CBSE Class 10th Mathematics
Basic Sample Paper - 08

Maximum Marks:
Time Allowed: 3 hours

General Instructions:

a. All questions are compulsory
b. The question paper consists of 40 questions divided into four sections A, B, C & D.
c. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises 6 questions of 4 marks each.
d. There is no overall choice. However internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

Section A

1. To represent 'the less than type' graphically, we plot the ____________ on the x – axis.

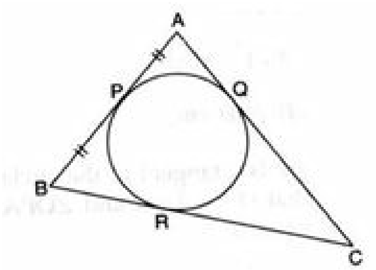
    a. class marks

    b. class size

    c. lower limits

    d. upper limits

2. In the given figure, if AP = PB, then AC =



a. AC = BC

b. AB = BC

c. AQ = QC

d. AC = AB

3. If d is the HCF of 56 and 72, the values of x, y satisfying d = 56x + 72y:

a. $x = 4, y = -3$

b. $x = -4, y = 3$

c. $x = 3, y = -4$

d. $x = -3, y = 4$

4. Which of the following is false?

a. $\text{H.C.F.}\,(p,q,r) \times \text{ L.C.M.}\,(p,q,r) = p \times q \times r$

b. $\text{H.C.F.}\,(p,q,r) = 1$; if p,q,r are prime numbers

c. $\text{H.C.F.}\,(a,b) \times \text{ L.C.M.}\,(a,b) = a \times b$

d. $\text{L.C.M.}\,(p,q,r) = p \times q \times r$; if p,q,r are prime numbers

5. Every positive even integer is of the form ____ for some integer ‘q’.

a. 2q + 1

b. none of these

c. 2q – 1

d. 2q

6. The maximum number of zeroes that a polynomial of degree 3 can have is

    a. Zero

    b. One

    c. Two

    d. Three

7. A polynomial of degree ____ is called a quadratic polynomial.

    a. 1

    b. 3

    c. 2

    d. 0

8. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, then the number of blue balls is

    a. 8

    b. 10

    c. 5

    d. 12

9. If the distance between the points (p, – 5) and (2, 7) is 13 units, then the value of ‘p’ is

    a. – 3, – 7

    b. 3, – 7

    c. 3, 7

    d. – 3, 7

10. (0, 3), (4, 0) and (– 4, 0) are the vertices of

   a. a right triangle

   b. an isosceles triangle

   c. a scalene triangle

   d. an equilateral triangle

11. Fill in the blanks:

   In $\triangle$ABC, $DE \parallel BC$, if $DE = \frac{2}{3}BC$ and area of $\triangle$ABC = 81cm^2, then the area of $\triangle$ADE is ________.

12. Fill in the blanks:

   The distance between the points (10 cos 30^o, 0) and (0, 10 cos 60^o) is ________.

   OR

   Fill in the blanks:

   The distance of point P(3, 4) from the origin is ________.

13. Fill in the blanks:

   If $\sin\theta = \frac{24}{25}$, then the value of $\sin^2\theta + \cos^2\theta$ is ________.

14. Fill in the blanks:

   If x tan 45^{o}cos60^o = sin60^o cot 60^o, then the value of x is ________.

15. Fill in the blanks:

   Two angles are said to be ________ if their sum is equal to 90^o.

16. If the angles of elevation of the top of a tower from two points distant a and b (a > b) from its foot and in the same straight line from it are respectively 30^o and 60^o, then find the height of the tower.

OR

A ladder, leaning against a wall, makes an angle of 60^o with the horizontal.If the foot of the ladder is 2.5 m away from the wall, then find the length of the ladder.

17. If k, (2k-1) and (2k+ 1) are the three successive terms of an AP, find the value of k.

18. If the circumference of two concentric circles forming a ring are 88 cm and 66 cm , then find the width of the ring.

19. If ΔABC ~ ΔDEF such that AB = 1.2 cm and DE = 1.4 cm. Find the ratio of areas of ΔABC and ΔDEF.

20. A black die and a white die are thrown at the same time. Write all the possible outcomes. What is the probability that the numbers obtained have a product less than 16?

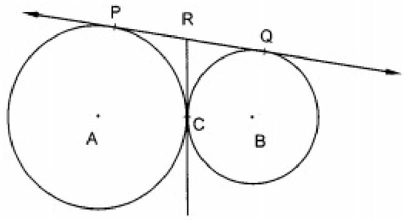
Section B

21. If α and β are the zeros of the quadratic polynomial f(x) = x^2 - p(x + 1) - c, show that $(\alpha+1)(\beta+1)=1-c.$

22. A point P is 25 cm away from the centre of a circle and the length of tangent drawn from P to the circle is 24 cm. Find the radius of the circle.

OR

In figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.



23. An equilateral triangle is inscribed in a circle of radius 6 cm. Find its side.

OR

If $\sin\alpha=\frac{1}{2}$, prove that $3\cos\alpha-4\cos^3\alpha=0.$

24. A circular park, 42 m in diameter, has a path 3.5 m wide running round it on the outside. Find the cost of gravelling the path at Rs 20 per m^2.

25. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish. What is the probability that the fish taken out is a male fish?

26. In a simultaneous throw of a pair of dice, find the probability of getting 5 as the sum.

Section C

27. On dividing a polynomial $3x^3 + 4x^2 + 5x - 13$ by a polynomial g(x), the quotient and the remainder are (3x + 10) and (16x - 43) respectively. Find g(x).

28. Construct a $\triangle$ABC in which BC = 9 cm, $\angle$B = 60° and AB = 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ of the corresponding sides of $\triangle$ABC.

OR

Construct a $\triangle$ABC in which BC = 9 cm, $\angle$B = 60° and AB = 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ of the corresponding sides of $\triangle$ABC.

29. A right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled in cones of height 12 cm and diameter 6 cm having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

30. Prove the trigonometric identity:
if $T_n = \sin^n \theta + \cos^n \theta$, prove that $\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$

OR

If sin 3θ = cos (θ - 6°), where 3θ and θ - 6 ° are both acute angles, find the value of θ.

