Chapter 7

FINANCIAL MATHEMATICS

Learning objectives

After studying this chapter, the students will be able to understand

- Annuities - types of annuities
- Future and present values of annuities
- The concept of gain or loss in the sale and purchase of a stock.
- Brokerage in share transactions
- Effective rate of return

Introduction

In our day-to-day life we have lot of **money** transactions. In many transactions payment is made in single payment or in equal installments over a certain period of time. The amounts of these installments are determined in such a way that they compensate for their waiting time. In other cases, in order to meet future planned expenses, a regular saving may be done. (i.e) at regular time intervals, a certain amount may be kept aside, on which the person gains interest. In such cases the concept of annuity is used.

7.1 Annuities

A sequence of equal payments made/ received at equal intervals of time is called **annuity**. The amount of regular payment of an annuity is called **periodic payment**. The time interval between two successive payments is called **payment interval** or **payment period**. Note that, the payment period may be annual, half yearly, quarterly, monthly (or) any fixed duration of time. The time interval between the first payment and the last payment of an annuity is called **term of an annuity**.

The sum of all payments made and interest earned on them at the end of the term of annuities is called future value of an annuity. The present or capital value of an annuity is the sum of the present values of all the payments of the annuity at the beginning of the annuity of purchase the payments due in future. Here we note that unless mentioned specifically, the payment means yearly payment.

7.1.1 Types of annuities

a) Based on the number of periods:

(i) Certain annuity: An annuity payable for a fixed number of years is called certain annuity.

Installments of payment for a plot of land, Bank security deposits, purchase of domestic durables are examples of certain annuity. Here the buyer knows the specified dates on which installments are to be made.

(ii) Annuity contingent: An annuity payable at regular interval of time till the happening of a specific event or the date

of which cannot be accurately foretold is called annuity contingent.

For example the endowment funds of trust, where the interest earned is used for welfare activities only. The principal remains the same and activity continues forever.

All the above types of annuities are based on the number of their periods. An annuity can also be classified on the basis of mode of payment as under.

b) Based on the mode of payment :

i) **Ordinary annuity:** An annuity in which payments of installments are made at the end of each period is called ordinary annuity (or immediate annuity)

For example repayment of housing loan, vehicle loan etc.,

ii) **Annuity due:** An annuity in which payments of installments are made in the beginning of each period is called annuity due.

In annuity due every payment is an investment and earns interest. Next payment will earn interest for one period less and so on, the last payment will earn interest of one period.

For example saving schemes, life insurance payments, etc.

Deferred annuity: An annuity which is payable after the lapse of a number of periods is called deferred annuity.

The derivation of the following formulae are given for better understanding and are exempted from examination.

(i) Amount of immediate annuity (or) Ordinary annuity (or) Certain annuity

Let 'a' be the ordinary annuity and i percent be the rate of interest per period. In ordinary annuity, the first installment is paid after the end of first period. Therefore it earns interest for $(n-1)$ period, second installment earns interest for $(n-2)$ periods and so on. The last installment earns for $(n-n)$ periods. (i.e) earns no interest.

For $(n-1)$ periods,

The amount of first annuity = $a(1+i)^{n-1}$

The amount of second annuity = $a(1+i)^{n-2}$

The amount of third annuity = $a(1+i)^{n-3}$

and so on.

$\therefore$ The total amount of annuity A for n period at i percent rate of interest is

$A = a(1+i)^{n-1}+a(1+i)^{n-2}+...a(1+i)+a$

$= a[(1+i)^{n-1}+(1+i)^{n-2}+...+(1+i)+1]$

$= a[1+(1+i)+(1+i)^2+...+(1+i)^{n-1}]$

$= a[1+r+r^2+...+r^{n-1}]$, where $1+i=r$

$= a[\frac{r^n-1}{r-1}]$, G.P with common ratio $r > 1$

$= a\left[\frac{(1+i)^n-1}{1+i-1}\right]$

$\boldsymbol{A = \frac{a}{i}[(1+i)^n-1]}$

(ii) Present Value of immediate annuity (or ordinary annuity)

Let 'a' be the annual payment of an ordinary annuity, n be the number of years and

i percent be the interest on one rupee per year and P be the present value of the annuity. In the case of immediate annuity, payments are made periodically at the end of specified period. Since the first installment is paid at the end of first year, its present value is $\frac{a}{1+i}$, the present value of second installment is $\frac{a}{(1+i)^2}$ and so on. If the present value of last installment is $\frac{a}{(1+i)^n}$, then we have

$$P = \frac{a}{1+i} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \cdots \frac{a}{(1+i)^n}$$

$$= \frac{a}{(1+i)^n} + \frac{a}{(1+i)^{n-1}} + \ldots + \frac{a}{(1+i)}$$

$$= \frac{a}{r^n}\left[1 + r + r^2 + \ldots + r^{n-1}\right], \text{ taking } 1 + i = r$$

$$= \frac{a}{r^n}\left[\frac{r^n - 1}{r - 1}\right], \text{ G.P with common ratio, } r > 1$$

$$= \frac{a}{(1+i)^n}\left[\frac{(1+i)^n - 1}{1 + i - 1}\right]$$

$$= \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$\boldsymbol{P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]}$$

(iii) Amount of annuity due at the end of *n* period

Annuity due is an annuity in which the payments are made at the beginning of each payment period. The first installment will earn interest for n periods at the rate of 'i' percent per period. Similarly second installment will earn interest for $(n - 1)$ periods, and so on the last interest for on period.

