Fourier Series:

→ It is an approximation process where a non-sinusoidal waveform is converted to sinusoidal such that all the Periodic signal are represented as in unique form.

→ Sinusoids and Complex exponents are eigen functions of LTI system.
(same o/p as input)

→ For a signal to have fourier series, orthogonality is must condition. (Two signal $x_1(t)$ and $x_2(t)$ are orthogonal if

$$\int_{t_1}^{t_2} x_1(t) \cdot x_2(t)\, dt = 0.$$

→ Means area under inner product inner product of two signal is zero.

Frequency \ Time	Continuous	Discrete
Continuous	C.T.F.T.	D.T.F.T.
Discrete	C.F.S.	D.F.S / D.F.T

Aim: one period $\longrightarrow$ many freq.

$\omega_0 \longrightarrow n\omega_0$.

→ There are 2 reasons for evaluating the F.S.

1. To obtain an expression for $f(t)$ that applies everywhere, rather than only over a single period.

2. To obtain phasor, which indirectly tell how much power is available at each harmonic of the waveform.

* Trignometric Fourier series (T.F.S.).

→ $0, \omega_0, 2\omega_0, 3\omega_0, \cdots$

$$x(t) = a_0 \cos(0 \cdot \omega_0 t) + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + a_3 \cos 3\omega_0 t + a_4 \cos 4\omega_0 t + \dots + b_0 \sin(0 \cdot \omega_0 t) + b_1 \sin \omega_0 t + b_2 \sin(2\omega_0 t) + b_3 \sin 3\omega_0 t + \dots$$

$$\therefore \boxed{x(t) = \underbrace{a_0}_{d.c.} + \underbrace{\sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin \omega_0 n t}_{a.c.}}$$

$$\Rightarrow \int_0^T \cos\omega_0 nt\, dt = \int_0^T \sin\omega_0 nt \cdot dt = 0.$$

$n \neq 0.$

$$\Rightarrow \int_0^T \sin\omega_0 mt \cdot \sin\omega_0 nt \cdot dt = T/2 \; ; \; m = n$$
$$= 0 \; ; \; m \neq n.$$

$$\Rightarrow \int_0^T \cos\omega_0 mt \cdot \cos\omega_0 nt \cdot dt = T/2 \; ; \; m = n$$
$$= 0 \; ; \; m \neq n.$$

$$\Rightarrow \int_0^T \cos\omega_0 mt \cdot \sin\omega_0 nt \cdot dt = 0 \; ; \; \forall\, m, n$$

$\Rightarrow$ By integrating both side of the eqn (1)

$$\int_0^T g(t) \cdot dt = \int_0^T a_0\, dt + \sum_{n=1}^{\infty} \cancelto{0}{\int a_n \cos\omega_0 nt\, dt} + \cancelto{0}{\int b_n \sin\omega_0 nt\, dt}$$

$$\therefore \int_0^T g(t) \cdot dt = a_0 \cdot T$$

$$\Rightarrow \boxed{a_0 = \frac{1}{T}\int_0^T g(t) \cdot dt} = \text{d.c. value (or) avg. value.}$$

$$\Rightarrow \boxed{a_0 = \frac{\text{Area of } g(t) \text{ over a period}}{\quad}}$$

$\Rightarrow$ Multiply both side of eq^n - ① by $\cos \omega_0 nt$ & integrating both side of eq^n - ① then we get,

$$\int_0^T g(t).\cos \omega_0 nt\, dt = \cancelto{0}{\int_0^T a_0 . \cos \omega_0 nt . dt}$$

$$+ \sum_{n=1}^{\infty} \cancelto{T/2 \cdot a_n.}{\int_0^T a_n . \cos \omega_0 nt . \cos \omega_0 nt\, dt}$$

$$+ \cancelto{0}{\int_0^T b_n . \cos \omega_0 nt . \sin \omega_0 nt . dt.}$$

$$\therefore \int_0^T g(t).\cos \omega_0 nt\, dt = a_n . \left(\frac{T}{2}\right)$$

$$\therefore \boxed{a_n = \frac{2}{T} \times \int_0^T g(t).\cos \omega_0 nt . dt.}$$

* Polar form:

$$\Rightarrow \quad g(t) = \cancel{a_n}\, d_0 + \sum_{n=1}^{\infty} d_n \cos (\omega_0 nt + \theta_n).$$

$$(\text{or})\ d_0 + \sum_{n=1}^{\infty} d_n \sin (\omega_0 nt + \theta_n).$$

fore convienent we will use cosine.

$$\therefore\ g(t) = d_0 + \sum_{n=1}^{\infty} d_n \cos\omega_0 n t \cdot \cos\theta_n - d_n \sin\omega_0 n t \cdot \sin\theta_n.$$

$a_n = d_n \cos\theta_n, \quad b_n = -d_n \cdot \sin\theta_n.$

$a_0 = d_0.$

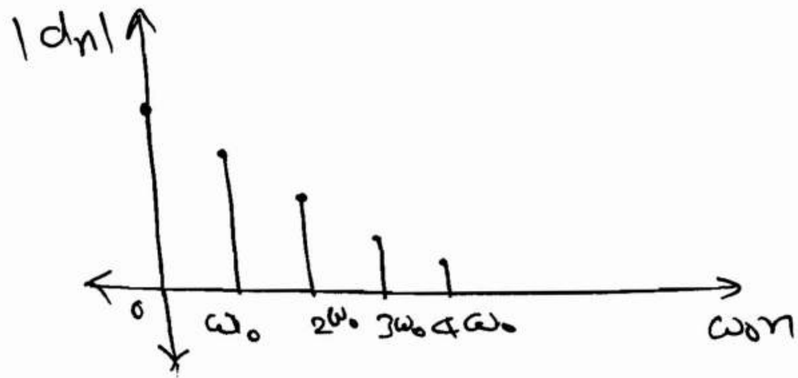
$$\boxed{\begin{aligned} \therefore\ |d_n| &= \sqrt{a_n^2 + b_n^2} \\ \therefore\ \theta_n &= \tan^{-1}\left(-\frac{b_n}{a_n}\right) \end{aligned}}$$

$|d_n| = \sqrt{a_n^2+b_n^2} \longrightarrow$ magnitude spectrum

$\theta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right) \longrightarrow$ phase spectrum.

Amplitude Spectrum.

$\Rightarrow$

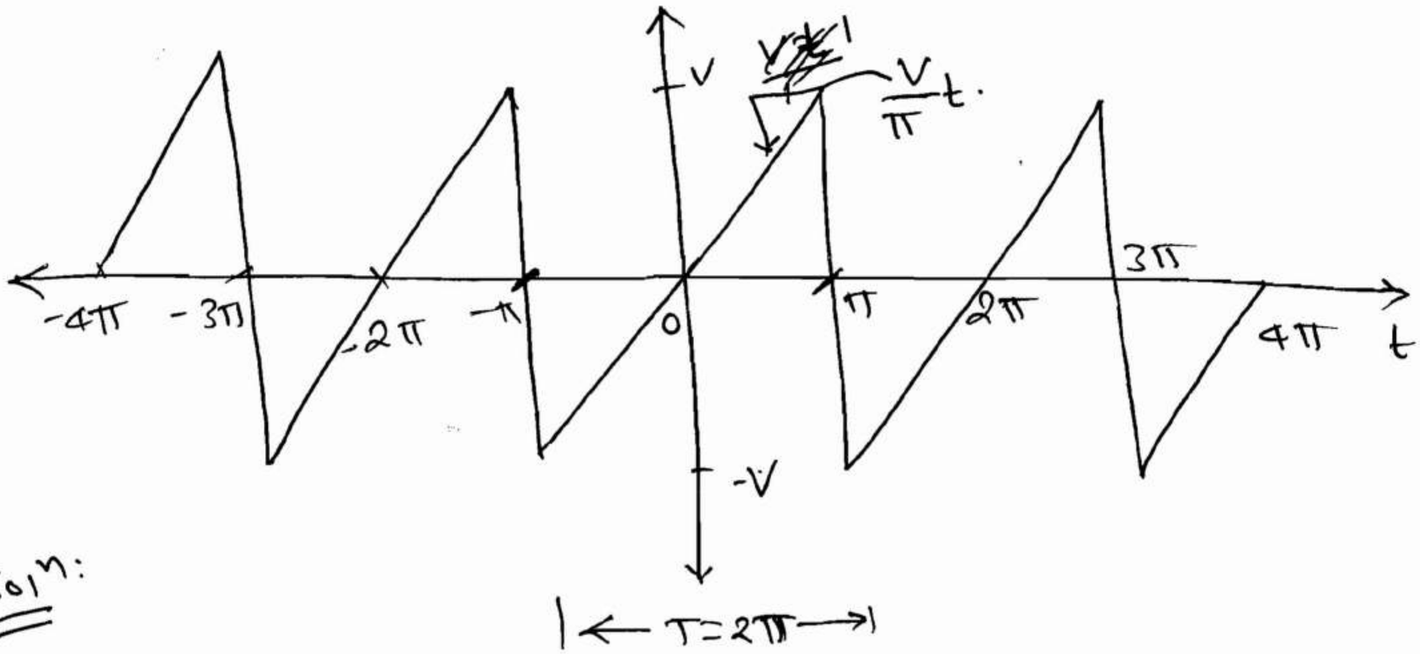


<u>Note</u>: From the magnitude spectrum we will inform that for what range of freq. the max. power is concentrated.

* Effect of Symmetry on Fourier ~~transform~~ coefficient:-

Symmetry	Condition	a_0	a_n	b_n
Even	$g(t) = g(-t)$	?	?	0
odd	$g(t) = -g(-t)$	0	0	?
Half-	[illegible]	[illegible]	$= 0$; n-even	$= 0$, n-even

[a] Find the TFS representation for the periodic waveform shown below.



Sol^n:

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1 \text{ rad/sec.}$$

→ Here, odd-symmetry.

So, $a_0 = 0$, $a_n = 0$.

$$x(t) = \frac{V}{\pi}t \quad , \quad -\pi \le t \le \pi.$$

$$b_n = \frac{2}{T}\int_0^T x(t).\ \sin\omega_0 n t\cdot dt$$

$$= \frac{2}{2\pi}\int_0^{2\pi} x(t).\ \sin\omega_0 nt\cdot dt$$

$$= \frac{1}{\pi}\cdot\int_{-\pi}^{\pi} x(t).\ \sin\omega_0 n\, t\, dt$$

$$= \frac{2}{\pi}\times\int^{\pi} \frac{V}{\pi}t.\ \sin\omega_0 n t\ dt.$$

$$= \frac{2V}{\pi^2} \times \left[(t) \cdot \left(-\frac{\cos \omega_0 n t}{\omega_0 n} \right) - (1) \left(-\frac{\sin \omega_0 n t}{(\omega_0 n)^2} \right) \right]_0^{\pi}$$

$\omega_0 = 1$ rad/sec.

$$= \frac{2V}{\pi^{\cancel{2}}} \times \left(-\frac{\cancel{\pi}}{n} \cdot \cos n\pi + 0 - 0 \right)$$

$$= \frac{2V}{\pi n} \times (-1) \times (-1)^n . \qquad (\because \cos n\pi = (-1)^n).$$

$$b_n = \frac{2V}{\pi n} \cdot (-1)^{n+1} .$$

$$\therefore \quad b_n = \frac{2V}{\pi n} , \quad n - \text{odd}.$$

$$= -\frac{2V}{\pi n} , \quad n - \text{even}.$$

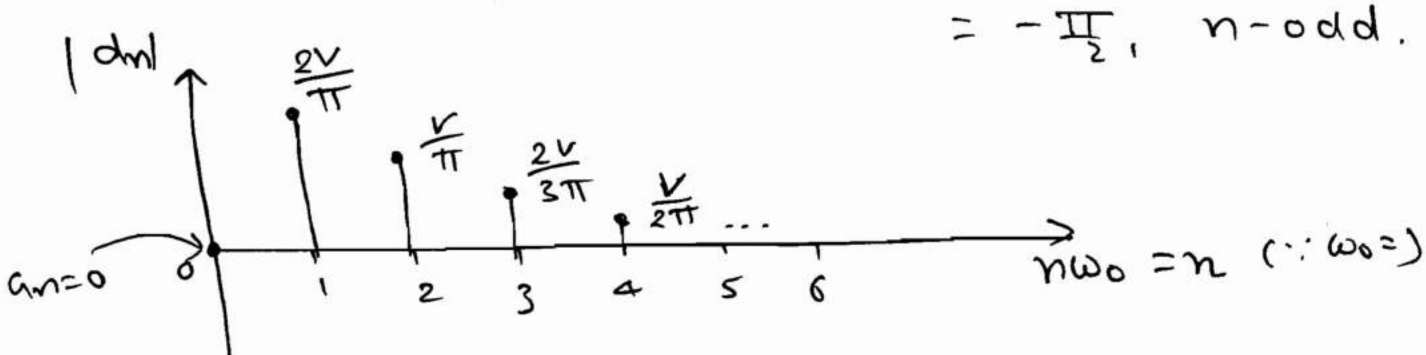
<u>Polar form:</u>

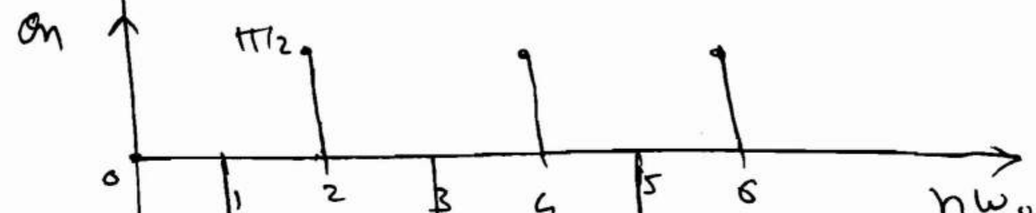
$$|d_n| = \sqrt{a_n^2 + b_n^2} = 0 + |b_n| .$$

