CAT 2020 Question Paper Slot 2

Quant

51. The distance from B to C is thrice that from A to B. Two trains travel from A to C via B. The speed of train 2 is double that of train 1 while traveling from A to B and their speeds are interchanged while traveling from B to C. The ratio of the time taken by train 1 to that taken by train 2 in travelling from A to C is

A 5:7

B 4:1

C 1:4

D 7:5

52. John takes twice as much time as Jack to finish a job. Jack and Jim together take one-thirds of the time to finish the job than John takes working alone. Moreover, in order to finish the job, John takes three days more than that taken by three of them working together. In how many days will Jim finish the job working alone?

53. Let $f(x) = x^2 + ax + b$ and $g(x) = f(x+1) - f(x-1)$. If $f(x) \geq 0$ for all real x, and $g(20) = 72$. then the smallest possible value of b is

A 16

B 4

C 1

D 0

54. For the same principal amount, the compound interest for two years at 5% per annum exceeds the simple interest for three years at 3% per annum by Rs 1125. Then the principal amount in rupees is

55. Let C be a circle of radius 5 meters having center at O. Let PQ be a chord of C that passes through points A and B where A is located 4 meters north of O and B is located 3 meters east of O. Then, the length of PQ, in meters, is nearest to

A 8.8

B 7.8

C 6.6

D 7.2

56. In a car race, car A beats car B by 45 km. car B beats car C by 50 km. and car A beats car C by 90 km. The distance (in km) over which the race has been conducted is

A 475

B 450

C 500

D 550

57. How many 4-digit numbers, each greater than 1000 and each having all four digits distinct, are there with 7 coming before 3?

58. The sum of the perimeters of an equilateral triangle and a rectangle is 90cm. The area, T, of the triangle and the area, R, of the rectangle, both in sq cm, satisfying the relationship $R = T^2$. If the sides of the rectangle are in the ratio 1:3, then the length, in cm, of the longer side of the rectangle, is

A 27

B 21

C 24

D 18

59. A sum of money is split among Amal, Sunil and Mita so that the ratio of the shares of Amal and Sunil is 3:2, while the ratio of the shares of Sunil and Mita is 4:5. If the difference between the largest and the smallest of these three shares is Rs.400, then Sunil's share, in rupees, is

60. The value of $\log_a(\frac{a}{b}) + \log_b(\frac{b}{a})$, for $1 < a \leq b$ cannot be equal to

A 0

B -1

C 1

D -0.5

61. Students in a college have to choose at least two subjects from chemistry, mathematics and physics. The number of students choosing all three subjects is 18, choosing mathematics as one of their subjects is 23 and choosing physics as one of their subjects is 25. The smallest possible number of students who could choose chemistry as one of their subjects is

A 22

B 21

C 20

D 19

62. In a group of 10 students, the mean of the lowest 9 scores is 42 while the mean of the highest 9 scores is 47. For the entire group of 10 students, the maximum possible mean exceeds the minimum possible mean by

A 5

B 4

C 3

D 6

63. A and B are two points on a straight line. Ram runs from A to B while Rahim runs from B to A. After crossing each other. Ram and Rahim reach their destination in one minute and four minutes, respectively. if they start at the same time, then the ratio of Ram's speed to Rahim's speed is

A $\frac{1}{2}$

B $\sqrt{2}$

C 2

D $2\sqrt{2}$

64. Two circular tracks T1 and T2 of radii 100 m and 20 m, respectively touch at a point A. Starting from A at the same time, Ram and Rahim are walking on track T1 and track T2 at speeds 15 km/hr and 5 km/hr respectively. The number of full rounds that Ram will make before he meets Rahim again for the first time is

A 5

B 3

C 2

D 4

65. Let C1 and C2 be concentric circles such that the diameter of C1 is 2cm longer than that of C2. If a chord of C1 has length 6cm and is a tangent to C2, then the diameter, in cm, of C1 is