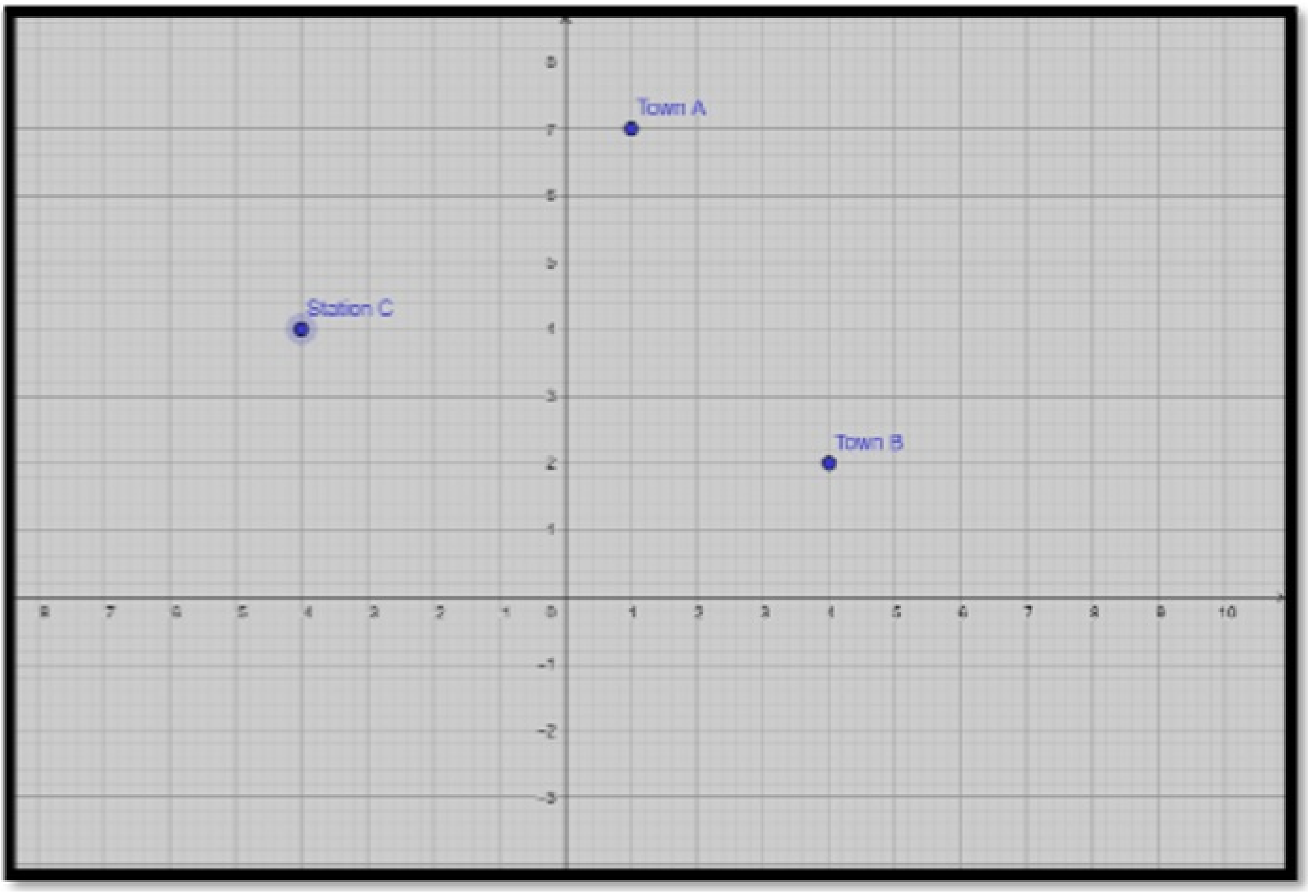
31. Prove that (3 - $\sqrt{5}$) is an irrational number.

OR

Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time they next toll together?

32. A point P is 26 cm from the centre of the circle. The length of the tangent drawn from P to the circle is 24 cm. Find the radius of the circle.

33. Two friends Seema and Aditya work in the same office in Delhi. In the Christmas vacations, both decided to go their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi. Based on the given situation answer the following questions:



i. Who will travel more distance, Seema or Aditya, to reach to their hometown?
ii. Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the point represented by Town A and Town B. Find the coordinates of the point represented by the point D.
iii. Find the area of the triangle formed by joining the points represented by A, B and C.

34. A man on the deck of a ship, 12 m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Find the distance of the cliff from the ship and the height of the cliff. [Use $\sqrt{3}$ = 1.732]

Section D

35. Check whether the equation $5x^2 - 6x - 2 = 0$ has real roots and if it has, finds them by the method of completing the square. Also, verify that roots obtained to satisfy the given equation.

36. If 8th term of an A.P. is half of its second term and 11th term exceeds one third of its fourth term by 1. Find the 15th term.

OR

Find the sum of all integers from 1 to 500 which are multiples of 2 or 5.

37. Solve for x and y : $\frac{2}{3x+2y}$ + $\frac{3}{3x-2y}$ = $\frac{17}{5}$; $\frac{5}{3x+2y}$ + $\frac{1}{3x-2y}$ = 2

38. In ΔABC, DE is parallel to base BC, with D on AB and E on AC. If $\frac{AD}{DB} = \frac{2}{3}$, find $\frac{BC}{DE}$.

OR

If a line is drawn parallel to one side of a triangle to intersect the other two sides in two points, then the other two sides are divided in the same ratio. Prove it.

39. A solid consisting of a right cone standing on a hemisphere is placed upright in a right circular cylinder full of water and touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm, the radius of the hemisphere is 60 cm and height of the cone is 120 cm, assuming that the hemisphere and the cone have common base.

OR

A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 cm high embankment. Find the width of the embankment.

40. In annual day of a school, age-wise participation of students is shown in the following frequency distribution:

Age of student (in years)	5-7	7-9	9-11	11-13	13-15	15-17	17-19
Number of students	20	18	22	25	20	15	10

Draw a less than type’ ogive for the above data and from it find the median age.

Solution

Section A

1. (d) upper limits
   Explanation:

   To represent 'the less than type' graphically, we plot the upper limits on the x-axis.

   e.g marks obtained by students are represented in grouped data as (0 - 10) , (10 - 20), (20 - 30) , (30 - 40) ....

   only upper limits such as 10, 20, 30, 40 .... are taken for the x-axis

2. (a) AC = BC
   Explanation:

   Since Tangents from an external point to a circle are equal.
   $\therefore$ PB = BR ....(i)
   PA = AQ ....(ii)
   CQ = CR ....(iii)
   Adding eq. (i) and (iii), we get
   PB + CQ = BR + CR
   $\Rightarrow$ AP + CQ = BC [Given: PB = AP]
   $\Rightarrow$AQ + CQ = BC [From eq. (ii) AP = AQ]
   $\Rightarrow$ AC = BC

3. (a) $x = 4, y = -3$

   Explanation:

   Since, HCF of 56 and 72, by Euclid's division lemma,

   72 = 56 $\times$ 1 + 16 ....(i)

   56 = 16 $\times$3 + 8 ....(ii)

16 = 8$\times$ 2 + 0 ....(iii)

$\therefore$ HCF of 56 and 72 is 8.

$\therefore$ 8 = 56 – 16$\times$ 3 (from eq. (ii))

8 = 56 – (72 – 56 $\times$1) $\times$3 [From eq. (i) : 16 = 72 – 56$\times$ 1]

8 = 56 – 3 $\times$72 + 56$\times$ 3

8 = 56 $\times$ 4 + (–3) $\times$ 72

$\therefore x=4, y=-3$

4. (a) $\text{H.C.F.}\,(p, q, r) \times \text{ L.C.M.}\,(p, q, r)=p \times q \times r$

   Explanation:

   $\text{H.C.F.}\,(p, q, r) \times \text{ L.C.M.}\,(p, q, r) \neq p \times q \times r.$

   This condition is applied on HCF and LCM of two numbers (either (p , q) or (q , r) or (r , p))

   For three numbers p,q and r the formula is different.

