Chapter 3

Theory of Stresses in Beams

CHAPTER HIGHLIGHTS

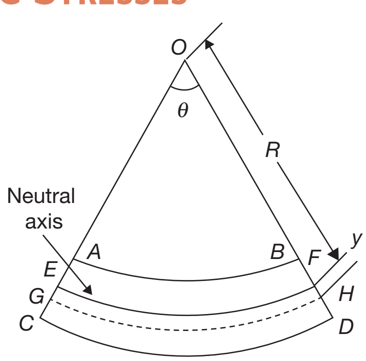
- *Introduction*
- *Bending stresses*
- *Centroid and moment of inertia of some plane figures*
- *Shearing stresses in beams*

Introduction

Stresses in Beams

When beams are subjected to bending moment and shear forces internal stresses are developed in them. Simple bending theory deals with finding stresses due to pure moment alone. When a beam is subjected to bending moment it sags or hogs. When it sags fibres at the bottom are stretched and fibres at top are compressed. In other words, tensile stresses are developed at the bottom and compressive stresses are developed at the top. When it hogs, the reverse happens. We can identify a layer in between called neutral layer on which there shall be neither compression nor tension, and consequently stresses are zero.

Bending Stresses



After bending, cross-sections AC and BD subtend an angle θ at O. Let, R be the radius of the neutral plane represented by EF. Then, $EF = R\theta$.

Now, consider the layer GH at the bottom of the neutral plane at a distance y from it:

$$GH = (R + y)\theta$$

Please note that before bending, its length is equal to $EF = R\theta$.

Therefore,

$$\text{Strain in } GH = \frac{\text{Final length} - \text{Original length}}{\text{Original length}}$$

$$= \frac{(R + y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

If f = The bending stress

$$\frac{\text{Stress}}{\text{Strain}} = E = \text{Young's modulus}$$

That is,
$$\frac{f}{\frac{y}{R}} = E$$

or $$\frac{f}{E} = \frac{y}{R}$$

or $$f = \frac{E}{R}y$$

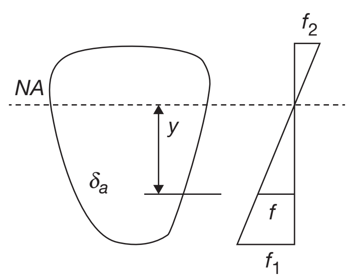
Neutral Axis

After bending, there is tension in the convex surface and compression in the concave surface. Therefore, between top and bottom, there is a layer at which there is no strain. This layer is known as neutral layer. The line of intersection of cross-section with the neutral layer is called 'neutral axis'.

It is seen that stress is varying linearly with distance y from the neutral axis. It can be shown that the neutral axis coincides with the centroid of the cross-section.

Position of Neutral Axis

Consider a beam with arbitrary cross-section as shown in the following figure.



Stress is varying linearly across the depth. Consider an element area δa at a distance y from the neutral axis. Let f be the stress on this area.

Force on the elemental area $= f\,\delta a$

Total force on the cross-section $= \Sigma f\,\delta a$

But, $f = \frac{E}{R}y$

$\therefore$ Total force $= \Sigma \frac{E}{R} y\,\delta a = \frac{E}{R}\Sigma y\delta a$

Since there are no axial forces acting on the beam, for equilibrium:

$$\frac{E}{R}\sum y\delta a = 0,$$

or $$\Sigma y\delta a = 0.$$

If A is the total area,

$$\frac{\Sigma y\delta a}{A} = 0$$

But, $\frac{\Sigma y\delta a}{A}$ is the distance of centroid from the neutral axis.

Therefore, the neutral axis coincides with the centroid of the cross-section.

Centroid or Centre of Area

Centroid or centre of area is the point where the whole area is assumed to be concentrated.

Moment of Inertia

Area moment of inertia is the second moment of the area with respect to an axis. With respect to X-axis, it is $I_{xx} = \int dAx^2$, and with respect to Y-axis, it is $I_{yy} = \int dAy^2$.

Where, x and y are the distance from the corresponding axes.

Mass moment of inertia,

$$I = mk^2$$

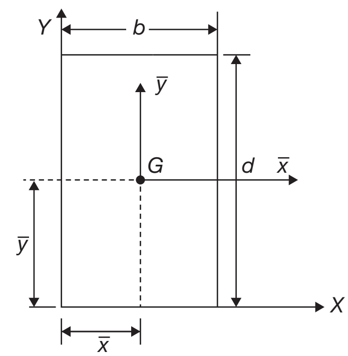
Where

m = Mass

k = Radius of gyration

Centroid and Moment of Inertia of Some Plane Figures

Rectangle



Position of centroid G:

$$\bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{d}{2}$$

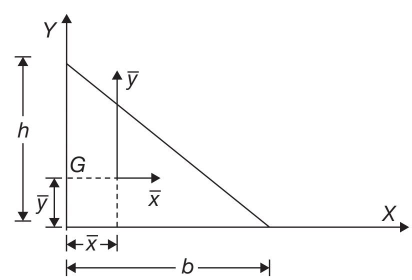
Moment of inertia about horizontal axis passing through G:

$$I_{xx} = \frac{bd^3}{12}$$

Moment of inertia about vertical axis passing through G:

$$I_{yy} = \frac{db^3}{12}$$

Triangle

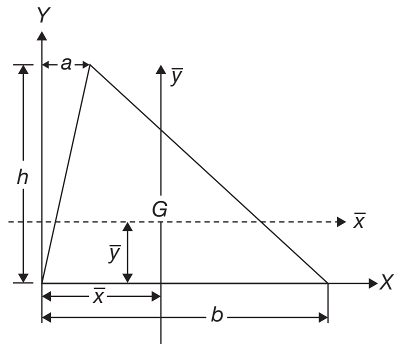


$$\bar{x} = \frac{b}{3}, \qquad \bar{y} = \frac{h}{3}$$

Moment of inertia about axes through G:

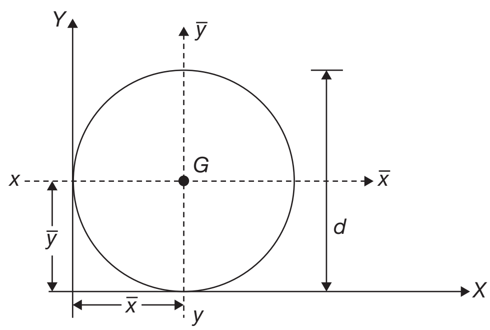
$$I_{\bar{x}} = \frac{bh^3}{36}, \qquad I_{\bar{y}} = \frac{hb^3}{36}$$

For any triangle as shown in the following figure:



$$\bar{x} = \frac{(a+b)}{3} \qquad \bar{y} = \frac{h}{3} \qquad I_{\bar{x}} = \frac{bh^3}{36}.$$

Circle

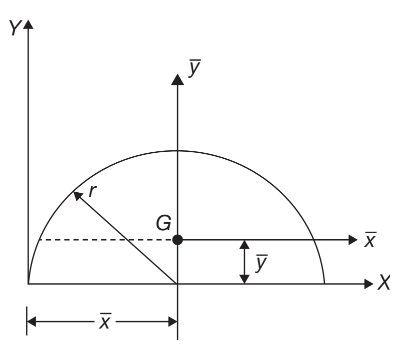


$$\bar{x} = \frac{d}{2}, \bar{y} = \frac{d}{2}$$

$$I_{\bar{x}} = I_{\bar{y}} = \frac{\pi d^4}{64} = \frac{\pi r^4}{4},$$

where, $r = \text{radius} = \frac{d}{2}$.

Semi-circle



$$\bar{x} = r, \qquad \bar{y} = \frac{4r}{3\pi}$$

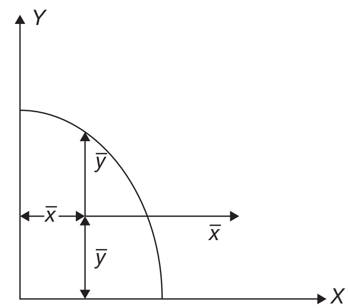
$$I_{\bar{y}} = \frac{\pi r^4}{8}$$

$$I_{\bar{x}} = (9\pi^2 - 64)\frac{r^4}{72\pi} \approx 0.11 r^4$$

Moment of inertia about X-axis:

$$I_x = \frac{\pi r^4}{8}.$$

Quadrant



$$\bar{x} = \bar{y} = \frac{4r}{3\pi}$$

$$I_{\bar{x}} = I_{\bar{y}} = \frac{(9\pi^2 - 64)r^4}{144\pi} \approx \frac{0.11}{2} r^4 \approx \frac{0.1}{2} r^4$$

$$I_x = I_y = \frac{\pi r^4}{16}.$$

Centroids of Solid Figures

1. Solid right-circular cone of height h:

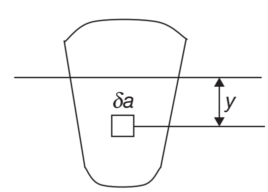
   Height of centroid from base $= \frac{h}{4}$.
2. Hollow right-circular cone of height h:

   Height of centroid from base $= \frac{h}{3}$.
3. Solid hemisphere of radius R:

   Height of centroid from base $= \frac{3R}{8}$.
4. Hollow hemisphere of radius R:

   Height of centroid from base $= \frac{R}{2}$.

Relationship between Bending Moment and Radius of Curvature



Considering an element of area δa at a distance y from the neutral axis,

Stress on the element, $f = \frac{E}{R} y$

Force on the element $= f\delta a = \frac{E}{R} y \delta a$

Moment of the force about neutral axis $= \frac{E}{R} y^2 \delta a$

Total moment of resistance $= M' = \sum \frac{E}{R} y^2 \delta a$

$$= \frac{E}{R} \sum y^2 \delta a$$

But,

$\Sigma y^2 da = I =$ Moment of inertia, or second moment of area about centroid.

$$\therefore M' = \frac{E}{R} \cdot I$$

But, $M' = M$, the applied moment. So,

$$\boxed{\frac{M}{I} = \frac{E}{R} = \frac{f}{y}}$$

Moment of Resistance of a Section

The stress is maximum on the extreme end or the cross-section where y is maximum.

Let, f_p be the maximum permissible stress of the material. Then the maximum stress should not exceed the maximum permissible stress,

or $\quad f_{\max} \le f_p,$

or $\quad \frac{M}{I} y_{\max} \le f_p,$

or $\quad M = \frac{I}{y_{\max}} f_p,$

where M is the moment carrying capacity of the section.