$$\therefore A = a(1+i)^n + a(1+i)^{n-1} + \ldots + a(1+i)^1$$

$$= a(1+i)[(1+i)^{n-1} + (1+i)^{n-2}) + \ldots + 1]$$

$$= a(1+i)[1 + (1+i) + (1+i)^2 + \ldots + (1+i)^{n-1}]$$

$$= ar[1 + r + r^2 + \ldots + r^{n-1}] \text{ , put } 1 + i = r$$

$$= ar\left[\frac{r^n - 1}{r - 1}\right], \text{ G.P with common ratio, } r > 1$$

$$= a(1+i)\left[\frac{(1+i)^n - 1}{1 + i - 1}\right]$$

$$= \frac{a(1+i)}{i}[(1+i)^n - 1]$$

$$\boldsymbol{A = \frac{a(1+i)}{i}[(1+i)^n - 1]}$$

(iv) Present value of annuity due

Since the first installment is paid at the beginning of the first period (year), its present value will be the same as 'a', the annual payment of annuity due. The second installment is paid in the beginning of the second year, hence its present value is given by $\frac{a}{(1+i)}$ and so on. The last installment is paid in the beginning of nth year, hence its present value is given as $\frac{a}{(1+i)^{n-1}}$

If P denotes the present value of annuity due, then

$$P = a + \frac{a}{1+i} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \ldots + \frac{a}{(1+i)^{n-1}}$$

$$= a\left[1 + \frac{1}{1+i} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \ldots + \frac{1}{(1+i)^{n-1}}\right]$$

$$= a\left[\frac{1 - \left(\frac{1}{1+i}\right)^n}{1 - \frac{1}{1+i}}\right]$$

$$= a\left[\frac{\frac{(1+i)^n - 1}{(1+i)^n}}{\frac{(1+i) - 1}{1+i}}\right]$$

$$= \frac{a(1+i)}{i}\left[\frac{(1+i)^n - 1}{(1+i)^n}\right]$$

$$\boldsymbol{P = \frac{a(1+i)}{i}\left[1 - \frac{1}{(1+i)^n}\right]}$$

(v) Perpetual Annuity

Perpetual annuity is an annuity whose payment continuous for ever. As such the amount of perpetuity is undefined as the amount increases without any limit as time passes on. We know that the present value P of immediate annuity is given by

$$P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

Now as per the definition of perpetual annuity as $n \to \infty$, we know that $\frac{1}{(1+i)^n} \to 0$ since $1+i > 1$.

Here $P = \frac{a}{i}[1-0]$

$$P = \frac{a}{i}$$

NOTE

In all the above formulae the period is of one year. Now if the payment is made more than once in a year then 'i' is replaced by $\frac{i}{k}$ and n is replaced by nk, where k is the number of payments in a year.

Example 7.1

A person pays ₹ 64,000 per annum for 12 years at the rate of 10% per year. Find the amount of an ordinary annuity $[(1.1)^{12}=3.3184]$.

Solution:

Here $a = 64{,}000$, $n = 12$ and $i = \frac{10}{100} = 0.1$

Amount of ordinary annuity $(A) = \frac{a}{i}[(1+i)^n - 1]$

$= \frac{64000}{0.1}[(1+0.1)^{12} - 1]$

$= 6{,}40{,}000[(1.1)^{12} - 1]$

$= 6{,}40{,}000[3.3184 - 1]$

$= 6{,}40{,}000\ [2.3184]$

$= 64 \times 23184$

$\therefore$ A = ₹ 14,83,776

Example 7.2

What amount should be deposited annually in an ordinary annuity scheme, so that after 16 years, a person receives ₹1,67,160 if the interest rate is 15% $[(1.15)^{16}=9.358]$.

Solution:

Here $A = 1{,}67{,}160$, $n = 16$ $i = \frac{15}{100} = 0.15$ $a = ?$

To find : a

Now $A = \frac{a}{i}[(1+i)^n - 1]$

$1{,}67{,}160 = \frac{a}{0.15}[(1+0.15)^{16} - 1]$

$= \frac{a}{0.15}[(1.15)^{16} - 1]$

$\Rightarrow a = \frac{1{,}67{,}160 \times 0.15}{(1.15)^{16} - 1}$

$= \frac{1{,}67{,}160 \times 0.15}{9.358 - 1} = \frac{1{,}67{,}160 \times 0.15}{8.358} = 3{,}000$

Therefore a = ₹ 3,000

Example 7.3

The age of the girl is 2 years. Her father wants to get ₹20,00,000 when his ward becomes 22 years. He opens an account with a bank at 10% rate of compound interest. What amount should he deposit at the end of every month in this recurring account? $[(1.0083)^{240}=6.194]$.

Solution:

Here $A = 20{,}00{,}000$; $i = \frac{10}{100} = 0.1$ $n = 20$ and $k = 12$

$A = \frac{a}{i/k}[(1+\frac{i}{k})^{nk} - 1]$

$20{,}00{,}000 = \frac{a}{\frac{0.1}{12}}\left[\left(1 + \frac{0.1}{12}\right)^{20 \times 12} - 1\right]$

$= \frac{12a}{0.1}\left[\left(1 + \frac{0.1}{12}\right)^{240} - 1\right]$

$= 120a\left[\left(\frac{12.1}{12}\right)^{240} - 1\right]$

$= 120a\ [(1.0083)^{240} - 1]$

$= 120a\ [6.194 - 1]$

$= 120a\ (5.194)$

$\Rightarrow a = \frac{20.00{,}000}{120 \times 5.194} = 3208.83$

$\therefore a \approx 3{,}209$

₹3,209 is to be deposited at the end of every month.

