Class: IX

SESSION : 2022-2023

SUBJECT: Mathematics

SAMPLE QUESTION PAPER - 6

with SOLUTION

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. Area of an isosceles triangle ABC with AB = a = AC and BC = b is **[1]**

a) $\frac{1}{4}b\sqrt{4a^2-b^2}$ b) $\frac{1}{4}b\sqrt{a^2-b^2}$

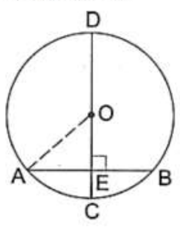
c) $\frac{1}{2}b\sqrt{4a^2-b^2}$ d) $\frac{1}{2}b\sqrt{a^2-b^2}$

2. Abscissa of a point is negative in **[1]**

a) quadrant IV only b) quadrant II and III

c) quadrant I and IV d) quadrant I only

3. In the given figure, CD is the diameter of a circle with centre O and CD is perpendicular to chord AB. If AB = 12 cm and CE = 3 cm, then radius of the circles is **[1]**



a) 7.5 b) 9 cm

c) 6 cm d) 8 cm

4. $(625)^{0.16} \times (625)^{0.09} =$ **[1]**

a) 625 b) 5

c) 125 | d) 25

5. In a bar graph, 0.25 cm length of a bar represents 100 people. Then, the length of bar which represents 2000 people is **[1]**

a) 4.5 cm
b) 4 cm
c) 5 cm
d) 3.5 cm

6. Any solution of the linear equation 2x + 0y + 9 = 0 in two variables is of the form **[1]**

a) $\left(-\frac{9}{2}, m\right)$
b) (-9 , 0)
c) $\left(0, -\frac{9}{2}\right)$
d) $\left(n, -\frac{9}{2}\right)$

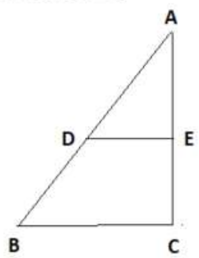
7. An exterior angle of a triangle is 108° and its interior opposite angles are in the ratio 4 : 5. The angles of the triangle are. **[1]**

a) 50°, 60°, 70°
b) 48°, 60°, 72°
c) 2°, 56°, 72°
d) 42°, 60°, 76°

8. $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} =$ **[1]**

a) 8
b) -10
c) 10
d) -8

9. D and E are the mid-points of the sides AB and AC. Of △ABC. If BC = 5.6cm, find DE. **[1]**



a) 2.8 cm
b) 3 cm
c) 2.9 cm
d) 2.5 cm

10. When $x^3 - 2x^2 + ax - b$ is divided by $x^2 - 2x - 3$, the remainder is x - 6 . The values of a and b are respectively. **[1]**

a) -2 and 6
b) -2, -6
c) 2 and -6
d) 2 and 6

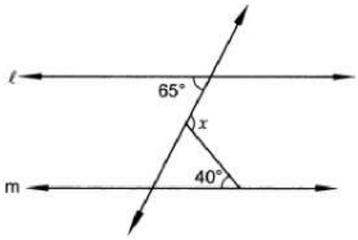
11. The value of $x - y^{x-y}$ when x = 2 and y = -2, is **[1]**

a) 14
b) -18

c) 18

d) -14

12. In Fig. if $l \parallel m$, then $x =$ **[1]**



a) 65°

b) 25°

c) 105°

d) 40°

13. If $\sqrt{5} = 2.236$, then $\frac{1}{\sqrt{5}}$ **[1]**

a) 44.72

b) 0.4472

c) 0.04472

d) 4.472

14. Which of the following is not a solution of 2x – 3y = 12? **[1]**

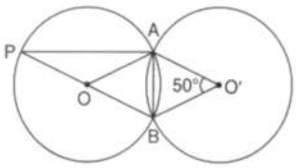
a) (0, -4)

b) (2, 3)

c) (6, 0)

d) (3, -2)

15. The given figure shows two congruent circles with centre O and O' intersecting at A and B. If $\angle AO'B = 50^o,$ then the measure of $\angle APB$ is **[1]**



a) 40^o

b) 25^o

c) 50^o

d) 45^o

16. How many linear equations can be satisfied by x = 2 and y = 3? **[1]**

a) only one

b) none of these

c) many

d) two

17. The ordinate of any point on x-axis is **[1]**

a) 0

b) any number

c) -1

d) 1

18. **Assertion (A):** If the diagonals of a parallelogram ABCD are equal, then ∠ABC **[1]**

$= 90^{o}$
Reason (R): If the diagonals of a parallelogram are equal, it becomes a rectangle.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

19. The expanded form of $(3x - 5)^3$ is **[1]**

a) none of these

b) $27x^3 + 135x^2 + 225x - 125$

c) $27x^3 - 135x^2 + 225x - 125$

d) $27x^3 + 135x^2 - 225x - 125$

20. **Assertion (A):** $2 + \sqrt{6}$ is an irrational number. **[1]**
Reason (R): Sum of a rational number and an irrational number is always an irrational number.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Read the following two statements which are taken as axioms: **[2]**

i. If two lines intersect each other, then the vertically opposite angles are not equal.

ii. If a ray stands on a line, then the sum of two adjacent angles so formed is equal to 180°.