66. Anil buys 12 toys and labels each with the same selling price. He sells 8 toys initially at 20% discount on the labeled price. Then he sells the remaining 4 toys at an additional 25% discount on the discounted price. Thus, he gets a total of Rs 2112, and makes a 10% profit. With no discounts, his percentage of profit would have been

A 50

B 60

C 54

D 55

67. The number of integers that satisfy the equality $(x^2 - 5x + 7)^{x+1} = 1$ is

A 3

B 2

C 4

D 5

68. The number of pairs of integers (x, y) satisfying $x \geq y \geq -20$ and $2x + 5y = 99$

69. From an interior point of an equilateral triangle, perpendiculars are drawn on all three sides. The sum of the lengths of the three perpendiculars is s. Then the area of the triangle is

A $\frac{\sqrt{3}s^2}{2}$

B $\frac{2s^2}{\sqrt{3}}$

C $\frac{s^2}{2\sqrt{3}}$

D $\frac{s^2}{\sqrt{3}}$

70. Let the m-th and n-th terms of a geometric progression be $\frac{3}{4}$ and 12. respectively, where $m < n$. If the common ratio of the progression is an integer r, then the smallest possible value of $r + n - m$ is

A 6

B 2

C -4

D -2

71. In May, John bought the same amount of rice and the same amount of wheat as he had bought in April, but spent ₹ 150 more due to price increase of rice and wheat by 20% and 12%, respectively. If John had spent ₹ 450 on rice in April, then how much did he spend on wheat in May?

A Rs.560

B Rs.570

C Rs.590

D Rs.580

72. If x and y are non-negative integers such that $x + 9 = z, y + 1 = z$ and $x + y < z + 5$, then the maximum possible value of $2x + y$ equals

73. Aron bought some pencils and sharpeners. Spending the same amount of money as Aron, Aditya bought twice as many pencils and 10 less sharpeners. If the cost of one sharpener is ₹ 2 more than the cost of a pencil, then the minimum possible number of pencils bought by Aron and Aditya together is

A 33

B 27

C 30

D 36

74. For real x, the maximum possible value of $\frac{x}{\sqrt{1+x^4}}$ is

A $\frac{1}{2}$

B 1

C $\frac{1}{\sqrt{3}}$

D $\frac{1}{\sqrt{2}}$

75. In how many ways can a pair of integers (x , a) be chosen such that $x^2 - 2 \mid x \mid + \mid a - 2 \mid = 0$?

A 6

B 5

C 4

D 7

76. if x and y are positive real numbers satisfying $x + y = 102$, then the minimum possible valus of $2601(1 + \frac{1}{x})(1 + \frac{1}{y})$ is

Answers

Quant

51.**A**	52.**4**	53.**B**	54.**90000**	55.**A**	56.**B**	57.**315**	58.**A**
59.**800**	60.**C**	61.**C**	62.**B**	63.**C**	64.**B**	65.**10**	66.**A**
67.**A**	68.**17**	69.**D**	70.**D**	71.**A**	72.**23**	73.**A**	74.**D**
75.**D**	76.**2704**						

Explanations

Quant

51. **A**
Let the distance from A to B be "x", then the distance from B to C will be 3x. Now the speed of Train 2 is double of Train 1. Let the speed of Train 1 be "v", then the speed of Train 2 will be "2v" while travelling from A to B.

Time taken by Train 1 = (x/v)

Time taken by Train 2 = (x/2v)

Now from B to C distance is "3x" and the speed of Train 2 is (v) and the speed of Train 1 is (2v).

Time taken by Train 1 = 3x/2v

Time taken by Train 2 = 3x/v

Total time taken by Train 1 = x/v(1+(3/2)) = (5/2)(x/v)

Total time taken by Train 2 = x/v(3+(1/2))= (7/2)(x/v)

Ratio of time taken = $\frac{\frac{5}{2}}{\frac{7}{2}} = \frac{5}{7}$

52. **4**
Let Jack take "t" days to complete the work, then John will take "2t" days to complete the work. So work done by Jack in one day is (1/t) and John is (1/2t) .

