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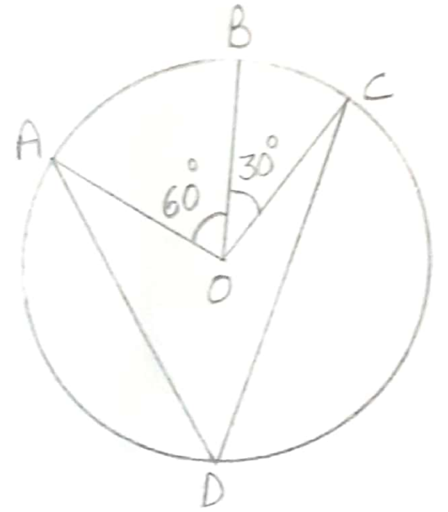
Exercise- 10.5

1. In fig A, B and C are three points on a circle with centre O such that $\angle BOC = 30°$ and $\angle AOB = 60°$. If D is a point on the circle other than arc ABC, find $\angle ADC$.



Sol:- $\angle AOC = \angle AOB + \angle BOC$
$= 60° + 30° = 90°$

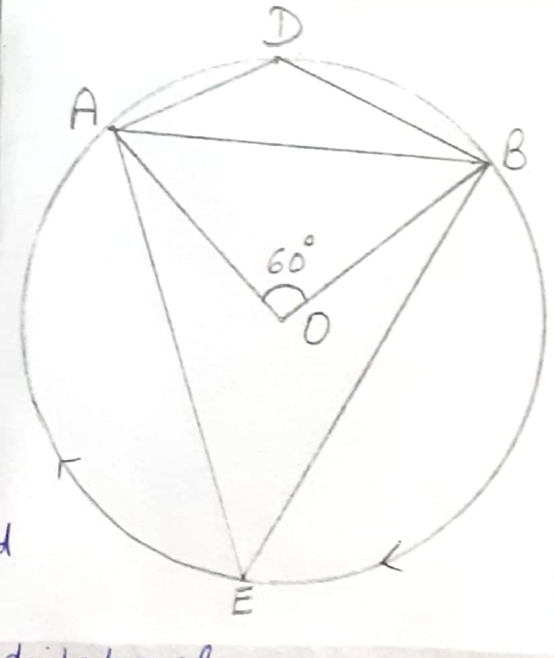
$\angle AOC = 2\angle ADC$

(∵ Angle subtended by an arc at centre is double the angle subtended by it at any remaining part of circle)

$\therefore \angle ADC = \frac{1}{2} \times \angle AOC = \frac{1}{2} \times 90° = 45°$

$\therefore \angle ADC = 45°$.

2. A chord of circle is equal to the radius of the circle. Find angle subtended by the chord at a point on minor arc and also at point on the major arc.



Sol:- Let AB is chord, which is equal to the radius. Join OB and OC.

Now $AB = OB = OA$

$\therefore \triangle OBA$ is an equilateral triangle

$\therefore \angle BOA = 60°$ or $\angle AOB = 60°$.

$\therefore \angle AEB = \frac{1}{2} \times \angle AOB = \frac{1}{2} \times 60° = 30°$

[∵ Angle subtended by an arc at centre is doubled the angle subtended by it at remaining part of circle]

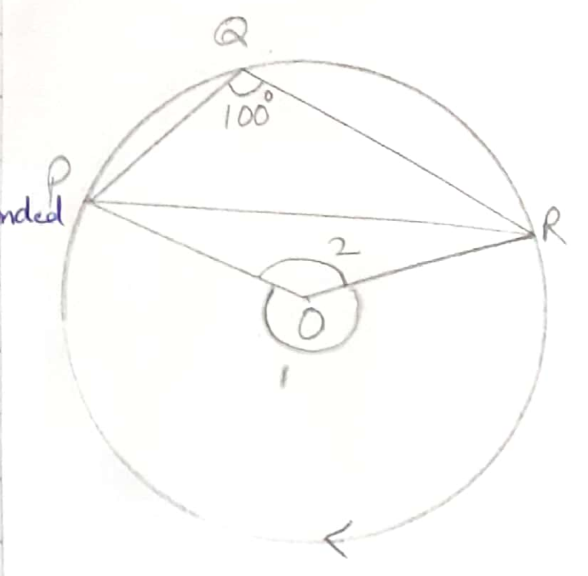
Here, AEBD is a cyclic quadrilateral

$\therefore \angle AEB + \angle ADB = 180°$ [∵ In cyclic quadrilateral sum of opposite angle is 180°].