Is this system of axioms consistent? Justify your answer.

22. Look at the Fig. Show that length AH > sum of lengths of AB + BC + CD. **[2]**

23. Insert five rational numbers between $-\frac{2}{3}$ and $\frac{3}{4}$. **[2]**

OR

If $\sqrt{2} = 1.414, \sqrt{3} = 1.732$ then find the value of $\frac{4}{3\sqrt{3}-2\sqrt{2}} + \frac{3}{3\sqrt{3}+2\sqrt{2}}$

24. The surface areas of two spheres are in the ratio 1 : 4. Find the ratio of their volumes. **[2]**

OR

Find the capacity in litres of a conical vessel with height 12 cm, slant height 13 cm.

25. Name the quadrant in which the following points lie: (i) A(2, 9) (ii) B(–3, 5) (iii) **[2]**

C(–4, –7) (iv) D(3, –2)

Section C

26. Draw a histogram of the following distribution: **[3]**

Height (in cm)	Number of students
150 - 153	7
153 - 156	8
156 - 159	14
159 - 162	10
162 - 165	6
165 - 168	5

27. Rationalise the denominator: $\frac{1}{\sqrt{7}+\sqrt{6}-\sqrt{13}}$ **[3]**

28. Find the solution of the linear equation x + 2y = 8 which represents a point on **[3]**

i. The x-axis

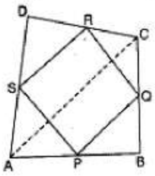
ii. The y-axis

29. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA. AC is a diagonal. Show that **[3]**

i. SR || AC and SR = $\frac{1}{2}$AC

ii. PQ = SR

iii. PQRS is a parallelogram.



30. Factorise: $3x^3 - x^2 - 3x + 1$ **[3]**

31. The following data on the number of girls (to the nearest ten) per thousand boys in different sections of the society is given below : **[3]**

Section	**Number of girls per thousand boys**
Scheduled caste	940
scheduled tribe	970
Non SC/ST	920
Backward districts	950
Non-backward districts	920
Rural	930

Section	Number of girls per thousand boys
Urban	910

i. Represent the information above by a bar graph.

ii. In the classroom discuss what conclusion can be arrived at from the graph.

OR

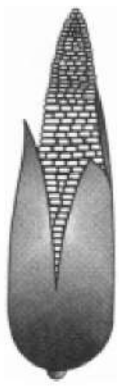
The following is the monthly expenditure (Rs.) of ten families of the particular area: 145, 115, 129, 135, 139, 158, 170, 175, 188, 163

i. Make a frequency distribution table by using the following class interval: 100 - 120, 120 - 140, 140 - 160, 160 - 180, 180 - 200.

ii. Construct a frequency polygon for the above frequency distribution.

Section D

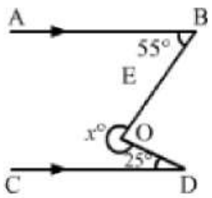
32. A corn cob (see Fig.), shaped somewhat like a cone, has the radius of its broadest end as 2.1 cm and length as 20 cm. If each 1 cm^2 of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob? **[5]**



33. Prove that if the arms of an angle are respectively perpendicular to the arms of another angle, then the angles are either equal or supplementary. **[5]**

OR

In each of the figures given below, AB || CD. Find the value of x°



34. Using factor theorem, factorize the polynomial: $2x^4 - 7x^3 - 13x^2 + 63x - 45$ **[5]**

35. If each side of a triangle is doubled, then find the ratio of area of new triangle thus formed and the given triangle. **[5]**

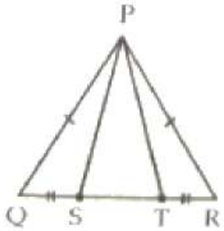
OR

Two sides of a triangular field are 85 m and 154 m in length and its perimeter is 324 m. Find the area of the field.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A children's park is in the shape of isosceles triangle said PQR with PQ = PR, S and T are points on QR such that QT = RS.



(i) Which rule is applied to prove that congruency of $\triangle PQS$ and $\triangle PRT$.

(ii) Name the type of $\triangle PST$.

OR

If $\angle QPR = 80^o$ find $\angle PQR$?

(iii) If PQ = 6 cm and QR = 7 cm, then find perimeter of $\triangle PQR$.

37. **Read the text carefully and answer the questions:** **[4]**

Reeta was studying in the class 9th C of St. Surya Public school, Mehrauli, New Delhi-110030

Once Ranjeet and his daughter Reeta were returning after attending teachers' parent meeting at Reeta's school. As the home of Ranjeet was close to the school so they were coming by walking.

Reeta asked her father, "Daddy how old are you?"

Ranjeet said, "Sum of ages of both of us is 55 years, After 10 years my age will be double of you.

(i) What is the second equation formed?

(ii) What is the present age of Reeta in years?

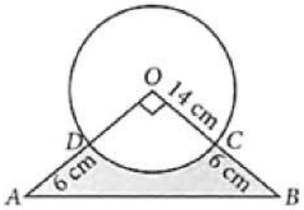
(iii) What is the present age of Ranjeet in years?

OR

If the ratio of age of Reeta and her mother is 3 : 7 then what is the age of Reeta's mother in years?

