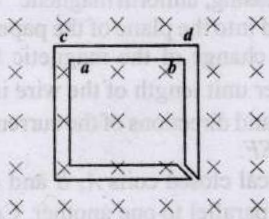
Chapter
Electromagnetic Induction

Topic-1: Magnetic Flux, Faraday's and Lenz's Law

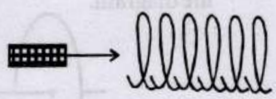
1 MCQs with One Correct Answer

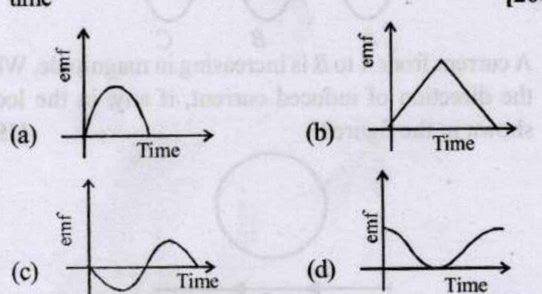
1 The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments **ab** and **cd**. Then, **[2009]**



(a) $I_1 > I_2$
(b) $I_1 < I_2$
(c) I_1 is in the direction *ba* and I_2 is in the direction *cd*
(d) I_1 is in the direction *ab* and I_2 is in the direction *dc*

2. A small bar magnet is being slowly inserted with constant velocity inside a solenoid as shown in figure. Which graph best represents the relationship between emf induced with time **[2004S]**





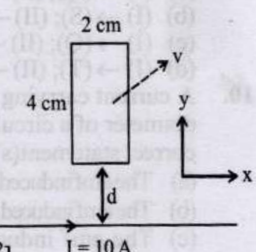
3. A thin circular ring of area *A* is held perpendicular to a uniform magnetic field of induction *B*. *A* small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is *R*. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is **[1995S]**

(a) $\frac{BR}{A}$ (b) $\frac{AB}{R}$ (c) ABR (d) $\frac{B^2A}{R^2}$

2 Integer Value Answer

4. A rectangular conducting loop of length 4 cm and width 2 cm is in the xy-plane, as shown in the figure. It is being moved away from a thin and long conducting wire along the direction $\frac{\sqrt{3}}{2}\hat{x}+\frac{1}{2}\hat{y}$ with a constant speed v. The wire is carrying a steady current I = 10 A in the positive x-direction. A current of 10 μA flows through the loop when it is at a distance d = 4 cm from the wire. If the resistance of the loop is 0.1 Ω, then the value of v is ________ ms^{-1}. **[Adv. 2023]**

[Given: The permeability of free space $\mu_0 = 4\pi \times 10^{-7}\,NA^{-2}$]



5. A series R-C combination is connected to an AC voltage of angular frequency ω = 500 radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is **[2011]**

4 Fill in the Blanks

6. In a straight conducting wire, a constant current is flowing from left to right due to a source of emf. When the source is switched off, the direction of the induced current in the wire will .... **[1993 - 1 Marks]**

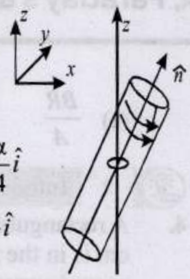
5 True / False

7. A coil of metal wire is kept stationary in a non-uniform magnetic field. An e.m.f. is induced in the coil. **[1986 - 3 Marks]**

8. An e.m.f. can be induced between the two ends of a straight copper wire when it is moved through a uniform magnetic field. **[1980]**

6 MCQs with One or More than One Correct Answer

9. A small circular loop of area A and resistance R is fixed on a horizontal xy-plane with the center of the loop always on the axis $\hat{n}$ of a long solenoid. The solenoid has m turns per unit length and carries current I counterclockwise as shown in the figure. The magnetic field due to the solenoid is in $\hat{n}$ direction. List-I gives time dependences of $\hat{n}$ in terms of a constant angular frequency ω. List-II gives the torques experienced by the circular loop at time $t = \frac{\pi}{6\omega}$. Let $\alpha = \frac{A^2\mu_0^2 m^2 I^2 \omega}{2R}$. **[Adv. 2022]**



List-I	List-II
(I) $\frac{1}{\sqrt{2}}(\sin\omega t\,\hat{j} + \cos\omega t\,\hat{k})$	(P) 0
(II) $\frac{1}{\sqrt{2}}(\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j})$	(Q) $-\frac{\alpha}{4}\hat{i}$
(III) $\frac{1}{\sqrt{2}}(\sin\omega t\,\hat{i} + \cos\omega t\,\hat{k})$	(R) $\frac{3\alpha}{4}\hat{i}$
(IV) $\frac{1}{\sqrt{2}}(\cos\omega t\,\hat{j} + \sin\omega t\,\hat{k})$	(S) $\frac{\alpha}{4}\hat{j}$
	(T) $-\frac{3\alpha}{4}\hat{i}$

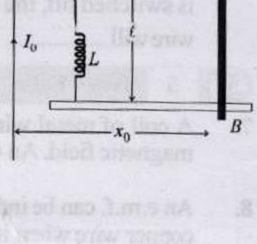
Which one of the following options is correct?

(a) (I) → (Q); (II) → (P); (III) → (S); (IV) → (T)
(b) (I) → (S); (II) → (T); (III) → (Q); (IV) → (P)
(c) (I) → (Q); (II) → (P); (III) → (S); (IV) → (R)
(d) (I) → (T); (II) → (Q); (III) → (P); (IV) → (R)

10. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it, the correct statement(s) is(are) **[2012]**
(a) The emf induced in the loop is zero if the current is constant.
(b) The emf induced in the loop is finite if the current is constant.
(c) The emf induced in the loop is zero if the current decreases at a steady rate.
(d) The emf induced in the loop is infinite if the current decreases at a steady rate.

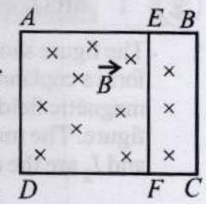
10 Subjective Problems

11. A metal bar AB can slide on two parallel thick metallic rails separated by a distance ℓ. A resistance R and an inductance L are connected to the rails as shown in the figure. A long straight wire carrying a constant current I_0 is placed in the plane of the rails and perpendicular to them as shown. The bar AB is held at rest at a distance x_0 from the long wire. At $t = 0$, it is made to slide on the rails away from the wire. Answer the following questions. **[2002 - 5 Marks]**

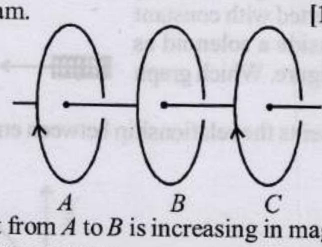


(a) Find a relation among i, $\frac{di}{dt}$ and $\frac{d\phi}{dt}$, where i is the current in the circuit and ϕ is the flux of the magnetic field due to the long wire through the circuit.
(b) It is observed that at time $t = T$, the metal bar AB is at a distance of $2x_0$ from the long wire and the resistance R carries a current i_1. Obtain an expression for the net charge that has flown through resistance R from $t = 0$ to $t = T$.
(c) The bar is suddenly stopped at time T. The current through resistance R is found to be $\frac{i_1}{4}$ at time $2T$. Find the value of $\frac{L}{R}$ in terms of the other given quantities.

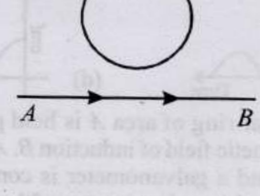
12. A rectangular frame $ABCD$, made of a uniform metal wire, has a straight connection between E and F made of the same wire, as shown in Fig. $AEFD$ is a square of side 1m, and $EB = FC =$ 0.5m. The entire circuit is placed in steadily increasing, uniform magnetic field directed into the plane of the paper and normal to it. The rate of change of the magnetic field is 1T/s. The resistance per unit length of the wire is $1\Omega/\text{m}$. Find the magnitudes and directions of the currents in the segments AE, BE and EF. **[1993-5 Marks]**



13. Three identical closed coils A, B and C are placed with their planes parallel to one another. Coils A and C carry equal currents as shown in Fig. Coils B and C are fixed in position and coil A is moved towards B with uniform motion. Is there any induced current in B ? If no, give reasons. If yes mark the direction of the induced current in the diagram. **[1982 - 2 Marks]**



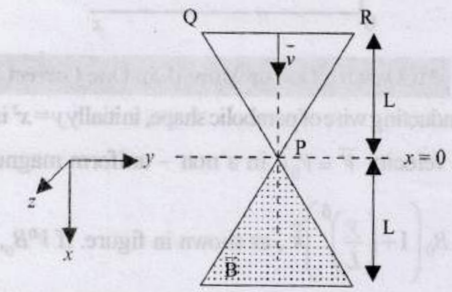
14. A current from A to B is increasing in magnitude. What is the direction of induced current, if any, in the loop as shown in the figure? **[1979]**



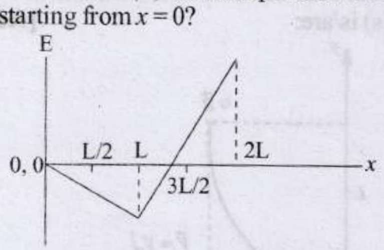
Topic-2: Motional and Static EMI and Application of EMI

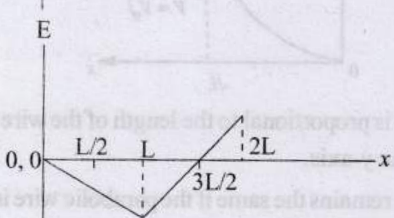
1 MCQs with One Correct Answer

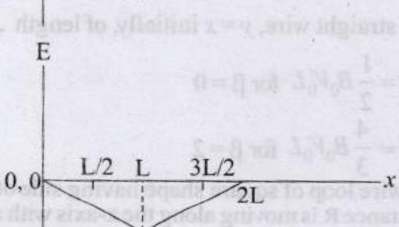
1. A region in the form of an equilateral triangle (in x-y plane) of height L has a uniform magnetic field $\vec{B}$ pointing in the +z-direction. A conducting loop PQR, in the form of an equilateral triangle of the same height L, is placed in the x-y plane with its vertex P at $x = 0$ in the orientation shown in the figure. At t = 0, the loop starts entering the region of the magnetic field with a uniform velocity $\vec{v}$ along the +x-direction. The plane of the loop and its orientation remain unchanged throughout its motion.

