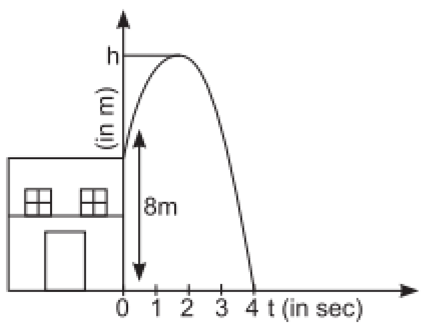
Polynomials

Case Study Based Questions

Case Study 1

Sukriti throws a ball upwards, from a rooftop which is 8 m high from ground level. The ball reaches to some maximum height and then returns and hit the ground. If height of the ball at time t (in sec) is represented by h(m), then equation of its path is given as $h=-t^2+2t+8$.



Based on the given information, solve the following questions:

Q1. The maximum height achieved by ball is:

a. 7 m

b. 8 m

c. 9 m

d. 10 m

Q2. The polynomial represented by above graph is:

a. linear polynomial

b. quadratic polynomial

c. constant polynomial

d. cubic polynomial

Q3. Time taken by ball to reach maximum height is: is given, is:

a. 2 sec

b. 4 sec

c. 1 sec

d. 2 min

Q4. Number of zeroes of the polynomial whose graph

a. 1

b. 2

c. 0

d. 3

Q5. Zeroes of the polynomial are:

a. 4

b. -2,4

c. 2,4

d. 0,4

Solutions

1. Given, $h=-t^2+2t+8$
At $t=0$, $h=-(0)^2+2(0)+8=8$
$t = 1$, $h=-(1)^2+2(1) +8=9$
$t=2$, $h=-(2)^2+2(2)+8=8$
$t=3$, $h=-(3)^2+2(3) +8=5$
$t=4$, $h=-(4)^2+2(4)+8=0$
Hence, the maximum height achieved by the ball is 9m.
So, option (c) is correct.

2. The polynomial represented by given graph is a quadratic polynomial. So, option (b) is correct.

3. The time taken by ball to reach maximum height is 1 sec.
So, option (c) is correct.

4. The number of zeroes of the polynomial whose graph is given, is 1. So, option (a) is correct.

5. Since, at $t=4,h=0$
Hence, zeroes of the polynomial is 4.
So, option (a) is correct.

Case Study 2

Asana is a body posture, originally and still a general term for a sitting meditation pose and later extended in hatha yoga and modern yoga as exercise, to any type of pose or position, adding reclining, standing, inverted, twisting and balancing poses. In the figure, one can observe that poses can be related to representation of quadratic polynomial.

Based on the above information, solve the following questions:

Q1. The shape of the poses shown is:

a. spiral

b. ellipse

c. linear

d. parabola

Q2. The graph of parabola opens downward, if:

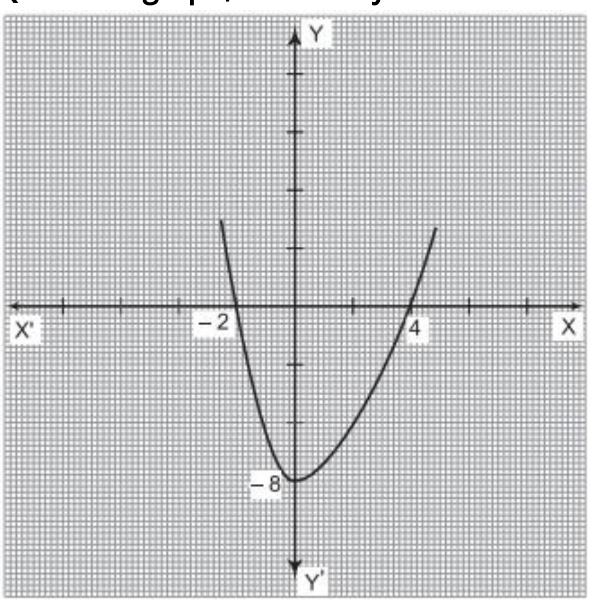
a. $a \geq 0$

b. $a = 0$

c. $a < 0$

d. $a > 0$

Q3. In the graph, how many zeroes are there for the polynomial?



a. 0
b. 1
c. 2
d. 3

Q4. The quadratic polynomial of the two zeroes in the above shown graph are:

a. $k(x^2-2x-8)$
b. $k(x^2+2x-8)$
c. $k(x^2+2x+8)$
d. $k(x^2-2x+8)$

Q5. The zeroes of the quadratic polynomial

$4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are:

a. $\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$
b. $-\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$
c. $\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$
d. $-\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$

Solutions

1. The shape of the poses shown is parabola which is a plane curve, mirror symmetrical and approximately
$\cup$ or $\cap$ shaped.

So, option (d) is correct.

2. If $a < 0$ in $f(x) = ax^2 + bx + c$, the parabola opens downward.
So, option (c) is correct.

3. Here, the graph cuts X-axis at two distinct points -2 and 4. Therefore, two zeroes are there for the polynomial. So, option (c) is correct.

4. In the given graph, the graph cuts X-axis at points -2 and 4, which are the zeroes of the graph. Now, the required polynomial
$= k [x^2$ - (Sum of zeroes) x + Product of zeroes)

$= k(x^2-(-2+4)x+(-2x4)]$

$= k(x^2-2x-8)$, where k is an arbitrary constant. So, option (a) is correct.

5. Given, polynomial = $4\sqrt{3}x^2+5x-2\sqrt{3}$

$= 4\sqrt{3}x^2+(8-3)x-2\sqrt{3}$

(by splitting the middle term)

$= 4\sqrt{}\sqrt{3}x^2+8x-3x-2\sqrt{3}$

$=4x(\sqrt{3}x+2)-\sqrt{}(\sqrt{3}x+2)$

$=(\sqrt{3}x+2)(4x-\sqrt{3})$

For the zeroes of the polynomial,

$(\sqrt{3}x+2)(4x-\sqrt{3})=0$

$\Rightarrow \sqrt{3}x+2=0$ or $4x-\sqrt{3}=0$

$\Rightarrow x=\frac{-2}{\sqrt{3}}$ or $x=\frac{\sqrt{3}}{4} \Rightarrow x=\frac{-2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

So, option (b) is correct.

Case Study 3

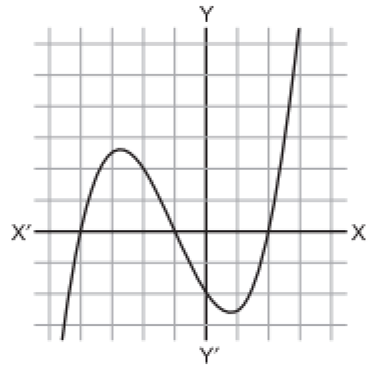
Roller Coaster Polynomials

Polynomials are everywhere. They play a key role in the study of algebra, in analysis and on the whole many mathematical problems involving them.

Since, polynomials are used to describe curves of various types, engineers use polynomials to graph the curves of roller coasters. [CBSE SQP 2021 Term-1]

Based on the given information, solve the following questions:

Q1. If the Roller Coaster is represented by the following graph y= p(x), then name the type of the polynomial it traces.



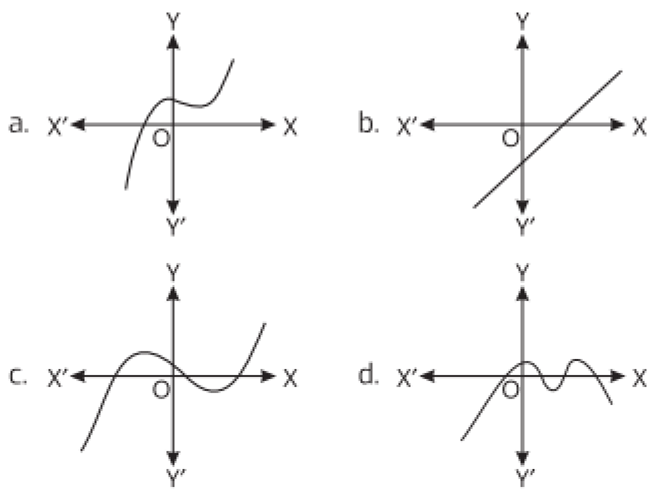
a. Linear

b. Quadratic

c. Cubic

d. Bi-quadratic

Q2. The Roller Coasters are represented by the following graphs y = p(x). Which Roller Coaster has more than three distinct zeroes?