$\frac{I}{y_{\max}} = z$ is the section modulus of the cross-section.

$$\boxed{\therefore M = f_p \cdot z}$$

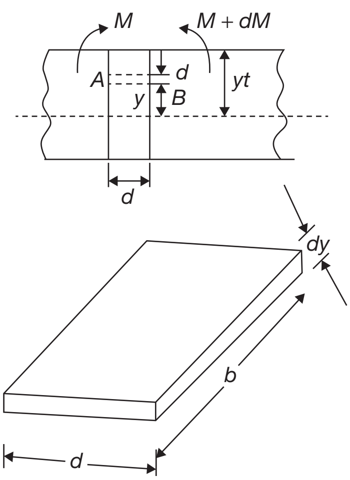
Section modulus of different section:

1. Rectangular section: $\frac{bd^2}{6}$
2. Hollow section: $\frac{1}{6}\frac{(BD^3 - bd^3)}{D}$
3. Circular section: $\frac{\pi d^3}{32}$
4. Triangular section: $\frac{bh^2}{24}$

Application of Bending Equation

The equation, $\frac{M}{I} = \frac{E}{R} = \frac{f}{y}$, is on the assumption that bending moment M is constant in a section throughout and no shear force is acting on the sections. But this is not the actual situation. There shear force and bending moment varies. So, application of bending equation has some limitations. As shear force is zero when bending moment is maximum, the equation can be applied at this situation.

Shearing Stresses in Beams



In a beam, consider an elemental length of length dx. Moments acting at the two sides of this element $= M$ and $M + dM$.

Bending stress at the left side of the element $= \frac{My}{I}$.

Corresponding force on the element $= \frac{M}{I} y\, b\, dy,$

where, $b =$ Breadth of the beam.

Force on the right side of the element due to the moment

$M + dM = \frac{M + dM}{I} y\, b\, dy.$

Unbalanced force towards the right

$$= \frac{M + dM}{I} y\, b\, dy - \frac{M}{I} y\, b\, dy = \frac{dM}{I} yb\, dy.$$

Total unbalanced forces acting above the section $AB = \int_y^{yt} \frac{dM}{I} y\, b\, dy.$

This force is resisted by shearing stresses in the plane at y, i.e., at AB. If the intensity of shearing stress is, q, $qb\, dx$

$$= \int_y^{yt} \frac{dM}{I} yb\, dy.$$

$$q = \frac{dM}{dx}\frac{1}{bI}\int_{y}^{yt} da \text{, where } da = bdy = \text{area of the element.}$$

But,

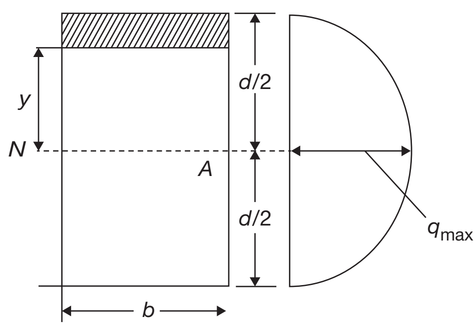
$\int_{y}^{yt} da = a\bar{y}$ = Moment of area above the section about the neutral axis and $\frac{dM}{dx} = F$.

$$\therefore \qquad q = \frac{F}{bI}a\bar{y}$$

Shear Stress Distribution across a Rectangular Section

We know that $q = \frac{F}{bI}a\bar{y}$.

Consider a rectangular section as shown in the following figure.



In the above expression, q, the shear stress is at a distance y from the neutral axis $a\bar{y}$ is the moment of area above this section.

That is, $a = \left(\frac{d}{2} - y\right)b$

$$\bar{y} = y + \left(\frac{d}{2} - y\right)\frac{1}{2} = \frac{1}{2}\left(\frac{d}{2} + y\right)$$

$$I = \frac{1}{12}bd^3$$

By substituting, we get: $q = \frac{6F}{bd^3}\left(\frac{d^2}{4} - y^2\right)$

Variation of shear stress is parabolic as shown in the figure.

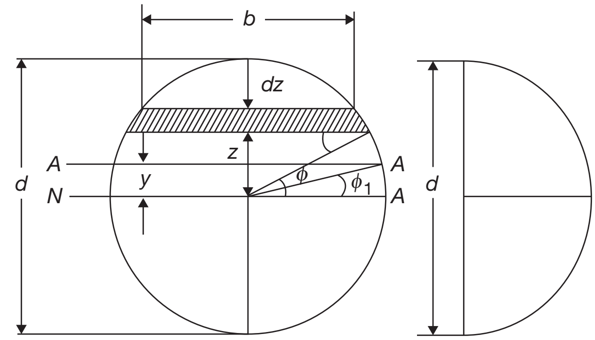
When $y = \pm\frac{d}{2}$, $q = 0$, also when $y = 0$, q is maximum

$$\int q_{max} = \frac{6F}{bd^3}\frac{d^2}{4}$$

$$= 1.5\frac{F}{bd} = 1.5q_{av}$$

$$\left(\frac{F}{bd} = \frac{\text{Shearing force}}{\text{Area}} = \text{Average shearing stress}\right)$$

Shear Stress Distribution across a Circular Section



In the given circular section of diameter d, shear stress is to be determined at a section AA at a distance y from the neutral axis NA. For finding the moment of the area above AA, an element of thickness dz at a vertical distance z from neutral axis is considered.