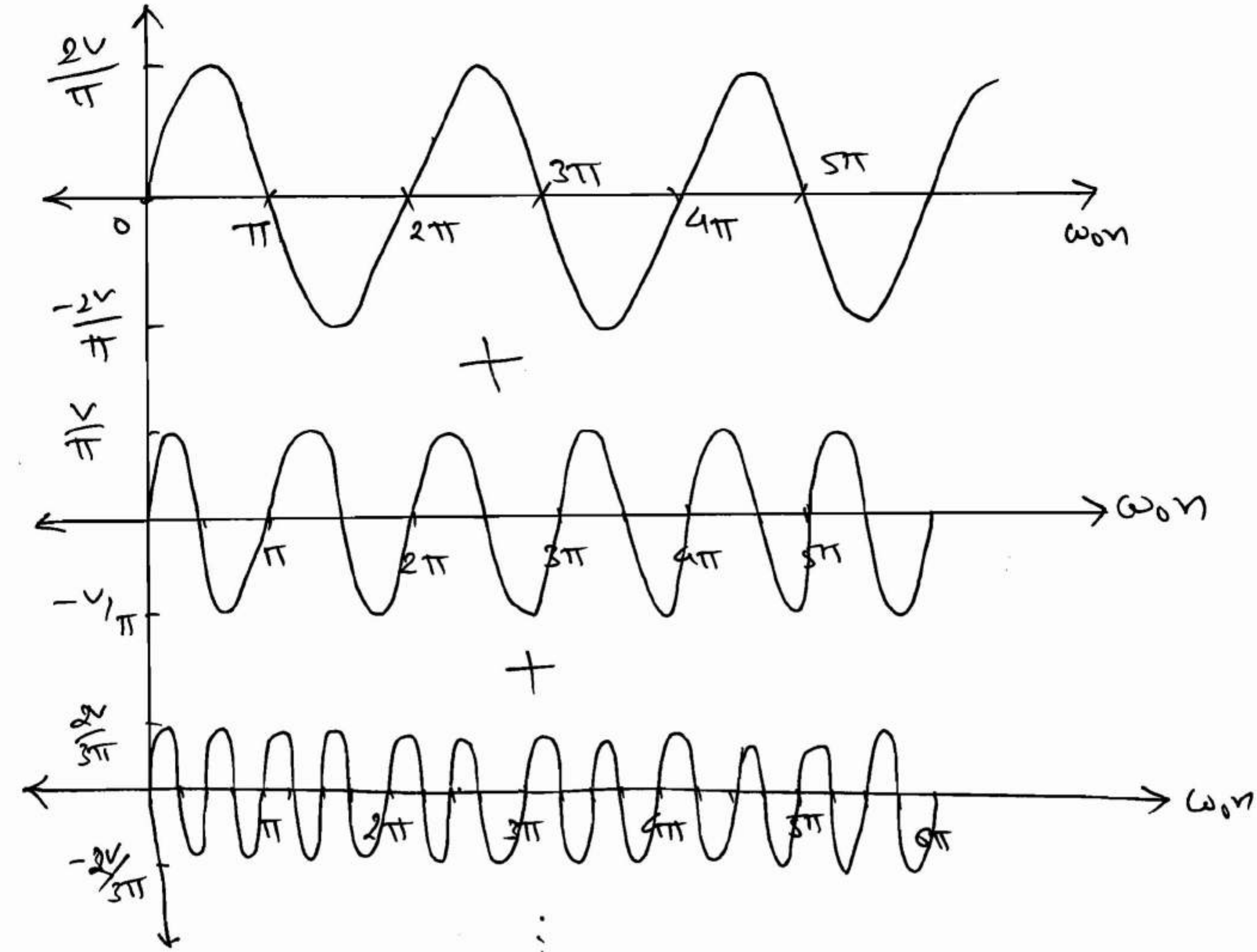
$$\therefore |d_n| = |b_n| = \left| \frac{2V}{\pi n} \right| .$$

$$\therefore \quad \cancel{\phi} \; \theta_n = \tan^{-1}(-b_n/a_n) = \frac{\pi}{2} , \quad n - \text{even}$$

$$= -\frac{\pi}{2} , \quad n - \text{odd}.$$







*

		Phase
Real & Even	$\Rightarrow$	$0°$ & $\pm 180°$
Real & Odd	$\Rightarrow$	$\pm \pi/2$.

$\boxed{\text{P 3.2.1.}}$ A periodic signal is given by

$x(t) = 3\sin(4t + 30°) - 4\cos(12t - 60°)$ find the amplitude of second harmonic?

Soln:

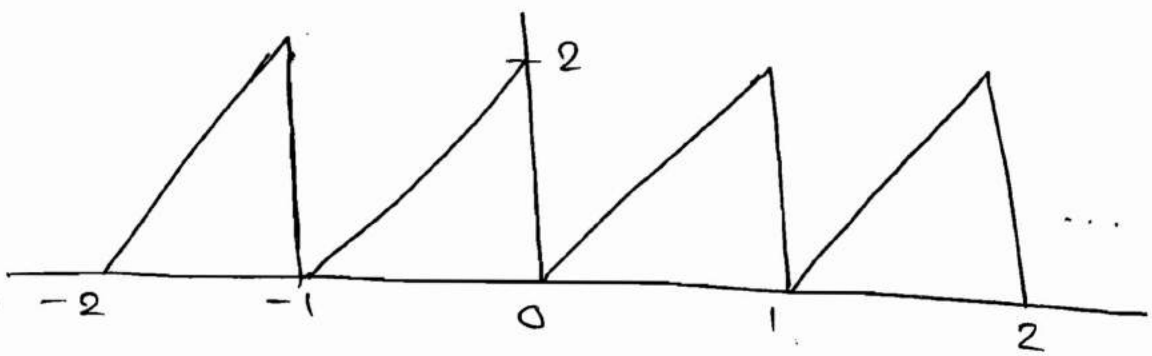
$$x(t) = 3\sin(\underset{\text{1st har.}}{4t} + 30°) - 4\underset{\text{3rd har.}}{\cos}(12t - 60°).$$

So, Amp. II^{nd} har = 0.

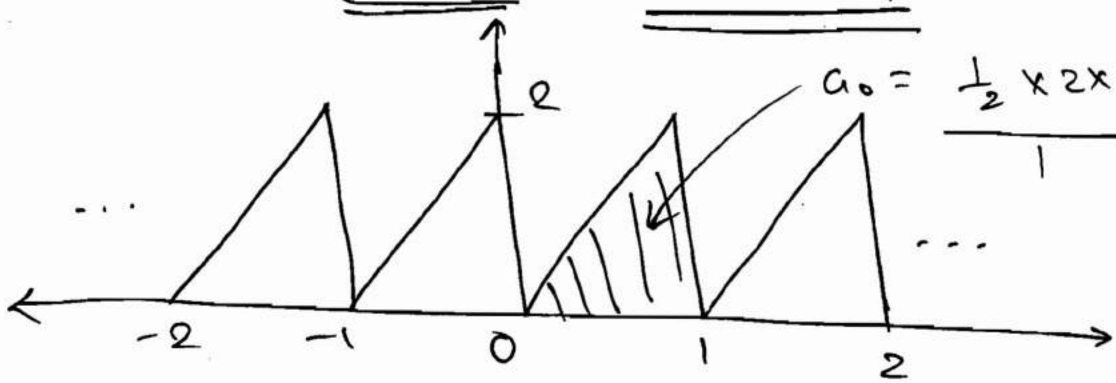
$\therefore$ phase of $\underline{III}^{rd}$ harmonic $= -60 \pm 180^\circ$

$$1 + j0 \Rightarrow 0^\circ$$
$$-1 + j0 \Rightarrow \pm 180^\circ.$$
$$+j \Rightarrow \pm 90^\circ.$$

$\boxed{P.\,3.2.3}$ Find the T.F.S. representation of the periodic signal $x(t)$ shown in fig 3.2.37.



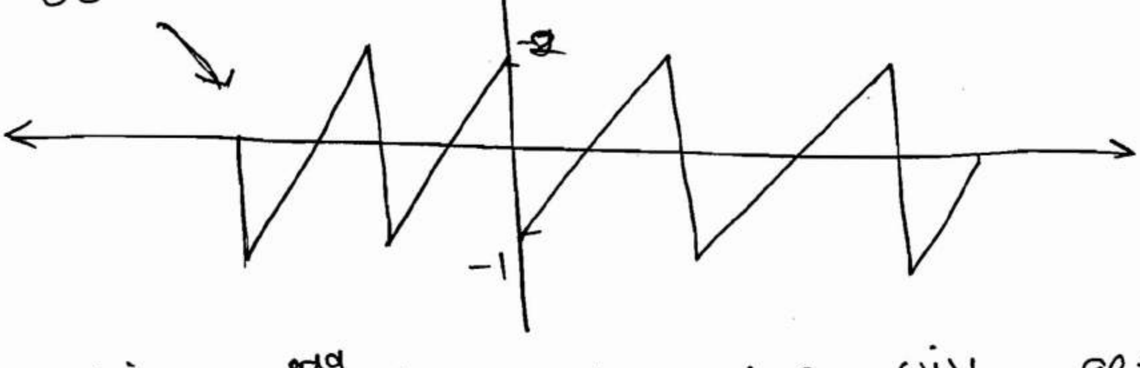
Sol^n: It is a <u>hidden</u> <u>symmetry,</u>



$$a_0 = \frac{\frac{1}{2} \times 2 \times 1}{1} = 1.$$

$$-1$$

odd.



$\Rightarrow$ a hidden odd symmetry we will get.

$\boxed{\text{Hidden odd : } a_0 \text{ \& } b_n.}$

Here, $\boxed{a_0 = 1}$ $\quad x(t) = 2t, \quad 0 \le t \le 1.$

$$b_n = \frac{2}{1}\int_0^1 2t \cdot \sin \omega_0 n \, . dt$$

Here, $T = 1 \Rightarrow \omega_0 = 2\pi$

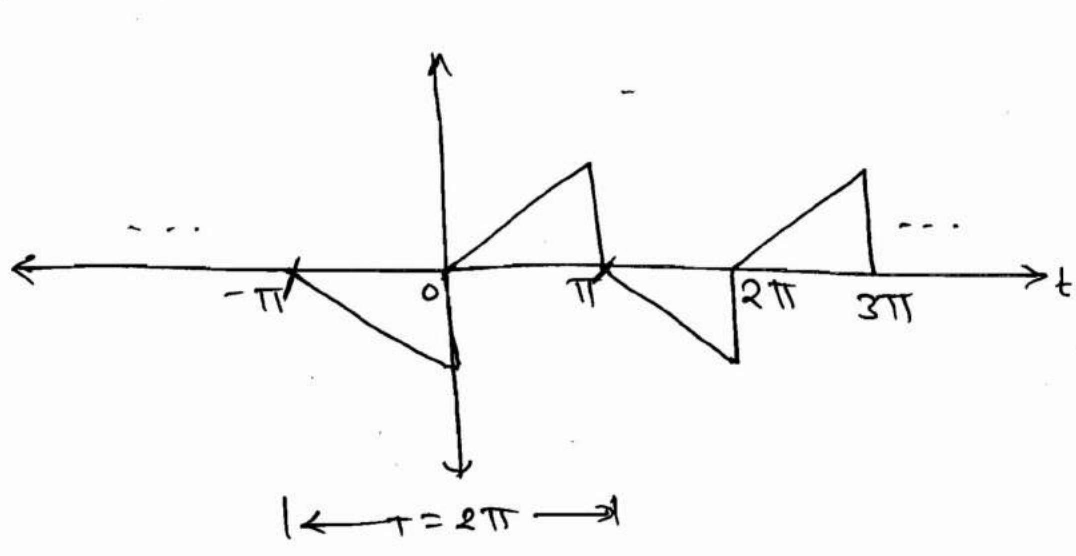
$$= \frac{2}{1} \times 2 \cdot \left[(t)\cdot\left(-\frac{\cos\omega_0 n}{\omega_0 n}\right) - (1)\left(-\frac{\sin\omega_0 n}{(\omega_0 n)^2}\right)\right]_0^1$$

$$= \overset{2}{\cancel{4}}\left[-\frac{1\cdot\cos 2\pi n}{\cancel{2}\pi n} + 0 - 0 - 0\right].$$

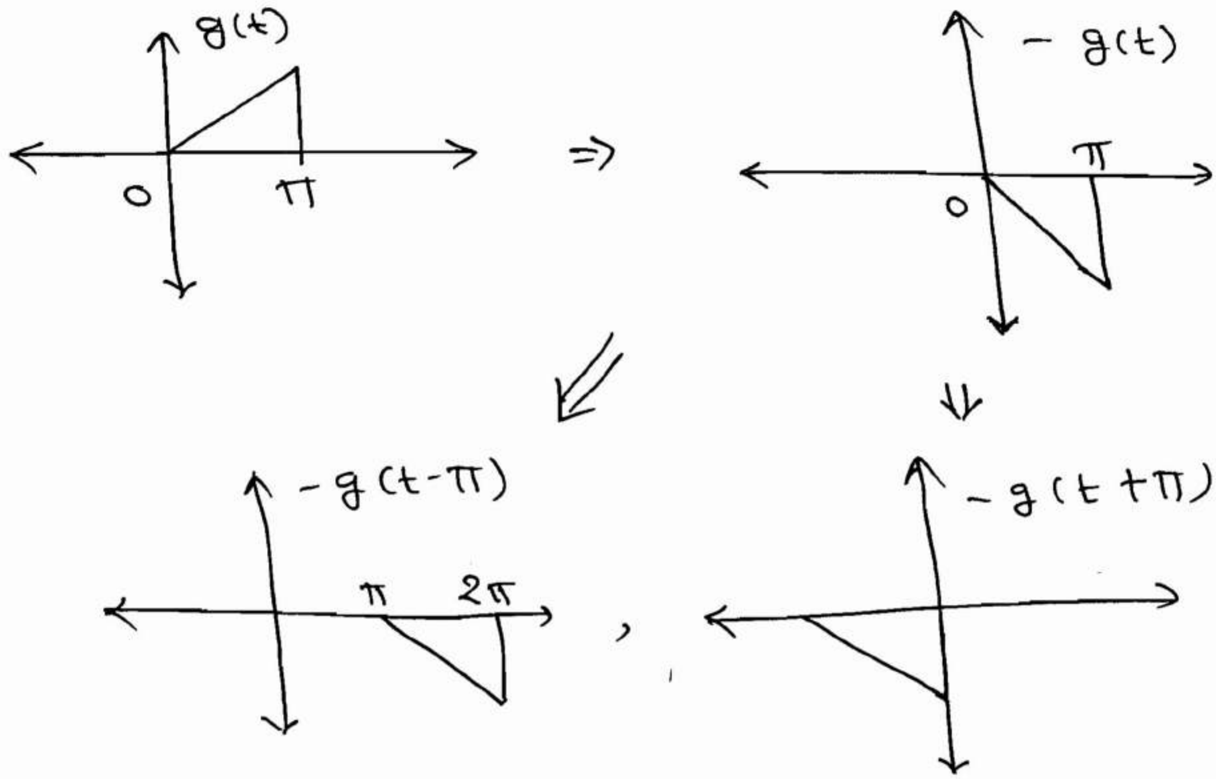
$$\boxed{b_n = \frac{-2}{\pi n}}$$

$\boxed{\text{P 3.2.4.}}$ For the Periodic waveforms show in figure what ~~exact~~ freq. Components are present in trignometric series expansion.

(P)



Sol^n:

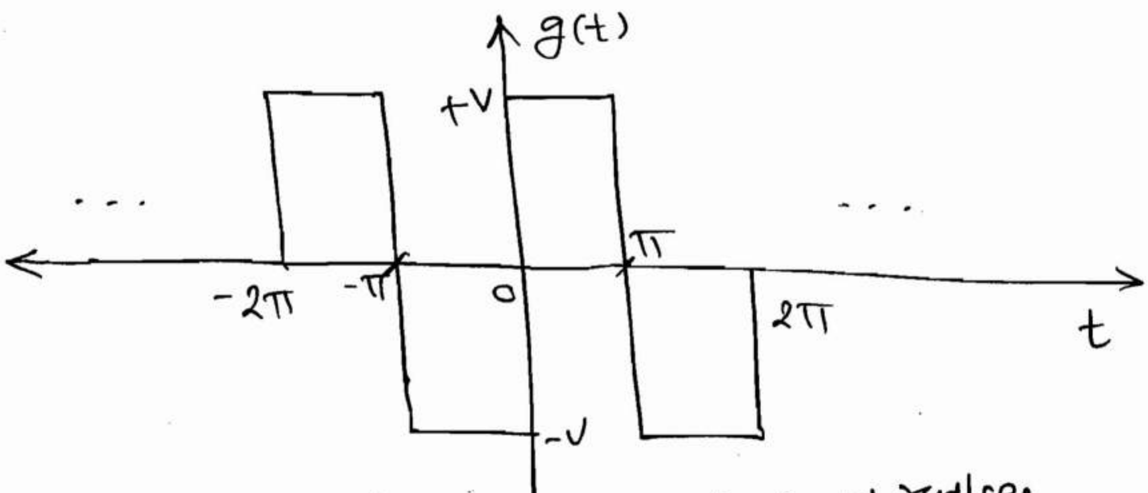


So, $g(t) = -g(t \pm T/2)$.