Q3. If the Roller Coaster is represented by the cubic polynomial $t(x) = px^3 + qx^2 + rx + s$, then which of the following is always true?

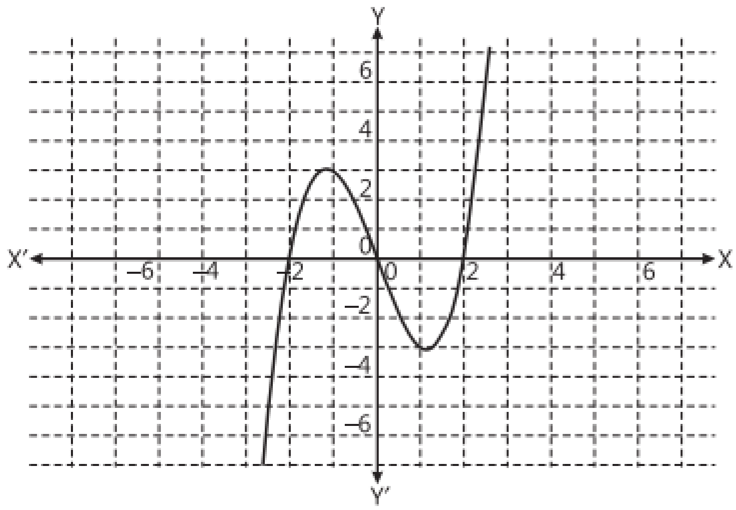
a. s #0

b. r #0

c. q#0

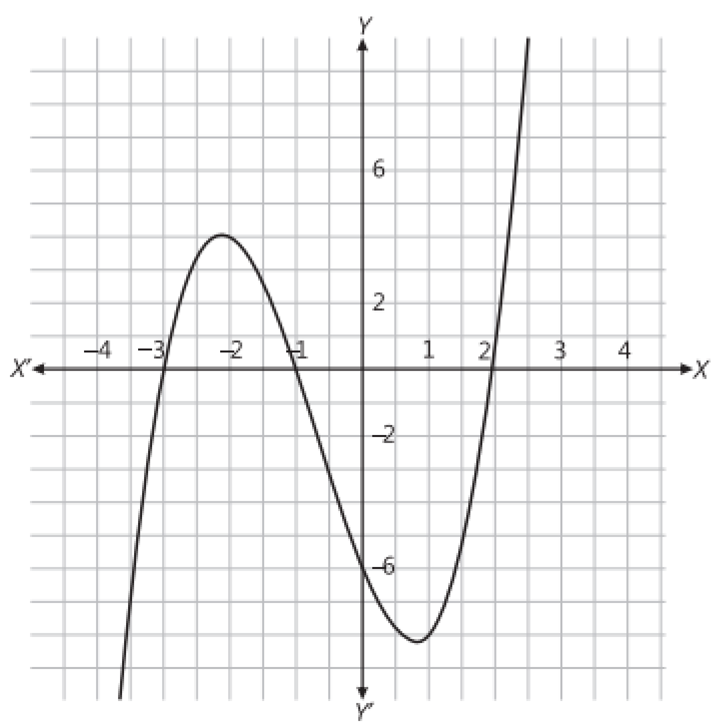
d. p #0

Q4. If the path traced by the Roller Coaster is represented by the following graph y = p(x), find the number of zeroes.



a. 0
b. 1
c. 2
d. 3

Q5. If the path traced by the Roller Coaster is represented by the following graph y = p(x), find its zeroes.



a. -3, -6, -1
b. 2, -6, -1
c. -3, -1, 2
d. 3, 1, -2

Solutions

1. Since the graph of the given polynomial y = p(x) cuts the X-axis at three points. So, the number of zeroes of the polynomial shown in the graph is three. This means the polynomial y = p(x) is cubic. So, option (c) is correct.

2. Polynomial of graph (a) cuts the X-axis at one point.
So, the number of zero is one.
Polynomial of graph (b) cuts the X-axis at one point.
So, the number of zeroes is one.
Polynomial of graph (c) cut the X-axis at three points.
So, the number of zeroes is three.
Polynomial of graph (d) cut the X-axis at four points.
So, the number of zeroes is four.
Thus, Roller Coaster of option (d) has more than
three distinct zeroes.
So, option (d) is correct.

3. A cubic polynomial $t(x) = px^3 + qx^2+rx+s$ is always true when p, q, rand s are real numbers and p = 0.
So, option (d) is correct.

4. Since the graph of the given polynomial y= p(x) cut the X-axis at three points. So, the number of zeroes of the polynomial shown in the graph is three. So, option (d) is correct.

5. (c)
The graph of the polynomial y = p(x), cut the X-axis at three points-3, -1 and 2, which are also called the
zeroes of y = p(x).
So, option (c) is correct.

Case Study 4

Ramesh was asked by one of his friends Anirudh to find the polynomial whose zeroes are $\frac{-2}{\sqrt{3}}$ and $\frac{\sqrt{3}}{4}$. He obtained the polynomial by following steps which are as shown below:

Let $\alpha = \frac{-2}{\sqrt{3}}$ and $\beta = \frac{\sqrt{3}}{4}$

Then, $\alpha + \beta = \frac{-2}{\sqrt{3}} + \frac{\sqrt{3}}{4} = \frac{-8+1}{4\sqrt{3}} = \frac{-7}{4\sqrt{3}}$

and $\alpha\beta = \frac{-2}{\sqrt{3}} \times \frac{\sqrt{3}}{4} = \frac{-1}{2}$

$$\therefore \text{ Required polynomial } = x^2 - (\alpha + \beta)x + \alpha\beta$$
$$= x^2 - \left(\frac{-7}{4\sqrt{3}}\right)x + \left(\frac{-1}{2}\right)$$
$$= x^2 + \frac{7x}{4\sqrt{3}} - \frac{1}{2}$$
$$= 4\sqrt{3}x^2 + 7x - 2\sqrt{3}$$

His another friend Kavita pointed out that the polynomial obtained is not correct.
Based on the above information, solve the following questions:

Q1. Is the claim of Kavita correct?

Q2. If given polynomial is incorrect, then find the correct quadratic polynomial.

Q 3. Find the value of $\alpha^2 + \beta^2$.

OR

If correct polynomial $p(x)$ is a factor of $(x - 2)$, then find $f(2)$.

Solutions

1. Given, $\alpha = -\frac{2}{\sqrt{3}}$ and $\beta = \frac{\sqrt{3}}{4}$

$\therefore \quad \alpha + \beta = \frac{-2}{\sqrt{3}} + \frac{\sqrt{3}}{4} = \frac{-8+3}{4\sqrt{3}} = \frac{-5}{4\sqrt{3}}$

and $\quad \alpha\beta = \frac{-2}{\sqrt{3}} \times \frac{\sqrt{3}}{4} = -\frac{1}{2}$

Yes, because value of (a + ß) calculated by Anirudh is incorrect.

2. Required polynomial = $k(x^2-(a + B)x+aß)$

$= k\left(x^2 + \frac{5x}{4\sqrt{3}} - \frac{1}{2}\right)$

$= \frac{k}{4\sqrt{3}}(4\sqrt{3}x^2 + 5x - 2\sqrt{3})$

$= (4\sqrt{3}x^2 + 5x - 2\sqrt{3})$

where $k = 4\sqrt{3}$

3. $a^2+ß^2=(a+ß)2-2αß$

$= \left(\frac{-5}{4\sqrt{3}}\right)^2 - 2 \times \left(\frac{-1}{2}\right) = \frac{25}{48} + 1 = \frac{73}{48}$

Alternate method:

$\alpha^2 + \beta^2 = \left(\frac{-2}{\sqrt{3}}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2 = \frac{4}{3} + \frac{3}{16} = \frac{64+9}{48} = \frac{73}{48}$

OR

We have, $p(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

Since, $p(x)$ is a factor of $(x-2)$, then

$p(2) = 4\sqrt{3}(2)^2 + 5(2) - 2\sqrt{3}$

$= 16\sqrt{3} + 10 - 2\sqrt{3} \quad = 14\sqrt{3} + 10$

Hence, remainder is $14\sqrt{3} + 10$.