5. (d) 2q
   Explanation:

   Let a be any positive integer and b=2

   Then by applying Euclid's Division Lemma, we have,

   a = 2q + r where $0 \leqslant r<2$ r = 0 or 1

   Therefore, a = 2q or 2q + 1

   Thus, it is clear that a = 2q

   i.e., a is an even integer in the form of 2q

6. (d) Three

Explanation:

The maximum number of zeroes that a polynomial of degree 3 can have is three because the number of zeroes of a polynomial is equals to the degree of that polynomial.

7. (c) 2

   Explanation:

   A polynomial of degree two is called a quadratic polynomial. An equation involving a quadratic polynomial is called a quadratic equation. A quadratic equation is an equation that can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$. This form is called the standard form of a quadratic equation. A quadratic equation can have two, one or no real number solutions.

8. (b) 10

   Explanation:

   Let number of blue balls be x.

   $\therefore$ Number of total outcomes = $5 + x$

   Now, P (getting red ball) = $\frac{5}{5+x}$

   $\therefore$ P (getting blue ball) = $2\left(\frac{5}{5+x}\right)$

   Also P (getting blue ball) = $\frac{x}{x+5}$

   $\therefore 2\left(\frac{5}{x+5}\right) = \frac{x}{x+5} \Rightarrow x = 10$

9. (d) – 3, 7

   Explanation:

   Let point A be $(p, -5)$ and point B (2, 7) and distance between A and B = 13 units

   $\therefore\ 13 = \sqrt{(2-p)^2 + (7+5)^2}$

   $\Rightarrow 13 = \sqrt{4 + p^2 - 4p + 144}$

   $\Rightarrow 13 = \sqrt{p^2 - 4p + 148}$

$\Rightarrow 169 = p^2 - 4p + 148$
$\Rightarrow p^2 - 4p - 21 = 0$

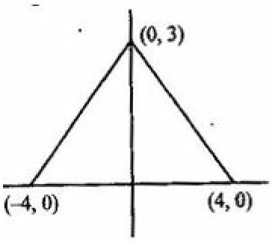
= p^2 - 7p + 3p - 21= 0

= p(p - 7) + 3(p - 7) = 0
$\Rightarrow (p - 7)(p + 3) = 0$
$\Rightarrow p = 7, p = -3$

10. (b) an isosceles triangle

    Explanation:

    Let vertices of a triangle ABC are A (0, 3), B$(-4, 0)$ and C (4, 0).
    $\therefore$ AB = $\sqrt{(-4-0)^2 + (0-3)^2}$
    = $\sqrt{16+9}$ = $\sqrt{25}$ = 5 units
    BC = $\sqrt{(4+4)^2 + (0-0)^2}$ = $\sqrt{64+0}$ = $\sqrt{64}$ = 8 units
    AC = $\sqrt{(4-0)^2 + (0-3)^2}$ = $\sqrt{16+9}$ = $\sqrt{25}$ = 5 units
    Since, two sides are equal, therefore, ABC is an isosceles triangle.



11. 36cm^2

12. 10 units

    OR

    5 units

13. 1

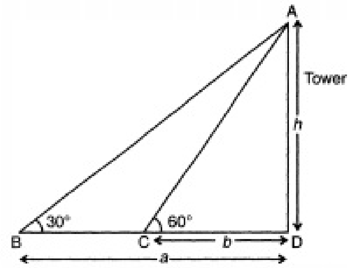
14. x = 1

15. complementary

16. Let the height of tower be h.

From $\triangle ABD$ $\frac{h}{a} = \tan 30^\circ$

$\therefore h = a \times \frac{1}{\sqrt{3}} = \frac{a}{\sqrt{3}} \quad \ldots \text{(i)}$



From $\triangle ACD$, $\frac{h}{b} = \tan 60^\circ$

$h = b \times \sqrt{3} = b\sqrt{3} \quad \ldots \text{(ii)}$

From (i) $a = \sqrt{3}h$

From (ii) $b = \frac{h}{\sqrt{3}}$

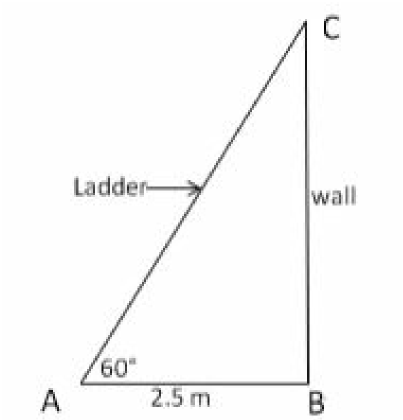
$\therefore \quad a \times b = \sqrt{3}h \times \frac{h}{\sqrt{3}}$

$\Rightarrow \quad h^2 = ab$

$\Rightarrow \quad h = \sqrt{ab}$

Hence, the height of the tower = $\sqrt{ab}$

OR



Let AC be the ladder of length $h\ m$ and BC be the wall.

Then, AB = 2.5 m and $\angle CAB = 60^o$

In right-angled $\triangle ABC$,

$\sec 60^o = \frac{H}{B} = \frac{AC}{AB}$

$\sec 60^o = \frac{h}{2.5}$

$\Rightarrow 2 = \frac{h}{2.5}\ [\because\ Sec\ 60^o = \ 2]$

$\Rightarrow h = 5\ m$

Therefore, find the length of the ladder = 5 m

17. So, $(2k-1)-k=(2k+1)-(2k-1)$
$k-1=2$
$k=2+1$
$k=3$

18. Circumference of the outer circle 2nr$_1$ = 88 cm
$\therefore \quad r_1=\frac{88\times 7}{2\times 22}=14\mathrm{cm}$
Circumference of the inner circle 2nr$_2$ = 66 cm
$\therefore r_2=\frac{66\times 7}{2\times 22}=\frac{21}{2}\mathrm{cm}=10.5\mathrm{cm}$
Width of the ring = r$_1$- r$_2$

= 14 - 10.5 cm

= 3. 5 cm

19. Given: $\Delta ABC\sim\Delta DEF$
According to theorem, the ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.
$\therefore \quad \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)}=\left(\frac{AB}{DE}\right)^2=\left(\frac{AC}{DF}\right)^2=\left(\frac{BC}{EF}\right)^2$
$\frac{Area(\Delta ABC)}{Area(\Delta DEF)}$ = $\left(\frac{AB}{DE}\right)^2$
$=\left(\frac{1.2}{1.4}\right)^2$
$=\left(\frac{6}{7}\right)^2$
= $\frac{36}{49}$
Hence, $\frac{area(\Delta ABC)}{area(\Delta DEF)}=\frac{36}{49}$

20. Consider the set of ordered pairs
{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)
(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)
(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)
(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)
(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)}
Clearly, there are 36 elementary events.