38. **Read the text carefully and answer the questions:** **[4]**

Director of a company selected a round glass trophy for awarding their employees an annual function. The design of each trophy is made as shown in the figure, where its base ABCD is golden plated from the front side at the rate of ₹6 per cm^2.



(i) Find the area of sector ODCO.

(ii) Find the area of $\triangle AOB$.

OR

Find the area of major sector formed in the given figure.

(iii) Find the total cost of golden plating.

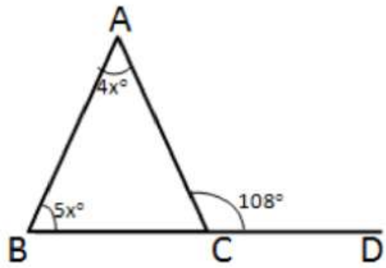
SOLUTION

Section A

1. **(a)** $\frac{1}{4}b\sqrt{4a^2-b^2}$
   Explanation: Here s = $\frac{a+a+b}{2}=\frac{2a+b}{2}$
   Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
   $=\sqrt{\frac{2a+b}{2}\left(\frac{2a+b}{2}-a\right)\left(\frac{2a+b}{2}-a\right)\left(\frac{2a+b}{2}-b\right)}$
   $=\sqrt{\frac{2a+b}{2}\left(\frac{b}{2}\right)\left(\frac{b}{2}\right)\left(\frac{2a-b}{2}\right)}$
   $=\frac{b}{4}\sqrt{4a^2-b^2}$
2. **(b)** quadrant II and III
   Explanation: The abscissa (x-axis) is -ve in 2nd and 3rd quadrant only because,
   Sign of point in 2nd quadrant is (-, +), and in 3rd quadrant, it is (-, -).
3. **(a)** 7.5
   Explanation: Let OA = OC = r cm.
   Then OE = (r - 3) cm and AE = $\frac{1}{2}$ AB = 6cm
   Now, in right ΔOAE, we have:
   $OA^2 = OE^2 + AE^2$ [Using paythagoras theorem]
   $\Rightarrow (r)^2 = (r - 3)^2 + 6^2$
   $\Rightarrow r^2 = r^2 + 9 - 6r + 36$
   $\Rightarrow 6r = 45$
   $\Rightarrow r=\frac{45}{6}=7.5\text{cm}$
   Hence, the required radius of the circle is 7.5 cm.
4. **(b)** 5
   Explanation: $(625)^{0.16}\times(625)^{0.09}$
   $= (625)^{0.16 + 0.09}$
   $= (625)^{0.25}$ or $(625)^{\frac{1}{4}}$
   But $625 = 5^4$
   So, $\left(5^4\right)^{\frac{1}{4}} = 5$
5. **(c)** 5 cm
   Explanation: Use unitary method
   0.25 cm - 100 people
   So 1 cm - 400 people
   So for 2000 people:
   $\frac{2000}{400}= 5$ cm
6. **(a)** $\left(-\frac{9}{2}, m\right)$
   Explanation: 2x + 9 = 0
   $\Rightarrow x=\frac{-9}{2}$ and y = m, where m is any real number
   Hence, $\left(-\frac{9}{2}, m\right)$ is the solution of the given equation.

7. **(b)** 48°, 60°, 72°

Explanation:



From figure, we have

$\angle A + \angle B = \angle ACD$

$\Rightarrow 4x° + 5x° = 180°$

$\Rightarrow 9x° = 108°$

$\Rightarrow x = 12°$

So, $\angle A = 48°$, $\angle B = 60°$

$\Rightarrow \angle C = 180° - 48° - 60° = 72°$

8. **(c)** 10

Explanation: $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow \frac{(\sqrt{3}+\sqrt{2})^2+(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$

$\Rightarrow \frac{(3+2+2\sqrt{6})+3+2-2\sqrt{6}}{3-2}$

$\Rightarrow 10$

9. **(a)** 2.8 cm

Explanation: By using Mid-Point theorem,

DE = Half of BC

Hence, DE = $0.5 \times 5.6 = 2.8$ cm

10. **(a)** -2 and 6

Explanation: If the remainder (x - 6) is subtracted from the given polynomial $f(x) = x^3 - 2x^2 + ax - b$,

then rest part of this polynomial is exactly divisible by $x^2 - 2x - 3$.

Therefore, $p(x) = x^3 - 2x^2 + ax - b - (x - 6)$

Now,

$x^2 - 2x - 3 = x^2 - 3x + x - 3$

$x^2 - 2x - 3 = (x + 1)(x - 3)$

Therefore, $(x + 1)(x - 3)$ are factors of polynomial p(x).