So, Half-wave symmetry.

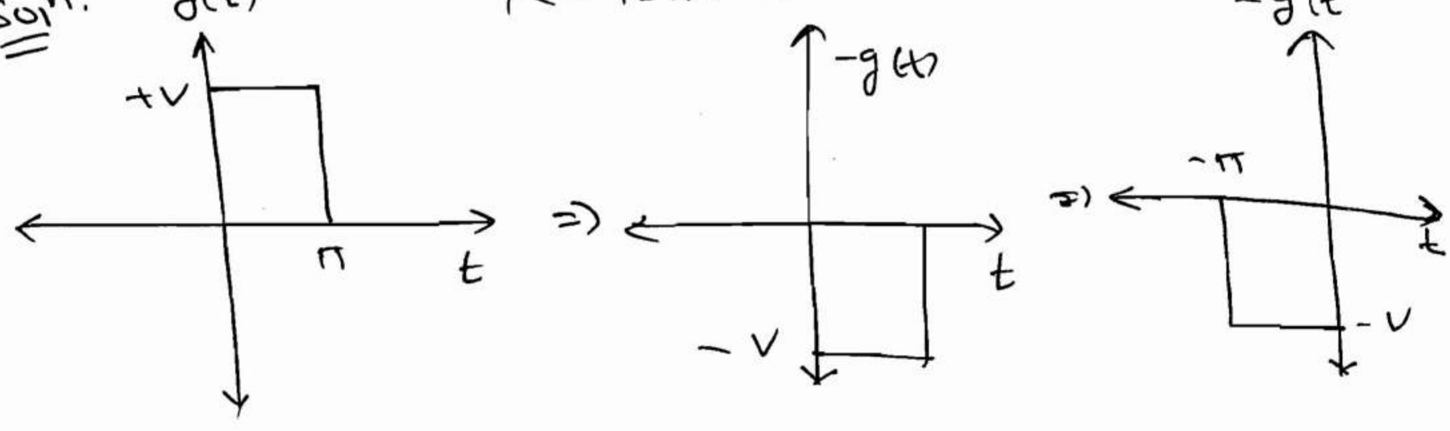
Hence, only odd harmonics.

$\boxed{a}$



$|\leftarrow T = 2\pi \rightarrow| \Rightarrow \omega_0 = 1$ rad/sec.

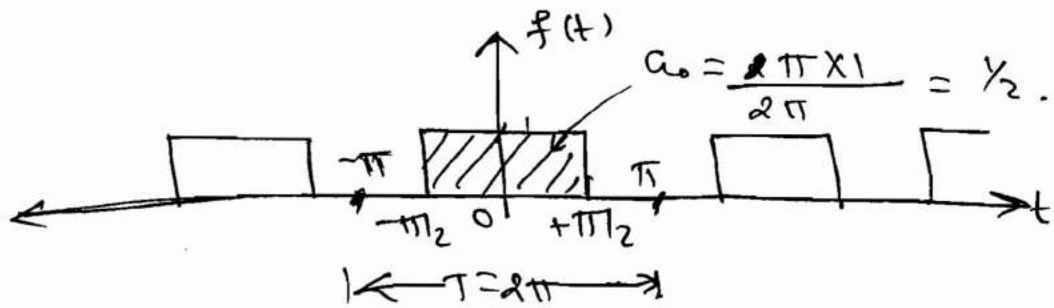
Sol^n:



→ Therefor given $g(t)$ has odd and half-wave symmetry both.

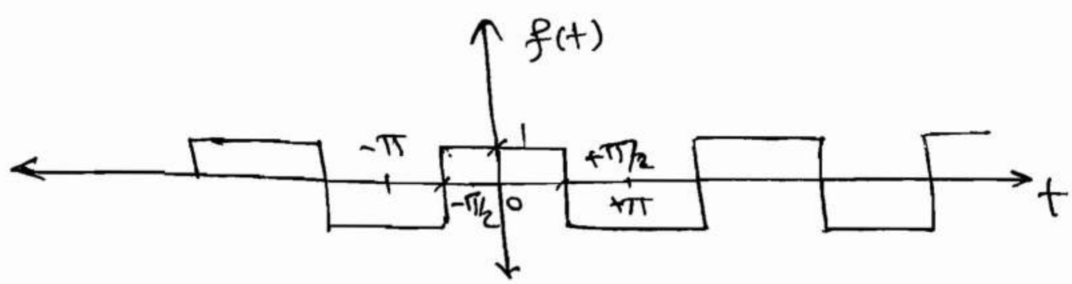
Hence, sine terms with odd harmonics.

(R)



Sol^n:

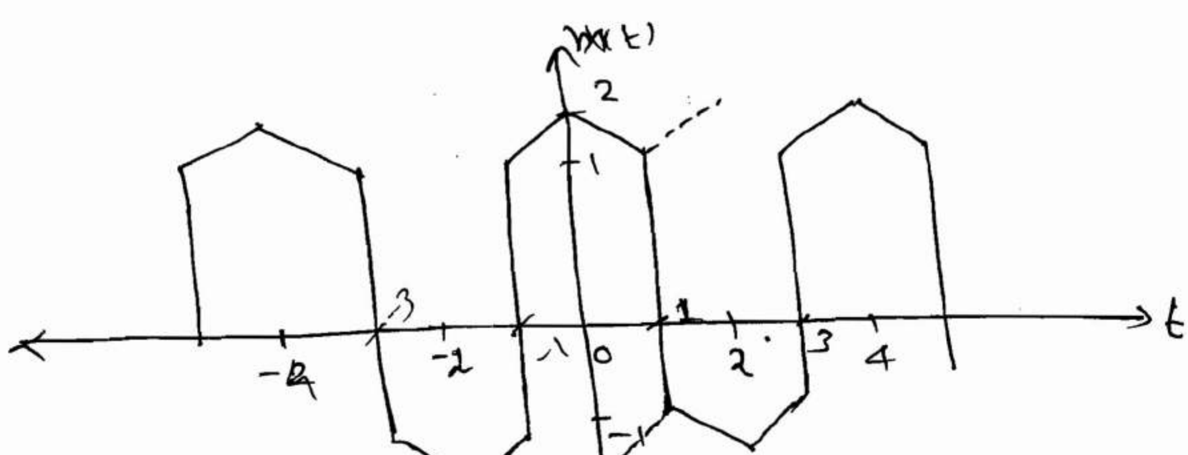
$\omega_0 = 1$ rad/sec.



So, It is a half-wave symmetry and also even symmetry.

So, ~~DC~~ cosine terms with odd harmonics.

(S)



Sol^n: Here, given $x(t)$ has even symmetry as

$\boxed{P\ 3.2.5.}$ Consider the signal $x(t) = 10\cos(10\pi t + \pi/7) + 4\sin(30\pi t + \pi/8)$. It's Power lying within the frequency band 10 Hz to 20 Hz is ____.

Sol^n^:

$\omega_0 = \text{G.C.D.}(10\pi, 30\pi)$

$\omega_0 = 10\pi$

$f = 5\text{ Hz}.$

$$x(t) = 10\cos(\underbrace{10\pi t}_{f = 5\text{ Hz}} + \pi/7) + 4\sin(\underbrace{30\pi t}_{15\text{ Hz}} + \pi/8)$$

So, Power in the freq. band 10 Hz to 20 Hz

is $\frac{4^2}{2} = 8\text{W}.$

$\boxed{Q-3.2.6}$ Consider the trignometric series, which holds true $\forall t$, given by

$x(t) = \sin\omega_0 t + \frac{1}{3}\sin 3\omega_0 t + \frac{1}{5}\sin 5\omega_0 t + \frac{1}{7}\sin 7\omega_0 t + \cdots$

At $\omega_0 = \frac{\pi}{2}$ the series converges to ____.

Sol^n^: at $\omega_0 = \pi/2$

$x(t) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

$= \pi/4.$

$\tan^{-1}x = x - \frac{x^2}{3} + \frac{x^3}{5} + - \frac{x^4}{7} + \frac{x^5}{15} + \cdots$

$x = 1$

$\therefore$

$\tan^{-1}(1) = \frac{\pi}{4} = 1 - \frac{1}{3} + 1/5 - 1/7 + \frac{1}{15} + \cdots$

$\boxed{P\ 3.2.7}$ A function is given by

$f(t) = \sin^2 t + \cos 2t$. which of the following is TRUE?

Sol^n:

$$f(t) = \sin^2 t + \cos 2t.$$

$$\Rightarrow f(t) = \frac{1 - \cos 2t}{2} + \cos 2t.$$

$$= \underbrace{\frac{1}{2}}_{f=0} + \underbrace{\frac{\cos 2t}{2}}_{\omega_0 = 2 \Rightarrow f = 1/\pi}.$$

So, f has frequency components at 0 and $1/\pi$ Hz.

$\boxed{P\ 3.2.8.}$ (a) The fundamental freq. of the composite signal.

$$x(t) = 2 + 3\cos(0.2t) + \cos(0.25t + \pi/2) + 4\cos(0.3t - \pi) \text{ is } \_\_.$$

Sol^n:

$$\omega_0 = \text{G.C.D.}(0.2, 0.25, 0.3).$$

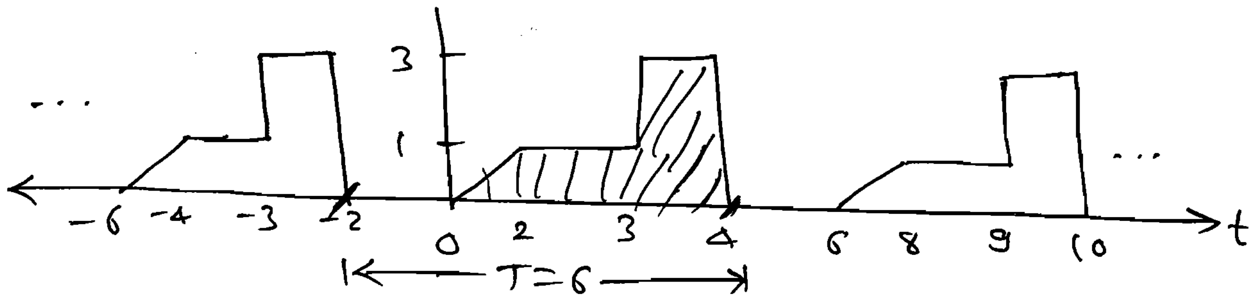
$$= 0.05\ (4, 5, 6).$$

So, $\boxed{\omega_0 = 0.05 \text{ rad/sec.}}$

(b) For a periodic signal $v(t) = 30\sin 100t + 10\cos 300t + 6\sin(500t + \pi/4)$, the fundamental freq. in rad/s is.

Sol^n: $\omega_0 = \text{G.C.D.}(100, 300, 500).$

$\boxed{P\,3.2.9.}$ The average value of the periodic signal $x(t)$ shown in figure is,



Soln:

avg. value = $\dfrac{\text{Area under curve over a period}}{\text{fundamental period.}}$

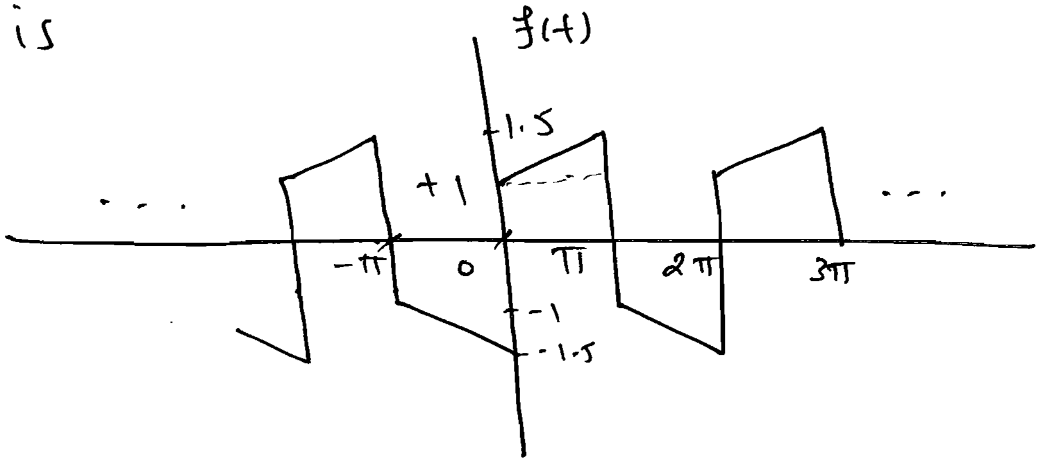
$\Rightarrow$ avg. value = $\dfrac{(\frac{1}{\cancel{2}}\times\cancel{2}\times1)+(1\times1)+(3\times1)}{6}$

$= 5/6.$

$\boxed{P\,3.2.10}$ $f(t)$, shown in fig. is represented by

$f(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos nt + b_n \sin nt.$ The value

of a_0 is



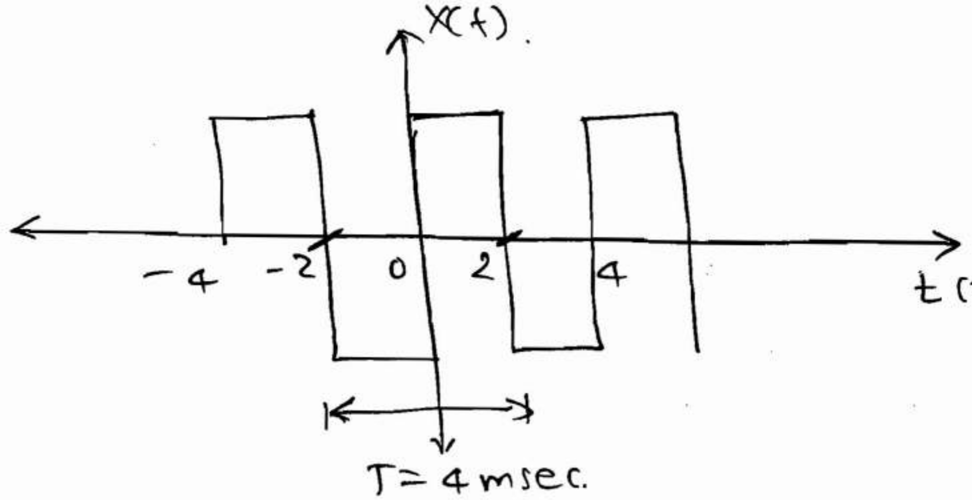
Soln:

$f(t)$ has odd symmetry hence,

$\boxed{a_0 = 0.}$

$\boxed{P\,3.2\text{-}11}$ A periodic rectangular signal $x(t)$ has

⇒



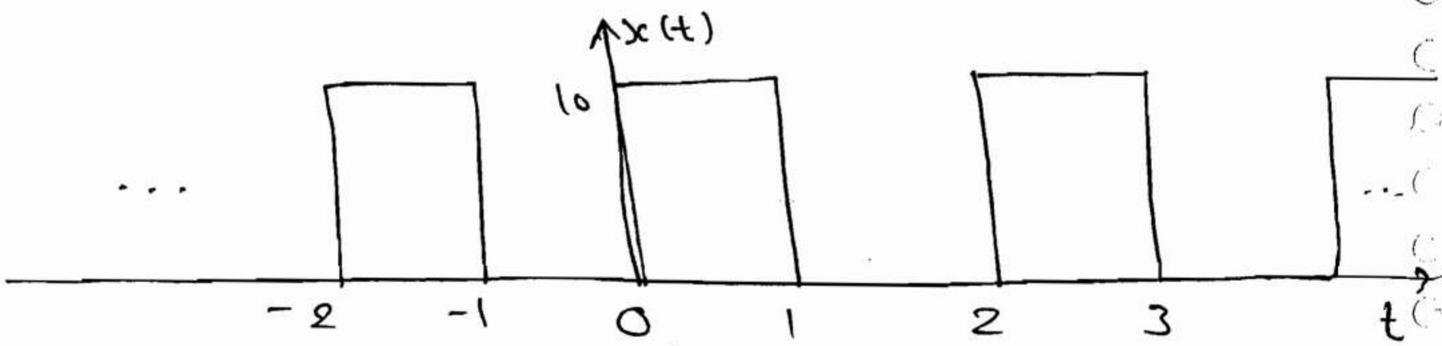
$\therefore \; f_0 = \frac{1}{T} = \frac{1000}{4} = 250 \text{ Hz}$

$\therefore$ freq. of the fifth harmonic is

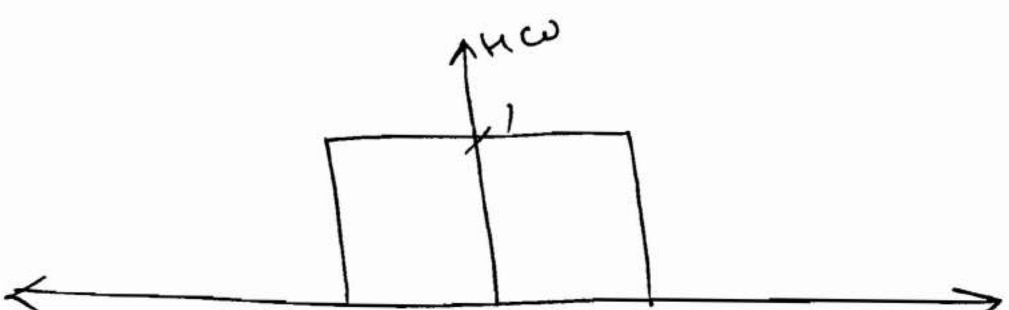
$$f_5 = 5 \times f_0 = 1250 \text{ Hz}.$$

$\boxed{P\,3.2.12}$ A periodic input signal x(t) shown below is applied to an L.T.I. system with freq. response

$$H(\omega) = \begin{cases} 1; & |\omega| < 4\pi \\ 0; & |\omega| > 4\pi \end{cases}$$ find the o/p?



Sol:



$\Rightarrow$ we should find $x(t)$ in fer. domain.

$$T = 2 \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{2} = \pi$$

$$a_0 = \frac{10 \times 1}{2} = 5.$$

$$a_n = \frac{2}{T} \int_0^T x(t) . \cos \omega_0 n . t \, dt$$

$$= \frac{2}{2} \int_0^2 x(t) . \cos \omega_0 n . t . \, dt.$$

$$= \int_0^1 10 . \cos \omega_0 n t . \, dt .$$

$$= 10 \int_0^1 . \cos \pi n t . \, dt$$

$$= 10 . \left[\frac{\sin \pi n t}{\pi n} \right]_0^1 = 0.$$

$$b_n = \frac{2}{2} \int_0^1 10 . \sin \omega_0 n t \, dt .$$

$$= 10 . \left[- \frac{\cos \pi n t}{\pi n} \right]_0^1$$

$$= 10 \left[\frac{1 - \cos \pi n}{\pi n} \right].$$

$$b_n = \frac{10}{\pi n} \left[1 - (-1)^n \right].$$

$$\therefore b_n = \frac{20}{\pi n}, \quad \text{'n-odd'}$$

$$b_n = 0, \quad \text{'n-even'}.$$

$$\therefore \quad x(t) = 5 + \sum_{n=1}^{\infty} \frac{10}{\pi n}\left[1-(-1)^n\right] . \sin \omega_0 n .$$

$$\therefore \quad x(t) = 5 + \frac{20}{\pi}\cos \omega_0 n + \frac{20}{3\pi}\cos 3\omega_0 n + \frac{20}{5\pi}\cos 5\omega_0 n + \cdots$$

$\omega_0 = \pi$

$$x(t) = 5 + \frac{20}{\pi}\cos \pi n + \frac{20}{3\pi}\cos 3\pi n + \frac{20}{5\pi}\cos 5\pi n + \cdots$$

So, first 3-terms passed through given filter.

* Exponential Fourier Series (or) Complex Fourier series:

$\Rightarrow$

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \omega_0 nt + b_n \sin \omega nt .$$

$$= a_0 + \sum_{n=1}^{\infty} a_n\left[\frac{e^{j\omega_0 nt} + e^{-j\omega_0 nt}}{2}\right] + b_n\left[\frac{e^{j\omega_0 nt} - e^{-j\omega_0 nt}}{2j}\right]$$

$$= a_0 + \sum^{\infty} e^{j\omega_0 nt}\left[a_n \mp jb_n\right] + e^{-j\omega_0 nt}\left[\ldots\right]$$

$$\therefore\ g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot e^{j\omega_0 n t} + \sum_{n=1}^{\infty} C_{-n} \cdot e^{-j\omega_0 n t}.$$

Put $n = -m$

$$\sum_{m=-1}^{-\infty} C_m \cdot e^{+j\omega_0 m t}.$$

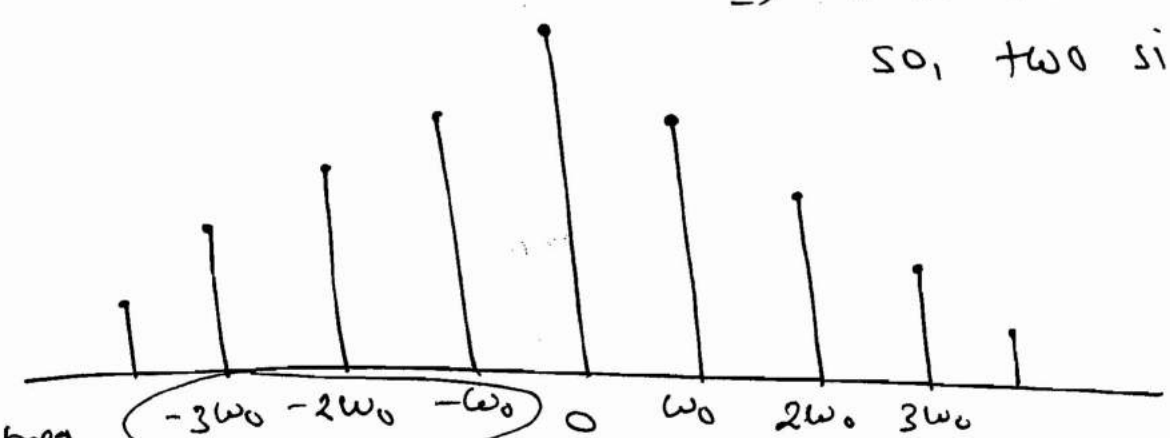
$$= \sum_{n=-1}^{-\infty} C_n \cdot e^{j\omega_0 n t}.$$

$$\therefore\ g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot e^{j\omega_0 n t} + \sum_{n=-\infty}^{-1} C_n \cdot e^{+j\omega_0 n t}.$$

$$\therefore\ \boxed{g(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{j\omega_0 n t}.}$$

$\Rightarrow$ +ve and -ve freqs. denotes opposite direction of rotation i.e. opposite phase angle.

$\Rightarrow -\infty$ to $+\infty$
so, two sided.



☆ T.F.S. to E.F.S. :

$$\Rightarrow \boxed{C_0 = a_0}$$

$$C_n = \frac{a_n - jb_n}{2}$$

$$\rightarrow \boxed{C_n = \frac{1}{T}\int_0^T g(t)\cdot e^{-jn\omega_0 t}\cdot dt}$$

$$\rightarrow \boxed{C_{-n} = \frac{a_n + jb_n}{2} = \frac{1}{T}\int_0^T g(t)\cdot e^{j\omega_0 n t}\cdot dt}$$

☆ E.F.S. to T.F.S.

$$\Rightarrow \boxed{\begin{aligned} a_0 &= C_0 \\ a_n &= C_n + C_{-n}. \\ b_n &= C_{-n} - C_n. \end{aligned}}$$

$\boxed{P\ 3.2.15}$ obtain the E.F.S. representation of periodic signal shown, hence find the T.F.S. ?

Soln: $x(t) = t, \quad 0 < t < 1.$

$\boxed{\omega_0 = 2\pi}$

$$C_n = \frac{1}{T} \int_0^T x(t) \cdot e^{-j\omega_0 n t} \cdot dt.$$

$$\therefore C_n = \frac{1}{1} \int_0^1 t \cdot e^{-j2\pi n t} \cdot dt.$$

$$C_n = \left[(t) \left(\frac{e^{-j2\pi n t}}{-j2\pi n} \right) - (1) \left(\frac{e^{-j2\pi n t}}{(2\pi n)^2} \right) \right]_0^1$$

$$C_n = \frac{(1) \cdot e^{-j2\pi n}}{-j2\pi n} - \frac{e^{-j2\pi n}}{(2\pi n)^2} + \frac{e^0}{(2\pi n)^2}.$$

$$= \frac{j}{2\pi n} - \cancel{\frac{1}{(2\pi n)^2}} + \cancel{\frac{1}{(2\pi n)^2}}$$

$$\boxed{C_n = \frac{j}{2\pi n}}, \quad \boxed{C_{-n} = \frac{-j}{2\pi n}}$$

$\Rightarrow \quad C_0 = \frac{1}{2} \times (1) \times (1) = \frac{1}{2}.$

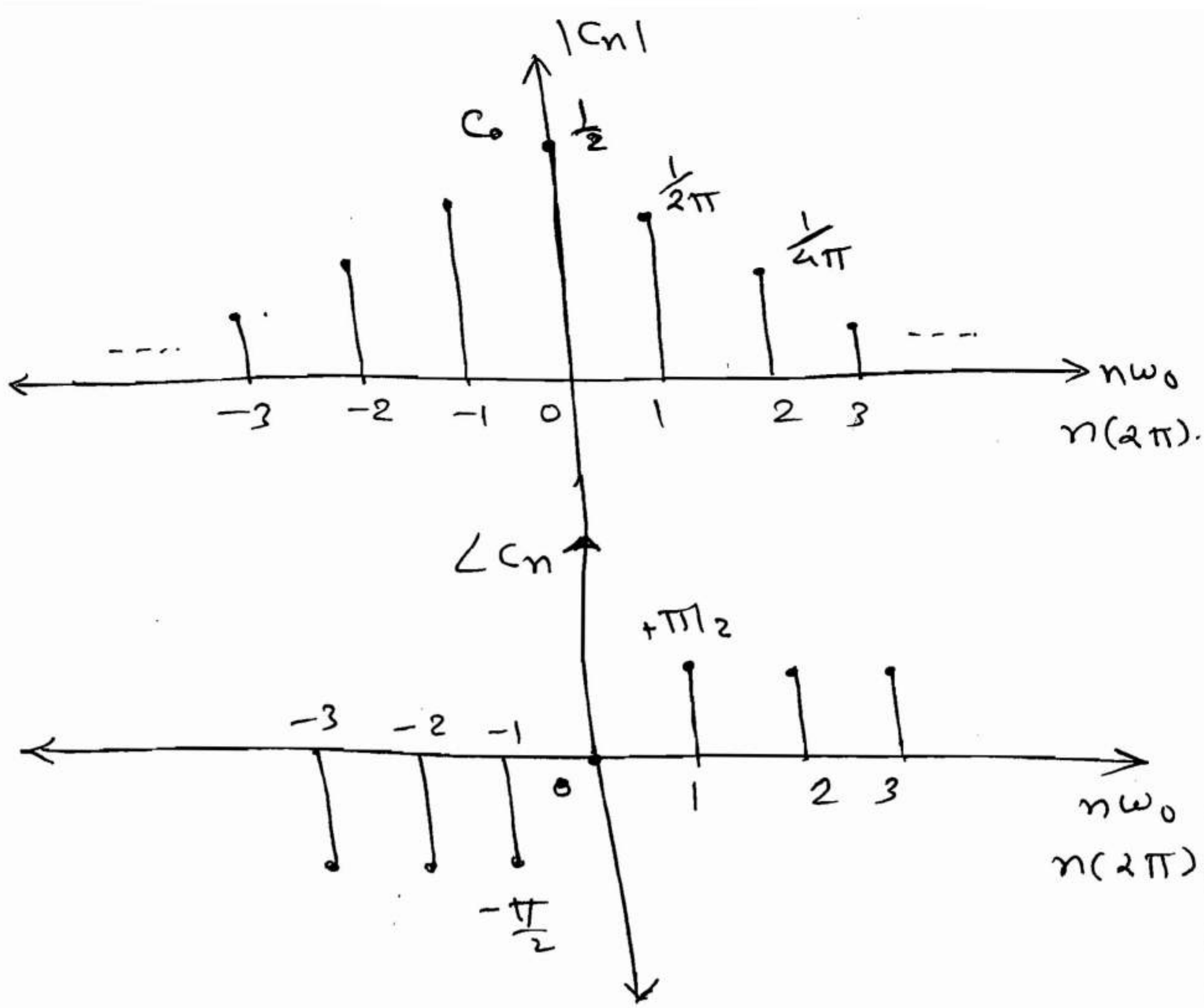
$\boxed{C_0 = \frac{1}{2}}$

$\angle C_n = \frac{\pi}{2}, \quad n \to +ve.$

$= -\frac{\pi}{2}, \quad n \to -ve.$

$\Rightarrow \quad |C_n| = \left| \frac{j}{2\pi n} \right|$

$|C_n| = \frac{1}{2\pi n}, \quad |C_0| = \frac{1}{2}.$



$$\Rightarrow \quad a_n = \frac{C_n + C_{-n}}{2}$$

$$a_n = \frac{\frac{j}{2\pi n} - \frac{j}{2\pi n}}{2} = 0$$

$$\boxed{a_n = 0}$$

$$\Rightarrow \quad j\, b_n = \frac{C_{-n} - C_n}{\cancel{2}\; 1} = \frac{-\frac{j}{2\pi n} - \frac{j}{2\pi n}}{\cancel{2}\; 1}.$$