$\therefore$ n(Total number of throws) = 36

Number of pairs such that the numbers obtained have a product less than 16 can be selected as listed below:

{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)

(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)

(3,1)(3,2)(3,3)(3,4)(3,5)

(4,1)(4,2)(4,3)

(5,1)(5,2)(5,3)

(6,1)(6,2)}

Therefore, n(Favourable events) =25

P(the number obtained appearing have a product less than 16)=

$\frac{number\ obtained\ have\ a\ product\ less\ than\ 16}{Total\ number\ throws} = \frac{25}{36}$

Section B

21. Since, α and β are the zeroes of the quadratic polynomial

    f(x) = x^2 - p (x+1) - c=x^2-px-(p+c)

    So A=1 B=-p,C=-(p+c)

    Sum of the zeroes α + β = $-\frac{B}{A}$ =P

    Product of the zeroes αβ = $\frac{C}{A}$ =- (p + c)

    (α + 1)(β + 1)

    = αβ + α + β + 1

    = αβ + (α + β) + 1

    =-(p+c)+p+1

    =-p-c-p+1

    =1-c

    Hence proved

22. OP = 25cm.

    Let TP be the tangent, so that TP = 24cm

    Join OT where OT is radius.

    Now, tangent drawn from an external point is perpendicular to the radius at the point of contact.

    $\therefore OT \perp PT$

In $\triangle OTP$,

By Pythagoras theorem, $OT^2 + TP^2 = OP^2$

$OT^2 + 24^2 = 25^2$

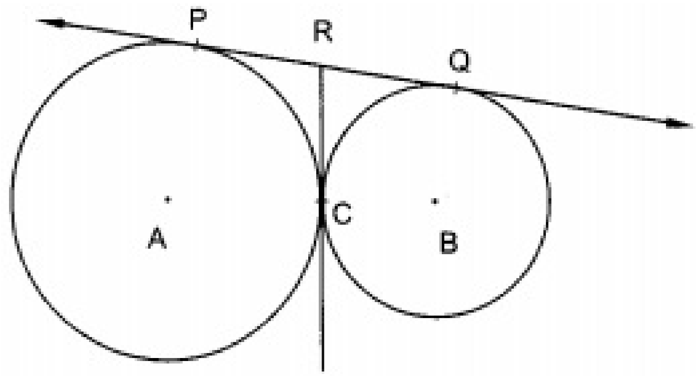
$OT^2 = 625 - 576$

$OT^2 = 49$

OT = 7

The radius of the circle will be 7cm.

OR

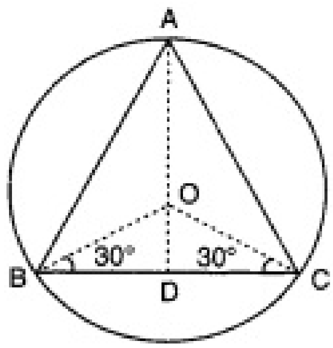


We know that the tangents drawn from an external point to a circle are equal.

$\therefore$ RP = RC and RC = RQ

$\Rightarrow$ RP = RQ

$\Rightarrow$ R is the mid-point of PQ.

23. 

Let ABC be an equilateral triangle inscribed in a circle of radius 6 cm.

Let O be the centre of the circle. Then,

OA = OB = OC = 6 cm.

Let OD be perpendicular from O on the side BC. Then,

D is the mid-point of BC and OB and OC are bisectors of $\angle B$ and $\angle C$ respectively.

$\therefore\ \angle OBD = 30^\circ$

In $\triangle OBD$, right angled at D, we have

$\angle OBD = 30^\circ$ and OB = 6 cm.

$\therefore\ \cos\angle OBD = \frac{BD}{OB}$

$\Rightarrow \quad \cos 30^\circ = \frac{BD}{6}$

$\Rightarrow \quad BD = 6\cos 30^\circ$

$= 6 \times \frac{\sqrt{3}}{2}$ (since, $\cos 30^\circ = \frac{\sqrt{3}}{2}$)

$= 3\sqrt{3}\text{cm}$

$\Rightarrow \quad BC = 2BD$ (since D is the mid-point of BC)

$= 2(3\sqrt{3})$

$= 6\sqrt{3}\text{cm}.$

Hence, the side of the equilateral triangle is $6\sqrt{3}\text{cm}$.

OR

$\sin\alpha = \frac{1}{2} \Rightarrow \sin^2\alpha = \frac{1}{4}$

$\therefore \cos^2\alpha = 1 - \sin^2\alpha = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow \cos\alpha = \frac{\sqrt{3}}{2}$

$\therefore$ L.H.S = $3\cos\alpha - 4\cos^3\alpha$

$= 3 \times \frac{\sqrt{3}}{2} - 4\left(\frac{\sqrt{3}}{2}\right)^3$

$= \frac{3\sqrt{3}}{2} - 4 \times \frac{3\sqrt{3}}{8}$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$

= 0

= R.H.S

24. Diameter of circular park=42 m.

$\Rightarrow$Radius of the circular park = 21 m.

Since width of the path=3.5 m.

Therefore, radius of the outer circle = 21 m + 3.5 m = 24.5 m.

Area of the path = $\pi \times \left[(24.5)^2 - (21)^2\right] \text{m}^2$

$= \frac{22}{7} \times (24.5 + 21)(24.5 - 21)\text{m}^2$

$= \left(\frac{22}{7} \times 45.5 \times 3.5\right) \text{m}^2$

$= 500.5\text{m}^2$

$\therefore$ cost of gravelling the path = Rs (500.5×20)

= Rs 10010.

25. There are 13 (= 8 + 5) fish out of which one can be chosen in 13 ways.
Total number of elementary events = 13
There are 5 male fish out of which one male fish can be chosen in 5 ways.
Favourable number of elementary events = 5
Hence, required probability $= \frac{5}{13}$

26. In a simultaneous throw of a pair of dice,we have to find the probability of getting 5 as the sum.

Consider the set of ordered pairs
{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)
(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)
(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)
(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)
(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)
(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)}
Clearly, there are 36 elementary events.
$\therefore$ n(Total number of throws) = 36
Number of pairs in getting 5 as the sum can be selected as listed below:
{(2,3)(3,2)(4,1)}
Therefore, n(Favourable evnets) = 4
P(getting 5 as the sum)=$\frac{number\ of\ pairs\ in\ getting\ 5\ as\ the\ sum}{total\ number\ of\ throws}=\frac{4}{36}=\frac{1}{9}$.

