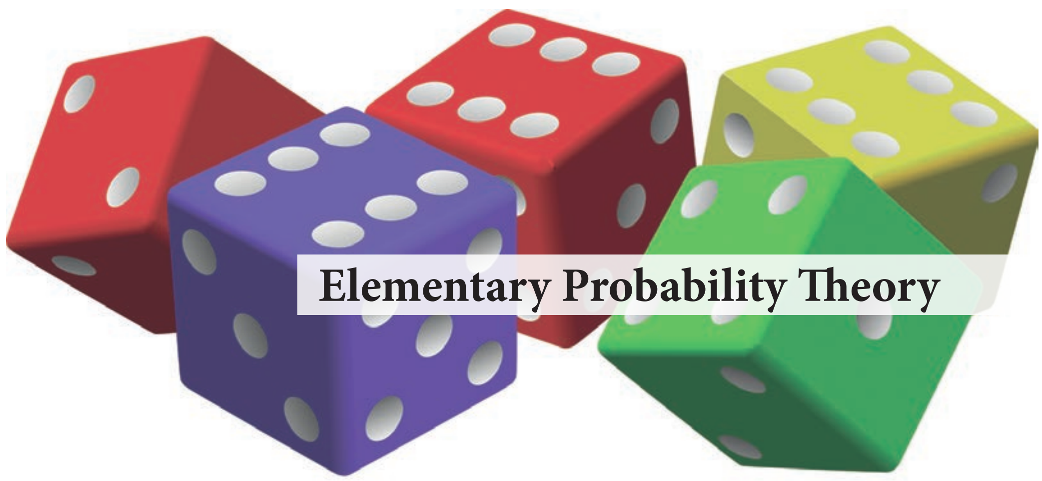
Chapter

8



Elementary Probability Theory

Blaise Pascal

(19 June, 1623 - 19 August, 1662)

Blaise Pascal was a French mathematician, who laid the foundation for the modern theory of probabilities. He was a child prodigy.

In 1642, while still a teenager, he started some pioneering work on calculating machines. After three years of effort, he built 20 finished machines (called Pascal's calculators) over the following 10 years, establishing him as one of the first two inventors of the mechanical calculator.

Pascal was an important mathematician, he wrote a significant treatise on the subject geometry and projective geometry at the age of 16, and later corresponded with Pierre de Fermat on probability theory, strongly influencing the development of modern economics and social science. Pascal's results caused many disputes before being accepted.

'The scientific imagination always restrains itself within the limits of probability'.

- Thomas Huxley

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Learning Objectives

- Knows about random experiments, trials, outcomes, events, sample space
- Understands the theorems on probability and applies in problems.
- Understands Independent events and multiplication theorem on probability.
- Explains conditional probability.
- Knows the application of Bayes' Theorem.

Introduction

Every scientific experiment conducted to investigate the patterns in natural phenomenon may result with "events" which may or may not happen. Most of the events in real life have uncertainty in their happening. For example,

Disintegration of a given atom of radium in a given time interval may or may not disintegrate;

A plant may or may not be infected by species during rainy season; The event of increase in the gold price under an economic condition in a country; A drug administered to a cancer patient for curing a disease in a period of time.

In all these cases, there is an amount of uncertainty prevails. Even for a student, asking a particular question in the examination from a particular portion of a subject is uncertain. Yet, the student is compelled to take a decision during the preparation of examination, whether to go for an in-depth study or leaving the question in choice. In a nutshell, one has to take a wise decision under the conditions of uncertainty. In such a situation, knowledge about the chance or probability for occurrence of an event of interest is vital and calculation of probability for happening of an event is imperative.

It is very much essential to determine a quantitative value to the chance or probability for the occurrence of random events in many real life situations. Before defining probability, students gain knowledge on the following essential terms.

8.1 Random experiment, Sample space, Sample point

Experiment: In Statistics, by the word experiment it means 'an attempt to produce a result'. It need not be a laboratory experiment.

Random Experiment: If an experiment is such that

(i) all the possible outcomes of the experiment are predictable, in advance

(ii) outcome of any trial of the experiment is not known, in advance, and

(iii) it can be repeated any number of times under identical conditions, is called a random experiment.

Sample space: The set of all possible outcomes of a random experiment is called the sample space of the experiment and is usually denoted by S (or Ω). If S contains only finite number of elements, it is termed as finite sample space. If S contains countable number of elements, S may be called as countable sample space or discrete sample space. Otherwise, S is called an uncountable sample space.

Sample Point: The outcome of a random experiment is called a sample point, which is an element in S.

Example 8.1

Consider the random experiment of tossing a coin once "Head" and "Tail" are the two possible outcomes. The sample space is S = {H, T}. It is a finite sample space which is presented in fig. 8.1.

OUT COMES

HEAD (H) **TAIL (T)**

Fig. 8.1. Tossing of a Coin Once

Example 8.2

Suppose that a study is conducted on all families with one or two children. The possible outcomes, in the order of births, are: boy only, girl only, boy and girl, girl and boy, both are boys and both are girls. Then, the sample space is S = {b,g,bg,gb,bb,gg}. It is also a finite sample space. Here, 'b' represents the child is a boy and 'g' represents the child is a girl.

Example 8.3

Consider the experiment of tossing a coin until head appears. Then, the sample space of this experiment is S = { (H), (T,H), (T,T,H), (T,T,T,H), … }. This is a countable sample space. If head appears in the first trial itself, then the element of S is (H); if head appears in the second attempt then the element of S is (T,H); if head appears in the third attempt then the element of S is (T,T,H) and so on.

Example 8.4

In the experiment of observing the lifetime of any animate or inanimate things, the sample space is

$$S = \{x: x \geq 0\},$$

where x denotes the lifetime. It is an example for uncountable sample space.

Event: A subset of the sample space is called an event. In this chapter, events are denoted by upper case English alphabets and the elements of the subsets by lower case English alphabets.

In Example 8.2, the event that the eldest child in the families is a girl is represented as

$$A = \{g, gb, gg\}$$

The event that the families have one boy is represented as

$$B = \{b, bg, gb\}.$$

In Example 8.4, A refers to the event that the refrigerator works to a maximum of 5000 hours. Then A = $\{ x: 0 < x \leq 5000 \}$ is the subset of

8.1.1 Mutually exclusive events

Two or more events are said to be mutually exclusive, when the occurrence of any one event excludes the occurrence of other event. Mutually exclusive events cannot occur simultaneously.

In particular, events A and B are said to be mutually exclusive if they are disjoint, that is, $A \cap B = \varnothing$

Consider the case of rolling a die. Let A = {1, 2, 3} and B = {4, 5, 6} be two events. Then we find $A \cap B = \varnothing$. Hence A and B are said to be mutually exclusive events.

8.2 Definitions of Probability

Probability is a measure of uncertainty. There are three different approaches to define the probability.

8.2.1 Mathematical Probability (Classical / a priori Approach)

If the sample space S, of an experiment is finite with all its elements being equally likely, then the probability for the occurrence of any event, A, of the experiment is defined as

$$\mathrm{P(A)} = \frac{No.of\ elements\ favourable\ to\ A}{No.\ of\ elements\ in\ S}$$

$$\mathrm{P(A)} = \frac{n(A)}{n(S)}\ .$$

The above definition of probability was used until the introduction of the axiomatic method. Hence, it is also known as classical definition of probability. Since this definition enables to calculate the probability even without conducting the experiment but using the prior knowledge about the experiment, it is also called as a priori probability.

Example 8.5

What is the chance of getting a king in a draw from a pack of 52 cards?

