Maximum spacing of distribution bars ≯ 5d or 450 mm which even is less

≯ 5(130) or 450

≯ 650 or 450

= 450 mm

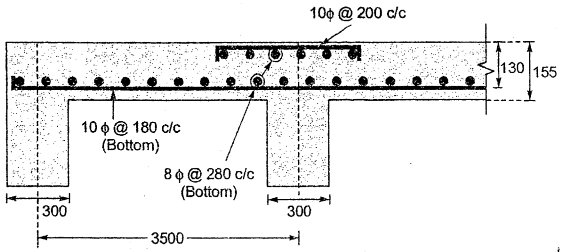
≯ 280 mm (OK)

Maximum main bar dia. ≯ $\frac{D}{8} = \frac{155}{8} = 19.375$ mm

> 10 mm (OK) (main bars)

> 8 mm (OK) (distribution bars)

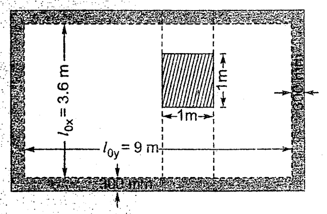
Provide 8ϕ bars @ 280 c/c as distribution reinforcement.



Example 4.17 A simply supported slab of size 3.6 m × 9.0 m (clear) is subjected to a live load of 8 kN/m². Floor finishing is 60 mm thick of cement concrete flooring. The slab is supported over 300 mm thick masonry wall supports. Design the slab using LSM. Use M 20 concrete and Fe 500 steel.

Solution:

Effective span:



Effective depth as per deflection criterion

$$d = \frac{\text{span}}{20}$$

$$\Rightarrow \quad d = \frac{3600}{20} = 180 \text{ mm}$$

Assuming effective cover of 30 mm,

Overall depth of the slab (D) = 180 mm + 30 mm = 210 mm

Effective span along x, l_x = clear span + d = 3.6 + 0.18 m
= clear span + c/c distance between the supports } whichever is less

= 3.78 m
= 3.6 + 0.3 m = 3.9 m } whichever is less

= 3.78 m

Effective span along, y l_y = clear span + d = 9.0 + 0.18 m
= clear span + c/c distance between the supports } whichever is less

= 9.18 m
= 9.0 + 0.3 m = 9.3 m } whichever is less

= 9.18 m

$$\frac{l_y}{l_x} = \frac{9.18}{3.78} = 2.42 > 2$$

∴ it is a one way slab.

Load Calculation

Live load = 8 kN/m² (Given)

Live load for 1 m × 1 m slab area = 8 × 1 × 1 kN = 8 kN

Dead load

Self weight of the slab = 0.21 × 1 × 1 × 25 kN
= 5.25 kN

Dead load of floor finishing = 0.06 × 1 × 1 × 24 kN = 1.44 kN

Total load, w = 8 + 5.25 + 1.44 kN
= 14.69 kN = 14.69 kN/m

Factored load, w_u = 1.5 × 14.69 kN/m = 22.04 kN/m

Design Bending Moment

Factored bending moment, $M_u = \frac{w_u l_x^2}{8} = \frac{22.04 \times 3.78^2}{8}$ kNm = 39.36 kN

Slab depth required

$Q = 0.36 f_{ck} k(1 - 0.42 k)$

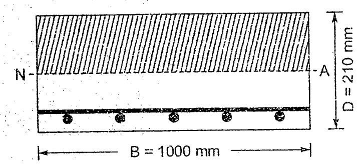
$= 0.36 \times 20 \times 0.46 (1 - 0.42 \times 0.46) = 2.67$ (k = 0.46 for Fe 500)

$$d = \sqrt{\frac{M_u}{QB}} = \sqrt{\frac{39.36 \times 10^6}{2.67 \times 1000}} = 121.41 \text{ mm}$$

But depth required from deflection criterion = 180 mm

∴ Adopt effective depth of 180 mm.

∴ Overall depth of the slab (D) = 180 + 30 mm
= 210 mm



Area of steel required

$$A_{st} = \frac{M_u}{0.87 f_y j d}$$

$$= \frac{39.36 \times 10^6}{0.87 \times 500 \times 0.8 \times 180} \text{ mm}^2 = 628.35 \text{ mm}^2$$

Alternatively, area of steel can be calculated as,

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.598 M_u}{f_{ck} B d^2}}\right] bd$$

$$= \frac{0.5 \times 20}{500}\left[1 - \sqrt{1 - \frac{4.598 \times 39.36 \times 10^6}{20 \times 1000 \times 180^2}}\right] \times 1000 \times 180 \text{ mm}^2$$

$= 544.04$ mm²

Using 10 mm dia. bars, spacing of bars required

$$= \frac{1000 A_\phi}{A_{st}} = \frac{1000}{A_{st}} \times \frac{\pi}{4} \times \phi^2 = \frac{1000}{544.04} \times \frac{\pi}{4} \times 10^2$$