Example 7.4

A person deposits ₹4,000 in the beginning of every year. If the rate of compound interest is 14% then ,find the amount after 10 years . $[(1.14)^{10}=3.707]$.

Solution :

Here $a = 4{,}000$; $i = 0.14$ and $n = 10$.

$A = (1+i)\ \frac{a}{i}[(1+i)^n-1]$

$= (1+0.14)\ \frac{4000}{0.14}\ [(1+0.14)^{10}-1]$

$= (1.14)\frac{4000}{0.14}\ [(1.14)^{10}-1]$

$= 1.14\times\frac{4000}{0.14}(3.707-1)$

$= 1.14\times\frac{4000}{0.14}(2.707) = 88{,}170.86$

$\therefore A \approx$ ₹88,171

Example 7.5

A person purchases a machine on 1st January 2009 and agrees to pay 10 installments each of ₹12,000 at the end of every year inclusive of compound rate of 15%. Find the present value of the machine. $[(1.15)^{10}=4.016]$.

Solution:

Here $n=10$, $a = 12{,}000$ and $i = 0.15$

Now $P = \frac{a}{i}\left[1-\frac{1}{(1+i)^n}\right]$

$= \frac{12{,}000}{0.15}\left[1-\frac{1}{(1+0.15)^{10}}\right]$

$= \frac{12{,}000}{0.15}\left[1-\frac{1}{(1.15)^{10}}\right]$

$= \frac{12{,}00{,}000}{15}\left[1-\frac{1}{4.016}\right]$

$= 80{,}000\left[\frac{4.016-1}{4.016}\right]$

$= 80{,}000\left[\frac{3.016}{4.016}\right] \approx 60{,}080$

$\therefore P =$ ₹ 60,080

Example 7.6

A photographer purchases a camera on installments. He has to pay 7 annual installments each of ₹36,000 right from the date of purchase. If the rate of compound interest is 16% then find the cost price (present value) of the camera. $[(1.16)^7=2.828]$

Solution:

Here $a = 36{,}000$; $n = 7$ and $i = 0.16$

Now $P = (1+i)\ \frac{a}{i}\left[1-\frac{1}{(1+i)^n}\right]$

$= (1+0.16)\ \frac{36{,}000}{0.16}\left[1-\frac{1}{(1+0.16)^7}\right]$

$= \frac{1.160}{0.16}(36{,}000)\left[1-\frac{1}{(1+0.16)^7}\right]$

$= \frac{116\times 36000}{16}\left[1-\frac{1}{2.828}\right]$

$= \frac{116\times 36000}{16}\left[\frac{2.828-1}{2.828}\right]$

$= \frac{116\times 36{,}000\times 1.828}{16\times 2.828}$

$= \frac{116\times 36{,}000\times 1828}{16\times 2828} \approx 1{,}68{,}709.$

$\therefore$ P = ₹ 1,68,709

Example 7.7

A person has taken a loan of ₹ 7,00,000 at 16% rate of interest from a finance company. If the repayment period is of 15 years then find the installment he has to pay at the beginning of each month. $[(1.0133)^{180}=9.772]$.

Solution:

Here $P = 7{,}00{,}000$; $n = 15$; $i = 0.16$ and $k = 12$.

$P = \left(1+\frac{i}{k}\right)\left(\frac{a}{\frac{i}{k}}\right)\left[1-\frac{1}{\left(1+\frac{i}{k}\right)^{nk}}\right]$

$700000=\left(1+\frac{0.16}{12}\right)\left(\frac{a}{\frac{0.16}{12}}\right)\left[1-\frac{1}{\left(1+\frac{0.16}{12}\right)^{15\times 12}}\right]$

$= \left(\frac{12.16}{12}\right)\left(\frac{12a}{0.16}\right)\left[1-\frac{1}{\left(\frac{12.16}{12}\right)^{180}}\right]$

$$= \frac{(1216)a}{16}\left[1 - \frac{1}{(1.0133)^{180}}\right]$$

$$= \frac{1216}{16}a\left[1 - \frac{1}{9.772}\right]$$

$$= \frac{1216}{16}a\left[\frac{9.772 - 1}{9.772}\right]$$

$$= \frac{1216}{16}a\left[\frac{8.772}{9.772}\right]$$

$$\Rightarrow a = \frac{7,00,000 \times 16 \times 9772}{1216 \times 8772} \approx ₹\ 10,261$$

Example 7.8

The chairman of a society wishes to award a gold medal to a student getting highest marks in Business Mathematics and Statistics. If this medal costs to ₹ 9,000 every year and the rate of compound interest is 15%, then what amount is to be deposited now.

Solution:

Here a = 9,000 and i = 0.15

$$P = \frac{a}{i}$$

$$= \frac{9000}{0.15}$$

$$= \frac{9,00,000}{15}$$

= 60,000.

Therefore the amount to be deposited is ₹ 60,000.