Case Study 5

A group of school friends went on an expedition to see caves. One person remarked that the entrance of the caves resembles a parabola and can be represented by a quadratic

polynomial
f(x)= ax^2 + bx + c, a# 0, where a, b and c are real numbers.

Based on the given information, solve the following questions:
Q1. Draw a neat labelled figure to show above situation diagrammatically.

Q2. If one of the zeroes of the quadratic polynomial
(p-1)x^2 + px + 1 is 4. Find the value of p.

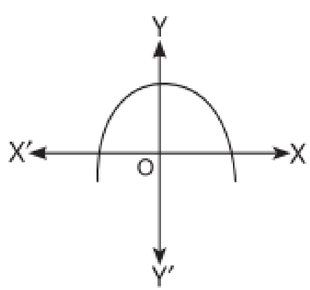
Q3. Find the quadratic polynomial whose zeroes are 5 and -12.

OR

If one zero of the polynomial f(x) = 5x^2 + 13x + m is reciprocal of the other, then find the value of m.

Solutions

1. We have, f(x) = ax^2 + bx + c, a<0
It means a figure is a shape of parabola which open downwards.



2. Since, x 4 is one of the zero of the polynomial (p-1)x^2+px + 1.

$$\therefore \quad (p-1)(4)^2 + p(4) + 1 = 0$$
$$\Rightarrow \quad 16p - 16 + 4p + 1 = 0$$
$$\Rightarrow \quad 20p = 15$$
$$\Rightarrow \quad p = \frac{3}{4}$$

3. Required quadratic polynomial

$= k\,[x^2$ - (Sum of zeroes)x + (Product of zeroes)]

$= k(x^2-(-12+5)x+(-12)(5))$

$= k(x^2+7x-60)$, where k is any arbitrary constant.

OR

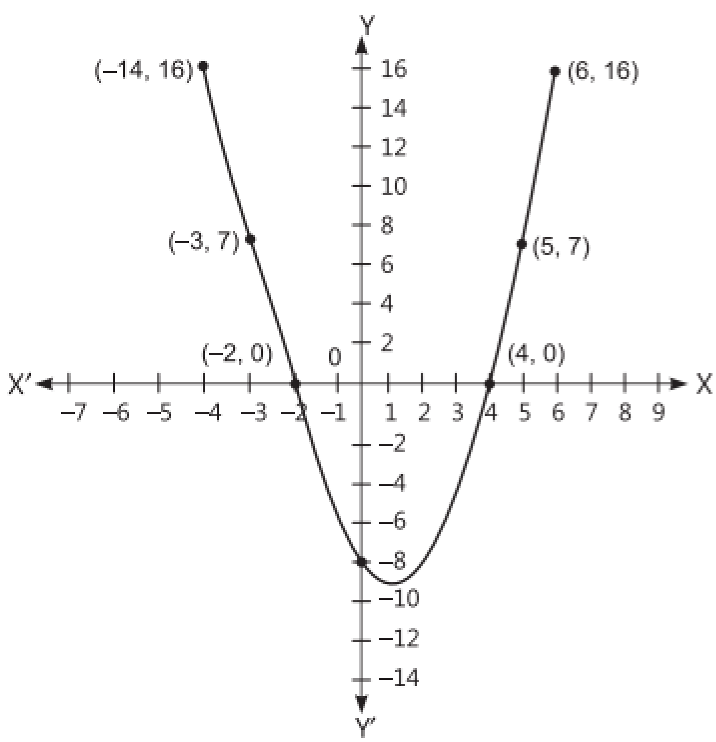
Let the zeroes of the quadratic polynomial be α. Since, one zero of the polynomial f(x) is reciprocal of the other.

$$\therefore \text{ Product of zeroes } = (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\Rightarrow \qquad \alpha \cdot \frac{1}{\alpha} = 1 \cdot \frac{m}{5} \quad \Rightarrow m = 5$$

Case Study 6

A student was given a task to prepare a graph of quadratic polynomial $p(x)=-8-2x+x^2$. To draw this graph, he take seven values of y corresponding to different values of x. After plotting the points on the graph paper with suitable values, he obtain the graph as shown below.



Based on the above graph, solve the following questions:

Q1. What is the shape of graph of a quadratic polynomial?

Q2. Find the zeroes of given quadratic polynomial.

Q3. The graph of the given quadratic polynomial cut at which points on the X-axis?

OR

The graph of the given quadratic polynomial cut at which point on Y-axis?

Solutions

1. The graph of a quadratic polynomial is a parabola which open upwards.

2. The zeroes of the quadratic polynomial
$p(x) = -8 -2x + x^2$ are x-coordinates of the points where the graph intersects the X-axis.
From the given graph, -2 and 4 are the x-coordinates of the points where the graph of $p(x)=-8-2x+x^2$ intersects the X-axis.
Hence, -2 and 4 are zeroes of $p(x)=-8-2x+x^2$.

3. The graph of the given quadratic polynomial cut
X-axis at points (-2, 0) and (4,0).

OR

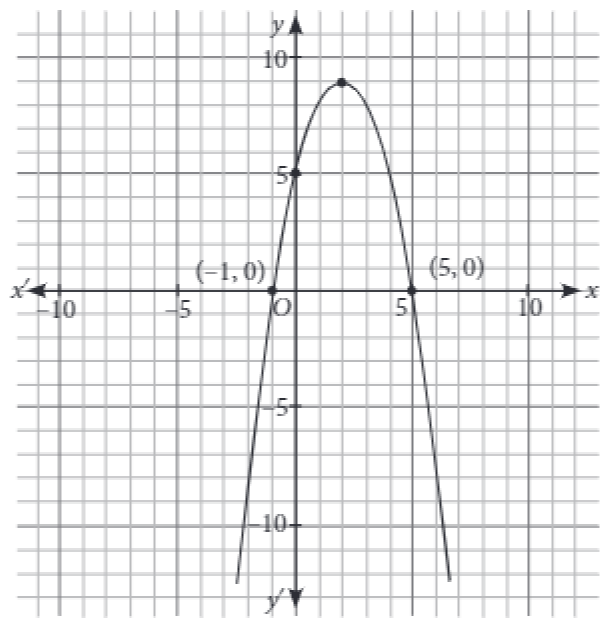
The graph of the given quadratic polynomial cut
Y-axis at point (0, -8).

Solutions for Questions 7 to 21 are Given Below

Case Study 7

Construction of Humps

ABC construction company got the contract of making speed humps on roads. Speed humps are parabolic in shape and prevents overspeeding, minimise accidents and gives a chance for pedestrians to cross the road. The mathematical representation of a speed hump is shown in the given graph.



Based on the above information, answer the following questions.

(i) The polynomial represented by the graph can be ______ polynomial.

(a) Linear (b) Quadratic

(c) Cubic (d) Zero

(ii) The zeroes of the polynomial represented by the graph are

(a) 1, 5 (b) 1, –5

(c) –1, 5 (d) –1, –5

(iii) The sum of zeroes of the polynomial represented by the graph are

(a) 4 (b) 5 (c) 6 (d) 7

(iv) If α and β are the zeroes of the polynomial represented by the graph such that $\beta > \alpha$, then $|8\alpha + \beta| =$

(a) 1 (b) 2 (c) 3 (d) 4

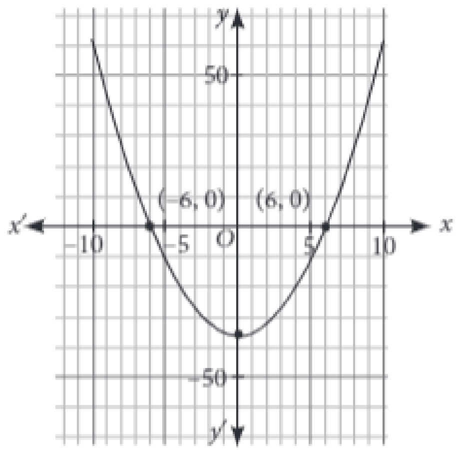
(v) The expression of the polynomial represented by the graph is

(a) $-x^2 - 4x - 5$ (b) $x^2 + 4x + 5$ (c) $x^2 + 4x - 5$ (d) $-x^2 + 4x + 5$

Case Study 8

Honeycomb

While playing in garden, Sahiba saw a honeycomb and asked her mother what is that. She replied that it's a honeycomb made by honey bees to store honey. Also, she told her that the shape of the honeycomb formed is parabolic. The mathematical representation of the honeycomb structure is shown in the graph.