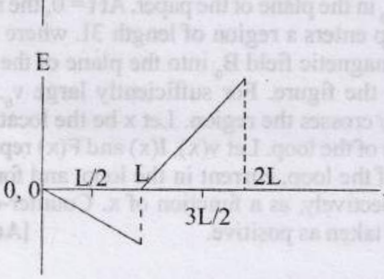


Which of the following graph best depicts the variation of the induced emf (E) in the loop as a function of the distance (x) starting from $x = 0$? **[Adv. 2024]**

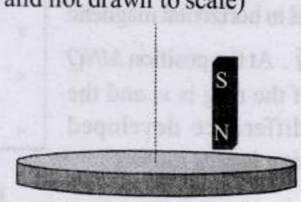
(a) 

(b) 

(c) 

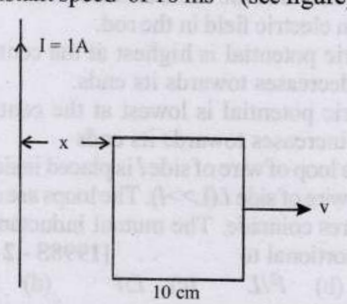
(d) 

2. A light disc made of aluminium (a nonmagnetic material) is kept horizontally and is free to rotate about its axis as shown in the figure. A strong magnet is held vertically at a point above the disc away from its axis. On revolving the magnet about the axis of the disc, the disc will (figure is schematic and not drawn to scale) **[Adv. 2020]**



(a) rotate in the direction opposite to the direction of magnet's motion
(b) rotate in the same direction as the direction of magnet's motion
(c) not rotate and its temperature will remain unchanged
(d) not rotate but its temperature will slowly rise

3. A square frame of side 10 cm and a long straight wire carrying current 1 A are in the plate of the paper. Starting from close to the wire, the frame moves towards the right with a constant speed of 10 ms^{-1} (see figure). **[Adv. 2020]**



The e.m.f induced at the time the left arm of the frame is at x = 10 cm from the wire is:

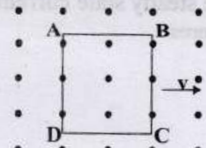
(a) 2 μV (b) 1 μV (c) 0.75 μV (d) 0.5 μV

4. Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be **[2001S]**

(a) (b) (c)

(a) maximum in situation (a)
(b) maximum in situation (b)
(c) maximum in situation (c)
(d) the same in all situations

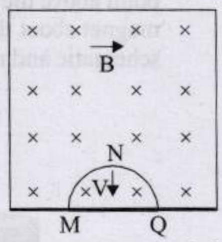
5. A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. An electric field is induced **[2001S]**



(a) in AD, but not in BC (b) in BC, but not in AD
(c) neither in AD nor in BC (d) in both AD and BC

6. A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 V battery. The current in the coil is 1.0 A at approximately the time **[1999S - 2 Marks]**
(a) 500 s (b) 25 s (c) 35 ms (d) 1 ms

7. A thin semi-circular conducting ring of radius R is falling with its plane vertical in horizontal magnetic induction $\overrightarrow{B}$. At the position MNQ the speed of the ring is v, and the potential difference developed across the ring is

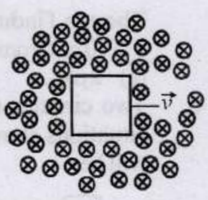


(a) zero **[1996 - 2 Marks]**
(b) $Bv\pi R^2/2$ and M is at higher potential
(c) πRBv and Q is at higher potential
(d) $2RBv$ and Q is at higher potential.

8. A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant, uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement from the following **[1998S - 2 Marks]**
(a) The entire rod is at the same electric potential.
(b) There is an electric field in the rod.
(c) The electric potential is highest at the centre of the rod and decreases towards its ends.
(d) The electric potential is lowest at the centre of the rod, and increases towards its ends

9. A small square loop of wire of side l is placed inside a large square loop of wire of side $L(L>>l)$. The loops are co-planar and their centres coincide. The mutual inductance of the system is proportional to **[1998S - 2 Marks]**
(a) l/L (b) l^2/L (c) L/l (d) L^2/l

10. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular and into the plane of the loop exists everywhere.



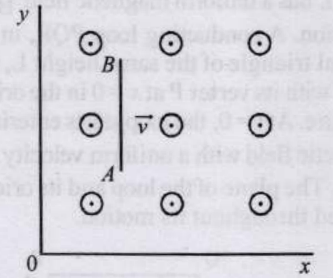
The current induced in the loop is: **[1989 - 2 Marks]**
(a) BLv/R clockwise (b) BLv/R anticlockwise
(c) $2BLv/R$ anticlockwise (d) zero.

4 Fill in the Blanks

11. A uniformly wound solenoidal coil of self inductance 1.8×10^{-4} henry and resistance 6 ohm is broken up into two identical coils. These identical coils are then connected in parallel across a 15-volt battery of negligible resistance. The time constant for the current in the circuit is .... seconds and the steady state current through the battery is .... amperes. **[1989 - 2 Marks]**

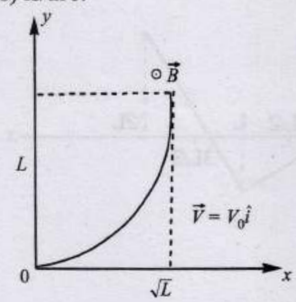
5 True / False

12. A conducting rod AB moves parallel to the x-axis (see Fig.) in a uniform magnetic field pointing in the positive z-direction. The end A of the rod gets positively charged. **[1987 - 2 Marks]**



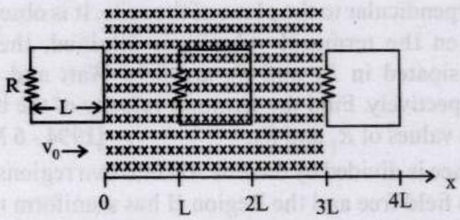
6 MCQs with One or More than One Correct Answer

13. A conducting wire of parabolic shape, initially $y = x^2$ is moving with velocity $\vec{V} = V_0\hat{i}$ in a non – uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V^0B_0, L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are: **[Adv. 2019]**

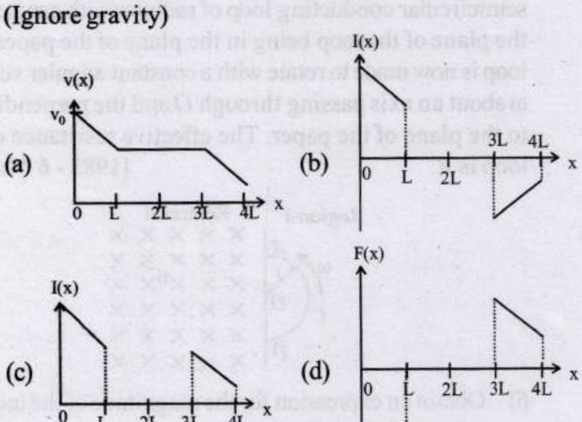


(a) $|\Delta\phi|$ is proportional to the length of the wire projected on the y-axis.
(b) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, $y = x$ initially, of length $\sqrt{2}L$
(c) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for $\beta = 0$
(d) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for $\beta = 2$

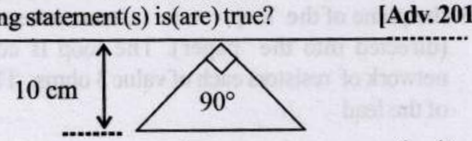
14. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At t = 0, the right edge of the loop enters a region of length 3L where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let v(x), I(x) and F(x) represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive. **[Adv. 2016]**



Which of the following schematic plot(s) is (are) correct? (Ignore gravity)



15. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 A s^{-1}. Which of the following statement(s) is(are) true? **[Adv. 2016]**



(a) The magnitude of induced *emf* in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(b) If the loop is rotated at a constant angular speed about the wire, an additional *emf* of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

(c) The induced current in the wire is in opposite direction to the current along the hypotenuse

(d) There is a repulsive force between the wire and the loop

16. The *SI* unit of inductance, the henry, can be written as **[1998S - 2 Marks]**

(a) weber/ampere (b) volt-second/ampere
(c) joule/$(\text{ampere})^2$ (d) ohm-second

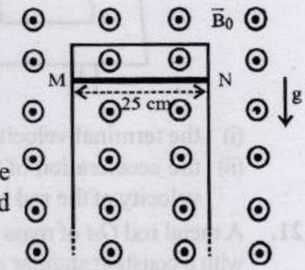
17. Two different coils have self-inductances $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then: **[1994 - 2 Marks]**

(a) $\frac{i_1}{i_2}=\frac{1}{4}$ (b) $\frac{i_1}{i_2}=4$ (c) $\frac{W_1}{W_2}=\frac{1}{4}$ (d) $\frac{V_1}{V_2}=4$

7 Match the Following

18. A thin conducting rod MN of mass 20 gm, length 25 cm and resistance 10 Ω is held on frictionless, long, perfectly conducting vertical rails as shown in the figure. There is a uniform magnetic field $B_0 = 4$ T directed perpendicular to the plane of the rod–rail arrangement. The rod is released from rest at time t = 0 and it moves down along the rails. Assume air drag is negligible. Match each quantity in List-I with an appropriate value from List-II, and choose the correct option. **[Adv. 2023]**

[Given: The acceleration due to gravity g = 10 m s^{-2} and $e^{-1} = 0.4$]

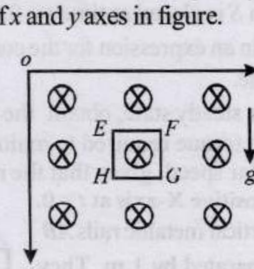


List–I	List–II
(P) At t = 0.2 s, the magnitude of the induced emf in Volt	(1) 0.07
(Q) At t = 0.2 s, the magnitude of the magnetic force in Newton	(2) 0.14
(R) At t = 0.2 s, the power dissipated as heat in Watt	(3) 1.20
(S) The magnitude of terminal velocity of the rod in m s^{-1}	(4) 0.12
	(5) 2.00

(a) P → 5, Q → 2, R → 3, S → 1
(b) P → 3, Q → 1, R → 4, S → 5
(c) P → 4, Q → 3, R → 1, S → 2
(d) P → 3, Q → 4, R → 2, S → 5

10 Subjective Problems

19. A magnetic field $B = B_0\,(y/a)\,\hat{k}$ is into the paper in the +z direction. B_0 and a are positive constants. A square loop *EFGH* of side a, mass *m* and resistance *R*, in $x-y$ plane, starts falling under the influence of gravity see figure) Note the directions of *x* and *y* axes in figure. **[1999 - 10 Marks]**



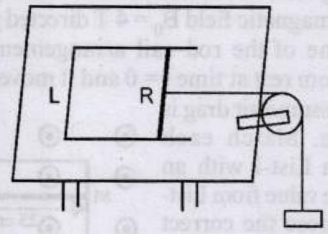
Find

(a) the induced current in the loop and indicate its direction.

(b) the total Lorentz force acting on the loop and indicate its direction, and

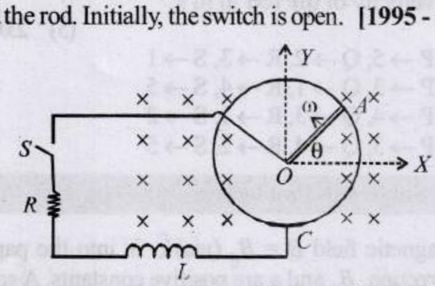
(c) an expression for the speed of the loop, $v(t)$ and its terminal value.

20. A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m, tied to the other end of the string hangs vertically. A constant magnetic field B exists perpendicular to the table. If the system is released from rest, calculate. **[1997 - 5 Marks]**



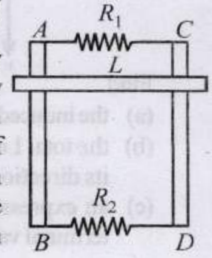
(i) the terminal velocity achieved by the rod, a nd
(ii) the acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity.

21. A metal rod OA of mass 'm' and length 'r' is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction $\vec{B}$ is applied perpendicular and into the plane of rotation as shown in the figure below. An inductor L and an external resistance R are connected through a switch S between the point O and a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open. **[1995 - 10 Marks]**

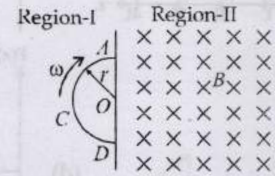


(a) What is the induced emf across the terminals of the switch?
(b) The switch S is closed at time $t = 0$.
  (i) Obtain an expression for the current as a function of time.
  (ii) In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed, given that the rod OA was along the positive X-axis at $t = 0$.

22. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at two ends by resistances R_1 and R_2 as shown in Figure. A horizontal metallic bar L of mass 0.2 kg slides without friction vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 Tesla perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0.76 Watt and 1.2 watt respectively. Find the terminal velocity of the bar L and the values of R_1 and R_2. **[1994 - 6 Marks]**

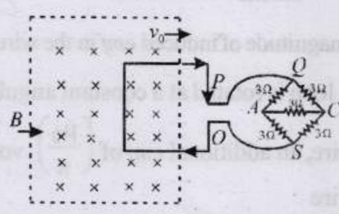


23. Space is divided by the line AD into two regions. Region I is field free and the Region II has a uniform magnetic field B directed into the plane of the paper. ACD is a semicircular conducting loop of radius r with centre at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and the perpendicular to the plane of the paper. The effective resistance of the loop is R. **[1985 - 6 Marks]**



(i) Obtain an expression for the magnitude of the induced current in the loop.
(ii) Show the direction of the current when the loop is entering into the Region II.
(iii) Plot a graph between the induced e.m.f and the time of rotation for two periods of rotation.

24. A square metal wire loop of side 10 cms and resistance 1 ohm is moved with a constant velocity v_0 in a uniform magnetic field of induction $B = 2$ webers/m^2 as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to a network of resistors each of value 3 ohms. The resistances of the lead



wires OS and PQ are negligible. What should be the speed of the loop so as to have a steady current of 1 milliampere in the loop ? Give the direction of current in the loop. **[1983 - 6 Marks]**

25. The two rails of a railway track, insulated from each other and the ground, are connected to a milli voltmeter. What is the reading of the milli voltmeter when a train travels at a speed of 180 km/hour along the track, given that the vertical component of earth's magnetic field is 0.2×10^{-4} weber/m^2 & the rails are separated by 1 meter? **[1981- 4 Marks]**

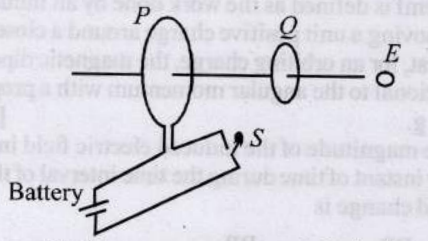
Topic-3: Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

1. An infinitely long cylinder is kept parallel to an uniform magnetic field B directed along positive z-axis. The direction of induced current as seen from the z-axis will be **[2005S]**
(a) zero
(b) anticlockwise of the +ve z axis
(c) clockwise of the +ve z axis
(d) along the magnetic field

2. A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be **[2002S]**
(a) halved (b) the same
(c) doubled (d) quadrupled

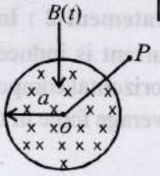
3. As shown in the figure, P and Q are two coaxial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q2} flows in Q. Then the direction IQ_1 and IQ_2 (as seen by E) are **[2002S]**



(a) respectively clockwise and anti-clockwise
(b) both clockwise
(c) both anti-clockwise
(d) respectively anti-clockwise and clockwise

4. A coil of wire having inductance and resistance has a conducting ring placed coaxially within it. The coil is connected to a battery at time $t = 0$, so that a time-dependent current $I_1(t)$ starts flowing through the coil. If $I_2(t)$ is the current induced in the ring, and $B(t)$ is the magnetic field at the axis of the coil due to $I_1(t)$, then as a function of time ($t > 0$), the product $I_2(t)\, B(t)$ **[2000S]**
(a) increases with time
(b) decreases with time
(c) does not vary with time
(d) passes through a maximum

5. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region **[2000S]**
(a) is zero
(b) decreases as $1/r$
(c) increases as r
(d) decreases as $1/r^2$



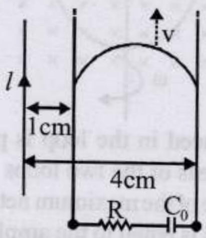
6. Two identical circular loops of metal wire are lying on a table without touching each other. Loop-A carries a current which increases with time. In response, the loop-B
(a) remains stationary **[1999S - 2 Marks]**
(b) is attracted by the loop-A
(c) is repelled by the loop-A
(d) rotates about its CM, with CM fixed

6 MCQs with One or More than One Correct Answer

7. A long straight wire carries a current, $I = 2$ ampere. A semi-circular conducting rod is placed beside it on two conducting parallel rails of negligible resistance. Both the rails are parallel to the wire. The wire, the rod and the rails lie in the same horizontal plane, as shown in the figure. Two ends of the semi-circular rod are at distances 1 cm and 4 cm from the wire. At time t = 0, the rod starts moving on the rails with a speed v = 3.0 m/s (see the figure).

A resistor R = 1.4 Ω and a capacitor C_0 = 5.0 μF are connected in series between the rails. At time t = 0, C_0 is uncharged. Which of the following statement(s) is(are) correct ?

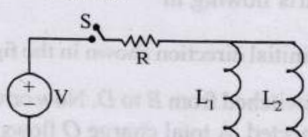
[$\mu_0 = 4\pi \times 10^{-7}$ SI units. Take ln 2 = 0.7]



[Adv. 2021]

(a) Maximum current through R is 1.2×10^{-6} ampere
(b) Maximum current through R is 3.8×10^{-6} ampere
(c) Maximum charge on capacitor C_0 is 8.4×10^{-12} coulomb
(d) Maximum charge on capacitor C_0 is 2.4×10^{-12} coulomb

8. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At t = 0, the switch is closed and current begins to flow. Which of the following options is/are correct? **[Adv. 2017]**



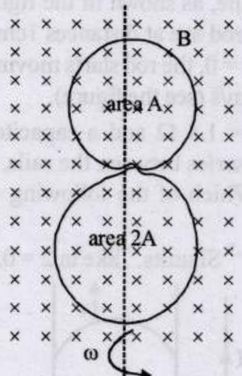
(a) After a long time, the current through L_1 will be $\dfrac{V}{R}\dfrac{L_2}{L_1 + L_2}$

(b) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(c) The ratio of the currents through L_1 and L_2 is fixed at all times $(t > 0)$

(d) At $t = 0$, the current through the resistance R is $\frac{V}{R}$

9. A circular insulated copper wire loop is twisted to form two loops of area A and 2A as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At t = 0, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct? **[Adv. 2017]**



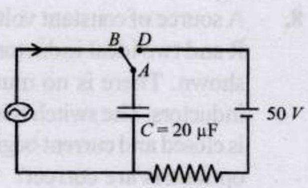
(a) The emf induced in the loop is proportional to the sum of the areas of the two loops

(b) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

(c) The net emf induced due to both the loops is proportional to cos ωt

(d) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

10. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0\cos(\omega t)$, with $I_0 = 1$ A and $\omega = 500$ rad s^{-1} starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\ \mu F$, $R = 10\ \Omega$ and the battery is ideal with emf of 50 V, identify the correct statement(s). **[Adv. 2014]**



(a) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C

(b) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise

(c) Immediately after A is connected to D, the current in R is 10 A

(d) $Q = 2 \times 10^{-3}$ C

11. L, C and R represent the physical quantities, inductance, capacitance and resistance respectively. The combination(s) which have the dimensions of frequency are **[1984- 2 Marks]**

(a) $1/RC$ (b) R/L

(c) $1/\sqrt{LC}$ (d) C/L

8 Comprehension/Passage Based Questions

Passage

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity w. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant g. **[Adv. 2013]**

12. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is

(a) $\frac{BR}{4}$ (b) $\frac{BR}{2}$ (c) BR (d) 2BR

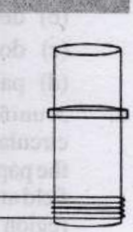
13. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is

(a) $-gBQR^2$ (b) $-\gamma\frac{BQR^2}{2}$

(c) $\gamma\frac{BQR^2}{2}$ (d) γBQR^2

9 Assertion and Reason Type Questions

14. Statement-1 : A vertical iron rod has coil of wire wound over it at the bottom end. An alternating current flows in the coil. The rod goes through a conducting ring as shown in the figure. The ring can float at a certain height above the coil.

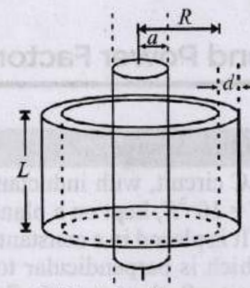


Statement-2 : In the above situation, a current is induced in the ring which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction. **[2007]**

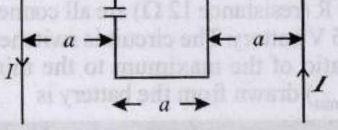
(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

10 Subjective Problems

15. A long solenoid of radius a and number of turns per unit length n is enclosed by cylindrical shell of radius R. thickness d ($d << R$) and length L. A variable current $i = i_0 \sin\omega t$ flows through the coil. If the resistivity of the material of cylindrical shell is ρ, find the induced current in the shell. **[2005 - 4 Marks]**

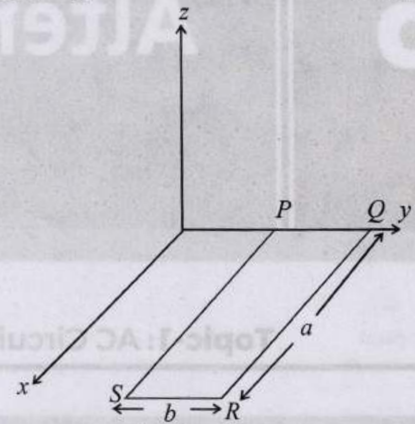


16. A square loop of side 'a' with a capacitor of capacitance C is located between two current carrying long parallel wires as shown. The value of I in the wires is given as $I = I_0 \sin\omega t$. **[2003 - 4 Marks]**



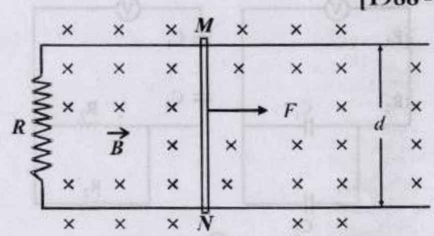
(a) Calculate maximum current in the square loop.
(b) Draw a graph between charges on the upper plate of the capacitor vs timez.

17. A rectangular loop $PQRS$ made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis (see figure). Take the vertically upward direction as the z-axis. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k})B_0$ exists in the region. The loop is held in the x-y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium. **[2002 - 5 Marks]**



(a) What is the direction of the current I in PQ?
(b) Find the magnetic force on the arm RS.
(c) Find the expression for I in terms of B_0, a, b and m.

18. Two long parallel horizontal rails, a distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (see figure). There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to the rod MN such that, as the rod moves, a constant current flows through R. **[1988 - 6 Marks]**



(i) Find the velocity of the rod and the applied force F as function of the distance x of the rod from R.
(ii) What fraction of the work done per second by F is converted into heat ?