Now let Jim take "m" days to complete the work. According to question, $\frac{1}{t} + \frac{1}{m} = \frac{3}{2t}\ or\ \frac{1}{m} = \frac{1}{2t} or\ m = 2t$
Hence Jim takes "2t" time to complete the work.

Now let the three of them complete the work in "p" days. Hence John takes "p+3" days to complete the work.

$\frac{1}{2t}\left(m+3\right) = \left(\frac{4}{2t}\right)m$

$\frac{1}{2t}\left(m+3\right) = \left(\frac{4}{2t}\right)m$

or m=1. Hence JIm will take (1+3)=4 days to complete the work. Similarly John will also take 4 days to complete the work

53. **B**

$f\left(x\right) =\ x^2 + ax + b$

$f\left(x+1\right) = x^2 + 2x + 1 + ax + a + b$

$f\left(x-1\right) = x^2 - 2x + 1 + ax - a + b$

$g(x) = f(x+1) - f(x-1) = 4x + 2a$

Now $g(20) = 72$ from this we get $a = -4$; $f\left(x\right) = x^2 - 4x\ + b$

For this expression to be greater than zero it has to be a perfect square which is possible for $b \geq\ 4$

Hence the smallest value of '**b**' is 4.

54. **90000**
For two years the compound interest is $\frac{PR(1)}{100} + \frac{PR(1)}{100}\left(1 + \frac{PR(1)}{100}\right)$

For three years the simple interest is $\frac{9PR}{100}$

Now R(1)= 5% and R=3%

Hence $\frac{5P}{100} + \frac{5P}{100}\left(1.05\right) - \frac{9P}{100} = 1125$

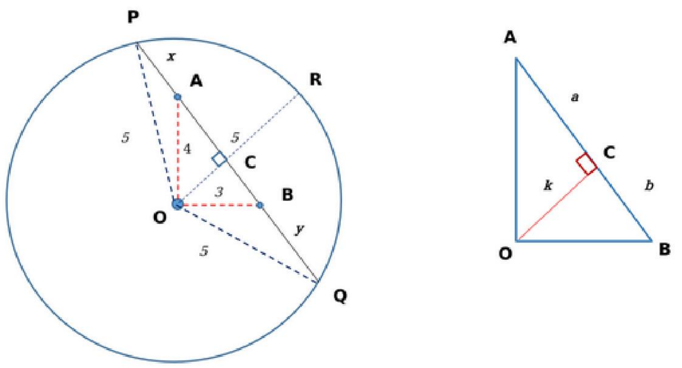
$\frac{-4P}{100} + \frac{5.25P}{100} = 1125$

$\frac{1.25P}{100} = 1125$

Solving we get P= 90000

55. **A**

We can form the following figure based on the given information:



Since OA = 4 m and OB=3 m; AB = 5 m. OR bisects the chord into PC and QC.

Since AB = 5 m, we have $a + b = 5 \quad ...(i)$ Also, $4^2 - k^2 = a^2 ...(ii)$ and $3^2 - k^2 = b^2 ...(iii)$

Subtracting (iii) from (ii), we get: $a^2 - b^2 = 7...(iv)$

Substituting (i) in (iv), we get $a - b = 1.4 \quad ...(v)$; $\left[(a+b)(a-b) = 7;\ \therefore\ (a-b) = \frac{7}{5}\right]$

Solving (i) and (v), we obtain the value of $a = 3.2$ and $b = 1.8$

Hence, $k^2 = 5.76$

Moving on to the larger triangle $\triangle POC$, we have $5^2 - k^2 = (x+a)^2$;

Substituting the previous values, we get: $(25 - 5.76) = (x + 3.2)^2$

$\sqrt{19.24} = (x + 3.2)$ or $x = 1.19m$

Similarly, solving for y using $\triangle QOC$, we get $y = 2.59m$

Therefore, $PQ = 5 + 2.59 + 1.19 = 8.78 \approx 8.8m$

Hence, Option A is the correct answer.