Example 7.9

A limited company wants to create a fund to help their employees in critical circumstances. The estimated expenses per month is ₹18,000. Find the amount to be deposited by the company if the rate of compound interest is 15%.

Solution:

Here a = 18,000 ; i = 0.15 and k = 12.

$$P = \frac{a}{i/k} = \frac{18,000}{0.15/12}$$

$$= \frac{18,000}{0.15} \times 12$$

$$= \frac{18,00,000 \times 12}{15}$$

= 14,40,000

Therefore the amount to be deposited is ₹14,40,000.

Example 7.10

Find the annual rate of interest, to get a perpetuity of ₹675 for every half yearly from the present value of ₹30,000.

Solution :

Here P=30,000 ; a = 675 ; k = 2, i = ?

$$P = \frac{a}{i/k}$$

$$30,000 = \frac{675}{\frac{i}{2}}$$

$$= \frac{1350}{i}$$

$$\Rightarrow i = \frac{1350}{30,000} = \frac{135}{3000} = 0.045$$

Rate of interest (i)= 0.045 × 100% = 4.5%

Exercise 7.1

1. Find the amount of an ordinary annuity of ₹3,200 per annum for 12 years at the rate of interest of 10% per year. [$(1.1)^{12}$ = 3.1384].
2. If the payment of ₹2,000 is made at the end of every quarter for 10 years at the rate of 8% per year, then find the amount of annuity. [$(1.02)^{40}$ =2.2080].
3. Find the amount of an ordinary annuity of 12 monthly payments of ₹1, 500 that earns interest at 12% per annum compounded monthly. [$(1.01)^{12}$ = 1.1262].

4. A bank pays 8% per annum interest compounded quarterly. Find the equal deposits to be made at the end of each quarter for 10 years to have ₹30,200? $[(1.02)^{40} = 2.2080]$.

5. A person deposits ₹2,000 at the end of every month from his salary towards his contributory pension scheme. The same amount is credited by his employer also. If 8% rate of compound interest is paid, then find the maturity amount at end of 20 years of service.
   $[(1.0067)^{240} = 4.9661]$.

6. Find the present value of ₹2,000 per annum for 14 years at the rate of interest of 10% per annum. If the payments are made at the end of each payment period. $[(1.1)^{-14} = 0.2632]$.

7. Find the present value of an annuity of ₹900 payable at the end of 6^{th} month for 6 years. The money compounded at 8% per annum. $[(1.04)^{-12} = 0.6252]$.

8. Find the amount at the end of 12 years of an annuity of ₹5,000 payable at the beginning of each year, if the money is compounded at 10% per annum. $[(1.1)^{12} = 3.1384]$.

9. What is the present value of an annuity due of ₹1,500 for 16 years at 8% per annum? $[(1.08)^{-16} = 0.2919]$.

10. What is the amount of perpetual annuity of ₹50 at 5% compound interest per year?

7.2 Stocks, Shares, Debentures and Brokerage

To start any big business, a large sum of money is needed. It is generally not possible for an individual to manage such a large sum. Therefore the total sum of money can be divided into equal parts called **shares**. The holder of shares are called **shareholders**.

7.2.1 Types of shares:

There are two type shares namely **common** (or equity) and **preferred**.

Generally, the profit gained by the company is distributed among their share holders The preferred share holders have a first claim on dividend. When they have been paid, the remaining profit is distributed among the common share holders.

7.2.2 Definitions

(i) **Capital stock** is the total amount invested to start a company.

(ii) The share purchased by the individual is also called **stock**.

(iii) The persons who buy the shares are also called **stock holders**

(iv) **Face value :** The original value of a share at which the company sells it to investors is called a face value or nominal value or par value. It is to be noted that the original value of share is printed on the share certificate.

(v) Market value : The price at which the stock is bought or sold in the market is called the market value (or cash value).

DO YOU KNOW?

The market value of a share keeps on changing from time to time.

Remarks:

(i) If the market value of a share is greater than the face value then, the share is said to be above par (or at premium).

(ii) If the market value of a share is the same as its face value then, the share is said to be at par.

(iii) If the market value of a share is less than the face value then, the share is said to be below par (or at discount).

Dividend :

The profit gained by a company, distributed among their share holders is called dividend. It is calculated on the face value of the share.

Dividend is expressed as percentage.

Some useful results:

(i) Investment:

Money invested = number of shares × market value of a share.

(ii) Income:

Annual income = number of shares × face value of a share × rate of dividend.

(iii) Return percentage (or yield percentage):

Percentage of return

$$= \frac{Income}{Investment} \times 100$$

(iv) Number of shares:

Number of shares purchased

$$= \frac{Investment}{market\ value\ of\ a\ share}$$

Stock exchange:

The place where the shares are traded is called the stock exchange (or) stock market.

Brokerage:

A broker who executes orders to buy and sell shares through a stock market is called Stock Broker. A fee or commission for their service is called the brokerage.

Brokerage is generally based on the face value and expressed as a percentage.

NOTE

(i) When the share is purchased, brokerage is added to its market price.

(ii) When the share is sold, brokerage is subtracted from the market price.

Example 7.11

Find the market value of 325 shares of face value ₹100 at a premium of ₹18.

Solution:

Face value of a share = ₹100

Premium per share = ₹18

M.V. of a share = ₹118

Market value of 325 shares = number of shares × M.V of a share

= 325 × 118 = ₹38,350

Therefore market value of 325 shares is ₹38,350.