Answer Key

Topic-1 : Magnetic Flux, Faraday's and Lenz's Law

1. (d) **2.** (c) **3.** (b) **4.** (4) **5.** (4) **7.** (False) **8.** (True) **9.** (a, c) **10.** (c) **11.** (a)

Topic-2 : Motional and Static EMI and Application of EMI

1. (a) **2.** (b) **3.** (b) **4.** (a) **5.** (d) **6.** (d) **7.** (d) **8.** (b) **9.** (b) **10.** (d) **12** (True) **13.** (a, b, d) **14.** (a, b) **15.** (a, d) **16.** (a, b, c, d) **17.** (a, c) **18.** (d)

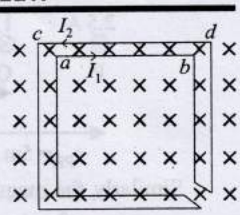
Topic-3 : Miscellaneous (Mixed Concepts) Problems

1. (a) **2.** (b) **3.** (d) **4.** (b) **5.** (b) **6.** (c) **7.** (a, c) **8.** (a, b, c) **9.** (b, d) **10.** (c, d) **11.** (a,b,c) **12.** (b) **13.** (b) **14.** (a)

Hints & Solutions

Topic-1: Magnetic Flux, Faraday's and Lenz's Law

1. **(d)** The magnetic field is increasing in the downward direction. Therefore, according to Lenz's law the current I_1 will flow in the direction ab and I_2 in the direction dc.



2. **(c)** Polarity of emf will be opposite when the magnet enters and leaves the coil.
Only graph (c) shows these characteristic.

3. **(b)** Current induced $i = \frac{|e|}{R} \Rightarrow i = \frac{1}{R}\frac{d\phi}{dt}$

But $i = \frac{dq}{dt}$ $\therefore$ $\frac{dq}{dt} = \frac{1}{R}\frac{d\phi}{dt} \Rightarrow \int dq = \frac{1}{R}\int d\phi \therefore q = \frac{BA}{R}$

4. **(4)** Here e.m.f., $\varepsilon = (B_1 - B_2)bv_y$

Current, $i = \frac{\varepsilon}{R} = \frac{\mu_0 I}{2\pi R}\left(\frac{I}{d} - \frac{1}{d+a}\right)bv_y$

$\Rightarrow 10^{-5} = \frac{2\times10^{-7}\times10}{0.1}\left[\frac{1}{4} - \frac{1}{8}\right]\times 2.v_y$

[Given: $I = 10A, R = 0.1\Omega, i = 10\mu A = 10^{-5} A$]

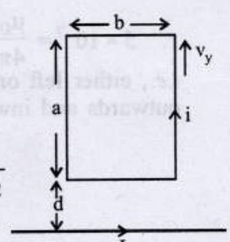
$\therefore v_y = 2$

$\tan\theta = \frac{v_y}{vx} \Rightarrow \frac{1}{\sqrt{3}} = \frac{2}{v_x}$

$\therefore v_x = 2\sqrt{3}$

$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{(2\sqrt{3})^2 + (2)^2}$

$= \sqrt{16} = 4\ ms^{-1}$



5. **(4)** Time constant, T = RC

Impedance $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

Given $Z = R\sqrt{1.25}$ $\therefore$ $R\sqrt{1.25} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

$\therefore RC = \frac{2}{\omega} = \frac{2}{500}\times 1000\ ms$ $\therefore RC = 4\ ms$

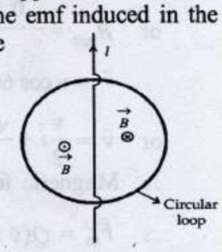
6. When the source is switched off, the current left to right decreases to zero. The induced current opposes the cause or change as per Lenz's law. Therefore, the induced current will be from left to right.

7. **False.** The coil of metal wire is kept stationary in a non-uniform magnetic field. And for induced emf to develop in a coil, the magnetic flux through the coil must change. But in this case magnetic flux linked with the coil is not changing.

8. **True.** when the wire is in motion, the electrons have a net velocity in the direction of motion.
A charged particle moving in a magnetic field experiences a force $\vec{F} = q(\vec{v}\times\vec{B})$.
Here also each electron experiences a force and therefore, electrons will move towards one end creating an emf between the two ends of a straight copper wire.

9. **(a, c)** If the current is constant, the emf induced in the loop zero. Emf will be induced in the circular wire loop when flux through it changes with time.

$e = -\frac{\Delta\phi}{\Delta t}$



when the current is constant, the flux changing through it will be zero.
Also, if the current decreases at steady rate, the emf. induced in the loop is zero.
When the current is decreasing at a steady rate then the change in the flux (decreasing inwards) on the right half of the wire is equal to the change in flux (decreasing outwards) on the left half of the wire such that $\Delta\phi$ through the circular loop is zero.

10. **(c)** (I) $\vec{B} = \frac{\mu_0 mI}{\sqrt{2}}\left(\sin\omega t\,\hat{j} + \cos\omega t\,\hat{k}\right)$

$\phi = \vec{B}\cdot\vec{A} = \frac{\mu_0 mI}{\sqrt{2}}\cos(\omega t)A$

$\varepsilon = -\frac{d\phi}{dt} = \frac{\mu_0 mI\,\omega A}{\sqrt{2}}\sin(\omega t)$

$i = \frac{\varepsilon}{R} = \frac{\mu_0 mI\,\omega A}{\sqrt{2}\,R}\sin(\omega t)$

So, $\vec{M} = i\vec{A} = \frac{\mu_0 mI\,\omega A^2}{\sqrt{2}R}\sin(\omega t)\,\hat{k}$

$\vec{\tau} = \vec{M}\times\vec{B} = \frac{\mu_0^2 m^2 I^2 \omega A^2}{2R}\sin^2(\omega t)(-\hat{i})$

For $t = \frac{\pi}{6\omega}$

$\vec{\tau} = \frac{\mu_0^2 m^2 I^2 \omega A^2}{2R}\times\frac{1}{4}(-\hat{i})$

$= \frac{A^2\mu_0^2 m^2 I^2 \omega}{2R}\times\frac{1}{4}(-\hat{i}) = \frac{\alpha}{4}(-\hat{i})$. So (I) $\to$ Q

(II) Solving as (I), $\phi = 0, \varepsilon = 0, i = 0, \vec{M} = 0$. So (II) $\to$ P

Similarly, (III) $\rightarrow$ S
(IV) $\rightarrow$ R

11. **(a)** To find the relation among $i, \frac{di}{dt}$ and $\frac{d\phi}{dt}$

Applying Kirchhoff's second law,

$$\frac{d\phi}{dt} - iR - L\frac{di}{dt} = 0$$

or $\frac{d\phi}{dt} = iR + L\frac{di}{dt}$ ...(i)

(b) Expression for the net charge through R from $t = 0$ to $t = T$.

$$d\phi = iRdt + Ldi \text{ (from eq.(}i\text{))}$$

Integrating, we get

$$\Delta\phi = R\Delta q + Li_1$$

$$\Delta q = \frac{\Delta\phi}{R} - \frac{Li_1}{R} \quad ...(ii)$$

Here, $\Delta\phi = \phi_f - \phi_i = \int_{x=2x_0}^{x=x_0} \frac{\mu_0}{2\pi}\frac{I_0}{x} l dx = \frac{\mu_0 I_0 l}{2\pi}\ln(2)$

So, from Eq. (ii) charge flown through the resistance upto time t = T, when current i_1, is

$$\Delta q = \frac{1}{R}\left[\frac{\mu_0 I_0 l}{2\pi}\ln(2) - Li_1\right]$$

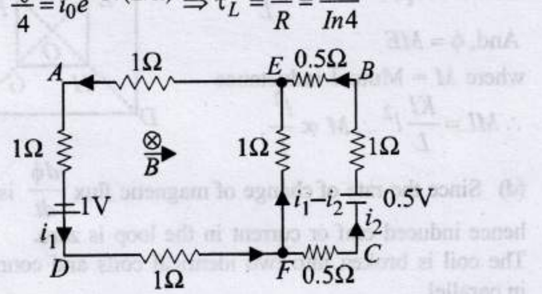
$i = i_0 e^{-t/\tau L}$

Here, $i = \frac{i_1}{4}, i_0 = i_1, t = (2T - T) = T$ and $\tau_L = \frac{L}{R}$

Substituting these values $i = i_0 e^{-t/\tau L}$

$$\frac{i_0}{4} = i_0 e^{-T/(L/R)} \Rightarrow \tau_L = \frac{L}{R} = \frac{T}{\ln 4}$$

12.



The entire circuit is placed in steadily increasing uniform magnetic field directed into the plane of paper and normal to it, an induced emf will be produced in the *AEFD* as well as in the circuit *EBCF* such that the current flowing in the loop creates magnetic lines of force in the upward direction (to the plane of paper).

Therefore, the current should flow in the anticlockwise direction in both the loops.

Induced emf in loop *AEFD*

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}BA = -A\frac{dB}{dt} = -1 \times 1 = -1\text{V}$$

Induced emf in loop *EBCF*

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}BA' = -A'\frac{dB}{dt} = -0.5 \times 1 = -0.5\text{ V}$$

Applying junction law at F, we get current in branch FE $(i_1 - i_2)$

Applying Kirchhoff's law in loop *EADFE*

$$-1 \times i_1 - 1 \times i_1 + 1 - 1 \times i_1 - 1(i_1 - i_2) = 0$$

$$\Rightarrow 4i_1 - i_2 = 1 \quad ...(i)$$

Applying Kirchhoff's law in loop *EBCFE*

$$+0.5i_1 - 0.5 + 1i_2 + 0.5i_2 - 1(i_1 - i_2) = 0$$

$$-i_1 + 3i_2 = 0.5 \quad ...(ii)$$

Solving eq. (i) and (ii)

$$11i_1 = 3.5 \Rightarrow i_1 = 3.5/11 = \frac{7}{22}A$$

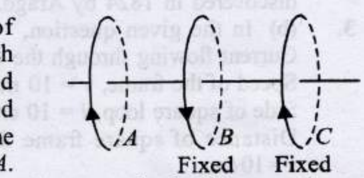
Also $11i_2 = 3 \Rightarrow i_2 = 3/11\, A = \frac{6}{22}A$

$\therefore$ Current in segment, $AE = i_1 = \frac{7}{22}A$; $BE = i_2 = \frac{6}{22}A$

and $EF = (i_1 - i_2) = \frac{1}{22}A$

13. Yes, When the coil A moves towards B, the number of magnetic lines of force passing through B changes. Therefore, an induced emf and hence induced current is produced in B.

The direction of current in B will be such as to oppose the field change in B and therefore, will be in the opposite direction of A.



14. The direction of induced current in the loop as shown below.

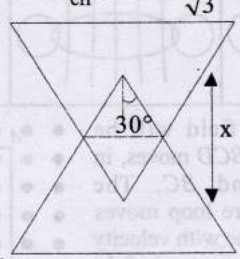
As the current increases, the number of magnetic lines of force passing through the loop increases in the outward direction. To oppose this change, the current will flow in the clockwise direction as per Lenz's law.