56. **B**

Now car A beats car B by 45km. Let the speed of car A be v(a) and speed of car B be v(b).

$\frac{v(a)}{v(b)} = \frac{m}{m-45}$ .....(1)where "'m" is the entire distance of the race track.

Moreover $\frac{v(b)}{v(c)} = \frac{m}{m-50}$.......(2)

and finally $\frac{v(a)}{v(c)} = \frac{m}{m-90}$......(3)

Multiplying (1) and (2) we get (3). $\frac{m}{m-90} = \frac{m}{m-45}\left(\frac{m}{m-50}\right)$

Solving we get m=450 which is the length of the entire race track

57. **315**

Here there are two cases possible

Case 1: When 7 is at the left extreme

In that case 3 can occupy any of the three remaining places and the remaining two places can be taken by (0,1,2,4,5,6,8,9)

So total ways 3(8)(7)= 168

Case 2: When 7 is not at the extremes

Here there are 3 cases possible. And the remaining two places can be filled in 7(7) ways.(Remember 0 can't come on the extreme left)

Hence in total 3(7)(7)=147 ways

Total ways 168+147=315 ways

58. **A**
Let the sides of the rectangle be "a" and "3a" m. Hence the perimeter of the rectangle is 8a.

Let the side of the equilateral triangle be "m" cm. Hence the perimeter of the equilateral triangle is "3m" cm. Now we know that 8a+3m=90......(1)

Moreover area of the equilateral triangle is $\frac{\sqrt{3}}{4}m^2$ and area of the rectangle is $3a^2$

According to the relation given $\left(\frac{\sqrt{3}}{4}m^2\right)^2 = 3a^2$

$\frac{3}{16}m^4 = 3a^2 \; or \; a^2 = \frac{m^4}{16}$

$a = \frac{m^2}{4}$

Substituting this in (1) we get $2m^2 + 3m - 90 = 0$ solving this we get m=6 (ignoring the negative value since side can't be negative)

Hence a=9 and the longer side of the rectangle will be 3a=27cm

59. **800**
Let the amount of money with Amal and Sunil be 6x and 4x. Now the amount of money with Mita be 5x. Difference between the largest and smallest amount is ₹400 i.e. 6x-4x=400 or 2x=400 or x=200 . Amount of money with Sunil is 200(4)=₹800

60. **C**
On expanding the expression we get $1 - \log_a b + 1 - \log_b a$

$or \; 2 - \left(\log_a b + \frac{1}{\log_b a}\right)$

Now applying the property of AM>=GM, we get that $\frac{\left(\log_a b + \frac{1}{\log_b a}\right)}{2} \geq 1 \; or \; \left(\log_a b + \frac{1}{\log_b a}\right) \geq 2$ Hence from here we can conclude that the expression will always be equal to 0 or less than 0. Hence any positive value is not possible. So 1 is not possible.

61. **C**
Now 23 students choose maths as one of their subject.

This means (MPC)+ (MC) + (PC)=23 where MPC denotes students who choose all the three subjects maths, physics and chemistry and so on.

So MC + PM =5 Similarly we have PC+ MP =7

We have to find the smallest number of students choosing chemistry

For that in the first equation let PM=5 and MC=0. In the second equation this PC=2

Hence minimum number of students choosing chemistry will be (18+2)=20 Since 18 students chose all the three subjects.