$30° + \angle ADB = 180°$

$\angle ADB = 180° - 30° = 150°$.

3. In figure, $\angle PQR = 100°$, where P, Q and R are points on circle with centre O. Find $\angle OPR$



Sol:- major $\angle ROP = 2 \times \angle PQR$

$\angle 1 = 2 \times 100° = 200°$

[∵ Angle subtended by an arc at centre is doubled the angle subtended by it at remaining part of circle]

Now $\angle 1 + \angle 2 = 360°$

$200° + \angle 2 = 360°$

$\angle POR = \angle 2 = 360° - 200° = 160°$

In ΔPOR, $OP = OR \Rightarrow \angle ORP = \angle OPR$

$\angle OPR + \angle ORP + \angle POR = 180°$

$\angle OPR + \angle OPR + \angle POR = 180°$

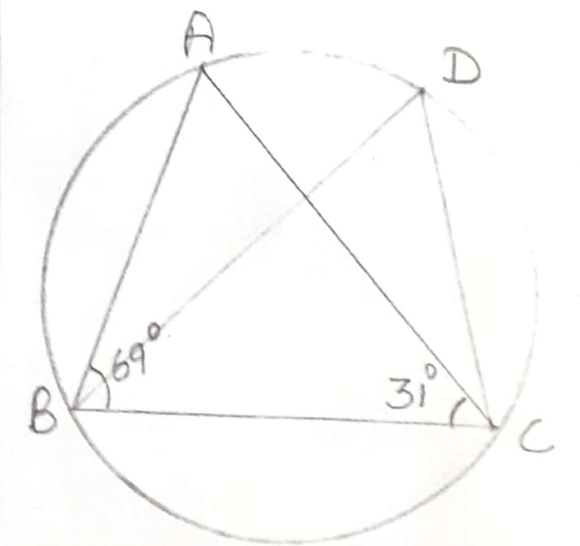
$2\angle OPR + 160° = 180°$

$2\angle OPR = 180° - 160°$

$2\angle OPR = 20°$

$\angle OPR = \frac{20°}{2} = 10°$.

4. In fig, $\angle ABC = 69°$, $\angle ACB = 31°$. Find $\angle BDC$.



Sol:- In ΔABC

$\angle BAC + \angle ABC + \angle ACB = 180°$

$\angle BAC + 69° + 31° = 180°$

$\angle BAC + 100° = 180°$

$\angle BAC = 180° - 100°$

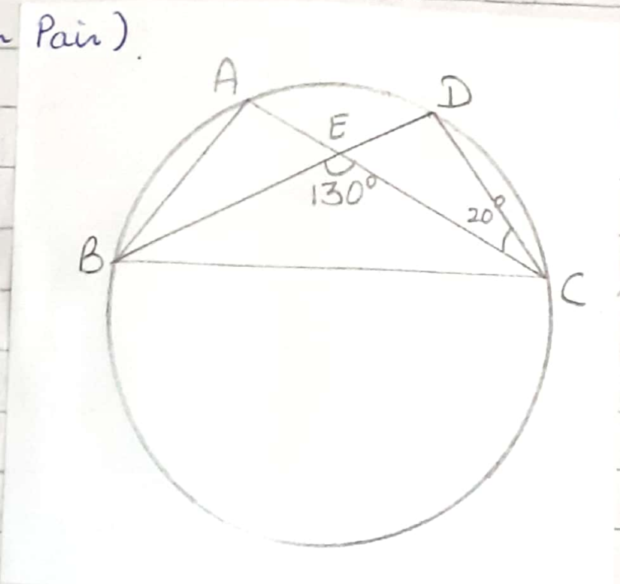
$\angle BAC = 80°$

Now $\angle BDC = \angle BAC$

[∵ Angles in the same segments are equal)

∴ $\angle BDC = 80°$.

5. In fig A, B and C are four points on a circle. AC and BD intersect at point E such that $\angle BEC = 130°$ and $\angle ECD = 20°$. Find $\angle BAC$.



