8. INDEFINITE INTEGRATION

1. If f & g are functions of x such that $g'(x) = f(x)$ then,

$\int f(x)\,dx = g(x) + c \Leftrightarrow \frac{d}{dx}\{g(x)+c\} = f(x)$, where c is called the **constant of integration**.

2. Standard Formula:

(i) $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,\ n \neq -1$

(ii) $\int \frac{dx}{ax+b} = \frac{1}{a}\,\ell n\,(ax+b) + c$

(iii) $\int e^{ax+b}\,dx = \frac{1}{a}\,e^{ax+b} + c$

(iv) $\int a^{px+q}\,dx = \frac{1}{p}\frac{a^{px+q}}{\ell n a} + c;\ a > 0$

(v) $\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + c$

(vi) $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c$

(vii) $\int \tan(ax+b)\,dx = \frac{1}{a}\,\ell n \sec(ax+b) + c$

(viii) $\int \cot(ax+b)\,dx = \frac{1}{a}\,\ell n \sin(ax+b) + c$

(ix) $\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + c$

(x) $\int \text{cosec}^2(ax+b)\,dx = -\frac{1}{a}\cot(ax+b) + c$

(xiii) $\int \sec x\,dx = \ell n(\sec x + \tan x) + c$ **OR** $\ell n \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + c$

(xiv) $\int \text{cosec}\,x\,dx = \ell n(\text{cosec}\,x - \cot x) + c$ **OR** $\ell n \tan\frac{x}{2} + c$ **OR** $-\ell n(\text{cosec}\,x + \cot x) + c$

(xv) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$

(xvi) $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

(xvii) $\int \frac{dx}{|x|\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + c$

(xviii) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ell n\left[x + \sqrt{x^2 + a^2}\right] + c$

(xix) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ell n\left[x + \sqrt{x^2 - a^2}\right] + c$

(xx) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\,\ell n\left|\frac{a+x}{a-x}\right| + c$

(xxi) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\,\ell n\left|\frac{x-a}{x+a}\right| + c$

(xxii) $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

(xxiii) $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\,\ell n\left(\frac{x + \sqrt{x^2 + a^2}}{a}\right) + c$

(xxiv) $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\,\ell n\left(\frac{x + \sqrt{x^2 - a^2}}{a}\right) + c$

3. Integration by Subsitutions

If we subsitute $f(x) = t$, then $f'(x)\,dx = dt$

4. Integration by Part :

$$\int (f(x)\,g(x))\,dx = f(x)\int (g(x))\,dx - \int\left(\frac{d}{dx}(f(x))\int (g(x))\,dx\right)dx$$

5. Integration of type $\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c}\,dx$

Make the substitution $x + \frac{b}{2a} = t$

6. Integration of type

$$\int \frac{px+q}{ax^2+bx+c}\,dx,\quad \int \frac{px+q}{\sqrt{ax^2+bx+c}}\,dx,\quad \int (px+q)\sqrt{ax^2+bx+c}\,dx$$

Make the substitution $x + \frac{b}{2a} = t$, then split the integral as some of two integrals one containing the linear term and the other containing constant term.

7. Integration of trigonometric functions

(i) $\int \frac{dx}{a + b\sin^2 x}$ **OR** $\int \frac{dx}{a + b\cos^2 x}$ **OR** $\int \frac{dx}{a\sin^2 x + b\sin x\cos x + c\cos^2 x}$ **put tan x = t.**

(ii) $\int \frac{dx}{a + b\sin x}$ **OR** $\int \frac{dx}{a + b\cos x}$ **OR** $\int \frac{dx}{a + b\sin x + c\cos x}$ **put** $\tan\frac{x}{2} = t$

(iii) $\int \frac{a.\cos x + b.\sin x + c}{\ell.\cos x + m.\sin x + n}$ **dx. Express** $Nr \equiv A(Dr) + B\frac{d}{dx}(Dr) + c$ **& proceed.**

8. Integration of type

$\int \frac{x^2 \pm 1}{x^4 + Kx^2 + 1}$ **dx where K is any constant.**

Divide Nr & Dr by x^2 & put $x \mp \frac{1}{x} = t$.

9. Integration of type

$\int \frac{dx}{(ax+b)\sqrt{px+q}}$ **OR** $\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$ **; put** $px + q = t^2$.

10. Integration of type

$\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$ **, put** $ax + b = \frac{1}{t}$ **;** $\int \frac{dx}{(ax^2+b)\sqrt{px^2+q}}$ **, put** $x = \frac{1}{t}$