62. **B**
Let x(1) be the least number and x(10) be the largest number. Now from the condition given in the question , we can say that

x(2)+x(3)+x(4)+........x(10)= 47*9=423..................(1)

Similarly x(1)+x(2)+x(3)+x(4)................+x(9)= 42*9=378..............(2)

Subtracting both the equations we get x(10)-x(1)=45

Now, the sum of the 10 observations from equation (1) is 423+x(1)

Now the minimum value of x(10) will be 47 and the minimum value of x(1) will be 2 . Hence minimum average 425/10=42.5

Maximum value of x(1) is 42. Hence maximum average will be 465/10=46.5

Hence difference in average will be 46.5-42.5=4 which is the correct answer

63. **C**
Let the speed of Ram be v(r) and the speed of Rahim be v(h) respectively. Let them meet after time "t" from the beginning.

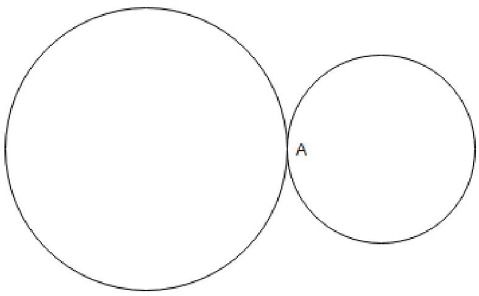
Hence Ram will cover v(r)(t) during that time and Rahim will cover v(h)t respectively.

Now after meeting Ram reaches his destination in 1 min i.e. Ram covered v(h)t in 1 minute or v(r)(1)= v(h)(t)

Similarly Rahim reaches his destination in 4 min i.e. Rahim covered v(r)t in 4 minutes or v(h)(4)= v(r)(t)

Dividing both the equations we get $\frac{v(r)}{4v(h)} = \frac{v(h)}{v(r)}\ or\ \frac{v(r)}{v(h)} = 2$ Hence the ratio is 2.

64. **B**



To complete one round Ram takes 100m/15kmph and Rahim takes 20m/5kmph

They meet for the first time after L.C.M of (100m/15kmph , 20m/5kmph) = 100m/5kmph=20m/kmph.

Distance traveled by Ram =20m/kmph * 15kmph =300m.

So, he must have ran 300/100=3 rounds.

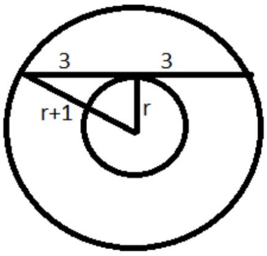
Note:

CAT gave both 2 and 3 as correct answers because of the word '**before**'.

65. **10**
Now we know that the perpendicular from the centre to a chord bisects the chord. Hence at the point of intersection of tangent, the chord will be divided into two parts of 3 cm each. As you can clearly see in the diagram, a right angled triangle is formed there.

Hence $(r+1)^2 = r^2 + 9$ or $r^2 + 1 + 2r = r^2 + 9\ or\ 2r = 8\ or\ r = 4cm$

Hence the radius of the larger circle is 5cm and diameter is 10cm.



66. **A**
Let the CP of the each toy be "x". CP of 12 toys will be "12x". Now the shopkeeper made a 10% profit on CP. This means that

12x(1.1)= 2112 or x=160 . Hence the CP of each toy is ₹160.

Now let the SP of each toy be "m". Now he sold 8 toys at 20% discount. This means that 8m(0.8) or 6.4m

He sold 4 toys at an additional 25% discount. 4m(0.8)(0.75)=2.4m Now 6.4m+2.4m=8.8m=2112 or m=240

Hence CP= 160 and SP=240. Hence profit percentage is 50%.

67. **A**
$(x^2-5x+7)^{x+1}=1$

There can be a solution when $\left(x^2-5x+7\right)=1$ or $x^2-5x\ +6=0$

or x=3 and x=2

There can also be a solution when x+1 = 0 or x=-1

Hence three possible solutions exist.