Solution:

In a pack there are 52 cards [n(s) = 52] which is shown in fig. 8.2

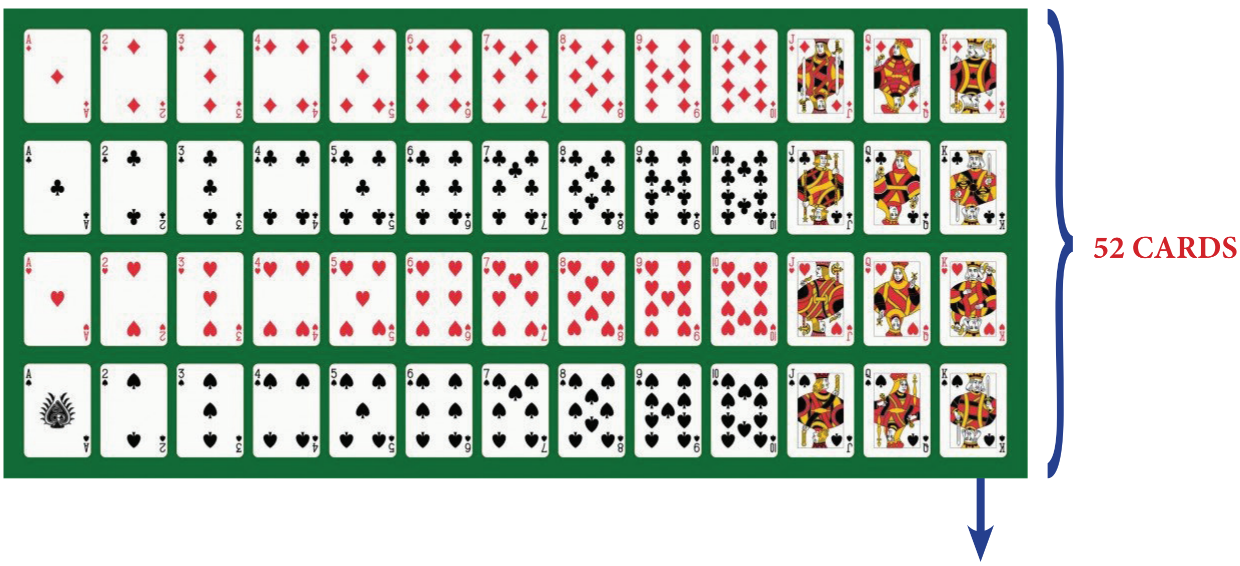


Fig. 8.2 A pack of cards

Let A be the event of choosing a card which is a king

In which, number of king cards $n(A) = 4$

Therefore probability of drawing a card which is king is = $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{4}{52}$

Example 8.6

A bag contains 7 red, 12 blue and 4 green balls. What is the probability that 3 balls drawn are all blue?

Solution:

Out of 23 balls 3 balls can be chosen $23C_3$ ways

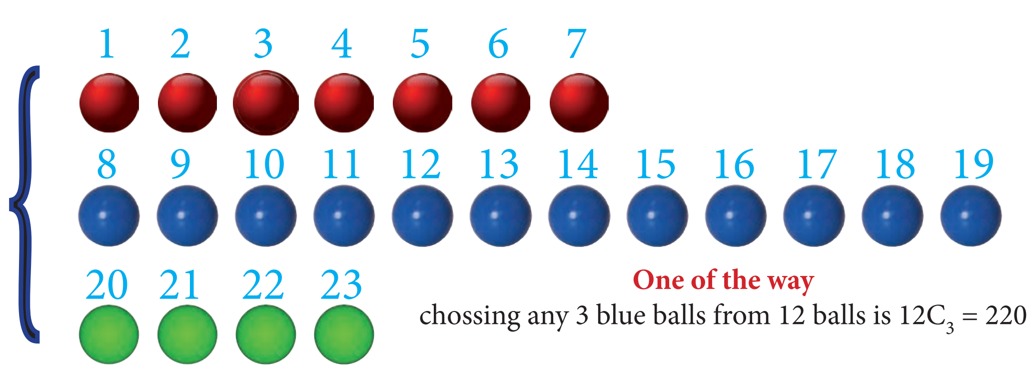


Fig. 8.3 Selection of 3 blue balls out of 23 balls

From the fig. 8.3 we find that:

Total number of balls = 7+12+14=23 balls

Out of 23 balls 3 balls can be selected in = $n(s)= 23C_3$ ways

Let A be the event of choosing 3 balls which is blue

Number of possible ways of drawing 3 out of 12 blue balls is = $n(A)=12C_3$ ways

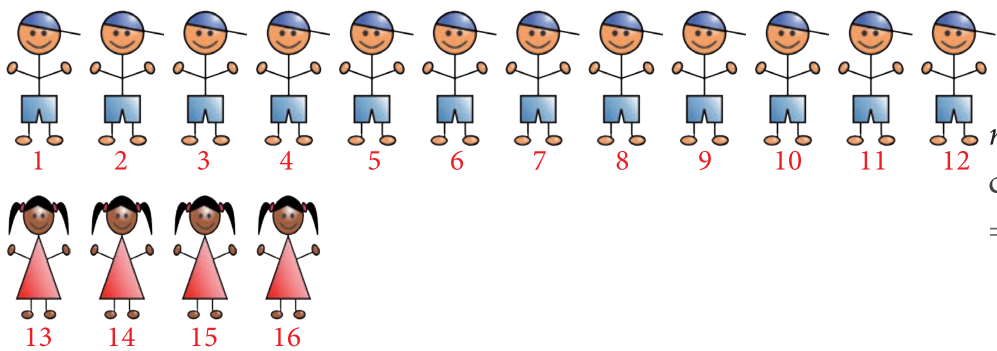
Therefore, $$P(A) = \frac{n(A)}{n(S)} = \frac{12C_3}{23C_3} = \frac{220}{1771}$$

$$= 0.1242$$

Example 8.7

A class has 12 boys and 4 girls. Suppose 3 students are selected at random from the class. Find the probability that all are boys.

Solution:



$n(s) = 16$

choosing 3 students

$= 16\,C_3$

chossing any 3 boys out of 12 boys is $12C_3 = 220$ ways

fig. 8.4 Selection of 3 Boys

From the fig 8.4, we find that:

Total number of students = 12+4=16

Three students can be selected out of 16 students in $16C_3$ ways

i.e. $n(s) = 16C_3 = \dfrac{16 \times 15 \times 14}{1 \times 2 \times 3} = 560$

Three boys can be selected from 12 boys in $12C_3$ ways

i.e. $n(A) = 12C3 = \dfrac{12 \times 11 \times 10}{1 \times 2 \times 3} = 220$

The required probability $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{220}{560} = \dfrac{11}{28}$

$= 0.392$

8.2.2 Statistical Probability (Relative Frequency/a posteriori Approach)

If the random experiment is repeated n times under identical conditions and the event A occurred in n(A) times, then the probability for the occurrence of the event A can be defined (Von Mises) as

$$P(A) = \lim_{n \to \infty} \frac{n(A)}{n}.$$

Since computation of probability under this approach is based on the empirical evidences for the occurrence of the event, it is also knows as relative frequency or a posteriori probability.

NOTE

It should be noted that repeating some experiments is not always possible. In such cases, relative frequency approach cannot be applied. Classical approach to compute probability requires that the sample space should be a finite set. It is seldom possible. Hence, we present in the next section axiomatic approach to probability which overcomes the limitations of both mathematical probability and statistical probability.

8.3 Axioms of Probability

(A.N. Kolmogorov)

Andrey Nikolaevich Kolmogorov (1903–1987) was a 20th-century Soviet mathematician who made significant contributions to the mathematics of probability theory, topology, intuitionistic logic, turbulence, classical ... Wikipedia

A.N. Kolmogorov proposed the axiomatic approach to probability in 1933. An axiom is a simple, indisputable statement, which is proposed without proof. New results can be found using axioms, which later become as theorems.

8.3.1 Axiomatic approach to probability

Let S be the sample space of a random experiment. If a number $P(A)$ assigned to each event $A \in S$ satisfies the following axioms, then $P(A)$ is called the probability of A.

Axiom-1 : $P(A) \geq 0$

Axiom-2 : $P(S) = 1$

Axiom-3 : If $\{A_1, A_2, \ldots\}$ is a sequence of mutually exclusive events i.e., $A_i \cap A_j = \phi$

when $i \neq j$, then

$$P\left(\cup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

Axiom-3 also holds for a set of finite number of mutually exclusive events. If $A_1, A_2, \ldots, A_n$ are mutually exclusive events in S and n is a finite positive integer, then

$$P(A_1 \cup A_2 \ldots A_n) = P(A_1) + P(A_2) + \ldots + P(A_n).$$

It may be noted that the previous two approaches to probability satisfy all the above three axioms.

8.3.2 Basic Theorems of Probability

Theorem 8.1: The probability of impossible event is 0 i.e., $P(\phi) = 0$.

Proof: Let $A_1 = S$ and $A_2 = \phi$. Then, A_1 and A_2 are mutually exclusive.