Example 7.12

A man buys 500 shares of face value ₹100 at ₹14 below par. How much money does he pay?

Solution:

Number of shares = 500

Face value of a share = ₹100

Discount = ₹14

Market value of a share = 100 – 14 = ₹86
[face value – discount]

Market value of 500 shares = Number of shares × market value of a share

= 500 × 86 = 43,000

Market value of 500 shares = ₹43,000

Example 7.13

A person buys 20 shares of par value of ₹10 of a company which pays 9% dividend at such a price that he gets 12% on his money. Find the market value of a share.

Solution:

Face value of a share = ₹10

Face value of 20 shares = ₹200

Dividend = $\frac{9}{100} \times 200 = ₹18$

$\left[S.I = \frac{PNR}{100}, N = 1\right]$

Investment = $\frac{18 \times 100}{1 \times 12} = ₹150$

$[P = \frac{100 \times SI}{N \times R}, \text{N} = 1]$

Market value of 20 shares = ₹150

The market value of one share = ₹$\frac{150}{20} = ₹7.50$

Example 7.14

If the dividend received from 10% of ₹25 shares is ₹2000. Find the number of shares.

Solution:

Let x be the number of shares.

Face value of 'x' shares = ₹25 x

Now $\frac{10}{100} \times 25x = ₹2{,}000$

$\Rightarrow x = \frac{2000 \times 100}{25 \times 10} = 800$

Hence the number of shares = 800

Example 7.15

Find the number of shares, which will give an annual income of ₹3,600 from 12% stock of face value ₹100.

Solution:

Let 'x' be the number of shares.

Face Value = ₹100

Face value of 'x' shares = ₹100x

$\frac{12}{100} \times 100x = ₹3600$

$12\,x = 3600 \Rightarrow x = 300$

Hence the number of shares = 300

Example 7.16

A man invest ₹96,000 on ₹100 shares at ₹80. If the company pays him 18% as dividend, find

(i) the number of shares he bought

(ii) the dividend

(iii) percentage of return.

Solution:

(i) Investment = ₹96,000

Face Value = ₹100

Market Value = ₹80

The number of shares bought

$= \frac{Investments}{M.V\ of\ one\ share}$

$= \frac{96{,}000}{80} = 1200$ shares

(ii) Total dividend

= No. of shares × Rate of dividend × Face value of a share

$= 1200 \times \frac{18}{100} \times 100 = ₹21{,}600$

(iii) Dividend on ₹ 96000 = ₹21600

Percentage of return = $\frac{21{,}600}{96{,}000} \times 100$

$= \frac{45}{2} = 22.5$

Thus return on the shares = 22.5%

Example 7.17

A person bought 12% stock for ₹54,000 at a discount of 17%. If he paid 1% brokerage, find the percentage of his income.

Solution:

Face value = ₹100

Market value = ₹(100 – 17 + 1) = ₹84

$\therefore$ percentage of his income = $\frac{(12\times100)}{84}$

$= \frac{100}{7} = 14\frac{2}{7}$

$\therefore$ % of income = $14\frac{2}{7}$ %

Example 7.18

Equal amounts are invested in 10% stock at 89 and 7% stock at 90 (1% brokerage paid in both transactions). If 10% stock bought ₹100 more by way of dividend income than the other, find the amount invested in each stock.

Solution:

Let x be the amount invested in each stock

Income on 10% stock at 89:

F.V. = ₹100, M.V. = ₹90

Number of shares = $\frac{investments}{M.V.} = \frac{x}{90}$

Annual income = $\frac{x}{90}\times\frac{10}{100}\times100$

$= \frac{x}{9}$ ... (1)

Income on 7% stock at 90:

M.V. = ₹91

Number of shares = $\frac{\text{investments}}{MV} = \frac{x}{91}$

Annual income = $\frac{x}{91}\times\frac{7}{100}\times100$

$= \frac{x}{13}$ ... (2)

Given : $\frac{x}{9}-\frac{x}{13} = 100 \Rightarrow x = ₹\ 2925$

$\therefore$ The amount invested in each stock $\approx$ ₹2,925.

Example 7.19

A capital of a company is made up of 1,00,000 preference shares with a dividend rate of 16% and 50,000 ordinary shares.The par value of each of preference and ordinary shares is ₹10.The total profit of a company is ₹ 3,20,000.If ₹40,000 were kept in reserve and ₹20,000 were kept in depreciation fund, what percent of dividend is paid to the ordinary share holders.

Solution:

F.V. = ₹10

Total face value of preference shares

= ₹1,00,000 × 10 = ₹10,00,000

Total face value of ordinary shares

= ₹50,000 × 10 = ₹5,00,000

Total dividend amount paid to shareholders

= ₹(3,20,000 – 40,000 – 20,000) = ₹2,60,000

Dividend for preference shares

$= \frac{16}{100}\times10,00,000 = ₹1,60,000$

Dividend to ordinary shares

= 2,60,000 – 1,60,000 = ₹1,00,000

Dividend rate for ordinary share

$= \frac{Income}{Investment}\times100\%$

Dividend % = $\frac{1,00,000}{5,00,000}\times100 = 20\%$

Example 7.20

A person sells a 20% stocks of face value ₹10,000 at a premium of 42%. With the money obtained he buys a 15% stock at a discount of 22%. What is the change in his income if the brokerage paid is 2%.