A ————→ B

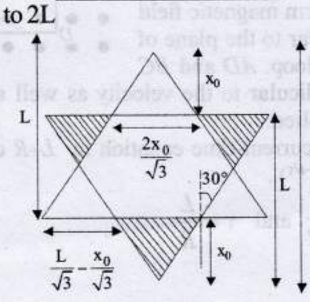
Topic-2: Motional and Static EMI and Application of EMI

1. **(a)** For X = 0 to L

Induced emf, $\varepsilon = Bl_{eff}v = B \times \frac{x}{\sqrt{3}}v$



For x = L to 2L



$$|\text{emf}| = B\left(\frac{L}{\sqrt{3}} - \frac{x_0}{\sqrt{3}}\right)v - B\frac{2x_0}{\sqrt{3}}v$$

$$= \frac{BvL}{\sqrt{3}} - \sqrt{3}Bvx_0 = Bv\left[\frac{L}{\sqrt{3}} - \sqrt{3}(x - L)\right]$$

$$= \frac{Bv}{\sqrt{3}}[L - 3x + 3L] = \frac{Bv}{\sqrt{3}}[4L - 3x]$$

At $x = \frac{4L}{3} \Rightarrow \text{emf} = 0$

Therefore graph option (a) best depicts the variation of the induced emf(E) as a function of the distance (x)

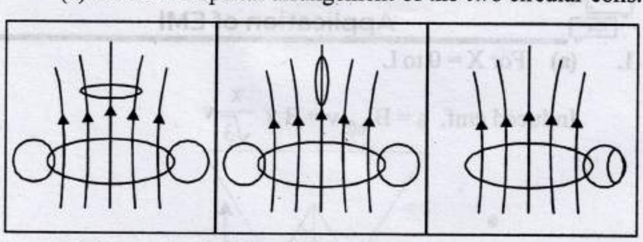
2. **(b)** Due to motion of magnet above the disc, the plate moves through the magnetic flux, due to which an EMF is generated in the plate and eddy currents are induced. These currents are such that it opposes the relative motion so disc will rotate in the same direction as the direction of magnet's motion.
This apparatus is called Arago's disk and the effect was discovered in 1824 by Arago.

3. **(b)** In the given question,
Current flowing through the wire, I = 1A
Speed of the frame, v = 10 ms^{-1}
Side of square loop, l = 10 cm
Distance of square frame from current carrying wires x = 10 cm.
We have to find, e.m.f induced e = ?
According to Biot-Savart's law

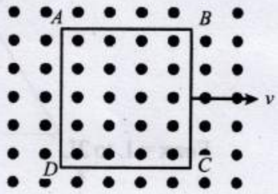
$$B = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{x^2} = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{1\times10^{-1}}{(10^{-1})^2} = 10^{-6}$$

Induced e.m.f. e = Blv = $10^{-6} \times 10^{-1} \times 10 = 1$ μv

4. **(a)** When current flows in any of the coils, the flux linked with the other coil is maximum when surface area to receive flux is maximum. Clearly the flux linkage is maximum in case (a) due to the spatial arrangement of the two circular coils.



5. **(d)** Electric field will be induced, as *ABCD* moves, in both *AD* and *BC*. The metallic square loop moves in its own plane with velocity *v* in a uniform magnetic field perpendicular to the plane of the square loop. *AD* and *BC* are perpendicular to the velocity as well as perpendicular to field applied.



6. **(d)** Using current-time equation in *L–R* circuit,
$I = I_0(1 - e^{-t/\tau})$

But $I_0 = \frac{V}{R}$ and $\tau = \frac{L}{R}$

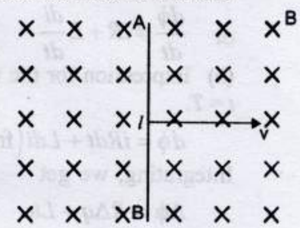
$\therefore\ I = \frac{V}{R}(1 - e^{-Rt/L}) = \frac{12}{6}\left[1 - e^{-6t/8.4\times10^{-3}}\right] = 1$ (given)

$\therefore\ t = 0.97 \times 10^{-3}$ s ≈ 1ms

7. **(d)** Induced emf produced across *MNQ* will be same as the induced emf produced in straight wire *MQ*.

$\therefore\ e_{MNQ} = e_{MQ} = Bv\ell = Bv \times 2R$ with Q at higher potential.

8. **(b)** In this case there is an electric field in the rod. A motional emf, e = Blv is induced in the rod. Or we can say a potential difference is induced between the two ends of the rod AB, with A at higher potential and B at lower potential. Due to this potential difference, there is an electric field in the rod.



9. **(b)** We know magnetic field due to a current flowing in a wire of finite length magnetic

$$B = \frac{\mu_0 I}{4\pi R}(\sin\alpha + \sin\beta)$$

Applying the above formula for *AB* for finding the magnetic field at centre *O*, of the loop

$$B = \frac{\mu_0 I}{4\pi(L/2)}(\sin 45° + \sin 45°) = \frac{\mu_0 I}{\sqrt{2}\pi L}$$

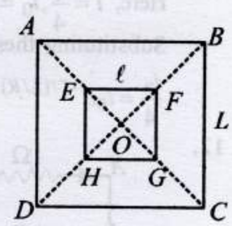
or $B \propto \frac{I}{L}$ or, $B = K\frac{I}{L}(K = \text{constant})$

Magnetic flux linked with the smaller loop, $\phi = B.A = \frac{KI}{L}l^2$

And, $\phi = ME$

where *M* = Mutual inductance

$\therefore MI = \frac{KI}{L}l^2 \therefore M \propto \frac{l^2}{L}$.



10. **(d)** Since the rate of change of magnetic flux $\frac{d\phi}{dt}$ is zero, hence induced emf or current in the loop is zero.

11. The coil is broken into two identical coils and connected in parallel.

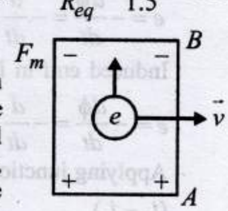
$$\therefore L_{eq} = \frac{L/2 \times L/2}{L/2 + L/2} = \frac{L}{4} = \frac{1.8\times10^{-4}}{4} = 0.45 \times 10^{-4}\text{ H},$$

$$\therefore R_{eq} = \frac{R/2 \times R/2}{R/2 + R/2} = \frac{R}{4} = \frac{6}{4} = 1.5\Omega$$

Time constant, $\tau = \frac{L_{eq}}{R_{eq}} = \frac{0.45\times10^{-4}}{1.5} = 0.3 \times 10^{-4}$s

Steady state current through the battery, $I = \frac{E}{R_{eq}} = \frac{15}{1.5} = 10$ A.

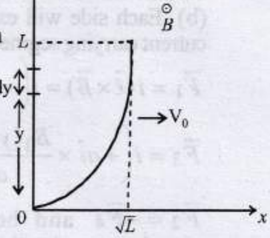
12. **True.** According to fleming's left hand rule, when conduction rod AB moves parallel to x-axis in a uniform magnetic field pointing in the positive *z*-direction, then the electrons will experience a force towards B. Hence, the end A will become positive due to deficiency of electrons at A.



13. **(a, b, d)** Given: $B = B_0\left[1+\left(\frac{y}{L}\right)^{\beta}\right]\hat{K}\ \vec{V} = V_0\hat{i}$

We now consider a infinite small length of wire dy at a distance y from the origin.

Emf induced across the length dy $|\Delta\phi| = B(dy)V_0$



$$|\Delta\phi| = B_0\left[1+\left(\frac{y}{L}\right)^{B}\right]V_0 dy$$

$\therefore$ Induced emf across the complete projection

$$|\Delta\phi| = B_0V_0\int_0^L\left[1+\left(\frac{y}{L}\right)^{\beta}\right]V_0dy = B_0V_0L\left[1+\frac{1}{\beta+1}\right]$$

For $\beta = 0, |\Delta\phi| = 2B_0V_0L$. Clearly, $|\Delta\phi| \propto L$

For $\beta = 2\ |\Delta\phi| = \frac{4}{3}B_0V_0L$

For a straight wire of length $\sqrt{2L}$ placed along y = x then the value of $|\Delta\phi|$ will remain the same as its projection of y-axis is same L as that of previous.

14. **(a, b)** $i = \frac{e}{R} = \frac{BLv}{R}$ – (i) [Counter–clockwise direction while entering, Zero when completely inside and clockwise while exiting]

$F = iLB = \frac{B^2L^2v}{R}$ – (ii) [Toward left while entering and exiting and zero when completely inside]

$\therefore\ -mV\frac{dv}{dx} = \frac{B^2L^2v}{R}$

$\therefore\ \int_{v_0}^{v} dV = -\frac{B^2L^2}{mR}\int_0^x dx \Rightarrow V - V_0 = -\frac{B^2L^2}{mR}x$

$\therefore\ V = V_0 - \frac{B^2L^2x}{mR}$ ... (iii)

[V decreases from x = 0 to x = L, remains constant for x = L to x = 3L again decreases from x=3L to x=4L hence graph (a) is correct]

From (i) and (iii)

$$i = \frac{BL}{R}\left[V_0 - \frac{B^2L^2x}{mR}\right]$$

[i decreases from x=0 to x=L i becomes zero from x = L to x = 3Li changes direction and decreases from x = 3L to x = 4L] Hence graph (b) is correct.

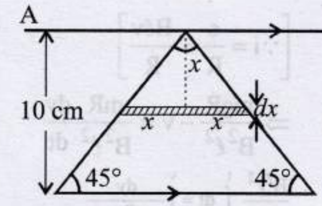
15. **(a, d)**

The flux passing through the triangular wire if i current flows through the inifinitely long conducting wire

$$d\phi = \int_0^{0.1}\frac{\mu_0 i}{2\pi x}\times 2\pi dx$$

$$\phi = \frac{\mu_0 i}{10\pi} = Mi$$

$$\therefore\ M = \frac{\mu_0}{10\pi}$$



$\left(\frac{dI}{dt} = 10\,AS^{-1}\text{ given}\right)$

Induced emf in the wire, $e = M\frac{di}{dt} = \frac{\mu_0}{10\pi}\times 10 = \frac{\mu_0}{\pi}V$

There will be no extra induced emf in the wire because there is no change in the magnetic.
Flux due to rotation of loop.
As the current in the triangular wire is decreasing the induced current in AB is in the same direction as the current in the hypotenuse of the triangular wire. Therefore force will be repulsive.

16. **(a, b, c, d)** (a) Inductance, $L = \frac{\phi}{i}$ or henry $= \frac{\text{weber}}{\text{ampere}}$

(b) Induced *emf*, $e = -L\left(\frac{di}{dt}\right)$

$\therefore\ L = -\frac{e}{(di/dt)}$ or henry $= \frac{\text{volt} - \text{second}}{\text{ampere}}$

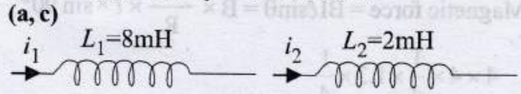
(c) Energy, $U = \frac{1}{2}Li^2$

$\therefore\ L = \frac{2U}{i^2}$ or henry $= \frac{\text{joule}}{(\text{ampere})^2}$

(d) $U = \frac{1}{2}Li^2 = i^2Rt$

$\therefore\ L = R.t$ or henry = ohm-second.

