Permutations and Combinations

INTRODUCTION

We often come across questions such as the following:

1. In how many ways can four bottles be arranged in a row?
2. In how many ways can five students be seated at a round table?
3. In how many ways can a group of five people be selected out of a gathering of ten people?
4. In how many ways can 5 maps be selected out of 8 and displayed in a row?

Answers to these questions and many other important and more difficult ones can often be given without actually writing down all the different possibilities. In this chapter, we will study some basic principles of the art of counting without counting which will enable us to answer such questions in an elegant manner.

FACTORIAL NOTATION

The continued product of first n natural numbers is called n factorial or factorial n and is denoted by or $n!$

Thus, $\lfloor n$ or $n! = 1 \times 2 \times 3 \times 4 \ldots (n-1)n$

$= n \times (n-1) \times (n-2) \ldots 3 \times 2 \times 1$

(in reverse order)

Notes:

1. When n is a negative integer or a fraction, $n!$ is not defined. Thus, $n!$ is defined only for positive integers.
2. According to the above definition, 0! makes no sense. However, we define $0! = 1$.
3. $n! = n(n-1)!$
4. $(2n)! = 2^n \times n! \, [1 \times 3 \times 5 \times 7 \ldots (2n-1)]$.

Illustration 1 Evaluate

(i) $\dfrac{30!}{28!}$ (ii) $\dfrac{9!}{5!3!}$

(iii) $\dfrac{12!-10!}{9!}$ (iv) $\dfrac{1}{3!}+\dfrac{1}{4!}+\dfrac{1}{5!}$

Solution: (i) $\dfrac{30!}{28!} = \dfrac{30 \times 29 \times 28!}{28!}$

$= 30 \times 29 = 870$

(ii) $\dfrac{9!}{5!3!} = \dfrac{9 \times 8 \times 7 \times 6 \times 5!}{5! \times 3 \times 2} = 504$

(iii) $\dfrac{12!-10!}{9!} = \dfrac{12 \times 11 \times 10! - 10!}{9!}$

$= \dfrac{10!}{9!}[132-1]$

$= 10 \times 131$

$= 1310$

(iv) $\dfrac{1}{3!}+\dfrac{1}{4!}+\dfrac{1}{5!} = \dfrac{4 \times 5}{3! \times 4 \times 5} + \dfrac{5}{4! \times 5} + \dfrac{1}{5!}$

$= \dfrac{20}{5!} + \dfrac{5}{5!} + \dfrac{1}{5!}$

$= \dfrac{26}{5!} = \dfrac{13}{60}$

Illustration 2 Convert into factorials:

(i) 4.5.6.7.8.9.10.11. (ii) 2.4.6.8.10.

Solution: (i) $4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11$

$= \dfrac{1.2.3.4.5.6.7.8.9.10.11}{1.2.3.}$

$= \dfrac{11!}{3!}$

(ii) $2 \times 4 \times 6 \times 8 \times 10$

$= (2 \times 1) \times (2 \times 2) \times (2 \times 3) \times (2 \times 4) \times (2.5)$

$= (2 \times 2 \times 2 \times 2 \times 2) \times (1 \times 2 \times 3 \times 4 \times 5)$

$= 2^5 \times 5!$

Fundamental Principle of Counting

Multiplication Principle If an operation can be performed in 'm' different ways; following which a second operation can be performed in 'n' different ways, then the two operations in succession can be performed in $m \times n$ different ways.

Illustration 3 How many numbers of two digits can be formed out of the digits 1, 2, 3, 4, no digit being repeated?

Solution: The first digit can be any one of the four digits 1, 2, 3, 4, that is the first digit can be chosen in four ways. Having chosen the first digit, we are left with three digits from which the second digit can be chosen. Therefore, the possible ways of choosing the two digits are 12 ways of choosing both the digits. Thus, 12 numbers can be formed.

Since the first digit can be chosen in four ways and for each choice of the first digit there are three ways of choosing the second digit, therefore, there are 4×3

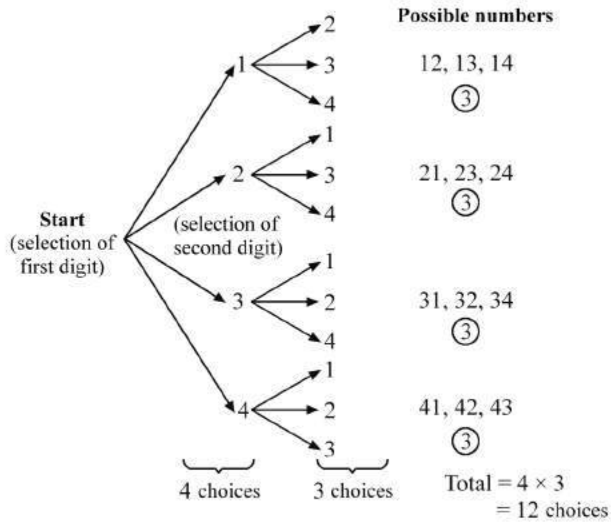


Illustration 4 Anu wishes to buy a birthday card for her brother Manu and send it by post. Five different types of cards are available at the card shop, and four different types of postage stamps are available at the post office. In how many ways can she choose the card and the stamp?

Solution: She can choose the card in five ways. For each choice of the card she has four choices for the stamp. Therefore, there are 5×4 ways, that is, 20 ways of choosing the card and the stamp.

Illustration 5 Mohan wishes to go from Agra to Chennai by train and return from Chennai to Delhi by air. There are six different trains from Agra to Chennai and five different flights from Chennai to Agra. In how many ways can he perform the journey?

Solution: Since he can choose any one of the six trains for going to Chennai, and for each such choice he has five choices for returning to Agra, he can perform the journey in 6×5 ways, that is, 30 ways.

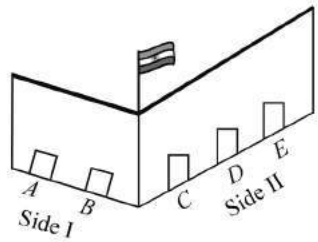
Addition Principle

If an operation can be performed in 'm' different ways and another operation, which is independent of the first operation, can be performed in 'n' different ways, then either of the two operations can be performed in $(m + n)$ ways.

Note:

The above two principles can be extended for any finite number of operations.