Width of the element:

$$b = d\cos\phi,\ z = \frac{d}{2}\sin\phi,\ dz = \frac{d}{2}\cos\phi\, d\phi$$

Area of element $= bdz = d\cos\phi \times \frac{d}{2}\cos\phi\, d\phi$

$$= \frac{d^2}{2}\cos^2\phi\, d\phi$$

Moment of area of the element = (Area) · z

$$= \frac{d^2}{2}\cos^2\phi\, d\phi \times \frac{d}{2}\sin\phi.$$

Moment of area above section AA about the neutral axis:

$$a\bar{y} = \int_{\phi_1}^{\pi/2}\frac{d^3}{4}\cos^2\phi\sin\phi\, d\phi$$

$$= \frac{d^3}{4}\left[\frac{-\cos^3\phi}{3}\right]_{\phi_1}^{\pi/2}$$

$$= \frac{d^3}{12}\left[-\cos^3\frac{\pi}{2} + \cos^3\phi_1\right]$$

$$= \frac{d^3}{12}\cos^3\phi_1$$

Shearing stress of on AA is: $q = \frac{F}{bI}a\bar{y}$

$$= \frac{16}{3}\times\frac{F}{\pi d^2}\cos^2\phi_1$$

$$= \frac{16}{3}\times\frac{F}{\pi d^2}(1 - \sin^2\phi_1)$$

$$= \frac{16}{3}\times\frac{F}{\pi d^2}\left[1 - \left(\frac{y}{\frac{d}{2}}\right)^2\right] = \frac{16}{3}\times\frac{F}{\pi d^2}\left(1 - \frac{4y^2}{d^2}\right)$$

That is, shear stress varies parabolically. Its value is maximum when $y = 0$, i.e., at neutral axis and is given by:

$$q_{\max} = \frac{16}{3}\frac{F}{\pi d^2} = \frac{4}{3}\frac{F}{\frac{\pi}{4}d^2} = \frac{4}{3}\frac{F}{A} = \frac{4}{3}q_{av},$$

Where

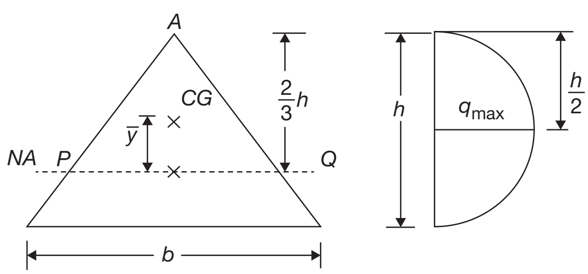
A = Area of cross-section

Q_{av} = Average shear stress

Shear Stress Distribution across Triangular Cross-sections

Let width at neutral axis PQ be b'.

Then, $q_{NA} = \frac{E}{b'I}a\bar{y}$.



Area above the neutral axis (NA):

$$A = \frac{1}{2}PQ \times \frac{2}{3}h$$

$$= \frac{1}{2} \times \frac{2}{3}b \times \frac{2}{3}h \;\; = \frac{2}{9}bh$$

$$\bar{y} = \frac{2}{3}h \times \frac{1}{3} = \frac{2}{9}h$$

$$b' = \frac{2}{3}b \qquad I = \frac{bh^3}{36}$$

$$\therefore q_{NA} = \frac{F \times \frac{2}{9}bh \times \frac{2}{9}h}{\frac{2}{3}b \times \frac{bh^3}{36}}$$

$$= \frac{4}{3} \times \frac{F}{\left(\frac{1}{2}bh\right)} = \frac{4}{3}q_{av}$$

It can be shown that the shear stress distribution is:

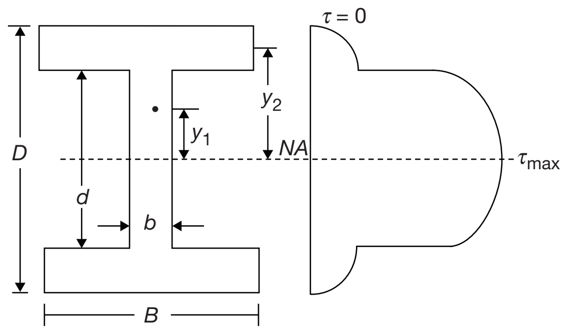
$$q = \frac{12F}{bh^3}y(h-y),$$

where, y is the distance from the top fibre

$$q_{\max} \text{ occurs at } y = \frac{h}{2} \text{ and } q_{\max} = 1.5\, q_{av}$$

Shear Stress Distribution across Symmetric

I–Section



Shear Stress Distribution

The *I*–section is symmetric about the neutral axis and shear stress q is maximum at the neutral axis (NA).

In the formula for shear stress:

$$q = \frac{F}{b'I}a\bar{y}$$

b' is the width where the shear stress is to be calculated. For calculation of shear stress at neutral axis, $b' = b$.

Moment of inertia for the section is:

$$I = \frac{BD^2}{12} - \frac{(B-b)d^2}{12}$$

$$a\bar{y} = a_1y_1 + a_2y_2,$$

Where

$$a_1 = \frac{bd}{2}$$

$$y_1 = \frac{d}{4}$$

$$a_2 = B\frac{(D-d)}{2}$$

$$y_2 = \frac{d}{2} + \frac{(D-d)}{4} = \frac{D+d}{4}$$

$$\therefore a\bar{y} = \frac{bd}{2}\cdot\frac{d}{4} + \frac{B}{2}(D-d)\frac{(D+d)}{4}$$

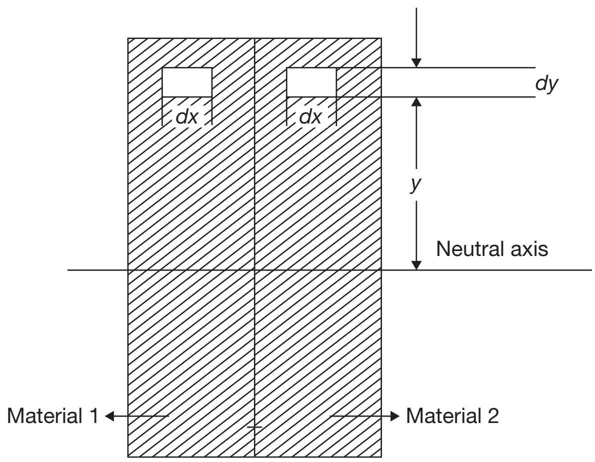
$$= \frac{1}{8}[bd^2 + B(D^2-d^2)]$$

$$q_{\max} = q_{NA} = \frac{F[B(D^2-d^2)+bd^2]}{8bI}$$

At $y = \frac{d}{2}$, there are two widths B and b, and correspondingly there are two values for the shear stress. The shear stress distribution for the sections will be as shown in the given figure.