Solution:

Step 1: For 20% stocks:

F.V. = ₹100

Income = $\frac{20}{100}\times10000$

= ₹ 2,000 ... (1)

Investment = ₹10,000

Face value = ₹100

Market value = ₹100 + 42 – 2 = 140

Number of shares = $\frac{investments}{FV}$

$= \frac{10,000}{100} = 100$

Sales proceeds = 100 × 140 = 14,000

Step 2: For 15% stocks:

M.V. = ₹100 – 22 + 2 = 80

Number of shares = $\frac{investments}{FV}$

$= \frac{14000}{80} = 175$

Income = $175 \times \frac{15}{100} \times 100$ = ₹2625

Step 3: Change of income:

Change of income = ₹ 2625 – ₹ 2000 = ₹ 625

Example 7.21

Which is better investment: 12% ₹20 shares at ₹ 16 (or) 15% ₹ 20 shares at ₹24 .

Solution:

Let the investment in each case be ₹ (16× 24)

Step 1: Income on 12% shares:

Income from 12 % ₹20 shares at ₹ 16

$= \frac{12}{16} \times (16 \times 24)$ = ₹ 288

Step 2: Income on 15% shares:

Income from 15% ₹20 Shares at ₹ 24

$= \frac{15}{24} \times (16 \times 24)$ = ₹ 240

Hence, the first investment is better.

Exercise 7.2

1. Find the market value of 62 shares available at ₹132 having the par value of ₹100.
2. How much will be required to buy 125 of ₹25 shares at a discount of ₹7.
3. If the dividend received from 9% of ₹20 shares is ₹1,620, then find the number of shares.
4. Mohan invested ₹29,040 in 15% of ₹100 shares of a company quoted at a premium of 20%. Calculate
   (i) the number of shares bought by Mohan
   (ii) his annual income from shares
   (iii) the percentage return on his investment
5. A man buys 400 of ₹10 shares at a premium of ₹2.50 on each share. If the rate of dividend is 12%, then find
   (i) his investment
   (ii) annual dividend received by him
   (iii) rate of interest received by him on his money
6. Sundar bought ₹ 4,500, 12% of ₹10 shares at par. He sold them when the price rose to ₹23 and invested the proceeds in ₹25 shares paying 10% per annum at ₹18. Find the change in his income.
7. A man invests ₹13,500 partly in 6% of ₹100 shares at ₹140 and the remaining in 5% of ₹100 shares at ₹125. If his total income is ₹560, how much has he invested in each?
8. Babu sold some ₹100 shares at 10% discount and invested his sales proceeds in 15% of ₹50 shares at ₹33. Had he sold his shares at 10% premium instead of 10% discount, he would have earned ₹450 more. Find the number of shares sold by him.
9. Which is better investment? 7% of ₹100 shares at ₹120 (or) 8% of ₹100 shares at ₹135.

10. Which is better investment? 20% stock at 140 (or) 10% stock at 70.

Exercise 7.3

Choose the correct answer

1. The dividend received on 200 shares of face value ₹100 at 8% is
   (a) ₹ 1600
   (b) ₹ 1000
   (c) ₹ 1500
   (d) ₹ 800

2. What is the amount realised on selling 8% stock of 200 shares of face value ₹100 at ₹50.
   (a) ₹ 16,000 (b) ₹ 10,000
   (c) ₹ 7,000 (d) ₹ 9,000

3. A man purchases a stock of ₹ 20,000 of face value ₹ 100 at a premium of 20%, then investment is
   (a) ₹ 20,000 (b) ₹ 25,000
   (c) ₹ 24,000 (d) ₹ 30,000

4. If a man received a total dividend of ₹ 25,000 at 10% dividend rate on a stock of face value ₹100, then the number of shares purchased.
   (a) 3500 (b) 4500
   (c) 2500 (d) 300

5. The brokerage paid by a person on the sale of 400 shares of face value ₹100 at 1% brokerage
   (a) ₹ 600 (b) ₹ 500
   (c) ₹ 200 (d) ₹ 400

6. Purchasing price of one share of face value ₹100 available at a discount of $9\frac{1}{2}$ % with brokerage $\frac{1}{2}$ % is
   (a) ₹89 (b) ₹90
   (c) ₹91 (d) ₹95

7. A person brought 100 shares of 9% stock of face value ₹100 at a discount of 10%, then the stock purchased is
   (a) ₹ 9000 (b) ₹ 6000
   (c) ₹ 5000 (d) ₹ 4000

8. The % of income on 7 % stock at ₹80 is
   (a) 9% (b) 8.75%
   (c) 8% (d) 7%

9. The annual income on 500 shares of face value ₹100 at 15% is
   (a) ₹ 7,500 (b) ₹ 5,000
   (c) ₹ 8,000 (d) ₹ 8,500

10. ₹ 5000 is paid as perpetual annuity every year and the rate of C.I. 10 %. Then present value P of immediate annuity is
    (a) ₹ 60,000 (b) ₹ 50,000
    (c) ₹ 10,000 (d) ₹ 80,000

11. If 'a' is the annual payment, 'n' is the number of periods and 'i' is compound interest for ₹1 then future amount of the ordinary annuity is
    (a) A= $\frac{a}{i}(1+i)[(1+i)^n-1]$
    (b) $A=\frac{a}{i}[(1+i)^n-1]$
    (c) $P=\frac{a}{i}$
    (d) $P=\frac{a}{i}(1+i)[1-(1+i)^{-n}]$

