SECTION - I

Straight Objective Type

1. $(x-a)^2$ is a factor of x^3+px+q

$$x^3-2cx+a^2\left|\begin{array}{l}\cancel{x^3}+0+px+q\\ \underline{\cancel{x^3}+2ax^2\pm a^2x}\end{array}\right|x+2a$$

$$2ax^2+(p-a^2)x+q$$

$$\underline{2ax^2\mp 4a^2x\mp 2a^3}$$

$$\underline{0}$$

$$\left(p-a^2+4a^2\right)x=0$$

$$p-3a^2=0$$

$$p=3a^2$$

$$q-2a^3=0$$

$$q=2a^3$$

Hence (c) is the correct option.

2. The homogeneous function of the second degree

$$=ax^2+bxy+cy^2$$

$2x-y$ is a factor of $ax^2+bxy+cy^2$

$$\Rightarrow (2x-y)(ax+by)=Ax^2+Bxy+Cy^2$$

$$x=y=1$$

$$(2\times 1-1)(a+b)=Ax^2+Bxy+Cy^2$$

$$a+b=A+B+C$$

Hence (b) is the correct option.

3. $15-x-6x^2=-\left(6x^2+x-15\right)$

$$=-\left(6x^2-9x+10x-15\right)$$

$= -(3x(2x-3)+5(2x-3)$

$= -(3x+5)(2x-3)$

$= (3x+5)(3-2x)$

Hence (c) is the correct option.

4. Let $f(x) = 7x^3 + 6x^2 - x + 1$

$f(2) = 7(2)^3 + 6(2)^2 - 2 + 1$

= 56 + 24 – 2 + 1

= 81 – 2 = 79

Hence the remainder is 79.

If $f(x)$ is divided by $n-a$ then the remainder is $f(a)$

Hence (a) is the correct option.

5. If $f(x) = x^2$ $\qquad g(x) = x^3$

then the value of $\dfrac{f(b)-f(a)}{g(b)-g(a)}$

$$= \frac{b^2-a^2}{b^3-a^3} = \frac{(b-a)(b+a)}{(b-a)(b^2+ab+a^2)}$$

$$= \frac{a+b}{a^2+ab+b^2}$$

Hence (b) is the correct option.

6. Let $f(x) = 5x^3 - (p+4)x^2 - px - (p+y)$

$x-4$ is a factor of f(x)

$\Rightarrow$ $f(4) = 0$

$\Rightarrow$ $f(4) = 5\left(4^3\right) - (p+4)4^2$

$-p(4) - (p+4) = 0$

$320 - 16p - 64 - 4p - p - 4 = 0$

$320 - 21p - 68 = 0$

$252 - 21p = 0$

$\Rightarrow \quad 21p = 252$

$$p = \frac{252}{21} = 12$$

Hence (b) is the correct option,

7. $x = \frac{1}{x} = 7$

$$\Rightarrow \quad \left(x - \frac{1}{x}\right)^3 = 7^3$$

$$\Rightarrow \quad x^3 - \frac{1}{x^3} - 3.x.\frac{1}{x}\left(x - \frac{1}{x}\right) = 343$$

$$x^3 \frac{1}{x^3} - 3.(7) = 343$$

$$x^3 \frac{1}{x^3} - 343 + 21 = 364$$

Hence (c) is the correct option.

8. If $x^2 - 3x + 2$ is a factor of the expression $x^4 - ax^2 + b$

$$x^2 + 3x + 2 \left| \begin{array}{l} x^4 + 0 + ax^2 + 0 + b \\ x^4 + 3x^3 + 2x^2 \end{array} \right.$$

$$-3x^3 - 2x^2 + 0$$

$$\mp 3x^3 \mp 9x^2 \mp 6x$$

$$[(a-2)+9]x^2 = 0$$

$$(a+7)x^2 = 0$$

$$a = -7$$

Hence (d) is the correct option.