$S = A_1 \cup A_2 = S \cup \phi$

Thus, by Axiom -3,

$$P(S) = P(S) + P(\phi)$$

Since by Axiom-2, $P(S) = 1$,

$$1 = 1 + P(\phi)$$

Hence, $P(\phi) = 0$.

Theorem 8.2: If S is the sample space and A is any event of the experiment, then $P(\bar{A}) = 1 - P(A)$.

Proof:

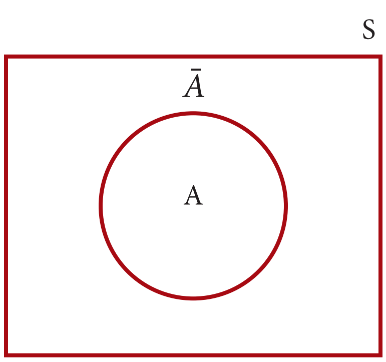


fig. 8.5 Venn diagram

Since A and $\bar{A}$ are mutually exclusive,

$$A \cup \bar{A} = S. \text{ (By Axiom -3)}$$

$$P(A \cup \bar{A}) = P(A) + P(\bar{A}) \text{ (see figure 8.4)}$$

$$P(S) = P(A) + P(\bar{A})$$

$$1 = P(A) + P(\bar{A}) \quad \text{(by Axiom-2)}$$

It implies that $P(\bar{A}) = 1 - P(A)$.

NOTE

Since $\bar{A}$ is an event, by Axiom -1, $P(\bar{A}) \geq 0$.
i.e., $1-P(A) \geq 0 \quad P(A) \leq 1$.

Thus, the probability for the occurrence of any event is always a real number between 0 and 1 i.e., $0 \leq P(A) \leq 1$.

Theorem 8.3: If A and B are two events in an experiment such that $A \subset B$, then $P(B-A) = P(B) - P(A)$.

Proof:

It is given that $A \subset B$.

The event B can be expressed as

$B = A \cup (B-A)$ (see Figure 8.6)

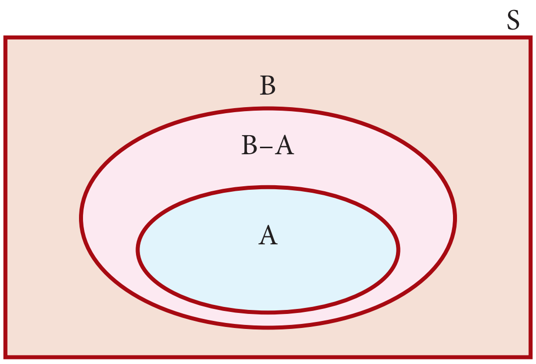


fig. 8.6 Venn diagram

Since $A \cap (B-A) = \varphi$,

$$P(B) = P(A \cup (B-A))$$

Hence, by Axiom-3,

$$\Rightarrow \quad P(B) = P(A) + P(B-A)$$

Therefore, $P(B-A) = P(B) - P(A)$.

Corollary: If $A \subset B$, then $P(A) \leq P(B)$.

Proof:

Since, by Axiom-1, $P(B-A) \geq 0$, it follows that

$P(B) - P(A) \geq 0$

$P(B) \geq P(A)$

$\Rightarrow \quad P(A) \leq P(B)$.

Example 8.8

In the experiment of tossing an unbiased coin (or synonymously balanced or fair coin), the sample space is $S = \{H, T\}$. What is the probability of getting head or tail?

Solution :

If the events A_1 and A_2 are defined as $A_1 = \{H\}$ and $A_2 = \{T\}$, then $S = A_1 \cup A_2$. Here, the events A_1 and A_2 are mutually exclusive, because they cannot occur together. Hence, by using Axiom-3, it can be written as

$$P(S) = P(A_1)+P(A_2)$$

Since the number of elementary events in S in favour of the occurrence of A_1 and A_2 is one each, they have equal chances to occur. Hence, $P(A_1) = P(A_2)$. Substituting this, it follows that

$$1 = P(A_1)+P(A_2) = 2P(A_1)$$

It gives that $P(A_1) = \frac{1}{2}$ and therefore $P(A_2) = \frac{1}{2}$.

$$\Rightarrow P(\text{Getting a Head}) = \frac{1}{2} = P(\text{Getting a Tail}).$$

Thus, a coin is called unbiased, if the probability of getting head is equal to that of getting tail.

Aliter: (Applying Classical approach)

Since $n(A_1) = 1 = n(A_2)$ and $n(S) = 2$,

$$P(A_1) = \frac{n(A_1)}{n(S)} = \frac{1}{2}.$$

Similarly, $P(A_2) = \frac{1}{2}$.

Remark:

If a biased coin is tossed and the outcome of head is thrice as likely as tail, then $P(A_1) = 3P(A_2)$.

Substituting this in $P(S) = P(A_1)+P(A_2)$, it follows that

$$1 = P(A_1)+P(A_2) = 4P(A_2)$$

It gives that $P(A_2) = \frac{1}{4}$ and hence $P(A_1) = \frac{3}{4}$.

It should be noted that the probabilities for getting head and tail differ for a biased coin.

Example 8.9

Ammu has five toys which are identical and one of them is underweight. Her sister, Harini, chooses one of these toys at random. Find the probability for Harini to choose an underweight toy?

fig. 8.7 Identical toys with on underweight toy

Solution :

It is seen from fig. 8.7, the sample space is S = {a_1, a_2, a_3, a_4, a_5}.Define the events A_1, A_2, A_3, A_4 and A_5 as

A : Harini chooses the underweight toy

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{5}$$

Therefore, the probability for Harini to choose an underweight toy is 1/5.

Example 8.10

A box contains 3 red and 4 blue socks. Find the probability of choosing two socks of same colour.

= 7 socks (3 Red + 4 Blue)

i.e., $n = 7$

A_1 : choosing 2 socks from 3 socks
$n(A_1) = 3\,C_2$ ways = 3 ways

A_2 : choosing 2 socks from 4 socks
$n(A_2) = 4\,C_2$ ways = 6 ways

fig.8.8 choosing 2 socks of same colour

Solution :

From fig. 8.8, total number of socks = 3 + 4 = 7

If two socks are drawn at random, then

No. of ways of selecting 2 socks = $7C_2 = 21$

A_1 = Selection of black socks,

$$n(A_1) = 3C_2 \quad = 3$$

$$P(A_1) = \frac{n(A_1)}{n(S)} \quad = \frac{3}{21}$$

A_2 = Selection of blue socks,

$$n(A_2) = 4C_2 \quad = 6$$

$$P(A_2) = \frac{n(A2)}{n(S)} \quad = \frac{6}{21}$$

then $A_1 \cup A_2$ represents the event of selecting 2 socks of same colour. Since the occurrence of one event excludes the occurrence of the other, these two events are mutually exclusive. Then, by Axiom-3,

$$P(A_1 \cup A_2) = P(A_1) + P(A_2)$$

Therefore, $P(A_1 \cup A_2) = \frac{3}{21} + \frac{6}{21} = \frac{9}{21} = \frac{3}{7}$

Thus, the probability of selecting two socks of same colour is 3/7.

Example 8.11

Angel selects three cards at random from a pack of 52 cards. Find the probability of drawing:

(i) 3 spade cards.

(ii) one spade and two knave cards

(iii) one spade, one knave and one heart cards.

Solution:

Total no. of ways of drawing 3 cards = n(S) = $52\,C_3$ = 22100

(a) Let A_1 = drawing 3 spade cards.