$$\boxed{b_n = \frac{1}{\pi n}}. \qquad a_0 = C_0$$

$$\boxed{a_0 = 1/2}$$

$$\Rightarrow \quad |d_n| = \sqrt{a_n^2 + b_n^2}$$

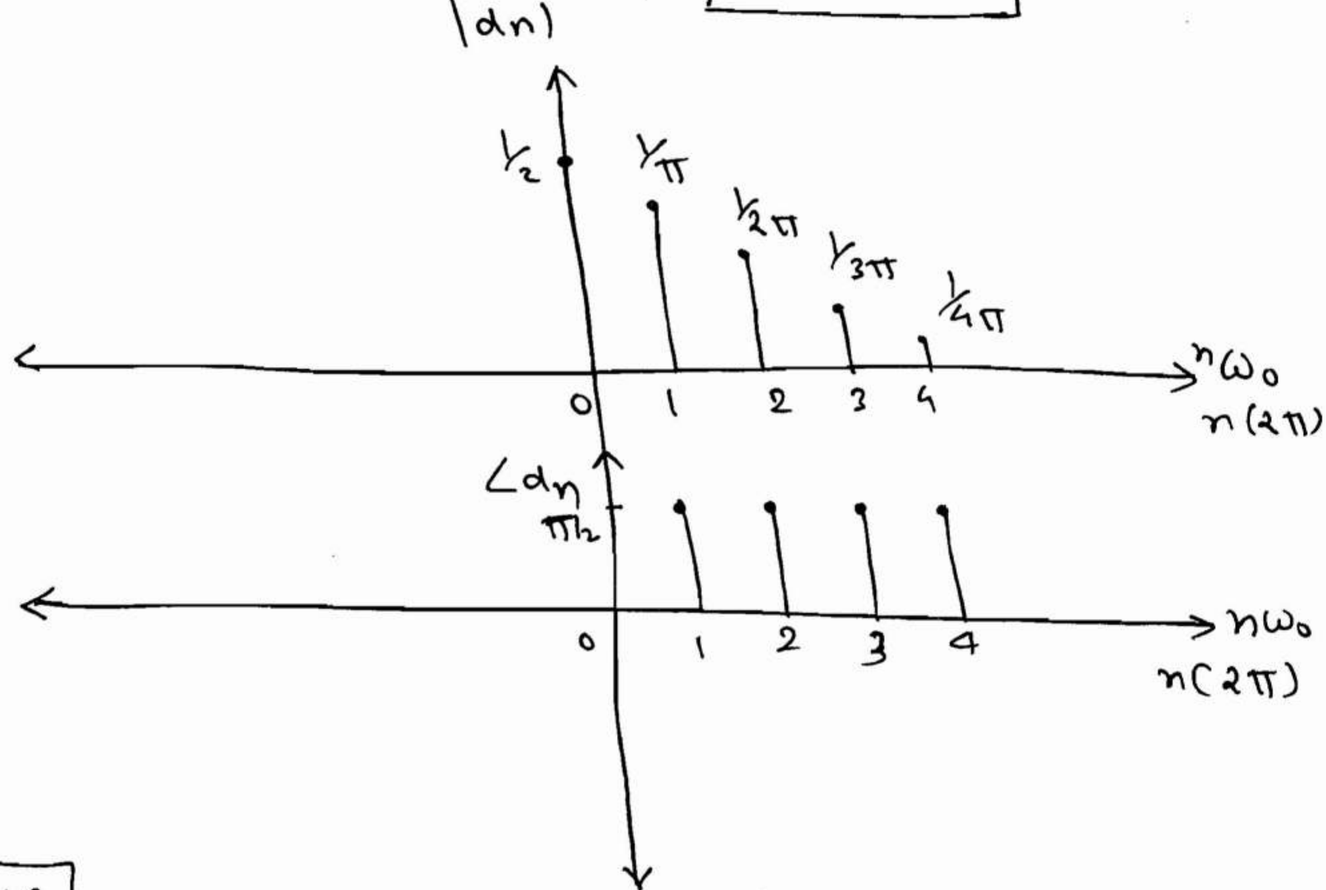
$$|d_n| = \sqrt{0 + \left(\frac{1}{\pi n}\right)^2}$$

$$\therefore \boxed{|d_n| = \frac{1}{\pi n}}$$

$$\angle d_n = +\tan^{-1}(-b_n/a_n).$$

$$\angle d_n = \tan^{-1}\left(+\frac{1}{\pi n}\Big/0\right).$$

$$\boxed{\angle d_n = \pi/2}$$



P3.2.14

(a) For the periodic signal

$$x(t) = 2 + \cos\left(\frac{2\pi t}{3}\right) + 4\sin\left[\frac{5\pi t}{3}\right], \text{ find}$$

the E.F.S. coefficients?

Soln:

$$x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{+j\omega_0 n t}.$$

$$= 2 + e^{j(2)(\frac{\pi}{3})t} + e^{-j(2)(\frac{\pi}{3})t} + 4\left[e^{j5(\frac{\pi}{3})t} \; - e^{-j5(\frac{\pi}{3})t}\right.$$

So, $\boxed{C_0 = 2}$ $\boxed{C_2 = C_{-2} = \frac{1}{2}}$

$$C_5 = \frac{4}{2j} \qquad \boxed{C_{-5} = +2j}$$

$\boxed{C_5 = -2j}$

$\boxed{Q}$ The F.S. Representation of periodic signal is $x(t) = \sum_{n=-\infty}^{+\infty} \frac{3}{4+(n\pi)^2} \cdot e^{jn\pi t}$ if one of the component is $a\cos 3\pi t$, Find a.

Soln: here, $\boxed{\omega_0 = \pi}$

$$A\cos 3\pi t$$

$\longrightarrow$ 3rd harmonic.

$$\therefore \quad a_n = C_n + C_{-n}.$$

$$\therefore \quad = C_3 + C_{-3}.$$

$$\Rightarrow \quad C_n = \frac{3}{4+(n\pi)^2}$$

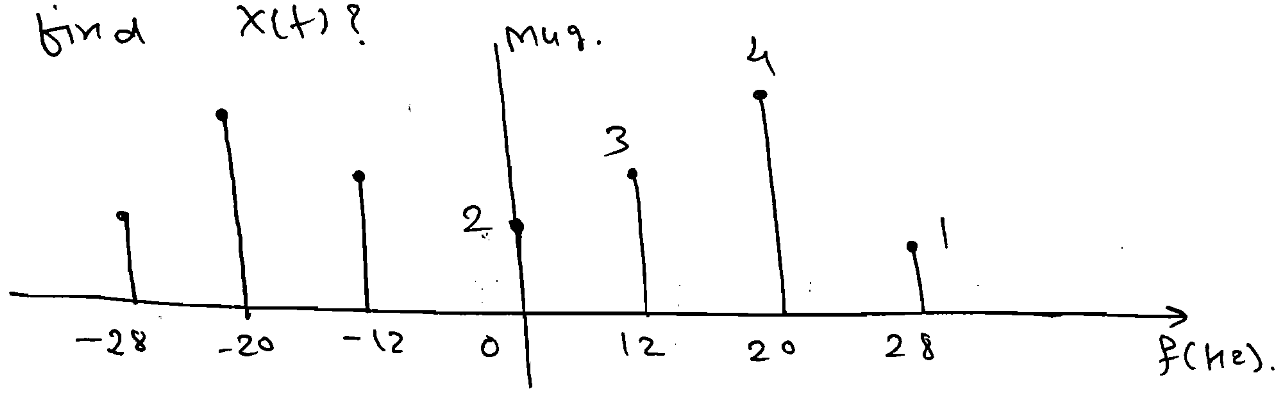
$$C_3 = \frac{3}{4+(3\pi)^2}$$

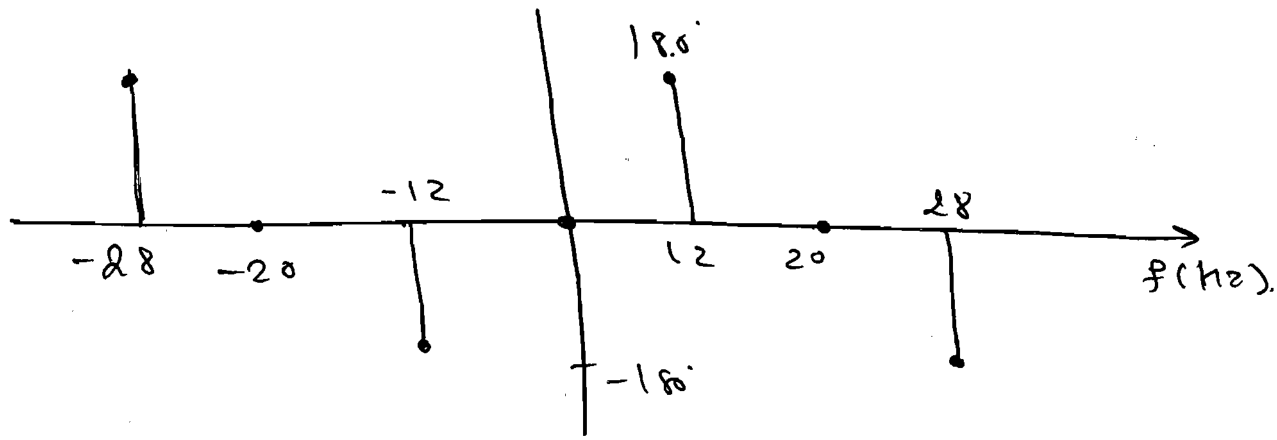
$$C_{-3} = \frac{3}{4+9\pi^2}.$$

$$\therefore \quad a_n = \frac{3}{4+9\pi^2} + \frac{3}{4+9\pi^2}.$$

Note: cos f^n corresponds to a_n.
sin f^n corresponds to b_n.

$\boxed{\text{P 3.2.16}}$ Consider the two-sided signal spectrum shown in figure for signal $x(t)$, find $x(t)$?





Sol:

$f_0 =$ GCD (12, 20, 28)

$\boxed{f_0 = 4 \text{ Hz}}$

⇒ Polar form:

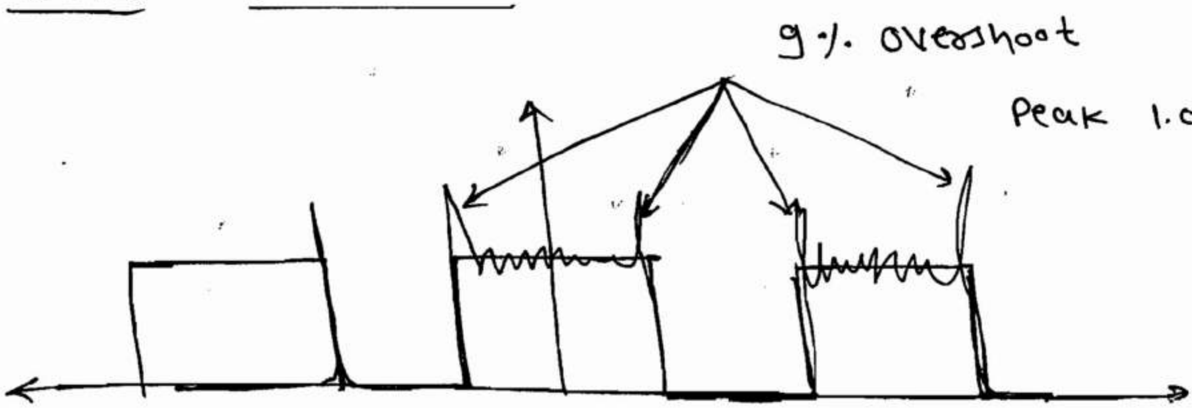
$$x(t) = d_0 + \sum_{n=-\infty}^{+\infty} d_n \cos(\omega_0 n t + \theta_n).$$

$$x(t) = 2 + 6\cos(2\pi(12)t + 180°)$$
$$+ 8\cos(2\pi(20)t + 0°).$$

* Convergence of FS:

⇒ $\left.\begin{matrix} a_0, a_n, b_n \\ c_0, c_n \end{matrix}\right\} < \infty \Rightarrow$ finite

⇒ Gibbs phenomena:

⇒ 

⇒ Irrespective of the no. of harmonics we are adding always at the point of discontinuity 9 % overshoot is observed i.e. gibbs phenomena. (Truncation in time corresponds to ripples in freq. domain).

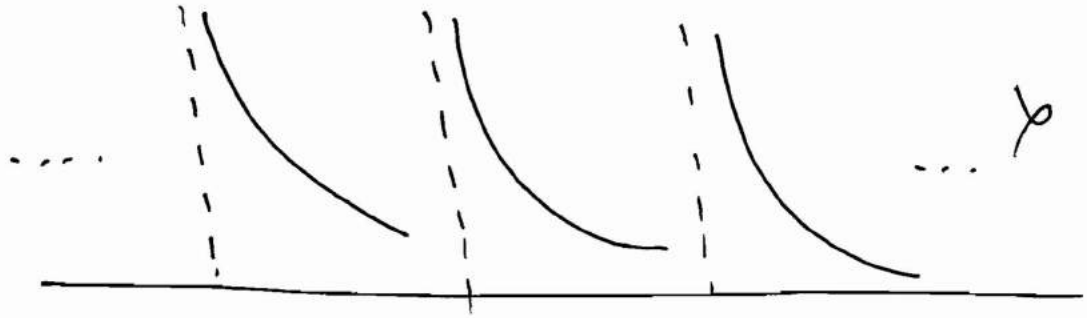
⇒ Dirchlet Condition:

(1) x(t) is absolutely integrable.

i.e. $\int_0^T |x(t)|\,dt < \infty$.

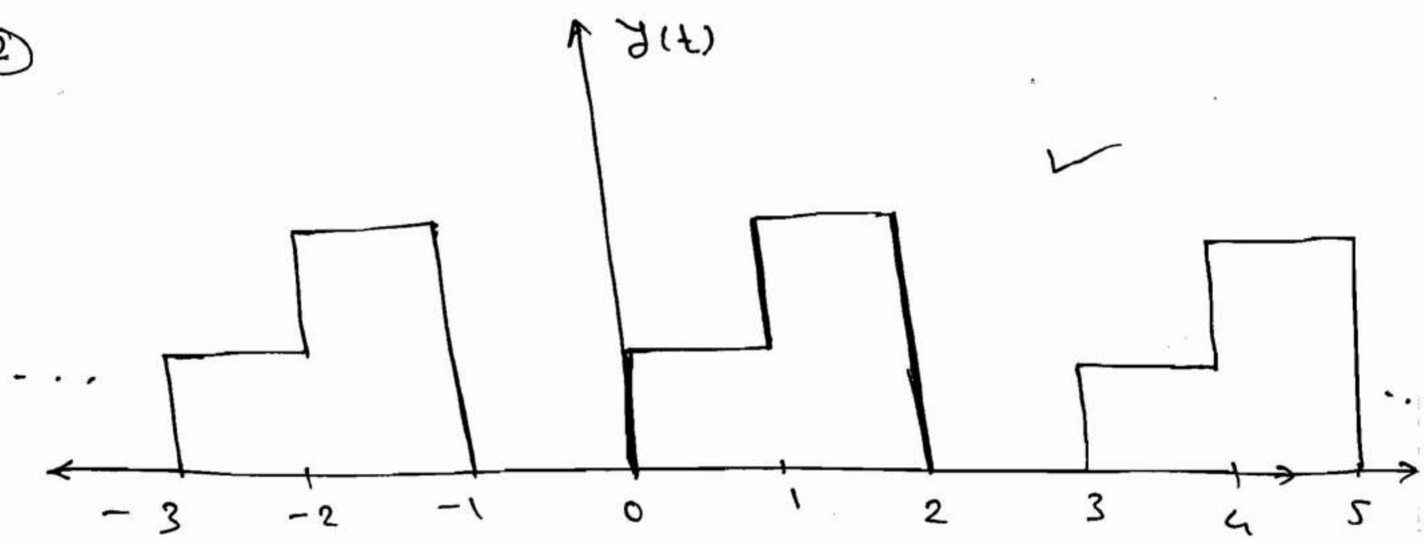
(2) x(t) has only finite number of maxima and minima.

e.g. ① $x(t) = \frac{1}{t}$, $0 < t < 1$.