Composite Beams

Composite beams are beams of more than one material rigidly connected, so that there is no slip at the common faces. When subjected to stresses, the strain in each part will be the same. These are also called 'fletched beams'.



Since the strains are same, $\dfrac{f_1}{E_1} = \dfrac{f_2}{E_2}$ or

$$f_1 = f_2 \times \frac{E_1}{E_2} = f_2 \times m,$$

where $m = \dfrac{E_1}{E_2}$ the modular ratio.

Comparing the moments of resistances of the elemental identical areas:

$$\delta M_1 = (f_1 dxdy)y = (f_2 mdxdy)y = m(f_2 dxdy)y$$

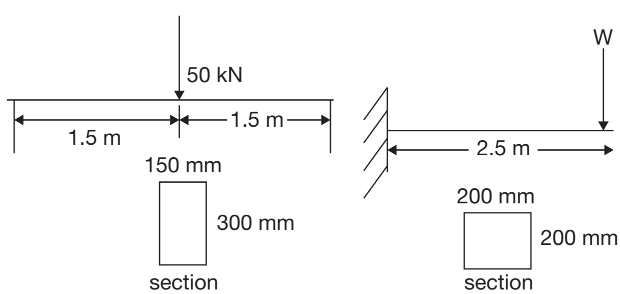
But, $\delta M_2 = (f_2 dxdy)y$.

It is observed that δM_1 is m times of δM_2.

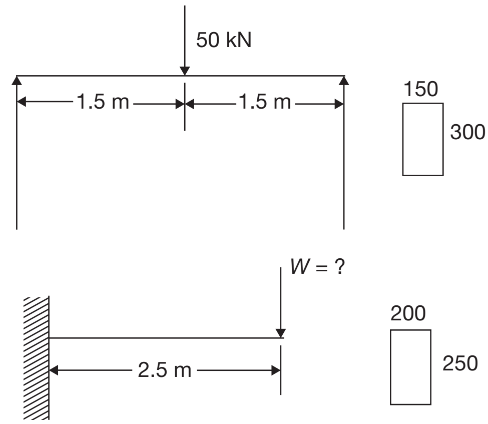
SOLVED EXAMPLES

Example 1

A beam of cross-section 150 × 300 mm can support a maximum load of 50 kN at its centre when it is used as a simply supported beam of 3 m span. If the same material is used for a cantilever of length 2.5 m and cross-section 200 × 250 mm, shown in the figure, then what is the maximum load it can support at its free end?



Solution



Maximum moment in simply supported beam $= \dfrac{WL}{4}$

$$= \frac{50 \times 3}{4} = 37.5\,\text{kN} - M = 37.5 \times 10^6\ \text{N/mm}.$$

If f is the stress at failure, $M = fz$

That is, $37.5 \times 10^6 = f \times \dfrac{1}{6} bd^2$

$$= f \times \frac{150 \times 300^2}{6}$$

$$\therefore f = \frac{37.5 \times 10^6 \times 6}{150 \times 300^2}$$

$$= \frac{375 \times 6}{15 \times 9} = 16.67\ \text{N/mm}^2$$

Maximum load the cantilever can take is calculated using the above stress.

Max moment = WL

$$= W \times 2.5 \times 10^6\ \text{Nmm}\ (W \text{ in kN}) = fz$$

$$z = \frac{1}{6} \times 200 \times 250^2$$

$$\therefore W_x\ 2.5 \times 10^6 = 16.67 \times \frac{1}{6} \times 200 \times 250^2$$

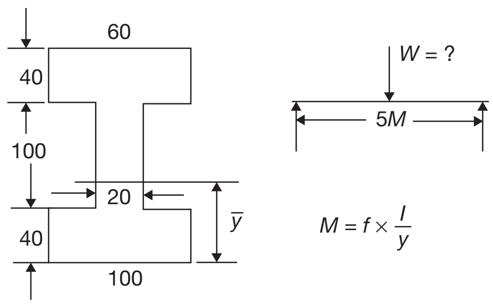
$$W \times 250 = \frac{16.67}{6} \times 2 \times 25^2$$

$$W = 13.89\ \text{kN}$$

Example 2

A cast iron beam of cross-section shown in the figure, with 5 m length, is simply supported at the ends. Find out the maximum concentrated central load it can take if permissible stresses are 28 N/mm^2 (tensile) and 80 N/mm^2 (compressive).

Solution



Moment carrying capacity of the beam is to be found out. For this, distance y from the centroid is to be found out for both tension and compression. So, position of the centroid is to be located first.