Since there are 13 Spades cards in a pack of cards,

No. of ways of drawing 3 spade cards = $n(A_1) = 13\,C_3 = 286$

Therefore, $P(A_1) = \frac{n(A_1)}{n(S)} = \frac{286}{22100}$

(b) Let A_2 = drawing one spade and two knave cards

No. of ways of drawing one spade card = $13C_1 = 13$

No. of ways of drawing two knave cards = $13C_2 = 78$

Since drawing a spade and 2 knaves should occur together,

No. of ways drawing one spade and two knave cards = $n(A_2)$ = 13×78 =1014

$$\text{Therefore, } P(A_2) = \frac{n(A_2)}{n(S)} = \frac{13 \times 78}{22100}$$

$$\text{Hence, } P(A_2) = \frac{1014}{22100} = \frac{507}{11050}$$

(c) Let A_3 = drawing one spade, one knave and one heart cards

No. of ways of drawing one spade, one knave and one heart cards is

$$n(A_3) = 13C_1 \times 13C_1 \times 13C_1 = 13 \times 13 \times 13$$

$$\text{Therefore, } P(A_3) = \frac{n(A_3)}{n(S)} = \frac{13 \times 13 \times 13}{22100}$$

$$\text{Hence, } P(A_3) = \frac{2197}{22100}.$$

8.4 Addition Theorem of Probability

Theorem 8.4 : (Addition Theorem of Probability for Two Events)

If A and B are any two events in a random experiment, then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Proof:

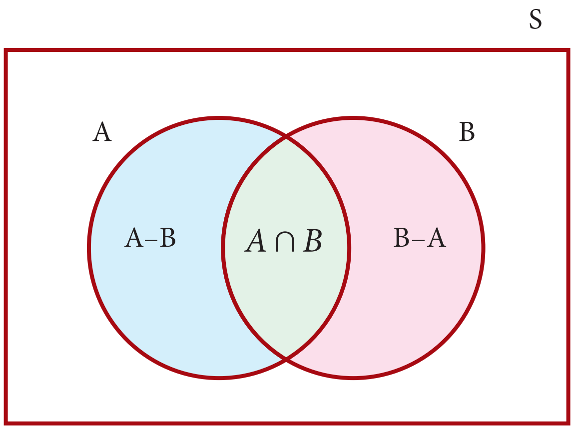


fig. 8.9 Venn diagram

For any two events A and B, the shaded region in fig. 8.9 represents the event $A \cup B$.

$A \cup B = A \cup (B - (A \cap B))$

The events A and B-(A ∩B) are mutually exclusive.

Using Axiom 3,

$$P(A \cup B) = P(A \cup [B-(A \cap B)])$$

$$= P(A) + P[B-(A \cap B)] \qquad (8.1)$$

Since $(A \cap B) \subset B$, ,

$$B = (A \cap B) \cup (B - (A \cap B))$$

The events on the right hand side are disjoints. Hence by axiom 3

$$P(B) = P(A \cap B) + P(B - (A \cap B))$$

i.e. $$P[B - (A \cap B)] = P(B) - P(A \cap B) \qquad (8.2)$$

Substituting (8.2) in (8.1), it follows that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Corollary: If A, B and C are any three events, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Example 8.12

In the Annual sports meet, among the 260 students in XI standard in the school, 90 participated in Kabadi, 120 participated in Hockey, and 50 participated in Kabadi and Hockey. A Student is selected at random. Find the probability that the student participated in (i) Either Kabadi or Hockey, (ii) Neither of the two tournaments, (iii) Hockey only, (iv) Kabadi only, (v) Exactly one of the tournaments.

Solution:

$n(s)=260$

Let A : the event that the student participated in Kabadi

B : the event that the student participated in Hockey.

$n(A) = 90$; $n(B) = 120$; $n(A \cap B) = 50$

$$P(A) = \frac{n(A)}{n(S)} = \frac{90}{260}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{120}{260}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{50}{260}$$

(i) The probability that the student participated in either Kabadi or Hockey is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{90}{260} + \frac{120}{260} - \frac{50}{260} = \frac{160}{260} = \frac{8}{13}$$

(ii) The probability that the student participated in neither of the two tournaments in

$$P(\overline{A}\cap\overline{B}) = P(\overline{A\cup B}) \text{ (By De Morgan's law } \overline{A\cup B}=\overline{A}\cap\overline{B}\text{)}$$

$$= 1 - P(A\cup B)$$

$$= 1 - \frac{8}{13} = \frac{5}{13}$$

(iii) The probability that the student participated in Hockey only is

$$P(\overline{A}\cap B) = P(B) - P(A\cap B)$$

$$= \frac{120}{260} - \frac{50}{260} = \frac{70}{260} = \frac{7}{26}$$

(iv) The probability that the student participated in Kabadi only

$$P(A\cap\overline{B}) = P(A) - P(A\cap B)$$

$$= \frac{90}{260} - \frac{50}{260} = \frac{40}{260} = \frac{2}{13}$$

(v) The probability that the student participated in exactly one of the tournaments is

$P[(A\cap\overline{B})\cup(\overline{A}\cap B)] = P(A\cap\overline{B}) + P(\overline{A}\cap B)$ [$\because A\cap\overline{B}, \overline{A}\cap B$ are mutually exclusive events]

$$= \frac{70}{260} + \frac{40}{260} = \frac{110}{260} = \frac{11}{26}$$

8.5 Conditional Probability

Consider the following situations:

(i) two events occur successively or one after the other (e.g) A occurs after B has occurred and

(ii) both event A and event B occur together.

Example 8.13

There are 4000 people living in a village including 1500 female. Among the people in the village, the age of 1000 people is above 25 years which includes 400 female. Suppose a person is chosen and you are told that the chosen person is a female. What is the probability that her age is above 25 years?

Solution:

Here, the event of interest is selecting a female with age above 25 years. In connection with the occurrence of this event, the following two events must happen.

A: a person selected is female

B: a person chosen is above 25 years.

Situation1:

We are interested in the event B, given that A has occurred. This event can be denoted by $B|A$. It can be read as "B given A". It means that first the event A occurs then under that condition, B occurs. Here, we want to find the probability for the occurrence of $B|A$ i.e., $P(B|A)$. This probability is called conditional probability. In reverse, the probability for selecting a female given that a person has been selected with age above 25 years is denoted by $P(A|B)$.

Situation 2:

Suppose that it is interested to select a person who is both female and with age above 25 years. This event can be denoted by $A \cap B$.

Calculation of probabilities in these situations warrant us to have another theorem namely Multiplication theorem. It is derived based on the definition of conditional probability.

$$P(A) = P(\text{Selecting a female}) = \frac{1500}{4000}$$

$$P(A \cap B) = P(\text{Selecting a female with age above 25 years}) = \frac{400}{1500}$$

Hence, $$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{400}{1500} \times \frac{4000}{1500} = \frac{160}{225} = \frac{32}{45}.$$

8.5.1 Definition of Conditional of Probability

If $P(B) > 0$, the conditional probability of A given B is defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad .$$

If $P(B) = 0$, then $P(A \cap B) = 0$. Hence, the above formula is meaningless when $P(B) = 0$. Therefore, the conditional probability $P(A|B)$ can be calculated only when $P(B)>0$.

The need for the computation of conditional probability is described in the following illustration.

Illustration

A family is selected at random from the set of all families in a town with one twin pair. The sample space is

$$S = \{(\text{boy, boy}), (\text{boy, girl}), (\text{girl, boy}), (\text{girl, girl})\}.$$

Define the events

A: the randomly selected family has two boys, and

B: the randomly selected family has a boy.

Let us assume that all the families with one twin pair are equally likely. Since

$$A = \{(\text{boy, boy})\},$$

$$B = \{(\text{boy, boy}), (\text{boy, girl}), (\text{girl, boy})\},$$

$$A \cap B = A = \{(\text{boy, boy})\}.$$

Applying the classical definition of probability, it can be calculated that

$$P(B) = \frac{3}{4} \text{ and } P(A \cap B) = \frac{1}{4}.$$

Suppose that the randomly selected family has a boy. Then, the probability that the other child in the pair is a girl can be calculated using conditional probability as

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{3/4} = 1/3.$$

Example 8.14

A number is selected randomly from 11 through 19. Consider the events

$A = \{ 11,14, 16, 18, 19 \}$ $B = \{ 12, 14, 18, 19 \}$ $C = \{ 13, 15, 18, 19 \}$.

