3 Matrices

Fastrack Revision

- A rectangular array of mn numbers arranged in m rows and n columns is called a matrix of order $m \times n$. The numbers are called the elements or entries of the matrix.
- The element of ith row and jth column is represented by a_{ij}.
- Two matrices are said to be equal, if the order of two matrices is equal and corresponding elements are also equal.
- A matrix having the same number of rows and columns is called a square matrix.
- A square matrix in which every non-diagonal element is zero, is called a diagonal matrix.
- A square matrix in which every non-diagonal element is 0 and every diagonal element is 1, is called an identity or unit matrix.
- A square matrix $A=[a_{ij}]$ is said to be upper triangular matrix, if $a_{ij}=0 \;\forall\; i>j$.
- A square matrix $A=[a_{ij}]$ is said to be lower triangular matrix, if $a_{ij}=0 \;\forall\; i<j$.
- If A and B are two matrices of same order, then their sum $(A+B)$ is a matrix of same order which is obtained by adding the corresponding elements of A and B.
- If A and B are two matrices of same order, then their difference $(A-B)$ is equal to $A+(-B)$, *i.e.,* sum of matrix A and the matrix $(-B)$.
- The product AB of two matrices A and B can be determined only when number of columns in matrix A = number of rows in column B.
- If A and B are square matrices of the same order, say 'n', then both the product AB and BA are defined and each is a square matrix of order 'n'.
- In the matrix product AB, the matrix A is called pre-multiplier (pre-factor) and B is called post-multiplier (post-factor).
- **Properties of Matrix Multiplication**
  (i) $AB \neq BA$
  (ii) $A(BC)=(AB)C$
  (iii) $A(B+C)=AB+AC$
  (iv) $(A+B)C=AC+BC$
  (v) $AI=A=IA$
- The matrix obtained by interchanging rows into columns and columns into rows of matrix A is called the transpose matrix of matrix A, denoted by A^T or A'.
- If the order of matrix A is $m \times n$, then
  order of $A^T = n \times m$
- **Some Results on Transpose of Matrices**
  (i) $(A^T)^T = A$
  (ii) $(A+B)^T = A^T + B^T$
  (iii) $(kA)^T = kA^T$ (where k is a constant)
  (iv) $(AB)^T = B^T A^T$.
- A square matrix of order $n \times n$ is said to be orthogonal, if $AA' = I_n = A'A$.
- A square matrix A is said to be symmetric matrix, if $A^T = A$.
- If A and B are symmetric matrices of the same order, then the product AB is symmetric, iff $BA = AB$.
- A square matrix A is said to be skew-symmetric matrix, if $A^T = -A$.
- All elements of main diagonal of a skew-symmetric matrix are zero.
- Every square matrix can be expressed uniquely as a sum of a symmetric and a skew-symmetric matrices.
  If A is a square matrix, then $A=P+Q$; where, $P=\frac{1}{2}(A+A^T)$ is a symmetric matrix and $Q=\frac{1}{2}(A-A^T)$ is a skew-symmetric matrix.
- A square matrix A of order m is called invertible, if there exists a square matrix B of order m such that
  $$AB = BA = I_m$$
  Also, B is called the inverse matrix of A and is denoted by A^{-1}.
- The inverse of every invertible matrix is unique.
- If A and B are two invertible square matrices of same order then
  $$(AB)^{-1} = B^{-1}A^{-1}$$

Practice Exercise

Multiple Choice Questions

Q 1. The number of all possible matrices of order 2×3 with each entry 1 or 2 is: *(CBSE 2021 Term-1)*

a. 16 b. 6
c. 64 d. 24

Q 2. If $A=[a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, & \text{when } i \neq j \\ 0, & \text{when } i = j \end{cases}$, then A^2 is:

(CBSE SQP 2023-24, 21 Term-1, CBSE 2023)

a. $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q 3. A matrix $A=[a_{ij}]_{3\times3}$ is defined by:

$$a_{ij}=\begin{cases}2i+3j, & i<j\\ 5, & i=j\\ 3i-2j, & i>j\end{cases}$$

The number of elements in A which are more than 5, is: *(CBSE 2021 Term-1)*

a. 3 b. 4
c. 5 d. 6

Q 4. If $x\begin{bmatrix}1\\2\end{bmatrix}+y\begin{bmatrix}2\\5\end{bmatrix}=\begin{bmatrix}4\\9\end{bmatrix}$, then: *(CBSE 2023)*

a. $x=1, y=2$ b. $x=2, y=1$
c. $x=1, y=-1$ d. $x=3, y=2$

Q 5. If $\begin{bmatrix}2a+b & a-2b\\ 5c-d & 4c+3d\end{bmatrix}=\begin{bmatrix}4 & -3\\ 11 & 24\end{bmatrix}$, then the value of $a+b-c+2d$ is: *(CBSE SQP 2021 Term-1)*

a. 8 b. 10 c. 4 d. −8

Q 6. If $A=\begin{bmatrix}0 & 2\\ 3 & -4\end{bmatrix}$ and $kA=\begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}$, then the values of k, a and b respectively are: *(CBSE SQP 2021 Term-1)*

a. −6, −12, −18 b. −6, −4, −9
c. −6, 4, 9 d. −6, 12, 18

Q 7. The product $\begin{bmatrix}a & b\\ -b & a\end{bmatrix}\begin{bmatrix}a & -b\\ b & a\end{bmatrix}$ is equal to: *(CBSE 2023)*

a. $\begin{bmatrix}a^2+b^2 & 0\\ 0 & a^2+b^2\end{bmatrix}$ b. $\begin{bmatrix}(a+b)^2 & 0\\ (a+b)^2 & 0\end{bmatrix}$
c. $\begin{bmatrix}a^2+b^2 & 0\\ a^2+b^2 & 0\end{bmatrix}$ d. $\begin{bmatrix}a & 0\\ 0 & b\end{bmatrix}$

Q 8. If $A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$, $B=\begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}$ and $A=B^2$, then x equals: *(CBSE 2023)*

a. ±1 b. −1
c. 1 d. 2

Q 9. If $A=\begin{bmatrix}4 & 1 & 0\\ 1 & -2 & 2\end{bmatrix}$, $B=\begin{bmatrix}2 & 0 & -1\\ 3 & 1 & x\end{bmatrix}$, $C=\begin{bmatrix}1\\2\\1\end{bmatrix}$ and $D=\begin{bmatrix}15+x\\ 1\end{bmatrix}$ such that $(2A-3B)\,C=D$, then $x=$

a. 3 b. −4 c. −6 d. 6

Q 10. If $\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\3\\2\end{bmatrix}$, then the value of $(2x+y-z)$ is: *(CBSE 2023)*

a. 1 b. 2 c. 3 d. 5

Q 11. If $\begin{bmatrix}4\\2\\3\end{bmatrix}[1\ \ 3\ \ -3]\begin{bmatrix}x\\y\\z\end{bmatrix}=O$, then $x+3y-3z$ is:

a. 1 b. 3 c. 4 d. 0

Q 12. If $A=\begin{bmatrix}1 & -3\\ 2 & k\end{bmatrix}$ and $A^2-4A+10I=A$, then k is equal to:

a. 0 b. −4
c. 4 and not 1 d. 1 or 4

Q 13. If for a square matrix A, $A^2-A+I=O$, then A^{-1} equals: *(CBSE 2023)*

a. A b. $A+I$
c. $I-A$ d. $A-I$

Q 14. If the matrix $\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is commutative with the matrix $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, then:

a. $a=0, b=c$ b. $b=0, c=d$
c. $c=0, d=a$ d. $d=0, a=b$

Q 15. If a matrix $A=[1\ \ 2\ \ 3]$, then the matrix AA' (where A' is the transpose of A) is: *(CBSE 2023)*

a. 14 b. $\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$
c. $\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 3 & 1 & 2\end{bmatrix}$ d. [14]

Q 16. If $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$ and $A^2=3I$, then: *(CBSE SQP 2021 Term-1)*

a. $1+\alpha^2+\beta\gamma=9$ b. $1-\alpha^2-\beta\gamma=0$
c. $3-\alpha^2-\beta\gamma=0$ d. $3+\alpha^2+\beta\gamma=0$

Q 17. If matrices A and B are of order $3\times n$ and $m\times5$ respectively, then the order of matrix $C=5A+3B$ is: *(CBSE SQP 2021 Term-1)*

a. 3×5 and $m=n$ b. 3×5
c. 3×3 d. 5×5

Q 18. Matrix A has m rows and $(n+5)$ columns, matrix B has m rows and $(11-n)$ columns. If both AB and BA exists, then:

a. AB and BA are square matrices
b. AB and BA are of orders 8×8 and 3×13 respectively
c. $AB=BA$
d. None of the above

Q 19. If A is 3×4 matrix and B is a matrix such that $A'B$ and BA' are both defined, then the order of matrix B is: *(CBSE 2023)*

a. 3×4 b. 3×3
c. 4×4 d. 4×3

Q 20. For the matrix $X=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$, (X^2-X) is: *(CBSE 2021 Term-1)*

a. $2I$ b. $3I$ c. I d. $5I$

Q 21. If $U=[2\ \ -3\ \ 4]$, $V=\begin{bmatrix}3\\2\\1\end{bmatrix}$, $X=[0\ \ 2\ \ 3]$ and $Y=\begin{bmatrix}2\\2\\4\end{bmatrix}$, then the value of $UV+XY$ is:

a. 20 b. [−20] c. −20 d. [20]

Q 22. If $A=\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$, then A^{100} is equal to:

a. $2^{100}A$ b. $2^{99}A$
c. $100A$ d. $299A$

Q 23. If $A+B=\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ and $A-2B=\begin{bmatrix}-1 & 1\\0 & -1\end{bmatrix}$, then A is: *(CBSE 2020)*

a. $\frac{1}{2}\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$ b. $\frac{1}{3}\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$
c. $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$ d. $\frac{1}{4}\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$

Q 24. If $A=\begin{bmatrix}5 & x\\y & 0\end{bmatrix}$ and $A=A^T$, where A^T is the transpose of the matrix A, then: *(CBSE 2023)*

a. $x=0, y=5$ b. $x=y$
c. $x+y=5$ d. $x=5, y=0$

Q 25. If a matrix A is both symmetric and skew-symmetric, then A is necessarily a/an: *(NCERT EXEMPLAR; CBSE 2021 Term-1)*

a. diagonal matrix b. zero square matrix
c. square matrix d. identity matrix

Q 26. If A and B are symmetric matrices of the same order, then:

a. AB is a symmetric matrix
b. $A-B$ is a skew-symmetric matrix
c. $AB+BA$ is a symmetric matrix
d. $AB-BA$ is a symmetric matrix

Q 27. If a square matrix $A=[a_{ij}]$, $a_{ij}=i^2-j^2$ is of even order, then:

a. A is a skew-symmetric matrix
b. A is a symmetric matrix
c. Both a. and b.
d. A is neither symmetric nor skew-symmetric

Q 28. If $A=\begin{bmatrix}3 & x-1\\2x+3 & x+2\end{bmatrix}$ is a symmetric matrix, then $x=$

a. 4 b. 3
c. −4 d. −3

Q 29. If A and B are symmetric matrices and $AB=BA$, then $A^{-1}B$ is a:

a. symmetric matrix
b. skew-symmetric matrix
c. unit matrix
d. None of the above

Q 30. If $A=[a_{ij}]$ is a skew-symmetric matrix of order n, then: *(CBSE SQP 2022-23)*

a. $a_{ij}=\frac{1}{a_{ji}}\ \forall i,j$ b. $a_{ij}\neq 0\ \forall i,j$
c. $a_{ij}=0$, where $i=j$ d. $a_{ij}\neq 0$, where $i=j$