$$\bar{y} = \frac{\Sigma ay}{A} = \frac{(60\times 40\times 160)+(100\times 20\times 90)+(100\times 40\times 20)}{60\times 40+100\times 20+40\times 100}$$

$$= 76.67 \text{ mm}$$

$$I = \frac{1}{12}\times 60\times 40^3 + (160-76.67)^2\times 60\times 40$$

$$+\frac{1}{12}\times 20\times 100^3 + (90-76.67)^2\times 100\times 20$$

$$+\frac{1}{12}\times 100\times 40^3 + (20-76.67)^2\times 100\times 40$$

$$= 16{,}985{,}333.36 + 2{,}022{,}044.47 + 13{,}379{,}288.93$$

$$= 32{,}386{,}666.76,\quad M = f\times\frac{I}{y}$$

For tensile stress,

$$M = \frac{28\times 32{,}386{,}667}{76.67} = 11{,}827{,}660$$

For compressive stress,

$$M = \frac{80\times 32{,}386{,}667}{(180-76.67)} = 25{,}074{,}358$$

Choosing the smaller,

$$M = 11{,}827{,}660 \text{ N/mm} = \frac{WL}{4},$$

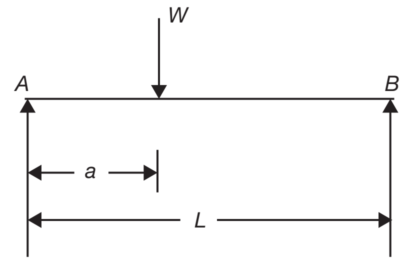
where, W = Load and L = Span = 5 m

$$\therefore W = \frac{11{,}827{,}660\times 4}{10^3\times 5}\text{ N} = 9462.13 \text{ N}.$$

Example 3

A wooden beam 200 mm × 250 mm is simply supported over a span of 6 m. When a concentrated load of 'W' is placed at a distance 'a' from the support, the maximum bending stress is 12 N/mm^2, and maximum shear stress is 0.8 N/mm^2. Determine values of 'W' and 'a'.

Solution



$$R_A = \frac{W(L-a)}{L}$$

$$R_B = \frac{Wa}{L}$$

R_A will be greater than R_B, if $a < L-a$.

Maximum shear: $F = \dfrac{W(L-a)}{L}$

Maximum bending moment: $M = \dfrac{w(L-a)}{L}a$

Maximum shear stress = 1.5 × Average shear stress

$$= 1.5\times\frac{F}{\text{Area}} = 1.5\times\frac{F}{200\times 250}$$

$$= 0.8 \text{ N/mm}^2 \text{ (given)}$$

$$\therefore F = \frac{0.8\times 200\times 250}{1.5}$$

That is,

$$\frac{W(L-a)}{L} = \frac{0.8\times 200\times 250}{1.5} = 26{,}667 \qquad (1)$$

Maximum moment $M = fz$, where

f = Maximum bending stress

= 12 N/mm^2 (given)

z = Section modulus $= \dfrac{bd^2}{6}$

That is,

$$\frac{W(L-a)a}{L} = \frac{12\times 200\times (250)^2}{6} = 25\times 10^6 \qquad (2)$$

By dividing Eq. (2) by Eq. (1):

$$a = \frac{25\times 10^6}{26{,}667} = 937.49 \text{ mm}$$

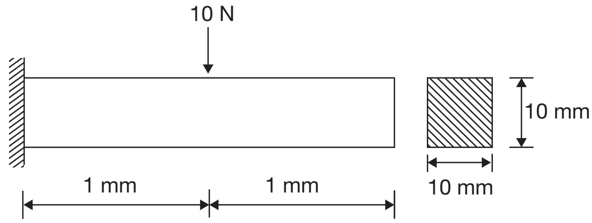
Substituting in Eq. (1):

$$W\frac{(6000-937.49)}{6000} = 26{,}667,$$

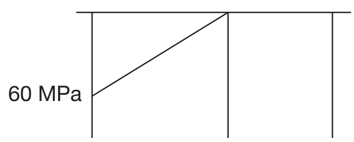
or $\qquad W = 31{,}605$ N.

Exercises

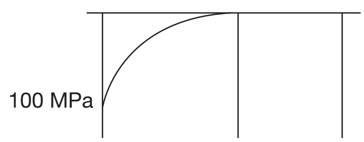
1. A cantilever beam has the square cross-section of 10 mm × 10 mm. It carries a transverse load of 10 N. Considering only the bottom fibres of the beam, the correct representation of the longitudinal variation of the bending stress is



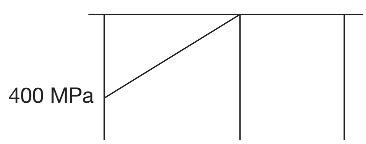
(A)



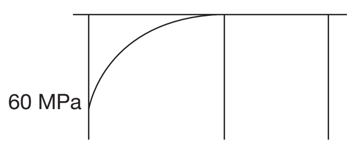
(B)



(C)



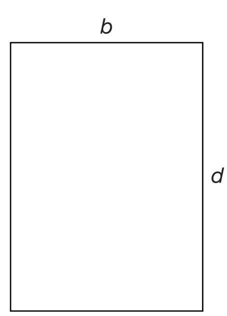
(D)



2. A simply supported beam with rectangular cross-section 100 mm × 200 mm has a span of 5 m. The permissible bending and shearing stress are 12 N/mm^2 and 0.8 N/mm^2 respectively. The maximum uniformly distributed load it can carry is

(A) 3.15 kN/m (B) 2.56 kN/m
(C) 2.82 kN/m (D) 5.33 kN/m

3. For the section shown in the following figure, second moment of the area about an axis $d/4$ distance above the bottom of the area is



(A) $\dfrac{bd^3}{48}$ (B) $\dfrac{bd^3}{12}$
(C) $\dfrac{7bd^3}{48}$ (D) $\dfrac{bd^3}{3}$