Illustration 6 Suppose there are 5 gates to a stadium, 2 on one side and 3 on the other. Sohan has to go out of the stadium. He can go out from any one of the 5 gates. Thus, the number of ways in which he can go out is 5. Hence, the work of going out through the gates on one side will be done in 2 ways and the work of going out through the gates on other side will be done in 3 ways. The work of going out will be done when Sohan goes out from side I or side II. Thus the work of going out can be done in $(2 + 3) = 5$ ways.



Note:

Addition theorem of counting is also true for more than two opeartions.

Permutation

Each of the different arrangements which can be made by taking some or all of given number of things or objects at a time is called a *permutation*.

Note:

Permutation of things means arrangement of things. The word arrangement is used if order of things is taken into account. Thus, if order of different things changes, then their arrangement also changes.

Notation

Let r and n be positive integers such that $1 \le r \le n$. Then, the number of permutations of n different things, taken r at a time, is denoted by the symbol ${}^nP_r$ or P, (n, r).

SOME BASIC RESULTS

1. ^nP_r or, $P(n, r) = \dfrac{n!}{(n-r)!} = n(n-1)(n-2)$ $\ldots[n-(r+1)], 0 \le r \le n.$

Illustration 7 Evaluate the following:

(i) $P(6, 4)$, (ii) $P(15, 3)$, (iii) $P(30, 2)$.

Solution: (i) We have

$$P(6, 4) = \frac{6!}{(6-4)!} = \frac{6.5.4.3.2.1}{2.1} = 360$$

(ii) We have

$$P(15, 3) = \frac{15!}{(15-3)!} = \frac{15!}{12!}$$

$$= \frac{(15\cdot14\cdot13)(12!)}{12!} = 2730$$

(iii) $P(30, 2) = \dfrac{30!}{(30-2)!} = \dfrac{30!}{28!}$

$$= \frac{(30.29)(28!)}{28!}$$

$= 870$

2. The number of permutations of n things, taken all at a time, out of which p are alike and are of one type, q are alike and are of second type and rest are all different $= \dfrac{n!}{p!\,q!}$.

Illustration 8 There are 5 red, 4 white and 3 blue marbles in a bag. They are drawn one by one and arranged in a row. Assuming that all the 12 marbles are drawn, determine the number of different arrangements.

Solution: Here, $n = 12, p_1 = 5, p_2 = 4$ and, $p_3 = 3$

$\therefore$ The required number of different arrangements

$$= \frac{n!}{p_1!\,p_2!\,p_3!} = \frac{12!}{5!4!3!}$$

$$= \frac{12\times11\times10\times9\times8\times7\times6}{4\times3\times2\times1\times3\times2\times1} = \frac{11\times10\times9\times8\times7}{2}$$

$= 990 \times 4 \times 7 = 27720$

3. The number of permutations of n different things taken r at a time when each thing may be repeated any number of times is n^r.

Illustration 9 In how many ways can 5 apples be distributed among 4 boys, there being no restriction to the number of apples each boy may get?

Solution: The required number of ways $= 4^5$

4. Permutations Under Restrictions

(*a*) Number of permutations of n different things, taken r at a time, when a particular thing is to be always included in each arrangement, is

$$r \times {}^{n-1}P_{r-1}.$$

(*b*) Number of permutations of n different things, taken r at a time, when s particular things are to be always included in each arrangement, is

$$s!\,(r-(s-1)) \times {}^{n-s}P_{r-s}.$$

(*c*) Number of permutations of n different things, taken r at a time, when a particular thing is never taken in each arrangement, is

$$^{n-1}P_r.$$

(*d*) Number of permutations of n different things, taken all at a time, when m specified things always come together, is $m! \times (n-m+1)!$

(*e*) Number of permutations of n different things, taken all at a time, when m specified things never come together, is $n! - m! \times (n-m+1)!$.

5. Circular Permutations

(*a*) Number of circular arrangements (permutations) of n different things $= (n-1)!$.

Illustration 10 In how many ways can eight people be seated at a round table?

Solution: Required number of ways $= (8-1)!$

$= 7! = 5040$

(*b*) Number of circular arrangements (permutations) of n different things when clockwise and anticlockwise arrangements are not different, that is, when observation can be made from both sides $= \dfrac{1}{2}(n-1)!$.

Illustration 11 Find the number of ways in which n different beads can be arranged to form a necklace.

Solution: Required number of arrangements

$$= \frac{1}{2}(5-1)! = \frac{1}{2} \times 4! = 12$$

Combination

Each of the different groups or selections which can be made by taking some or all of a number of things (irrespective of order) is called a *combination*.

Note:

Combination of things means selection of things. Obviously, in selection of things order of things has no importance. Thus, with the change of order of things selection of things does not change.

Notations The number of combinations of n different things taken r at a time is denoted by ${}^nC_r$ or, $C(n, r)$.

Thus, $${}^nC_r = \frac{n!}{r!(n-r)!} \quad (0 \le r \le n)$$

$$= \frac{{}^nP_r}{r!}$$

$$= \frac{n(n-1)(n-2)\dots(n-r+1)}{r(r-1)(r-2)\dots 3\cdot 2\cdot 1}$$

If $r > n$, then, ${}^nC_r = 0$.

Illustration 12 Evaluate:

(i) ${}^{11}C_3$ (ii) ${}^{10}C_8$ (iii) ${}^{100}C_{98}$

Solution: (i) $${}^{11}C_3 = \frac{11!}{3!(11-3)!} = \frac{11!}{3!8!}$$

$$= \frac{11 \times 10 \times 9 \times 8!}{3 \times 2 \times 8!} = 165$$

(ii) $${}^{10}C_8 = \frac{10!}{8!2!} = \frac{10 \times 9 \times 8!}{8! \times 2} = 45$$

(iii) $${}^{100}C_{98} = \frac{100!}{98!2!}$$

$$= \frac{100 \times 99 \times 98!}{98! \times 2} = 4950$$

Key Points to Remember

1. ${}^nC_r = {}^nC_{n-r}$
2. ${}^nC_0 = {}^nC_n = 1, {}^nC_1 = n$
3. If ${}^nC_x = {}^nC_y$ then either $x = y$ or, $y = n - x$ that is, $x + y = n$
4. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
5. $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$.
6. If n is even then the greatest value of ${}^nC_r$ is ${}^nC_{n/2}$.
7. If n is odd then the greatest value of ${}^nC_r$ is

$${}^nC_{\frac{n+1}{2}} \text{ or, } {}^nC_{\frac{n-1}{2}}.$$

8. $${}^nC_r = \frac{r \text{ decreasing numbers starting with } n}{r \text{ increasing numbers starting with } 1}$$

$$= \frac{n(n-1)(n-2)\dots(n-r+1)}{1 \times 2 \times 3 \dots r}.$$

9. ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$.
10. ${}^nC_0 + {}^nC_2 + {}^nC_4 + \dots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots = 2^{n-1}$.
11. Number of combinations of n different things taken r at a time
    (*a*) when p particular things are always included $= {}^{n-p}C_{r-p}$
    (*b*) when p particular things are never included $= {}^{n-p}C_r$
    (*c*) when p particular things are not together in any selection $= {}^nC_r - {}^{n-p}C_{r-p}$.

