Ratio and Proportion

13.1 Daljeet and Kanha are two cattle keepers. They own 50 cows and 200 cows respectively. If we compare the number of cows owned by them, we find Kanha has 200 - 50 = 150 cows more than Daljeet. This is one way of comparison by the taking difference. Similarly, in our daily life, many a times we compare two quantities of the same type.

We know that weight of an adult elephant is generally in between 2000kgs to 5000kgs. Where as weight of an adult human is generally in between 50kgs to 100kgs.

In the given picture, weights of the elephant and the man are 2550kgs and 75kgs respectively. Now, let's compare the weights of the elephant and the man by the taking difference.

2550kgs – 75kgs = 2475kgs.

Here, we observe that there is much difference between their weights. In such situations, taking the difference does not express the comparison. The elephant's weight is too much as compared to the human weight. So, let's compare these weights by division.

$$\frac{\text{Elephant's weight}}{\text{Human's weight}} = \frac{2550}{75} = \frac{34}{1}$$

We can say that the elephant's weight is 34 times the human's weight.

Thus, in certain situations, comparison by division makes better sense than comparison by taking the difference. The comparison by division is the Ratio. In the next section, we shall learn more about 'Ratios'.

13.2 The Concept of Ratio

Consider the following:

Hakim Khan's weight is 25 kg and his father's weight is 75 kg. How many times Father's weight is of Hakim Khan's weight?

$$\frac{75 \text{ Kg}}{25 \text{ Kg}} = \frac{3}{1}$$ It is threee times. In the above examples, we compared the two quantities in terms of 'how many times'. This comparison is known as the ratio. We denote, ratio by using the symbol ':'

Do and Learn ◆

1. Cost of a pen is Rs 10 and cost of a pencil is Rs 2. What will be the ratio of a pen's cost to a pencil's cost?

2. Ravi walks 6 km in an hour while Suraj walks 4 km in an hour. What is the ratio of the distance covered by Ravi to the distance covered by Suraj?

Meena and Shehnaaz have Rs. 15/- and Rs. 30/- respectively.

1. How many times the money owned by Meena is the money owned by Shehnaaz?

$$\frac{Money\ owned\ by\ Meena}{Money\ owned\ by\ Shehnaaz} = \frac{15}{30} = \frac{1}{2} = Half$$

Or 1 : 2

Hence, Meena owns half the money as Shehnaaz.

2. How many times the money owned by Shehnaaz is the money owned by Meena?

$$\frac{Money\ owned\ by\ Shehnaaz}{Money\ owned\ by\ Meena} = \frac{30}{15} = \frac{2}{1} = Twice$$

Or 2 : 1

Hence, Shehnaaz owns twice the money as Meena.

Observe that the two ratios 1 : 2 and 2 : 1 are different from each other. Thus, the order in which quantities are taken to express their ratio is important. Read the table carefully and fill in the blanks:

1st quantity	2nd quantity	How many times 2nd quantity is to 1st quantity?	Ratio	How many times 1st quantity is to 2nd quantity?	Ratio
2 apples	6 apples	Three times	3 : 1	One third	1 : 3
500 gms Jaggery	1000gms Jaggery	....	....	....	....
T Shirt Rs.200/-	Jacket Rs.1000/-	....	....	....	....

Hamid has 5 goats and Kaalu has 8 bicycles. Can we express these quantities in ratio? Think about it.

Goats and bicycles are not of same category. Therefore we can't establish any relation between them. Goats and bicycles can't be compared to each other as they belong to different categories.

For establishing a relation or comparison by ratio, the two quantities must be of the same category and in the same unit. If they are not, they should be expressed in the same unit before the ratio is taken.

Example 1 Saurabh takes 15 minutes to reach school from his house and Sachin takes one hour to reach school from his house. Find the ratio of the time taken by Saurabh to the time taken by Sachin.

Solution Since the time taken by Saurabh and Sachin are given in different units, we first need to convert them into same unit.