Q 31. If A is square matrix such that $A^2=A$, then $(I+A)^3-7A$ is equal to: *(CBSE SQP 2021 Term-1)*

a. A b. $I+A$ c. $I-A$ d. I

Q 32. If A is a square matrix and $A^2=A$, then $(I+A)^2-3A$ is equal to: *(CBSE 2023)*

a. I b. A c. $2A$ d. $3I$

Q 33. If $A^2=A$, then $(I+A)^4$ is equal to:

a. $I+A$ b. $I+4A$
c. $I+15A$ d. None of these

Q 34. If $A^3=O$, then $A^2+A+I=$

a. $I-A$ b. $(I-A)^{-1}$
c. $(I+A)^{-1}$ d. $I+A$

Q 35. If $AB=A$ and $BA=B$, then:

a. $B=I$ b. $A=I$ c. $A^2=A$ d. $B^2=I$

Q 36. If $AB=A$ and $BA=B$, then A^2+B^2 is equal to:

a. $A+B$ b. $A-B$
c. $2A+B$ d. None of these

Q 37. If A and B are square matrices of size $n\times n$ such that $A^2-B^2=(A-B)(A+B)$, then which of the following will be always true?

a. Either A or B is a zero matrix
b. Either A or B is an identity matrix
c. $A=B$
d. $AB=BA$

Q 38. If A,B are non-singular square matrices of the same order, then $(AB^{-1})^{-1}=$ *(CBSE SQP 2022-23)*

a. $A^{-1}B$ b. $A^{-1}B^{-1}$
c. BA^{-1} d. AB

Q 39. If $A=\begin{bmatrix}1 & -1 & 0\\2 & 3 & 4\\0 & 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 2 & -4\\-4 & 2 & -4\\2 & -1 & 5\end{bmatrix}$, then: *(CBSE SQP 2021 Term-1)*

a. $A^{-1}=B$ b. $A^{-1}=6B$ c. $B^{-1}=B$ d. $B^{-1}=\frac{1}{6}A$

Q 40. The inverse of the matrix $X=\begin{bmatrix}2 & 0 & 0\\0 & 3 & 0\\0 & 0 & 4\end{bmatrix}$ is: *(CBSE 2021 Term-1)*

a. $24\begin{bmatrix}1/2 & 0 & 0\\0 & 1/3 & 0\\0 & 0 & 1/4\end{bmatrix}$ b. $\frac{1}{24}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

c. $\frac{1}{24}\begin{bmatrix}2 & 0 & 0\\0 & 3 & 0\\0 & 0 & 4\end{bmatrix}$ d. $\begin{bmatrix}1/2 & 0 & 0\\0 & 1/3 & 0\\0 & 0 & 1/4\end{bmatrix}$

Q 41. If $A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & a & 1\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1/2 & -1/2 & 1/2\\ -4 & 3 & c\\ 5/2 & -3/2 & 1/2\end{bmatrix}$, then the values of a and c are:

a. 1, 1 b. 1, −1
c. 1, 2 d. −1, 1

Q 42. If A and B are square matrices of the same order and $AB=3I$, then A^{-1} is equal to:

a. $3B$ b. $\frac{1}{3}B$
c. $3B^{-1}$ d. $\frac{1}{3}B^{-1}$

Q 43. If A and B are square matrices of the same order such that $(A+B)(A-B)=A^2-B^2$, then $(ABA^{-1})^2$ is equal to:

a. B^2 b. I
c. A^2B^2 d. A^2

Q 44. If $A^2-A+I=O$, then the inverse of A is:

a. $I-A$ b. $A-I$ c. A d. $A+I$

Assertion & Reason Type Questions

Directions (Q. Nos. 45-56): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false and Reason (R) is true

Q 45. Assertion (A): Matrix $3\times 3, a_{ij}=\frac{i-j}{i+2j}$ cannot be expressed as a sum of symmetric and skew-symmetric matrices.

Reason (R): Matrix $3\times 3, a_{ij}=\frac{i-j}{i+2j}$ is neither symmetric nor skew-symmetric.

Q 46. Assertion (A): Scalar matrix $A=[a_{ij}]=\begin{cases}k; & i=j\\ 0; & i\neq j\end{cases}$ where, k is a scalar, is an identity matrix when $k=1$.

Reason (R): Every identity matrix is not a scalar matrix.

Q 47. Assertion (A): $\begin{bmatrix}3 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 7\end{bmatrix}$ is a diagonal matrix.

Reason (R): $A=[a_{ij}]$ is a square matrix such that $a_{ij}=0, \forall\, i\neq j$, then A is called diagonal matrix.

Q 48. Assertion (A): $B=\begin{bmatrix}-\frac{1}{2} & \sqrt{5} & 2 & 3\end{bmatrix}_{1\times 4}$ is a row matrix.

Reason (R): If $B=[b_{ij}]_{1\times n}$ is a row matrix, then its order is $1\times n$.

Q 49. Assertion (A): If $\begin{bmatrix}xy & 4\\ z+5 & x+y\end{bmatrix}=\begin{bmatrix}4 & w\\ 0 & 4\end{bmatrix}$, then $x=2, y=2, z=-5$ and $w=4$.

Reason (R): Two matrices are equal, if their orders are same and their corresponding elements are equal.

Q 50. Assertion (A): The product of two diagonal matrices of order 3×3 is also a diagonal matrix.

Reason (R): Matrix multiplication is always non-commutative.

Q 51. Let A be a square matrix of order 3 satisfying $AA'=I$.

Assertion (A): $A'=A^{-1}$.

Reason (R): $(AB)'=B'A'$.

Q 52. Assertion (A): Let $A=[a_{ij}]$ be an $m\times n$ matrix and O be an $m\times n$ zero matrix, then $A+O=O+A=A$. In other words, O is the additive identity for matrix addition.

Reason (R): Let $A=[a_{ij}]_{m\times n}$ be any matrix, then we have another matrix as $-A=[-a_{ij}]_{m\times n}$ such that $A+(-A)=(-A)+A=O$. Then, $-A$ is the additive inverse of A or negative of A.

Q 53. Assertion (A): For multiplication of two matrices A and B, the number of columns in A should be less than the number of rows in B.

Reason (R): For getting the elements of the product matrix, we take rows of A and columns of B, multiply them elementwise and take the sum.

Q 54. Assertion (A): If $A=\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then $(A+B)^2=A^2+B^2+2AB$.

Reason (R): For the matrices A and B given in Assertion (A), $AB=BA$.

Q 55. Assertion (A): If $A=\frac{1}{3}\begin{bmatrix}1 & -2 & 2\\ -2 & 1 & 2\\ -2 & -2 & -1\end{bmatrix}$, then $A(A^T)=I$.

Reason (R): For any square matrix A, $(A^T)^T=A$.

Q 56. For any square matrix A with real number entries, consider the following statements:

Assertion (A): $A+A'$ is a symmetric matrix.

Reason (R): $A-A'$ is a skew-symmetric matrix.

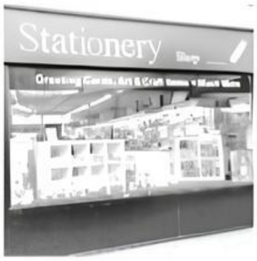
Answers

1. (c)	2. (d)	3. (b)	4. (b)	5. (a)	6. (b)	7. (a)	8. (c)	9. (c)	10. (d)
11. (d)	12. (c)	13. (c)	14. (c)	15. (d)	16. (c)	17. (b)	18. (a)	19. (a)	20. (a)
21. (d)	22. (b)	23. (b)	24. (b)	25. (b)	26. (c)	27. (a)	28. (c)	29. (a)	30. (c)
31. (d)	32. (a)	33. (c)	34. (b)	35. (c)	36. (a)	37. (d)	38. (c)	39. (d)	40. (d)
41. (b)	42. (b)	43. (a)	44. (a)	45. (d)	46. (c)	47. (a)	48. (a)	49. (a)	50. (c)
51. (b)	52. (b)	53. (d)	54. (a)	55. (b)	56. (b)				

Case Study Based Questions

Case Study 1

A manufacturer produces three stationery products pencil, eraser and sharpener which he sells in two markets. Annual sale is mentioned below:



Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
X	15,000	6,000	8,000
Y	9,500	17,000	12,000

If the unit sale price of pencil, eraser and sharpener are ₹ 3.50, ₹ 1.75 and ₹ 2.00 respectively, and unit cost of the above three commodities are ₹ 3.25, ₹ 1.50 and ₹ 0.75 respectively.

Based on the above information, solve the following questions:

Q 1. Total revenue of market X is:

a. ₹ 64,000 b. ₹ 60,000

c. ₹ 79,000 d. ₹ 81,000

Q 2. Total revenue of market Y is:

a. ₹ 35,000 b. ₹ 87,000

c. ₹ 53,000 d. ₹ 81,000

Q 3. Cost incurred in market X is:

a. ₹ 13,000 b. ₹ 30,100

c. ₹ 47,400 d. ₹ 63,750

Q 4. Profit in markets X and Y respectively are:

a. ₹ 15,250 and ₹ 21,625 b. ₹ 17,000 and ₹ 15,000

c. ₹ 10,000 and ₹ 20,000 d. ₹ 51,000 and ₹ 71,000

Q 5. Gross profit in both market is:

a. ₹ 23,000 b. ₹ 32,000

c. ₹ 36,875 d. ₹ 40,200

Solutions

1. Given data can be written in matrix form as below:

$$\text{Market}\ \begin{array}{c} X \\ Y \end{array}\overset{\begin{array}{ccc}\text{Pencil} & \text{Eraser} & \text{Sharpener}\end{array}}{\begin{bmatrix} 15{,}000 & 6{,}000 & 8{,}000 \\ 9{,}500 & 17{,}000 & 12{,}000 \end{bmatrix}_{2\times 3}} \quad \overset{\text{Sale Price}}{\begin{bmatrix} 3.50 \\ 1.75 \\ 2.00 \end{bmatrix}_{3\times 1}} \quad \overset{\text{Cost Price}}{\begin{bmatrix} 3.25 \\ 1.52 \\ 0.75 \end{bmatrix}_{3\times 1}}$$

Let $A = \begin{bmatrix} 15{,}000 & 6{,}000 & 8{,}000 \\ 9{,}500 & 17{,}000 & 12{,}000 \end{bmatrix}$,

$B = \begin{bmatrix} 3.50 \\ 1.75 \\ 2.00 \end{bmatrix}$, $C = \begin{bmatrix} 3.25 \\ 1.50 \\ 0.75 \end{bmatrix}$

Now, $AB = \begin{bmatrix} 15{,}000 & 6{,}000 & 8{,}000 \\ 9{,}500 & 17{,}000 & 12{,}000 \end{bmatrix} \begin{bmatrix} 3.50 \\ 1.75 \\ 2.00 \end{bmatrix}$

$$\Rightarrow \quad AB = \begin{bmatrix} 15{,}000 \times 3.50 + 6{,}000 \times 1.75 + 8{,}000 \times 2 \\ 9{,}500 \times 3.50 + 17{,}000 \times 1.75 + 12{,}000 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 52{,}500 + 10{,}500 + 16{,}000 \\ 33{,}250 + 29{,}750 + 24{,}000 \end{bmatrix} = \begin{bmatrix} 79{,}000 \\ 87{,}000 \end{bmatrix}$$

and $AC = \begin{bmatrix} 15{,}000 & 6{,}000 & 8{,}000 \\ 9{,}500 & 17{,}000 & 12{,}000 \end{bmatrix} \begin{bmatrix} 3.25 \\ 1.50 \\ 0.75 \end{bmatrix}$

$$= \begin{bmatrix} 15{,}000 \times 3.25 + 6{,}000 \times 1.50 + 8{,}000 \times 0.75 \\ 9{,}500 \times 3.25 + 17{,}000 \times 1.50 + 12{,}000 \times 0.75 \end{bmatrix}$$

$$= \begin{bmatrix} 48{,}750 + 9{,}000 + 6{,}000 \\ 30{,}875 + 25{,}500 + 9{,}000 \end{bmatrix} = \begin{bmatrix} 63{,}750 \\ 65{,}375 \end{bmatrix}$$

TiP

Total revenue is the sum of the product of each commodity with corresponding unit sale price and total cost incurred is the sum of the product of each commodity with corresponding unit cost price.