Illustration 13 In how many ways can 5 members forming a committee out of 10 be selected so that

(i) two particular members must be included.

(ii) two particular members must not be included.

Solution: (i) When two particular members are included, then, we have to select $5 - 2 = 3$ members out of $10 - 2 = 8$

∴ The required no. of ways

$$= C(8, 3) = \frac{8!}{3!5!} = \frac{8 \times 7 \times 6}{6} = 56$$

(ii) When 2 particular members are not included, then, we have to select 5 members out of $10 - 2 = 8$

∴ The required no. of ways

$$= C(8, 5) = \frac{8!}{5!3!}$$

$$= \frac{8 \times 7 \times 6}{6} = 56$$

12. (*a*) Number of selections of r consecutive things out of n things in a row $= n - r + 1$.

(*b*) Number of selections of r consecutive things out of n things along a circle

$$\begin{cases} n, \text{ when } r < n \\ 1, \text{ when } r = n \end{cases}.$$

13. (*a*) Number of selections of zero or more things out of n different things

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$$

(*b*) Number of combinations of n different things selecting at least one of them is

$${}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n - 1.$$

(*c*) Number of selections of r things $(r \le n)$ out of n identical things is 1.

(*d*) Number of selections of zero or more things out of n identical things $= n + 1$.

(*e*) Number of selections of one or more things out of n identical things $= n$.

(*f*) If out of $(p + q + r + t)$ things, p are alike of one kind, q are alike of second kind, r are alike of third kind and t are different, then the total number of selections is

$(p + 1)(q + 1)(r + 1)2^t - 1$.

(g) The number of ways of selecting some or all out of $p + q + r$ items where p are alike of one kind, q are alike of second kind and rest are alike of third kind is $[(p + 1)(q + 1)(r + 1)] - 1$.

14. (*a*) Number of ways of dividing $m + n$ different things in two groups containing m and n things, respectively $(m \ne n)$:

$${}^{m+n}C_m = \frac{(m+n)!}{m!\,n!}.$$

(*b*) Number of ways of dividing $m + n + p$ different things in three groups containing m, n and p things, respectively $(m \ne n \ne p)$:

$$\frac{(m+n+p)!}{m!\,n!\,p!}.$$

Some Useful Short-cut Methods

1. The number of triangles which can be formed by joining the angular points of a polygon of n sides as vertices are $\frac{n(n-1)(n-2)}{6}$.

Illustration 14 Find the number of triangles formed by joining the vertices of an octagon.

Solution: The required number of triangles

$$= \frac{n(n-1)(n-2)}{6}$$

$$= \frac{8(8-1)(8-2)}{6} = \frac{8 \times 7 \times 6}{6} = 56$$

2. The number of diagonals which can be formed by joining the vertices of a polygon of n sides are $\frac{n(n-3)}{2}$.

Illustration 15 How many diagonals are there in a decagon?

Solution: The required number of diagonals

$$= \frac{n(n-3)}{2} = \frac{10(10-3)}{2} = \frac{10 \times 7}{2} = 35$$

3. If there are 'm' horizontal lines and 'n' vertical lines then the number of different rectangles formed are given by $({}^mC_2 \times {}^nC_2)$.

Illustration 16 In a chess board, there are 9 vertical and 9 horizontal lines. Find the number of rectangles formed in the chess board.

Solution: The require number of rectangles

$$= {}^9C_2 \times {}^9C_2 = 36 \times 36 = 1296$$

4. These are 'n' points in a plane out of which 'm' points are collinear. The number of triangles formed by the points as vertices are given by ${}^nC_3 - {}^mC_3$.

Illustration 17 There are 14 points in a plane out of which 4 are collinear. Fine the number of triangles formed by the points as vertices.

Solution: The required number of triangles

$$= {}^{14}C_3 - {}^4C_3 = 364 - 4 = 360$$

5. There are 'n' points in a plane out of which 'm' points are collinear. The number of straight lines formed by joining them are given by

$$({}^nC_2 - {}^mC_2 + 1).$$

Illustration 18 There are 10 points in a plane out of which 5 are collinear. Find the number of straight lines formed by joining them.

Solution: The required number of straight lines

$$= {}^nC_2 - {}^mC_2 + 1$$
$$= {}^{10}C_2 - {}^5C_2 + 1 = 45 - 10 + 1 = 36$$

> **6.** If there are 'n' points in a plane and no three points are collinear, then the number of triangles formed with 'n' points are given by $\dfrac{n(n-1)(n-2)}{6}$.

Illustration 19 Find the number of triangles that can be formed with 14 points in a plane of which no three points are collinear.

Solution: The required number of triangles

$$= \frac{n(n-1)(n-2)}{6} = \frac{14\times13\times12}{6} = 364$$

> **7.** The number of quadrilaterals that can be formed by joining the vertices of a polygon of n sides are given by $\dfrac{n(n-1)(n-2)(n-3)}{24}$, where $n > 3$.

Illustration 20 Find the number of quadrilaterals that can be formed by joining the vertices of a septagon.

Solution: The required number of quadrilaterals

$$= \frac{n(n-1)(n-2)(n-3)}{24}$$
$$= \frac{7(7-1)(7-2)(7-3)}{24}$$
$$= \frac{7\times6\times5\times4}{24} = 35$$

> **8.** There are n points in a plane and no points are collinear, then the number of straight lines that can be drawn using these 'n' points are given by $\dfrac{n(n-1)}{2}$.

Illustration 21 How many straight lines can be drawn with 18 points on a plane of which no points are collinear?