Time taken by Saurabh = 15 minutes

Time taken by Sachin = 1 hour = 60 minutes

Hence, required ration is

$$15 \text{ minutes} : 60 \text{ minutes} = \frac{15}{60} = \frac{3}{12} = \frac{1}{4}$$

So, the ratio of time taken by Saurabh and Sachin is 1 : 4.

Example 2 Out of 45 students in 6th class, 25 are girls and remaining are boys.

(a) What is the ratio of number of girls to the number of boys?
(b) What is the ratio of number of boys to the number of girls?

Solution

Number of girls = 25
Number of boys = 45 - 25 = 20

(a) Ratio of number of girls to the number of boys = 25 : 20 = 5 : 4
(b) Ratio of number of boys to the number of girls = 20 : 25 = 4 : 5

Observe that the two ratios 5 : 4 and 4 : 5 are different from each other.

Do and Learn ◆

The numbers of vehicles passing on the road between 9am to 10am are recorded in the table below.

Vehicle Type	Number of vehicles
Two wheeler	24
Car	60
Truck	40

1. What is the ratio of number of two wheelers to the number of cars?

2. What is the ratio of number of total vehicles to the number of trucks?

3. Weight of a car is 3300 pounds and weight of abicycle is 22 pounds. What is the ratio of the weight of bicycle to the weight of car?

13.2.1 Ratio in different situations

Rahul and Khushi started a business and invested money in the ratio 4 : 5. After one year the total profit was Rs 54,000.

Rahul said "we will divide it equally", Khushi said "I should get more as I have invested more". It was then decided that profit will be divided in the ratio of their investment.

Here, the two terms of the ratio 4 : 5 are 4 and 5. Sum of these terms = 4 + 5 = 9

What does this mean?

This means if the profit is Rs 9 then Rahul should get Rs 4 and Khushi should get Rs 5.

Or, we can say that Rahul gets 4 parts and Khushi gets 5 parts out of the 9 parts, i.e., Rahul should get 4/9 of the total profit and Khushi should get 5/9 of the total profit.

If the total profit were Rs 54,000, could you find the share of each?

Here, total profit is Rs 54,000 which is divided into 9 parts = $\dfrac{54000}{9}$

$$\text{Rahul's share} = \dfrac{54000}{9} \times 4 = \text{Rs } 24000.$$

$$\text{Khushi's share} = \dfrac{54000}{9} \times 5 = \text{Rs } 30000.$$

$$\text{Observe that } 4:5 \text{ or } \dfrac{4}{5} \text{ and}$$

$$24000 : 30000 \text{ or } \dfrac{24000}{30000} = \dfrac{4}{5} \quad \text{Why?}$$

Example 3 Divide Rs 45 in the ratio 1 : 2 between Riya and Kanchan.

Solution The two parts are 1 and 2.

Therefore, sum of the parts = 1 + 2 = 3.

This means if there are Rs 3, Riya will get Re 1 and Kanchan will get Rs 2. Or, we can say that Riya gets 1 part and Kanchan gets 2 parts out of every 3 parts.

Therefore, Riya's share = $\dfrac{1}{3} \times 45 = \text{Rs } 15.$

And Kanchan's share = $\dfrac{2}{3} \times 45 = \text{Rs } 30.$

Do and Learn ◆

(1) Find the ratio of number of doors and the number of windows in your classroom.
(2) Draw any rectangle and find the ratio of its length to its breadth.

13.2.2 Equivalent ratios:

Consider the following :

1. Length of a room is 30 m and its breadth is 20 m. So, the ratio of length of the room to the breadth of the room $= \dfrac{30}{20} = \dfrac{3}{2} = 3 : 2$

2. Length of a board in Gaurav's school is 360 cm and its breadth is 240 cm. So, the ratio of length of the board to the breadth of the board $= \dfrac{360}{240} = \dfrac{3}{2} = 3 : 2$

The ratio in both the examples is 3 : 2.
Note the ratios 30 : 20 and 360 : 240 in lowest form are same as 3 : 2.
These are **equivalent ratios** You have read about like fractions in the chapter of of fractions.
Can you think of some more examples having the ratio 3 : 2?

Let's understand equivalent ratios with one more example.