Sol:- $\angle CED + \angle BEC = 180°$ (Linear Pair)

$\angle CED + 130° = 180°$

$\angle CED = 180° - 130° = 50°$

In ΔECD

$\angle ECD + \angle CDE + \angle DEC = 180°$

$20° + \angle CDE + 50° = 180°$

$70° + \angle CDE = 180°$

$\angle CDE = 180° - 70° = 110°$

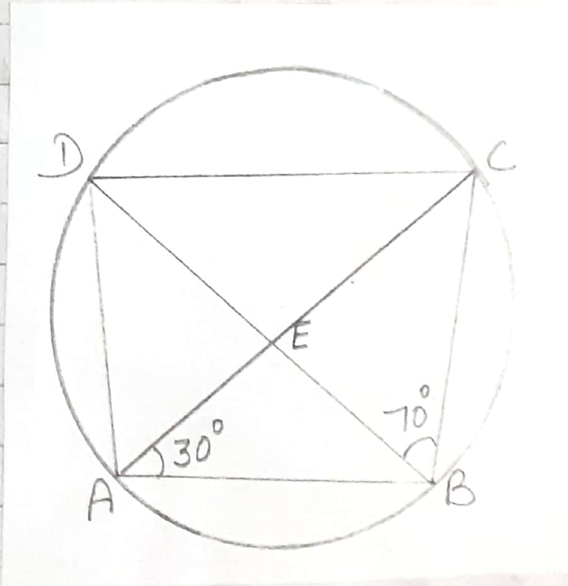
Now $\angle CDB = \angle CDE = 110°$.

Now $\angle BAC = \angle CDB$

[$\because$ Angles in the same segments are equal]

$\therefore \angle BAC = 110°$.

6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70°$, $\angle BAC$ is $30°$. find $\angle BCD$. Further if $AB = BC$, find $\angle ECD$.



Sol. $\angle DAC = \angle DBC = 70°$

[$\because$ Angles in the same segment are equal]

$\therefore \angle DAC = 70°$

Now $\angle DAB = \angle DAC + \angle BAC$

$= 70° + 30° = 100°$

In cyclic quadrilateral ABCD

$\angle DAB + \angle BCD = 180°$

$100° + \angle BCD = 180°$

$\angle BCD = 180° - 100° = 80°$

Now $AB = BC$

$\therefore$ In ΔABC given $AB = BC$, $\therefore \angle ACB = \angle BAC = 30°$

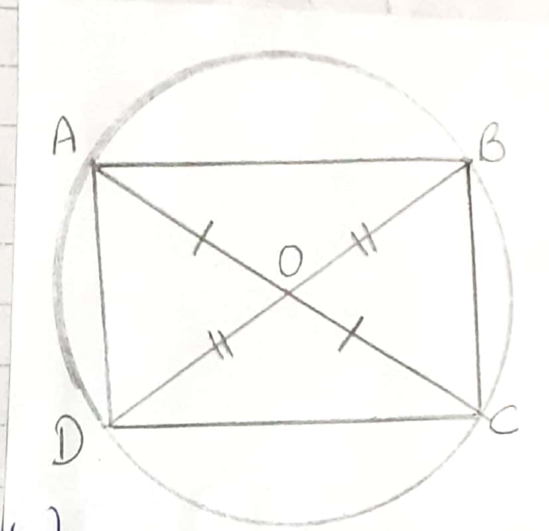
[$\because$ Angles opposite to equal side in triangle are equal]

Now $\angle BCD = \angle ECB + \angle ECD$

$80° = 30° + \angle ECD$

$\therefore \angle ECD = 80° - 30° = 50°$.

7. If diagonals of cyclic quadrilateral are diameters of the circle through the vertices of quadrilateral, Prove that it a rectangle.



Sol:- AC is diameter

$\therefore \angle B = \angle D = 90°$ —(1)

[$\because$ Angle in a semi-circle is a right angle]

IIly BD is a diameter.

$\therefore \angle A = \angle C = 90°$ —(2)

Now In ΔAOD and $\angle COB$

AO = CO (Radius of circle)

$\angle AOD = \angle COB$ (Vertically opp. angles)

DO = BO (Radius of circle)

$\therefore \Delta AOD \cong \Delta COD$ (SAS congruence Rule)

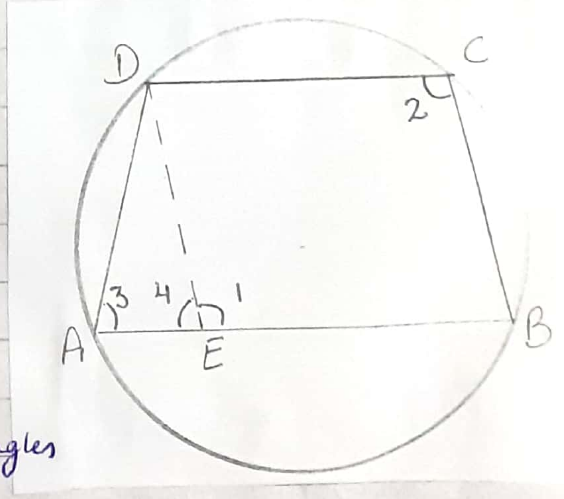
$\therefore$ AD = BC (by c.p.c.t) —(3).