Solution: The required number of straight lines

$$= \frac{18(18-1)}{2} = \frac{18\times17}{2} = \frac{n(n-2)}{2} = 153$$

> **9.** In a party, every person shakes hands with every other person. If there was a total of H handshakes in the party, then the number of persons 'n' who were present in the party can be calculated from the equation:
>
> $$\frac{n(n-1)}{2} = H.$$

Illustration 22 In a party every person shakes hands with every other person. If there was a total of 105 handshakes in the party, find the number of persons who were present in the party.

Solution: Let 'n' be the number of persons present in the party.

We have the equation

$$\frac{n(n-1)}{2} = H$$

$$\Rightarrow \quad \frac{n(n-1)}{2} = 105$$

$$\Rightarrow \quad n(n-1) = 15 \times (15-1)$$

$$\Rightarrow \quad n = 15$$

Practice Exercises

Difficulty Level-1 (Based on Memory)

1. There are 6 boxes numbered 1, 2, ... 6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutively numbered. The total number of ways in which this can be done is:

(*a*) 5 (*b*) 21
(*c*) 33 (*d*) 60

[Based on CAT, 2003]

2. A graph may be defined as a set of points connected by lines called edges. Every edge connects a pair of points. Thus, a triangle is a graph with 3 edges and 3 points. The degree of a point is the number of edges connected to it. For example, a triangle is a graph with three points of degree 2 each. Consider a graph with 12 points. It is possible to reach any point from any other point through a sequence of edges. The number of edges, e, in the graph must satisfy the condition:

(*a*) $11 \le e \le 66$ (*b*) $10 \le e \le 66$

(*c*) $11 \le e \le 65$ (*d*) $0 \le e \le 11$

[Based on CAT, 2003]

3. The letters of the word PROMISE are arranged so that no two of the vowels should come together. Find the total number of arrangements.

(*a*) 49 (*b*) 1440

(*c*) 7 (*d*) 1898

[Based on MAT, 2003]

4. In an examination paper there are two groups, each containing 4 questions. A candidate is required to attempt 5 questions but not more than 3 questions from any group. In how many ways can 5 questions be selected?

(*a*) 24 (*b*) 48

(*c*) 96 (*d*) None of these

[Based on MAT, 2002]

5. A box contains 10 balls out of which 3 are red and the rest are blue. In how many ways can a random sample of 6 balls be drawn from the bag so that at the most 2 red balls are included in the sample and no sample has all the 6 balls of the same colour?

(*a*) 105 (*b*) 168

(*c*) 189 (*d*) 120

[Based on MAT, 2002]

6. A cricket team of 11 players is to be formed from 20 players including 6 bowlers and 3 wicketkeepers. The number of ways in which a team can be formed having exactly 4 bowlers and 2 wicketkeepers is:

(*a*) 20790 (*b*) 6930

(*c*) 10790 (*d*) 360

[Based on MAT, 2002]

7. Three boys and three girls are to be seated around a table in a circle. Among them the boy *X* does not want any girl neighbour and the girl *Y* does not want any boy neighbour. How many such arrangements are possible?

(*a*) 5 (*b*) 6

(*c*) 4 (*d*) 2

[Based on MAT, 2002]

8. In a hockey championship, three were 153 matches played. Every two teams played one match with each other. The number of teams participating in the championship is:

(*a*) 18 (*b*) 19

(*c*) 17 (*d*) 16

[Based on MAT, 2002]

9. Seven points lie on a circle. How many chords can be drawn by joining these points?

(*a*) 22 (*b*) 21

(*c*) 23 (*d*) 24

[Based on MAT, 2005]

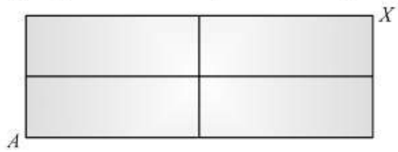
10. The value of ${}^{10}C_4 + {}^{10}C_5$ is:

(*a*) 462 (*b*) 466

(*c*) 469 (*d*) 465

[Based on MAT, 2005]

11. In the figure below, how many paths are there from *A* to *X* if the only ways to move are up and to the right?



(*a*) 4 (*b*) 5

(*c*) 6 (*d*) 9

[Based on MAT (Dec), 2006]

12. All letters of the word 'AGAIN' are permuted in all possible ways and the words so formed (with or without meaning) are written as in a dictionary. Then, the 50th word is:

(*a*) NAAGI (*b*) NAAIG

(*c*) IAANG (*d*) INAGA

[Based on MAT (Feb), 2011]

13. An examination paper contains 8 questions of which 4 have 3 possible answers each, 3 have 2 possible answers each and the remaining one question has 5 possible answers. The total number of possible answers to all the questions is:

(*a*) 1278 (*b*) 1728

(*c*) 1306 (*d*) 3240

[Based on MAT (Feb), 2011]

14. There are ten steps in a staircase and a person has to take those steps. At every step, the person has got a choice of taking one step or two steps or three steps. The number of ways in which a person can take those steps is:

(*a*) 3^{10} (*b*) 3^9

(*c*) 3^8 (*d*) None of these

[Based on MAT (Feb), 2011]

15. What is the number of six-digit telephone numbers in a city if atleast one of their digits is repeated and zero cannot initiate, the number?

(*a*) 763920 (*b*) 453621

(*c*) 145698 (*d*) 781243

[Based on MAT (Dec), 2010]

16. Eight balls of different colours are to be placed in three boxes of different sizes. Each box can hold all the eight balls. What is the number of ways the balls can be placed so that no box remains empty?

(*a*) 7968 (*b*) 6796

(*c*) 3652 (*d*) 846720

[Based on MAT (Dec), 2010]

17. There are 5 different green dyes, 4 different blue dyes and 3 different red dyes. How many combinations of dyes can be chosen taking atleast one green and one blue dye?

(*a*) 3720 (*b*) 1253
(*c*) 2546 (*d*) 2373

[Based on MAT (Dec), 2010]

18. What is the number of ways in which 5 identical balls can be distributed among 10 identical boxes, if not more than one ball can go into a box?

(*a*) 230 (*b*) 204
(*c*) 250 (*d*) None of these

[Based on MAT (Dec), 2010]

19. Three boys and three girls are to be seated around a table in a circle. Among the boys, *X* does not want any girl adjacent to him and the girl *Y* does not want any boy adjacent to her. How many such arrangements are possible?