Now,

$p(-1) = 0$

And

$p(3) = 0$

$p(-1) = (-1)^2 + a(-1) - b - (-1 - 6) = 0$

$= -1 - 2 - a - b + 1 + 6 = 0$

$= -a - b + 4 = 0$

$a + b = 4$ ..... (i)

and

$p(3) = (3)^3 - 2(3)^3 + a(3) - b(3 - 6) = 0$

$= 27 - 18 + 3a - b + 3 = 0$
$3a - b = -12$ ..... (ii)
Solving (i) and (ii) we get
$a = -2, b = 6$

11. **(d)** -14

Explanation: $x = 2, y = -2$
$x - y^{x-y} = 2 - (-2)^{2-(-2)}$
$= 2 - (-2)^{2+2}$
$= 2 - (-2)^4$
$= 2- (+16)$
$= 2 - 16$
$= -14$

12. **(c)** $105°$

Explanation: Given that,
$l \parallel m$ and n cuts them
Let,
$\angle 1 = 65^o$
$\angle 2 = x$
$\angle 3 = 40^o$
$\angle 1 = \angle 4 = 65^o$ (Alternate angle) (i)
$\angle 3 + \angle 4 + \angle 5 = 180^o$ (Angle sum property)
$40^o + 65^o + \angle 5 = 180^o$
$\angle 5 = 75^o$
Now,
$\angle 2 + \angle 5 = 180^o$ (Linear pair)
$x + 75^o = 180^o$
$x = 105^o$

13. **(b)** 0.4472

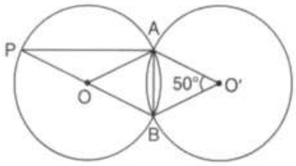
Explanation: $\sqrt{5} = 2.236$
So, $\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5} = \frac{2.236}{5}$
$= 0.4472$

14. **(b)** (2, 3)

Explanation: We have to check (2, 3) is a solution of $2x – 3y = 12$ if (2, 3) satisfy the equation then (2, 3) solution of $2x – 3y = 12$
LHS = $2x - 3y$
$2 \times 2 - 3 \times 3$
$4 - 9 = -5$
RHS = -5
LHS $\neq$ RHS
So (2, 3) is not a solution of $2x - 3y = 12$

15. **(b)** 25^o

Explanation:



Since both the triangles are congruent,
So, OA = O'A,
OB = O'B
AB = AB (Common)
Hence, $\triangle AOB \cong \triangle AO'B$
Thus, $\angle AOB = \angle AO'B = 50^0$
Since, PB is a straight line, therefore:-
$\angle AOP + \angle AOB = 180^o$
$\Rightarrow \angle AOP = 180^o - 50^o = 130^o$
Again, In triangle OPA,
$\Rightarrow \angle P = \angle A$
$\Rightarrow \angle A + \angle P + \angle O = 180^o$
$\Rightarrow 2\angle P + 130^o = 180^o$
$\Rightarrow \angle P = \frac{50^0}{2} = 25^0$
Thus, $\angle OPA = 25^0$

16. **(c)** many
 Explanation: There are infinite many eqution which satisfy the given value x = 2, y = 3 for example
 x + y = 5
 x - y = -1
 3x - 2y = 0
 etc.......
17. **(a)** 0
 Explanation: The ordinate of any point on x-axis is always zero. This means that this point hasn't covered any distance on y-axis.
18. **(a)** Both A and R are true and R is the correct explanation of A.
 Explanation: Both A and R are true and R is the correct explanation of A.
19. **(c)** $27x^3 - 135x^2 + 225x - 125$
 Explanation: $(3x - 5)^3$
 $= (3x)^3 - (5)^3 - 3 \times 3x \times 5(3x - 5)$
 $= 27x^3 - 125 - 45x(3x - 5)$
 $= 27x^3 - 125 - 135x^2 + 225x$
 $= 27x^3 - 135x^2 + 225x - 125$
20. **(a)** Both A and R are true and R is the correct explanation of A.
 Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. It is known that, if two lines intersect each other, then the vertically opposite angles are equal. It is a theorem, therefore,given Statement I is false and not an axiom.
 Also, we know that, if a ray stands on a line, then the sum of two adjacent angles so formed is equal to 180°. It is an axiom. Therefore, given statement parallel is true and an axiom.

Thus, in given statements, first is false and second is an axiom. Therefore, given system of axioms is not consistent.

22. From the given figure, we have
AB + BC + CD = AD [AB, SC and CD are the parts of AD] Here, AD is also the parts of AH.
By Euclid's axiom, the whole is greater than the part. i.e., AH > AD.
Therefore, length AH > sum of lengths of AB + BC + CD.

23. $-\frac{2}{3} = \frac{-2\times 4}{3\times 4} = \frac{-8}{12}$
$\frac{3}{4} = \frac{3\times 3}{4\times 3} = \frac{9}{12}$
So, the five rational numbers between $-\frac{2}{3}$ and $\frac{3}{4}$ are $\frac{8}{12}$, $\frac{7}{12}$, $\frac{6}{12}$, $\frac{5}{12}$ **and** $\frac{4}{12}$

OR

Given, $\frac{4}{3\sqrt{3}-2\sqrt{2}} + \frac{3}{3\sqrt{3}+2\sqrt{2}}$
$= \frac{4(3\sqrt{3}+2\sqrt{2})+3(3\sqrt{3}-2\sqrt{2})}{(3\sqrt{3}-2\sqrt{2})(3\sqrt{3}+2\sqrt{2})}$
$= \frac{12\sqrt{3}+8\sqrt{2}+9\sqrt{3}-6\sqrt{2}}{(3\sqrt{3})^2-(2\sqrt{2})^2}$
$= \frac{21\sqrt{3}+2\sqrt{2}}{27-8}$
$= \frac{21\sqrt{3}+2\sqrt{2}}{19}$
$= \frac{21(1.732)+2(1.414)}{19}$
$= \frac{36.372+2.828}{19}$
= 2.063

24. Suppose that the radii of the spheres are r and R.
We have:
$\frac{4\pi r^2}{4\pi R^2} = \frac{1}{4}$
$\Rightarrow \frac{r}{R} = \sqrt{\frac{1}{4}} = \frac{1}{2}$
Now, ratio of the volumes = $\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \left(\frac{r}{R}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$
Therefore, The ratio of the volumes of the spheres is 1 : 8.