Example 4   Give two equivalent ratios of 3 : 2.

Solution:

Ratio $3 : 2 = \dfrac{3}{2} = \dfrac{3 \times 2}{2 \times 2} = \dfrac{6}{4}$.

Therefore, 6 : 4 is an equivalent ratio of 3 : 2

Similarly, the ratio $9 : 6 = \dfrac{9 \div 3}{6 \div 3} = \dfrac{3}{2}$

So, 9 : 6 is another equivalent ratio of 3 : 2.

Therefore, we can get equivalent ratios by multiplying or dividing the numerator and denominator by the same number.

Example 5   Fill in the missing numbers:

$$\dfrac{12}{18} = \dfrac{\square}{3} = \dfrac{6}{\square}$$

Solution   In order to get the first missing number, we consider the fact that $18 = 3 \times 6$. i.e. when we divide 18 by 6 we get 3. This indicates that to get the missing number of second ratio, 12 must also be divided by 6.

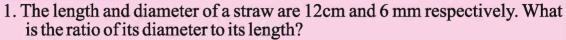When we divide, we have, $12 \div 6 = 2$

Hence, the second ratio is $\dfrac{2}{3}$.

Similarly, to get third ratio we multiply both terms of second ratio by 3. (Why?)

Hence, the third ratio is $\dfrac{6}{9}$.

Therefore, $\dfrac{12}{18} = \dfrac{2}{3} = \dfrac{6}{9}$ [These are all equivalent ratios.]

Do and Learn. ◆

1. The length and diameter of a straw are 12cm and 6 mm respectively. What is the ratio of its diameter to its length?
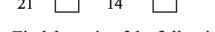
2. Anand takes 25 minutes to reach school from his house and Anshul takes one hour to reach school from his house. Find the ratio of the time taken by Anand to the time taken by Anshul.

>──○○○◄ Exercise 13.1 ►○○○──<

1. A social awareness camp was organized in the summer vacations this year. 25 girls and 15 boys participated in the camp and put water bowl for birds.

(a) What is the ratio of number of girls to the number of boys?

(b) What is the ratio of number of girls to the number of participants?

2. During a Tree Plantation programme in a school, students of 6th class planted 8 Neem trees, 13 mango trees and 19 Guava trees.

(a) What is the ratio of number of Neem trees planted to the number of Mango trees planted?

(b) What is the ratio of total number of trees planted to the number of Neem trees planted?

3. See the figure and find the ratio of .

(a) Number of triangles to the number of circles.

(b) Number of squares to all the figures.

(c) Number of triangles to all the figures.

4. Fill in the following blanks:

$$\frac{15}{21} = \frac{5}{\square} = \frac{\square}{14} = \frac{25}{\square}$$

5. Find the ratio of the following :

(i) 25 to 150   (ii) 72 to 36
(iii) 55kms to 121kms   (iv) 35 minutes to 55 minutes

6. Find the ratio of the following :

(i) 60 paisa and 3 Rupees   (ii) 800gms and 5kgs
(iii) 15 minutes and 1 hour   (iv) 1 liter and 750ml

7. During a year, a cowshed had received donations worth Rs.3,25,000 out of which Rs.3,00,000 were spent on the welfare of the cows. Find the ratio of donations received to the expenditure incurred.

8. Mahesh studies 4 hours daily and Laxmi studies 180 minutes daily. Find the ratio of study time of Mahesh to study time of Laxmi. (1 hour = 60 minutes)

9. Out of 720 students in a school, 360 students stay at a hostel. Find the ratio of number of students staying at the hostel to the total number of students.

10. Talisma and Gurumit started a business and invested money in the ratio 2 : 5. After one year the total profit was Rs 35,000. Find the shares of profit for Talisma and Gurumit.

11. Consider the statement: Ratio of breadth and length of a hall is 3 : 4. Complete the following table that shows some possible breadths and lengths of the hall.

Breadth of the hall (in meters)	6	......	24	......
Length of the hall (in meters)	8	16	....	....