(*a*) 8 (*b*) 4
(*c*) 6 (*d*) 2

[Based on MAT (Sept), 2010 (Dec), 2009]

20. In a monthly test, the teacher decides that there will be three questions; one each from Exercises 7, 8 and 9 of the text book. There are 12 questions in Exercise 7, 18 in Exercise 8 and 9 in Exercise 9. In how many ways can the three questions be selected?

(*a*) 1944 (*b*) 2094
(*c*) 1894 (*d*) 2194

[Based on MAT (Sept), 2010]

21. If there are 12 persons in a party and each of them shakes hands with each other, how many handshakes happen in the party?

(*a*) 77 (*b*) 66
(*c*) 44 (*d*) 55

[Based on MAT (Sept), 2010]

22. There are 6 boxes numbered 1, 2, ..., 6. Each box is to be filled up either with a red or a green ball in such a way that atleast 1 box contains a green ball and the boxes containing green balls are consecutively numbered. The total number of ways in which this can be done is:

(*a*) 33 (*b*) 21
(*c*) 5 (*d*) 60

[Based on MAT (May), 2010]

23. A man has 5 friends and his wife has 4 friends. They want to invite either of their friends, one or more to a party. In how many ways can they do so?

(*a*) 9 (*b*) 18
(*c*) 31 (*d*) 46

[Based on MAT (Feb), 2010]

24. Chanda Negi got a 4-digit pass code (which is formed out of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9) of her ATM card from SBI Bank. But after the 50th day, she lost the pass code and also forgot the number. How many maximum number of trials she many have to take to get the right number? (0 can be the beginning of the code number.)

(*a*) 10^4 (*b*) 10!
(*c*) 9^4 (*d*) 9!

[Based on MAT (Sept), 2009]

25. A number lock on a suitcase has 3 wheels each labelled with 10 digits from 0 to 9. If opening of the lock is a particular sequence of three digits with no repeats, how many such sequences will be possible?

(*a*) 720 (*b*) 760
(*c*) 680 (*d*) 780

[Based on MAT (Sept), 2009]

26. There are 6 multiple choice questions in an examination. How many sequences of answers are possible, if the first three questions have 4 choices each and the next three have 5 each?

(*a*) 4000 (*b*) 5000
(*c*) 6000 (*d*) 8000

[Based on MAT (May), 2009]

27. The average age of 3 children in a family is 20% of the average age of the father and the eldest child. The total age of the mother and the youngest child is 39 years. If the father's age is 26 years, what is the age of the second child?

(*a*) 20 years (*b*) 18 years
(*c*) 15 years (*d*) Cannot be determined

[Based on MAT (May), 2009]

28. The number of ways in which a committee of 3 ladies and 4 gentlemen can be appointed out of 8 ladies and 7 gentlemen, if Mrs *X* refuses to serve in a committee of which Mr *Y* is a member, is

(*a*) 1540 (*b*) 1960
(*c*) 3240 (*d*) None of these

[Based on MAT (Feb), 2009]

29. A student is to answer 10 out of 13 questions in an examination such that he must choose atleast four from the first five questions. The number of choices available to him is:

(*a*) 140 (*b*) 196
(*c*) 280 (*d*) 346

[Based on MAT (May), 2008]

30. The number of ways in which a team of eleven players can be selected from 22 players including 2 of them and excluding 4 of them is

(*a*) ${}^{16}C_{11}$ (*b*) ${}^{16}C_5$
(*c*) ${}^{16}C_9$ (*d*) ${}^{20}C_9$

[Based on MAT (May), 2008]

31. An examination paper contains 8 questions of which 4 have 3 possible answers each, 3 have 2 possible answers each and the remaining 1 question has 5 possible answers. The total number of possible answers to all the questions is:

(*a*) 2880 (*b*) 78
(*c*) 94 (*d*) 3240

[Based on MAT (May), 2008]

32. The number of ways in which a committee of 5 can be chosen from 10 candidates so as to exclude the youngest, if it includes the oldest is:

(*a*) 178 (*b*) 196

(*c*) 292 (*d*) None of these

[Based on MAT (Feb), 2008]

33. 11 persons decide to spend an afternoon in two groups. A group of them decides to go to a theatre and the remaining decide to play tennis. In how many ways can the group for tennis be formed, if there must be atleast four persons in each group?

(*a*) 1682 (*b*) 1584

(*c*) 1884 (*d*) 1782

[Based on MAT (Dec), 2007]

34. A committee of 5 is to be formed from a group of 12 students consisting of 8 boys and 4 girls. In how many ways can the committee be formed if it consists of exactly 3 boys and 2 girls?

(*a*) 436 (*b*) 336

(*c*) 548 (*d*) 356

[Based on MAT (Dec), 2007]

35. In how many ways can a student choose a programme of 5 courses, if 9 courses are available and 2 courses are compulsory for every student?

(*a*) 45 ways (*b*) 55 ways

(*c*) 35 ways (*d*) 65 ways

[Based on MAT (Dec), 2007]

36. A family consists of a grandfather, 5 sons and daughters and 8 grandchildren. They are to be seated in a row for dinner. The grandchildren wish to occupy the 4 seats at each end and the grandfather refuses to have a grandchild on either side of him. The number of ways in which the family can be made to sit is:

(*a*) 11360 (*b*) 11520

(*c*) 21530 (*d*) None of these

[Based on MAT (Dec), 2006]

37. A father with 8 children takes 3 children at a time to the zoological garden, as often as he can without taking the same 3 children together more than once. Then

(*a*) number of times he will go to the zoological garden is 56

(*b*) number of times each child will go to the zoological garden is 21

(*c*) number of times a particular child will not go to the zoological garden is 35

(*d*) All of the above

[Based on MAT (Dec), 2006]

38. There are 4 candidates for the post of a lecturer in Mathematics and one is to be selected by votes of 5 men. The number of ways in which the votes can be given is:

(*a*) 1048 (*b*) 1072

(*c*) 1024 (*d*) None of these

[Based on MAT (Feb), 2006]

39. The number of ways in which 6 men and 5 women can dine at a round table, if no two women are to sit together is given by:

(*a*) $6! \times 5!$ (*b*) $5! \times 4!$

(*c*) 30 (*d*) $7! \times 5!$

[Based on MAT (Feb), 2006]

40. A number of points are marked on a plane and are connected pairwise by a line segment. If the total number of line segments is 10, how many points are marked on the plane?