= 144.36 mm c/c ≈ 140 mm c/c

$\not>$ 3d (3 × 180 mm = 540 mm)

$\not>$ 400 mm (OK)

Distribution reinforcement

Distribution reinforcement = 0.12% of $bD = \frac{0.12}{100} \times 1000 \times 210$ = 252 mm²

Using 8 mm dia. bars, spacing of bars required

$$= \frac{1000A_\phi}{A_{st}} = \frac{1000}{A_{st}} \times \frac{\pi}{4} \times \phi^2 = \frac{1000}{252} \times \frac{\pi}{4} \times 8^2$$

$$= 199.46 \text{ mm c/c} \approx 190 \text{ mm c/c}$$

Check for shear

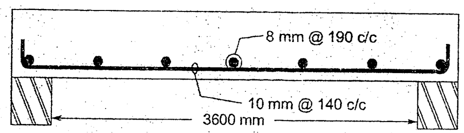
Factored shear force $\quad V_u = \frac{w_u l}{2} = \frac{22.04 \times 37.5}{2} \text{kN} = 41.66 \text{ kN}$

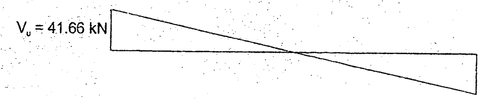
Nominal shear stress, $\quad \tau_{ve} = \frac{V_u}{Bd} = \frac{41.66 \times 10^3}{1000 \times 180} \text{N/mm}^2 = 0.23 \text{ N/mm}^2$

$$< \tau_c (= 0.28 \text{ N/mm}^2)$$

So the slab is safe in shear.

Check for bond (is covered in more detail later chapters)





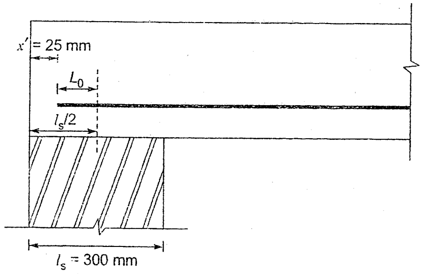
$$\tau_{bd\,(dev)} = \frac{V_u}{\Sigma o jd} = \frac{V_u}{n\phi\pi jd}$$

$$= \frac{41.66 \times 10^3}{\left(\frac{1000}{140}\right) \times \pi \times 10 \times 0.8 \times 180} \text{N/mm}^2 = 1.29 \text{ N/mm}^2$$

$$\tau_{bd\,(per)} = 1.2 \times 1.60 \text{ N/mm}^2 \text{ (for tension in LSM)} = 1.92 \text{ N/mm}^2$$

$$> 1.29 \text{ N/mm}^2 \qquad \text{(OK)}$$

Check for development length (covered in more detail in later chapters)



$$L_0 = \frac{l_s}{2} - x' = \frac{300}{2} - 25 \text{ mm} = 125 \text{ mm}$$

$$M_{u_1} = 0.87 f_y A_{st}\, jd = \left(\frac{1000}{140}\right) \times \frac{\pi}{4} \times (10)^2 \times 0.87 \times 500 \times 0.8 \times 180 \text{ Nmm}$$

$$= 35.14 \text{ kNm}$$

$$V_u = 41.66 \text{ kN}$$

Development length, $\quad L_d = \frac{0.87 f_y \phi}{4\tau_{bd(per)}} = \frac{0.87 \times 500 \times 10}{4 \times 1.92} \text{ mm} = 566.4 \text{ mm}$

Now, $\quad 1.3\frac{M_{u_1}}{V_u} + L_0 = 1.3 \times \frac{35.14 \times 10^6}{41.66 \times 10^3} + 125 \text{mm} = 1221.57 \text{ mm}$

$\therefore \quad L_d \not> 1.3\frac{M_{u_1}}{V_u} + L_0 \qquad$ (OK)

Objective Brain Teasers

Q.1 When the neutral axis depth (x_u) of a beam section is more than the limiting depth ($x_{u\,lim}$), then stress in tension steel reinforcement:
(a) $> 0.87f_y$ (b) $< 0.87f_y$
(c) $= 0.87f_y$ (d) $0.67f_y$

Q.2 The limiting percentage of tension reinforcement in a beam made up of M35 concrete and reinforced with Fe 415 steel is:
(a) 3.09% (b) 1.53%
(c) 1.41% (d) 1.68%

Q.3 An inverted T-beam subjected to gravity loading will act as:
(a) T beam (b) Rectangular beam
(c) L beam (d) Data insufficient