9. If $a + b + c = 6$

$bc + ca + ab = 11$

$abc=6$

$(1-a)(1-b)(1-c)$

$=1-(a+b+c)+(ab+bc+ca)-abc$

$=1-6+11-6$

$=0$

Hence (c) is the correct option

10. If $x=\dfrac{a}{b+c}, y=\dfrac{b}{c+a}, z=\dfrac{c}{a+b}$

$$\frac{1}{x}=\frac{b+c}{a}$$

$$\frac{1}{y}=\frac{c+a}{b}$$

$$\frac{1}{z}=\frac{a+b}{c}$$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$$

$$=\frac{bc(b+c)+ac(a+c)-ab(a+b)}{abc}$$

Hence (a) is the correct option.

11. $8x^3+25y^3$

$=(2x)^3+(5y)^3$

$=(2x+5y)+\left(4x^2+25y^2+10xy\right)$

Hence (a) is the correct option.

12. If a polynomial $p(x)$ is divided by $x-a$ then $p(a)$ is the remainder

If $p(a)=0$

$\Rightarrow$ $x-a$ is a factor of $p(x)$ when

$p(x)=x^4-3x^2+2x+1$ is divided by

$x-1$ then the remainder is p(1)

$p(1) = 1^4 - 3(1)^2 + 2.1 + 1$

$1 - 3 + 2 + 1 = 1$

Hence (b) is the correct option

13. Let $f(x) = ax^3 + 4x^2 + 3x - 4$

$g(x) = x^3 - 4x + a$

When f(x) is divided by $x - 3$ the remainder is $f(3)$.

When $g(x)$ is divided by $x - 3$ the remainder is $g(3)$

$f(3) = g(3)$

$\Rightarrow a(3)^3 + 4(3)^2 + 3(3) - 4$

$= 3^3 - 4(3) + a$

$\Rightarrow 27a + 36 + 9 - 4 = 27 - 12 + a$

$\Rightarrow 27a + 41 = 15 + a$

$\Rightarrow 26a = -26$

$a = -1$

Hence (c) is the correct option

14. $f(x) = x^4 - 2x^3 + 3x^3 - ax + b$

If $f(x)$ is divided by $x - 1$ then the remainder is $f(1)$ and $f(x)$ is divided by $x + 1$ then the remainder is $f(-1)$

$f(1) = 1^4 - 2(1)^3 + 3(1)^2 - a(1) + b = 5$

$1 - 2 + 3 - a + b = 5$

$-a + b = 5$

$f(-1) = (-1)^4 - 2(-1)^3$

$+3(-1)^2 - a(-1) + b = 9$

$1 + 2 + 3 + a + b = 9$

$a + b = 3$

$-a + b = 3$

$\underline{\ a + b = 3\ }$

$2b = 6$

$b=3$
$a=0$
then $f(x)=x^4-2x^3+3x^2+3$
when $f(x)$ is divided by $x-2$ then/the remainder is $f(2)$
$f(2)-2^4-2.2^4+3(2)^2+3$
$=16-16+12+3$
$=15$
Hence (d) is the correct option.

15. $x^3-3x^2+4x-12$
$x^3-3x^2+4x-12$

$$\begin{array}{c|cccc} & 1 & -3 & 4 & -12 \\ 3 & 0 & 3 & 0 & 12 \\ \hline & 1 & 0 & 4 & \boxed{0} \end{array}$$

$\Rightarrow x^3-3x^2+4x-12=(x-3)(x^2+4)$
Hence, $x-3$ is a factor.
Hence (a) is the correct option

16. Let $f(x)=3x^2+k$
$x+3$ is a factor of f(x)
$\Rightarrow f(-3)=0$
$f(-3)=3(-3)^2+k=0$
$k+27=0$
$k=-27$
Hence (c) is the correct option

17. x^3+10x^2+ax+6 is exactly divisible
by $x-1$ as well as $x-2$

$$x^2-3x+2\left|\begin{array}{l} \cancel{x^3}+10x^2+ax+6 \\ \cancel{x^3}\mp 3x^3\pm 2x \end{array}\right| x+7x^2+(a-2)x+6$$

Hence (a) is the correct option.