(*a*) 14 (*b*) 10

(*c*) 5 (*d*) 9

[Based on MAT, 1999]

41. There are 6 parallel vertical lines and 7 parallel horizontal lines. These two groups of parallel lines intersect each other. How many parallelograms will be formed?

(*a*) 294 (*b*) 42

(*c*) 315 (*d*) None of these

[Based on MAT, 1999]

42. How many numbers between 1000 and 10,000 contain the digits 1, 3, 5 and 7?

(*a*) 16 (*b*) 24

(*c*) 12 (*d*) None of these

[Based on MAT, 1999]

43. From among 36 teachers in a school, one principal and one vice-principal are to be appointed. In how many ways can this be done?

(*a*) 1260 (*b*) 1250

(*c*) 1240 (*d*) 1800

[Based on MAT, 1999]

44. A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Chemistry part II unless Chemistry part I is also borrowed. In how many ways can he choose the three books to be borrowed?

(*a*) 56 (*b*) 27

(*c*) 26 (*d*) 41

[Based on MAT, 1999]

45. The number of 3-digit numbers exactly divisible by 5 is:

(*a*) 181 (*b*) 180

(*c*) 179 (*d*) 199

[Based on MAT, 1999]

46. In how many ways can six different rings be worn on four fingers of one hand?

(*a*) 10 (*b*) 12

(*c*) 15 (*d*) 16

[Based on MAT, 2000]

47. How many arrangements can be formed out of the letters of the word EXAMINATION so that vowels always occupy odd places?

(*a*) 72000 (*b*) 86400

(*c*) 10800 (*d*) 64000

[Based on SNAP, 2007]

48. There are 10 stations on a railway line. The number of different journey tickets that are required by the authorities is:

(*a*) 101 (*b*) 90

(*c*) 81 (*d*) 10

[Based on SNAP, 2008]

49. In how many ways can four letters of the word 'SERIES' be arranged?

(*a*) 24 (*b*) 42

(*c*) 84 (*d*) 102

[Based on IIFT, 2010]

50. Amit has 11 friends: 7 boys and 4 girls. In how many ways, can Amit invite them, if there have to be exactly 4 boys in the invitees?

(*a*) 560 (*b*) 450

(*c*) 650 (*d*) 820

[Based on JMET, 2011]

Directions (Q. Nos. 51 to 52) *Read the following information and answer the questions.*

Out of 6 ruling and 5 opposition party members, 4 are to be selected for a delegation.

51. In how many ways can this be done so as to include exactly one ruling party member?

(*a*) 50 (*b*) 80

(*c*) 45 (*d*) 60

[Based on JMET, 2011]

52. In how many ways can this be done so as to include at least one opposition member?

(*a*) 300 (*b*) 315

(*c*) 415 (*d*) 410

[Based on JMET, 2011]

Directions (Q. 53–54): *Answer the questions based on the following information.*

On a shelf there are 3 books on Psychology, 3 books on Management and 4 books on Economics.

53. In how many ways can the books be arranged, if the books on only Management are to be arranged together?

(*a*) 62540 (*b*) 320

(*c*) 40320 (*d*) None of these

[Based on IRMA, 2008]

54. In how many ways can all the books be arranged at random?

(*a*) 3628800 (*b*) 41520

(*c*) 30020 (*d*) 4164840

[Based on IRMA, 2008]

55. How many different ways can 2 students be seated in a row of 4 desks, so that there is always at least one empty desk between the students?

(*a*) 4 (*b*) 3

(*c*) 2 (*d*) 6

[Based on NMAT, 2005]

56. A cultural committee of 6 is to be formed from 7 men and 4 women. In how many ways the committee can be formed with at least 2 women in the committee?

(*a*) 210 (*b*) 140 (*c*) 300 (*d*) 371

[Based on ATMA, 2006]

57. The highest power of 10 that will divide 100! is:

(*a*) 2 (*b*) 24 (*c*) 4 (*d*) 25

[Based on ATMA, 2008]

58. The total number of possible proper three-digit integers that can be formed using 0, 1, 3, 4 and 5 without repetition such that they are divisible by 5 are:

(*a*) 20 (*b*) 21

(*c*) 22 (*d*) 24

[Based on JMET, 2009]

59. There are 6 equally spaced points *A*, *B*, *C*, *D*, *E* and *F* marked on a circle with radius *R*. How many convex pentagons of distinctly different areas can be drawn using these points as vertices?

(*a*) ${}^6P_5$ (*b*) 6

(*c*) 55 (*d*) None of these

60. Find the number of ways in which ten different flowers can be strung together to make a garland in such a way that three particular flowers are always together?

(*a*) 30240 (*b*) 30420

(*c*) 23400 (*d*) None of these

[Based on MAT, 2012]

61. A committee of 3 experts is to be selected our of a panel of 7 persons. Three of them are engineers, three are managers and one is both engineer and manager. In how many ways can the committee be selected, if it must have atleast one engineer and one manager?

(*a*) 33 (*b*) 22 (*c*) 11 (*d*) 66

[Based on MAT, 2012]

62. A cricket team of 11 players is to be formed from a pool of 16 players that includes 4 bowlers and 2 wicket keepers. In how many different ways can a team be formed, so that the team has atleast 3 bowlers and one wicket keeper?

(*a*) 2472 (*b*) 2274

(*c*) 2427 (*d*) 1236

[Based on MAT, 2012]

63. A student is to answer 10 out of 13 questions in an examination such that he must choose atleast 4 from the first five questions. The number of choices available to him is:

(*a*) 346 (*b*) 140

(*c*) 196 (*d*) 280

[Based on MAT, 2013]

64. In how many ways can 10 boys $a_{1,}$ a_2, $a_{3,\ldots}$ a_{10} can be seated in a row such that a_2 always sits before a_5 and a_5 always sits before a_7?

(*a*) $\frac{9!}{3}$ (*b*) 10_{p_3}

(*c*) ${}^{10}C_3 \times 90$ (*d*) None of these

[Based on MAT, 2013]

65. In a tennis tournament, eight players must be divided into four teams of two players each. The number of different ways in which this can be done is:

(*a*) 120 (*b*) 35

(*c*) 30 (*d*) 105

[Based on MAT, 2013]

66. Two circles touch each other internally. The sum of their areas is 116π cm^2 and distance between their centres is 6 cm. Find the radii of the circles.

