Chapter 15. Probability

Question-1

(iv) P (A) = 1/2, P (B) = 1/4, P (C) = 1/8, P (D) = 0.46

Solution:

(i) P(A)+ P(B)+ P(C)+ P(D) = 0.37+ 0.17+ 0.14 + 0.32 = 1.00

Total Probability = 1

∴ Permissible.

(ii) P(A)+ P(B)+ P(C)+ P(D) = 0.30 + 0.28 + 0.26 + 0.18 = 1.02 > 1

∴ not permissible.

(iii) Not possible since P (C) is negative.

(iv) P (A) + P (B) + P (C) + P (D) = $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$

$= \frac{8+4+2+1}{16} = \frac{15}{16} \neq 1$

Since the total probability ≠ 1. It is not permissible.

Question-2

(iii) a sum of 9 or 11

Solution:

(i) Sample space = {(1,1) (1,2), (1,3), (1,4), (1,5), (1,6), (2,1) (2,2), (2,3), (2,4), (2,5), (2,6), (3,1) (3,2), (3,3), (3,4), (3,5), (3,6), (4,1) (4,2), (4,3), (4,4), (4,5), (4,6), (5,1) (5,2), (5,3), (5,4), (5,5), (5,6), (6,1) (6,2), (6,3), (6,4), (6,5), (6,6)}

n(S) = 36

A = obtaining sum less than 5

= {(1,1) (1,2), (2,1), (3,1), (2,2), (1,3)}

n(A) = 6

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(ii) B = obtaining sum greater than 10 = {(5,6) (6,5) (6,6)}

∴ n(B) = 3

∴ P(B) = $\frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

(iii) C = a sum of 9 or 11

C = {(4,5) (5,4) (3,6) (6,3) (6,5) (5,6)}

∴ n (C) = 6

∴ $P(C) = \frac{n(C)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

Question-3

Three coins are tossed once. First the probability of getting

(i) exactly two heads

(ii) atleast two heads

(iii) almost two heads

Solution:

n (S) = 8

Probability of getting head = $\frac{1}{2}$ and Probability of getting tail = $\frac{1}{2}$

(i) Probability of getting exactly two heads = $3C_2 \left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 = \frac{3}{8}$

(ii) Probability of getting atleast two heads

= P(2) + P(3)

$= 3C_2 \left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 + 3C_3\left(\frac{1}{2}\right)^3$

$= \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

(iii) Probability of getting almost two heads = P(0)+ P(1)+ P(2)

$= 3C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 + 3C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 + \frac{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{3}\right)^1$

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$

Question-4

(iii) the card is either queen or 7?

Solution:

Probability of the card is jack or king

= Probability (jack) + Probability (king)

$= \frac{4}{52} + \frac{4}{52} = \frac{2}{13}$

Probability of getting the card that will be 5 or smaller = $\frac{4+4+4+4}{52} = \frac{16}{52} = \frac{4}{13}$

Probability of getting queen or 7 = $\frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$

Question-5

A bag contains 5 white and 7 black balls, 3 balls are drawn at random. Find the probability that

(i) all are white

(ii) one white and 2 black.

Solution:

(i) n (S) = $10C_3$

A = event of getting all the three are white balls

∴ n (A) = $5C_3$

\ $P(A) = \frac{5C_3}{10C_3} = \frac{5\times 4\times 3}{1\times 2\times 3} \times \frac{1\times 2\times 3}{10\times 9\times 8} = \frac{1}{12}$

(ii) B = event of getting one white and 2 black balls

n (B) = $5C_1 \times 7C_2$

∴ $P(B) = \frac{5C_1 \times 7C_2}{10C_3} = 5 \times \frac{7\times 6}{1\times 2} \times \frac{1\times 2\times 3}{10\times 9\times 8} = \frac{7}{8}$

Question-6

In a box containing 10 bulbs, 2 are defective. What is the probability that among 5 bulbs chosen at random, none is defective.

Solution:

Probability of getting a defective bulb = $\frac{2}{10}$ = $\frac{1}{5}$ ∴ Probability of getting a non defective bulb = $\frac{4}{5}$

While 5 bulbs are chosen at random, the probability of getting none of the defective bulb is = $10C_5\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^5$

= $\frac{10\times 9\times 8\times 7\times 6}{1\times 2\times 3\times 4\times 5}\times\frac{4\times 4\times 4\times 4\times 4}{5\times 5\times 5\times 5\times 5}$ = $10C_5\left(\frac{4}{5}\right)^5$

Question-7

(i) one is a mango and the other is an apple

(ii) both are of the same variety.

Solution:

n(S) = $7C_2$

A = getting 1 mango and 1 apple

n(A) = $4C_1 \times 3C_1 = 4 \times 3$

∴ P(A) = $\frac{4\times 3}{7\times 6}\times 1\times 2$ = $\frac{4}{7}$

B = getting both as same variety

n(B) = $4C_2 + 3C_2$ = $\frac{4\times 3}{7\times 6}+\frac{3\times 2}{1\times 2}$ = 6 + 3 = 9

∴ P(B) = $\frac{9}{7\times 6}\times 1\times 2$ = $\frac{3}{7}$

Question-8

Out of 10 outstanding students in a school there are 6 girls and 4 boys. A team of 4 students is selected at random for a quiz program. Find the probability that are atleast 2 girls.