$\therefore$ Total revenue of market X is ₹ 79,000.

So, option (c) is correct.

2. From the above data, total revenue of market Y is ₹ 87,000.

So, option (b) is correct.

3. From the above data, cost incurred in market X is ₹ 63,750.

So, option (d) is correct.

4.

TiP

Two matrices can be subtracted, if they are of the same order.

From the above data, profit $= AB - AC$

$$= \begin{bmatrix} 79{,}000 \\ 87{,}000 \end{bmatrix} - \begin{bmatrix} 63{,}750 \\ 65{,}375 \end{bmatrix} = \begin{bmatrix} 79{,}000 - 63{,}750 \\ 87{,}000 - 65{,}375 \end{bmatrix} = \begin{bmatrix} 15{,}250 \\ 21{,}625 \end{bmatrix}$$

$\because$ Total revenue of market X is ₹ 79,000 and cost incurred in market X is ₹ 63,750.

$\therefore$ Profit in market X = ₹ (79,000 − 63,750) = ₹ 15,250

$\because$ Total revenue of market Y is ₹ 87,000 and cost incurred in market Y is ₹ 65,375.

$\therefore$ Profit in market Y = ₹ (87,000 − 65,375) = ₹ 21,625

So, option (a) is correct.

5. Gross profit in both markets

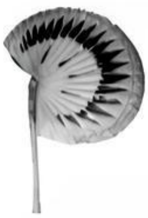
= Profit in market X + Profit in market Y

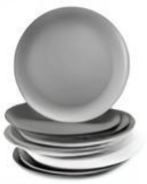
= ₹ (15,250 + 21,625) = ₹ 36,875

So, option (c) is correct.

Case Study 2

Three schools A, B and C organised a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold are given:





School/Article	A	B	C
Hand-fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Based on the above information, solve the following questions:

Q 1. The fund collected by school A if they sold 45 hand-fans, 40 mats and 25 plates, is:

a. ₹ 6,375 b. ₹ 14,000 c. ₹ 21,000 d. ₹ 18,000

Q 2. The fund collected by school B and C is:

a. ₹ 14,000 b. ₹ 18,000 c. ₹ 21,000 d. ₹ 6,375

Q 3. The total fund collected by all the schools is:

a. ₹ 6,375 b. ₹ 14,000 c. ₹ 18,000 d. ₹ 21,000

Q 4. If the number of hand-fans and mats are interchanged for all the schools, what is the total fund collected by all schools?

a. ₹ 21,000 b. ₹ 18,000 c. ₹ 14,000 d. ₹ 6,375

Q 5. The total number of all articles sold is:

a. 230 b. 280 c. 330 d. 350

Solutions

1. As we have to find the funds collected by each school. We write table as:

$$\begin{array}{ccc} & \text{Hand-fan} \quad \text{Mats} \quad \text{Plates} & \\ \begin{bmatrix} A \\ B \\ C \end{bmatrix} = & \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} & \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} 1{,}000 + 5{,}000 + 1{,}000 \\ 625 + 4{,}000 + 1{,}500 \\ 875 + 5{,}000 + 2{,}000 \end{bmatrix} = \begin{bmatrix} 7{,}000 \\ 6{,}125 \\ 7{,}875 \end{bmatrix}$$

Funds collected by schools A, B and C are ₹ 7,000, ₹ 6,125 and ₹ 7,875 respectively.

Fund collected by school A if they sold 45 hand-fans, 40 mats and 25 plates

$= 45 \times 25 + 40 \times 100 + 25 \times 50$

$= 1{,}125 + 4{,}000 + 1{,}250 =$ ₹ 6,375

So, option (a) is correct.

2. Fund collected by schools B and C

$= 6{,}125 + 7{,}875 =$ ₹ 14,000

So, option (a) is correct.

3. Total fund collected by all the schools

$= 7{,}000 + 6{,}125 + 7{,}875 =$ ₹ 21,000

So, option (d) is correct.

4. According to the given condition,

$$\begin{bmatrix} 50 & 40 & 20 \\ 40 & 25 & 30 \\ 50 & 35 & 40 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 1{,}250 + 4{,}000 + 1{,}000 \\ 1{,}000 + 2{,}500 + 1{,}500 \\ 1{,}250 + 3{,}500 + 2{,}000 \end{bmatrix} = \begin{bmatrix} 6{,}250 \\ 5{,}000 \\ 6{,}750 \end{bmatrix}$$

Total fund collected by all schools

$= 6{,}250 + 5{,}000 + 6{,}750 =$ ₹ 18,000

So, option (b) is correct.

5. Total number of all articles sold

$= (40 + 25 + 35) + (50 + 40 + 50) + (20 + 30 + 40)$
$= 330$

So, option (c) is correct.

Case Study 3

Two farmers Ramakishan and Gurucharan Singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in rupees) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B.


September sales (in ₹)

$$A=\begin{array}{c}\begin{bmatrix}10{,}000 & 20{,}000 & 30{,}000\\ 50{,}000 & 30{,}000 & 10{,}000\end{bmatrix}\end{array}\begin{array}{l}\text{Ramakishan}\\ \text{Gurucharan}\end{array}$$

October sales (in ₹)

$$B=\begin{array}{c}\begin{bmatrix}5{,}000 & 10{,}000 & 6{,}000\\ 20{,}000 & 10{,}000 & 10{,}000\end{bmatrix}\end{array}\begin{array}{l}\text{Ramakishan}\\ \text{Gurucharan}\end{array}$$

Based on the above information, solve the following questions:

Q 1. The total sales in September and October for each farmer in each variety can be represented as:

a. $A+B$ b. $A-B$ c. $A>B$ d. $A<B$

Q 2. What is the value of A_{23}?

a. 10,000 b. 20,000 c. 30,000 d. 40,000

Q 3. The decrease in sales from September to October is given by:

a. $A+B$ b. $A-B$ c. $A>B$ d. $A<B$

Q 4. If Ramakishan receives 2% profit on gross sales, compute his profit for each variety sold in October.

a. ₹ 100, ₹ 200 and ₹ 120 b. ₹ 100, ₹ 200 and ₹ 130

c. ₹ 100, ₹ 220 and ₹ 120 d. ₹ 110, ₹ 200 and ₹ 120

Q 5. If Gurucharan receives 2% profit on gross sales, compute his profit for each variety sold in September.

a. ₹ 100, ₹ 200, ₹ 120 b. ₹ 1,000, ₹ 600, ₹ 200

c. ₹ 400, ₹ 200, ₹ 120 d. ₹ 1,200, ₹ 200, ₹ 120

Solutions

1. Total sales in September and October for each farmer in each variety can be represented as $A+B$.

So, option (a) is correct.

2. The value of A_{23} in $A=10{,}000$

TiP

A_{23} means, the element in matrix A represented by intersection of second row and third column.

So, option (a) is correct.

3. The decrease in sales from September to October is given by $A-B$.

So, option (b) is correct.

4. 2% of $B=\frac{2}{100}\times B=0.02\times B$

$$=0.02\begin{bmatrix}5{,}000 & 10{,}000 & 6{,}000\\ 20{,}000 & 10{,}000 & 10{,}000\end{bmatrix}$$

$$=\begin{bmatrix}100 & 200 & 120\\ 400 & 200 & 200\end{bmatrix}\begin{array}{l}\text{Ramakishan}\\ \text{Gurucharan}\end{array}$$

$\therefore$ Required profit of Ramakishan for each variety sold in October are ₹ 100, ₹ 200 and ₹ 120.

So, option (a) is correct.

5. 2% of $A=\frac{2}{100}\times A=0.02\times A$

$$=0.02\begin{bmatrix}10{,}000 & 20{,}000 & 30{,}000\\ 50{,}000 & 30{,}000 & 10{,}000\end{bmatrix}$$

$$=\begin{bmatrix}200 & 400 & 600\\ 1{,}000 & 600 & 200\end{bmatrix}\begin{array}{l}\text{Ramakishan}\\ \text{Gurucharan}\end{array}$$

$\therefore$ Required profit of Gurucharan for each variety sold in September are ₹1,000, ₹ 600 and ₹ 200.

So, option (b) is correct.

Case Study 4

To promote the making of toilets for women, an organisation tried to generate awareness through (i) house calls, (ii) emails and (iii) announcements.

The cost for each model per attempt is given below:

(i) ₹ 50 (ii) ₹ 20 (iii) ₹ 40

The number of attempts made in the villages X, Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Also, the chance of making of toilets corresponding to one attempt of given model is:

(i) 2% (ii) 4% (iii) 20%

Based on the above information, solve the following questions:

Q 1. The cost incurred by the organisation on village X is:

a. ₹ 10,000 b. ₹ 15,000 c. ₹ 30,000 d. ₹ 20,000

Q 2. The cost incurred by the organisation on village Y is:

a. ₹ 25,000 b. ₹ 18,000 c. ₹ 23,000 d. ₹ 28,000

Q 3. The cost incurred by the organisation on village Z is:

a. ₹ 19,000 b. ₹ 39,000

c. ₹ 45,000 d. ₹ 50,000

Q 4. The total number of toilets that can be expected after the promotion in village X, is:

a. 20 b. 30 c. 40 d. 50

Q 5. The total number of toilets that can be expected after the production in village Z, is:

a. 26 b. 36

c. 46 d. 56

Solutions

1. Let ₹ A, ₹ B and ₹ C be the cost incurred by the organisation for villages X, Y and Z respectively. Then A, B, C will be given by the following matrix equation:

$$\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}\begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 400 \times 50 + 300 \times 20 + 100 \times 40 \\ 300 \times 50 + 250 \times 20 + 75 \times 40 \\ 500 \times 50 + 400 \times 20 + 150 \times 40 \end{bmatrix}$$

$$= \begin{bmatrix} 20{,}000 + 6{,}000 + 4{,}000 \\ 15{,}000 + 5{,}000 + 3{,}000 \\ 25{,}000 + 8{,}000 + 6{,}000 \end{bmatrix} = \begin{bmatrix} 30{,}000 \\ 23{,}000 \\ 39{,}000 \end{bmatrix}$$

The cost incurred by the organisation on village X is ₹ 30,000.

So, option (c) is correct.

2. From the above data, the cost incurred by the organisation on village Y is ₹ 23,000.

So, option (c) is correct.

3. From the above data, the cost incurred by the organisation on village Z is ₹ 39,000.

So, option (b) is correct.

4. Total number of toilets that can be expected in each village is given by the following matrix:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}\begin{bmatrix} 2/100 \\ 4/100 \\ 20/100 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 8 + 12 + 20 \\ 6 + 10 + 15 \\ 10 + 16 + 30 \end{bmatrix} = \begin{bmatrix} 40 \\ 31 \\ 56 \end{bmatrix}$$

The total number of toilets that can be expected after promotion in village X is 40.

So, option (c) is correct.

5. The total number of toilets that can be expected after the production in village Z is 56.

So, option (d) is correct.

Case Study 5

Assume X, Y, Z, W and P are matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $p \times k$, respectively.