Similarly AB = DC —(4).

From (1), (2), (3) and (4), In quadrilateral each angle is of 90° and opposite sides are equal.

$\therefore$ ABCD is a rectangle

8. If non-parallel sides of trapezium are equal, prove that it is cyclic.



Sol. given:- In trapezium ABCD

AB ∥ CD and AD = BC

To Prove:- ABCD is cyclic quad.

Construction:- Draw DE ∥ CB

Proof:- DE ∥ CB and EB ∥ DC

$\therefore$ EBCD is a parallelogram.

$\therefore$ DE = CB and $\angle 1 = \angle 2$ —(1).

($\because$ In parallelogram opposite angles are equal)

Now AD = BC (given and BC = DE $\Rightarrow$ AD = DE.

$\therefore \angle 4 = \angle 3$ (In ΔADE, angles opposite to equal sides) (2).

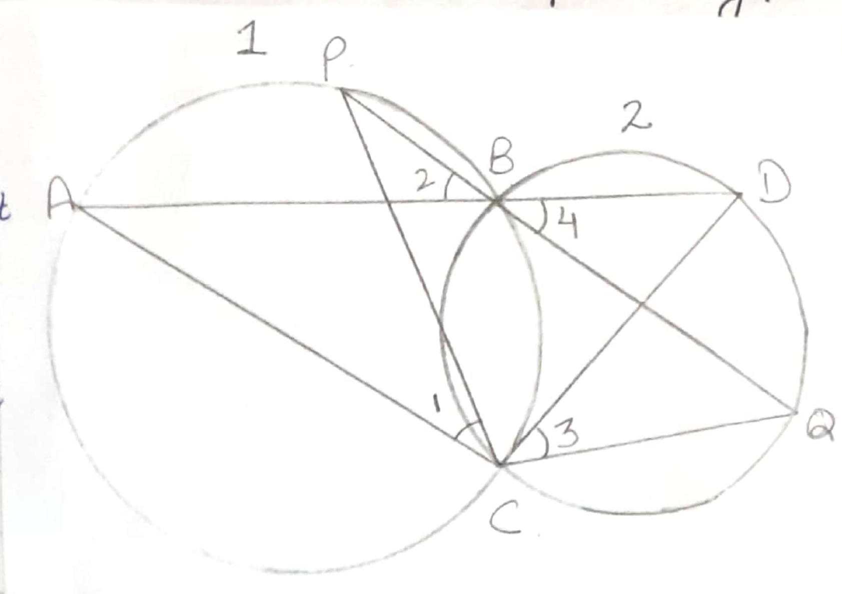
$\angle 1 + \angle 4 = 180°$ (linear pair

$\angle 2 + \angle 3 = 180°$ (from (1) & (2)).

$\angle C + \angle A = 180°$

$\therefore$ ABCD is a cyclic quadrilateral [opposite angles of cyclic quadrilateral are supplementary)

9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively. Prove that $\angle ACP = \angle QCD$



Sol:- In circle 1

$\angle 1 = \angle 2$ — (1)

($\because$ Angles on same segment of circle are equal)

IIly. $\angle 3 = \angle 4$ — (2)

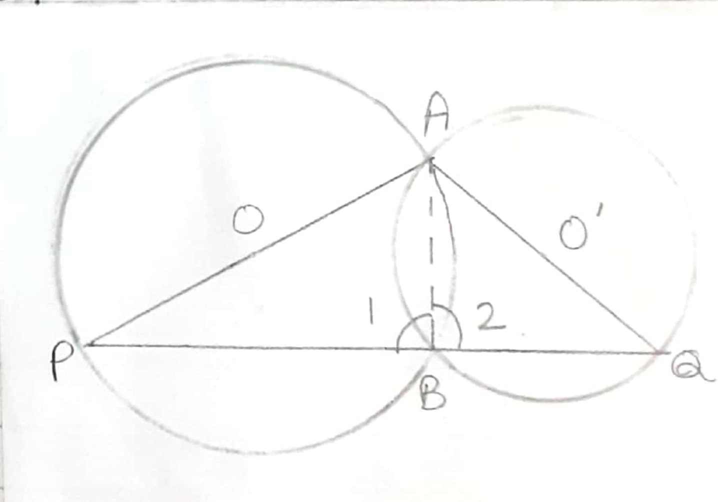
$\angle 2 = \angle 4$ (vertically opposite angles) — (3)

