Chapter – 15

Probability

Introduction to Probability

Introduction

We often hear these statements in our daily life like:

- It will probably rain in the afternoon.
- Sunrisers Hyderabad has a good chance of winning the IPL series this year.
- Jack is probably right.

In all of the above statements, there is uncertainty about the occurrence of the event.

The word 'probability' means there is uncertainty about the happening of the event.

Therefore, the probability is the measure of uncertainty of an event, numerically. It can have the value from 0 to 1.

Applications

Probability is widely used in

- Weather forecasting
- Developing sports strategies
- Stock markets
- Physical Sciences
- Biological Sciences
- Medical Sciences

Important terms related to Probability

Experiment	An operation that produces some well-defined outcomes. E.g. tossing a coin, rolling a dice.
Outcome	The possible result of an experiment is called an outcome. E.g. getting heads when a coin is tossed.
Trial	A trial is an action that results in one or several outcomes. E.g. If a coin is tossed 30 times then each toss of the coin is a trial.
Event	It is the collection of some outcomes of the experiment. It is generally denoted by E. E.g. getting an odd number on a rolled dice.
Elementary Event	An event having only one outcome of the experiment is called an elementary event. E.g. getting heads or tails on tossing a coin
Compound Event	It is obtained by combining two or more elementary events associated with an experiment
Equally Likely Outcomes	When each outcome of an experiment is as likely to occur as the other, then the outcomes of the experiment are said to be equally likely. E.g. When a die is thrown then the six outcomes, 1, 2, 3, 4, 5 and 6 are equally likely to appear.

Probability - A Theoretical Approach

Probability – A theoretical approach

There are mainly two types of probability:

- Experimental Probability
- Theoretical Probability

Experimental Probability

Experimental Probabilities are based on the results of actual experiments and adequate recordings of the happening of the events. These probabilities are only estimates.

Let n be the total number of trials. The experimental probability or empirical probability P(E) of an event happening, is given by

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{The total number of trials}}$$

Empirical probability can be applied to every event associated with an experiment which can be repeated a large number of times.

However, there are certain limitations associated with an empirical probability

• Certain experiments cannot be repeated as they may be very expensive or unfeasible in many situations. Like repeating the experiment of launching a satellite to calculate the empirical probability of its failure during launching.
• Repetition of the phenomenon of an earthquake to compute the empirical probability of a multi-storied building getting destroyed in an earthquake.

Theoretical Probability

There are many experiments where we can make certain assumptions to avoid the repetition of the experiment. These assumptions help us to calculate the exact (theoretical) probability.

The theoretical Probability (also called classical probability) of an event E, written as P(E), is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Number of all possible outcomes of the experiment}}$$

where we assume that the outcomes of the experiment are equally likely.

So, in theoretical probability, we try to predict what will happen without actually experimenting.

Important Points
• The sum of the probabilities of all the elementary events of an experiment is 1.
Example, if there are three elementary events A, B and C in the experiment, then $P(A) + P(B) + P(C) = 1$
• The event $\bar{E}$, representing 'notE' is called the complement of the event E. We also say that E and $\bar{E}$ are complementary events.

$$P(E) + P(\bar{E}) = 1 \Rightarrow P(\bar{E}) = 1 - P(E)$$

• The probability of an event that is impossible to occur is 0 and such an event is called an impossible event.

• The probability of an event that is sure (or certain) to occur is 1. Such an event is called a sure event or a certain event.

• The probability of any event can be $0 \leq P(E) \leq 1$.

Example: An unbiased die is thrown. What is the probability of getting:

i) a multiple of 3
ii) an even number or a multiple of 3
When a die is thrown, the possible outcomes are 1, 2, 3, 4, 5 and 6.

i) Let E be the event 'getting a multiple of 3'.

Number of possible outcomes = 6

The outcomes favourable to E are 3 and 6.

The number of outcomes favourable to E = 2

$$P(E) = \frac{Number\ of\ outcomes\ favourable\ to\ E}{Number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$$

$$P(E) = \frac{2}{6} = \frac{1}{3}$$

ii) Let E be the event 'getting an even number or a multiple of 3'.

Number of possible outcomes = 6

The outcomes favourable to E are 2, 3, 4 and 6

The number of outcomes favourable to E = 4

$$P(E) = \frac{Number\ of\ outcomes\ favourable\ to\ E}{Number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$$

$$P(E) = \frac{4}{6} = \frac{2}{3}$$

Example: One card is drawn from a well-shuffled deck of 52 cards.

Find the probability of getting

i) a king of black colour

ii) a face card

iii) the jack of heart

iv) the queen of the diamond

Total number of cards in a deck = 52

∴ Total number of outcomes = 52

i) Let E be the event 'getting a king of black colour'

Now, there are four kings in a deck of playing cards out of which

two are black and two are red.