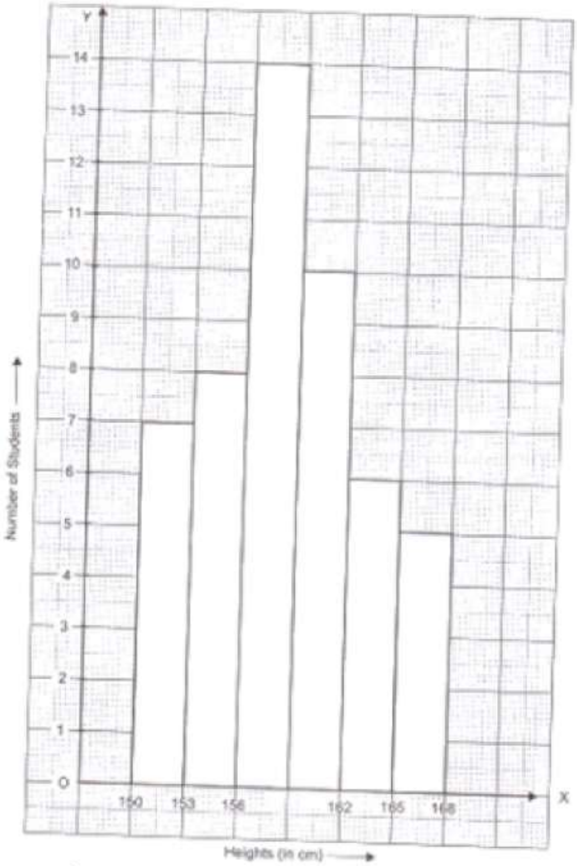
OR

12 cm, l = 13 cm.
$r^2 + h^2 = l^2$
$\Rightarrow r^2 + (12)^2 = (13)^2$
$\Rightarrow r^2 + 144 = 169$
$\Rightarrow r^2 = 169 - 144$
$\Rightarrow r^2 = 25$
$\Rightarrow r = \sqrt{25}$
$\Rightarrow$ r = 5 cm
$\therefore$ Capacity $= \frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12$
$= \frac{2200}{7}\ cm^3 = \frac{2200}{7000} l$
$= \frac{11}{35} l$

25. (i) I quadrant
(ii) II quadrant
(iii) III quadrant
(iv) IV quadrant

Section C

26. Histogram which represent the given frequency distribution is shown below:



27. $\frac{1}{\sqrt{7}+\sqrt{6}-\sqrt{13}}$

$\frac{1}{(\sqrt{7}+\sqrt{6})-\sqrt{13}} \times \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(\sqrt{7}+\sqrt{6})+\sqrt{13}}$

$= \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(\sqrt{7}+\sqrt{6})^2-\sqrt{13}^2}$ [$\because a^2 - b^2 = (a + b)(a - b)$]

$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{(7+6+2\sqrt{42})-13}$

$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{13+2\sqrt{42}-13}$

$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{2\sqrt{42}}$

$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{2\sqrt{42}} \times \frac{\sqrt{42}}{\sqrt{42}}$

$= \frac{\sqrt{7\times 42}+\sqrt{6\times 42}+\sqrt{13\times 42}}{2(\sqrt{42})^2}$

$= \frac{\sqrt{7\times 7\times 6}+\sqrt{6\times 6\times 7}+\sqrt{546}}{2\times 42}$

$= \frac{7\sqrt{6}+6\sqrt{7}+\sqrt{546}}{84}$

28. i. On x-axis y = 0

$\Rightarrow \ x + 2 \times 0 = 8 \Rightarrow \ x = \ 8$

Therefore, the required point is (8, 0).

ii. On y-axis x = 0

$\Rightarrow \ 0 + 2y = 8$

$\Rightarrow\ y = \frac{8}{2} \Rightarrow\ y = 4$

Thus, the required point is (0, 4).

29. Given: ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA. AC is a diagonal.

To Prove :

i. SR || AC and SR = $\frac{1}{2}$AC

ii. PQ = SR

iii. PQRS is a parallelogram

Proof :

i. In $\triangle$DAC,

As S is the mid-point of DA and R is the mid-point of DC

$\therefore$ SR || AC and SR = $\frac{1}{2}$AC . . . [Mid point theorem]

ii. In $\triangle$BAC,

As P is the mid-point of AB and Q is the mid-point of BC

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$AC . . . [Mid point theorem]

But from (i) SR = $\frac{1}{2}$AC

$\therefore$ PQ = SR

iii. PQ || AC . . .[From (i)]

SR || AC . . .[From (i)]

$\therefore$ PQ || SR . . .[Two lines parallel to the same line are parallel to each other]

Similarly, PQ = SR . . .[From (ii)]

$\therefore$ PQRS is a parallelogram . . . [A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length]

30. Let f(x) = $3x^3 - x^2 - 3x + 1$ be the given polynomial. The factors of the constant term + 1 are ±1. The factor of coefficient of x^3 is 3. Hence, possible rational roots of f(x) are: $\pm\frac{1}{3}$.