Find (i) P(A/B) (ii) P(A/C) (iii) P(B/C) (iv) P (B/A)

Solution:

given A $= \{ 11,14, 16, 18, 19 \}$ $B = \{ 12, 14, 18, 19 \}$ $C = \{ 13, 15, 18, 19 \}$.

$$A \cap B = \{ 14, 18, 19 \} \quad A \cap C = \{ 18, 19 \} = B \cap C$$

$$P(A) = \frac{5}{9} \qquad P(B) = \frac{4}{9} = P(C)$$

$$P(A \cap B) = \frac{3}{9} \qquad P(A \cap C) = \frac{2}{9} = P(B \cap C)$$

Therefore, the probability for the occurrence of A given that B has occurred is

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{3}{9}}{\frac{4}{9}}$$

$$P(A/B) = \frac{3}{4}.$$

The probability for the occurrence of A given that C has occurred is

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{2}{9}}{\frac{4}{9}}$$

$$P(A/C) = 1/2.$$

Similarly, the conditional probability of B given C is

$$P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{\frac{2}{9}}{\frac{4}{9}}$$

$$P(B/C) = \frac{1}{2}$$

and the conditional probability of B given A is

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{3}{9}}{\frac{5}{9}}$$

$$P(B/A) = \frac{3}{5}.$$

Example 8.15

A pair of dice is rolled and the faces are noted. Let

A: sum of the faces is odd, B: sum of the faces exceeds 8, and

C: the faces are different then find (i) P (A/C) (ii) $P(B/C)$

Solution:

The outcomes favourable to the occurrence of these events are

A = { (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (3,6)
(4,1), (4,3), (4,5), (5,2), (5,4), (5,6), (6,1), (6,3), (6,5) }

B = { (3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6) }

C = { (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5) }

Since A and B are proper subsets of C, $A \cap C = A$ and $B \cap C = B$.

Here, $P(A) = \frac{18}{36} = \frac{1}{2}$

$P(B) = \frac{10}{36} = \frac{5}{9}$

$P(C) = \frac{30}{36} = \frac{5}{6}$.

Since, $A \cap C = A$. Hence,

$$P(A \cap C) = P(A) = \frac{1}{2}$$

$$P(B \cap C) = P(B) = \frac{5}{9}.$$

Hence, the probability for the sum of the faces is an odd number given that the faces are different is

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{2}}{\frac{5}{6}} = \frac{3}{5}$$

Similarly, the probability for the sum of the faces exceeds 8 given that the faces are different is

$$P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{\frac{2}{9}}{\frac{5}{6}} = \frac{4}{15}$$

8.5.2 Axioms

The conditional probabilities also satisfy the same axioms introduced in Section 8.3.

If S is the sample space of a random experiment and B is an event in the experiment, then

(i) $P(A/B) \geq 0$ for any event A of S.

(ii) $P(S/B) = 1$

(iii) If $A_1, A_2, \ldots$ is a sequence of mutually exclusive events, then

$$P(\bigcup_{i=1}^{\infty} A_i | B) = \sum_{i=1}^{\infty} P(A_i | B)$$

In continuation of conditional probability, another property of events, viz., independence can be studied. It is discussed in the next section. Also, multiplication theorem, a consequence of conditional probability, will be studied later.

8.6 Independent Events

For any two events A and B of a random experiment, if $P(A/B) = P(A)$, then knowledge of the event B does not change the probability for the occurrence of the event A. Such events are called independent events.

$$\text{If} \qquad P(A/B) = P(A), \text{ then}$$

$$\frac{P(A \cap B)}{P(B)} = P(A).$$

$$\Rightarrow \qquad P(A \cap B) = P(A) \times P(B).$$

Similarly, the relation $P(B/A) = P(B)$ also indicates the independence of the events A and B.

Definition :

Two events A and B are said to be independent of one another, if

$$P(A \cap B) = P(A) \times P(B).$$

NOTE

The greater the value of β_2, the more peaked the distribution.

(i) If A and B are not independent events, they are called dependent events.

(ii) The above definition of independence may be extended to finite number of events. If the events $A_1, A_2, \ldots, A_n$ satisfy

$$P(A_1 \cap A_2 \cap \ldots\ldots A_n) = P(A_1) \times P(A_2) \times \ldots \times P(A_n),$$

then $A_1, A_2, \ldots, A_n$ are called independent events.

Example 8.16

In tossing a fair coin twice, let the events A and B be defined as A: getting head on the first toss, B: getting head on the second toss. Prove that A and B are independent events.

Solution:

The sample space of this experiment is

$S = \{HH, HT, TH, TT\}$.

The unconditional probabilities of A and B are $P(A) = \frac{1}{2} = P(B)$.

The event of getting heads in both the tosses is represented by $A \cap B$. The outcome of the experiment in favour of the occurrence of this event is HH. Hence, $P(A \cap B) = \frac{1}{4}$.

$\therefore$ $P(A \cap B) = P(A) \times P(B)$ holds. Thus, the events A and B are independent events.

Example 8.17

In the experiment of rolling a pair of dice, the events A, B and C are defined as A : getting 2 on the first die, B : getting 2 on the second die, and C : sum of the faces of dice is an even number. Prove that the events are pair wise independent but not mutually independent?

Solution:

$$S = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$$

$$n(S) = 36$$

The outcomes which are favourable to the occurrence of these events can be listed below:

$$A = \{ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \}$$

$$n(A) = 6$$

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

$$B = \{ (1,2), (2,2), (3,2), (4,2), (5,2), (6,2) \}$$

$$n(B) = 6$$

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

$$C = \{ (1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5),$$
$$(4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6) \}$$

$$n(C) = 18$$

$$P(C) = \frac{18}{36} = \frac{1}{2} \; .$$

$$A \cap B = \{ (2,2) \}$$

$$n(A \cap B) = 1$$

$$P(A \cap B) = \frac{1}{36}$$

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$$A \cap C = \{ (2,2), (2,4), (2,6) \}$$

$$n(A \cap C) = 3$$

$$P(A \cap C) = \frac{3}{36}$$

$$B \cap C = \{ (2,2), (4,2), (6,2) \}$$

$$n(B \cap C) = 3$$

$$P(B \cap C) = \frac{3}{36}$$

$$A \cap B \cap C = \{ (2,2) \}$$

$$n(A \cap B \cap C) = 1$$

$$P(A \cap B \cap C) = \frac{1}{36}$$

NOTE

If A and B are mutually exclusive events, $P(A \cap B) = 0$

The following relations may be obtained from these probabilities

$$P(A \cap B) = P(A) \times P(B)$$

$$\frac{1}{36} = \frac{1}{6} \times \frac{1}{6}$$

$$P(A \cap C) = P(A) \times P(C)$$

$$\frac{3}{36} = \frac{1}{6} \times \frac{1}{2}$$

$$P(B \cap C) = P(B) \times P(C)$$

$$\frac{3}{36} = \frac{1}{6} \times \frac{1}{2}$$

$$P(A \cap B \cap C) \neq P(A) \times P(B) \times P(C).$$

$$\frac{1}{36} \neq \frac{1}{6} \times \frac{1}{6} \times \frac{1}{2}$$

NOTE

If A and B are independent events, $P(A \cap B) = P(A) \times P(B)$

The above relations show that when the events A, B and C are considered in pairs, they are independent. But, when all the three events are considered together, they are not independent.

8.7 Multiplication Theorem on Probability

Theorem 8.5: (Multiplication Theorem of Probability)

If A and B are any two events of an experiment, then

$$P(A \cap B) = \begin{cases} P(A)P(B \mid A), & \text{if } P(A) > 0 \\ P(B)P(A \mid B), & \text{if } P(B) > 0 \end{cases}$$

Proof:

If $P(B) > 0$, multiplying both sides of $P(A/B)= \dfrac{P(A \cap B)}{P(B)}$ by $P(B)$, we get

$P(A \cap B) = P(B)\, P(A/B)$

If $P(A) > 0$, interchanging A and B in $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$, it also follows that

$P(A \cap B)= P(A)P(B/A)$.

Hence, the theorem is proved.

NOTE

If A and B are any two events of an experiment, then $P(A \cap B)$ can be calculated using the conditional probability of A given B or of B given A. Multiplication theorem of probability can be extended to compute $P(A \cap B \cap C)$ for the events A, B and C as follows:

$$P(A \cap B \cap C) = P(A)P(B/A)P(C/A \cap B).$$

Example 8.18

A box contains 7 red and 3 white marbles. Three marbles are drawn from the box one after the other without replacement. Find the probability of drawing three marbles in the alternate colours with the first marble being red.

Solution:

The event of interest is drawing the marbles in alternate colours with the first is red. This event can occur only when the marbles are drawn in the order (Red , White , Red)

If A and C represent the events of drawing red marbles respectively in the first and the third draws and B is the event of drawing white marble in the second draw, then the required event is $A \cap B \cap C$. The probability for the occurrence of $A \cap B \cap C$ can be calculated applying

$$P(A \cap B \cap C) = P(A)P(B/A)P(C/A \cap B)$$

Since there are 7 red and 3 white marbles in the box for the first draw,

$$P(A) = \frac{7}{10}$$

Now, there will be 6 red and 3 white marbles in the box for the second draw if the event A has occurred. Hence,

$$P(B/A) = \frac{3}{9}$$

Similarly, there will be 6 red and 2 white marbles in the box for the third draw if the events A and B have occurred. Hence,

$$P(C/A \cap B) = \frac{6}{8}.$$

$\therefore \quad P(A \cap B \cap C) = \frac{7}{10} \times \frac{3}{9} \times \frac{6}{8} = \frac{7}{40}$ is the required probability of drawing three marbles in the alternate colours with the first marble being red.