Based on the above information, solve the following questions: (NCERT EXERCISE)

Q 1. The restriction on n, k and p, so that define the order of $PY + WY$.

Q 2. If $n = p$, then find the order of the matrix $7X - 5Z$.

Solutions

1. Given, order of the matrix $P = p \times k$

order of the matrix $Y = 3 \times k$

and order of the matrix $W = n \times 3$

PY is defined when,

Number of columns of matrix P = Number of rows of matrix Y

$$\Rightarrow \quad k = 3 \quad \quad ...(1)$$

Also, WY is defined when,

Number of columns of matrix W = Number of rows of matrix Y

$$\Rightarrow \quad 3 = 3 \quad \quad \text{(True)}$$

Now, $PY + WY$ is defined when both PY and WY have same order.

$\therefore$ Order of matrix $PY = p \times 3$

and order of matrix $WY = n \times k$

Here, restriction for $PY + WY$ are $p = n$ and $k = 3$.

2. Matrix X is of the order $2 \times n$.

Therefore, matrix $7X$ is also of the same order.

Matrix Z is of the order $2 \times p$ i.e., $2 \times n$ [Since, $n = p$]

Therefore, matrix $5Z$ is also of the same order.

Now, both the matrices $7X$ and $5Z$ are of the order $2 \times n$.

Thus, matrix $7X - 5Z$ is well-defined and is of order $2 \times n$.

Case Study 6

Sanjeev, Amit and Nitika were given the task of creating a square matrix of order 3. X, Y and Z are the matrices created by Sanjeev, Amit and Nitika respectively, which is given below:

$$X = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ -1 & 0 & 2 \end{bmatrix}, Y = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 3 \\ 2 & 1 & -1 \end{bmatrix}, Z = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Based on the above information, solve the following questions:

Q 1. If $a = 5$ and $b = -3$, then find the value of $(bX)^T + (aZ)^T$.

Q 2. Find the value of $(XY - YZ)$.

Q 3. If $a = -4$ and $b = -2$, then find the value of $(a - b)(YZ)^T$.

Solutions

1. Here, $X^T = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 0 \\ -1 & 1 & 2 \end{bmatrix}$ and $Z^T = \begin{bmatrix} 3 & -1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

Now, $(bX)^T + (aZ)^T = bX^T + aZ^T$

$$= (-3)\begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 0 \\ -1 & 1 & 2 \end{bmatrix} + 5\begin{bmatrix} 3 & -1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 & 3 \\ -6 & -9 & 0 \\ 3 & -3 & -6 \end{bmatrix} + \begin{bmatrix} 15 & -5 & 0 \\ 5 & 10 & 5 \\ 0 & 5 & 10 \end{bmatrix} = \begin{bmatrix} 12 & -5 & 3 \\ -1 & 1 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

2. Here, $XY = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 3 \\ 2 & 1 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 2+2-2 & 1+0-1 & -1+6+1 \\ 0+3+2 & 0+0+1 & 0+9-1 \\ -2+0+4 & -1+0+2 & 1+0-2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 6 \\ 5 & 1 & 8 \\ 2 & 1 & -1 \end{bmatrix}$$

and $YZ = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 3 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 6-1+0 & 2+2-1 & 0+1-2 \\ 3+0+0 & 1+0+3 & 0+0+6 \\ 6-1+0 & 2+2-1 & 0+1-2 \end{bmatrix} = \begin{bmatrix} 5 & 3 & -1 \\ 3 & 4 & 6 \\ 5 & 3 & -1 \end{bmatrix}$$

$$\therefore XY - YZ = \begin{bmatrix} 2 & 0 & 6 \\ 5 & 1 & 8 \\ 2 & 1 & -1 \end{bmatrix} - \begin{bmatrix} 5 & 3 & -1 \\ 3 & 4 & 6 \\ 5 & 3 & -1 \end{bmatrix} = \begin{bmatrix} -3 & -3 & 7 \\ 2 & -3 & 2 \\ -3 & -2 & 0 \end{bmatrix}$$

3. Here, $YZ = \begin{bmatrix} 5 & 3 & -1 \\ 3 & 4 & 6 \\ 5 & 3 & -1 \end{bmatrix}$ [from Q. 2]

Now, $(a-b)(YZ)^T = (-4+2)\begin{bmatrix} 5 & 3 & -1 \\ 3 & 4 & 6 \\ 5 & 3 & -1 \end{bmatrix}^T$

$$= -2\begin{bmatrix} 5 & 3 & 5 \\ 3 & 4 & 3 \\ -1 & 6 & -1 \end{bmatrix} = \begin{bmatrix} -10 & -6 & -10 \\ -6 & -8 & -6 \\ 2 & -12 & 2 \end{bmatrix}$$

Case Study 7

If $A = [a_{ij}]$ is $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A.

A square matrix $A = [a_{ij}]$ is said to be symmetric, if $A^T = A$ for all possible values of i and j.

A square matrix $A = [a_{ij}]$ is said to be skew-symmetric, if $A^T = -A$ for all possible values of i and j.

Based on the above information, solve the following questions:

Q 1. Evaluate $(ABC)^T$, by using transpose properties.

Q 2. What is the relation between symmetric and skew-symmetric matrices?

Q 3. For any square matrix A with real number entries, show that $(A+A)^T$ is symmetric matrix and $(A-A)^T$ is a skew-symmetric matrix.

Or

If $A^T = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$, then evaluate $(2A+B)^T$.

Solutions

1. $(ABC)^T = \{(AB)C\}^T = C^T(AB)^T = C^T B^T A^T$

2. Any square matrix can be expressed as sum of a symmetric and skew-symmetric matrices.

3. $(A + A^T)^T = (A)^T + (A^T)^T = A^T + A$ $\quad [\because (A^T)^T = A]$

$$= (A + A^T)$$

and $(A - A^T)^T = (A)^T - (A^T)^T = A^T - A = -(A - A^T)$

So, $(A + A^T)$ is symmetric matrix and $(A - A^T)$ is a skew-symmetric matrix.

Or

Given, $A^T = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$

$\therefore$ $(A^T)^T = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}^T$

$\Rightarrow$ $A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$

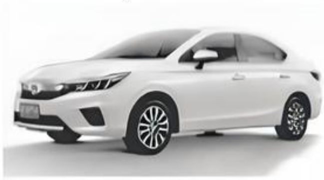
$\therefore$ $2A + B = 2\begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$

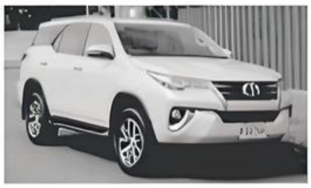
$$= \begin{bmatrix} 0 & 4 \\ -2 & 6 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -1 & 10 \end{bmatrix}$$

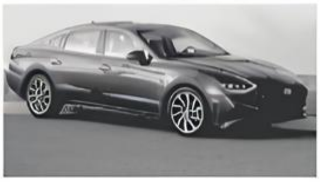
$\Rightarrow$ $(2A + B)^T = \begin{bmatrix} 2 & 3 \\ -1 & 10 \end{bmatrix}^T = \begin{bmatrix} 2 & -1 \\ 3 & 10 \end{bmatrix}$

Case Study 8

Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.







Based on the above information, solve the following questions:

Q 1. Find the matrix summarising sales data of 2019 and 2020.

Q 2. Find the matrix form of the total number of cars sold in two given years, by each dealer.

Or

Find the matrix form of the increase in sales from 2019 to 2020.

Q 3. If each dealer receive profit of ₹ 50,000 on sale of a Hatchback, ₹ 1,00,000 on sale of a Sedan and ₹ 2,00,000 on sale of a SUV, then find the matrix form of the amount of profit received in the year 2020 by each dealer.

Solutions

1. In 2019, dealer A sold 120 Hatchback, 50 Sedan and 10 SUV;
dealer B sold 100 Hatchback, 30 Sedan and 5 SUV
and dealer C sold 90 Hatchback, 40 Sedan and 2 SUV.
$\therefore$ Required matrix, say P, is given by

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc}\text{Hatchback} & \text{Sedan} & \text{SUV}\end{array} \\ \begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix}\end{array}$$

In 2020, dealer A sold 300 Hatchback, 150 Sedan, 20 SUV
dealer B sold 200 Hatchback, 50 Sedan, 6 SUV
and dealer C sold 100 Hatchback, 60 Sedan, 5 SUV.
$\therefore$ Required matrix, say Q, is given by

$$Q = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc}\text{Hatchback} & \text{Sedan} & \text{SUV}\end{array} \\ \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}\end{array}$$

2. Total number of cars sold in two given years, by each dealer, is given by

$$P + Q = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc}\text{Hatchback} & \text{Sedan} & \text{SUV}\end{array} \\ \begin{bmatrix} 120+300 & 50+150 & 10+20 \\ 100+200 & 30+50 & 5+6 \\ 90+100 & 40+60 & 2+5 \end{bmatrix}\end{array}$$

$$= \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc}\text{Hatchback} & \text{Sedan} & \text{SUV}\end{array} \\ \begin{bmatrix} 420 & 200 & 30 \\ 300 & 80 & 11 \\ 190 & 100 & 7 \end{bmatrix}\end{array}$$

Or

The increase in sales from 2019 to 2020 is given by

$$Q - P = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc}\text{Hatchback} & \text{Sedan} & \text{SUV}\end{array} \\ \begin{bmatrix} 300-120 & 150-50 & 20-10 \\ 200-100 & 50-30 & 6-5 \\ 100-90 & 60-40 & 5-2 \end{bmatrix}\end{array}$$

$$= \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc}\text{Hatchback} & \text{Sedan} & \text{SUV}\end{array} \\ \begin{bmatrix} 180 & 100 & 10 \\ 100 & 20 & 1 \\ 10 & 20 & 3 \end{bmatrix}\end{array}$$

3. The amount of profit in 2020 received by each dealer is given by the matrix

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc}\text{Hatchback} & \text{Sedan} & \text{SUV}\end{array} \\ \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}\end{array}\begin{array}{c} \\ \begin{bmatrix} 50{,}000 \\ 1{,}00{,}000 \\ 2{,}00{,}000 \end{bmatrix}\end{array}$$

$$= \begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 1{,}50{,}00{,}000 + 1{,}50{,}00{,}000 + 40{,}00{,}000 \\ 1{,}00{,}00{,}000 + 50{,}00{,}000 + 12{,}00{,}000 \\ 50{,}00{,}000 + 60{,}00{,}000 + 10{,}00{,}000 \end{bmatrix}$$

$$= \begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 3{,}40{,}00{,}000 \\ 1{,}62{,}00{,}000 \\ 1{,}20{,}00{,}000 \end{bmatrix}$$

Very Short Answer Type Questions

Q 1. If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements? *(NCERT EXERCISE)*

Q 2. How many number of matrices are possible of order 3×3 with each entry 0 or 1? *(NCERT EXERCISE)*

Q 3. Construct a 3×2 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i - 3j|$. *(NCERT EXERCISE)*

Q 4. Find the values of x, y and z from the:

$$\begin{bmatrix} x+y+z \\ x+y \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}.$$ *(NCERT EXERCISE)*

Q 5. If $A = \begin{bmatrix} 2+i & -i \\ 3 & 4i \end{bmatrix}$ and $B = \begin{bmatrix} 1+i & 2i \\ 2i & 3 \end{bmatrix}$, then find $A + B$.

Q 6. If $A = \begin{bmatrix} 4 & 2 & 13 \\ 0 & 5 & 7 \\ 6 & 8 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 3 \\ 3 & 10 & 5 \\ 5 & 7 & 0 \end{bmatrix}$, then find $(3A - 2B)$.