12. Present age of father is 45 years and that of his son is 15 years. Find the ratio of
    (a) Present age of father to the present age of son.
    (b) Age of the father to the age of son, when son was 10 years old.
    (c) Age of father after 5 years to the age of son after 5 years.
    (d) Age of father to the age of son when father will be 60 years old.

13.3 Proportion

Two friends Dinky and Priti went to market to purchase hair clips. They purchased 20 hair clips for Rs 30. Dinky gave Rs 18 and Priti gave Rs 12. After they came back home, Priti asked Dinky to give 10 hair clips to her. But Dinky said, "since I have given more money so I should get more clips. You should get 8 hair clips and I should get 12".

Can you tell who is correct, Dinky or Priti? Why?

Ratio of money given by Dinky to the money given by Priti = Rs 18 : Rs 12 = 3 : 2

According to Priti's suggestion, the ratio of the number of hair clips for Dinky to the number of hair clips for Priti = 10 : 10 = 1 : 1

According to Dinky's suggestion, the ratio of the number of hair clips for Dinky to the number of hair clips for Priti = 12 : 8 = 3 : 2

Now, notice that according to Priti's distribution, ratio of hair clips and the ratio of money given by them is not the same.

But according to the Dinky's distribution the two ratios are the same. Hence, we can say that Dinky's distribution is correct.

If two ratios are equal, we say that they are in proportion and use the symbol '::' or '=' to equate the two ratios.

18 : 12 :: 12 : 8 or 18 : 12 = 12 : 8

Meaning of Ratios in Proportion:

Consider the following examples:

1. Divakar purchased 3 pens for Rs 15 and Anu purchased 10 pens for Rs 50. Whose pens are more expensive?

   Ratio of number of pens purchased by Divakar to the number of pens purchased by Anu = 3 : 10. Ratio of their costs = 15 : 50 = 3 : 10

   Both the ratios 3 : 10 and 15 : 50 are equal, i.e., 15 : 50 = 3 : 10

   Therefore, the pens were purchased for the same price by both.

2. A man travels 35 km in 2 hours. Another man travels 70 km in 4 hours. Is the speed of first man is in proportion with the speed of second man?

   The ratio of distances travelled by both = 35 : 70 = 1 : 2

   The ratio of time taken by both = 2 : 4 = 1 : 2

   Both ratios are equal.

   i.e., 35 : 70 :: 2 : 4

   Or 35 : 70 = 2 : 4

   It is read as 35 is to 70 as 2 is to 4. And, we can say that the four numbers 35, 70, 2 and 4 are in proportion.

   Let us consider another example.

3. Cost of 3 kg of apples is Rs 360 and 15 kg of watermelons cost Rs 90. Now, ratio of the weight of apples to the weight of watermelon is 3 : 15 = 1: 5

   And ratio of the cost of apples to the cost of the watermelon is 360 : 90 = 4 : 1.

   Here, the two ratios 3 : 15 and 360 : 90 are not equal, i.e. 3 : 15 ≠ 360 : 90

   Therefore, the four quantities 3, 15, 360 and 90 are not in proportion.

Do and Learn. ◆

Determine if the following are in proportion.

(i)   3 : 5 and 1 : 15   (ii) 4 : 12 and 9 : 27

(iii) Rs 10 is to Rs 15 and 4 is to 6

Example 6 Are the ratios 30 cm : 36 cm and 10 m : 12 m in proportion?

Solution

30 cm : 36 cm = 5 : 6

10 m : 12 m = 5 : 6

So, 30 : 36 = 10 : 12

Hence, the ratios 30 cm : 36 cm and 10 m : 12 m in proportion.

i.e., 30 : 36 :: 10 : 12

In a statement of proportion, the four quantities involved when taken in order are known as respective terms. First and fourth terms are known as **extreme terms**. Second and third terms are known as **middle terms**. In the above example, 30 and 12 are the extreme terms. 36 and 10 are the middle terms.

Extreme Terms

30:36::10:12

Middle Terms

$30 \times 12 = 36 \times 10$ i.e.,

$360 = 360.$

Therefore, the four quantities 30, 36, 10 and 12 are in proportion.