Example 8.19

Three cards are drawn successively from a well-shuffled pack of 52 playing cards. Find the probability all three cards drawn successively is ace without replacing the card after each draw.

Solution:

Let A: all the three cards drawn are aces

At the first draw, there will be 4 aces among 52 cards. Having drawn an ace in the first draw, there will be 3 aces among 51 cards. Similarly, there will be 2 aces among 51 cards for the third draw.

Then, as discussed in Example 8.20, by Theorem 8.5

$$P(A) = \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50}$$

$$P(A) = \frac{1}{5525}.$$

Example 8.20

There are 13 boys and 6 girls in a class. Four students are selected randomly one after another from that class. Find the probability that: (i) all are girls, (ii) first two are boys and next are girls

Solution:

(i) Suppose that

B: all the randomly selected students are girls

There will be 6 girls among 19 students, in total, while selecting the first student; there will be 5 girls among 18 students, in total, while selecting the second student; 4 girls among 17 students, in total, while selecting the third student; and 3 girls among the remaining 16 students, in total, while selecting the fourth student.

Then, by applying the Theorem-8.5 for simultaneous occurrence of these four events, it follows that

$$P(B) = \frac{6}{19} \times \frac{5}{18} \times \frac{4}{17} \times \frac{3}{16}$$

$$P(B) = \frac{5}{1292}.$$

(ii) Suppose that

C: In the randomly selected students the first two are boys and the next are girls

There will be 13 boys among the 19 students, in total, while selecting the first student; there will be 12 boys among 18 students, in total, while selecting the second student; 6 girls among 17 students, in total, while selecting the third student; and 5 girls among the remaining 16 students, in total, while selecting the fourth student.

Then, by applying the Theorem 8.5 for simultaneous occurrence of these four events, it follows that

$$P(C) = \frac{13}{19} \times \frac{12}{18} \times \frac{6}{17} \times \frac{5}{16} = \frac{65}{1292}$$

8.8 Bayes' Theorem and its Applications

In some cases, probability for the occurrence of an event of interest A may be difficult to compute from the given information. But, it may be possible to calculate its conditional probabilities $P(A/B)$ and $P(A/\bar{B})$ for some other event B of the same experiment. Then, $P(A)$ can be calculated applying the law of total probability. This theorem is a prelude for Bayes' theorem.

Theorem 8.6 (Law of Total Probability)

If $B_1, B_2, \ldots, B_n$ are mutually exclusive events such that $\bigcup_{j=1}^{n} B_j = S$ and $P(B_j) > 0$ for $j = 1, 2, \ldots, n$, Then for any event A

$$P(A) = P(A/B_1)P(B_1) + P(A/B_2)P(B_2) + \ldots + P(A/B_n)P(B_n).$$

In real life situations, decision making is an ongoing process. Situations may arise where we are interested in an event on an ongoing basis. Every time some new information may be available and based on this the probability of the event should be revised. This revision of probability with additional information is formalized in probability theory in the theorem known as Bayes' Theorem.

Theorem 8.7 (Bayes' Theorem)

Let $B_1, \dots, B_n$ be n mutually exclusive events such that where S is the sample space of the random experiment. If $P(B_j) > 0$ for $j = 1, 2, \dots, n$, then for any event A of the same experiment with $P(A) > 0$,

$$P(B_j/A) = \frac{P(A/B_j)P(B_j)}{P(A/B_1)P(B_1) + P(A/B_2)P(B_2) + \dots + P(A/B_n)P(B_n)},\ j = 1, 2, \dots, n.$$

[This Theorem is due to Rev. Thomas Bayes (1701-1761), an English philosopher and a priest. This work was published posthumously by his friend Richard Price during 1763 in the name of Bayes.]

Proof:

For each event B_j, $i = 1, 2, \dots, n$, by the definition of conditional probability

$$P(B_j/A) = \frac{P(B_j \cap A)}{P(A)}$$

$$= \frac{P(A/B_j)P(B_j)}{P(A)} \quad \text{for } j = 1, 2, \dots, n$$

Then, by the generalization of Theorem 8.5,

$$P(B_j/A) = \frac{P(A/B_j)P(B_j)}{P(A/B_1)P(B_1) + P(A/B_2)P(B_2) + \dots + P(A/B_n)P(B_n)},\ j = 1, 2, \dots, n.$$

Example 8.21

Mr. Arivazhagan, Mr. Ilavarasan and Mr. Anbarasan attended an interview conducted for appointing a Physical Teacher in a school. Mr. Arivazhagan has 45% chance for selection, Mr. Ilavarasan has 28% chance and Mr.Anbarasan has 27% chance. Also, the chance for implementing monthly Mass Drill (MD) programme in the school is 42% if Mr.Arivazhagan is appointed; 40% if Mr.Ilavarasan is appointed; and 48% if Mr.Anbarasan is appointed.

Find the probability that the mass drill is implemented by if:

(i) Mr.Arivazhagan is appointed as the Physical Education Teacher.

(ii) Mr.Ilavarasan is appointed as the Physical Education Teacher.

(iii) Mr.Anbarasan is appointed as the Physical Education Teacher.

Solution:

Let MD denote the event that the monthly Mass Drill programme is implemented in the school. Also, let

A: Mr.Arivazhagan is appointed

B: Mr.Ilavarasan is appointed

C: Mr.Anbarasan is appointed.S

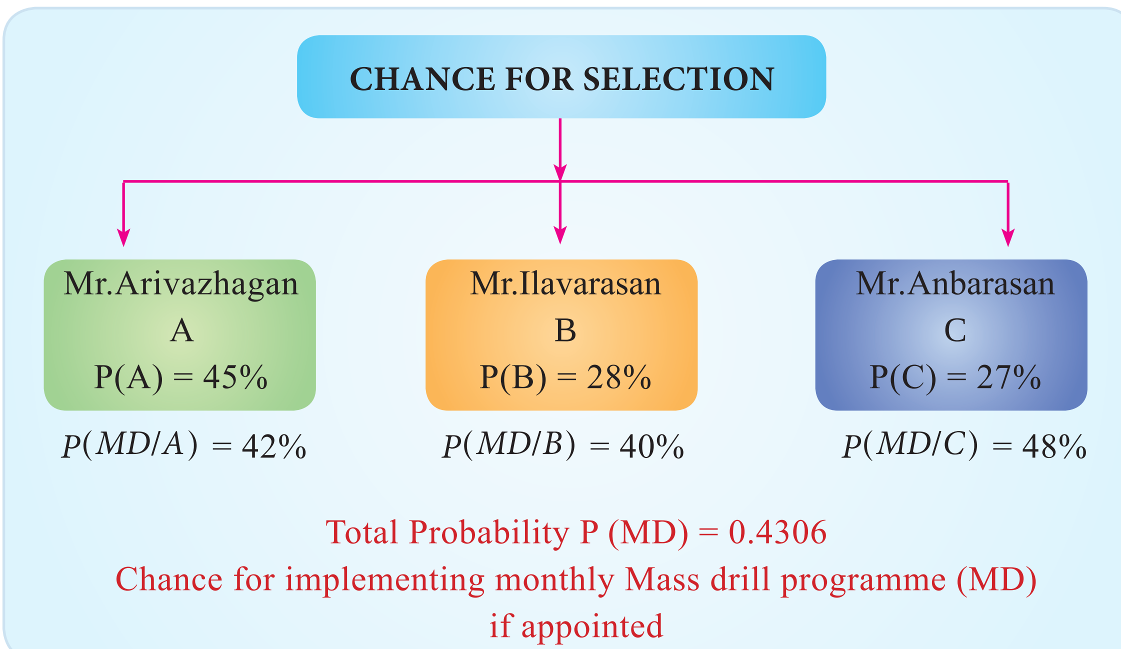


Fig. 8.10 Appointment of Physical Teacher

From fig. 8.10 the information given about these events are

$P(A) = 0.45$ $\qquad P(B) = 0.28$ $\qquad P(C) = 0.27$

$P(MD/A) = 0.42$ $\qquad P(MD/B) = 0.40$ $\qquad P(MD/C) = 0.48$

With these, the probability for implementing monthly Mass Drill programme in the school can be computed using total probability as

$$
\begin{aligned}
P(MD) &= P(MD/A)P(A) + P(MD/B)P(B) + P(MD/C)P(C) \\
&= (0.42\times 0.45) + (0.40\times 0.28) + (0.48\times 0.27) \\
&= 0.189 + 0.112 + 0.1296 \\
\therefore \quad P(MD) &= 0.4306.
\end{aligned}
$$