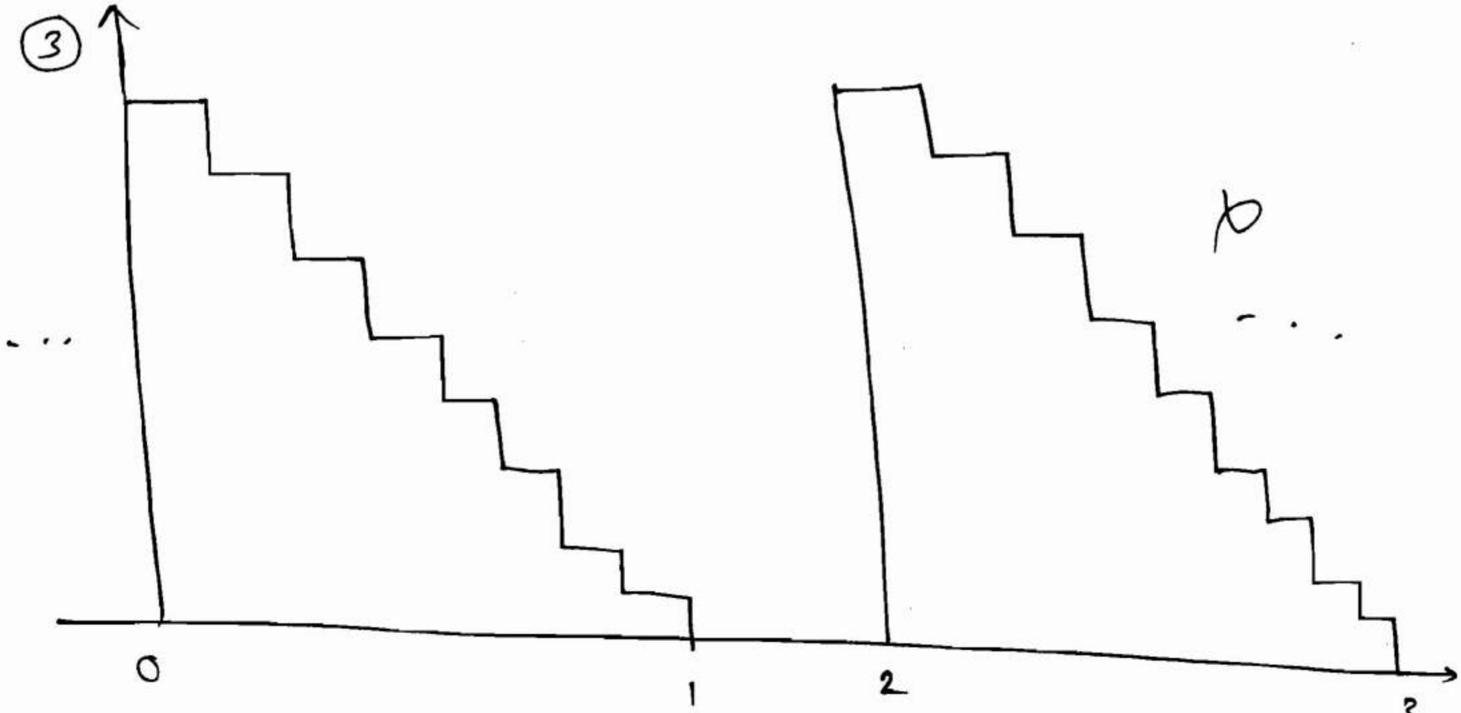
Not absolutely Integrable.

②

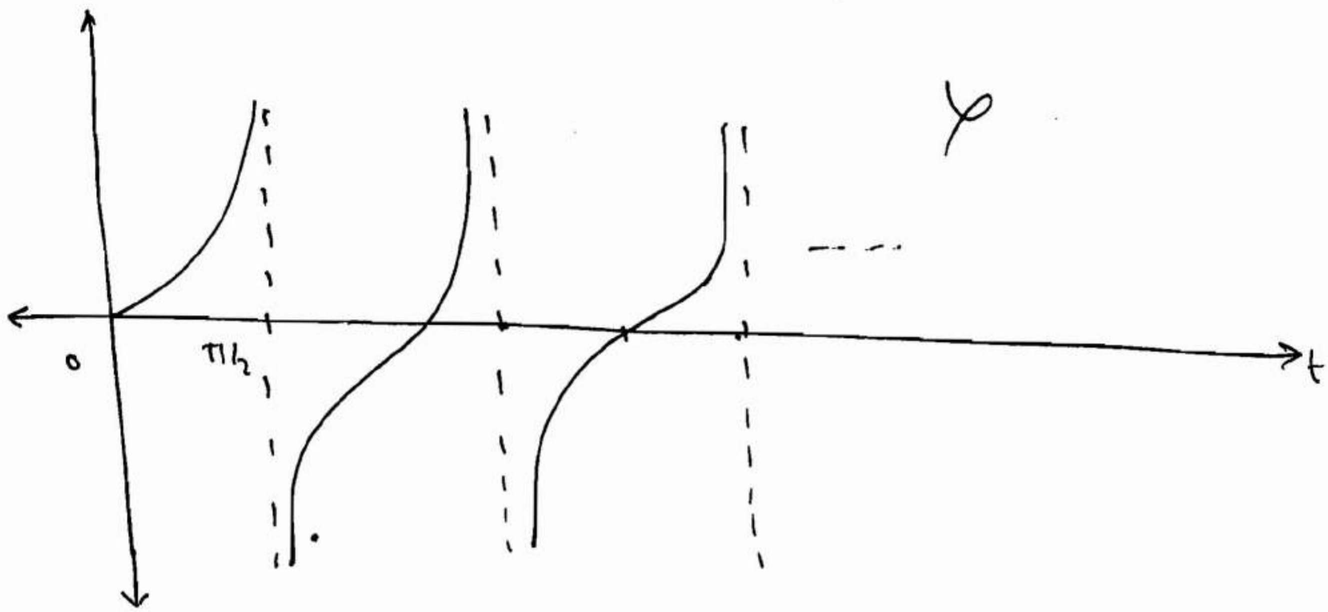


=> Valid because finite(3) discontinuity (3 to 4)

③

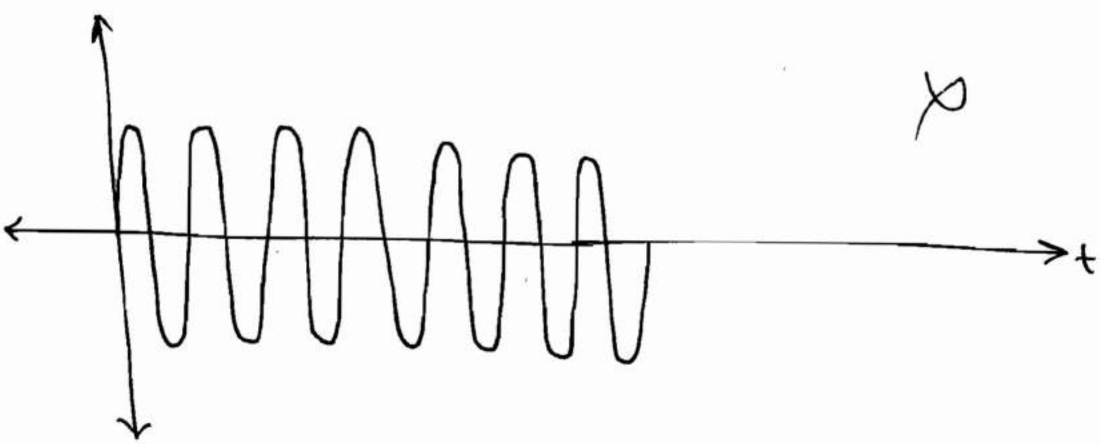


④ $x(t) = \tan t, \quad 0 < t < \pi/2.$



Not valid because within the limit it has infinite discontinuity.

⑤ $x(t) = \sin\left(\frac{2\pi}{t}\right), \quad 0 < t < 1.$



$\Rightarrow$ Not valid more no. of maxima & minima.

* Properties of F.S :-

(1) Linearity:

$\Rightarrow$ If $x_1(t) \longrightarrow C_n$

$x_2(t) \longrightarrow d_n$

then $\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha C_n + \beta d_n$.

(2) Time Shift:

$\Rightarrow$ If $x(t) \longrightarrow C_n$

the $x(t-t_0) \longrightarrow e^{-j\omega_0 n t_0} \cdot C_n$.

$\rightarrow$ when we shift in the time-domain, it changes the phase of each harmonic in proportion to its freq. $n\omega_0$.

(3) Frequency Shift :-

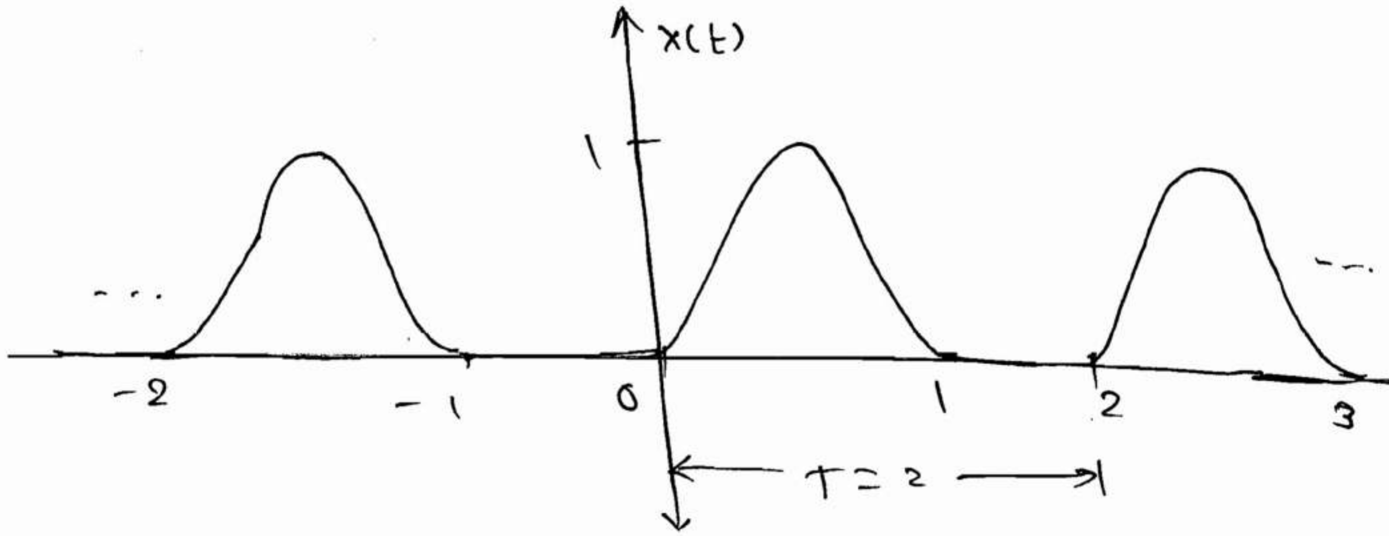
$\Rightarrow$ If $x(t) \longrightarrow C_n$

the $e^{j\omega_0 M t} \cdot x(t) \longrightarrow C_{n-M}$.

Note: Shifting one domain corresponds to multiplication by exponential term in

P 3.3.1 The F.S. coefficient of signal $x(t)$ shown in fig (a) are $c_0 = \frac{1}{\pi}$, $c_1 = -j0.25$

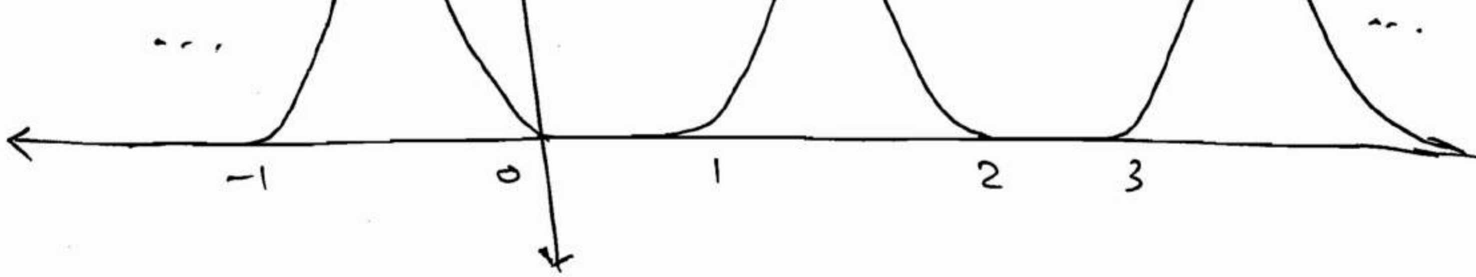
$c_n = \frac{1}{\pi(1-n^2)}$ (n even) Find F.S. coefficient of $y(t)$, $f(t)$ and $g(t)$?



$\Rightarrow \omega_0 = \frac{2\pi}{T} = \pi$

Soln:

①



$\Rightarrow y(t) = x(t-1)$.

$x(t) \longrightarrow c_n$.

$\therefore$

$\Rightarrow x(t-1) = y(t) \longrightarrow e^{-j\omega_0(1)\cdot n} \cdot c_n = d_n$

$\therefore \quad d_n = (-1)^n . C_n.$

$\therefore \quad d_0 = (-1)^0 \cdot C_0 = C_0$

$$\boxed{d_0 = 1/\pi}$$

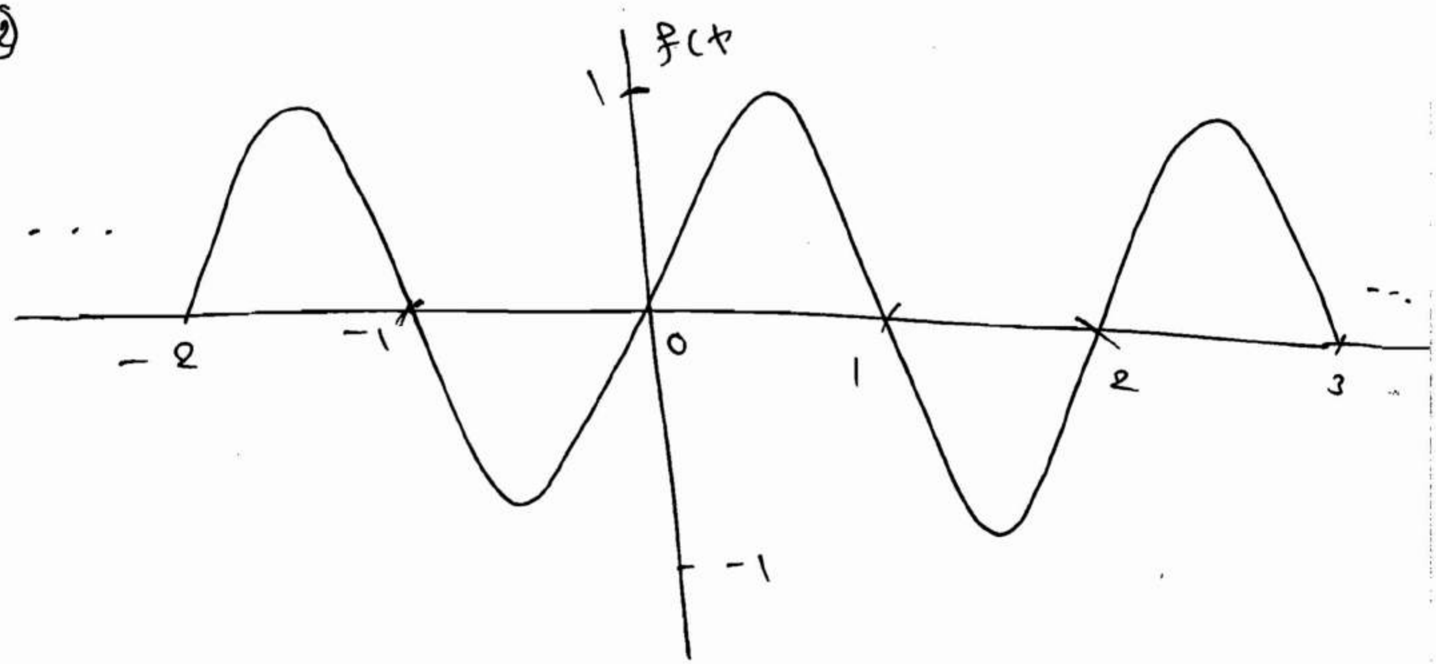
$\therefore \quad d_1 = (-1)^1 . C_1 = +j0.25$

$$\boxed{d_1 = +j0.25}$$

$\therefore \quad d_n = \dfrac{(-1)^n \times \pi}{(1-n^2)} \quad (n \text{ even}).$

$$\therefore \quad \boxed{d_n = C_n. \quad (n \text{ even})}$$

(2)