Q.4 The moment of resistance of a beam with minimum amount of tension reinforcement ($A_{st\,min}$) is approximately same as:
(a) Cracking torque of the beam
(b) Cracking moment of the beam
(c) Modulus of rupture of concrete
(d) None of the above

Q.5 The minimum grade of concrete recommended for severe climatic condition as per IS 456: 2000 is
(a) M40 (b) M30
(c) M35 (d) M20

Q.6 The minimum nominal cover to reinforcement recommended for very severe climatic condition as per IS 456: 2000 is:
(a) 30 mm (b) 75 mm
(c) 45 mm (d) 50 mm

Q.7 The minimum diameter of hanger bars recommended for holding the stirrups is:
(a) 6 mm (b) 8 mm
(c) 10 mm (d) 12 mm

Q.8 A beam of width 300 mm and effective depth 500 mm of M25 concrete is subjected to a factored flexural moment of 100 kNm. The depth of neutral axis of the beam is:
(a) 79 mm (b) 265.5 mm
(c) 240 mm (d) 50 mm

Q.9 The limiting l/D ratio for the end beam of a continuous span whose one end is simply supported and other continuous is:
(a) 7 (b) 23
(c) 20 (d) 10

Q.10 For an under-reinforced beam, the moment of resistance of the beam can be computed from:
(a) Tension side formula
(b) Compression side formula
(c) Both tension and compression side formula
(d) None of (a) or (b)

Q.11 A continuous beam of span 3 m is considered as a deep beam if:
(a) Beam depth < 1200 mm
(b) Beam depth < 1500 mm
(c) Beam depth > 1200 mm
(d) Beam depth > 1350 mm

Q.12 The limit on span to depth ratio is specified in IS 456: 2000 because
(a) it ensures the limit or tensile crack width
(b) it ensures safety against shear failure
(c) it limits the maximum deflection of beam
(d) it limits the maximum tensile stress in steel

Q.13 The depth of rectangular portion of stress block of concrete in compression is
(a) $\frac{3}{7}x_u$ (b) $\frac{4}{7}x_u$
(c) $\frac{2}{7}x_u$ (d) x_u

Q.14 The size of the test specimen to determine the modulus of rupture of concrete is
(a) 150 mm × 150 mm × 700 mm
(b) 100 mm × 100 mm × 700 mm
(c) 150 mm × 150 mm × 150 mm
(d) 100 mm × 100 mm × 100 mm

Q.15 With increase in the rate of loading during testing, compressive strength of concrete
(a) decrease
(b) increase
(c) may increase or decrease depending on concrete grade
(d) data insufficient

Q.16 In order to ensure durability for similar conditions of exposure, water cement ratio in reinforced concrete as compared to plain concrete is
(a) less (b) more
(c) equal (d) data insufficient

Q.17 Consider the following statements:
(i) Compressive strength of concrete decreases with increase in w/c ratio of the concrete
(ii) Water is needed in concrete for hydration of cement and workability
(iii) Creep and shrinkage of concrete are independent of w/c ratio of concrete
of the above statement, true statement(s) is are:
(a) (i) and (ii) (b) (i) and (iii)
(c) (ii) and (iii) (d) (i), (ii) and (iii)

Q.18 Maximum strain in extreme fibre of concrete and in tension steel of Fe 415 grade ($E_s = 2 \times 10^5$ N/mm²) in a balance section in limit state of flexure are respectively
(a) 0.0035 and 0.0031
(b) 0.0035 and 0.0038
(c) 0.002 and 0.0042
(d) 0.002 and 0.0038

Q.19 The modulus of elasticity of M30 concrete as per IS 456: 1978 and IS 456: 2000 respectively are
(a) 31220 N/mm², 27386 N/mm²
(b) 27386 N/mm², 31220 N/mm²
(c) 27386 N/mm², 27386 N/mm²
(d) None of these

Q.20 The main reinforcement in RC slab consists of 10 mm diameter bars at 100 mm c/c spacing. When 10 mm bars are replaced by 12 mm diameter bars then spacing would be
(a) 175 mm (b) 160 mm
(c) 150 mm (d) 140 mm

Q.21 A RC beam is subjected to following flexural moments
DL moment = 30 kNm
LL moment = 40 kNm
Seismic moment = 20 kNm
The design flexural moment for limit state of collapse is
(a) 105 kNm (b) 108 kNm
(c) 135 kNm (d) 90 kNm

Q.22 The assumption that "plane sections remain plane before and after the bending" leads to
(a) uniform stress over the beam section
(b) uniform strain over the beam section
(c) linearly varying stress over the beam section
(d) linearly varying strain over the beam section

Q.23 The basic principle of structural design is based on
(i) strong column weak beam concept
(ii) strong footing weak column concept
Which of the above statement(s) is/are true?
(a) (i) only (b) (ii) only
(c) Both (i) and (ii) (d) Neither (i) nor (ii)