18. If $2x^3+ax^2+11x+a$ is exactly divisible by $2x-1$ then $f\left(\frac{1}{2}\right)=0$

$\Rightarrow f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3$

$+a\left(\frac{1}{2}\right)^2+11\left(\frac{1}{2}\right)+a+3=0$

$\Rightarrow \frac{1}{4}+\frac{a}{4}+\frac{11}{2}+a+3=0$

$\Rightarrow 1+a+22+4a+12=0$

$5a+35=0$

$\Rightarrow \quad a=-7$

Hence (b) is the correct option.

19. $x-2$ and $x-\frac{1}{2}$ are factors of px^2+5x+r then

$(x-2)\left(x-\frac{1}{2}\right)=px^2+5x+r$

$x^2-x\left(2+\frac{1}{2}\right)+1=px^2+5x+r$

$\Rightarrow p=1, r=1$

$\Rightarrow p=r$

Hence (a) is the correct option.

20. x^2-1 is a factor of

$ax^4+bx^3+cx^2+dx+a$

$x-1$ and $x+1$ are factor of

$ax^4+bx^3+cx^2+dx+c$

$a+b+c+d+e=0$

and $a+c+e=b+d$

Since If $x+1$ is a factor of $f(x)$ then the sum of coefficient of all

even terms is equal to sum of odd coefficients.
Hence (b) is the correct option.

21. If $f(\mathrm{x}) = \mathrm{x}^3 - 6x^2 + 2x - 4$ is divided by

$3x - 1$ then the remainder is $f\left(\frac{1}{3}\right)$

$$f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 - 6\left(\frac{1}{3}\right)^2 + 2\left(\frac{1}{3}\right) - 4$$

$$= \frac{1}{27} - \frac{6}{9} + \frac{2}{3} - 4 = 0$$

$$= \frac{1 - 18 + 18 + 108}{27} = \frac{-107}{27}$$

Hence (c) is the correct option

22. Let $f(\mathrm{x}) = \mathrm{ax}^3 + 3x^2 - 13$

$g(\mathrm{xc}) = 2\mathrm{x}^3 - 5x + a$

$f(x)$ is divided by $x + 2$ then the remainder is $f(-2)$ and $g(x)$ divided by $x + 2$ then the remainder is $g(-2)$

$f(-2) = \mathrm{g}(-2)$

$a(-2)^3 + 3(-2)^2 - 13$

$= 2(-2)^3 - 5(-2) + \mathrm{a}$

$-8a + 12 - 13 = -16 + 10 + a$

$-99 = -5$

$a = \frac{5}{9}$

Hence (b) is the correct option.

23. Let $f(\mathrm{x}) = \mathrm{x}^3 + 2x^2 - 5ax - 7$

$g(\mathrm{x}) = \mathrm{x}^3 + ax^2 - 12x + 6$

$f(\mathrm{x})$ is divided by x $+1$ then the remainder is $f(-1)$

$\Rightarrow f(-1) = R_1$

If g(x) is divided by $x-2$ then the

Remainder is $g(2)$

$\Rightarrow g(2) = R_2$

$1+2(1)^2 - 5a(1) - 7 = R_1$

$\Rightarrow 1+2-5a-7 = R_1$

$\Rightarrow -5a-4 = R_1$

$2^3 + a(2)^2 - 12(2) + 6 - R_2$

$8+4a-24+6 = R_2$

$4a-10 = R_2$

$2R_1 + R_2 = 10a - 8 + 4a - 10 = 6$

$-6a - 18 = 6$

$-6a = 24$

Hence (d) is the correct option

24. $x^n - y^n$ is always divisible by $x-1$ for all 'n' belongs to Natural numbers

Hence (a) is the correct option

25. $x+3$ is a factor of $3x^2 + kx + 6$

$\Rightarrow 3(-3)^2 + k(-3) - 6 = 0$

$\Rightarrow -27 + 3k - 6 = 0$

$\Rightarrow -3k - 33 = 0$

$k = -11$

Hence (b) is the correct option

26. Let $f(x) = 2x^2 + ax^2 + 11x + a + 3$ is exactly divisible by $2x-1$

$\Rightarrow f\left(\frac{1}{2}\right) = 0$

$$\Rightarrow 2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + 11\left(\frac{1}{2}\right) + a + 3 = 0$$

$$\frac{1}{4} + \frac{a}{4} + \frac{11}{2} + a + 3 = 0$$

$$\frac{1 + a + 22 + 4a + 12}{4} = 0$$

$$5a + 35 = 0$$

$\Rightarrow$ $5a = -35$

$a = -7$

Hence (a) is the correct option

27. Let $f(x) = a(b^2 - c^2)$

$+b(c^2 - a^2) + c(a^2 - b^2)$

$\Rightarrow a - b$ is a factor $f(x)$

Hence (a) is the correct option

28. Let $f(x) = x^3 + ax^2 - 2x + a + 4$

$x + a$ is a factor of $f(x)$ (x)