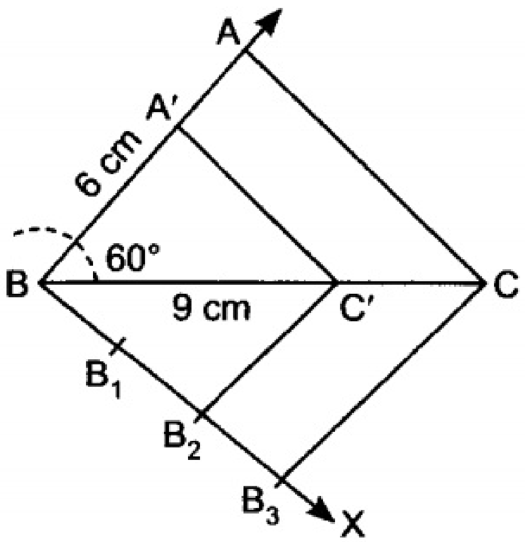
Section C

27. Dividend = $3x^3 + 4x^2 + 5x - 13$
Divisor = g(x)
Quotient = (3x + 10)
Remainder = (16x - 43)
Dividend = Divisor $\times$ Quotient + Remainder
$3x^3 + 4x^2 + 5x - 13 = (3x + 10)g(x) + (16x - 43)$
$g(x)(3x + 10) = (3x^3 + 4x^2 + 5x - 13) - (16x - 43)$

$$\Rightarrow g(x) = \frac{3x^3+4x^2-11x+30}{3x+10}$$

$$
\begin{array}{r}
x^2 - 2x + 3 \\
3x + 10 \,\overline{)\, 3x^3 + 4x^2 - 11x + 30} \\
3x^3 + 10x^2 \qquad\qquad\quad \\
- \quad - \qquad\qquad\qquad \\
\hline
-6x^2 - 11x \qquad \\
-6x^2 - 20x \qquad \\
+ \quad + \qquad\quad \\
\hline
9x + 30 \\
9x + 30 \\
- \quad - \\
\hline
0 \\
\hline
\end{array}
$$

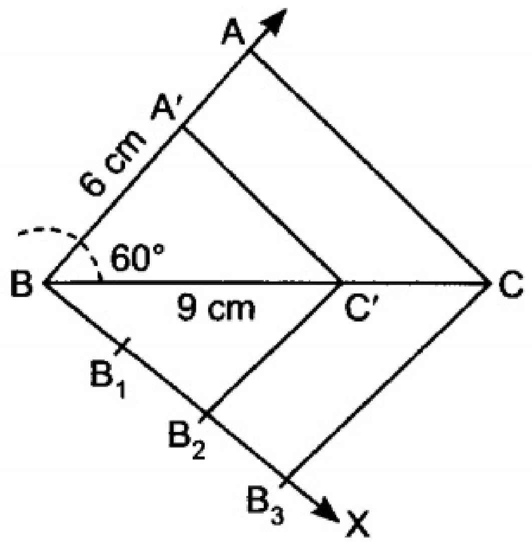
Hence, g(x) = x^2 - 2x + 3

28. 

Steps of Construction:

i. A line segment BC = 9 cm is drawn.
ii. ∠ABC = 60° is constructed at B.
iii. An arc of 6 cm radius to be drawn with B as centre, cutting BA at A.
iv. A and C are joined. Then triangle ABC is constructed.
v. An acute angle CBX is drawn below BC.
vi. Points B_1, B_2, B_3 are taken on BX, such that $BB_1 = B_1B_2 = B_2B_3$.
vii. B_3 and C are joined.
viii. B_2C' is drawn parallel to B_3C, meeting BC at C'.
ix. C'A' is drawn parallel to CA, meeting BA at A'.
x. A'BC' is the required triangle similar to $\triangle$ABC whose sides are $\frac{2}{3}$ of the corresponding sides of $\triangle$ABC.

OR



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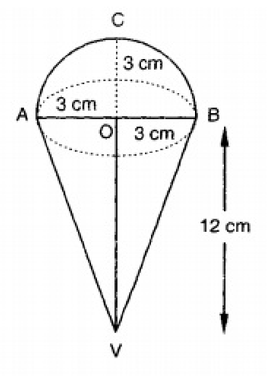
29. We have, r = Radius of the cylinder = 6 cm, h = Height of the cylinder = 15 cm



$\therefore$ Volume of the cylinder = $\pi r^2 h = \pi \times 6^2 \times 15\text{cm}^3 = 540\pi\text{cm}^3$

It is given that

r_1 = Radius of the ice-cream cone = 3 cm and, h_1 = Height of the ice-cream cone = 12 cm

$\therefore$ Volume of teh concial part of ice-cream cone

$\frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3} \times \pi \times 3^2 \times 12\text{cm}^3 = 36\pi\text{cm}^3$

Volume of the hemispherical top of the ice-cream cone =

$\frac{2}{3}\pi r_1^3 = \frac{2}{3} \times \pi \times 3^3 = 18\pi\text{cm}^3$

Total volume of the ice-cream cone = $(36\pi + 18\pi)\text{cm}^3 = 54\pi\text{cm}^3$

$\therefore$ Number of ice-cream cones = $\frac{\text{Volume of the cylinder}}{\text{Total volume of ice-cream cone}} = \frac{540\pi}{54\pi} = 10$

30. We know,

$T_n = \sin^n\theta + \cos^n\theta$

$\therefore T_3 = \sin^3\theta + \cos^3\theta$

$T_5 = \sin^5\theta + \cos^5\theta$

$T_1 = \sin\theta + \cos\theta$

$T_7 = \sin^7\theta + \cos^7\theta$+ cot$\theta$ = m and cosecθ - cotθ = n,

LHS $= \frac{T_3 - T_5}{T_1}$

$= \frac{\sin^3\theta+\cos^3\theta-\left(\sin^5\theta+\cos^5\theta\right)}{\sin\theta+\cos\theta}$

$= \frac{\sin^3\theta+\cos^3\theta-\sin^5\theta-\cos^5\theta}{\sin\theta+\cos\theta}$

$= \frac{\sin^3\theta-\sin^5\theta+\cos^3\theta-\cos^5\theta}{\sin\theta+\cos\theta}$

$= \frac{\sin^3\theta(1-\sin^2\theta)+\cos^3\theta(1-\cos^2\theta)}{\sin\theta+\cos\theta}$

$= \frac{\sin^3\theta\cos^2\theta+\cos^3\theta\sin^2\theta}{\sin\theta+\cos\theta} \begin{bmatrix} \because 1-\sin^2\theta = \cos^2\theta \\ 1-\cos^2\theta = \sin^2\theta \end{bmatrix}$

$= \frac{\sin^2\theta\cos^2\theta(\sin\theta+\cos\theta)}{(\sin\theta+\cos\theta)}$

$= \sin^2\theta\cos^2\theta$

RHS $= \frac{T_5 - T_7}{T_3}$

$= \frac{\sin^5\theta+\cos^5\theta-(\sin^7\theta+\cos^7\theta)}{\left(\sin^3\theta+\cos^3\theta\right)}$

$= \frac{\sin^5\theta+\cos^5\theta-\sin^7\theta-\cos^7\theta}{\left(\sin^3\theta+\cos^3\theta\right)}$

$= \frac{\sin^5\theta-\sin^7\theta+\cos^5\theta-\cos^7\theta}{\sin^3\theta+\cos^3\theta}$

$= \frac{\sin^5\theta(1-\sin^2\theta)+\cos^5\theta(1-\cos^2\theta)}{\sin^3\theta+\cos^3\theta}$
$= \frac{\sin^5\theta\cos^2\theta+\cos^5\theta\sin^2\theta}{\sin^3\theta+\cos^3\theta}$
$= \frac{\sin^2\theta\cos^2\theta(\sin^3\theta+\cos^3\theta)}{(\sin^3\theta+\cos^3\theta)}$
$= \sin^2\theta\cos^2\theta$

LHS = RHS

Hence proved.