Based on the above information, answer the following questions.

(i) Graph of a quadratic polynomial is _______ in shape.

(a) straight line (b) parabolic

(c) circular (d) None of these

(ii) The expression of the polynomial represented by the graph is

(a) $x^2 - 49$ (b) $x^2 - 64$ (c) $x^2 - 36$ (d) $x^2 - 81$

(iii) Find the value of the polynomial represented by the graph when $x = 6$.

(a) -2 (b) -1 (c) 0 (d) 1

(iv) The sum of zeroes of the polynomial $x^2 + 2x - 3$ is

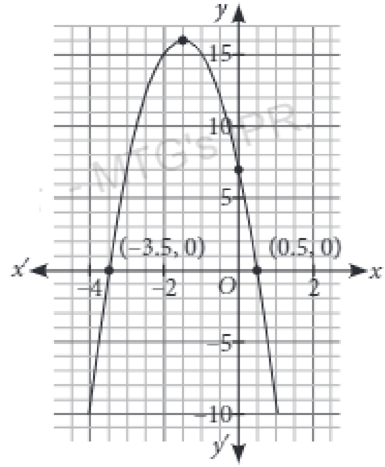
(a) -1 (b) -2 (c) 2 (d) 1

(v) If the sum of zeroes of polynomial $at^2 + 5t + 3a$ is equal to their product, then find the value of a.

(a) -5 (b) -3 (c) $\frac{5}{3}$ (d) $\frac{-5}{3}$

Case Study 9

Just before the morning assembly a teacher of kindergarten school observes some clouds in the sky and so she cancels the assembly. She also observes that the clouds has a shape of the polynomial. The mathematical representation of a cloud is shown in the figure.



(i) Find the zeroes of the polynomial represented by the graph.

(a) $-1/2, 7/2$ (b) $1/2, -7/2$ (c) $-1/2, -7/2$ (d) $1/2, 7/2$

(ii) What will be the expression for the polynomial represented by the graph?

(a) $p(x) = 12x^2 - 4x - 7$ (b) $p(x) = -x^2 - 12x + 3$ (c) $p(x) = 4x^2 + 12x + 7$ (d) $p(x) = -4x^2 - 12x + 7$

(iii) What will be the value of polynomial represented by the graph, when $x = 3$?

(a) 65 (b) –65 (c) 68 (d) –68

(iv) If α and β are the zeroes of the polynomial $f(x) = x^2 + 2x - 8$, then $\alpha^4 + \beta^4 =$

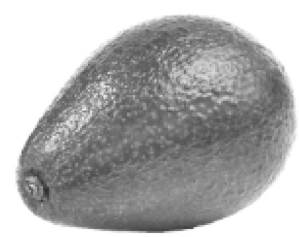
(a) 262 (b) 252 (c) 272 (d) 282

(v) Find a quadratic polynomial where sum and product of its zeroes are $0, \sqrt{7}$ respectively.

(a) $k(x^2 + \sqrt{7})$ (b) $k(x^2 - \sqrt{7})$ (c) $k(x^2 + \sqrt{5})$ (d) none of these

Case Study 10

Avocado



Pankaj's father gave him some money to buy avocado from the market at the rate of $p(x) = x^2 - 24x + 128$. Let α, β are the zeroes of $p(x)$.

Based on the above information, answer the following questions.

(i) Find the value of α and β, where $\alpha < \beta$.

(a) –8, –16 (b) 8, 16 (c) 8, 15 (d) 4, 9

(ii) Find the value of $\alpha + \beta + \alpha\beta$.

(a) 151 (b) 158 (c) 152 (d) 155

(iii) The value of $p(2)$ is

(a) 80 (b) 81 (c) 83 (d) 84

(iv) If α and β are zeroes of $x^2 + x - 2$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/5

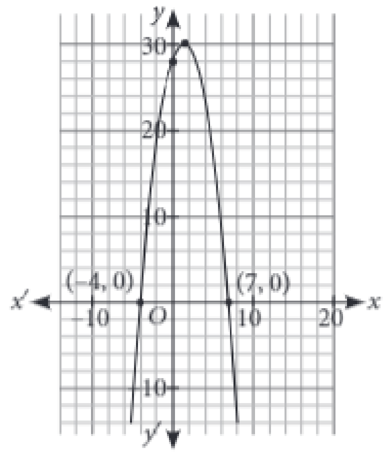
(v) If sum of zeroes of $q(x) = kx^2 + 2x + 3k$ is equal to their product, then $k =$

(a) 2/3 (b) 1/3 (c) –2/3 (d) –1/3

Case Study 11

Mountain Trekking

Two friends Trisha and Rohan during their summer vacations went to Manali. They decided to go for trekking. While trekking they observes that the trekking path is in the shape of a parabola. The mathematical representation of the track is shown in the graph.



Based on the above information, answer the following questions.

(i) The zeroes of the polynomial whose graph is given, are

(a) 4, 7 (b) –4, 7 (c) 4, 3 (d) 7, 10

(ii) What will be the expression of the given polynomial $p(x)$?

(a) $x^2 - 3x + 28$ (b) $-x^2 + 4x + 28$ (c) $x^2 - 4x + 28$ (d) $-x^2 + 3x + 28$

(iii) Product of zeroes of the given polynomial is

(a) –28 (b) 28 (c) –30 (d) 30

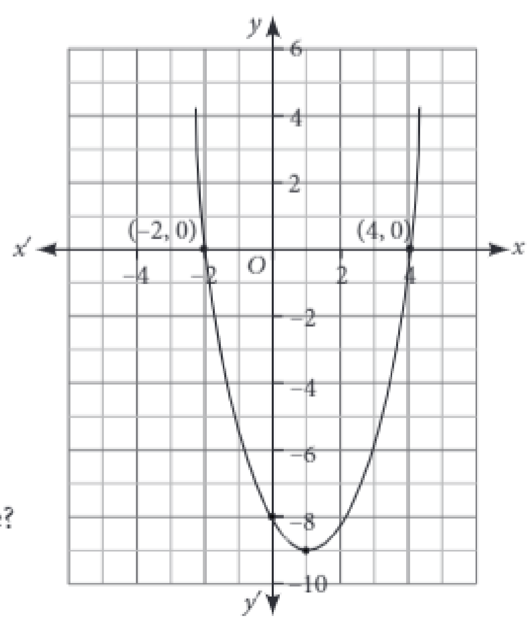
(iv) The zeroes of the polynomial $9x^2 - 5$ are

(a) $\frac{3}{\sqrt{5}}, \frac{-3}{\sqrt{5}}$ (b) $\frac{2}{\sqrt{5}}, \frac{-2}{\sqrt{5}}$ (c) $\frac{\sqrt{5}}{3}, \frac{-\sqrt{5}}{3}$ (d) $\frac{\sqrt{5}}{2}, \frac{-\sqrt{5}}{2}$

(v) If $f(x) = x^2 - 13x + 1$, then $f(4) =$

(a) 35 (b) –35 (c) 36 (d) –36

Case Study 12



Neeru saw a creeper on the boundary of her aunt's house which was in the shape as shown in the figure. Answer the following questions by considering that creeper has same mathematical shape as shown in the figure.

Based on the above information, answer the following questions.

(i) The shape represents a ______ polynomial.

(a) Linear (b) Cubic

(c) Quadratic (d) None of these

(ii) How many zeroes does the polynomial (shape of the creeper) have?

(a) 0 (b) 1

(c) 2 (d) 3

(iii) The zeroes of the polynomial, represented by the graph, are

(a) 4, –2 (b) –4, 2 (c) 4, 2 (d) –5, 6

(iv) The expression of the polynomial, represented by the graph, is

(a) $x^2 + 2x - 8$ (b) $x^2 - 2x - 8$ (c) $x^3 - x + 8$ (d) $x^3 - x^2 + 2x + 8$

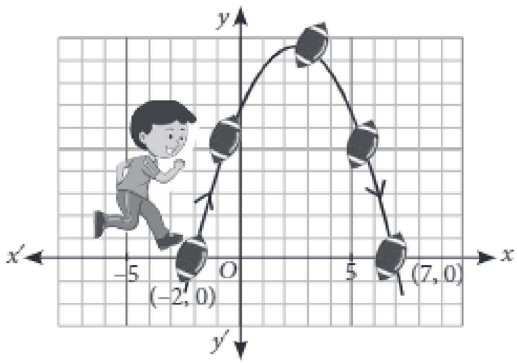
(v) For what value of x, the value of the polynomial, represented by the graph, is –5?