$\Rightarrow f(-a) = 0$

$(-a)^3 + a(-a)^2 - 2(-a) + a + a + 4 = 0$

$a^3 - a^3 + 2a + a + 4 = 0$

$-2a^3 + 3a + 4 = 0$

$\Rightarrow a = 0$

Hence (b) is the correct option.

29. $x + a$ us a factor of $x^n + a^n$ for any odd positive integer

Hence (b) is the correct option

30. Let $f(x) = (x - b)^5 + (b - a)^5$

$f(a) = (x - b)^5 + (b - a)^5$

$\Rightarrow$ $a - b$ is a factor of $f(x)$

Hence (a) is the correct option

SECTION - II
Assertion - Reason Questions

31. $8x^3+125y^3=(2x)^3+(5y)^3$

$=(2x+5y)\left(4x^2+10xy+25y^2\right)$

Hence (d) is the correct option.

32. $\left(\frac{x}{y}+\frac{y}{x}\right)^2=\frac{x^2}{y^2}+2.\frac{x}{y}.\frac{y}{x}+\frac{y^2}{x^2}$

$=\frac{x^2}{y^2}+2+\frac{y^2}{x^2}$

Hence (a) is the correct option.

33. $x-3$ is a factor of the polynomial $x^3-3x^2+4x-12$ Since

3	0	−3	4	−12
	0	3	0	12
	1	0	4	0

Hence (a) is the correct option.

34. $12-9+2+1=$

Sum of even term coefficients = Sum of odd terms

$\Rightarrow x+1$ is a coefficient of the polynomial

$a-b,\ b-c,\ c-a$ are factors of

$a\left(b^2-c^2\right)+b\left(c^2-a^2\right)+c\left(a^2-b^2\right)$

Hence (b) is the correct option.

35. $x^4+2x^3-13x^2-14x+24$

Sum of coefficient of all terms

$=1+2-13-14+24=0$

Hence $x-1$ is a factor

$$
\begin{array}{r|rrrrr}
 & 1 & 2 & -13 & -14 & 24 \\
1 & 0 & 1 & 3 & -10 & -24 \\
\hline
 & 1 & 3 & -10 & -24 & \underline{|0} \\
-2 & 0 & -2 & -2 & 24 & \\
\hline
 & 1 & +1 & -12 & \underline{|0} & \\
3 & 0 & 3 & 12 & & \\
\hline
 & 1 & 4 & \underline{|0} & &
\end{array}
$$

Hence

$x^4+2x^3+13x^2-14x+24$

$=(x+1)(x+2)(x-3)(x+4)$

Hence (a) is the correct explanation.

36. The highest power of 'n' in an algoric expression is called the degree of the polynomial.

$\Rightarrow 3x^3-5x^2+8x+9$ is a polynomial in x of degree 3.

Hence (a) is the correct option.

37. $x^3+6x+5=(x+5)(x+1)$

$\Rightarrow$ $(x+1)$ is a factor of x^2+6x+5

Hence (a) is the correct option.

38. $ax-by+bx-ay$

$=ax+bx-(by+ay)$

$=x(a+b)-y(a+b)$

$=(a+b)(x-y)$

$\Rightarrow$ $(a+b)$ is factor of

$ax-by+bx-ay$

$a^2-2ab+b^2=(a-b)^2$

Hence (c) is the correct option.