OR

Given,

$\sin 3\theta = \cos(\theta - 6°)$

$\cos(90° - 3\theta) = \cos(\theta - 6°)$

$90° - 3\theta = \theta - 6°$

$4\theta = 90° + 6° = 96°$

$\therefore \quad \theta = \frac{96°}{4} = 24°$

31. Let $3 - \sqrt{5} = \frac{p}{q}$

$\therefore 3 - \sqrt{5} = \frac{P}{q}$(where p and q are integers, co-prime and $q \neq 0$)

$\Rightarrow \quad 3 - \frac{p}{q} = \sqrt{5}$

$\Rightarrow \quad \frac{3q-p}{q} = \sqrt{5}$

$(3q - p)$ and q are integers, so $\left(\frac{3q-p}{q}\right)$ is a rational number, but $\sqrt{5}$ is an irrational number. This contradiction arises because of our wrong assumption.

So $(3 - \sqrt{5}$ is an irrational number

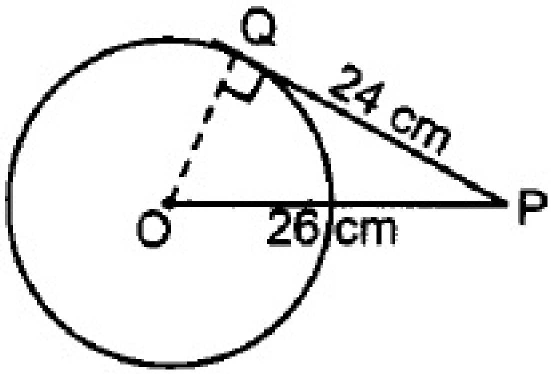
OR

These three balls will toll together after the intervals = LCM(9,12,15) minutes

$9 = 3 \times 3$
$12 = 2 \times 2 \times 3$
$15 = 3 \times 5$

So $LCM(9, 12, 15) = 2 \times 2 \times 3 \times 3 \times 5 = 180$

$\therefore$ The bells will toll together after 180 minutes.

32. 

According to the question, $OP = 26\ cm\ and\ PQ = 24\ cm$

In $\triangle$ OQP, we have $\angle$Q = 90°

$\mathrm{OP}^2 = \mathrm{OQ}^2 + \mathrm{PQ}^2$

$\Rightarrow (26)^2 = \mathrm{OQ}^2 + (24)^2$

$\Rightarrow \mathrm{OQ}^2 = 676 - 576 = 100$

$\Rightarrow \quad OQ = 10\mathrm{cm}$

$\therefore$ Radius of the circle $= 10cm$

33. i. A(1, 7), B(4, 2), C(-4, 4)

Distance travelled by Seema, AC = $\sqrt{[-4-1]^2 + [4-7]^2} = \sqrt{34}$ units

Distance travelled by Aditya, BC = $\sqrt{[-4-4]^2 + [4-2]^2} = \sqrt{68}$ units

$\therefore$ Aditya travels more distance

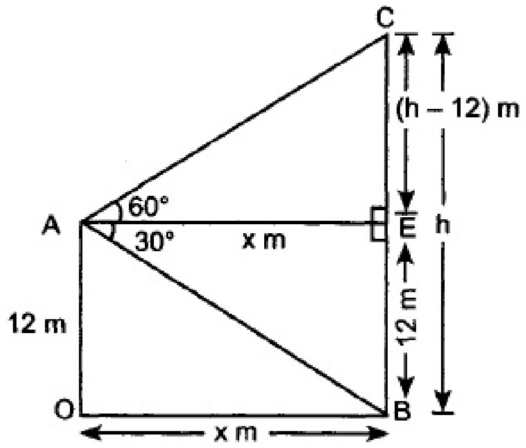
ii. By using mid-point formula,

Coordinates of D are $\left(\frac{1+4}{2}, \frac{7+2}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$

iii. $\mathrm{ar}(\Delta\mathrm{ABC}) = \frac{1}{2}$ [1(2 - 4) + 4(4 - 7) - 4(7 - 2)]

= 17 sq. units

34. A is the position of the man, OA = 12 m, BC is cliff.



BC = h m and CE = (h - 12)m

Let $\mathrm{AE}=\mathrm{OB}=x$ m

In right angled triangle AEB,

$\frac{\mathrm{AE}}{\mathrm{BE}}=\cot 30^{\circ} \Rightarrow \mathrm{AE}=12 \times \sqrt{3}$

$=12 \times 1.732 \mathrm{m}=20.78 \mathrm{m}$

$\therefore$ Distance of ship from cliff = 20.78 m.

In right angled triangle AEC,

$\frac{CE}{AE}=\tan 60^{\circ} \Rightarrow \frac{h-12}{12 \sqrt{3}}=\sqrt{3}$

$h-12=36 \Rightarrow h=48 \mathrm{m}$

Section D

35. We know that Discriminant = b^2-4ac

Here, a = 5, b = (- 6) and c = (- 2)

$=(-6)^2-4 \times 5 \times -2$

= 36 + 40 = 76 > 0

So the equation has real and two distinct roots

$5x^2-6x=2$

(dividing both the sides by 5)

$x^2-\frac{6}{5}x=\frac{2}{5}$

On adding square of the half of coefficient of x

$\frac{6}{5\times2} = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$

$x^2 - \frac{6}{5}x + \frac{9}{25} = \frac{2}{5} + \frac{9}{25}$

$\Rightarrow \quad x - \frac{3}{5} = \pm\frac{\sqrt{19}}{5}$

$\Rightarrow \quad x = \frac{3+\sqrt{19}}{5}$ or $\frac{3-\sqrt{19}}{5}$

Verification :

$5\left[\frac{3+\sqrt{19}}{5}\right]^2 - 6\left[\frac{3+\sqrt{19}}{5}\right] - 2$

$= \frac{9+6\sqrt{19}+19}{5} - \left(\frac{18+6\sqrt{19}}{5}\right) - 2$

$= \frac{28+6\sqrt{19}}{5} - \frac{18+6\sqrt{19}}{5} - 2$

$= \frac{28+6\sqrt{19}-18-6\sqrt{19}-10}{5}$

Similarly

$5\left[\frac{3-\sqrt{19}}{5}\right]^2 - 6\left[\frac{3-\sqrt{19}}{5}\right] - 2 = 0$

Hence, Verified.

36. Consider the first term and the common difference as a and d respectively.

Now $a_8 = \frac{1}{2}$ a_2 [Given]

$\Rightarrow$ a + (8 – 1)d = $\frac{1}{2}$ [a + (2 – 1)d [$\because a_n$ = a + (n – 1)d]

$\Rightarrow$ 2(a + 7d) = a + d

$\Rightarrow$ 2a + 14d – a – d = 0

$\Rightarrow$ a + 13d = 0 ...(i).