Q 7. Find the value of $x - y$, if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}.$$

(NCERT EXERCISE; CBSE 2019)

Q 8. If $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find the matrix A. *(CBSE 2019)*

Q 9. If $X + Y = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, $2X - Y = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, then find the value of X.

Q 10. If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, then find BA.

Q 11. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then show that AB is a zero matrix.

Q 12. Find the value of the matrices

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}_{3 \times 2} \times \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}_{2 \times 3}.$$

Q 13. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew-symmetric, find the values of a and b. *(CBSE 2018)*

Short Answer Type-I Questions

Q 1. If $A=\begin{bmatrix}\cos^2 x & \sin^2 x\\ \sin^2 x & \cos^2 x\end{bmatrix}$ and $B=\begin{bmatrix}\sin^2 x & \cos^2 x\\ \cos^2 x & \sin^2 x\end{bmatrix}$, then find $(A+B)$. *(NCERT EXERCISE)*

Q 2. If $A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$, then find $(A+B)$ and $(A-B)$.

Q 3. If $X+Y=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$, then find the values of X and Y.

Q 4. If $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$, then find (A^2-5A).

Q 5. Find a matrix A such that $2A-3B+5C=O$, where $B=\begin{bmatrix}-2 & 2 & 0\\ 3 & 1 & 4\end{bmatrix}$ and $C=\begin{bmatrix}2 & 0 & -2\\ 7 & 1 & 6\end{bmatrix}$. *(CBSE 2019)*

Q 6. If $A=\begin{bmatrix}0 & -1\\ 0 & 2\end{bmatrix}$ and $B=\begin{bmatrix}3 & 5\\ 0 & 0\end{bmatrix}$, then find the values of AB and BA. *(NCERT EXERCISE)*

Q 7. Show that all the diagonal elements of a skew-symmetric matrix are zero. *(CBSE 2017)*

Q 8. If $A=\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}2 & 4 & 2\\ 3 & 5 & 1\end{bmatrix}'$, then find AB.

Q 9. If $A=\begin{bmatrix}3 & \sqrt{3} & 2\\ 4 & 2 & 0\end{bmatrix}$, then prove that $(A')'=A$.

Q 10. If $A=\begin{bmatrix}2 & 4 & -1\\ -1 & 0 & 2\end{bmatrix}$ and $B=\begin{bmatrix}3 & 4 & 5\\ -1 & 2 & 7\\ 2 & 1 & 0\end{bmatrix}$, then find $(AB)'$.

Q 11. Given $A=\begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix}$, compute A^{-1} with the help of $2A^{-1}=9I-A$.

Short Answer Type-II Questions

Q 1. If $\begin{bmatrix}x+y & 2\\ 5+z & xy\end{bmatrix}=\begin{bmatrix}6 & 2\\ 5 & 8\end{bmatrix}$, then find the values of x, y and z. *(NCERT EXERCISE)*

Q 2. Simplify:

$$\cos\theta\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & \cos\theta\\ -\cos\theta & \sin\theta\end{bmatrix}$$

(NCERT EXERCISE)

Q 3. If $A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$, then prove that $A^3=4A$.

Q 4. If $A=\begin{bmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{bmatrix}$, then show that $A^3-23A-40I=O$. *(CBSE 2023)*

Q 5. If $A=\begin{bmatrix}1 & 2 & 5\\ 3 & 4 & 6\end{bmatrix}$ and $B=\begin{bmatrix}4 & 0\\ 2 & 1\\ 1 & 5\end{bmatrix}$, then find AB and BA. Is $AB=BA$?

Q 6. If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$, then prove that $F(x)\,F(y)=F(x+y)$. *(NCERT EXERCISE)*

Q 7. If $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$ is such that $A^2=I$, then prove that $1-\alpha^2-\beta\gamma=0$. *(NCERT EXERCISE)*

Q 8. Find matrix X so that $X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$. *(CBSE 2017)*

Q 9. Let $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}$ and $C=\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$, then find a matrix D such that $CD-AB=O$. *(CBSE 2017)*

Q 10. Find matrix A such that:

$$\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}_{3\times 2} A=\begin{bmatrix}-1 & -8\\ 1 & -2\\ 9 & 22\end{bmatrix}_{3\times 2}.$$

(CBSE 2017)

Q 11. Find the value of x from the following:

$$[x\ -5\ -1]\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix}=O.$$

Q 12. Using an example, prove that $(A+B)'=A'+B'$, where A and B are matrices of same order.

Q 13. If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ and $A+A'=I$, then find the value of α. *(NCERT EXERCISE)*

Q 14. If $A=\begin{bmatrix}2 & 1\\ 3 & 7\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2\\ 3 & 5\end{bmatrix}$, then prove that $(AB)'=B'A'$.

Q 15. If $A=\begin{bmatrix}-2\\ 4\\ 5\end{bmatrix}$, $B=[1\ \ 3\ \ -6]$, then verify that: $(AB)'=B'A'$. *(NCERT EXERCISE)*

Q 16. If A and B are symmetric matrices, then prove that $AB-BA$ is a skew-symmetric matrix.

Q 17. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute *i.e.*, $AB = BA$. *(NCERT EXERCISE)*

Q 18. Show that the matrix $B'AB$ is symmetric or skew-symmetric, if A is symmetric or skew-symmetric. *(NCERT EXERCISE)*

Q 19. Prove that every square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrices.

Long Answer Type Questions

Q 1. If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I = O$. Hence find A^5. *(NCERT EXEMPLAR)*

Q 2. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is an identity matrix of order 2, then prove that:

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}.$$ *(NCERT EXERCISE)*

Q 3. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then prove that:

$$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n \in N.$$ *(NCERT EXERCISE)*

Q 4. Find the values of x, y, z, if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A'A = I$. *(NCERT EXERCISE)*

Q 5. Write the matrix $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ in the form of sum of a symmetric matrix and a skew-symmetric matrix.

Solutions

Very Short Answer Type Questions

1. The possible orders of matrices containing 24 elements are:

$$1\times 24,\ 24\times 1,\ 2\times 12,\ 12\times 2,$$
$$3\times 8,\ 8\times 3,\ 4\times 6,\ 6\times 4$$

Possible orders of matrices containing 13 elements are:

$$1\times 13 \text{ and } 13\times 1$$

2. The number of elements in a matrix of order 3×3 are 9 in which 0 or 1 can be placed at each position. So, there are $2^9 = 512$ ways to fill the position of the matrix.

$\therefore$ Number of possible matrices $= 512$

3. In general, a 3×2 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

Now, $a_{ij} = \frac{1}{2}|i - 3j|, i = 1, 2, 3$ and $j = 1, 2$

$\therefore$ $a_{11} = \frac{1}{2}|1 - 3\cdot 1| = 1, a_{12} = \frac{1}{2}|1 - 3\cdot 2| = \frac{5}{2}$

$a_{21} = \frac{1}{2}|2 - 3\cdot 1| = \frac{1}{2}, a_{22} = \frac{1}{2}|2 - 3\cdot 2| = 2$

$a_{31} = \frac{1}{2}|3 - 3\cdot 1| = 0, a_{32} = \frac{1}{2}|3 - 3\cdot 2| = \frac{3}{2}$

Hence, required matrix $A = \begin{bmatrix} 1 & 5/2 \\ 1/2 & 2 \\ 0 & 3/2 \end{bmatrix}$.

4. We have, $\begin{bmatrix} x + y + z \\ x + y \\ y + z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

TR!CK

If two matrices are equal, then their corresponding elements are equal.

On comparing both sides, we get

$$x + y + z = 9 \quad ...(1)$$
$$x + y = 5 \quad ...(2)$$
and $$y + z = 7 \quad ...(3)$$

From eqs. (1) and (2), we get

$$5 + z = 9$$
$\Rightarrow$ $$z = 9 - 5 = 4$$

From eqs. (1) and (3), we get

$$x + 7 = 9$$
$\Rightarrow$ $$x = 9 - 7 = 2$$

Put the values of x and z in eq. (1), we get

$$2 + y + 4 = 9$$
$\Rightarrow$ $$y = 9 - 6 = 3$$
$\therefore$ $$x = 2, y = 3 \text{ and } z = 4$$

5.

TiP

Two matrices can be added only when they are of the same order.

Here, $A + B = \begin{bmatrix} 2+i & -i \\ 3 & 4i \end{bmatrix} + \begin{bmatrix} 1+i & 2i \\ 2i & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3+2i & i \\ 3+2i & 3+4i \end{bmatrix}$$

6.

TiP

If A is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each elements of A by a scalar k.

$$3A-2B=3\times\begin{bmatrix}4&2&13\\0&5&7\\6&8&9\end{bmatrix}-2\times\begin{bmatrix}2&0&3\\3&10&5\\5&7&0\end{bmatrix}$$

$$=\begin{bmatrix}12&6&39\\0&15&21\\18&24&27\end{bmatrix}-\begin{bmatrix}4&0&6\\6&20&10\\10&14&0\end{bmatrix}$$

$$=\begin{bmatrix}12-4&6-0&39-6\\0-6&15-20&21-10\\18-10&24-14&27-0\end{bmatrix}=\begin{bmatrix}8&6&33\\-6&-5&11\\8&10&27\end{bmatrix}$$

7. We have,

$$2\begin{bmatrix}1&3\\0&x\end{bmatrix}+\begin{bmatrix}y&0\\1&2\end{bmatrix}=\begin{bmatrix}5&6\\1&8\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2&6\\0&2x\end{bmatrix}+\begin{bmatrix}y&0\\1&2\end{bmatrix}=\begin{bmatrix}5&6\\1&8\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2+y&6\\1&2x+2\end{bmatrix}=\begin{bmatrix}5&6\\1&8\end{bmatrix}$$

TR!CK

Two matrices A and B are said to be equal, if:
- *order of A and B is same.*
- *corresponding elements of A and B are equal.*

On comparing both sides, we get

$$2+y=5\Rightarrow y=5-2=3$$

and $\quad 2x+2=8\Rightarrow 2x=8-2=6$

$$\Rightarrow \quad x=\frac{6}{2}=3$$

So, $\quad x-y=3-3=0$

8. We have,

$$3A-B=\begin{bmatrix}5&0\\1&1\end{bmatrix} \text{ and } B=\begin{bmatrix}4&3\\2&5\end{bmatrix}$$

TiP

Addition of two matrices is possible when its order are same.

$$\Rightarrow \quad 3A=\begin{bmatrix}5&0\\1&1\end{bmatrix}+B=\begin{bmatrix}5&0\\1&1\end{bmatrix}+\begin{bmatrix}4&3\\2&5\end{bmatrix}$$

$$\Rightarrow \quad 3A=\begin{bmatrix}9&3\\3&6\end{bmatrix}=3\begin{bmatrix}3&1\\1&2\end{bmatrix}$$

$$\therefore \quad A=\begin{bmatrix}3&1\\1&2\end{bmatrix}$$

9.

TiP

Two matrices can be added, if they are of the same order.

Adding the given matrix equations.

$$(X+Y)+(2X-Y)=\begin{bmatrix}2&1\\1&2\end{bmatrix}+\begin{bmatrix}1&2\\2&1\end{bmatrix}$$

$$\Rightarrow \quad 3X=\begin{bmatrix}3&3\\3&3\end{bmatrix}\Rightarrow X=\begin{bmatrix}1&1\\1&1\end{bmatrix}$$

10. $BA=\begin{bmatrix}1&3\\-2&5\end{bmatrix}\begin{bmatrix}2&4\\3&2\end{bmatrix}$

TR!CK

Product of two matrices is possible when number of columns of first matrix is equal to the number of rows of second matrix.

$$=\begin{bmatrix}2+9&4+6\\-4+15&-8+10\end{bmatrix}=\begin{bmatrix}11&10\\11&2\end{bmatrix}$$

11. $AB=\begin{bmatrix}1&-1\\-1&1\end{bmatrix}\begin{bmatrix}1&1\\1&1\end{bmatrix}=\begin{bmatrix}1\cdot1+(-1)\cdot1&1\cdot1+(-1)\cdot1\\(-1)\cdot1+1\cdot1&(-1)\cdot1+1\cdot1\end{bmatrix}$

$$=\begin{bmatrix}1-1&1-1\\-1+1&-1+1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

Clearly, AB is a zero matrix. **Hence proved.**

COMMON ERROR

Mostly students commit error while multiplying matrices.