(*a*) 10 cm, 4 cm

(*b*) 11 cm, 4 cm

(*c*) 9 cm, 5 cm

(*d*) 10 cm, 5 cm

[Based on MAT, 2014]

67. There is a number lock with four rings. How many attempts at the maximum would have to be made before getting the right number?

(*a*) 10^4 (*b*) 255

(*c*) $10^4 - 1$ (*d*) 256

[Based on MAT, 2014]

68. There are 10 persons P_1, P_2,…, P_9, P_{10}. Out of these 10 persons, 5 persons are to be arranged in a line such that in each arrangement P_1 must occur whereas P_4 and P_5 do not occur. The number of such possible arrangements are:

(*a*) ${}^7C_4 \times 5!$

(*b*) ${}^9C_5 \times 5!$

(*c*) ${}^8C_5 \times 5!$

(*d*) ${}^9C_5 \times 4!$

[Based on MAT, 2014]

69. There are five boys and three girls who are sitting together to discuss a management problem at a round table. In how many ways can they sit around the table so that no two girls are together?

(*a*) 1200 (*b*) 1400

(*c*) 1420 (*d*) 1440

[Based on SNAP, 2013]

70. The number of ways in which a committee of 3 ladies and 4 gentlemen can be appointed from a meeting consisting of 8 ladies and 7 gentlemen, if Mrs. X refuses to serve in a committee if Mr. Y is its member, is:

(*a*) 1960

(*b*) 3240

(*c*) 1540

(*d*) None of these

[Based on SNAP, 2013]

71. A family consists of a grandfather, 5 sons and daughters and 8 grandchildren. They are to be seated in a row for dinner. The grandfather wishes to occupy the 4 seats at each end and the grandfather refuses to have a grandchild on either side of him. The number of ways in which the family can be made to sit is:

(*a*) 1136 (*b*) 11520

(*c*) 21530 (*d*) None of these

[Based on SNAP, 2013]

72. The number of ways in which a committee of 3 ladies and 4 gentlemen can be appointed out of 8 ladies and 7 gentlemen, if Mrs. X refuses to serve in a committee of which Mr. Y is a member, is:

(*a*) 1,540 (*b*) 1,960

(*c*) 3,240 (*d*) None of these

[Based on SNAP 2012]

73. In a football championship, 153 matches were played. Every two teams played one match with each other. The number of teams, participating in the championship were:

(*a*) 18 (*b*) 14

(*c*) 16 (*d*) 22

(Based on MAT 2011)

74. In a certain city, all telephone numbers have six digits, the first two digits always being 41 or 42 or 46 or 62 or 64. The number of telephone numbers having all six digits distinct is:

(*a*) 9200 (*b*) 7200

(*c*) 8400 (*d*) 1200

(Based on MAT 2011)

75. There are six teachers. Out of them, two are primary teachers and two are secondary teachers. They are to stand in a row, so as the primary teachers, middle teachers and secondary teachers are always in a set. The number of ways in which they can do so, is:

(*a*) 52 (*b*) 48

(*c*) 34 (*d*) 34

(Based on MAT 2011)

76. A candidate is required to answer 7 questions out of 12 questions which are divided into two groups each containing 6 questions. He is not permitted to attempt more than 5 questions from each group. The number of ways in which he can choose the 7 questions is:

(*a*) 780 (*b*) 640

(*c*) 820 (*d*) 720

(Based on MAT (Feb), 2012)

77. If 20 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, the number of points in which they intersect each other is:

(*a*) 220 (*b*) 190

(*c*) 250 (*d*) 120

(Based on MAT (Feb), 2012)

78. A boy has 3 library tickets and 8 books of his interest are there in the library. Out of these 8, he does not want to borrow Chemistry Part II, unless Chemistry Part I is also borrowed. The number of ways in which he can choose the three books to be borrowed is:

(*a*) 61 (*b*) 32
(*c*) 51 (*d*) None of these

(Based on MAT (Feb), 2012)

79. What is the maximum sum of the terms in the arithmetic progression 25, 24½, 24, …?

(*a*) 637½
(*b*) 625
(*c*) 662½
(*d*) 650

(Based on SNAP 2013)

Difficulty Level-2
(Based on Memory)

1. How many numbers greater than a million can be formed by using the digits 7, 4, 6 and 0 if 4 has to be used twice, 6 has to be used thrice and the rest only once?

(*a*) 720 (*b*) $\frac{7!}{3!}$
(*c*) 360 (*d*) None of these

2. From a group of 8 students, 3 students have to be selected. Mr *X* is not selected unless Mr *Y* is also selected. In how many ways can the 3 students be selected?

(*a*) 41 (*b*) 21
(*c*) 20 (*d*) None of these

3. A box contains 5 white balls, and 7 red balls. In how many ways can 3 balls be drawn from the box if at least one black ball is to be included in the draw?

(*a*) 396 (*b*) 180
(*c*) 20 (*d*) 596

4. The total number of numbers from 1,000 to 9,999 (both inclusive) that do not have four different digits is:

(*a*) 1,236 (*b*) 2,325
(*c*) 3,574 (*d*) 4,464

5. What is the remainder left after dividing 1! + 2! + 3! + ... + 100! by 7?

(*a*) 0 (*b*) 5
(*c*) 21 (*d*) 14

[Based on CAT, 2003]

6. There are 5 boys and 3 girls in a family. They are photographed in groups of 2 boys and one girl. The number of different photographs will be:

(*a*) 360 (*b*) 180
(*c*) 30 (*d*) 60

7. There are 40 students in a class. A student is allowed to shake hand only once with a student who is taller than him or equal in height to him. He cannot shake hand with somebody who is shorter than him. Average height of the class is 5 feet. What is the difference between the maximum and minimum number of handshakes that can happen in such a class?

(*a*) $\left(\frac{^{40}C_2}{2} - 20\right)$ (*b*) 361
(*c*) $^{40}C_2 - 40$ (*d*) $^{40}C_2$

8. 8 lines in a set are parallel to each other, where the distance between any two adjacent parallel lines is 1 cm. 6 parallel lines in another set are there where the distance between any two adjacent lines is also 1 cm. These 6 lines of the second set intersect the lines of the previous set to form a number of parallelograms. How many of the parallelograms thus formed are not rhombuses?