We have, f(1) = $3(1)^3 - (1)^2 - 3(1) + 1 = 3 - 1 - 3 + 1 = 0$.

And f(-1) = $3(-1)^3 - (-1)^2 - 3(-1) + 1 = -3 - 1 + 3 + 1 = 0$

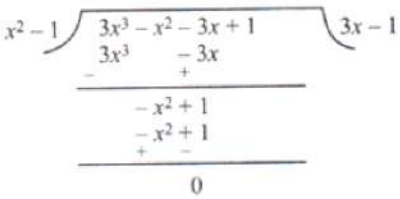
So, (x - 1) and (x + 1) are factors of f(x).

$\Rightarrow$ (x - 1)(x + 1) is also a factor of f(x).

$\Rightarrow$ $x^2 - 1$ is a factor of f(x).

Let us now divide f(x) = $3x^3 - x^2 - 3x + 1$ by $x^2 - 1$ to get the other factors of f(x).

By long division, we have

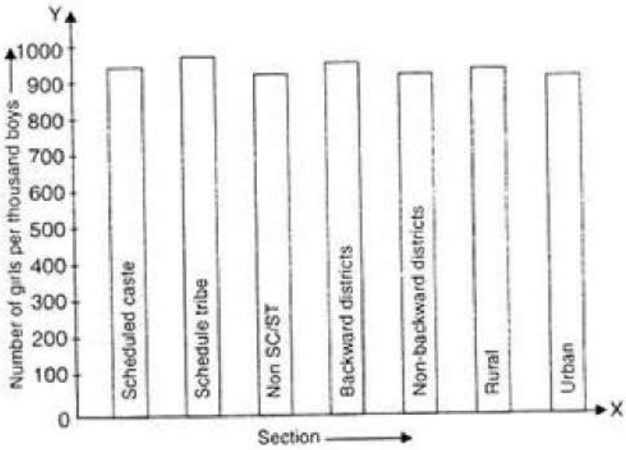


$\therefore$ $3x^3 - x^2 - 3x + 1 = (x^2 - 1)(3x - 1)$

$\Rightarrow$ $3x^3 - x^2 - 3x + 1 = (x - 1)(x + 1)(3x - 1)$

Hence, $3x^3 - x^2 - 3x + 1 = (x - 1)(x + 1)(3x - 1)$

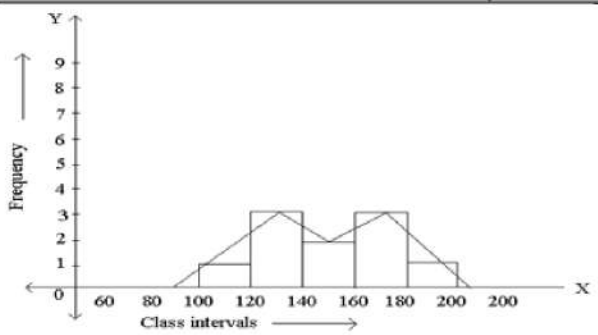
31.

i. 

ii. The two conclusions we can arrive at from the graph are as follows:
iii. The numbers of girls to the nearest ten per thousand boys is maximum in Scheduled Tribe section of the society and minimum in Urban section of the society.
iv. The number of girls to the nearest ten per thousand boys is the same for 'Non SC/ST' and 'Non-backward Districts' sections of the society.

OR

Frequency Distribution		
Class intervals	Tally Marks	Frequency
100-120	I	1
120-140	III	3
140-160	II	2
160-180	III	3
180-120	I	1
Total		10



Section D

32. Since the grains of corn are found on the curved surface of the corn cob.
So, Total number of grains on the corn cob = Curved surface area of the corn cob × Number of grains of corn on 1 cm^2
Now, we will first find the curved surface area of the corn-cob.
We have, r = 2.1 and h = 20
Let l be the slant height of the conical corn cob. Then,
$l = \sqrt{r^2 + h^2} = \sqrt{(2.1)^2 + (20)^2} = \sqrt{4.41 + 400} = \sqrt{404.41}$ = 20.11
$\therefore$ Curved surface area of the corn cub = πrl
$= \frac{22}{7} \times 2.1 \times 20.11$ cm^2
= 132.726 cm^2 = 132.73 cm^2

Hence, Total number of grains on the corn cob = 132.73 × 4 = 530.92

So, there would be approximately 531 grains of corn on the cob.