12. $\begin{bmatrix}2&1\\3&2\\-1&1\end{bmatrix}_{3\times2}\times\begin{bmatrix}1&0&1\\-1&2&1\end{bmatrix}_{2\times3}$

TiP

Product of two matrices is possible when number of columns of first matrix is equal to the number of rows of second matrix.

$$=\begin{bmatrix}2\times1+1\times(-1)&2\times0+1\times2&2\times1+1\times1\\3\times1+2\times(-1)&3\times0+2\times2&3\times1+2\times1\\(-1)\times1+1\times(-1)&-1\times0+1\times2&-1\times1+1\times1\end{bmatrix}_{3\times3}$$

$$=\begin{bmatrix}2-1&0+2&2+1\\3-2&0+4&3+2\\-1-1&0+2&-1+1\end{bmatrix}=\begin{bmatrix}1&2&3\\1&4&5\\-2&2&0\end{bmatrix}$$

13. Given that, matrix $A=\begin{bmatrix}0&a&-3\\2&0&-1\\b&1&0\end{bmatrix}$ is skew-symmetric matrix.

$$\therefore \quad A^T=-A$$

$$\Rightarrow \quad \begin{bmatrix}0&a&-3\\2&0&-1\\b&1&0\end{bmatrix}^T=-\begin{bmatrix}0&a&-3\\2&0&-1\\b&1&0\end{bmatrix}$$

TR!CK

Two matrices A and B are said to be equal, if:
- *order of A and B is same.*
- *corresponding elements of A and B are same*
  i.e., $a_{ij}=b_{ij}\ \forall\ i$ and j.

$$\Rightarrow \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & 3 \\ -2 & 0 & 1 \\ -b & -1 & 0 \end{bmatrix}$$

On comparing corresponding elements, we get

$$-a = 2 \Rightarrow a = -2$$

and $-3 = -b \Rightarrow b = 3$

Short Answer Type-I Questions

1. $$A + B = \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2 x + \cos^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \sin^2 x + \cos^2 x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

2. $$A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

and $$A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

3. Adding the given matrix equations, we have

$$(X + Y) + (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

TiP

Two matrices can be added/subtracted, if they are of the same order.

$$\Rightarrow \quad 2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

Again, $$X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} \Rightarrow Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - X$$

$$= \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

$$\Rightarrow \quad Y = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

4. We have, $$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

Now, $$A^2 = A \cdot A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1-0 & 1-3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\therefore A^2 - 5A = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix}$$

5. Given that, $$B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$$

and $$C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$$

TR!CK

Here, A must be taken of the order 2 × 3, because B and C are of the same order 2 × 3 and the sum of two or more matrices is possible only when they are of the same order.

Let $$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

We have, $2A - 3B + 5C = O$

$$\Rightarrow 2\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - 3\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} + 5\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

TR!CK

Here, O i.e., zero matrix must be taken of the order 2 × 3, because two matrices are said to be equal, if they are of the same order.

$$\Rightarrow \begin{bmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \end{bmatrix} + \begin{bmatrix} 6 & -6 & 0 \\ -9 & -3 & -12 \end{bmatrix} + \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a+16 & 2b-6 & 2c-10 \\ 2d+26 & 2e+2 & 2f+18 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

On comparing the corresponding elements, we get

$$2a + 16 = 0 \Rightarrow a = \frac{-16}{2} = -8,$$

$$2b - 6 = 0 \Rightarrow b = \frac{6}{2} = 3,$$

$$2c - 10 = 0 \Rightarrow c = \frac{10}{2} = 5,$$

$$2d + 26 = 0 \Rightarrow d = \frac{-26}{2} = -13,$$

$$2e + 2 = 0 \Rightarrow e = \frac{-2}{2} = -1$$

and $$2f + 18 = 0 \Rightarrow f = \frac{-18}{2} = -9$$

So, matrix $$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$$

6. $$AB = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

and $$BA = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0+0 & -3+10 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 7 \\ 0 & 0 \end{bmatrix}$$

7. Let $A=[a_{ij}]$ be a given matrix.
Since, it is skew-symmetric matrix.

$\therefore \quad A' = -A$

$\Rightarrow \quad a_{ji} = -a_{ij}$ for all i, j

$\Rightarrow \quad a_{ii} = -a_{ii}$ for all values of i

when $j = i$,

$\Rightarrow 2a_{ii} = 0$ for all values of $i \Rightarrow a_{ii} = 0$ for all values of i

$\Rightarrow \quad a_{11} = a_{22} = a_{33} = \ldots\ldots = a_{nn} = 0$

Hence, all the diagonal elements of a skew-symmetric matrix are zero (as diagonal elements are: $a_{11}, a_{22}, a_{33}, \ldots, a_{nn}$). **Hence proved.**

8. Given, $A=\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}, B=\begin{bmatrix}2 & 4 & 2\\ 3 & 5 & 1\end{bmatrix}' = \begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}$

TiP

Practice similar type of questions based on multiplication of two matrices.

$$\therefore \quad AB = \begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}\times\begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}$$

$$= \begin{bmatrix}2-8+6 & 3-10+3\\ -8+8+10 & -12+10+5\end{bmatrix} = \begin{bmatrix}0 & -4\\ 10 & 3\end{bmatrix}$$

9.

TiP

If $A = [a_{ij}]$ is $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is the transpose of A.

Given, $\quad A = \begin{bmatrix}3 & \sqrt{3} & 2\\ 4 & 2 & 0\end{bmatrix} \Rightarrow A' = \begin{bmatrix}3 & 4\\ \sqrt{3} & 2\\ 2 & 0\end{bmatrix}$

and $\quad (A')' = \begin{bmatrix}3 & \sqrt{3} & 2\\ 4 & 2 & 0\end{bmatrix} = A$

$\therefore \quad (A')' = A$ **Hence proved.**

10. $AB = \begin{bmatrix}2 & 4 & -1\\ -1 & 0 & 2\end{bmatrix}\begin{bmatrix}3 & 4 & 5\\ -1 & 2 & 7\\ 2 & 1 & 0\end{bmatrix}$

$$= \begin{bmatrix}6-4-2 & 8+8-1 & 10+28-0\\ -3+0+4 & -4+0+2 & -5+0+0\end{bmatrix}$$

$$= \begin{bmatrix}0 & 15 & 38\\ 1 & -2 & -5\end{bmatrix}$$

$$\Rightarrow \quad (AB)' = \begin{bmatrix}0 & 1\\ 15 & -2\\ 38 & -5\end{bmatrix}$$

11. $9I - A = 9\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix}$

$$= \begin{bmatrix}9 & 0\\ 0 & 9\end{bmatrix} - \begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix} = \begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$$

$\because \quad 2A^{-1} = 9I - A$ (Given)

$\therefore \quad A^{-1} = \frac{1}{2}(9I - A)$

$$= \frac{1}{2}\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix} = \begin{bmatrix}7/2 & 3/2\\ 2 & 1\end{bmatrix}$$

Short Answer Type-II Questions

1. We have,

$$\begin{bmatrix}x+y & 2\\ 5+z & xy\end{bmatrix} = \begin{bmatrix}6 & 2\\ 5 & 8\end{bmatrix}$$

Corresponding elements of equal matrices are equal. So put corresponding elements of two matrices equal.

$\therefore \quad x + y = 6 \quad \ldots(1)$

$xy = 8$

and $\quad 5 + z = 5$

$\Rightarrow \quad z = 0$

Now, $(x+y)^2 - (x-y)^2 = 4xy$

$\Rightarrow \quad (6)^2 - (x-y)^2 = 4 \times 8$

$\Rightarrow \quad (x-y)^2 = 36 - 32 = 4$

$\Rightarrow \quad x - y = \pm 2 \quad \ldots(2)$

Adding eqs. (1) and (2), we get

$(x+y)+(x-y) = 6 \pm 2$

$\Rightarrow \quad 2x = 6 \pm 2$

$\Rightarrow \quad 2x = 6 + 2$ or $2x = 6 - 2$

$\Rightarrow \quad x = 4$ or $x = 2$

Subtracting eq. (2) from eq. (1), we get

$(x+y)-(x-y) = 6 \mp 2$

$\Rightarrow \quad 2y = 6 - 2$ or $2y = 6 + 2$

$\Rightarrow \quad 2y = 4$ or $2y = 8$

$\Rightarrow \quad y = 2$ or $y = 4$

So, $x = 4, y = 2, z = 0$ or $x = 2, y = 4, z = 0$

COMMON ERR(!)R

Some students commit error while finding the values of x and y.

2. Given, $\cos\theta\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix} + \sin\theta\begin{bmatrix}\sin\theta & \cos\theta\\ -\cos\theta & \sin\theta\end{bmatrix}$

$$= \begin{bmatrix}\cos^2\theta & -\cos\theta\sin\theta\\ \sin\theta\cos\theta & \cos^2\theta\end{bmatrix} + \begin{bmatrix}\sin^2\theta & \sin\theta\cos\theta\\ -\sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2\theta+\sin^2\theta & -\cos\theta\sin\theta+\sin\theta\cos\theta\\ \sin\theta\cos\theta-\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

3.

TiP

The multiplication of two matrices A and B is defined, if the number of columns of A is equal to the number of rows of B.

$$A^2 = A\cdot A = \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1+1 & -1-1\\ -1-1 & 1+1\end{bmatrix} = \begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$$

$$A^3 = A\cdot A^2 = \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\cdot\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & -2-2 \\ -2-2 & 2+2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \quad ...(1)$$

and $$4A = 4\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \quad ...(2)$$

From eqs. (1) and (2), we get

$A^3 = 4A$ **Hence proved.**

4. We know that,

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+12 & 2-4+6 & 3+2+3 \\ 3-6+4 & 6+4+2 & 9-2+1 \\ 4+6+4 & 8-4+2 & 12+2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$A^3 = A \cdot A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 19+2+42 & 4+24+18 & 8+16+45 \\ 57-2+14 & 12-24+6 & 24-16+15 \\ 76+2+14 & 16+24+6 & 32+16+15 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

Now, $A^3 - 23A - 40I$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} + \begin{bmatrix} -23 & -46 & -69 \\ -69 & 46 & -23 \\ -92 & -46 & -23 \end{bmatrix} + \begin{bmatrix} -40 & 0 & 0 \\ 0 & -40 & 0 \\ 0 & 0 & -40 \end{bmatrix}$$

$$= \begin{bmatrix} 63-23-40 & 46-46+0 & 69-69+0 \\ 69-69+0 & -6+46-40 & 23-23+0 \\ 92-92+0 & 46-46+0 & 63-23-40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$ **Hence proved.**

COMMON ERR(!)R

Some students make square and cube the elements in A and obtain the answer.

5.