Therefore, the number of outcomes favourable to E = 2

$$P(E) = \frac{Number\ of\ outcomes\ favourable\ to\ E}{Number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$$

$$P(E) = \frac{2}{52} = \frac{1}{26}$$

ii) Let E be the event 'getting a face card'

There are 12 face cards in a deck of cards, namely 4 kings, 4 queens, 4 jacks

Therefore, the number of outcomes favourable to E = 12

$$P(E) = \frac{Number\ of\ outcomes\ favourable\ to\ E}{Number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$$

$$P(E) = \frac{12}{52} = \frac{3}{13}$$

iii) Let E be the event 'getting a jack'

There are four jacks in a deck of cards.

Therefore, the number of outcomes favourable to E = 4

P(E) = $\frac{Number\ of\ outcomes\ favourable\ to\ E}{Number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$

P(E) = $\frac{4}{52} = \frac{1}{13}$

iv) LetE be the event 'getting a queen of diamond'

There is only one queen of the diamond card in a deck of cards.

Therefore, the number of outcomes favourable to E = 1

P(E) = $\frac{Number\ of\ outcomes\ favourable\ to\ E}{Number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$

P(E) = $\frac{1}{52}$

Example: One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will

i) be an ace   ii) not be an ace

A well-shuffled deck of cards ensures equally likely outcomes.

Total number of cards in a deck = 52

∴ Total number of outcomes = 52

i) Let E be the event 'getting an ace'

There are four aces in a deck of cards.

Therefore, the number of outcomes favourable to E = 4

P(E) = $\frac{Number\ of\ outcomes\ favourable\ to\ E}{Number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$

P(E) = $\frac{4}{52} = \frac{1}{13}$

ii) Let $\bar{E}$ be the event 'not getting an ace'

We know, $P(\bar{E}) = 1 - P(E)$

$P(\bar{E}) = 1 - \frac{1}{13} = \frac{13-1}{13} = \frac{12}{13}$

Example: Savita and Hamida are friends. What is the probability that both will have

i) different birthdays?

ii) the same birthdays? (ignoring a leap year)

There are 365 days in a year. Now, Savita's birthday can be any day of the year and Hamida's birthday can also be any day of 365 days in the year.

i) If Savita's birthday is different from Hamida's,

Number of favourable outcomes for her birthday is 365 – 1 = 364

So, P (Savita's birthday is different from Hamida's) = $\frac{364}{365}$

ii) P (Savita and Hamida have the same birthday) = 1 - $\frac{364}{365}$

$= \frac{364-365}{365} = \frac{1}{365}$ (Using $P(\bar{E}) = 1 - P(E)$)

Example: A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green, is $\frac{2}{3}$. Find the number of blue marbles in the jar.

Total number of marbles = 24

P (getting a green marble) = $\frac{2}{3}$

P (getting a blue marble) = $1 - \frac{2}{3}$ (Using $P(\bar{E}) = 1 - P(E)$)

$= \frac{3-2}{3} = \frac{1}{3}$

P (getting a blue marble) $= \frac{Number\ of\ blue\ marbles}{Total\ Number\ of\ marbles}$

$\frac{1}{3} = \frac{Number\ of\ blue\ marbles}{24}$

Number of blue marbles $= \frac{24}{3} = 8$

Example: A pair of dice is thrown once. Find the probability of getting

i) doublet of prime numbers

ii) doublet of odd numbers

When a pair of dice is thrown simultaneously, the possible outcomes are (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6).

Total number of possible outcomes $= 6 \times 6 = 36$

i) Doublet of prime numbers are (2, 2), (3, 3), (5, 5)

Number of favourable outcomes = 3

P(doublet of prime numbers) $= \frac{Number\ of\ favourable\ outcomes}{Number\ of\ possible\ outcomes}$

$= \frac{3}{36} = \frac{1}{12}$

ii) Doublet of odd numbers are (1, 1), (3, 3), (5, 5)

Number of favourable outcomes = 3

P(doublet of prime numbers) = $\frac{Number\ of\ favourable\ outcomes}{Number\ of\ possible\ outcomes} = \frac{3}{36} = \frac{1}{12}$

Example: Three different coins are tossed together. Find the probability of getting

i) exactly two heads

ii) at least two heads

iii) at least two tails

Here, we denote 'heads' by H and 'tails' by T. When three coins are tossed simultaneously, the possible outcomes are (H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, H, H), (T, H, T), (T, T, H) and (T, T, T)

Number of possible outcomes = 8

i) Favourable outcomes of getting exactly two heads are (H, H, T), (H, T, H) and (H, T, H)

Number of Favourable outcomes = 3

P(getting exactly two heads) = $\frac{Number\ of\ favourable\ outcomes}{Number\ of\ possible\ outcomes} = \frac{3}{8}$

ii) Favourable outcomes of getting at least two heads are (H, H, H), (H, H, T), (H, T, H) and (T, H, H).

Number of Favourable outcomes = 4

P(getting at least two heads) = $\frac{Number\ of\ favourable\ outcomes}{Number\ of\ possible\ outcomes} = \frac{4}{8} = \frac{1}{2}$

iii) Favourable outcomes of getting at least two tails are(H, T, T), (T,H, T), (T, T, H) and (T, T, T)

Number of Favourable outcomes = 4

P(getting exactly two heads) = $\frac{Number\ of\ favourable\ outcomes}{Number\ of\ possible\ outcomes} = \frac{4}{8} = \frac{1}{2}$