39. $$\frac{\left(a^2-b^2\right)^3+\left(b^2-c^2\right)+\left(c^2-a^2\right)}{(a-b)^3+(b-c)^3+(c-a)^3}$$

If $a+b+c=0$ then

$a^3+b^3+c^3=3abc$

$a^2-b^2+b^2-c^2-a^2=0$

$\Rightarrow \left(a^2-b^2\right)^3+\left(b^2-c^2\right)^3+\left(c^2-a^2\right)^2$

$=3\left(a^2-b^2\right)\left(b^2-c^2\right)\left(c^2-a^2\right)$

$a-b+b-c+c-a=0$

$\Rightarrow (a-b)^3+(b-c)^3+(c-a)^3$

$=3(a-b)(b-c)+(c-a)$

$$\frac{\left(a^2-b^2\right)+\left(b^2-c^2\right)+\left(c^2-a^2\right)}{(a-b)^3+(b-c)^3+(c-a)^3}$$

$$=\frac{3\left(a^2-b^2\right)\left(b^2-c^2\right)\left(c^2-a^2\right)}{3(a-b)(b-c)(c-a)}$$

$=(a+b)(b+c)(c+a)$

$x-2y+2y-3z+3z-x=0$

$\Rightarrow (x-2y)^3+(2y-3z)^3+(3z-x)^3$

$=3(x-2y)(2y-3z)(3z-x)$

$\Rightarrow x-2y$ is a factor

Hence (b) is the correct option.

40. If a polynomial p(x) is divided by $ax+b$, the remainder is the value of $p(x)$ at $x=-b/a$ i.e, $p(-b/a)$

$(x-a)$ $(x-b)$ is a factor of a polynomial

$p(x)$ if $p(a)=0$ and $p(b)=0$

Hence (b) is the correct option.

Paragraph 41 to 43

41. Let $f(x) = ax^3 + bx^2 + x - 6$

$x + 2$ is a factor of $f(x)$

$\Rightarrow f(-2) = 0$

$f(-2) = a(-2)^3 + b(-2)^2 + (-2) - 6 = 0$

$-8a + 4b - 8 = 0$

$-8a + 4b = 8$

$-2a + b = 2$

when $f(x)$ is divided by $x - 2$ then it leaves the remainder 2

i.e., $f(2) = 4$

$f(2) = a(2)^3 + b(2)^2 + 2 - 6 = 4$

$8a + 4b = 8$

$2a + b = 2$

$-2a + b = 2$

$2a + b = 2$

$2b = 4$

$b = 2$

$a = 0$

Hence (b) is the correct option.

42. b = 2

Hence (c) is the correct option

43. If $a = 0, b = +2$

then $f(x) = 2x^2 + x = 6$

$2x^2 + x = 6$

$= 2x^2 + 4x - 3x - 2$

$= 2x(x+2) - 3(x+2)$

$= (2x-3)(x+2)$

$x+2$ is a factor

Hence (b) is the correct option

Paragraph 44 to 46

44. $\frac{x^2}{4y^2} - \frac{2}{3} + \frac{4y^2}{9x^2}$

$= \left(\frac{x}{2y}\right)^2 - 2 \cdot \frac{x}{2y} \cdot \frac{2y}{3x} + \left(\frac{2y}{3x}\right)^2$

$= \left(\frac{x}{2y} - \frac{2y}{3x}\right)^2$

Hence (a) is the correct option.

45. $25(3x-4y)^2 - k(9x^2-16y^2)$ is a

$+16(3x+4y)^2$

perfect square

$\Rightarrow 5(3x-4y)^2 - 2 \cdot 5 \cdot 4(3x-4y)$ is a

$(3x+4y) + 4(3x+4y)^2$

perfect square

$\Rightarrow k = 2 \times 5 \times 4$

$= 40$

Hence (c) is the correct option.

46. If $\left(x^2 + \frac{1}{x^2}\right) - 4\left(x + \frac{1}{x}\right) + k$ is a

perfect square

i.e., $\left(x^2 + \frac{1}{x}\right)^2 - 2 - 4\left(x + \frac{1}{x}\right) + 6$

$$\left(x^2+\frac{1}{x}\right)^2-4\left(x+\frac{1}{x}\right)+4=\left(x+\frac{1}{x}-2\right)^2$$

$\Rightarrow k=6$

Hence (b) is the correct option.