68. **17**
We have 2x + 5y = 99 or $x=\frac{(99-5y)}{2}$

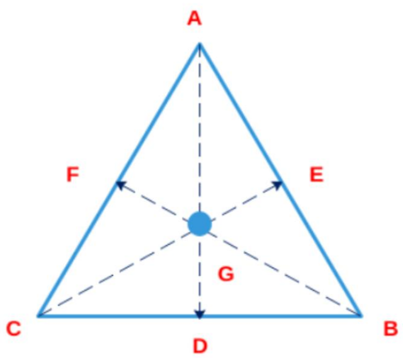
Now $x\geq\ y\ \geq\ -20$; So $\frac{(99-5y)}{2}\geq\ y\ ;\ 99\geq 7y\ or\ y\leq\approx\ 14$

So $-20\leq y\leq 14$. Now for this range of "y", we have to find all the integral values of "x". As the coefficient of "x" is 2,

then (99 - 5y) must be even, which will happen when "y" is odd. However, there are only 17 odd values of "y" be -20 and 14.

Hence the number of possible values is 17.

69. **D**



Based on the question: AD, CE and BF are the three altitudes of the triangle. It has been stated that {GD+GE+GF = s}

Now since the triangle is equilateral, let the length of each side be "a". So area of triangle will be

$\frac{1}{2}\times\ GD\times\ a+\ \frac{1}{2}\times\ GE\times\ a\ +\frac{1}{2}\times\ GF\times\ a=\frac{\sqrt{3}}{4}a^{2}$

Now $GD+GE+GF=\frac{\sqrt{3}a}{2}$ or $s=\frac{\sqrt{3}a}{2}\ or\ a=\frac{2s}{\sqrt{3}}$

Given the area of the equilateral triangle = $\frac{\sqrt{3}}{4}a^{2}$; substituting the value of 'a' from above, we get the area {in terms 's'}= $\frac{s^{2}}{\sqrt{3}}$

70. **D**

Let the first term of the GP be "a" . Now from the question we can show that

$ar^{m-1}=\frac{3}{4}\quad ar^{n-1}=12$

Dividing both the equations we get $r^{m-1-n+1}=\frac{1}{16}\ or\ r^{m-n}=16^{-1}\ or\ r^{n-m}=16$

So for the minimum possible value we take Now give minimum possible value to "r" i.e -4 and n-m=2

Hence minimum possible value of r+n-m=-4+2=-2

71. **A**

Let John buy "m" kg of rice and "p" kg of wheat.

Now let the price of rice be "r" in April. Price in May will be "1.2(r)"

Now let the price of wheat be "w" in April . Price in April will be "1.12(w)".

Now he spent ₹150 more in May , so 0.2(rm)+0.12(wp)=150

Its also given that he had spent ₹450 on rice in April. So (rm)=450

So 0.2(rm)= (0.2)(450)=90 Substituting we get (wp)=60/0.12 or (wp)=500

Amount spent on wheat in May will be 1.12(500)=₹560

72. **23**

We can write x=z-9 and y=z-1 Now we have x+y< z+5

Substituting we get z-9+z-1<z+5 or z<15

Hence the maximum possible value of z is 14

Maximum value of "x" is 5 and maximum value of "y" is 13

Now 2x+y = 10+13=23

73. **A**

Let the number of pencils bought by Aron be "p" and the cost of each pencil be "a".

Let the number of sharpeners bought Aron be "s" and the cost of each sharpener be "b".

Now amount spent by Aron will be (pa)+(sb)

Aditya bought (2p) pencils and (s-10) sharpeners. Amount spent will be (2pa)+(s-10)b

Amount spent in both the cases is same

pa + sb = 2pa + (s-10)b or pa=10b

Now its given in the question that cost of sharpener is 2 more than pencil i.e. b=a+2

pa= 10a+20 or a=20/(p-10)

Now the number of pencils has to be minimum, for that we have to find smallest "p" such that both "p" and "a" are integers. The smallest such value is p=11 . Total number of pencils bought will be p+2p=11+22=33

74. **D**

Now $\frac{x}{\sqrt{1+x^{4}}}=\frac{1}{\sqrt{\frac{1+x^{4}}{x^{2}}}}=\frac{1}{\sqrt{\frac{1}{x^{2}}+x^{2}}}$