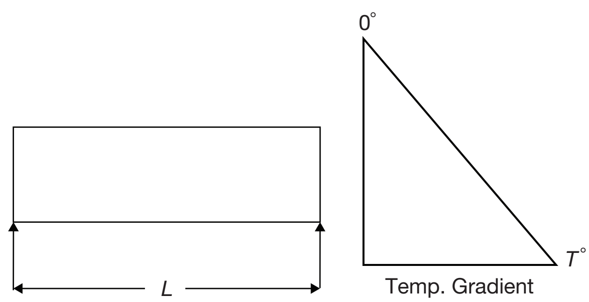
4. A homogeneous, simply supported prismatic beam of width B, depth D and span L is subjected to a concentrated load of magnitude P. The load can be placed anywhere along the span of the beam. The maximum flexural stress developed in beam is

(A) $\dfrac{2PL}{3BD^2}$ (B) $\dfrac{3PL}{4BD^2}$
(C) $\dfrac{4PL}{3BD^2}$ (D) $\dfrac{3PL}{2BD^2}$

5. The maximum bending stress induced in a steel wire of modulus of elasticity 200 kN/mm^2 and diameter 1 mm when would on a drum of diameter 1 m, is approximately equal to

(A) 50 N/mm^2 (B) 100 N/mm^2
(C) 200 N/mm^2 (D) 400 N/mm^2

6. A simply supported beam of uniform rectangular cross-section of width b and depth h is subjected to linear temperature gradient, 0° at the top and T° at the bottom, as shown in the figure. The coefficient of linear expansion of the beam material is α. The resulting vertical deflection at the mid-span of the beam is



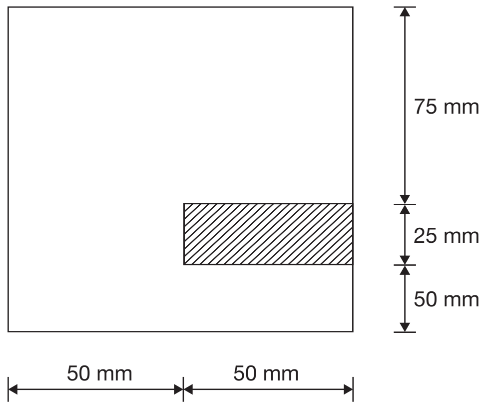
(A) $\dfrac{\alpha Th^2}{8L}$ upward

(B) $\dfrac{\alpha Th^2}{8h}$ upward

(C) $\dfrac{\alpha Th^2}{8L}$ downward

(D) $\dfrac{\alpha TL^2}{8h}$ downward

7. A beam with the cross-section shown in the following figure is subjected to a positive bending moment (causing compression at the top) of 16 kN-m acting around the horizontal axis. The tensile force acting on the hatched area of the cross-section is



(A) zero (B) 5.9 kN
(C) 8.9 kN (D) 17.8 kN

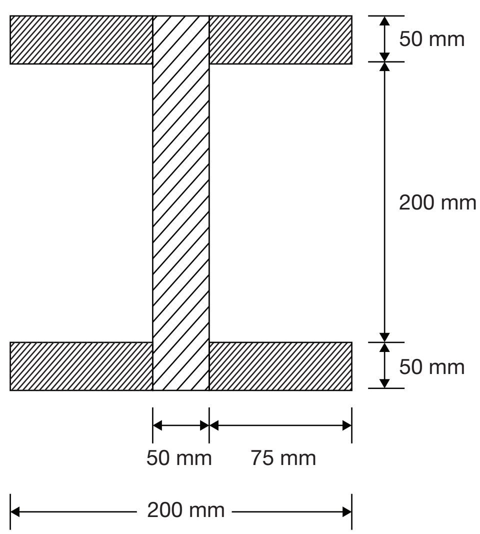
8. For a given shear force across a symmetrical 'I' section the intensity of shear stress is maximum at the
(A) extreme fibres.
(B) centroid of the section.
(C) at the junction of the flange and the web, but on the web.
(D) at the junction of the flange and the web, but on the flange.

9. If a beam of rectangular cross-section is subjected to a vertical shear force V, the shear force carried by the upper one-third of the cross-section is

(A) zero (B) $\frac{7V}{27}$

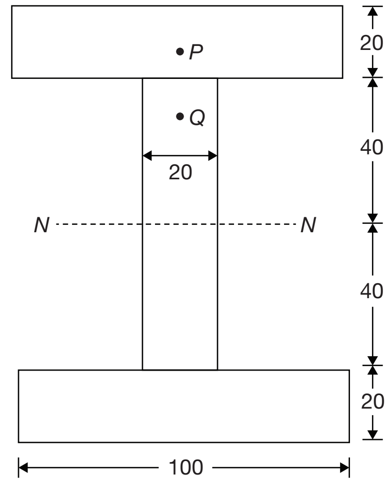
(C) $\frac{8V}{27}$ (D) $\frac{V}{3}$

10. I-section of a beam is formed by gluing wooden planks as shown in the following figure. If this beam transmits a constant vertical shear force of 3000 N, the glue at any of the four joints will be subjected to a shear force (in kN/m length) of



(A) 3.0 (B) 4.0
(C) 8.0 (D) 10.7

11. The given figure (all dimensions are in mm) shows an I-section of the beam.



The shear stress at point P (very close to the bottom of the flange) is 12 MPa. The stress at point Q in the web (very close to the flange) is
(A) indeterminable due to incomplete data.
(B) 60 MPa
(C) 18 MPa
(D) 12 MPa

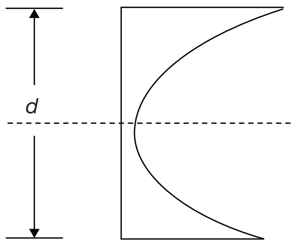
12. At a section of a beam, shear force is F with zero BM. The cross-section is square with side 'a'. Point A lies on neutral axis and point B is mid way between neutral axis and top edge, i.e., at distance $a/4$ above the neutral axis. If τ_A and τ_B denote shear stresses at points A and B, then what is the value of τ_A/τ_B?
(A) 0
(B) 3/4
(C) 4/3
(D) None of these

13. Two beams of same material have equal cross-sectional area. If one beam has square cross-section and the other has circular cross-section,
(A) both the beam will be equally strong.
(B) circular section will be stronger.
(C) square section will be stronger.
(D) strength depends on loading condition.