(a) $x = 3$ (b) $x = -1$ (c) Both (a) and (b) (d) Can't be determined

Case Study 13

Soccer Match

In a soccer match, the path of the soccer ball in a kick is recorded as shown in the following graph.



Based on the above information, answer the following questions.

(i) The shape of path of the soccer ball is a

(a) Circle (b) Parabola (c) Line (d) None of these

(ii) The axis of symmetry of the given parabola is

(a) y-axis (b) x-axis

(c) line parallel to y-axis (d) line parallel to x-axis

(iii) The zeroes of the polynomial, represented in the given graph, are

(a) –1, 7 (b) 5, –2 (c) –2, 7 (d) –3, 8

(iv) Which of the following polynomial has –2 and –3 as its zeroes?

(a) $x^2 - 5x - 5$ (b) $x^2 + 5x - 6$ (c) $x^2 + 6x - 5$ (d) $x^2 + 5x + 6$

(v) For what value of 'x', the value of the polynomial $f(x) = (x - 3)^2 + 9$ is 9?

(a) 1 (b) 2 (c) 3 (d) 4

Case Study 14

Slinky Spring Dog Toy

Prachi was playing with a slinky spring dog toy and asked her brother Rhythm, what is the shape thus formed called. Rhythm explained her that the shape formed is a parabola. He also explained her that parabola is the graphical representation of a quadratic polynomial.

Based on the above information, answer the following questions.

(i) The general form of polynomial representing the parabolic graph is

(a) $ax^2 + c, a \neq 0$ (b) $ax^2 + bx + c, b \neq 0$

(c) $ax^2 + bx + c$, a, b and $c \neq 0$ (d) $ax^2 + bx + c, a \neq 0$

(ii) Kavita drawn a parabola passing through (–4, 3), (–1, 0), (1, 8), (0, 3), (–3, 0) and (–2, –1) on the graph paper. Then zeroes of the polynomial representing the graph is

(a) 3 and –3 (b) –1 and –2 (c) –3 and –1 (d) 1 and 8

(iii) Which of the following is correct?

(a) A parabola intersects x-axis at maximum 2 points.

(b) A parabola intersects x-axis only at 1 point.

(c) A parabola intersects x-axis exactly at 2 points.

(d) A parabola intersects x-axis at least at 2 points.

(iv) The product of roots of the polynomial $5x(x - 6)$ is

(a) 3/2 (b) 2/3 (c) 3 (d) 0

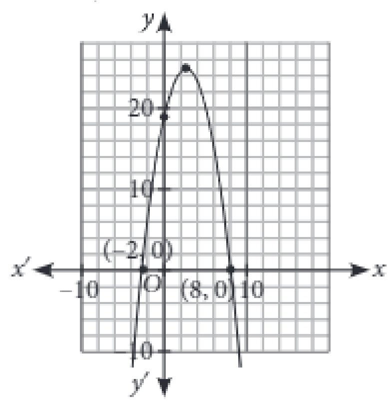
(v) The sum of zeroes of a quadratic polynomial $ax^2 + bx + c, a \neq 0$ is

(a) a/b (b) a/c (c) $-b/a$ (d) $-c/a$

Case Study 15

Application of Quadratic Polynomial–Highway Tunnel

Shweta and her husband Sunil who is an architect by profession, visited France. They went to see Mont Blanc Tunnel which is a highway tunnel between France and Italy, under the Mont Blanc Mountain in the Alps, and has a parabolic cross-section. The mathematical representation of the tunnel is shown in the graph.



Based on the above information, answer the following questions.

(i) The zeroes of the polynomial whose graph is given, are

(a) –2, 8 (b) –2, –8 (c) 2, 8 (d) –2, 0

(ii) What will be the expression of the polynomial given in diagram?

(a) $x^2 - 6x + 16$ (b) $-x^2 + 6x + 16$ (c) $x^2 + 6x + 16$ (d) $-x^2 - 6x - 16$

(iii) What is the value of the polynomial, represented by the graph, when $x = 4$?

(a) 22 (b) 23 (c) 24 (d) 25

(iv) If the tunnel is represented by $-x^2 + 3x - 2$, then its zeroes are

(a) –1, –2 (b) 1, –2 (c) –1, 2 (d) 1, 2

(v) If one zero is 4 and sum of zeroes is –3, then representation of tunnel as a polynomial is

(a) $x^2 - x + 24$ (b) $-x^2 - 3x + 28$ (c) $x^2 + x + 28$ (d) $x^2 - x + 28$

Case Study 16

Application of Polynomials–Architectural Structures

Quadratic polynomial can be used to model the shape of many architectural structures in the world. Pershing field of Jersey city in US is one such structure.

Based on the above information, answer the following questions.

(i) If the Arch is represented by $10x^2 - x - 3$, then its zeroes are

(a) $\frac{1}{2}, \frac{-3}{2}$ (b) $\frac{-1}{2}, \frac{3}{5}$ (c) $\frac{-1}{2}, \frac{1}{3}$ (d) $\frac{-1}{3}, \frac{2}{3}$

(ii) The zeroes of the polynomial are the points where its graph

(a) intersect the x-axis (b) intersect the y-axis

(c) intersect either of the axes (d) Can't say

(iii) The quadratic polynomial whose sum of zeroes is 0 and product of zeroes is 1 is given by

(a) $x^2 - x$ (b) $x^2 + x$ (c) $x^2 - 1$ (d) $x^2 + 1$

(iv) Which of the following has $\frac{-1}{2}$ and 2 as their zeroes?

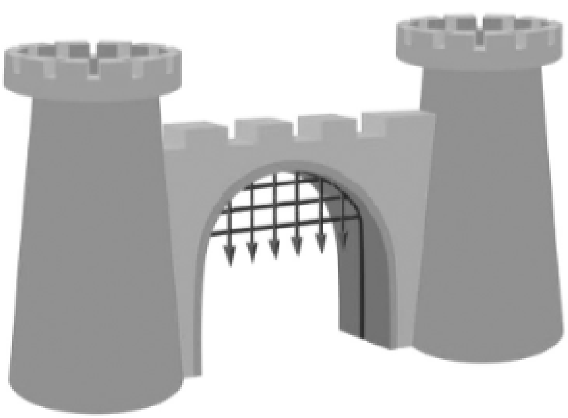
(a) $6x^2 - 4x + 6$ (b) $3x^2 - x + 2$ (c) $2x^2 - 7x + 2$ (d) $2x^2 - 3x - 2$

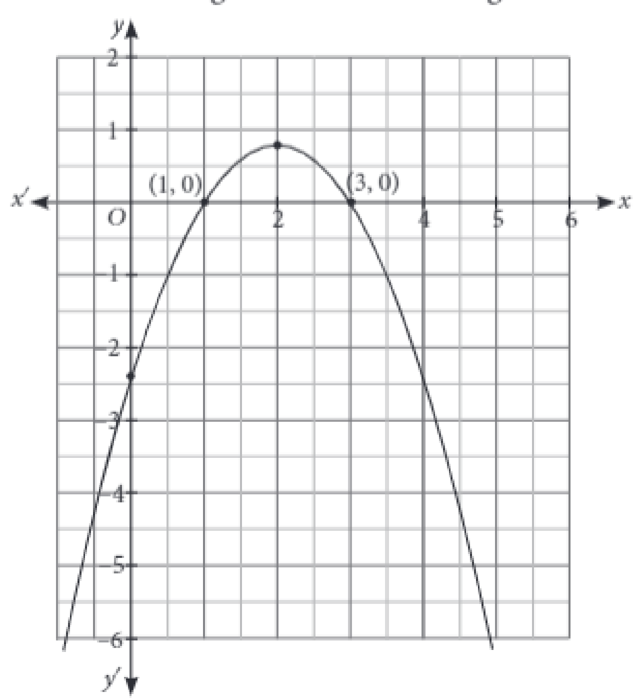
(v) The product of zeroes of the polynomial $\sqrt{3}x^2 - 14x + 8\sqrt{3}$ is

(a) 4 (b) 6 (c) 8 (d) 10

Case Study 17

Priya visited a temple in Gwalior. On the way she sees the Agra Fort. The entrance gate of the fort has a shape of quadratic polynomial (parabolic). The mathematical representation of the gate is shown in the figure.