> **If any four quantities are in proportion, then product of extreme terms = product of middle terms**

Example 7 Do the ratios 15 cm to 3 m and 10 sec to 2 minutes form a proportion?

Solution

Ratio of 15 cm to 3 m

$= 15 : 3 \times 100$ (1 m = 100 cm)

$= 1 : 20$

Ratio of 10 sec to 2 min

$= 10 : 2 \times 60$ (1 min = 60 sec)

$= 1 : 12$

Since, $1 : 20 \neq 1 : 12$, therefore, the given ratios do not form a proportion.

Exercise 13.2

1. Determine if the following are in proportion.

   (i) 8, 6, 48, 36   (ii) 12, 18, 20, 30

   (iii) 14, 20, 26, 32   (iv) 26, 65, 32, 60

2. Write True (T) or False (F) against each of the following statements :

   (i) 20 : 60 :: 15 : 45   (ii) 20 : 22 :: 32 : 42

   (iii) 0.9 : 0.36 :: 10 : 4   (iv) 5.2 : 26 :: 1.8 : 0.9

3. Are the following statements true?

   (i) 30 sec : 1 min :: 16 m : 32 m

   (ii) 2.5 litres : 5 litres :: 25 m : 50 m

   (iii) Rs 44 : Rs 20 :: 66 litres : 30 litres

   (iv) 12 m : 15 m :: 48 kg : 64 kg

4. Determine if the following ratios form a proportion. Also, write the middle terms and extreme terms where the ratios form a proportion.
   (i) 200 cm : 2.5 m and Rs 40 : Rs 5
   (ii) 1 kg : 250 g and Rs 40 : Rs 10
   (iii) 4 kg : 16 kg and 20 g : 400 g
   (iv) 39 persons : 65 persons and 9 litres : 15 litres

13.4 Unitary Method

Two friends Neelam and Punam went to market to purchase vegetables. Neelam purchased 2 kg potatoes for Rs 30. What is the price of 1Kg potatoes? Consider one more situation. Mohan travels 40 km in 4 hours. With the same speed how much distance would he be able to travel 5 hours?

These are examples of the kind of situations that we face in our daily life. How would you solve these?

Reconsider the first example:

Cost of 2 kg potatoes = Rs 30

Therefore, Cost of 1 kg potatoes = $\frac{30}{2}$ = Rs 15

Now, if you were asked to find cost of 5 kg potatoes.
It would be = Rs 15 × 5 = Rs 75
Reconsider the second example:
Now, we want to know the distance travelled by Mohan in 1 hour.
Since, the distance travelled in 4 hours = 40 km

Therefore, the distance travelled in 1 hour = $\frac{40}{4}$ km = 10 km

Therefore, the distance travelled in 5 hours = 10 × 5 km = 50 km

The method in which first we find the value of one unit and then the value of required number of units is known as Unitary Method.

Do and Learn: ◆
   Read the table and fill in the boxes.

Number of Books	Price paid by Reshma	Price paid by Seema
2 Books	Rs 50	Rs 70
1 Book	Rs 25	………..
5 books	………..	………..

Example 8   If the cost of a garland is Rs 7, what will be the cost of 8 such garlands?

Solution   Since the cost of 1 garland = Rs. 7

Therefore, cost of 8 garlands = Rs. 7 × 8 = Rs 56

Example 9   A car travels 60 km in 3 litres of petrol. How much distance will it cover in 1 litre of petrol?

Solution   Since the distance travelled in 3 litres of petrol = 60 km

Therefore, the distance travelled in 1 litre of petrol $= \dfrac{60}{3}$ km = 20 km

Example 10   If the cost of a dozensoaps is Rs 174, what will be the cost of 8 such soaps?

Solution   We know that 1 dozen = 12

Since, cost of 12 soaps = Rs 174

Therefore, cost of 1 soap = Rs $\dfrac{174}{12}$ = Rs 14.50

Therefore, cost of 8 soaps = Rs 14.50 × 8 = Rs 116

Thus, cost of 8 such soaps is Rs 116.