17. **(a, c)**



According to question, rate of change of current $\frac{di_1}{dt}$ = constant = $m(say)$
And according to faraday's law,

Induced emf $V_1 = -L_1\frac{di_1}{dt} = -8\times10^{-3}\times m$

$\therefore\ \frac{V_2}{V_1} = \frac{L_2}{L_1} = \frac{2\times10^{-3}}{8\times10^{-3}} = \frac{1}{4}$

Since Power given to the two coils are equal

$\therefore\ V_1i_1 = V_2i_2$

$\Rightarrow\ \frac{i_1}{i_2} = \frac{V_2}{V_1} = \frac{1}{4}$ ...(i)

Energy $W = \frac{1}{2}Li^2$

$$\therefore\ \frac{W_2}{W_1} = \frac{\frac{1}{2}L_2i_2^2}{\frac{1}{2}L_1i_1^2} = \frac{1}{4}\times4\times4 = 4$$

18. **(d)** From force equation, $mg - Bi\ell = ma$

$$= mg - Bi\ell = \frac{mdv}{dt} \Rightarrow mg - \frac{BBi\ell}{R}\times\ell = \frac{mdv}{dt}$$

$$\left[\because i=\frac{\varepsilon}{R}=\frac{B\ell v}{R}\right]$$

$$\Rightarrow \frac{mgR}{B^2\ell^2}-v=\frac{mR}{B^2\ell^2}\frac{dv}{dt}$$

$$\frac{B^2\ell^2}{mR}\int_{t=0}^{t}dt=\int_0^v\frac{dv}{\frac{mgR}{B^2\ell^2}-v}$$

$$\text{or, }\frac{B^2\ell^2}{mR}=\frac{16\times\frac{1}{16}}{20\times10^{-3}\times10}=\frac{1}{0.2}=5$$

$$\text{Now }\frac{mgR}{B^2\ell^2}=\frac{20\times10^{-3}\times10\times10}{16\times\frac{1}{16}}=2$$

$$\text{And }\frac{B^2\ell^2}{mR}=\frac{16\times\frac{1}{16}}{20\times10^{-3}\times10}=\frac{1}{0.2}=5$$

$$\therefore 5t=\left[-\ell n(2-v)\right]_0^v\Rightarrow -5t=\ell n\left[\frac{2-v}{v}\right]$$

$\therefore v = 2(1-e^{-5t})$

At $t = 0.2$ sec

$v = 2(1-e^{-5\times0.2})$

$v = 2(1-0.4)$

$v = 1.2$ m/s

At $t = 0.2$ s

Induced emf $\varepsilon = Bv\ell$

$\therefore \varepsilon = 4\times1.2\times\frac{1}{4} = 1.2$ Volt

Magnetic force $= BI\ell\sin\theta = B\times\frac{B\ell v}{R}\times\ell\times\sin 90°$

$$=\frac{4\times4\times\frac{1}{4}\times1.3\times\frac{1}{4}}{10}=0.12\,\text{N}$$

Power dissipated as heat $P = i^2R = \frac{v^2}{R}$

$$\therefore P=\frac{1.2\times1.2}{10}=0.144\text{ watt}$$

At terminal velocity, the net force become zero

$$\therefore mg=Bi\ell\Rightarrow mg=B\times\frac{B\ell v_t}{R}\times\ell$$

$$\therefore v_T=\frac{mgR}{B^2\ell^2}=\frac{20\times10^{-3}\times10\times10}{16\times\frac{1}{16}}=2\text{ m/s}$$

19. Given: $\beta = B_0\left(\frac{y}{a}\right)\hat{k}$ and side of square loop $= a$

Suppose at $t = 0$, $y = 0$ and $t = t$, $y = y$

(a) Total magnetic flux $\phi = \vec{B}.\vec{A}$

$$\therefore\quad \phi=\frac{B_0y}{a}.a^2=B_0ya$$

$$\text{Net emf., } e=-\frac{d\phi}{dt}=-B_0a\frac{dy}{dt}=-B_0av(t)$$

$$\therefore |i|=\frac{|e|}{R}=\frac{B_0av(t)}{R}\quad\text{[Anticlock wise]}$$

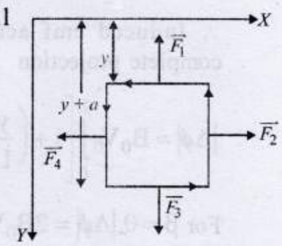
As loop goes down, magnetic flux linked with it increases, hence induced current flows in such a direction so as to reduce the magnetic flux linked with it. Therefore, induced current flows in anticlockwise direction.

(b) Each side will experience a force as shown in figure (A current carrying segment in a magnetic field experiences a force).

$$\vec{F}_1=i(\vec{\ell}\times\vec{B})=i\left(-a\hat{i}\times\frac{B_0y}{a}\hat{k}\right)=B_0y(i\times\hat{j});$$

$$\vec{F}_3=i\left(+a\hat{i}\times\frac{B_0(y+a)}{a}\hat{k}\right)=iB_0(y+a)\hat{j}$$

$\vec{F}_2 = -\vec{F}_4$ and hence will cancel out each other.



Net force,

$$\vec{F}=\vec{F}_1+\vec{F}_2+\vec{F}_3+\vec{F}_4$$

$$=-iB_0a\hat{j}=-\frac{B_0^2a^2v(t)}{R}\hat{j}$$

in upward direction.

(c) Net force on the loop,

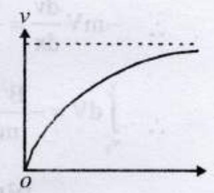
$$F=mg\hat{j}+\vec{F}=\left[mg-\frac{B_0^2a^2v(t)}{R}\right]\hat{j};$$

$$\therefore\quad m\frac{dv}{dt}=mg-\frac{B_0^2a^2v(t)}{R}$$

$$\text{Integrating it, we get, }\int_0^v\frac{dv}{g-\frac{B_0^2a^2v(t)}{mR}}=\int_0^t dt$$

$$\frac{\log\left[g-\frac{B_0^2a^2v(t)}{mR}\right]_0^{v(t)}}{\frac{-B_0^2a^2}{mR}}=t$$

$$\text{or }\log\left[\frac{g-\frac{B_0^2a^2v(t)}{mR}}{g}\right]=-\frac{B_0^2a^2t}{mR}$$



$$\text{or }\quad 1-\frac{B_0^2a^2v(t)}{mgR}=e^{-\left(B_0^2a^2t\right)/mR}$$

$$\text{or }\quad 1-e^{-\left(B_0^2a^2t\right)/mR}=\frac{B_0^2a^2}{mgR}v(t);$$

$$\therefore\quad v(t)=\frac{mgR}{B_0^2a^2}\left[1-e^{-\left(B_0^2a^2t\right)/mR}\right]$$

When terminal velocity is attained, $v(t)$ does not depend on t

$$\therefore\quad v(t)=\frac{mgR}{B_0^2a^2}$$

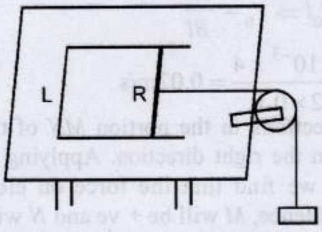
20. (i) Let v be the velocity of the rod at any time t,

$\therefore$ Induced emf $e = BvL$ and so induced current in the rod

$$I=\frac{\text{Induced e.m.f.}}{R}=\frac{BvL}{R}$$

Due to this current, the rod in the field B will experience a force

$F = BIL = \dfrac{B^2L^2v}{R}$ (opposite to its motion) ...(i)



So, net force in the system

$T - F = 0 \times a$, i.e., $T = F$ [$\because$ rod is massless]

$\Rightarrow mg - T = ma \Rightarrow a = g - \dfrac{T}{m} = g - \dfrac{B^2L^2v}{mR}$ ...(ii)

So rod will acquire terminal velocity when its acceleration is zero i.e.,

$g - \dfrac{B^2L^2v_T}{mR} = 0$ i.e. $v_T = \dfrac{mgR}{B^2L^2}$;

(ii) When the velocity of the rod is half of its terminal velocity

$v = \dfrac{v_T}{2} = \dfrac{mgR}{2B^2L^2}$

Substituting this value of velocity in eq. (ii)

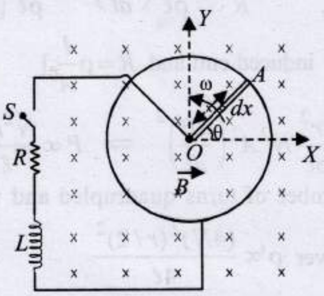
Acceleration, $a = g - \dfrac{B^2L^2}{mR} \times \dfrac{1}{2}\dfrac{mgR}{B^2L^2} = g - \dfrac{1}{2}g = \dfrac{g}{2}$

21. (a) (i) Let us consider a small element of length dx of metal rod OA at a distance x from the origin. Small amount of emf induced in this small length due to uniform magnetic field B

$de = B\,(dx)\,v$ ...(i)

where v is the velocity of small length dx

$v = x\omega$ ...(ii)



$\therefore$ The total emf acoss the whole metallic rod OA

$e = \int_0^r Bx\omega\,dx = B\omega\left[\dfrac{x^2}{2}\right]_0^r = \dfrac{Br^2\omega}{2}$

(b) (i) The above diagram can be reconstructed as follows. Switch s is a closed at time $t = 0$. Hence it is a case of growth of current in L-R circuit. Therefore current at any time 't'

$I = I_0(1 - e^{t/\tau})$

Here,

$I_0 = \dfrac{e}{R} = \dfrac{B\omega r^2}{2R}$

and $\tau = \dfrac{L}{R}$

$\therefore I = \dfrac{B\omega r^2}{2R}[1 - e^{-\left(\frac{R}{L}\right)t}]$

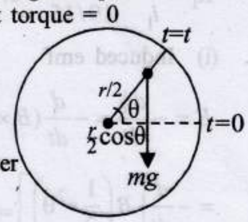
(ii) In steady state,

$I = \dfrac{B\omega r^2}{2R}$ [$\because$ t has a large value and $e^{-\left(\frac{R}{L}\right)t} \to 0$]

Since the rod is rotating in a vertical plane, work needs to be done to keep it at constant angular speed.

At constant angular speed, net torque = 0

Power loss due to current I



$P = I^2R = \left(\dfrac{Br^2\omega}{2R}\right)^2 R$

If torque required for this power is τ_1 then

$P = \tau_1\omega \Rightarrow \tau_1 = \dfrac{B^2r^4\omega}{4R}$

Torque of weight (mg) about centre

$\tau_2 = mg \times \dfrac{r}{2}\cos\theta$

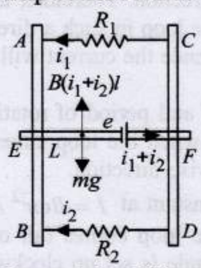
The total torque

$\tau = \tau_1 + \tau_2$ (Clockwise)

$\tau = \dfrac{B^2r^4\omega}{4R} + \dfrac{mgr}{2}\cos\omega t$

The required torque will be of same magnitude and in anticlockwise direction. The net torque will then be zero and the angular speed of the rod will be maintained constant.

22. The direction of induced current can be found with the help of Lenz's law.



Potential difference across parallel combinations remains the same

Also, $P_1 = ei_1 = 0.76$ W

and $P_2 = ei_2 = 1.2$ W

$\therefore \dfrac{i_1}{i_2} = \dfrac{1.76}{1.2} \Rightarrow i_1 = \dfrac{1.76}{1.2}i_2$ ...(ii)

The horizontal metallic bar L moves with a terminal velocity. i.e., the net force on the bar is zero.