Based on the above information, answer the following questions.

(i) Find the zeroes of the polynomial represented by the graph.

(a) $-1, 3$ (b) $1, 3$ (c) $1, -3$ (d) $0, 1$

(ii) What will be the expression for the polynomial represented by the graph?

(a) $x^2 + 4x - 5$ (b) $x^2 - 4x + 5$ (c) $-x^2 + 4x - 3$ (d) $x^2 + 5x - 4$

(iii) What will be the value of polynomial, represented by the graph, when $x = 4$?

(a) -2 (b) 3 (c) -3 (d) 2

(iv) If one zero of a polynomial $p(x)$ is 7 and product of its zeroes is -35, then $p(x) =$

(a) $-x^2 + 2x + 35$ (b) $x^2 + 2x + 35$ (c) $x^2 + 12x - 35$ (d) $x^2 - 12x - 35$

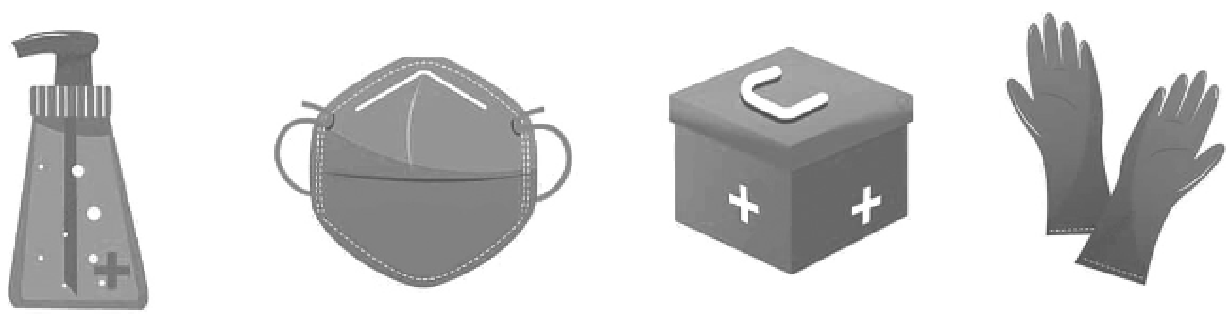
(v) If the gate is represented by the polynomial $-x^2 + 5x - 6$, then its zeroes are

(a) $2, -3$ (b) $2, 3$ (c) $-2, 3$ (d) $-2, -3$

Case Study 18

Social Service

Shray, who is a social worker, wants to distribute masks, gloves, and hand sanitizer bottles in his block. Number of masks, gloves and sanitizer bottles distributed in 1 day can be represented by the zeroes α, β, γ, $(\alpha > \beta > \gamma)$ of the polynomial $p(x) = x^3 - 18x^2 + 95x - 150$.



Based on the above information, answer the following questions.

(i) Find the value of α, β, γ.

(a) $-10, -5, -3$ (b) $3, 6, 5$

(c) $10, 5, 3$ (d) $4, 8, 9$

(ii) The sum of product of zeroes taken two at a time is

(a) 91 (b) 92 (c) 94 (d) 95

(iii) Product of zeroes of polynomial $p(x)$ is

(a) 150 (b) 160 (c) 170 (d) 180

(iv) The value of the polynomial $p(x)$, when $x = 4$ is

(a) 5 (b) 6 (c) 7 (d) 8

(v) If α, β, γ are the zeroes of a polynomial $g(x)$ such that $\alpha + \beta + \gamma = 3$, $\alpha\beta + \beta\gamma + \gamma\alpha = -16$ and $\alpha\beta\gamma = -48$, then $g(x) =$

(a) $x^3 - 2x^2 - 48x + 6$ (b) $x^3 + 3x^2 + +16x - 48$

(c) $x^3 - 48x^2 - 16x + 3$ (d) $x^3 - 3x^2 - 16x + 48$

Case Study 19

Barrier Chains

While playing badminton Ronit seeing the barrier chains hung between two posts at the edge of the walk way of a street. It is hung in the shape of the parabola. Parabola is the graphical representation of a particular type of polynomial.

Based on the above information, answer the following questions.

(i) Which of the following polynomial is graphically represented by a parabola?

(a) Linear polynomial (b) Quadratic polynomial

(c) Cubic polynomial (d) None of these

(ii) If a polynomial, represented by a parabola, intersects the x-axis at –3, 4 and y-axis at –2, then its zero(es) is/are

(a) –1, 2 and –2 (b) 2 and –2 (c) –1 (d) –3 and 4

(iii) If the barrier chains between two posts is represented by the polynomial $x^2 - x - 12$, then its zeroes are

(a) 4, 3 (b) –2, 5 (c) 4, –3 (d) 4, –5

(iv) The sum of zeroes of the polynomial $4x^2 - 9x + 2$ is

(a) 1/4 (b) 9/4 (c) 2/4 (d) –9/4

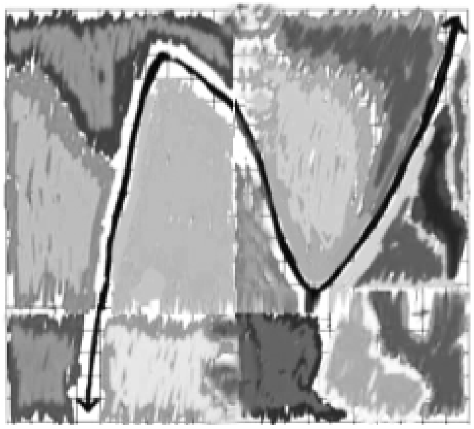
(v) The reciprocal of product of zeroes of the polynomial $x^2 - 9x + 20$ is

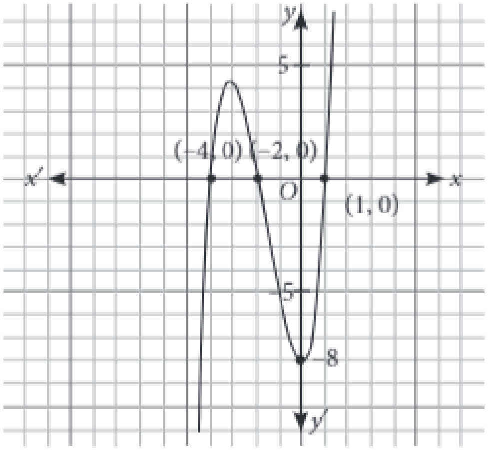
(a) 5 (b) 1/8 (c) 1/20 (d) 20

Case Study 20

Painting Exhibition

Shruti is very good in painting. So she thought of exhibiting her paintings in which she want to display her latest painting which is in the form of a graph of a polynomial as shown below :





Based on the above information, answer the following questions.

(i) The number of zeroes of the polynomial represented by the graph is

(a) 1 (b) 2 (c) 3 (d) can't be determined

(ii) The sum of zeroes of the polynomial represented by the graph is

(a) –4 (b) –3 (c) 2 (d) –5

(iii) Find the value of the polynomial represented by the graph when $x = 0$.

(a) –6 (b) –8 (c) 6 (d) 8

(iv) The polynomial representing the graph drawn in the painting by Shruti is a

(a) quadratic polynomial (b) cubic polynomial

(c) bi-quadratic polynomial (d) linear polynomial

(v) The sum of product of zeroes, taken two at a time, of the polynomial represented by the graph is

(a) 2 (b) 3 (c) –2 (d) –3

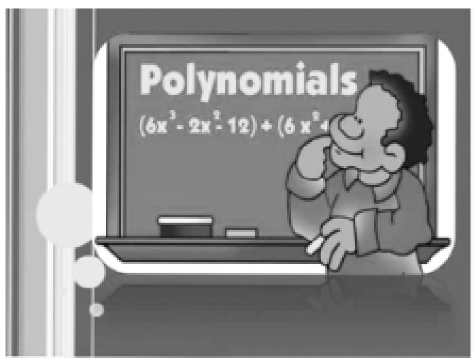
Case Study 21

The tutor in a coaching centre was explaining the concept of cubic polynomial as - A cubic polynomial is of the form $ax^3 + bx^2 + cx + d$, $a \neq 0$ and it has maximum three real zeroes. The zeroes of a cubic polynomial are namely the x-coordinates of the points where the graph of the polynomial intersects the x-axis. If α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then the relation between their zeroes and their coefficients are

$\alpha + \beta + \gamma = -b/a$

$\alpha\beta + \beta\gamma + \alpha\gamma = c/a$

$\alpha\beta\gamma = -d/a$



Based on the above information, answer the following questions.