33. i. In $\triangle BOD$,

$\angle OBD + \angle BOD + \angle ODB = 180^o$

(The sum of the three angles of a triangle is 180^o)

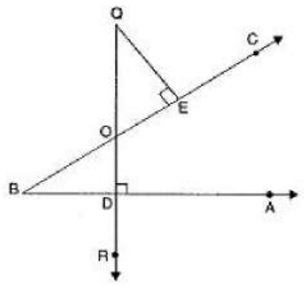
$\Rightarrow \angle OBD + \angle BOD + 90^o = 180^o$

$\Rightarrow \angle OBD + \angle BOD = 90^o$ .... (1)

In$\triangle OEQ$,

$\angle EQO + \angle QOE + \angle OEQ = 180^o$ .... (2)

(The sum of the three angles of a triangle is 180^o)



$\Rightarrow \angle EQO + \angle QOE + 90^o = 180^o$

$\Rightarrow \angle EQO + \angle QOE = 90^o$ .... (2)

From (1) and (2), we get

$\angle OBD + \angle BOD = \angle EQO + \angle QOE$

But $\angle BOD = \angle QOE$ (Vertically Opposite Angles)

$\therefore \angle OBD = \angle EQO$

ii. Join BQ

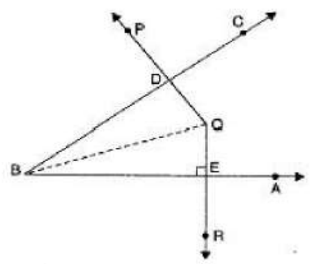
In$\triangle BDQ$,

$\angle DBQ + \angle BQD + \angle QDB = 180^o$

(The sum of the three angles of a triangle is 180^o)

$\Rightarrow \angle DBQ + \angle BQD + 90^o = 180^o$

$\Rightarrow \angle DBQ + \angle BQD = 90^o$ .... (1)



In$\triangle BQE$,

$\angle EBQ + \angle BQE + \angle BEQ = 180^o$

(The sum of the three angles of a triangle is 180^o)

$\Rightarrow \angle EBQ + \angle BQE + 90^o = 180^o$

$\Rightarrow \angle EBQ + \angle BQE = 90^o$

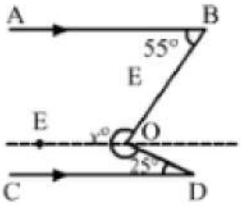
Adding (1) and (2), we get

$(\angle DBQ + \angle EBQ) + (\angle BQD + \angle BQE) = 90^o + 90^o = 180^o$

$\Rightarrow \angle DBE + \angle EQD = 180^{o}$

$\Rightarrow \angle DBE$ and $\angle EQD$ are supplementary.

OR



Draw EO || AB || CD

Then, $\angle EOB + \angle EOD = x^\circ$

Now, EO || AB and BO is the transversal.

$\therefore \angle EOB + \angle ABO = 180^\circ$ [Consecutive Interior Angles]

$\Rightarrow \angle EOB + 55^\circ = 180^\circ$

$\Rightarrow \angle EOB = 125^\circ$

Again, EO || CD and DO is the transversal.

$\therefore \angle EOD + \angle CDO = 180^\circ$ [Consecutive Interior Angles]

$\Rightarrow \angle EOD + 25^\circ = 180^\circ$

$\Rightarrow \angle EOD = 155^\circ$

Therefore,

$x^\circ = \angle EOB + \angle EOD$

$x^\circ = (125 + 155)^\circ$

$x^\circ = 280^\circ$

34. Given, $f(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$

The factors of constant term - 45 are $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45$

The factors of the coefficient of x^4 is 2. Hence possible rational roots of f(x) are

$\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm 1/2, \pm 3/2, \pm 5/2, \pm 9/2, \pm 15/2, \pm 45/2$

Let, $x - 1 = 0$

$\Rightarrow x = 1$

$f(1) = 2(1)^4 - 7(1)^3 - 13(1)^2 + 63(1) - 45$

$= 2 - 7 - 13 + 63 - 45$

$= 0$

Let, $x - 3 = 0$

$\Rightarrow x = 3$

$f(3) = 2(3)^4 - 7(3)^3 - 13(3)^2 + 63(3) - 45$

$= 162 - 189 - 117 + 189 - 45$

$= 0$

Let $x+3=0$

$\Rightarrow x=-3$

$f(-3)=2(-3)^4-7(-3)^3-13(-3)^2+63(-3)-45$

$=162+189-117-189-45=0$

Let $2x-5=0$

$\Rightarrow x=\frac{5}{2}$

$f(\frac{5}{2})=2(\frac{5}{2})^4-7(\frac{5}{2})^3-13(\frac{5}{2})^2+63(\frac{5}{2})-45$

$=\frac{625-875-650+1260-360}{2^3}$

$=\frac{1885-1885}{2^3}=0$

Therefore, (x-1),(x-3),(x+3) and (2x-5) are factors of f(x).

Since f(x) is a polynomial of degree 4, therefore it cannot have more than more factors.

Therefore, $2x^4 - 7x^3 - 13x^2 + 63x - 45 = (x - 1)(x - 3)(x + 3)(2x - 5)$

35. Let a, b, c be the sides of the given triangle and s be its semi-perimeter.

Then, $s = \frac{a+b+c}{2}$ ...(i)

$\therefore$ Area of the given triangle= $\sqrt{s(s-a)(s-b)(s-c)} = \Delta$ say

As per given condition, the sides of the new triangle will be 2a, 2b, and 2c.