Solution:

Possibilities	Girls 6	Boys 4	Combinations
1	2	2	$6C_2 \times 4C_2 = 90$
2	3	1	$6C_3 \times 4C_1 = 80$
3	4	0	$6C_4 \times 4C_0 = 15$
			Total = 185

$n(S) = 10C_4$

$\therefore$ Probability $= \frac{185}{10C_4} = \frac{185}{10 \times 9 \times 8 \times 7} 1 \times 2 \times 3 \times 4 = \frac{37}{42}$

Question-9

What is the chance that a leap year should have fifty three Sundays?

Solution:

An ordinary year consists of 365 days.

365 days = 52 weeks + 1 day. 52 weeks will have 52 Sundays.

Let one day be any one of the following Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday.

The probability that an ordinary year may contain 53 Sundays = $\frac{1}{7}$

A leap year contains 366 days

366 days = 52 weeks + 2 days

These two days may be any one of the following combinations

1) Monday + Tuesday
2) Tuesday + Wednesday
3) Wednesday + Thursday
4) Thursday + Friday
5) Friday + Saturday
6) Saturday + Sunday
7) Sunday + Monday

In the above seven, Sunday appears only on two.

Required Probability = $\frac{\text{No. of favourable events}}{\text{Total no. of events}} = \frac{2}{7} \left(\because \begin{matrix} n(A) = 2 \\ n(S) = 7 \end{matrix} \right)$

Question-10

An integer is chosen at random from the first fifty positive integers. What is the probability that the integer chosen is a prime or multiple of 4.

Solution:

n (S) = 50

A = {2,3,5,7, 11, 13,17, 19,23,29,31,37,41,43,47, 4,8,12,16,20,24,28,32,36,40,44,48}

n(A) = 27

$\therefore$ P(A) = $\frac{27}{50}$

Question-11

(i) P(B)

(ii) P($\bar{A} \cap \bar{B}$)

Solution:

(i) P(A) = 0.36, P(A ∪ B) = 0.9, P(A ∩ B) = 0.25

P(A ∪ B) = P(A)+ P(B) - P(A ∩ B)

0.9 = 0.36 + P (B) - 0.25

P(B) = 0.9 + 0.25- 0.36= 0.79

(ii) P($\bar{A} \cap \bar{B}$) =1 - P(A ∪ B)= 1-0.9=0.1

Question-12

(i) P($\bar{A}$)

(ii) P(A ∪ B)

(iii) ($A \cap \bar{B}$)

(iv) P($\bar{A} \cap \bar{B}$)

Solution:

(i) P(A) = 0.28, P (B) = 0.44. A and B are mutually exclusive.

P($\bar{A}$) = 1 - P (A) = 1 - 0.28 = 0.72

(ii) P(A ∪ B) = P (A) + P (B) = 0.28 + 0.44 = 0.72

(iii) ($A \cap \bar{B}$) = P (A) - P (A ∩ B) = 0.28 - 0 = 0.28

(iv) P($\bar{A} \cap \bar{B}$) = 1- P(A ∪ B)= 1- 0.72= 0.28

Question-13

Given P(A) = 0.5 , P(B) = 0.6 and P (A ∩ B) = 0.24. Find

(i) P(A ∪ B)

(ii) P($\bar{A} \cap B$)

(iii) P($A \cap \bar{B}$)

(iv) ($\bar{A} \cup \bar{B}$)

(v) P($\bar{A} \cap \bar{B}$)

Solution:

(i) P (A ∪ B) = P (A) + P (B) - P (A ∩ B) = 0.5 + 0.6 - 0.24 = 0.86

(ii) P($\bar{A} \cap B$) = P (B) - P (A ∩ B) = 0.6 - 0.24 = 0.36

(iii) P($A \cap \bar{B}$) = P(A)- P(A ∩ B) = 0.5- 0.24 = 0.26

(iv) P($\bar{A} \cup \bar{B}$) = 1 - P(A ∩ B) = 1 - 0.24 = 0.76

(v) P($\bar{A} \cap \bar{B}$) = 1 - P(A ∪ B) = 1 - 0.86 = 0.14

Question-13

Given P(A) = 0.5 , P(B) = 0.6 and P (A ∩ B) = 0.24. Find

(i) P(A ∪ B)

(ii) P($\bar{A} \cap B$)

(iii) P($A \cap \bar{B}$)

(iv) $(\bar{A} \cup \bar{B})$

(v) P($\bar{A} \cap \bar{B}$)

Solution:

(i) P (A ∪ B) = P (A) + P (B) - P (A ∩ B) = 0.5 + 0.6 - 0.24 = 0.86
(ii) P($\bar{A} \cap B$) = P (B) - P (A ∩ B) = 0.6 - 0.24 = 0.36
(iii) P($A \cap \bar{B}$) = P(A)- P(A ∩ B) = 0.5- 0.24 = 0.26
(iv) P$(\bar{A} \cup \bar{B})$ = 1 - P(A ∩ B) = 1 - 0.24 = 0.76
(v) P$(\bar{A} \cap \bar{B})$ = 1 - P(A ∪ B) = 1 - 0.86 = 0.14

Question-14

A die is thrown twice. Let A be the event. "First die shows 4' and B be the 'second die shows 4'. Find P(A ∪ B).