(i) Which of the following are the zeroes of the polynomial $x^3 - 4x^2 - 7x + 10$?

(a) –3, 1 and 3 (b) –1, 2 and –3

(c) 2, –1 and 5 (d) –2, 1 and 5

(ii) If $-\frac{1}{2}$, –2 and 5 are zeroes of a cubic polynomial, then the sum of product of zeroes taken two at a time is

(a) $\frac{23}{2}$ (b) $-\frac{1}{2}$

(c) –23 (d) $-\frac{23}{2}$

(iii) In which of the following polynomials the sum and product of zeroes are equal ?

(a) $x^3 - x^2 + 5x - 1$ (b) $x^3 - 4x$

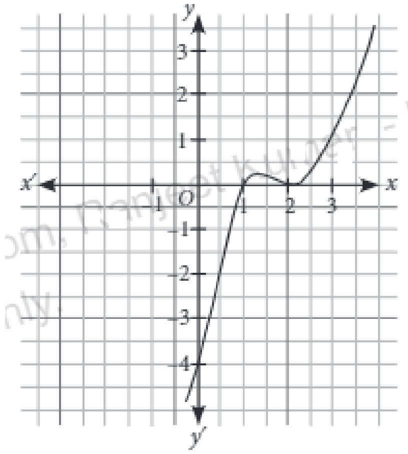
(c) $3x^3 - 5x^2 - 11x - 3$ (d) Both (a) and (b)

(iv) The polynomial whose all the zeroes are same is

(a) $x^3 + x^2 + x - 1$ (b) $x^3 - 3x^2 + 3x - 1$

(c) $x^3 - 5x^2 + 6x - 1$ (d) $3x^3 + x^2 + 2x - 1$

(v) The cubic polynomial, whose graph is as shown below, is



(a) $x^3 - 5x^2 + 8x - 4$ (b) $x^3 - 7x^2 + 11x + 9$

(c) $3x^3 - 4x^2 + x - 5$ (d) $x^3 - 9$

HINTS & EXPLANATIONS

7. **(i) (b):** Since, the given graph is parabolic is shape, therefore it will represent a quadratic polynomial.

[$\because$ Graph of quadratic polynomial is parabolic in shape]

(ii) (c): Since, the graph cuts the x-axis at –1, 5. So the polynomial has 2 zeroes *i.e.*, –1 and 5.

(iii) (a): Sum of zeroes = –1 + 5 = 4

(iv) (c): Since α and β are zeroes of the given polynomial and $\beta > \alpha$

$\therefore$ $\alpha = -1$ and $\beta = 5$.

$\therefore$ $|8\alpha + \beta| = |8(-1) + 5| = |-8 + 5| = |-3| = 3$.

(v) (d): Since the zeroes of the given polynomial are – 1 and 5.

$\therefore$ Required polynomial $p(x)$

$= k\{x^2 - (-1 + 5)x + (-1)(5)\} = k(x^2 - 4x - 5)$

For $k = -1$, we get

$p(x) = -x^2 + 4x + 5$, which is the required polynomial.

8. **(i) (b):** Graph of a quadratic polynomial is a parabolic in shape.

(ii) (c): Since the graph of the polynomial cuts the x-axis at (–6, 0) and (6, 0). So, the zeroes of polynomial are –6 and 6.

$\therefore$ Required polynomial is

$p(x) = x^2 - (-6 + 6)x + (-6)(6) = x^2 - 36$

(iii) (c): We have, $p(x) = x^2 - 36$

Now, $p(6) = 6^2 - 36 = 36 - 36 = 0$

(iv) (b): Let $f(x) = x^2 + 2x - 3$. Then,

$$\text{Sum of zeroes} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{(2)}{1} = -2$$

(v) (d): The given polynomial is $at^2 + 5t + 3a$

Given, sum of zeroes = product of zeroes

$$\Rightarrow \frac{-5}{a} = \frac{3a}{a} \quad \Rightarrow \quad a = \frac{-5}{3}$$

9. **(i) (b):** Since the graph of the polynomial intersect the x-axis at $x = \frac{1}{2}, \frac{-7}{2}$, therefore required zeroes of the polynomial are $\frac{1}{2}$ and $\frac{-7}{2}$.

(ii) (d): $\because \frac{1}{2}$ and $\frac{-7}{2}$ are the zeroes of the polynomial.

So, at $x = \frac{1}{2}, \frac{-7}{2}$, the value of the polynomial will be 0.

From options, required polynomial is

$p(x) = -4x^2 - 12x + 7$.

(iii) (b): We have, $p(x) = -4x^2 - 12x + 7$

$\therefore$ $p(3) = -4(3)^2 - 12(3) + 7 = -36 - 36 + 7 = -65$

(iv) (c): Here $f(x) = x^2 + 2x - 8$ and α, β are its zeroes.

$\therefore$ $\alpha + \beta = -2$ and $\alpha\beta = -8$

Now, $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$= ((\alpha + \beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2$

$= [(-2)^2 - 2(-8)]^2 - 2(-8)^2$

$= [4 + 16]^2 - 2(-8)^2 = (20)^2 - 2(64)$

$= 400 - 128 = 272$

(v) (a): We have sum of zeroes = 0 and product of zeroes = $\sqrt{7}$

So, required polynomial = $k(x^2 - 0 \cdot x + \sqrt{7})$
$= k(x^2 + \sqrt{7})$

10. (i) (b): Given, α and β are the zeroes of $p(x) = x^2 - 24x + 128.$
Putting $p(x) = 0$, we get
$x^2 - 8x - 16x + 128 = 0$
$\Rightarrow x(x-8) - 16(x-8) = 0$
$\Rightarrow (x-8)(x-16) = 0 \Rightarrow x = 8$ or $x = 16$
$\therefore \alpha = 8, \beta = 16$

(ii) (c): $\alpha + \beta + \alpha\beta = 8 + 16 + (8)(16)$
$= 24 + 128 = 152$

(iii) (d): $p(2) = 2^2 - 24(2) + 128 = 4 - 48 + 128 = 84$

(iv) (a): Since α and β are zeroes of $x^2 + x - 2$.
$\therefore \alpha + \beta = -1$ and $\alpha\beta = -2$
Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-1}{-2} = \frac{1}{2}$

(v) (c): Sum of zeroes $= \frac{-2}{k}$
Product of zeroes = $\frac{3k}{k} = 3$
According to question, we have $\frac{-2}{k} = 3$
$\Rightarrow k = \frac{-2}{3}$

11. (i) (b): Since, the graph intersects the x-axis at two points, namely $x = -4, 7$
So, $-4, 7$ are the zeroes of the polynomial.

(ii) (d): $p(x) = -x^2 + 3x + 28$

(iii) (a): Product of zeroes = $\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\therefore$ Required product of zeroes = $\frac{28}{-1} = -28$

(iv) (c): We have, $9x^2 - 5 = (3x)^2 - (\sqrt{5})^2$
$= (3x - \sqrt{5})(3x + \sqrt{5})$

$\therefore x = \frac{\sqrt{5}}{3}$ or $\frac{-\sqrt{5}}{3}$

(v) (b): Here, $f(x) = x^2 - 13x + 1$
$\therefore f(4) = 4^2 - 13(4) + 1 = 16 - 52 + 1 = -35$

12. (i) (c): The shape represents a quadratic polynomial.

(ii) (c): Since, the graph of polynomial cuts the x-axis at $(-2, 0)$ and $(4, 0)$. So, the polynomial has 2 zeroes.

(iii) (a): The zeroes of the polynomial are -2 and 4.