12. A invested some money in 10% stock at ₹96. If B wants to invest in an equally good 12% stock, he must purchase a stock worth of
    (a) ₹ 80 (b) ₹ 115.20
    (c) ₹ 120 (d) ₹ 125.40

13. An annuity in which payments are made at the beginning of each payment period is called

(a) Annuity due
(b) An immediate annuity
(c) perpetual annuity
(d) none of these

14. The present value of the perpetual annuity of ₹ 2000 paid monthly at 10 % compound interest is
(a) ₹ 2,40,000 (b) ₹ 6,00,000
(c) ₹ 20,40,000 (d) ₹ 2,00,400

15. Example of contingent annuity is
(a) Installments of payment for a plot of land
(b) An endowment fund to give scholarships to a student
(c) Personal loan from a bank
(d) All the above

Miscellaneous Problems

1. Find the amount of annuity of ₹2000 payable at the end of each year for 4 years of money is worth 10% compounded annually.
$[(1.1)^4 = 1.4641]$

2. An equipment is purchased on an installment basis such that ₹5000 on the signing of the contract and four yearly installments of ₹3000 each payable at the end of first, second, third and the fourth year. If the interest is charged at 5% p.a find the cash down price. $[(1.05)^{-4} = 0.8227]$

3. (i) Find the amount of an ordinary annuity of ₹500 payable at the end of each year for 7 years at 7% per year compounded annually.
$[(1.07)^7 = 1.6058]$

(ii) Calculate the amount of an ordinary annuity of ₹10,000 payable at the end of each half-year for 5 years at 10% per year compounded half-yearly. $[(1.05)^{10} = 1.6289]$

(iii) Find the amount of an ordinary annuity of ₹600 is made at the end of every quarter for 10 years at the rate of 4% per year compounded quarterly. $[(1.01)^{40} = 1.4889]$

(iv) Find the amount of an annuity of ₹2000 payable at the end of every month for 5 years if money is worth 6% per annum compounded monthly. $[(1.005)^{60} = 1.3489]$

4. Naveen deposits ₹250 at the end of each month in an account that pays an interest of 6% per annum compounded monthly, how many months will be required for the deposit to amount to atleast ₹6390?
$[\log(1.1278) = 0.0523, \log(1.005) = 0.0022]$

5. A cash prize of ₹1,500 is given to the student standing first in examination of Business Mathematics by a person every year. Find out the sum that the person has to deposit to meet this expense. Rate of interest is 12% p.a

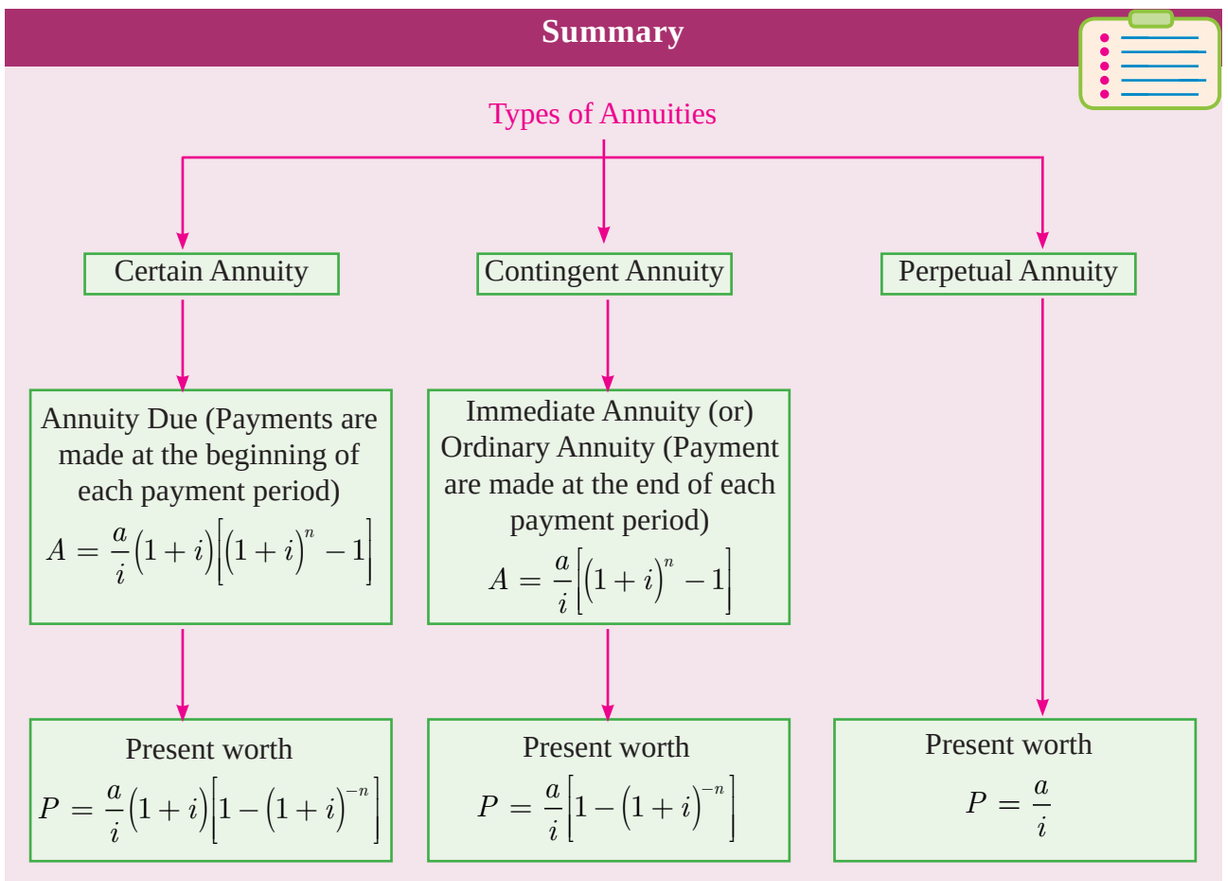
6. Machine A costs ₹15,000 and machine B costs ₹20,000. The annual income from A and B are ₹4,000 and ₹7,000 respectively. Machine A has a life of 4 years and B has a life of 7 years. Find which machine may be purchased. (Assume discount rate 8% p.a)
$[(1.08)^{-4} = 0.7350, (1.08)^{-7} = 0.5835]$