If it is known that the monthly Mass Drill programme is implemented in the school,

(i) the probability for Mr.Arivazhagan is appointed as the Physical Teacher can be calculated applying Theorem8.7 as

$$P(A/MD) = \frac{P(MD/A)P(A)}{P(MD/A)P(A)+P(MD/B)P(B)+P(MD/C)P(C)}$$

$$= \frac{0.42 \times 0.45}{(0.42 \times 0.45)+(0.40 \times 0.28)+(0.48 \times 0.27)}$$

$$= \frac{0.189}{0.4306}$$

$$\therefore \quad P(A/MD) = 0.4389.$$

(ii) the probability for Mr.Ilavarasan is appointed as the Physical Teacher can be calculated applying Theorem8.7 as

$$P(B/MD) = \frac{P(MD/B)P(B)}{P(MD/A)P(A)+P(MD/B)P(B)+P(MD/C)P(C)}$$

$$= \frac{0.40 \times 0.28}{(0.42 \times 0.45)+(0.40 \times 0.28)+(0.48 \times 0.27)}$$

$$\therefore \quad P(B/MD) = \frac{0.112}{0.4306}$$

$$= 0.2601.$$

(iii) the probability for Mr.Anbarasan is appointed as the Physical Teacher can be calculated applying Theorem8.7 as

$$\mathrm{P(C|MD)} = \frac{P(MD/C)P(C)}{P(MD/A)P(A)+P(MD/B)P(B)+P(MD/C)P(C)}$$

$$= \frac{0.4 \times 0.27}{(0.42 \times 0.45)+(0.40 \times 0.28)+(0.28 \times 0.27)}$$

$$= \frac{0.1296}{0.4306}$$

$$\mathrm{P(C|MD)} = 0.3010.$$

Example 8.22

Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coin, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold.

Solution:

In fig 8.11 given below if yellow colour denotes gold coin and grey colour denotes silver coin then:

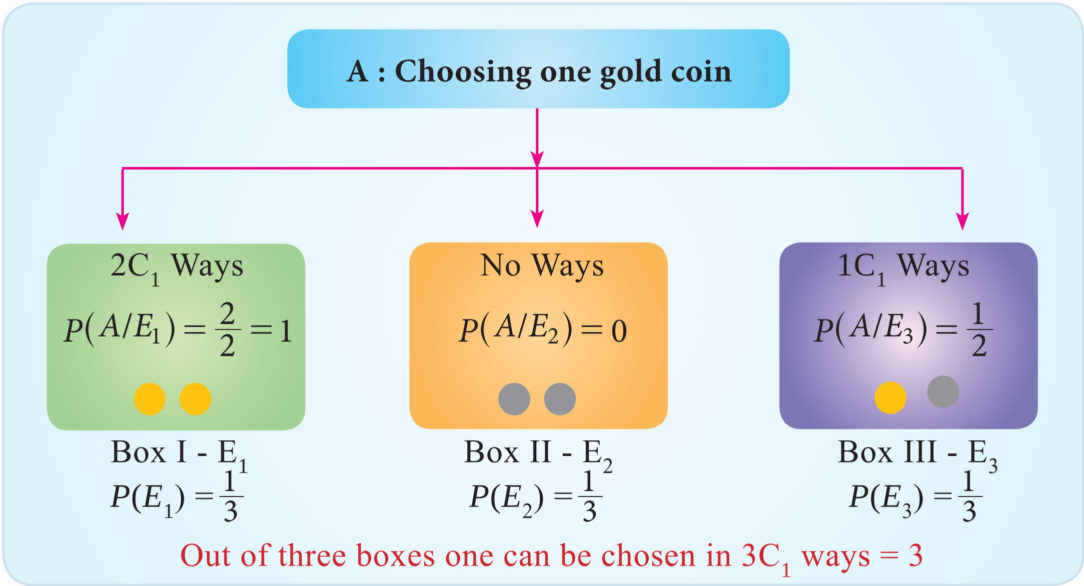


Fig. 8.11

If E_1, E_2 and E_3 be the events that the boxes I, II, III are chosen respectively.

Then $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

let A be the event that the gold coin is drawn

$P(A/E_1)$ = P (a gold coin from box 1) $= \frac{2}{2} = 1$

$P(A/E_2)$ = P (a gold coin from box 1I) $= \frac{0}{2} = 0$

$P(A/E_3)$ = P (a gold coin from box III) $= \frac{1}{2}$

Now, the probability that the other coin in the box is also gold is same as probability of choosing the box I and drawing a gold coin = $P(A/E_1)$

By Bayes' Theorem,

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{\sum_{i=1}^{3} P(E_i)\,P(A/E_i)}$$

$$= \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times 0 + \frac{1}{3} \times \frac{1}{2}}$$

$$= \frac{2}{3}$$

Points to Remember

- Probability of an event:

  (i) Classical Method: $P(A) = \frac{No.\ of\ outcomes\ in\ favour\ of\ A}{No.\ if\ elements\ in\ samplespace}$

  (ii) Relative Frequency Method: $P(A) = \lim_{n \to \infty} \frac{n(A)}{n}$

- Axioms

  (i) $P(A) \geq 0$

  (ii) $P(S) = 1$

  (iii) If $\{A_1, A_2, \dots\}$ is a sequence of mutually exclusive events, then

  $P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$

- Probability of complementary event: $P(\bar{A}) = 1 - P(A)$

- Addition theorem of probability: $P(A \cup B) = P(A) + P(B)$, if A and B are mutually exclusive events

  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- If $A \subset B$, then $P(A) \leq P(B)$ and $P(B-A) = P(B) - P(A)$

- Conditional probability $P(A/B) = \frac{P(A \cap B)}{P(B)}, if\ P(B) > 0$

- A and B are independent events if $P(A \cap B) = P(A) \times P(B)$

- Multiplication theorem of probability $P(A \cap B) = \begin{cases} P(A/B) \times P(B), if\ P(B) > 0 \\ P(B/A) \times P(A), if\ P(A) > 0 \end{cases}$

- Bayes' theorem $P(B_j/A) = \frac{P(A/B_j) \times P(B_j)}{\sum_{i=1}^{n} P(A/B_j) \times P(B_j)}$, $j = 1, 2, \dots n$

EXERCISE 8

I Choose the best answer:

1. Which one of the following is not related to random experiment?

   (a) outcomes are predictable in advance

   (b) outcomes is unknown, in advance

   (c) experiment repeatable finite number of times

   (d) experiment is repeatable any number of times.

2. Mathematical probability may also be termed as

   (a) statistical probability (b) classical probability

   (c) Empirical probability (d) None of the above

3. In rolling of a die until 4 appears, the sample space is

   (a) a null set (b) a countable finite set

   (c) a countable infinite set (d) an uncountable set

4. A patient who has undergone a difficult surgery will survive a minimum of 12 years. Then, his survival time, x (in years), can be represented by

   (a) $\{\underline{x} : 12 \leq x < \infty\}$ (b) $\{x : 12 \leq x < 24\}$

   (c) $\{x : 0 \leq x < 12\}$ (d) any interval along the real line.