TiP

Give ample practice on problems based on multiplication of two matrices.

$$AB = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 6 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 2 & 1 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4+5 & 0+2+25 \\ 12+8+6 & 0+4+30 \end{bmatrix} = \begin{bmatrix} 13 & 27 \\ 26 & 34 \end{bmatrix}$$

and $$BA = \begin{bmatrix} 4 & 0 \\ 2 & 1 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 8+0 & 20+0 \\ 2+3 & 4+4 & 10+6 \\ 1+15 & 2+20 & 5+30 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 & 20 \\ 5 & 8 & 16 \\ 16 & 22 & 35 \end{bmatrix}$$

Here, $AB \neq BA$

COMMON ERR(!)R

Some students commit an error while calculating.

6. Given, $$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ...(1)$$

$$\therefore \quad F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{[Replace } x \text{ by } y]$$

$$F(x)F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x\cos y - \sin x\sin y + 0 & -\cos x\sin y - \sin x\cos y + 0 & 0+0+0 \\ \sin x\cos y + \cos x\sin y + 0 & -\sin x\sin y + \cos x\cos y + 0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x\cos y - \sin x\sin y & -\cos x\sin y - \sin x\cos y & 0 \\ \sin x\cos y + \cos x\sin y & -\sin x\sin y + \cos x\cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

TR!CKS

- $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$
- $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Replace x by $(x+y)$ in eq. (1), we get

$$F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore \quad F(x)F(y) = F(x+y)$ **Hence proved.**

7. Given, $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$

TR!CK

If two matrix are equivalent, then its corresponding elements are equal.

$$A^2=AA=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$$
$$=\begin{bmatrix}\alpha^2+\beta\gamma & \alpha\beta-\alpha\beta\\ \alpha\gamma-\alpha\gamma & \beta\gamma+\alpha^2\end{bmatrix}$$
$$=\begin{bmatrix}\alpha^2+\beta\gamma & 0\\ 0 & \beta\gamma+\alpha^2\end{bmatrix}$$

Given; $A^2=I$

$$\Rightarrow \begin{bmatrix}\alpha^2+\beta\gamma & 0\\ 0 & \beta\gamma+\alpha^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

On comparing corresponding elements, we get

$$\alpha^2+\beta\gamma=1$$
$$\Rightarrow 1-\alpha^2-\beta\gamma=0$$

Hence proved.

8. Given,

$$X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}_{2\times 3}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}_{2\times 3} \quad ...(1)$$

Let $X=\begin{bmatrix}a & b\\ c & d\end{bmatrix}_{2\times 2}$

TR!CK

The product of two matrices are possible when number of columns of first matrix is equal to the number of rows of second matrix.
As, RHS have a matrix of order 2×3.
So, LHS must contain the matrix of order 2×3. For this X should be a matrix of order 2×2.
i.e., $2\times \boxed{2\ \ 2}\times 3=2\times 3$

From eq. (1), we get

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}a+4b & 2a+5b & 3a+6b\\ c+4d & 2c+5d & 3c+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

On comparing both sides, we get

$$a+4b=-7 \Rightarrow a=-7-4b \quad ...(2)$$
$$3a+6b=-9 \Rightarrow a+2b=-3 \quad ...(3)$$
$$c+4d=2 \Rightarrow c=2-4d \quad ...(4)$$
and $$3c+6d=6 \Rightarrow c+2d=2 \quad ...(5)$$

From eqs. (2) and (3), we get

$$-7-4b+2b=-3$$
$$\Rightarrow 2b=-4$$
$$\Rightarrow b=-2$$

From eq. (2),

$$a=-7-4(-2)$$
$$=-7+8=1$$
$$\Rightarrow a=1$$

From eqs. (4) and (5), we get

$$2-4d+2d=2 \Rightarrow d=0$$

From eq. (5),

$$c=2-4\times 0 \Rightarrow c=2$$
$$\therefore X=\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$$

9. Given, $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}$

and $C=\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$

TR!CK

The order of matrix D must be 2×2, as product of two matrices is possible when number of columns of first matrix is equal to the number of rows of second matrix. After that the addition or subtraction of two matrices is possible only when the order of both matrices is same.

Let, matrix $D=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

We have, $CD-AB=O$

$$\Rightarrow \begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}-\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

(Here, O is zero matrix)

$$\Rightarrow \begin{bmatrix}2a+5c & 2b+5d\\ 3a+8c & 3b+8d\end{bmatrix}-\begin{bmatrix}10-7 & 4-4\\ 15+28 & 6+16\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}2a+5c & 2b+5d\\ 3a+8c & 3b+8d\end{bmatrix}-\begin{bmatrix}3 & 0\\ 43 & 22\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}2a+5c-3 & 2b+5d\\ 3a+8c-43 & 3b+8d-22\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

On comparing both sides, we get

$$2a+5c-3=0 \Rightarrow 2a+5c=3 \quad ...(1)$$
$$2b+5d=0 \Rightarrow 2b=-5d \quad ...(2)$$
$$3a+8c-43=0 \Rightarrow 3a+8c=43 \quad ...(3)$$
and $$3b+8d-22=0 \Rightarrow 3b+8d=22 \quad ...(4)$$

From eqs. (2) and (4), we get

$$3\left(-\frac{5d}{2}\right)+8d=22$$
$$\Rightarrow -15d+16d=44$$
$$\Rightarrow d=44$$

From eq. (2), we get

$$2b=-5\times 44$$
$$\Rightarrow b=-5\times 22$$
$$\Rightarrow b=-110$$

From eqs. (1) and (3), we get

$$3\left(\frac{3-5c}{2}\right)+8c=43$$
$$\Rightarrow 9-15c+16c=86$$
$$\Rightarrow c=77$$

Put the value of c in eq. (1), we get

$$2a+5\times 77=3$$
$$\Rightarrow 2a=3-385=-382$$
$$\Rightarrow a=-191$$

So, matrix $D=\begin{bmatrix}-191 & -110\\ 77 & 44\end{bmatrix}$.

10.

TR!CK

Here, we will take matrix A of the order 2×2.
$\because$ RHS has a matrix of order 3×2.
$\therefore$ LHS should also be a matrix of order 3×2.
For this, A should be of the order 2×2.
i.e., $3 \times \boxed{2\ 2} \times 2 = 3 \times 2$

Let matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2 \times 2}$

Now, $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a-c & 2b-d \\ a+0 & b+0 \\ -3a+4c & -3b+4d \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$

Comparing on both sides,

$2a - c = -1$ ...(1)

and $2b - d = -8$ ...(2)

Also, $a = 1$ and $b = -2$

Put the value of 'a' in eq. (1), we get

$2(1) - c = -1$

$\Rightarrow c = 2 + 1 = 3$

Put the value of 'b' in eq. (2), we get

$2(-2) - d = -8$

$\Rightarrow d = -4 + 8 = 4$

So, $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$

11. From the given matrix equation,

$$[x-2 \quad -10 \quad 2x-5-3] \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

TiP

Give ample practice in problem based on multiplication of two matrices.

$\Rightarrow [x-2 \quad -10 \quad 2x-8] \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

$\Rightarrow [x(x-2) - 40 + (2x-8)] = 0$

$\Rightarrow x^2 - 2x - 40 + 2x - 8 = 0$

$\Rightarrow x^2 - 48 = 0$

$\Rightarrow x^2 = 48$

$\Rightarrow x^2 = 16 \times 3$

$\Rightarrow x = \pm 4\sqrt{3}$

12. Let $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$

Then, $A + B = \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & 2 \\ 3 & 2 \end{bmatrix}$

TiP

Students must remember this expression $(A+B)' = A' + B'$ as an identity or a property of Transpose of matrix.

$\Rightarrow (A+B)' = \begin{bmatrix} 3 & 2 \\ 0 & 2 \\ 3 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & 0 & 3 \\ 2 & 2 & 2 \end{bmatrix}$ ...(1)

Now, $A' = \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}' = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & -1 \end{bmatrix}$

and $B' = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}' = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix}$

$\Rightarrow A' + B' = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3 & 0 & 3 \\ 2 & 2 & 2 \end{bmatrix}$ ...(2)

From eqs. (1) and (2), we get

$(A+B)' = A' + B'$ **Hence proved.**

13. Given, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

Given, $A + A' = I$

$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos\alpha + \cos\alpha & -\sin\alpha + \sin\alpha \\ \sin\alpha - \sin\alpha & \cos\alpha + \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

TR!CK

If two matrix are equivalent, then its corresponding elements are equal.

$\Rightarrow 2\cos\alpha = 1$

$\Rightarrow \cos\alpha = \dfrac{1}{2}$

$\Rightarrow \cos\alpha = \cos\dfrac{\pi}{3}$

$\therefore \alpha = \dfrac{\pi}{3}$

14. $\because A = \begin{bmatrix} 2 & 1 \\ 3 & 7 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 2 & 3 \\ 1 & 7 \end{bmatrix}$

and $B = \begin{bmatrix} -1 & 2 \\ 3 & 5 \end{bmatrix} \Rightarrow B' = \begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 2 & 1 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} -2+3 & 4+5 \\ -3+21 & 6+35 \end{bmatrix} = \begin{bmatrix} 1 & 9 \\ 18 & 41 \end{bmatrix}$

Now, LHS $= (AB)' = \begin{bmatrix} 1 & 9 \\ 18 & 41 \end{bmatrix}' = \begin{bmatrix} 1 & 18 \\ 9 & 41 \end{bmatrix}$

and $\quad \text{RHS} = B'A' = \begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 7 \end{bmatrix}$

$$= \begin{bmatrix} -2+3 & -3+21 \\ 4+5 & 6+35 \end{bmatrix} = \begin{bmatrix} 1 & 18 \\ 9 & 41 \end{bmatrix}$$

Hence, $\quad$ LHS = RHS $\quad$ **Hence proved.**

COMMON ERR(!)R

Students commit error when multiplying the matrices together.

15. $AB = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}\begin{bmatrix} 1 & 3 & -6 \end{bmatrix} = \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix}$

TiP

Give ample practice in problem based on multiplication of two matrices.

$$\Rightarrow \quad (AB)' = \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix}'$$

$$= \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix} \quad ...(1)$$

Now, $\quad B' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}$ and $A' = \begin{bmatrix} -2 & 4 & 5 \end{bmatrix}$

$$\Rightarrow B'A' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}\begin{bmatrix} -2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix} \quad ...(2)$$

From eqs. (1) and (2), we get $(AB)' = B'A'$

Hence proved.

16. Given, A and B are symmetric matrices.

$\therefore \quad A' = A$ and $B' = B$

Now, $(AB - BA)' = (AB)' - (BA)' \quad [\because (X-Y)' = X' - Y']$

$= B'A' - A'B' \quad [\because (XY)' = Y'X']$

$= BA - AB \quad [\because B' = B, A' = A]$

$= -(AB - BA)$

$\therefore AB - BA$ is a skew-symmetric matrix. **Hence proved.**

17. Since, A and B are both symmetric matrices, therefore $A' = A$ and $B' = B$.

Let AB be symmetric, then

$(AB)' = AB$

$\Rightarrow \quad B'A' = AB$

$\therefore \quad BA = AB \quad [\because A' = A$ and $B' = B]$

Conversely, if $AB = BA$, then we shall show that AB is symmetric.

Now, $(AB)' = B'A' = BA = AB \quad [\because A$ and B are symmetric$]$

Hence, AB is symmetric. **Hence proved.**

18. (i) Let A be a symmetric matrix, then

$A' = A$

$\therefore \quad (B'AB)' = (B'(AB))'$

$= (AB)'(B')' = (B'A')B = B'AB$

$[\because (AB)' = B'A'$ and $A' = A]$

Hence, $B'AB$ is a symmetric matrix. **Hence proved.**

(ii) Let A be a skew-symmetric matrix, then

$A' = -A$

Now, $(B'(AB))' = (AB)'(B')'$

$= (B'A')B = B'(-A)B \quad [\because A' = -A]$

$= -B'AB = -(B'AB)$

Hence, $B'AB$ is a skew-symmetric matrix.