Example 11   Cost of 10 scout badges is Rs 50. What will be the cost of 32 such scout badges?

Solution   Since, Cost of 10 scout badges = Rs 50

Therefore, cost of 1 scout badge = Rs $\dfrac{50}{10}$ = Rs 5

Therefore, cost of 32 scout badges = Rs 5 x 32 = Rs 160

Thus, cost of 32 scout badges is Rs 160.

Example 12   If the cost of 2.5 litres of milk is Rs 100.

     (i)   What quantity of milk can be purchased in Rs 10?

     (ii) What will be the cost of 2 litres of milk?

Solution   (i) In this case, quantity of milk is unknown and cost is known. Therefore, we proceed as follows:

Since, quantity of milk which can be purchased in Rs 100 = 2500 ml

Therefore, quantity of milk which can be purchased in Rs 1 $= \dfrac{2500}{100}$ ml

Therefore, quantity of milk which can be purchased in Rs 10 $= \dfrac{2500}{100}$ ml × Rs 10

= 250 ml

(ii) In this case, cost of milk is unknown and quantity is known. Therefore, we proceed as follows:

Since, cost of 2.5 litres of milk = Rs 100

Therefore, cost of 1 litre of milk = Rs $\dfrac{100}{2.5}$ = Rs 40

Therefore, cost of 2 litres of milk = Rs 40 × 2 = Rs 80.

If the cost of one object is given, we can find out the cost of many objects by multiplying the cost of one object with the number of objects. If the cost of several objects are given, we can find out the cost of one object by dividing the cost of several objects by the number of objects.

Exercise 13.3

1. If the cost of 1 quintal of sugar is Rs 2700, find the cost of 1 kg of sugar.

2. The bus fare for 200 km is Rs. 150. What will be the fare for 500 km?

3. If the interest on Rs 700 is Rs 168 for a certain amount of time. What will be the interest on Rs 1500 for the same time and same rate of interest?

4. In a day, 6 girls were able to fence 18 plants. Then 15 girls can fence how many plants in a day?

5. 5 persons can save 15 litres of water if they take bath using a bucket instead of shower. Then 25 persons can save how many litres of water?

6. A motorbike travels 120 km in 2 litres of petrol. How many litres of petrol will it require to travel 300 km?

7. A train travels 130 km in 2 hours. How much time is required to cover 520 km with the same speed?

8. Cost of 4 chairs is Rs 900. How many such chairs can be purchased for Rs 33750?

9. Geeta pays Rs 10500 as rent for 3 months. How much does she has to pay for a whole year, (if the rent per month remains same?)

10. Rahim made 48 runs in 8 overs and Kabir made 54 runs in 6 overs. Who made more runs per over?

11. Cost of 3 kg of millet is Rs 49.50.

    (i) What will be the cost of 7 kg of millet?

    (ii) What quantity of millet can be purchased in Rs 165?

$$\bowtie\!\!\boxed{\textbf{We Learnt}}\!\!\bowtie$$

1. For comparing quantities of the same type, we commonly use the method of taking difference between the quantities.

2. In many situations, a more meaningful comparison between quantities is made by using division, i.e. by seeing how many times one quantity is to the other quantity. This method is known as comparison by ratio.

3. For comparison by ratio, the two quantities must be in the same unit. If they are not, they should be expressed in the same unit before the ratio is taken.

4. The same ratio may occur in different situations

5. Note that the ratio 3 : 5 is different from 5 : 3. Thus, the order in which quantities are taken to express their ratio is important.

6. A ratio may be treated as a fraction, thus the ratio 1 : 2 may be treated as $\frac{1}{2}$

7. Two ratios are equivalent, if the fractions corresponding to them are equivalent.

8. A ratio can be expressed in its lowest form.

9. Four quantities are said to be in proportion, if the ratio of the first and the second quantities is equal to the ratio of the third and the fourth quantities.

10. The order of terms in the proportion is important. 3, 10, 12 and 40 are in proportion, but 3, 10, 40 and 12 are not.

11. The method in which we first find the value of one unit and then the value of the required number of units is known as the unitary method.