From (1), (2) and (3)

$\angle 1 = \angle 3$

$\Rightarrow \angle ACP = \angle QCD$

10. If circles are drawn taking two sides of a triangle as diameters, prove that point of intersection of these circles lies on third side.



Given:- Two circles intersect at A and B. AP and AQ are diameters

To Prove:- Point B, lies on third side PQ of triangle.

~~Proof~~:- Construction:- Join A & B.

Proof:- AP is a diameter

$\therefore \angle 1 = 90°$ ($\because$ Angle in a semi-circle is a right angle)

AQ is a diameter

$\therefore \angle 2 = 90°$

Now $\angle 1 + \angle 2 = 90° + 90° = 180°$

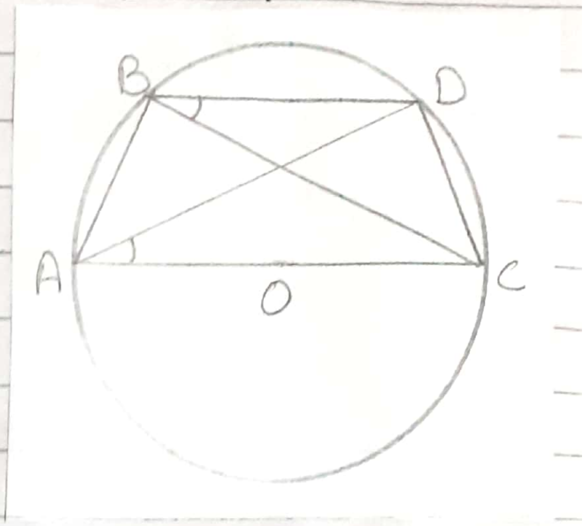
$\therefore \angle PBQ = 180°$

$\Rightarrow$ PBQ is a line segment.

$\therefore$ Point B, lies on third side PQ.

11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

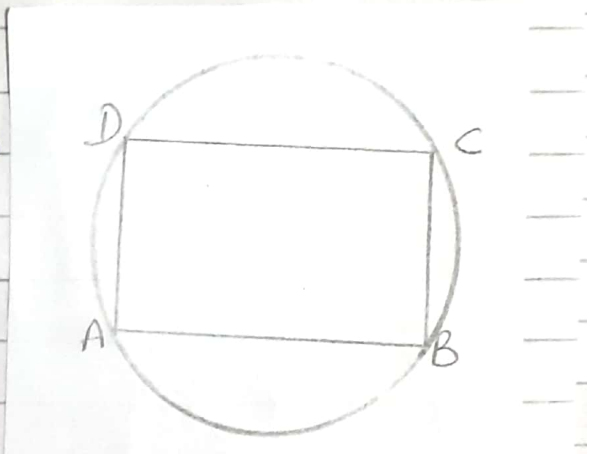
Sol:- Given AC is the common hypotenuse and $\angle ABC = \angle ADC$ (each 90°)



So, it can be said that they lie in the semi-circle.
So, triangles ABC and ADC are in the semi-circle and points A, B, C and D are concyclic.
Hence, CD is the chord of the circle with centre O
We know that the angles which are in same segment of circle are equal.
$\therefore \angle CAD = \angle CBD$.

12. Prove that a cyclic parallelogram is a rectangle.

Sol:- ABCD is a parallelogram.



$\therefore \angle B = \angle D$ — (1)
($\because$ In parallelogram opposite angles are equal)
ABCD is cyclic
$\therefore \angle B + \angle D = 180^\circ$
$\angle B + \angle B = 180^\circ$ (from (1))
$2\angle B = 180^\circ$
$\angle B = \frac{180^\circ}{2} = 90^\circ$

$\therefore \angle B = \angle D = 90^\circ$
In parallelogram ABCD $\angle B = \angle D = 90^\circ$.
$\therefore$ ABCD is a rectangle.