7. Vijay wants to invest ₹27,000 in buying shares. The shares of the following companies are available to him. ₹100 shares of company *A* at par value; ₹100 shares of company B at a premium of ₹25; ₹100 shares of company *C* at a discount of ₹10; ₹50 shares of company *D* at a premium of 20%. Find how many

shares will he get if he buys shares of company (i) *A* (ii) *B* (iii) *C* (iv) *D*

8. Gopal invested ₹8,000 in 7% of ₹100 shares at ₹80. After a year he sold these shares at ₹75 each and invested the proceeds (including his dividend) in 18% for ₹25 shares at ₹41. Find
   (i) his dividend for the first year
   (ii) his annual income in the second year
   (iii) The percentage increase in his return on his original investment
9. A man sells 2000 ordinary shares (par value ₹10) of a tea company which pays a dividend of 25% at ₹33 per share. He invests the proceeds in cotton textiles (par value ₹25) ordinary shares at ₹44 per share which pays a dividend of 15%. Find (i) the number of cotton textiles shares purchased and (ii) change in his dividend income.
10. The capital of a company is made up of 50,000 preferences shares with a dividend of 16% and 25,000 ordinary shares. The par value of each of preference and ordinary shares is ₹10. The company had a total profit of ₹1,60,000. If ₹20,000 were kept in reserve and ₹10,000 in depreciation, what percent of dividend is paid to the ordinary share holders

Summary



- **Endowment or Scholarship fund**

  (i) If the scholarship is awarded endlessly, then $P = \frac{a}{i}$

  (ii) If the scholarship is awarded for a fixed period, say, n years, then $P = \frac{a}{i}\left[1-\left(1+i\right)^{-n}\right]$

- **Face Value:** The original Value of the share is called its nominal values or face value or printed value.

- **Market Value:** The price at which the share is sold (or) purchased in the capital market through stock exchanges is called the market value.

- A Share is called at par if the market value of the share is equal to its face (or) nominal value.

- A share is said to be above par (or) at premium, if the market value of the share is more than its nominal value.

- A share is said to be below par (or) at discount, if the market value of the share is less than its nominal value.

- The part of the annual profit, which a share holder gets for his investment from the company is called dividend.

- Dividend is always declared on the face value of the share and the rate of dividend is expressed as a percentage of the nominal value of a share per annum.

- Annual income of a shareholders = $\frac{n \times r \times F.V}{100}$

  Where n = number of shares with the shareholders
  r = rate of dividend,

- Annual Return = $\frac{Annual\ income}{investment\ in\ shares} \times 100\%$

- Number of Shares held = $\frac{investment}{M.V(or)F.V.of\ one\ share(as\ the\ type\ of\ investment)}$

  (or)

  $= \frac{Annual\ income}{income\ from\ one\ share}$ (or) $= \frac{Total\ F.V}{F.V.of\ one\ share}$

- Which is better investment for two stocks of same face value

  Let the investment (each case) = (M.V of First stock × M.V. of Second Stock)

  Case (i)

  Income form $r_1\% = \frac{r_1}{(M.V.\ of\ r_1\%\ Stock)} \times$ Investment

  Case (ii)

  Income form $r_2\% = \frac{r_2}{(M.V.\ of\ r_2\%\ Stock)} \times$ Investment

GLOSSARY (கலைச்சொற்கள்)	
Brokerage	தரகு
Capital value	மூலதன மதிப்பு
Debentures	கடன் பத்திரங்கள்
Equity shares	சம பங்கு
Face value	முக மதிப்பு
Immediate annuity	தவணை பங்கீட்டு தொகை
Interest	வட்டி
Market price	சந்தை விலை
Payment interval	செலுத்தும் கால இடைவெளி
Periodic payment	காலமுறை செலுத்துதல்
Perpetual annuity	நிரந்தர தவணை பங்கீட்டு தொகை
Preference shares	முன்னுரிமை பங்குகள்
Selling price	விற்பனை விலை
Share holders	பங்குதாரர்கள்
Shares	பங்குகள்
Stock exchange	பங்குச் சந்தை
Stocks	சரக்கு முதல்கள்
Term of annuity	தவணை பங்கீட்டு தொகை காலம்
Transaction	பரிவர்த்தனை