Now, $a_{11} = \frac{1}{3}$ a_4 [Given]

$\Rightarrow a + (11 - 1)d = \frac{1}{3}[a + (4 - 1)d] + 1$

$\Rightarrow (a + 10d) = \frac{1}{3}(a + 3d) + 1$

$\Rightarrow 3(a + 10d) = a + 3d + 3$

$\Rightarrow 3a + 30d - a - 3d = 3$

$\Rightarrow 2a + 27d = 3$.......(ii).

Multiplying (i) by 2, we have

$2a + 26d = 0$........ (iii).

Now, subtracting (iii) from (ii), we get.

$$\begin{aligned} 2a + 27d &= 3 \quad \text{...(ii)} \\ 2a + 26d &= 0 \quad \text{(iii)} \\ - \quad - \quad & \quad - \\ \hline d &= 3 \end{aligned}$$

Now, $a + 13d = 0$ [From (i)].

$\Rightarrow a + 13 \times 3 = 0 \Rightarrow a = -39$.

Now, we know that $a_n = a + (n - 1)d$

$\Rightarrow a_{15} = -39 + (15 - 1)3$

$= -39 + 14 \times 3$

$= -39 + 42$

$\Rightarrow a_{15} = 3$.

Thus 15th term is 3

OR

Integers from 1 to 500 which are multiples of 2 are 2, 4, 6, 8......

This forms an A.P., with a = 2, d = 2 and l = 500

Let the number of these terms be n.

Then,

a_n = 500

$\Rightarrow$a + (n - 1)d = 500

$\Rightarrow$2 + (n - 1)(2) = 500

$\Rightarrow$(n - 1)(2) = 498

$\Rightarrow$n - 1 = 249

$\Rightarrow$n = 250

$\Rightarrow S_{250} = \frac{250}{2}[2 \times 2 + 249 \times 2]$

$= 125 \times [4 + 498]$

$= 125 \times 502$

= 62750

Integers from 1 to 500 which are multiples of 5 are 5, 10, 15, 20,.......

This forms an A.P., with a = 5, d = 5 and l = 500

Let the number of these terms be n.

Then,

a_n = 500

$\Rightarrow$a + (n - 1)d = 500

$\Rightarrow$5 + (n - 1)(5) = 500

$\Rightarrow$(n - 1)(5) = 495

$\Rightarrow$n - 1 = 99

$\Rightarrow$n = 100

$\Rightarrow S_{100} = \frac{100}{2}[2 \times 5 + 99 \times 5]$

$= 50 \times [10 + 495]$

$= 50 \times 505$

= 25250

Again, multiples of 10 are included in both i.e., multiple of 2 and multiple of 5 also.

Integers from 1 to 500 which are multiples of 10 are 10, 20.........., 500

This forms an A.P., with a = 10, d = 10 and l = 500

Let the number of these terms be n.

Then,

a_n = 500

$\Rightarrow$a + (n - 1)d = 500

$\Rightarrow$10 + (n - 1)(10) = 500

$\Rightarrow$(n - 1)(10) = 490

$\Rightarrow$n - 1 = 49

$\Rightarrow$n = 50

$\Rightarrow S_{50} = \frac{50}{2}[2 \times 10 + 49 \times 10]$

$= 25 \times [20 + 490]$

$= 25 \times 510$

= 12750

Now, Sum of all integers from 1 to 500 which are multiples of 2 or 5

= (Sum of all integers from 1 to 500 which are multiples of 2 + Sum of all integers from 1 to 500 which are multiples of 5 -Sum of all integers from 1 to 500 which are multiples of 2 and 5)

= 62750 + 25250 - 12750

= 75250

37. $\frac{2}{3x+2y}$ + $\frac{3}{3x-2y}$ = $\frac{17}{5}$; $\frac{5}{3x+2y}$ + $\frac{1}{3x-2y}$ = 2

Let $\frac{1}{3x+2y}$ = a, $\frac{1}{3x-2y}$ = b

$2a + 3b = \frac{17}{5}$...(i)

$5a + b = 2$...(ii)

On multiplying (ii) by 3

$15a + 3b = 6$...(iii)

Subtracting (iii) from (i)

$-13a = \frac{17}{5}$ - 6

-13a = $\frac{-13}{5} \Rightarrow$ a = $\frac{1}{5}$

i.e, $3x + 2y = 5$...(iv)

again b = 2 - 5· $\frac{1}{5}$ = 1,

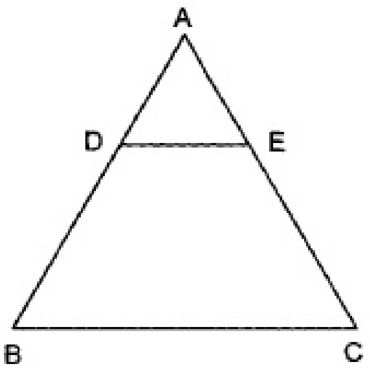
$\therefore 3x - 2y = 1$...(v)

Adding (iv) and (v),

$6x = 6 \Rightarrow$ x =1

Hence, $2y = 5 - 3 = 2 \Rightarrow$ y =1

38. In ΔABC, we have



$DE \| BC$

$\Rightarrow \quad \frac{AB}{AD} = \frac{AC}{AE}$ { using Thales theorm}

Thus, in triangles ABC and ADE, we have

$\frac{AB}{AD} = \frac{AC}{AE}$

and, $\angle$ A = $\angle$A

Therefore, by SAS-criterion of similarity, we have

$\Delta ABC \sim \Delta ADE$

$\Rightarrow \quad \frac{AB}{AD} = \frac{BC}{DE}$ ....(i)

It is given that

$\frac{AD}{DB} = \frac{2}{3}$

$\Rightarrow \quad \frac{DB}{AD} = \frac{3}{2}$

$\Rightarrow \quad \frac{DB}{AD} + 1 = \frac{3}{2} + 1 \Rightarrow \frac{DB+AD}{AD} = \frac{5}{2} \Rightarrow \frac{AB}{AD} = \frac{5}{2}$ ....(ii)

From (i) and (ii), we get $\frac{BC}{DE} = \frac{5}{2}$.

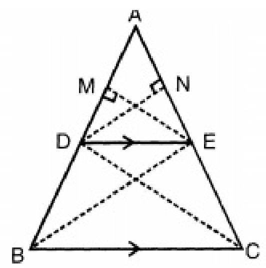
OR

Given: ABC is a triangle in which DE $\|$ BC.

To prove: $\frac{AD}{BD} = \frac{AE}{CE}$

Construction: Draw $DN \perp AE$ and $EM \perp AD$., Join BE and CD.