Hence proved.

19. Let A be a square matrix, then

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = P + Q$$

where, $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$

Now, $P' = \left\{\frac{1}{2}(A + A')\right\}' = \frac{1}{2}(A + A')' \quad [\because (kA)' = kA']$

$= \frac{1}{2}[A' + (A')'] \quad [\because (A+B)' = A' + B']$

$= \frac{1}{2}(A' + A) \quad [\because (A')' = A]$

$= \frac{1}{2}(A + A') \quad$ [From the commutative law of addition of matrices]

$= P$

So, P is a symmetric matrix.

Again $Q' = \left\{\frac{1}{2}(A - A')\right\}' = \frac{1}{2}(A - A')' = \frac{1}{2}[A' - (A')']$

$= \frac{1}{2}(A' - A) = -\frac{1}{2}(A - A') = -Q$

So, Q is a skew-symmetric matrix.

Hence, the square matrix A can be expressed as the sum of a symmetric matrix $\frac{1}{2}(A + A')$ and a skew-symmetric matrix $\frac{1}{2}(A - A')$. **Hence proved.**

Long Answer Type Questions

1.

TiP

Adequate practice is required in problems based on multiplication of two matrices.

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 6+6 \\ -2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$$

$$-4A = \begin{bmatrix} -8 & -12 \\ 4 & -8 \end{bmatrix}$$

and $\quad 7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$$\therefore \quad A^2 - 4A + 7I = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} + \begin{bmatrix} -8 & -12 \\ 4 & -8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1-8+7 & 12-12+0 \\ -4+4+0 & 1-8+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Hence proved.

$\Rightarrow \quad A^2 = 4A - 7I$

Now, $A^3 = A \cdot A^2 = A(4A - 7I) = 4A^2 - 7AI$

$= 4(4A - 7I) - 7A \quad [\because AI = A]$

$= 16A - 28I - 7A = 9A - 28I$

Again, $A^5 = A^3 \cdot A^2$

$= (9A - 28I) \cdot (4A - 7I)$

$= 36A^2 - 63A - 112A + 196I$

$= 36(4A - 7I) - 175A + 196I$

$= -31A - 56I$

$$= -31\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 56\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$$

COMMON ERROR

Most students find difficulty in finding the correct value of A^5.

2. Given, $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$

Let $\tan\frac{\alpha}{2} = t$.

We know that,

$$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1 + \tan^2\frac{\alpha}{2}}$$

$$\Rightarrow \quad \sin\alpha = \frac{2t}{1+t^2}$$

and $$\cos\alpha = \frac{1 - \tan^2\frac{\alpha}{2}}{1 + \tan^2\frac{\alpha}{2}}$$

$$\Rightarrow \quad \cos\alpha = \frac{1-t^2}{1+t^2}$$

$$\therefore \quad A = \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

and $$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}$$

Now, $$\text{LHS} = I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

and $$\text{RHS} = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}\begin{bmatrix} \frac{1-t^2}{1+t^2} & -\frac{2t}{1+t^2} \\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{bmatrix}$$

TiP

Use the correct formula in the right place.

$$= \begin{bmatrix} 1 \times \frac{(1-t^2)}{1+t^2} + t \times \frac{2t}{1+t^2} & 1 \times \frac{-2t}{1+t^2} + t \times \frac{1-t^2}{1+t^2} \\ -t \times \frac{1-t^2}{1+t^2} + 1 \times \frac{2t}{1+t^2} & -t \times \frac{-2t}{1+t^2} + 1 \times \frac{1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-t^2}{1+t^2} + \frac{2t^2}{1+t^2} & -\frac{2t}{1+t^2} + \frac{t(1-t^2)}{1+t^2} \\ -\frac{t(1-t^2)}{1+t^2} + \frac{2t}{1+t^2} & \frac{2t^2}{1+t^2} + \frac{1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t-t^3}{1+t^2} \\ \frac{-t+t^3+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-t(1+t^2)}{1+t^2} \\ \frac{t(1+t^2)}{1+t^2} & \frac{1+t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} = \text{LHS}$$

Hence, $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ **Hence proved.**

3. We shall prove it by using principle of mathematical induction.

Here, $P(n)$: If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Then, $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n \in N$

Now, $P(1): A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\therefore \quad A^1 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$\therefore \quad P(k)$: $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Then $A^k = \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}$

Now, we shall prove that the result holds for $n = k + 1$ also.

Now, $A^{k+1} = A \cdot A^k = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos\theta \cdot \cos k\theta - \sin\theta \cdot \sin k\theta & \sin k\theta \cdot \cos\theta + \cos k\theta \cdot \sin\theta \\ -\sin\theta \cdot \cos k\theta - \cos\theta \cdot \sin k\theta & -\sin k\theta \cdot \sin\theta + \cos\theta \cdot \cos k\theta \end{bmatrix}$$

TR!CKS

- *$\sin(A + B) = \sin A \cdot \cos B + \cos A \cdot \sin B$*
- *$\cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$*

$$= \begin{bmatrix} \cos(\theta + k\theta) & \sin(\theta + k\theta) \\ -\sin(k\theta + \theta) & \cos(\theta + k\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

Therefore, the result is true for $n = k + 1$ also.

Thus, by the principle of mathematical induction, $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$. holds for all natural numbers. **Hence proved.**

4. Given, $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

$\Rightarrow \quad A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

Also, $A'A = I$

$$\Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+yx-yx & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-zx+zx & 2yz-zy-zy & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

TR!CK

Two matrices A and B are said to be equal, if each element of A is equal to the corresponding element of B.

Comparing on both sides, we get

$2x^2 = 1 \quad \Rightarrow x = \pm \frac{1}{\sqrt{2}}$

$6y^2 = 1 \quad \Rightarrow y = \pm \frac{1}{\sqrt{6}}$

and $\quad 3z^2 = 1 \quad \Rightarrow z = \pm \frac{1}{\sqrt{3}}$

Hence, $x = \pm \frac{1}{\sqrt{2}}$, $y = \pm \frac{1}{\sqrt{6}}$ and $z = \pm \frac{1}{\sqrt{3}}$

COMMON ERR!R

Few students change the order while solving A′A as they write A as the first matrix and proceed further. This leads them to a wrong solution.

5. Given, $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$

TiP

Do not get confused between symmetric and skew-symmetric matrices.
Remember that:
- *if the matrix A is symmetric, then $A = A^T$.*
- *if the matrix A is skew-symmetric, then $A^T = -A$.*

Now,
$$\frac{A+A'}{2} = \frac{1}{2}\begin{bmatrix} 2+2 & -1-2 & 1-4 \\ -2-1 & 3+3 & -2+4 \\ -4+1 & 4-2 & -3-3 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2}(A+A')' = \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix}'$$

$$= \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix}$$

$$= \frac{1}{2}(A+A')$$

= Symmetric matrix

and
$$\frac{A-A'}{2} = \frac{1}{2}\begin{bmatrix} 2-2 & -2+1 & -4-1 \\ -1+2 & 3-3 & 4+2 \\ 1+4 & -2-4 & -3+3 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2}(A-A')' = \begin{bmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{bmatrix}'$$

$$= \begin{bmatrix} 0 & 1/2 & 5/2 \\ -1/2 & 0 & -3 \\ -5/2 & 3 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{bmatrix}$$

$$= -\frac{1}{2}(A-A')$$

= Skew-symmetric matrix

$$\therefore A = \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{bmatrix}$$

Chapter Test

Multiple Choice Questions

Q 1. If $a_{ij} = |2i + 3j^2|$, then matrix $A_{2\times 2} = [a_{ij}]$ will be:

a. $\begin{bmatrix} 5 & -14 \\ 7 & 16 \end{bmatrix}$ b. $\begin{bmatrix} 5 & 14 \\ -7 & 16 \end{bmatrix}$ c. $\begin{bmatrix} 5 & 14 \\ 7 & 16 \end{bmatrix}$ d. $\begin{bmatrix} 5 & 14 \\ 7 & -16 \end{bmatrix}$

Q 2. If $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^4 = I$, then:

a. $a = 1 = 2b$ b. $a = b$ c. $a = b^2$ d. $ab = 1$

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false and Reason (R) is true

Q 3. Assertion (A): Scalar matrix $A = [a_{ij}] = \begin{cases} k; & i = j \\ 0; & i \neq j \end{cases}$; where k is a scalar, is an identity matrix when $k = 1$.
Reason (R): Every identity matrix is not a scalar matrix.

Q 4. Let A and B be two symmetric matrices of order 3.
Assertion (A): $A(BA)$ and $(AB)A$ are symmetric matrices.
Reason (R): AB is symmetric matrix, if matrix multiplication of A with B is commutative.

Case Study Based Questions

Q 5. Case Study 1

A trust fund has ₹ 35,000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Woman Welfare Association).

Let A be 1×2 matrix and B be a 2×1 matrix, representing the investment and interest rate on each bond respectively.

Based on the given information, solve the following questions:

(i) If ₹ 15,000 is invested in bond X, then find the matrix summarising investment and interest rest of X and Y respectively.

(ii) If ₹ 15,000 is invested in bond X, then find the total amount of interest received on both bonds.

Or

If the trust fund obtains an annual total interest of ₹ 3,200, then find the investment in two bonds.

(iii) If the amount of interest given to old age home is ₹ 500, then find the amount of investment in bond Y.

Q 6. Case Study 2

In a city, there are two factories A and B. Each factory produces sports clothes for boys and girls. There are three types of clothes produced in both the factories, type I, II and III. For boys, the number of units of types I, II and III respectively are 80, 70 and 65 in factory A and 85, 65 and 72 are in factory B. For girls, the number of units of types I, II and III respectively are 80, 75, 90 in factory A and 50, 55, 80 are in factory B.

Based on the above information, solve the following questions:

(i) Find the matrix form of the total production of sports clothes of each type for boys.

(ii) Find the matrix form of the total production of sports clothes of each type for girls.

(iii) Let R be a 3×2 matrix that represents the total production of sports clothes of each type for boys and girls, then find the transpose of R.

Very Short Answer Type Questions

Q 7. If $A = \begin{bmatrix} 1 \\ 5 \\ -4 \end{bmatrix}$ and $B = [3 \quad -1 \quad 6]$, then find $(AB)'$.

Q 8. Let $A=\begin{bmatrix}1 & 2\\ 3 & -5\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$ and X be a matrix such that $A=BX$, then find X.

Short Answer Type-I Questions

Q 9. If $A=\begin{bmatrix}i & 0\\ 0 & i\end{bmatrix}$, $n \in N$, then find A^{4n}.

Q 10. If $\begin{bmatrix}1/25 & 0\\ x & 1/25\end{bmatrix}=\begin{bmatrix}5 & 0\\ -a & 5\end{bmatrix}^{-2}$, then find the value of x.

Short Answer Type-II Questions

Q 11. If $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$ is the sum of a symmetric matrix B and a skew-symmetric matrix C, then find C.

Q 12. If $A=\begin{bmatrix}b & 2 & a\\ 2 & 1 & -2\\ -2 & 2 & -1\end{bmatrix}$ is a matrix satisfying $AA^T=9I_3$, then find the values of a and b respectively.

Long Answer Type Questions

Q 13. Let $A=\begin{bmatrix}\cos^2\theta & \sin\theta\cos\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$ and $B=\begin{bmatrix}\cos^2\phi & \sin\phi\cos\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$. If $AB=O$, then find the value of θ.

Q 14. If $P=\begin{bmatrix}\sqrt{3}/2 & 1/2\\ -1/2 & \sqrt{3}/2\end{bmatrix}$, $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $Q=PAP^T$, then find $P^TQ^{2005}P$.