$\therefore B(i_1 + i_2) = mg$

$\Rightarrow i_1 + i_2 = \dfrac{mg}{B\ell} = \dfrac{0.2 \times 9.8}{0.6 \times 1} = \dfrac{49}{15}$ amp. ......(iii)

From eq. (ii) and (iii)

$\dfrac{1.76}{1.2}i_2 + i_2 = \dfrac{49}{15}$

$\Rightarrow i_2 = 2$ A. $\Rightarrow i_1 = \dfrac{19}{15}$ A. $\Rightarrow e = \dfrac{0.76}{19/15} = 0.6V$

Induced emf across L due to the movement of bar L in a magnetic field

$e = Bv_T L \Rightarrow v_T = \frac{e}{BL} = \frac{0.6}{0.6 \times 1} = 1\,m/s$

Also from eq. (i),

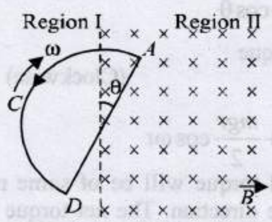
$R_1 = \frac{e}{i_1} = \frac{0.6}{19/15} = 0.47\Omega$ and $R_2 = \frac{e}{i_2} = \frac{0.6}{2} = 0.3\Omega$

23. (i) Induced emf

$$E = -\frac{d\phi}{dt} = -\frac{d}{dt}(B \times A)$$

$$= -\frac{d}{dt}\left[B\left(\frac{1}{2}r^2\theta\right)\right] = -\frac{1}{2}Br^2\frac{d\theta}{dt} = -\frac{1}{2}Br^2\omega \; [\because \theta = \omega t]$$

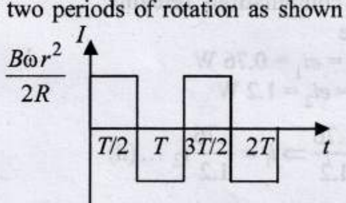
$$\therefore \; I = \frac{E}{R} = -\frac{1}{2}\frac{Br^2\omega}{R} \Rightarrow |I| = \frac{1}{2}\frac{Br^2\omega}{R}$$



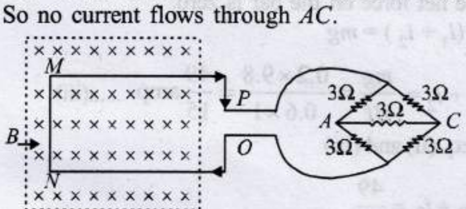
(ii) When the loop is entering in the magnetic field i.e., in region–II, magnetic lines of force passing through the loop is increasing in the downward direction. Therefore, as per Lenz's law current will flow in the loop in such a direction which will oppose the change. Hence the current will flow in the anti-clockwise direction.

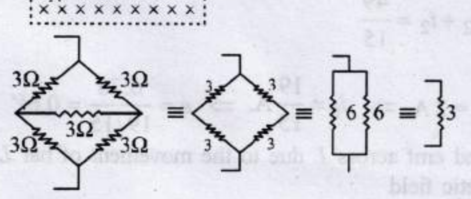
(iii) Graph between induced emf and period of rotation: For first half rotation, ($t = T/2$), when the loop enters the field, the current is in anticlockwise direction.

Magnitude of current remains constant at $I = B\omega r^2/2R$. For next half rotation, when the loop comes out of the field, current of the same magnitude is set up clockwise. Anticlockwise current is supposed to be positive. The I-t graph for two periods of rotation as shown below.



24. The network behaves like a balanced wheatstone bridge. So no current flows through AC.





$\therefore$ Equivalent resistance of the circuit $R = 3 + 1 = 4\Omega$

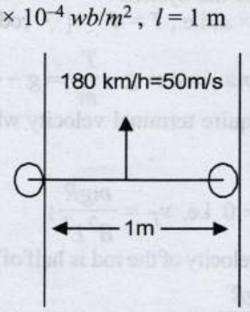
The emf induced, $e = BV_0 l$

$e = IR = BV_0 l \Rightarrow V_0 = \frac{IR}{Bl}$

or, $V_0 = \frac{1 \times 10^{-3} \times 4}{2 \times 0.1} = 0.02\,m/s$

The free electrons in the portion MN of the rod have a velocity v in the right direction. Applying Fleming's left hand rule, we find that the force on electron will be towards N. Hence, M will be + ve and N will be negative. Current will flow in clockwise direction.

25. Here, $B = 0.2 \times 10^{-4}\,wb/m^2$, $l = 1$ m



Here emf developed due to motional emf.

$e = vB\ell = 50 \times 0.2 \times 10^{-4} \times 1 = 10^{-3} = 1$ mv

The reading of milli voltmeter is $1mv$.

Topic-3: Miscellaneous (Mixed Concepts) Problems

1. **(a)** As cylinder is kept parallel to an uniform magnetic field, so no change in magnetic flux and hence induced current will be zero.

2. **(b)** Power, $P = \frac{E^2}{R} = \frac{\pi r^2}{\rho\ell}\left(\frac{d\phi}{dt}\right)^2 = \frac{\pi r^2}{\rho\ell}\left[\frac{d}{dt}(NBA)^2\right]$

[Here, E = induced emf and $R = \rho\frac{l}{A}$]

or, $P = \frac{\pi r^2}{\rho\ell}N^2A^2\left(\frac{dB}{dt}\right)^2 \Rightarrow P \propto \frac{N^2r^2}{\ell}$

When number of turns quadrupled and the wire radius haved Power $\rho' \propto \frac{(4N)^2(r/2)^2}{4\ell}$

$\therefore \frac{P}{\rho'} = \frac{1}{1}$ $\therefore$ Power remains the same.

3. **(d)** When switch S is closed, a magnetic field is set-up in the space around P. The field lines threading Q increases in the direction from right to left. According to Lenz's law, I_{Q_1} will flow so as to oppose the cause or chnage and flow in anticlockwise direction as seen by E. Opposite is the case when S is opened. I_{Q_2} will be clockwise.

4. **(b)** A conducting ring is placed coaxially within a coil and the coil has inductance as well as resistance. The magnetic field at the centre of the coil

$B(t) = \mu_0 n I_1$.

As the current increases, B will also increase with time till it reaches a maximum value (when the current becomes steady).

Induced emf in the ring

$$e=-\frac{d\phi}{dt}=-\frac{d}{dt}(\vec{B}.\vec{A})=-A\frac{d}{dt}(\mu_0 nI_1)$$

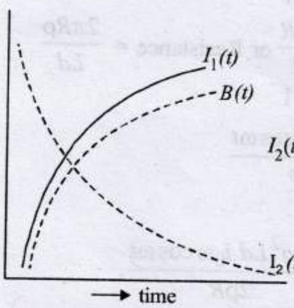
$\therefore$ Induced current in the ring

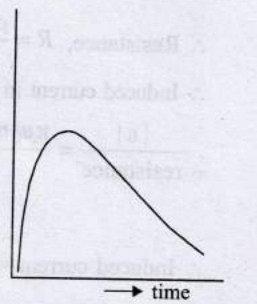
$$I_2(t)=\frac{|e|}{R}=\frac{\mu_0 nA}{R}\frac{dI_1}{dt}$$

[$\frac{dI_1}{dt}$ decreases with time and hence I_2 also decreases with time.]

Where $I_1=I_{max}(1-e^{-t/\tau})$

The relevant graphs are as follows.





Hence as a function of time (t>0) the product $I_2(t)$ B(t) decreases with time.

5. **(b)** $\oint\vec{E}.\vec{d\ell}=\frac{d\phi}{dt}=\frac{d}{dt}(\vec{B}.\vec{A})=\frac{d}{dt}(BA\cos 0°)=A\frac{dB}{dt}$

$$\Rightarrow E(2\pi r)=\pi a^2\frac{dB}{dt}\ for\ r\geq a$$

$$\Rightarrow E=\frac{a^2}{2r}\frac{dB}{dt}\Rightarrow E\propto\frac{1}{r}$$

Hence magnitude of the induced electric field at a distance r from centre of circular region decreases as $\frac{1}{r}$.

6. **(c)** Loop-A carries a current. And hen the current in the loop A increases with time the magnetic lines of force in loop B also increases as loop A is placed near loop B. This induces an emf in B in such a direction that current flows opposite in B as compared to A hence the loop B is repelled by loop A.

7. **(a, c)** EMF developed across the semi-circular rod,

$$\varepsilon=\int_1^4\frac{\mu_0 i}{2\pi r}drv=\frac{\mu_0 iv}{2\pi}\ln\frac{b}{a}=\frac{\mu_0 iv}{2\pi}\ln\frac{4}{1}=\frac{\mu_0 iv}{\pi}\ln 2$$

$$\therefore\ \frac{4\pi\times10^{-7}\times2\times3\times0.7}{\pi}=24\times7\times10^{-8}$$

$$=1.6\times10^{-6}\text{ V}$$

Therefore maximum current through R,

$$i_{max}=\frac{\varepsilon}{R}=\frac{1.68\times10^{-6}}{1.4}=1.2\times10^{-6}\text{ A}$$

And maximum charge on capacitor C_0

$$Q_{max}=C_0E=5\times10^{-6}\times1.68\times10^{-6}=8.4\times10^{-12}\text{ C}$$

8. **(a, b, c)** At t = 0 inductors L_1 and L_2 will offer infinite resistance hence current through circuit is zero.

After a long time the current through the resistor is constant I will divide into two parts L_1 and L_2 which are in parallel

$\therefore I_1L_1=I_2L_2$ $[I=I_1+I_2]$

$$I_1=\frac{V}{R}\left[\frac{L_2}{L_1+L}\right]$$

and $$I_2=\frac{V}{R}\left[\frac{L_1}{L_1+L_2}\right]$$

Also the ratio of currents through L_1 and L_2 is fixed at all times At t = 0, $I\approx0$

9. **(b, d)** The net magnetic flux through the loops at time t

$\phi=B.2A\cos\omega t-BA\cos\omega t=B\ (2A-A)\cos\omega t=BA\cos\omega t$

$$\therefore\left|\frac{d\phi}{dt}\right|=B\omega A\sin\omega t$$

So, $\left|\frac{d\phi}{dt}\right|$ is maximum when $\phi=\omega t=\pi/2$

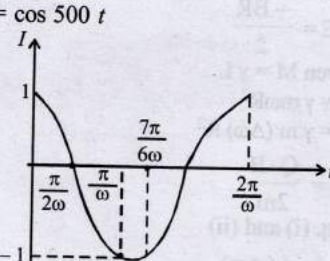
The emf induced in the smaller loop,

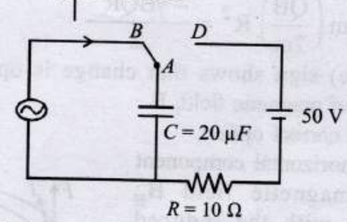
$$\varepsilon_{smaller}=-\frac{d}{dt}(BA\cos\omega t)=B\omega A\sin\omega t$$

$\therefore$ Amplitude of maximum net emf induced in both the loops = Amplitude of maxium emf induced in the smaller loop alone.

10. **(c, d)** For maximum charge on the capacitor, $\frac{dQ}{dt}=I=0$

$I=I_0\cos\omega t=\cos 500\ t$





Till $t=\frac{7\pi}{6\omega}$, the charge will be maximum at $\frac{\pi}{2\omega}$

$$Q'=\int_0^{\pi/2\omega}\cos 500t\ dt=\left[\frac{\sin 500t}{500}\right]_0^{\pi/2\omega}$$

$$=\frac{1}{500}\sin\left(500\times\frac{\pi}{2\times500}\right)=\frac{1}{500}\text{C}$$

i.e., $Q_{max}=\frac{1}{500}c=2\times10^{-3}C$

From the graph it is clear that just before $t=\frac{7\pi}{6\omega}$, the current is in anticlockwise direction.