Paragraph 47 to 49

47. When $f(\text{x})=\text{x}^2+4x+5$ is divided by $x-5$ then the remainder is $f(5)$

$f(5)=5^2+4(5)+5$

$=25+20+5$

$=45+5$

$=50$

Hence (c) is the correct option.

48. When $f(\text{x})=\text{x}^3+5x-3$ is divided by $2x-1$ then the remainder is $f\left(\frac{1}{2}\right)$

$$f\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3+5\left(\frac{1}{2}\right)-3$$

$$=\frac{1}{8}+\frac{5}{2}-3$$

$$=\frac{1+20-2y}{8}=\frac{-3}{8}$$

Hence (b) is the correct option.

49. Let $f(\text{x})=(\text{a}-\text{b})\text{x}^2+(\text{b}-\text{c})\text{x}+(\text{c}-\text{a})$ is divided by $x-1$ then the remainder is $f(1)$

$f(1)=(\text{a}-\text{b})1^2+(\text{b}-\text{c})1+(\text{c}-\text{c})=0$

Hence (c) is the correct option.

Paragraph 50 to 52

50. $x-y+y-z+z-x=0$

$\Rightarrow (x-y)^3+(y-z)^3+(z-x)^3$

$=3(x-y)(y-z)(z-x)$

$x-y$ is a factor of

$(x-y)^3+(y-z)+(z-x)^3$

Hence (a) is the correct option.

51. $p(q-r)+q(r-p)+r(p-q)=0$

$\Rightarrow p^3(q-r)^3+q^3(r-p)^3+r(p-q)^3$

$=3pqr(q-r)(r-p)(p-q)$

Hence p, $p-q$ are the factor of

$p^3(q-r)^3+q^3(r-p)^3+r^3(p-q)^3$

Hence (b) are correct options.

52. $a^2-b^2+b^2-c^2-a^2=0$

$\Rightarrow \left(a^2-b^2\right)+\left(b^2-c^2\right)^3+\left(c^2-a^2\right)^3$

$=3\left(a^2-b^2\right)\left(b^2-c^2\right)\left(c^2-a^2\right)=3(a-b)(a+b)(b-c)$

$(b+c)(c-a)(a+a)$

Hence option (a) are correct options.

Paragraph 53 to 55

53. If $\frac{x^2-1}{x}=4$

$\Rightarrow \frac{x-1}{x}=4$

$\Rightarrow \frac{x^6-1}{x^3}=x^3-\frac{1}{x^3}$

$$= \left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right)$$

$$= 4\left(\left(x + \frac{1}{x}\right)^2 + 2 + 1\right)$$

$$= 4(4^2 + 2 + 1)$$

$$= 4(16 + 3) = 76$$

Hence (a) is the correct option.

54. The value of

$$216 - 144x + 108x^2 - 27x^3$$

at $x = 3$ is

$$= 216 - 144 + 3 + 108\left(3^2\right) - 27(3^3)$$

$$= 216 - 372 + 972 - 729$$

$$= 27$$

Hence (b) is the correct option.

55. $(6a - 5b)^3 - (3a - 4b)^3$

$-3(3a - b)(6a - 5b)(3a - 4b)$

$$= (6a - 5b)^3 - (3a - 4b)^3 - 3(3a - b)$$

$(6a - 5b)(-3b + 4b + 6a + 5b)$

$$= [(6a - 5b) - (3a - 4b)]^3$$

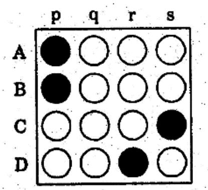
$$= [3a - b]^3$$

If $3a - b = 0$

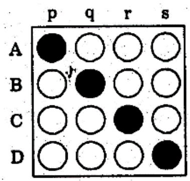
$\Rightarrow (3a - b)^3 = 0$

Hence (c) is the correct option.

56.



57.



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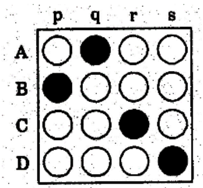
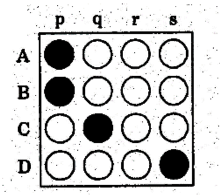


59.



60.