5. If A and B are mutually exclusive events them $P(A \cup B)$ is equal to

   (a) $P(A) + P(B)$ (b) $P(A) - P(B)$

   (c) $P(A) + P(B) - \underline{P}(A \cap B)$ (d) $P(A)P(B)$

6. If A={1, 2 } ; B ={3, 4, 5};C={5, 6} are events in S. Then the number sample point in S is

   (a)1 (b)4 (c) 3 (d) 6

7. Probability of not getting 3 when a die is thrown is

   (a) $\frac{1}{3}$ (b) $\frac{5}{6}$ (c) $\frac{1}{6}$ (d) $\frac{1}{4}$

8. If A_1 and A_2 are two events with $P(A_1)=\frac{4}{9}$ and $P(A_2)= \frac{3}{9}$ and $P(A_1 \cap A_2)=\frac{2}{9}$ then the probability that none of these two events occur is

   (a) $\frac{7}{9}$ (b) $\frac{4}{9}$ (c) $\frac{30}{81}$ (d) $\frac{2}{9}$

9. If A and B are independent events with $P(A) = P(B)$; and $P(A \cap B)$ = a ; then $P(B)$ is

(a) $2a$ (b) $\sqrt{a}$ (c) $\frac{a}{2}$ (d) a^2

10. If A and B are two events with P(A/B)= 0.3, P(B/A) = 0.2. thenP(A) is

(a) $\frac{3}{10}$ (b) $\frac{7}{10}$ (c) $\frac{6}{7}$ (d) $\frac{1}{7}$

II. Fill in the blanks:

11. The probability of the entire sample space is ______________

12. On throwing the single die, then the event of getting odd number or even number are ____________ event.

13. Probability of getting a Monday in a week is ____________

14. If A_1 and A_2 are independent events, then $P(A_1 \cup A_2) = p(A_1) +$ ______

15. If $A \subset B$, then $P(A)$___$P(B)$.

16. If probability for the occurrence of an event is 1, then the event is known as __________ event.

17. If $P(A) = 0$, then A is called _________ event.

18. Axiomatic approach to probability was proposed by __________

19. If A and B are two events with $P(A \cup B) = \frac{10}{15}$, then $P(\bar{A} \cap \bar{B}) =$ ___________

20. The conditional probability P(A/B) can be calculated using $P(B)$,if _________

III Very Short Answer Questions:

21. If E_1 and E_2 are two mutually exclusive events and Given that $P(E_2) = 0.5$ and $P(E_1 \cup E_2) = 0.7$ then find $P(E_1)$.

22. Find the probability that a leap year selected at random will contain 53 Fridays?

23. Two coins are tossed simultaneously. What is the probability of getting exactly two heads?

24. If $P(A \cap B) = 0.3$, $P(B)$= 0.7 find the value of P(A/B)

25. A box contains 5 red and 4 white marbles. Two marbles are drawn successively from the box without replacement and it is noted that the second one is white. What is the probability that the first is also white?

26. Define:
    (i) Event, (ii) Random Experiment, (iii) Sample space, (iv) Mutually exclusive events, (v) Exhaustive events (vi) Independent events, (vii) Dependent events.

27. Define mathematical probability.

28. Define statistical probability.

29. Define conditional probability.

30. State the Multiplication theorem on probability for any two events.

31. State the Multiplication theorem of probability for independent events.

IV. Short Answer Questions:

32. What is the chance that a non-leap year selected at random will contain 53 Sundays or 53 Mondays?

33. When two dice are thrown, find the probability of getting doublets. (same number on both dice).

34. A box containing 5 green balls land 3 red colour balls. Find the probability of selecting 3 green colour balls without replacement.

35. There are 5 items defective in a sample of 30 items. Find the probability that an item chosen at random from the sample is (i) defective (ii) non – defective

36. Given that $P(A) = 0.35$, $P(B) = 0.73$ and $P(A \cap B) = 0.14$, find $P(\overline{A} \cup \overline{B})$

37. State the axioms of probability.

38. State the theorem on total probability.

39. State Bayes' theorem.

40. A card is drawn at random from a well shuffled pack of 52 cards. What is the probability that it is (i) an ace (ii) a diamond card?

V Calculate the following:

41. State and prove addition theorem on probability.

42. An urn contains 5 red and 7 green balls. Another urn contains 6 red and 9 green balls. If a ball is drawn from any one of the two urns, find the probability that the ball drawn is green.

43. In a railway reservation office, two clerks are engaged in checking reservation forms. On an average, the first clerk (A_1) checks 55% of the forms, while the second clerk (A_2) checks the remaining. A_1 has an error rate of 0.03 and A_2 has an error rate of 0.02. A reservation form is selected at random from the total number of forms checked during the day and is discovered to have some errors. Find the probability that the form is checked by A_1 and A_2 respectively.

44. In a university, 30% of the students are doing a course in statistics use the book authored by A_1, 45% use the book authored by A_2 and 25% use the book authored by A_3. The proportion of the students who learnt about each of these books through their teacher are $P(A_1)$ =0.50, $P(A_2)$=0.30, and $P(A_3)$=0.20. One of the student selected at random revealed that he learned the book he is using through their teachers. Find the probability that the book used it was authored by A_1, A_2, and A_3 respectively.

45. A bolt manufacturing company has four machines *A*, *B*, *C* and *D* producing 20%, 15%, 25% and 40% of the total output respectively. 5%, 4%, 3% and 2% of their output (in the same order) are defective bolts. A bolt is chosen at random from the factory and is found defective what is the probability of getting a defective bolt.

46. A city is partitioned into districts *A, B, C* having 20 percent, 40 percent and 40 percent of the registered voters respectively. The registered voters listed as Democrats are 50 percent in A, 25 percent in B and 75 percent in C. A registered voter is chosen randomly in the city. Find the probability that the voter is a listed democrat.

ANSWERS

I.1. (*a*) 2. (*b*) 3. (c) 4. (a) 5. (a) 6. (d) 7. (b) 8. (b) 9. (b) 10. (c)

II. 11. one 12. mutually exclusive 13. $\frac{1}{7}$ 14. $P(A_1) + P(A_2)$

15. less than or equal to 16. sure event 17.impossible event

18. A.N. Kolmogorov 19. $\frac{1}{3}$ 20. greater than

III. 21. 0.2 22. $\frac{2}{7}$ 23. $\frac{1}{2}$ 24. $\frac{3}{7}$ 25. $\frac{4}{9}$

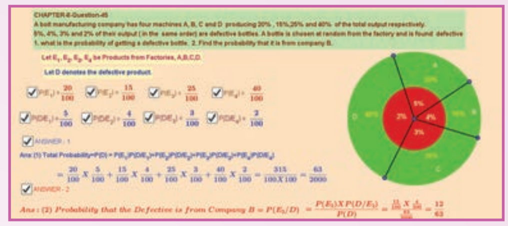
IV.32. $\frac{2}{7}$ 33. $\frac{1}{6}$ 34. $\frac{5}{28}$ 35.(i) $\frac{1}{6}$ (ii) $\frac{5}{6}$ 36. 0.86 40. (i) $\frac{1}{3}$ (ii) $\frac{1}{4}$

V.42. $\frac{71}{120}$ 43. 0.647, 0.353 44. 0.45, 0.40, 0.15 45. $\frac{63}{2000}$ 46. 50%

ICT CORNER

ELEMENTARY PROBABILITY THEORY-BAYES THEOREM PROBLEM

This activity is for conditional probability based Bayes theorem.
This helps to understand the problem.



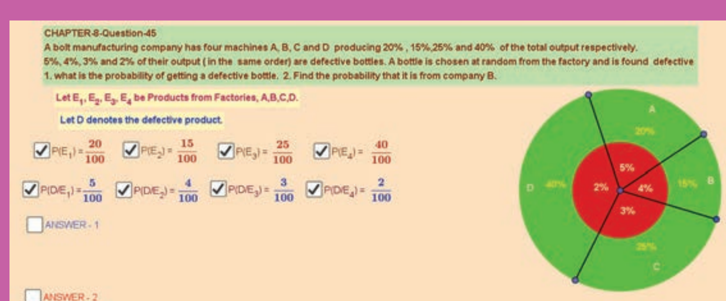
Steps:

- Open the browser and type the URL given (or) scan the QR code.
- GeoGebra work book called "**11th Standard Statistics**" will appear. In this several work sheets for Statistics are given, open the worksheet named "**Probability-Bayes theorem**"
- A text book exercise problem is explained step by step. Read the question carefully and compare with the diagram on right-hand side to assign the names as given and observe the check box names.
- Now click on the check boxes for separate probabilities and conditional probabilities. Now try to understand what conditional probability is, by observing the diagram.
- Now recall the formula for total probability **P(D)** and Bayes formula **P(E_2/D)** and work out.
- Now click on "**Answer-1**" and "**Answer-2**" and check your answer.

Step-1

Step-2

Step-3



Pictures are indicatives only*

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