(iv) (b): Required polynomial is
$p(x) = x^2 - (-2 + 4)x + (-2)(4) = x^2 - 2x - 8$

(v) (c): Consider, $p(x) = -5$
$\Rightarrow x^2 - 2x - 8 = -5 \Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow (x-3)(x+1) = 0 \Rightarrow x = -1$ or $x = 3$
So, at $x = 3$ and at $x = -1$, $p(x) = -5$.

13. (i) (b): The shape of the path of the soccer ball is a parabola.

(ii) (c): The axis of symmetry of the given curve is a line parallel to y-axis.

(iii) (a): The zeroes of the polynomial, represented in the given graph, are -2 and 7, since the curve cuts the x-axis at these points.

(iv) (d): A polynomial having zeroes -2 and -3 is
$p(x) = x^2 - (-2 - 3)x + (-2)(-3) = x^2 + 5x + 6$

(v) (c): We have, $f(x) = (x-3)^2 + 9$
Now, $9 = (x-3)^2 + 9$
$\Rightarrow (x-3)^2 = 0 \Rightarrow x - 3 = 0 \Rightarrow x = 3$

14. (i) (d): The general form of polynomial representing the parabolic graph is $ax^2 + bx + c, a \neq 0$.

(ii) (c): The zeroes of the polynomial are the points at which its graph intersects the x-axis, *i.e.*, whose y coordinate is 0.
$\therefore$ Zeroes are -3 and -1.

(iii) (a): A parabola intersects x-axis at maximum 2 points.

(iv) (d): The product of roots of the polynomial $5x^2 - 30x$ is 0. [$\because$ constant term = 0]

(v) (c): Sum of zeroes of quadratic polynomial $ax^2 + bx + c, a \neq 0$ is $\frac{-b}{a}$.

15. (i) (a): Since, the graph intersects the x-axis at two points, namely $x = 8, -2$.
So, $8, -2$ are the zeroes of the given polynomial.

(ii) (b): The expression of the polynomial given in diagram is $-x^2 + 6x + 16$.

(iii) (c): Let $p(x) = -x^2 + 6x + 16$
When $x = 4$, $p(4) = -4^2 + 6 \times 4 + 16 = 24$

(iv) (d): Let $f(x) = -x^2 + 3x - 2$
Now, consider $f(x) = 0 \Rightarrow -x^2 + 3x - 2 = 0$
$\Rightarrow x^2 - 3x + 2 = 0 \Rightarrow (x-2)(x-1) = 0$
$\Rightarrow x = 1, 2$ are its zeroes.

(v) (b): Let α and β are the zeroes of the required polynomial.
Given, $\alpha + \beta = -3$
If $\alpha = 4$, then $\beta = -7$
$\therefore$ Representation of tunnel is $-x^2 - 3x + 28$.

16. (i) (b): Put $10x^2 - x - 3 = 0$

$\Rightarrow 10x^2 - 6x + 5x - 3 = 0 \Rightarrow (2x+1)(5x-3) = 0$

$\Rightarrow x = \frac{-1}{2}$ or $\frac{3}{5}$

Thus, the zeroes are $\frac{3}{5}$ and $\frac{-1}{2}$.

(ii) (a): The zeroes of the polynomial are the points where its graph intersect the x-axis.

(iii) (d) (iv) (d)

(v) (c): Product of zeroes $= \frac{8\sqrt{3}}{\sqrt{3}} = 8$

17. (i) (b): Since, the graph of the polynomial intersect the x-axis at $x = 1, 3$ therefore required zeroes of the polynomial are 1 and 3.

(ii) (c)

(iii) (c): Let $f(x) = -x^2 + 4x - 3$

Then $f(4) = -4^2 + 4 \times 4 - 3$
$= -16 + 16 - 3 = -3$

(iv) (a): Clearly, other zero $= \frac{-35}{7} = -5$

Thus, the zeroes are 7 and –5.

From the options, 7 and –5 satisfies only $-x^2 + 2x + 35$.

So, $p(x) = -x^2 + 2x + 35$

(v) (b): Let $p(x) = -x^2 + 5x - 6$

For zeroes, consider $p(x) = 0$

$\Rightarrow -x^2 + 5x - 6 = 0 \Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow (x-3)(x-2) = 0 \Rightarrow x = 3, 2$

Thus, the required zeroes are 3 and 2.

18. (i) (c): For finding α, β, γ, consider $p(x) = 0$

$\Rightarrow x^3 - 18x^2 + 95x - 150 = 0$

$\Rightarrow (x-3)(x^2 - 15x + 50) = 0$

$\Rightarrow (x-3)(x-5)(x-10) = 0 \Rightarrow x = 10$ or $x = 5$ or $x = 3$

Thus $\alpha = 10, \beta = 5$ and $\gamma = 3$

(ii) (d): Here $\alpha = 10, \beta = 5$ and $\gamma = 3$

$\therefore$ Sum of product of zeroes taken two at a time
$= \alpha\beta + \beta\gamma + \gamma\alpha = (10)(5) + (5)(3) + (3)(10)$
$= 50 + 15 + 30 = 95$

(iii) (a): Product of zeroes of polynomial $p(x) = \alpha\beta\gamma$
$= (10)(5)(3) = 150$

(iv) (b): We have $p(x) = x^3 - 18x^2 + 95x - 150$

Now, $p(4) = 4^3 - 18(4)^2 + 95(4) - 150$
$= 64 - 288 + 380 - 150 = 6$

(v) (d): $g(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

$\Rightarrow g(x) = x^3 - 3x^2 - 16x - (-48) = x^3 - 3x^2 - 16x + 48$

19. (i) (b)

(ii) (d): Since, the parabola intersects the x-axis at –3 and 4. So, zeroes of the polynomial are –3 and 4.

(iii) (c): Let $f(x) = x^2 - x - 12$
$= x^2 - 4x + 3x - 12 = (x+3)(x-4)$

Consider $f(x) = 0 \Rightarrow (x+3)(x-4) = 0 \Rightarrow x = 4, -3$

(iv) (b): Sum of zeroes $= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$
$= -\frac{(-9)}{4} = \frac{9}{4}$

(v) (c): Product of zeroes $= \frac{20}{1} = 20$

$\therefore$ Reciprocal of product of zeroes $= \frac{1}{20}$

20. (i) (c): Since the graph intersect the x-axis at 3 points, therefore the polynomial has 3 zeroes.

(ii) (d): Clearly the graph intersect the x-axis at $x = -4$, $x = -2$ and $x = 1$, therefore the zeroes are –4, –2 and 1.

Now, the sum of zeroes $= -4 - 2 + 1 = -5$

(iii) (b): From the graph, it can be seen that

When $x = 0$, then $y = -8$.

(iv) (b): Since there are 3 zeroes, therefore the graph represents a cubic polynomial.

(v) (a): The sum of product of zeroes taken two at a time $= (-4)(-2) + (-2)(1) + (1)(-4) = 8 - 2 - 4 = 2$

21. (i) (d): For finding zeroes, check whether $x^3 - 4x^2 - 7x + 10$ is 0 for given zeroes.

Let $p(x) = x^3 - 4x^2 - 7x + 10$. Then,

Clearly $p(-2) = p(1) = p(5) = 0$

So, the zeroes are –2, 1 and 5.

(ii) (d): Here $\alpha = \frac{-1}{2}, \beta = -2$ and $\gamma = 5$

$\therefore$ Sum of product of zeroes taken two at a time
$= \alpha\beta + \beta\gamma + \gamma\alpha$

$= \left(\frac{-1}{2}\right)(-2) + (-2)(5) + (5)\left(\frac{-1}{2}\right) = 1 - 10 - \frac{5}{2} = \frac{-23}{2}$

(iii) (d): Consider $x^3 - x^2 + 5x - 1$

Sum of zeroes = 1 = Product of zeroes

Now, consider $x^3 - 4x$

Sum of zeroes = 0 = Product of zeroes.

(iv) (b): Let α, α, α, be the zeroes of the cubic polynomial. [$\because$ All zeroes are same]

Then, $\alpha^3 = 1 \Rightarrow \alpha = 1$ [Using given options]

So, the required polynomial is $(x-1)^3 = x^3 - 3x^2 + 3x - 1$

(v) (a): Clearly $x = 1$ and $x = 2$ are the zeroes of given polynomial, both of which satisfies $x^3 - 5x^2 + 8x - 4$.