$\Rightarrow \quad$ 4 $\quad f(t) = x(t) - x(t-1).$

$f(x) = x(t) - y(t).$

$\therefore \quad \downarrow$ linearity

$f_n = C_n - d_n.$

$f_0 = C_0 - d_0 = 1/\pi - 1/\pi = 0 \Rightarrow \boxed{f_0 = 0}$

$\therefore \quad F_1 = c_1 - d_1$

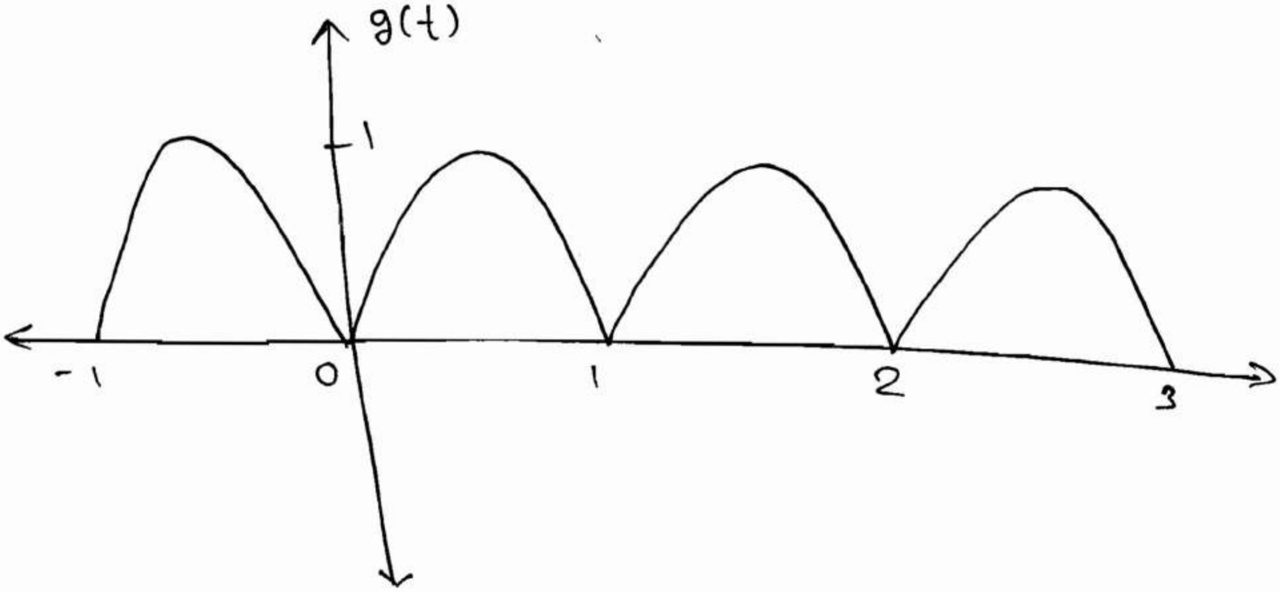
$= -j0.25 - j0.25$

$$\boxed{F_1 = -j0.5}$$

$\Rightarrow \quad F_n = c_n - d_n.$

$$\boxed{F_n = 0}.$$

(3)



$\Rightarrow \quad g(t) = x(t) + x(t-1) = x(t) + y(t)$

$\Downarrow$

$G_n = c_n + d_n.$

$\therefore \quad g_0 = c_0 + d_0 = 2/\pi$

$$\boxed{g_0 = 2/\pi}$$

$\Rightarrow \quad g_1 = c_1 + d_1$

$$\boxed{g_1 = 0}$$

$g_n = c_n + d_n$

$\boxed{P\ 3.3.2}$ Let $x(t)$ be a periodic signal with period T and F.S. coefficient C_n. Let, $y(t) = x(t-t_0) + x(t+t_0)$. The F.S. coefficient of $y(t)$

If $d_n = 0\ \forall$ odd n then t_0 can be

a) $T/8$ b) $T/4$ c) $T/2$ d) $2T$.

Soln:
$$y(t) = x(t-t_0) + x(t+t_0).$$
$$\Downarrow$$
$$\therefore d_n = e^{-j\omega_0 n t_0} \cdot C_n + e^{j\omega_0 n t_0} \cdot C_n.$$
$$d_n = 2C_n \left[\frac{e^{j\omega_0 n t_0} + e^{-j\omega_0 n t_0}}{2} \right].$$
$$\therefore d_n = 2C_n \cos \omega_0 n t_0.$$
$$\Rightarrow d_n = 0 \quad \text{when} \quad \omega_0 n t_0 = \frac{n\pi}{2},\ n \text{ odd}.$$
$$\therefore \omega_0 \cdot t_0 = \frac{\pi}{2}.$$
$$\therefore \frac{2\cancel{\pi}}{T} \cdot t_0 = \frac{\cancel{\pi}}{2}$$
$$\therefore \boxed{t_0 = T/4}$$

4) Time - Scaling:

$$\Rightarrow x(t) \longrightarrow C_n |_{T,\ \omega_0}$$

Time - Compressing by α changes frequency from ω_0 to $\alpha\omega_0$.

$$\rightarrow \quad x(t) = \sum_{n=-\infty}^{+\infty} C_n \cdot e^{-j\omega_0 n t}.$$

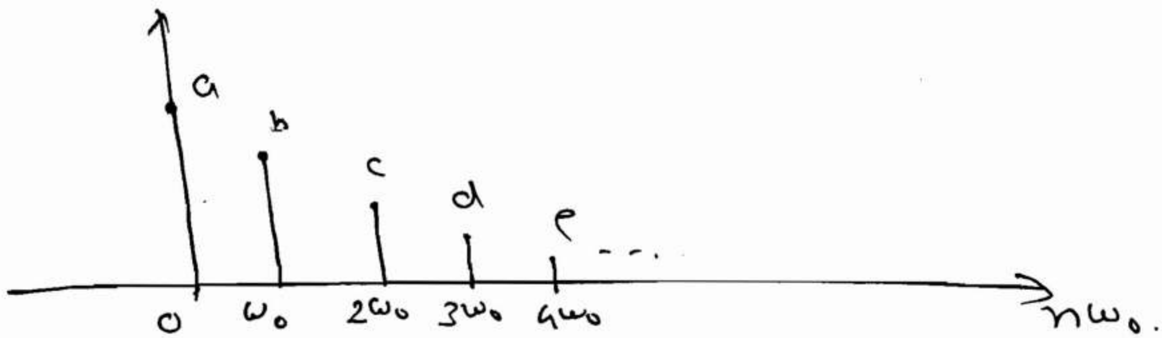
$$\therefore \quad x(\alpha t) = \sum_{n=-\infty}^{+\infty} C_n \cdot e^{-j\alpha\omega_0 n t}.$$

Now, $\boxed{\omega_0' = \alpha\omega_0.}$

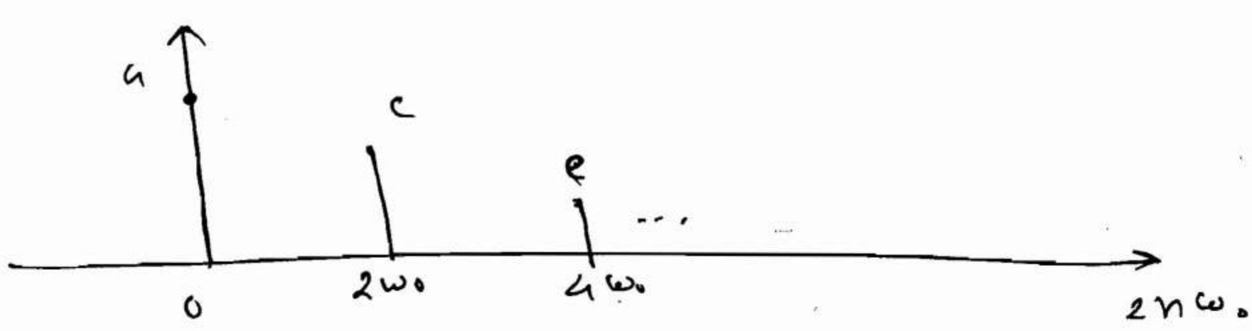
$$\frac{2\pi}{T'} = \frac{\alpha \times 2\pi}{T}.$$

$$\therefore \boxed{T' = T/\alpha}$$

e.g. $\quad x(t) \longrightarrow C_n|_{\omega_0}$



$\Rightarrow \quad x(2t) \longrightarrow C_n|_{2\omega_0}$



Note: Compression in time domain is expunssion in freq. domain.

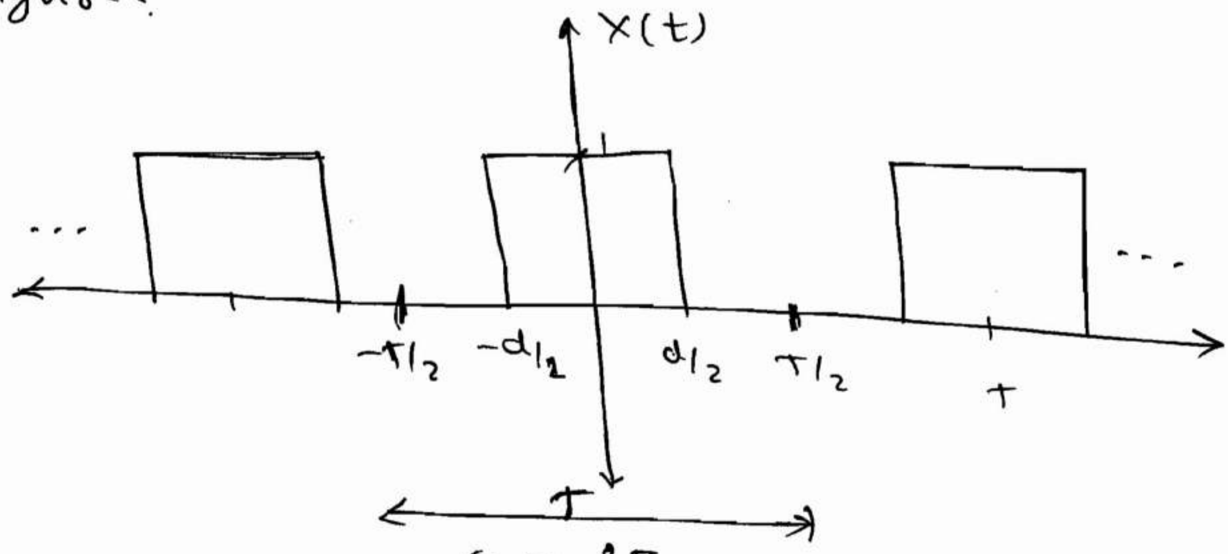
(5) Differentiation in time:

$\Rightarrow \quad x(t) \longleftrightarrow C_n.$

$$\therefore \quad \frac{dx(t)}{dt} \longleftrightarrow (j\omega_0 n)\, C_n.$$

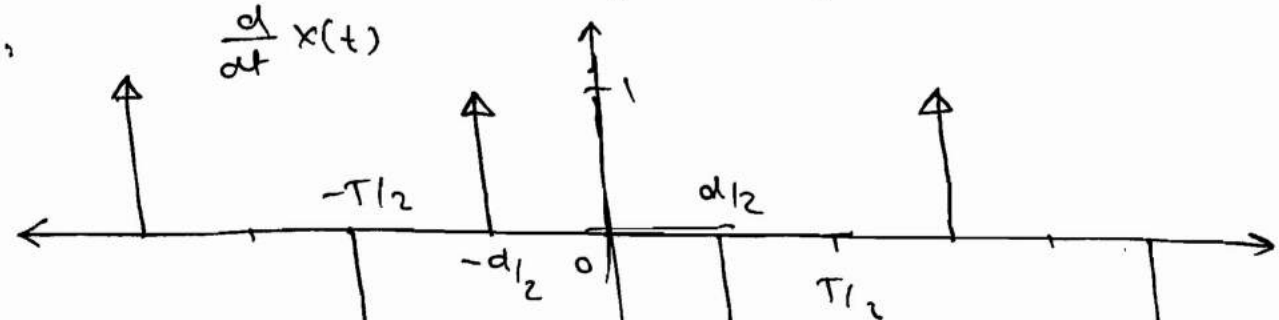
$$\therefore \quad \frac{d^K x(t)}{dt^K} \longleftrightarrow (j\omega_0 n)^K . C_n.$$

$\boxed{P\ 3.3.3}$ By using derivative method, find F.S. coefficient of the signal shown in figure?