Proof :



In $\triangle ADE$,

Area of $\Delta ADE = \frac{1}{2} \times AE \times DN$ ...(i)

In ΔDEC,

Area of $\Delta\, DCE = \frac{1}{2} \times CE \times DN$ ...(ii)

Dividing equation (l) by equation (ii),

$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$

$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEC)} = \frac{AE}{CE}$ ...(iii)

Similarly, In ΔADE,

Area of $\Delta\, ADE = \frac{1}{2} \times AD \times EM$ ...(iv)

In ΔDEB,

Area of $\Delta\, DEB = \frac{1}{2} \times EM \times BD$ ...(v)

Dividing equation (iv) by equation (v),

$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$

$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{AD}{BD}$ ...(vi)

ΔDEB and ΔDEC lie on the same base DE and between two parallel lines DE and BC.

∴ Area (ΔDEB) = Area (ΔDEC)

From equation (iii),

$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{AE}{CE}$.......(vii)

From equation (vi) and equation (vii),

$\frac{AE}{CE} = \frac{AD}{BD}$

∴ If a line is drawn parallel to one side of a triangle to intersect the other two sides in two points, then the other two sides are divided in the same ratio.

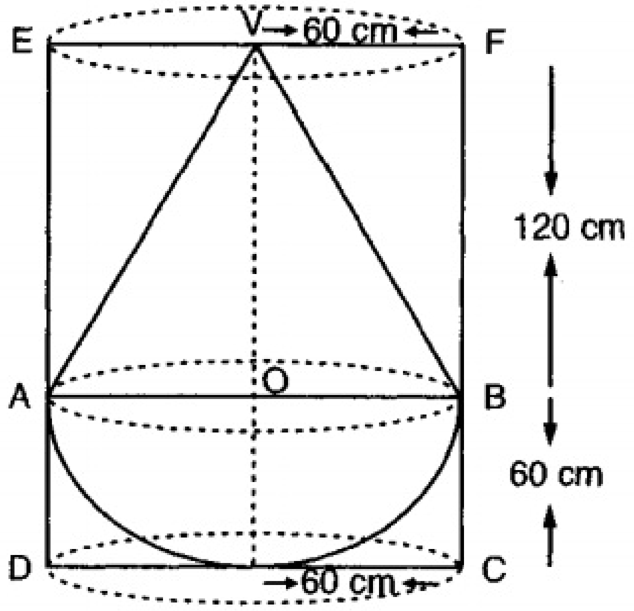
39. We have radius of cylinder = radius of cone = radius of hemisphere = 60 cm

Height of cone = 120 cm

∴ Height of cylindrical vessel = 120 + 60 =180 cm

∴ V = Volume of water that the cylinder contains = $\pi r^2 h = \left\{\pi \times (60)^2 \times 180\right\}$ cm^3

Let V_1 be the volume of the conical part. Then,



$V_1 = \frac{1}{3}\pi r^2 h_1$

$\Rightarrow \quad V_1 = \frac{1}{3} \times \pi \times 60^2 \times 120\text{cm}^3 = \left\{\pi \times 60^2 \times 40\right\}\text{cm}^3$

For hemispherical part r = Radius = 60 cm

Let V_2 be the volume of the hemisphere. Then,

$V_2 = \left\{\frac{2}{3}\pi \times 60^3\right\}\text{cm}^3$

$\Rightarrow \quad V_2 = \left\{2\pi \times 20 \times 60^2\right\}\text{cm}^3 = \left\{40\pi \cdot 60^2\right\}\text{cm}^3$

Let V_3 the the volume of the water left-out in the cylinder. Then,

$V_3 = V - V_1 - V_2$

$V_3 = \left\{\pi \times 60^2 \times 180 - \pi \times 60^2 \times 40 - 40\pi \times 60^2\right\}\text{cm}^3$

$V_3 = \pi \times 60^2 \times \{180 - 40 - 40\}\text{cm}^3$

$V_3 = \frac{22}{7} \times 3600 \times 100\text{cm}^3$

$\Rightarrow \quad V_3 = \frac{22 \times 360000}{7}\text{cm}^3 = \frac{22 \times 360000}{7 \times (100)^3}\text{m}^3 = \frac{22 \times 36}{700}\text{m}^3 = 1.1314\text{m}^3.$

OR

According to the question,the well of diameter 4 metre is dug 14 metre deep.

We are given that, Depth of well = 14 m, radius = 2 m.

Volume of earth taken out $= \pi r^2 h$

$= \frac{22}{7} \times 2 \times 2 \times 14$

$= 176m^3$

Let r be the width of embankment,then

the radius of outer circle of embankment = 2 + r

Area of upper surface of embankment $= \pi\left[(2+r)^2-(2)^2\right]$

Volume of embankment = Volume of earth taken out

or, $\pi\left[(2+r)^2-(2)^2\right]\times 0.4 = 176$

or, $\pi[4+r^2+4r-4]\times 0.4 = 176$

or, $r^2+4r = \frac{176\times 7}{0.4\times 22}$

or, $r^2+4r = 140$

or, $r^2+4r-140 = 0$

or, $(r+14)(r-10) = 0$

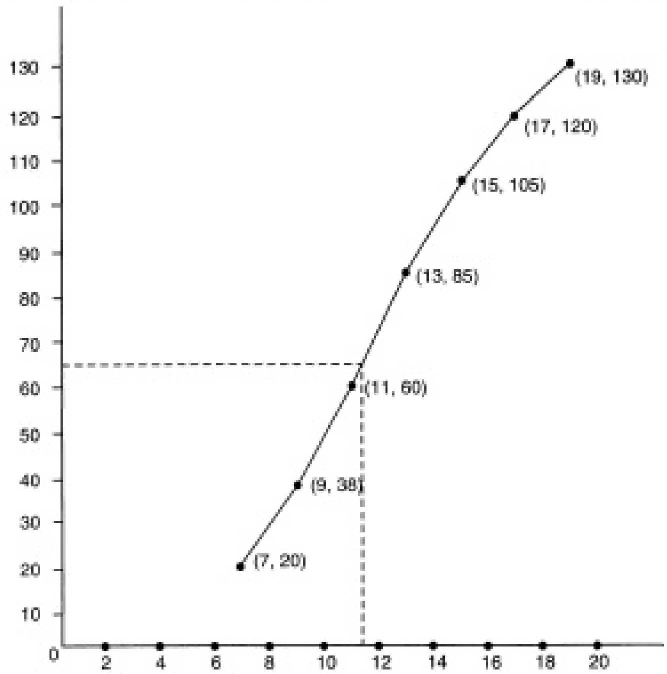
or, r = 10 m [as radius can't be negative]

Hence width of embankment = 10 m.

40.

Students	**c.f.**
Less than 7	20
Less than 9	38
Less than 11	60
Less than 13	85
Less than 15	105
Less than 17	120
Less than 19	130

Units: x-axis 1 cm = 2; y-axis 1 cm = 10



This curve is the required cumulative frequency curve or an ogive of the less than type.

Here, N = 130,

So, $\frac{N}{2} = \frac{130}{2}$ = 65

Now, we locate the point on the ogive whose ordinate is 65. The x-coordinate corresponding to this ordinate is 11.4.

Hence, the required median on the graph is 11.4.