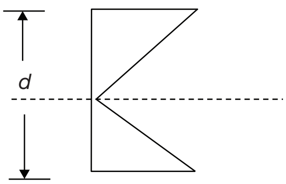
14. Two shafts 'A' and 'B' are made of same materials. The diameter of shaft 'A' is twice that of shaft 'B'. What is the ratio of power transmitted by shafts 'A' to that by 'B'?
(A) 2 : 1 (B) 4 : 1
(C) 8 : 1 (D) 16 : 1

15. The shear stress distribution diagram of a beam of rectangular cross-section, subjected to transverse loading will be

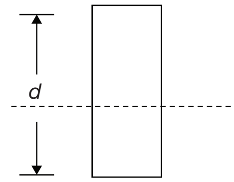
(A)



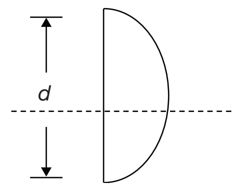
(B)



(C)



(D)



Where '*d*' is the depth of the beam

PREVIOUS YEARS' QUESTIONS

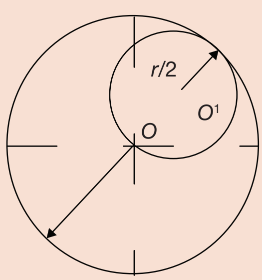
1. The shear stress at the neutral axis in a beam of triangular sections with a base of 40 mm and height of 20 mm, subjected to a shear force of 3 kN is **[GATE, 2007]**

(A) 3 MPa
(B) 6 MPa
(C) 10 MPa
(D) 20 MPa

2. The point within the cross sectional plane of a beam through which the resultant of the external on the beam has to pass through to ensure pure bending without twisting of the cross-section of the beam is called **[GATE, 2009]**

(A) moment center (B) centroid
(C) shear center (D) elastic center

3. A disc of radius '*r*' has a hole of radius '*r*/2' cut-out as shown. The centroid of the remaining disc (shaded portion) at a radial distance from the center '*O*' is **[GATE, 2011]**



(A) $\frac{r}{2}$ (B) $\frac{r}{3}$

(C) $\frac{r}{6}$ (D) $\frac{r}{8}$

4. The 'plane section remains plane' assumption in bending theory implies: **[GATE, 2013]**

(A) Strain profile is linear
(B) Stress profile is linear
(C) Both strain and stress profiles are linear
(D) Shear deformations are neglected

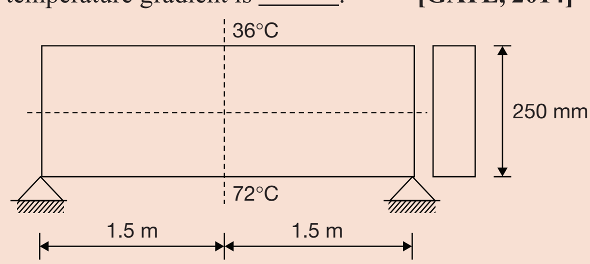
5. A symmetric I-section (with width of each flange = 50 mm, thickness of each flange = 10 mm, depth of web = 100 mm, and thickness of web = 10 mm) of steel is subjected to a shear force of 100 kN. Find the magnitude of the shear stress (in N/mm^2) in the web at its junction with the top flange _________. **[GATE, 2013]**

6. The first moment of area about the axis of bending for a beam cross-section is **[GATE, 2014]**

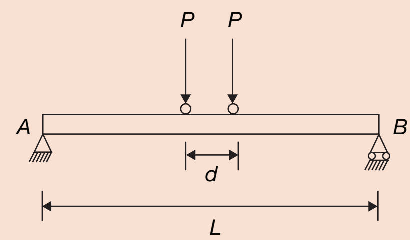
(A) moment of inertia.
(B) section modulus.
(C) shape factor.
(D) polar moment of inertia.

7. Polar moment of inertia (I_p), in cm^4, of a rectangular section having width, $b = 2$ cm and depth $d = 6$ cm is _______. **[GATE, 2014]**

8. The beam of an overall depth 250 mm as shown in the figure is used in a building subjected to two different thermal environments. The temperatures at the top and bottoms surfaces of the beam are 36°C and 72°C respectively. Considering coefficient of thermal expansion (α) as 1.50×10^{-5} per °C, the vertical deflection of the beam (in mm) at its mid-span due to temperature gradient is ______. **[GATE, 2014]**



9. A simply supported beam AB of span $L = 24$ m is subjected to two wheel loads acting at a distance, $d = 5$ m apart as shown in the figure. Each wheel transmits a load, $P = 3$ kN and may occupy any position along the beam. If the beam is an I-section having section modulus. $S = 16.2$ cm^3, the maximum bending stress (in GPa) due to the wheel loads is ___________. **[GATE, 2015]**



Answer Keys

Exercises

1. A **2.** B **3.** C **4.** D **5.** C **6.** D **7.** C **8.** B **9.** B **10.** B
11. B **12.** C **13.** C **14.** C **15.** D

Previous Years' Questions

1. C **2.** C **3.** C **4.** A **5.** 71.2 **6.** A **7.** 40 **8.** 2.43 **9.** 1.783