Immediately after A is connected to D.

At $t = \frac{7\pi}{6\omega}$, the charge on the upper plate of capacitor

$$\int_0^{\frac{7\pi}{6\omega}} \cos 500t\, dt = \frac{1}{500}\sin\left(500 \times \frac{7\pi}{6\times 500}\right)$$

$$= -\frac{1}{500}\times\frac{1}{2} = -10^{-3}\text{C}$$

Now applying KVL

$$50 + \frac{10^{-3}}{20\times 10^{-6}} - i\times 10 = 0 \Rightarrow i = 10\text{ A}$$

The maximum charge on C, $Q = CV = 20\times 10^{-6}\times 50 = 10^{-3}$C
Therefore, the total charge flown from the battery $= 2\times 10^{-3}$ C

11. **(a,b,c)** $\frac{1}{RC}$, R/L and $1/\sqrt{LC}$ have the dimensions of frequency i.e., $[M^\circ L^\circ T^{-1}]$

12. **(b)** $\int \vec{E}.\vec{dl} = \frac{-d\phi}{dt} = -\frac{d}{dt}(B\pi R^2) = -\pi R^2\frac{dB}{dt}$

$$= -\pi R^2 B$$

$$\therefore E\times 2\pi R = -\pi R^2 B$$

$$\therefore E = \frac{-BR}{2}$$

13. **(b)** Given $M = \gamma L$

$\therefore$ $M = \gamma m\omega R^2$

$\therefore$ $M = \gamma m(\Delta\omega)R^2$ ...(1)

But $\Delta\omega = \frac{Q\times B}{2m}$ ...(2)

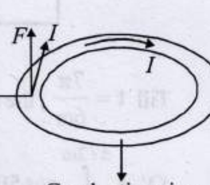
From eq. (i) and (ii)

$$\Delta M = -\gamma m\left(\frac{QB}{2m}\right)R^2 = \frac{-\gamma BQR^2}{2}$$

Here –(ve) sign shows that change is opposite to the direction of magnetic field, B.

(b) is the correct option.

14. **(a)** The horizontal component of the magnetic field B_H interacts with the induced current produced in the conducting ring which produces an average force in the upward direction. This is an accordance with Fleming's left hand rule.



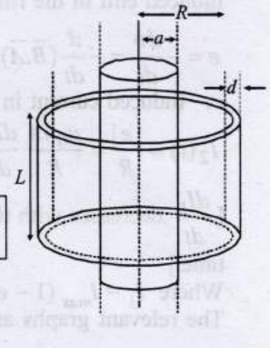
Conducting ring

15. A cylindrical shell of length L, thickness d and radius R surrounds a coaxial solenoid of radius a. The coil of the solenoid carries a variable current $i = i_0 \sin \omega t$.

Outside the solenoid, the magnetic field is zero.

Magnetic field inside the solenoid, $B = \mu_0 n i_0 \sin \omega t$.

$\therefore$ Magnetic flux linked with solenoid



$\phi = BA$

or $\phi = (\mu_0 n i_0 \sin\omega t)(\pi a^2)$

$$\therefore \frac{d\phi}{dt} = \pi\mu_0 n a^2 i_0 \omega co\ \omega t$$

$$\text{or } \varepsilon = -\pi\mu_0 n a^2 i_0\ \omega\cos\omega t \quad \left[\because e = -\frac{d\phi}{dt}\right]$$

Resistance of shell, $R = \frac{pl}{A}$

$\therefore$ Resistance, $R = \frac{\rho\times 2\pi R}{L\times d}$ or Resistance $= \frac{2\pi R\rho}{Ld}$

$\therefore$ Induced current in shell, I

$$= \frac{|\varepsilon|}{\text{resistance}} = \frac{\pi\mu_0 n a^2 i\omega\cos\omega t}{\frac{2\pi Rp}{Ld}}$$

$$\therefore \text{Induced current} = \frac{\mu_0 n a^2 Ld\ i_0\omega\cos\omega t}{2\rho R}.$$

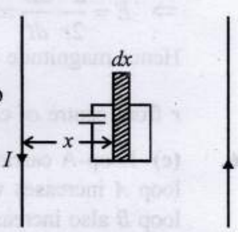
16. (a) Let us consider a small strip of thickness dx at a distance x from the left wire. As shown in the figure. The magnetic field at this strip due to both wires $B = B_A + B_B$ (Perpendicular to the plane of paper directed upwards)

$$B = \frac{\mu_0}{2\pi}\frac{I}{x} + \frac{\mu_0}{2\pi}\frac{I}{(3a-x)} = \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} + \frac{1}{3a-x}\right]$$

Small amount of magnetic flux passing through the strip of thickness dx

$$d\phi = B\times adx = \frac{\mu_0 Ia\times 3a\,dx}{2\pi x(3a-x)}$$

Total flux through the square loop



$$\phi = \int_a^{2a}\frac{\mu_0 I\times 3a^2}{2\pi}\frac{dx}{x(3a-x)}$$

$$= \frac{\mu_0 Ia}{\pi}\ln 2$$

$$\text{or, } \phi = \frac{\mu_0 a\ln(2)}{\pi}\ (I_0\sin\omega t)$$

The emf produced $e = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0 a I_0\omega}{\pi}\ln(2)\cos\omega t$

Charge stored in the capacitor

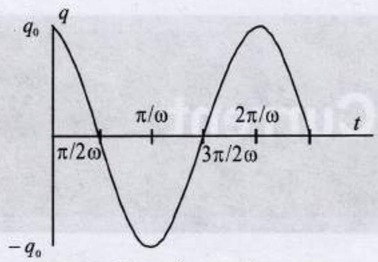
$$q = C\times e = C\times\frac{\mu_0 a I_0\omega}{\pi}\ln(2)\cos\omega t \quad \text{... (i)}$$

$\therefore$ Current in the loop

$$i = \frac{dq}{dt} = \frac{C\times\mu_0 a I_0\omega^2}{\pi}\ln(2)\sin\omega t$$

$$\therefore i_{max} = \frac{\mu_0 a I_0\omega^2 C\ln(2)}{\pi}$$

(b) From eq. (i), the graph between charge and time $(q-t)$ is as shown below



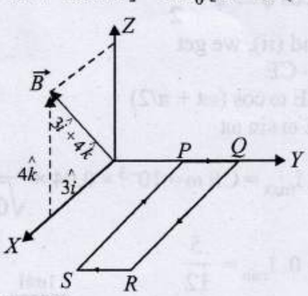
Charge on upper plate of capacitor, $q = q_0 \cos \omega t$

Here, $q_0 = \dfrac{C \times \mu_0 a I_0 \omega \ln(2)}{\pi}$

17. (a) Let us consider the current in the clockwise direction in loop *PQRS* i.e., *P* to *Q* in the wire *PQ*. Force on wire *QR*,

$$\vec{F}_{QR} = I(\vec{\ell} \times \vec{B}) = I[(a\hat{i}) \times (3\hat{i} + 4\hat{k})B_0]$$

$$= I B_0 [3a\hat{i} \times \hat{i} + 4a\hat{i} \times \hat{k}] = I B_0 [0 + 4a(-\hat{j})] = -4aB_0 I \hat{j}$$



Force on wire *PS*

$$\vec{F}_{PS} = I(\vec{\ell} \times \vec{B}) = I[a(-\hat{i}) \times (3\hat{i} + 4\hat{k})B_0] = 4aB_0 I \hat{j}$$

i.e., force on *QR* is equal and opposite to that on *PS* and balance each other.

Force on *RS*

$$\vec{F}_{RS} = I(\vec{\ell} \times \vec{B}) = I[b(-\hat{j}) \times (3\hat{i} + 4\hat{k})B_0]$$

$$= I bB_0 [3\hat{k} - 4\hat{i}] \quad \text{... (i)}$$

Torque about *PQ* by this force

$$\vec{\tau}_{RS} = \vec{r} \times \vec{F} = (\hat{i}a) \times (3\hat{k} - 4\hat{i}) IbB_0$$

$$= -I abB_0 (3\hat{j}) \quad \text{... (ii)}$$

Torque about *PQ* due to weight of the wire *PQRS*

$$\tau = mg\left(\frac{a}{2}\right) \quad \text{... (iii)}$$

(b) Magnetic force on Rs from eq (i), $F_{RS} = \text{Ib } B_0\left[3\hat{k} - 4\hat{i}\right]$

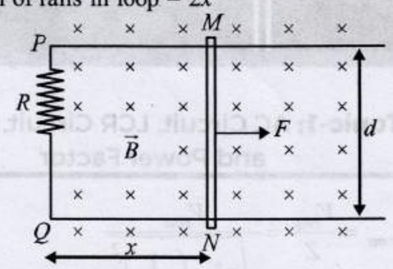
(c) For the wire loop to be horizontal,

$$3IabB_0 = mg\frac{a}{2} \Rightarrow I = \frac{mg}{6bB_0} \quad \text{... (iv)}$$

As because torque due to mg and current are in opposite directions. Therefore, current is from *P* to *Q*.

18. (i) As shown in figure, a variable force *F* is applied to the rod *MN* such that as the rod moves in the uniform magnetic field. A constant current flows through *R*. Consider the loop *MPQN*. Let *MN* be at a distance *x* from *PQ*.

Length of rails in loop = 2*x*



∴ Resistance of rails in loop = $2x\lambda$

∴ Total resistance of loop $R_{net} = R + 2\lambda x$

Let *V* be the velocity of rod at this instant, then due to the motion of the rod emf induced, $e = Bvd$

$$\therefore \text{ Induced current } (I) = \frac{e}{R_{net}} = \frac{Bvd}{R + 2\lambda x}$$

So for constant *I*, $v = \dfrac{(R + 2\lambda x)}{Bd} I$ ...(i)

And due to induced current *I* the wire will experience a force $F_M = BId$ opposite to its motion, the equation of motion of the wire

$F - F_M = ma \Rightarrow F = F_M + ma$

But as here $F_M = BId$ and from equation (i)

$$a = \frac{dv}{dt} = \frac{2\lambda I}{Bd}\frac{dx}{dt} = \frac{2\lambda I v}{Bd} = \frac{2\lambda I^2}{(Bd)^2}(R + 2\lambda x)$$

$$\therefore F = BId + \frac{2\lambda m I^2}{(Bd)^2}(R + 2\lambda x)$$

(ii) As the work done by force *F* per second.

$$\frac{dW}{dt} = P = Fv = \left[BId + \frac{2\lambda m I^2}{(Bd)^2}(R + 2\lambda x)\right]\left[\frac{R + 2\lambda x}{Bd}.I\right]$$

$$\text{i.e., } P = \left[I^2(R + 2\lambda x) + \frac{2\lambda m I^3}{B^3 d^3}(R + 2\lambda x)^2\right]$$

and heat produced per second,

$$H = I^2(R + 2\lambda x)$$

$$\therefore \quad f = \frac{H}{P} = \left[1 + \frac{2\lambda m I(R + 2\lambda x)}{B^3 d^3}\right]^{-1}$$