Solution:

n (S) = 36;

A = event of "first die shows 4"

n(A) = 6 ;

∴ P(A) = $\frac{6}{36}$

B = event of "second die shows 4"

n(B) = 6 ;

∴ P(A) = $\frac{6}{36}$

A ∩ B = event of first die showing 4 and second die showing 4

n(A ∩ B) = 1 ;

∴ P(A ∩ B) = $\frac{1}{36}$

∴ P (A ∪ B) = P (A) + P (B) - P (A ∩ B) = $\frac{6}{36}+\frac{6}{36}-\frac{1}{36} = \frac{11}{36}$

Question-15

The probability of event A occurring is 0.5 and event B occurring is 0.3. If A and B are mutually exclusive events, then find the probability of neither A nor B occurring.

Solution:

P(A) = 0.5 ; P (B) = 0.3 ; A ∩ B = φ (given)

$P(\bar{A} \cap \bar{B})$ = 1- P(A∪ B)

= 1-{P(A)+P(B)}

= 1- {0.5 + 0.3} = 0.2

Question-16

A card is drawn at random from a deck of 52 cards. What is the probability that the drawn card is
(i) a queen or club card?
(ii) a queen or a black card?

Solution:

(i) n(S) = 52

A = getting a queen

n(A) = 4 ; P (A) = $\frac{4}{52}$

B = getting a club

n(B) = 13 ; P(B) = $\frac{13}{52}$

A ∩ B = getting a club queen

n(A ∩ B) = 1 ; P(A ∩ B) = $\frac{1}{52}$

∴ P(A ∪ B) = $\frac{4}{52}+\frac{13}{52}-\frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

(ii) A = getting a queen

n (A) = 4; P(A) = $\frac{4}{52}$

B = getting a black card;

n(B) = 26 ; P (B) = $\frac{26}{52}$

A∩ B = getting a black queen

n(A∩ B) =2; P(A∩ B) = $\frac{2}{52}$

∴ P(A∪ B) = P (A) + P (B) - P (A ∩ B) = $\frac{4}{52}+\frac{26}{52}-\frac{2}{52} = \frac{28}{52} = \frac{7}{13}$

Question-17

The probability that a new ship will get an award for its design is 0.25, the probability that it will get an award for the efficient use of materials is 0.35 and that it will get both awards is 0.15. What is the probability, that
(i) it will get atleast one of the two awards?
(ii) it will get only one of the awards?

Solution:

(i) Probability for design award = 0.25 = P (A)
Probability for efficient award = 0.35 = P(B)
Probability for both the awards = 0.15 = $P(A \cap B)$
Probability that it will get atleast one award
$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.35 - 0.15 = 0.45$
(ii) $P(A \cap \bar{B}) + P(\bar{A} \cap B) = P(A) - P(A \cap B) + P(B) - P(A \cap B)$
$= 0.25 - 0.15 + 0.35 - 0.15$
$= 0.60 - 0.30$
$= 0.30$

Question-18

Define independent and mutually exclusive events. Can two events be mutually exclusive and independent simultaneously?

Solution:

Independent events
Two events A and B are said to be independent events if happening of one does not depend on happening of the other.

Mutually exclusive events
Two events A and B are said to be mutually exclusive if happening one prevents the happening of the other.
These two events cannot be mutually exclusive and independent simultaneously for non-empty events. Of course possible for any one being null event that is impossible event

Question-19

If A and B are independents, prove that $\bar{A}$ and $\bar{B}$ are independents.

Solution:

Given A and B are independents.

P(A ∩ B) = P(A) . P(B)

To prove that $\bar{A}$ and $\bar{B}$ and also independents.

That is $P(\bar{A} \cap \bar{B}) = P(\bar{A}).P(\bar{B})$

LHS $P(\bar{A} \cap \bar{B}) = 1 - P(\bar{A} \cup \bar{B})$

$= 1 - P(A) + P(B) - P(A \cap B)$

$= 1 - P(A) + P(B) - P(A). P(B)$

$=[1 - P(A)] [1 - P(B)]$

$= P(\bar{A}).P(\bar{B})$ = R.H.S

Question-20

If P(A) = 0.4, P(B) = 0.7 and P(B/A) = 0.5 find P(A/B) and P(A ∪ B).

Solution:

P (A) = 0.4, P(B) = 0.4 and P(B/A) = 0.5

$P(B/A) = \frac{P(A \cap B)}{P(A)}$

$0.5 = \frac{P(A \cap B)}{0.4}$

P (A∩ B) = 0.2

(i) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.7} = \frac{2}{7}$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{4}{10} + \frac{2}{10} - \frac{2}{10} = \frac{9}{10}$