$\omega_0 = \frac{2\pi}{T} = 2\pi/T$

Sol^n:



$\therefore$ ~~$d_{xxx/2}$~~

$$y(t) = \frac{d}{dt} x(t).$$

$$\therefore \quad d_n = (j\omega_0 n)\, c_n.$$

$$\therefore \quad c_n = \frac{d_n}{j\omega_0 n}.$$

$$\therefore \quad d_n = \frac{1}{T} \int_{0}^{T} y(t) \cdot e^{-j\omega_0 n t} \cdot dt.$$

$$\therefore \quad d_n = \frac{1}{T} \int_{0}^{T} \left[\delta(t + d/2) - \delta(t - d/2)\right] \cdot e^{-j\omega_0 n t} \cdot dt.$$

Now,
$$\int_{t_1}^{t_2} \delta(t - t_0) \cdot x(t)\, dt = x(t_0). \qquad t_1 \le t_0 \le t_2.$$

$$\therefore \quad d_n = \frac{1}{T}\left[e^{j\omega_0 n \frac{d}{2}} - e^{-j\omega_0 n d/2}\right]$$

$$\therefore \quad d_n = \frac{2j}{T}\left[\frac{e^{j\omega_0 n \frac{d}{2}} - e^{-j\omega_0 n d/2}}{2j}\right].$$

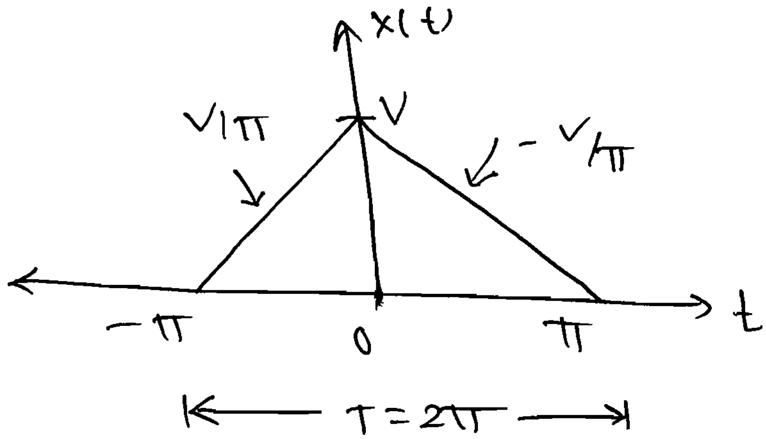
$$\therefore \quad d_n = \frac{2j}{T} \times \sin \omega_0 n \frac{d}{2}.$$

$$\therefore \quad \cancel{d_{n}}\ \ C_n = \frac{d_n}{j\omega_0 n}.$$

$$= \frac{2}{T \times \frac{2\pi}{T} \times n} \times \sin \frac{\omega_0 n d}{2}.$$

$$\boxed{C_n = \frac{\sin \frac{\omega_0 n d}{2}}{\pi n}}$$

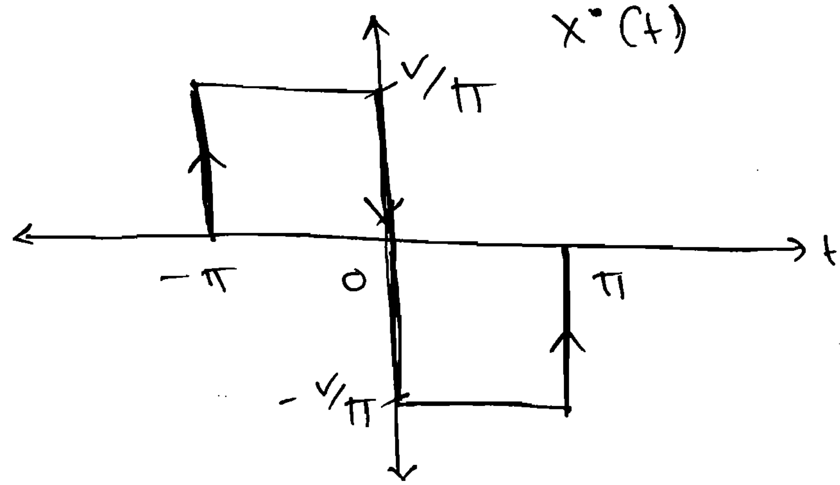
Q

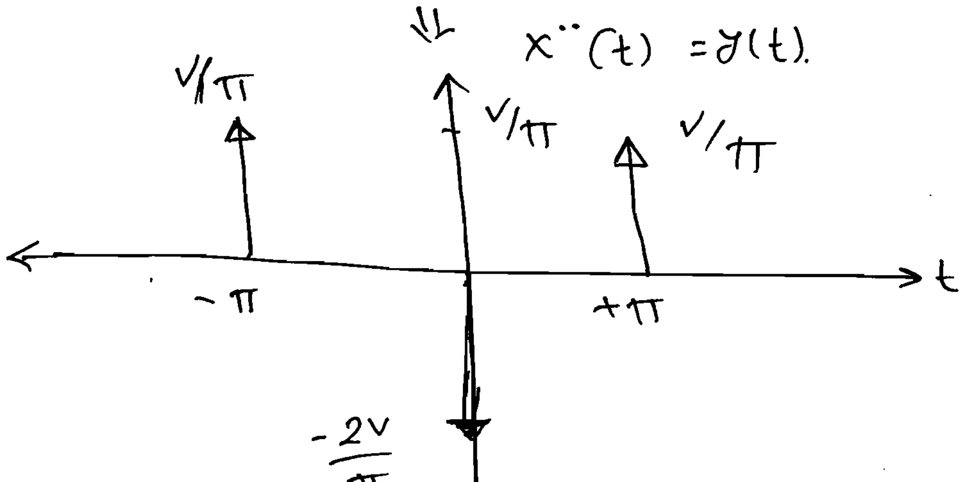


$\Rightarrow \omega_0 = 1$ rad/sec.

$\Downarrow$

$\Rightarrow$





~~Ex~~: $\Rightarrow$ $$y(t) = \frac{d^2 x(t)}{dt}.$$

$$\Downarrow$$

$$d_n = (j\omega_0 n)^2 \cdot C_n.$$

$$\therefore \quad C_n = \frac{d_n}{(j\omega_0 n)^2}.$$

$$\boxed{C_n = -\frac{d_n}{n^2}.} \Rightarrow \boxed{C_n \propto |n^{-2}|.} \quad (\because \omega_0 = 1 \text{ rad/sec}).$$

$\Rightarrow$ $$g(t) = \frac{dx(t)}{dt}.$$

$$g_n = (j\omega_0 n) \cdot C_n.$$

$$C_n = \frac{g_n}{(jn)}$$

$$\Rightarrow \boxed{C_n \propto |n^{-1}|}$$

$\boxed{P\ 3.3.5}$ A periodic signal has the F.S. representation $x(t) \xleftrightarrow[\omega_0=\pi]{F.S.} C_n = -n \cdot 2^{-|n|}$. Without finding $x(t)$, find the F.S. representation [d_n & ω_0] if (a) $y(t) = x(3t)$.

Soln: $$d_n = C_n \Big|_{\omega_0' = 3\omega_0}.$$

$$-|n| \Big|$$

(b) $y(t) = \frac{d}{dt} x(t)$.

Sol^n:

$$d_n = (j\omega_0 n) \cdot c_n \Big|_{\omega_0 = \pi}$$

$$\therefore \quad d_n = (j\pi n) \cdot (-n) \cdot 2^{-|n|}.$$

$$d_n = -j\pi n^2 \cdot 2^{-|n|} \Big|_{\omega_0 = \pi}.$$

(c) $y(t) = x(t-1)$.

Sol^n:

$$d_n = e^{-j\omega_0 n(1)} \cdot c_n.$$

$$= e^{-j\pi n} \cdot (-n) \cdot 2^{-|n|}.$$

$$= (-1)^n \cdot (-n) \cdot 2^{-|n|}.$$

$$d_n = n\,(-1)^{n+1} \cdot 2^{-|n|} \Big|_{\omega_0 = \pi}.$$

(d) $y(t) = \text{Re}\{x(t)\}$.

Sol^n: $x(t) = x_R(t) + j\,x_I(t)$.

$$x^*(t) = x_R(t) - j\,x_I(t).$$

$$\therefore \quad x_R(t) = \frac{x(t) + x^*(t)}{2}$$

$$\therefore \quad x_I(t) = \frac{x(t) - x^*(t)}{2}$$

$$\boxed{\begin{aligned} \therefore x_R(t) &= \frac{x(t)+x^*(t)}{2} \xleftrightarrow{F.S.} \frac{C_n + C^*_{-n}}{2}. \\ x_I(t) &= \frac{x(t)-x^*(t)}{2} \xleftrightarrow{F.S.} \frac{C_n - C^*_{-n}}{2}. \end{aligned}}$$

(e) $y(t) = \cos 4\pi t \; x(t).$

Sol^n:

$$y(t) = \left[\frac{e^{j4\pi t} + e^{-j4\pi t}}{2}\right] x(t).$$

$$y(t) = \frac{\overset{\leftarrow m=4}{e^{j4\pi t}} \cdot x(t) + \overset{\swarrow m=-4}{e^{-j4\pi t}} \cdot x(t)}{2}.$$

$$\therefore d_n = \frac{C_{n-4} + C_{n+4}}{2}.$$

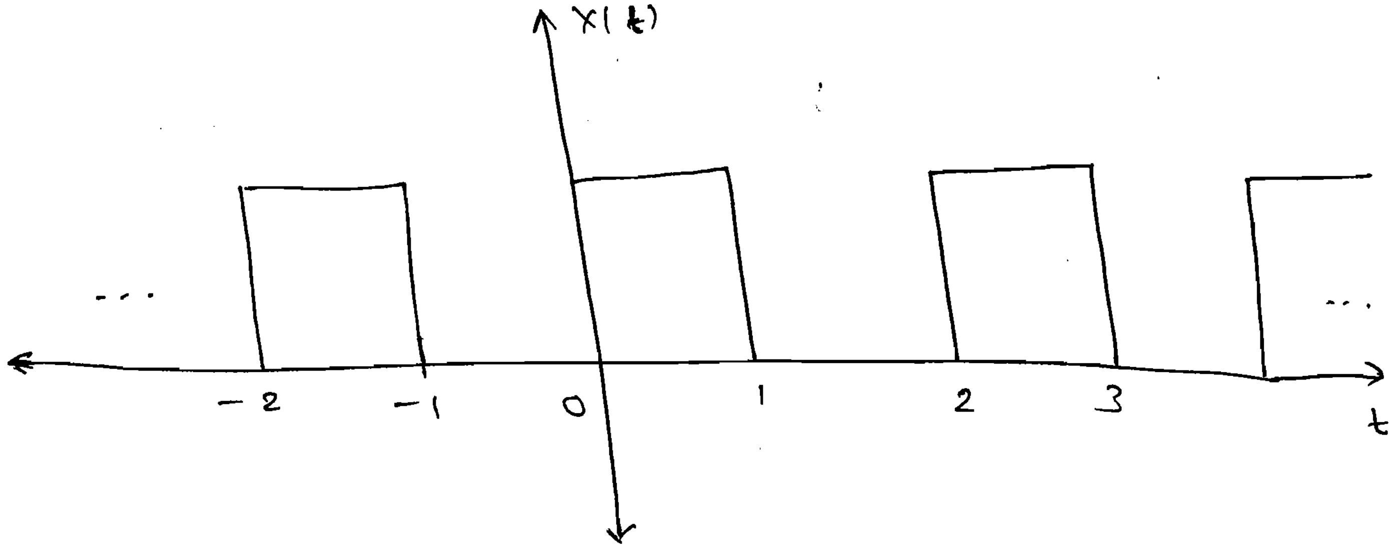
$$\therefore d_n = \frac{-(n-4)\cdot 2^{-|n-4|} + -(n+4)\cdot 2^{-|n+4|}}{2}.$$

6) <u>Parseval's</u> <u>Power</u> <u>theorem</u>:

⇒ The total average Power in Periodic signal is equal to the Sum of the Squared amplitude of each harmonics.

$x(t) \longleftrightarrow C_n$ then $\boxed{\frac{1}{T}\int^{T} |x(t)|^2 \, dt = \sum_{n=-\infty}^{+\infty} |C_n|^2.}$

P 3.3.6. Find the Power up to II harmonic for the periodic signal shown in figure.



Soln:

from Q : 3.2.12

$\Rightarrow$ Amp : 10 | $b_n = \frac{20}{n\pi}$ (odd n).

a_0 : 5

a_n : 0

Here, Amp = 1 So,

$C_0 = a_0 = \frac{1}{2}$.

$$C_n = \frac{a_n - jb_n}{2} = -\frac{j}{2}\left(\frac{2}{n\pi}\right) = -\frac{j}{n\pi} \quad \text{(odd n)}.$$

$\Rightarrow$ Power required upto II harmonic.

$$P = \sum_{n=-2}^{+2} |C_n|^2.$$

$$P = |C_{-2}|^2 + |C_{-1}|^2 + |C_0|^2 + |C_1|^2 + |C_2|^2$$

$$= \frac{1}{(2\pi)^2} + \frac{1}{(\pi)^2} + \left(\frac{1}{2}\right)^2 + \frac{1}{(\pi)^2} + \frac{1}{(2\pi)^2}.$$

Calculation of total power:

$$P = \frac{1}{T}\int_0^T |x(t)|^2 dt$$

$$= \frac{1}{2}\int_0^1 (1)^2 \cdot dt.$$

$$P = \frac{1}{2}$$

$$\boxed{P = 0.5 \text{ watts}}$$

<u>Note</u>: Maximum energy (or) Power of any signal is always there only in the low freq. region.

$\boxed{P\ 3.3.7}$ The F.S. coefficients, of a periodic signal $x(t)$ is expressed as $x(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_0 t}$ are given by

$C_{-2} = 2 - j1$; $C_{-1} = 0.5 + j0.2$; $C_0 = j2$;
$C_1 = 0.5 - j0.2$; $C_2 = 2 + j1$; $C_n = 0$ for $|n| > 2$.

Which of the following is TRUE ?

(a) $x(t)$ has finite energy because only finitely many coefficients are non zero.

(b) $x(t)$ has zero average value because it is periodic.

(d) the real part of x(t) is even.

Soln: → Every periodic signal is power signal. and it energy is ∞.

so option A is wrong.

→ As $C_0 = j2$ is given i.e x(t) has non zero average value and it it indicates it is complex. hence option b is also wrong.

→ $x(t) = x_R(t) + x_I(t)$.

$$\therefore \quad C_n = \frac{1}{T} \int_{-\infty}^{+\infty} x(t) . e^{-j\omega_0 n t} \, dt.$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} [x_R(t) + x_I(t)] . e^{-j\omega_0 n t} \, dt.$$

$$\therefore \quad C_0 = 0 + j2 \text{ (constant)}$$

So, real part of x(t) is 0.

hence, the imaginary part of x(t) is constant.

So, Ans → (c).

* System with periodic Inputs:

$\Rightarrow$ i/p $x(t) = e^{j\omega t} \xrightarrow{L.T.I.}$ o/p $y(t) = e^{j\omega t} \cdot H(\omega)$.

$$\rightarrow \quad y(t) = \int_{-\infty}^{+\infty} x(t-\tau) \cdot h(\tau)\, d\tau .$$

$$= \int_{-\infty}^{+\infty} e^{j\omega(t-\tau)} \cdot h(\tau)\, d\tau .$$

$$= e^{j\omega t} \cdot \int_{-\infty}^{+\infty} e^{-j\omega\tau} \cdot h(\tau) \cdot d\tau .$$

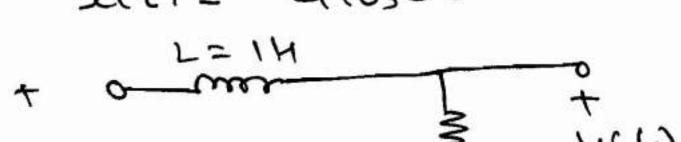
$$\boxed{y(t) = e^{j\omega t} \cdot H(\omega).}$$

$$\Rightarrow \quad x(t) = \sum_{n=-\infty}^{+\infty} C_n \cdot e^{jn\omega_0 t} .$$

$$\downarrow$$

$$y(t) = \sum_{n=-\infty}^{+\infty} C_n \cdot H(n\omega_0) \cdot e^{jn\omega_0 t} .$$

$\boxed{\text{P3.4.1}}$ Find the output Voltage of the system shown in figure, if the input Voltage is $x(t) = 4\cos 2t$



$$\therefore \frac{dy(t)}{dt} + y(t) = x(t).$$

$$\therefore j\omega e^{j\omega t} H(\omega) + e^{j\omega t} \cdot H(\omega) = e^{j\omega t}.$$

$$\therefore H(\omega) = \frac{1}{1 + j\omega}.$$

$$\therefore H(n\omega_0) = \frac{1}{1 + j\omega_0 n} = \frac{1}{1 + j2n}.$$

$(\because \omega_0 = 2)$.

$$\Rightarrow x(t) = 4\cos 2t$$

$$= 2e^{+j(1)(2)t} + 2e^{j(-1)(2)t}.$$

$$\therefore y(t) = 2 \cdot e^{j(1)(2)t} \cdot H(1) + 2e^{-j(2)t} \cdot H(-1).$$

$$\therefore y(t) = 2\left[\frac{1}{1+j2}\right] e^{j2t} + 2\left[\frac{1}{1-j2}\right] e^{-j2t}.$$

$$= k\cos(2t + \phi).$$

Note: Signal Analysis] F.S. & F.T.

System Analysis] L.T & Z.T.