Q.24 A RC beam is of effective span 5500 mm. The permissible deflection in beam including the effect of all heads along with temperature, creep and shrinkage is
(a) 15.71 mm (b) 22 mm
(c) 2.75 cm (d) 1.1 cm

Q.25 The direct tensile strength of concrete is determined from
(a) modulus of elasticity of concrete
(b) modulus of rupture of concrete
(c) creep coefficient of concrete
(d) shrinkage coefficient of concrete

Q.26 By controlling the span to depth ratio of a beam, the following can be controlled
(a) shear stress in beam
(b) flexural stress in beam
(c) deflection of beam
(d) All of the above

Answers

1. (b)	2. (d)	3 (b)	4. (b)	5. (b)
6. (d)	7. (c)	8. (a)	9. (b)	10. (a)
11. (c)	12. (c)	13. (a)	14. (a)	15. (b)
16. (a)	17. (d)	18. (b)	19. (a)	20. (d)
21. (b)	22. (d)	23. (c)	24. (b)	25. (b)
26. (c)				

Hints:

2. (d)

$$p_{tlim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{ulim}}{d}\right)$$

$$= 41.61\left(\frac{35}{415}\right)(0.48) = 1.68\%$$

8. (a)

Use $\frac{x_u}{d} = \left(1.2021 - \sqrt{1 - 4.598\frac{M_u}{bd^2 f_{ck}}}\right)$

or $M_u = 0.36\, f_{ck}\, B\, x_u\, (d - 0.42\, x_u)$

9. (b)
It will be average of 20 and 26 i.e. simply supported and continuous spans.

11. (c)
For continuous deep beams, $L/D < 2.5$ and for simply supported beams, this ratio should be < 2.

18. (b)

$$\epsilon_s = 0.002 + \frac{f_y}{1.15\,\epsilon_s}$$

$$= 0.002 + \frac{415}{1.15 \times 2 \times 10^5}$$

$$= 0.0038$$

19. (a)

$$\epsilon_C = 5700\sqrt{f_{ck}} \quad \text{(IS 456: 1978)}$$

$$= 5000\sqrt{f_{ck}} \quad \text{(IS 456: 2000)}$$

20. (d)

$$A_{st} = \frac{1000 A_\phi}{S}$$

$$\therefore \quad \frac{1000 \times \frac{\pi}{4} \times 10^2}{100} = \frac{1000 \times \frac{\pi}{4} \times 12^2}{S}$$

$$\Rightarrow \quad S = 100 \times \frac{12^2}{10^2} = 144 \simeq 140 \text{ mm(say)}$$

21. (b)

1.5 (DL + LL) = 1.5(30 + 40) = 105 kNm
1.5 (DL + EQL) = 1.5(30 + 20) = 75 kNm
1.2(DL + LL + EQL) = 1.2(30 + 40+20)=108 kNm

24. (b)
Permissible deflection

$$= \frac{5500}{250} = 22 \text{ mm}$$

Conventional Practice Questions

Q.1 A RC rectangular beam is one brick wide (i.e., 230 mm wide) and overall 400 mm deep. It is reinforced with 4 nos. of 12 mm dia. bars of Fe415 at an effective cover of 31 mm. The beam is simply supported over a span of 3.5 m. Calculate the moment of resistance of the beam and the maximum super-imposed uniformly distributed load that the beam can carry. Use M 20 concrete.

Ans. [Moment of resistance = 53.5 kNm, Maximum super-imposed load = 21 kN/m]

Q.2 Find the amount of tension steel required in a reinforced concrete beam of size 230 mm × 390 mm to resist an ultimate moment of 50 kNm. Assume effective cover as 40 mm and use M 20 concrete and Fe 500 steel.

Ans. [A_{st} = 386 mm^2]

Q.3 Determine the ultimate moment capacity of a rectangular beam of size 250 mm × 450 mm with 50 mm effective cover. The total area of tension steel provided is 3600 mm^2. Use M20 concrete and Fe415 steel.

Ans. [M_{ulim} = 110 kNm]

Q.4 A 150 mm thick one way reinforced concrete slab is reinforced with 10 mm dia. bars provided at 200 mm c/c. Assuming 25 mm effective cover, determine the ultimate moment of resistance of the section. Use M20 concrete and Fe415 steel.

Ans. [M_{ulim} = 16.6 kNm]

Q.5 A reinforced concrete beam of size 500 mm × 800 mm is provided with a duct of size 260 mm × 560 mm located concentrically with the beam. The centroid of tension steel is at a distance of 60 mm from the bottom face of beam. Determine the balanced moment of resistance of the section and amount of reinforcement required. Use M 20 concrete and Fe 415 steel.

Ans. [$M_{u\,lim}$ = 553 kNm, A_{st} = 2478 mm^2]