So, the semi-perimeter of the new triangle =

$s' = \frac{2a+2b+2c}{2} = a + b + c$ ...(ii)

From (i) and (ii), we get

s' = 2s

Area of new triangle = $\sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$

$= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

$= \sqrt{16s(s-a)(s-b)(s-c)}$

$= 4\sqrt{s(s-a)(s-b)(s-c)} = 4\Delta$

The required ratio = $4\Delta : \Delta$ = 4:1

Therefore the ratio of area of new triangle thus formed and the given triangle is 4 : 1.

OR

Let:

a = 85 m and b = 154 m

Given that perimeter = 324 m

Perimeter= 2s = 324 m

$\Rightarrow$ s = $\frac{324}{2}$ m

or, a + b + c = 324

$\Rightarrow$ c = 324 - 85 - 154 = 85 m

By Herons's formula, we have:

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{162(162-85)(162-154)(162-85)}$

$= \sqrt{162 \times 77 \times 8 \times 77}$

$= \sqrt{1296 \times 77 \times 77}$

$= \sqrt{36 \times 77 \times 77 \times 36}$

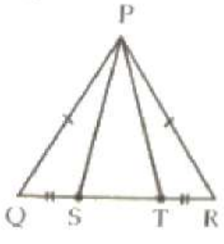
$= 36 \times 77$

= 2772 m^2

Section E

36. **Read the text carefully and answer the questions:**

A children's park is in the shape of isosceles triangle said PQR with PQ = PR, S and T are points on QR such that QT = RS.



(i) In $\triangle$PQS and $\triangle$PRT

PQ = PR (Given)

QS = TR (Given)

$\angle PQR = \angle PRQ$ (corresponding angles of an isosceles $\triangle$)
By SAS commence
$\triangle PQS \cong \triangle PRT$

(ii) $\triangle PQS \cong \triangle PRT$
$\Rightarrow$ PS = PT (CPCT)
So in $\triangle PST$
PS = PT
It is an isosceles triangle.

OR

Let $\angle Q = \angle R = x$ and $\angle P = 80^o$
In $\triangle PQR$, $\angle P + \angle Q + \angle R = 180^o$ (Angle sum property of $\triangle$)
$80^o + x + x = 180^o$
$2x = 180^o - 80$
$2x = 100^o$
$x = \frac{100^\circ}{2}$
$= 50^o$

(iii) Perimeter = sum of all 3 sides
PQ = PR = 6 cm
QR = 7 cm
So, P = (6 + 6 + 7) cm
= 19 cm

37. **Read the text carefully and answer the questions:**
Reeta was studying in the class 9th C of St. Surya Public school, Mehrauli, New Delhi-110030
Once Ranjeet and his daughter Reeta were returning after attending teachers' parent meeting at Reeta's school. As the home of Ranjeet was close to the school so they were coming by walking.
Reeta asked her father, "Daddy how old are you?"
Ranjeet said, "Sum of ages of both of us is 55 years, After 10 years my age will be double of you.

(i) $x - 2y = 10$
(ii) $x + y = 55$ ...(i) and $x - 2y = 10$ ...(ii)
Subtracting (ii) from (i)
$x + y - x + 2y = 55 - 10$
$\Rightarrow 3y = 45$

$\Rightarrow$ y = 15

So present age of Reeta is 15 years.

(iii) x + y = 55 ...(i) and x - 2y = 10 ...(ii)

Subtracting (ii) from (i)

x + y - x + 2y = 55 - 10

$\Rightarrow$ 3y = 45

$\Rightarrow$ y = 15

Put y = 15 in equation (i)

x + y = 55

$\Rightarrow$ x + 15 = 55

$\Rightarrow$ x = 55 − 15 = 40

So Ranjeet's present age is 40 years.

OR

Let Reeta;s mother age be 'z'.

Given Reeta age : Her mother age = 7 : 5

We know that Reeta age = 15 years

$\frac{Mother\ age}{Reeta\ age} = \frac{7}{5}$

$\Rightarrow$ z = $\frac{7}{3} \times y$

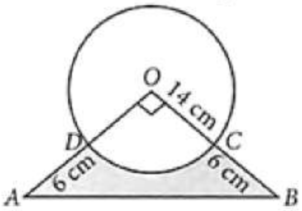
$\Rightarrow$ z = $\frac{7}{3} \times 15$

$\Rightarrow$ Here Mother age = 35 years

Hence Reeta's mother's age is 35 years.

38. **Read the text carefully and answer the questions:**

Director of a company selected a round glass trophy for awarding their employees an annual function. The design of each trophy is made as shown in the figure, where its base ABCD is golden plated from the front side at the rate of ₹6 per cm^2.



(i) Area of sector ODCO = $\frac{1}{4}\pi r^2$

= $\frac{1}{4} \times \frac{22}{7} \times 14 \times\ 14$ = 154 cm^2

(ii) Area of $\triangle$AOB = $\frac{1}{2} \times OA \times OB = \frac{1}{2}(20 \times 20)$

= 200 cm^2

OR

Area of major sector = area of circle - area of minor sector

= $\pi r^2 - \frac{1}{4}\pi r^2 = \frac{3\pi r^2}{4} = \frac{3}{4} \times \frac{22}{7} \times 14 \times 14$ = 462 cm^2

(iii) Area of region which is golden plated = area of $\triangle$OAB - area of sector ODCO.

= 200 - 154 = 46 cm^2

$\therefore$ Total cost of golden plating = ₹(6 × 46) = ₹276