(*a*) 385 (*b*) 365
(*c*) 350 (*d*) 335

9. 18 guests have to sit, half on each side of a long table. Four particular guests desire to sit on a particular side, three others on the other side. The number of ways in which the seating arrangement can be made is

(*a*) $\frac{11!9!}{5!6!}$ (*b*) $\frac{11!9!9!}{5!6!}$
(*c*) 11! 4! 3! (*d*) None of these

10. Five persons *A*, *B*, *C*, *D* and *E* occupy seats in a row such that *A* and *B* sit next to each other. In how many possible ways can these five people sit?

(*a*) 24 (*b*) 48
(*c*) 72 (*d*) None of these

[Based on IIT Joint Man. Ent. Test, 2004]

11. What is the number of ways in which triangles are formed in an octagon with each triangle having the centre point of the octagon as one vertex and vertices of octagon as the other?

(*a*) 25 (*b*) 28
(*c*) 30 (*d*) 32

Directions (Q. 12 and 13): *Refer to the data and answer the questions that follow: Five couples, Bansals, Kansals, Singhals, Sharmas and Khannas go for dinner. They want to sit around a round table.*

12. If the couples want to sit together, number of ways in which they can sit around the table is:

(*a*) $\frac{10}{2^5}$ (*b*) $\frac{9!}{2^5}$

(*c*) $4!\,2^5$ (*d*) $9!\,2^5$

13. From a group of 7 men and 6 women 5 people are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

(*a*) 756 (*b*) 735

(*c*) 564 (*d*) 645

[Based on IRMA, 2002]

14. In how many different ways can the letters of the word TRAINER be arranged so that the vowels always come together?

(*a*) 1440 (*b*) 120

(*c*) 720 (*d*) 360

[Based on IRMA, 2002]

15. A polygon has 44 diagonals, then the number of its sides are:

(*a*) 11 (*b*) 9

(*c*) 7 (*d*) 5

[Based on FMS (Delhi), 2002]

16. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is:

(*a*) 19670 (*b*) 39758

(*c*) 69760 (*d*) 99748

[Based on FMS (Delhi), 2002]

17. A five-digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways in which this can be done is:

(*a*) 211 (*b*) 216

(*c*) 221 (*d*) 311

[Based on FMS (Delhi), 2002]

18. How many parallelograms will be formed if 7 parallel horizontal lines intersect 6 parallel vertical lines?

(*a*) 42 (*b*) 294

(*c*) 315 (*d*) None of these

[Based on I.P. Univ., 2002]

19. Which of the following is equal to $\frac{1\times3\times5\ldots(2n-1)}{2\times4\times6\ldots(2n)}$?

(*a*) $(2n)! \div (2^n(n!))^2$ (*b*) $(2n)! \div n!$

(*c*) $(2n-1) \div (n-1)!$ (*d*) 2^n

[Based on SCMHRD, 2002]

20. How many four-digit numbers, each divisible by 4 can be formed using the digits 1, 2, 3, 4 and 5, repetitions of digits being allowed in any number?

(*a*) 100 (*b*) 150

(*c*) 125 (*d*) 75

[Based on SCMHRD, 2002]

21. How many 6 digit numbers can be formed using the digit 2 two times and the digit 4 four times?

(*a*) 16 (*b*) 15

(*c*) 18 (*d*) 6^4

22. Mr Greg Chappel has to select the Indian team for the finals among 8 bowlers, 6 batsmen and 3 wicketkeepers. How many possible combinations does Greg has at his disposal if he has to choose 11 players consisting of 5 bowlers, 5 batsmen and a wicketkeeper?

(*a*) 504 ways

(*b*) 1,008 ways

(*c*) 108 ways

(*d*) 1,196 ways

23. How many four-letter computer passwords can be formed using only the symmetric letters (no repetition allowed)?

(*a*) 7920 (*b*) 330

(*c*) 14640 (*d*) 419430

[Based on CAT, 2002]

24. How many three-letter computer passwords can be formed (no repetition allowed) with at least one symmetric letter?

(*a*) 990 (*b*) 2730

(*c*) 12870 (*d*) 1560000

[Based on CAT, 2002]

25. In how many ways is it possible to choose a white square and a black square on a chess board so that the squares must not lie in the same row or column?

(*a*) 56 (*b*) 896

(*c*) 60 (*d*) 768

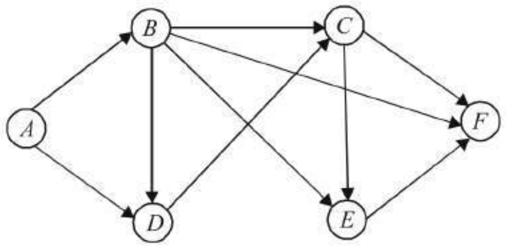
[Based on CAT, 2002]

26. There are 5 red, 6 black and 4 white balls in a box. First ball drawn is kept back after noting down its colour. In how many ways one can draw the second ball having different colour than that of the previously selected ball?

(*a*) 154 ways (*b*) 120 ways

(*c*) 148 ways (*d*) 89 ways

27. The figure below shows the network connecting cities *A*, *B*, *C*, *D*, *E* and *F*. The arrows indicate permissible direction of travel. What is the number of distinct paths from *A* to *F*?



(*a*) 9 (*b*) 10
(*c*) 11 (*d*) None of these

[Based on CAT, 2001]

28. In how many ways seven girls and six boys can sit around a round table so that no two boys sit together?

(*a*) $(6!)^2$ (*b*) $6! \times 7!$
(*c*) $(7!)^2$ (*d*) $5! \times 6!$

29. There are 6 tasks and 6 persons. Task 1 cannot be assigned either to person 1 or to person 2, task 2 must be assigned to either person 3 or person 4. Every person is to be assigned one task. In how many way can the assignment be done?

(*a*) 144 (*b*) 180
(*c*) 192 (*d*) 360

[Based on CAT, 2006]

30. Find the remainder when $\left[(6!)^{7!}\right]^{13333}$ is divided by 13.

(*a*) 1 (*b*) 5
(*c*) 8 (*d*) None of these

31. In how many ways can the letters of the word ABACUS be rearranged such that the vowels always appear together?

(*a*) 6!/2! (*b*) $3! \times 3!$
(*c*) $(3! \times 3!)/2!$ (*d*) $(4! \times 3!)/2!$

[Based on SNAP, 2009]

32. A five-digit number divisible by 3 is to be formed using numerical 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways this can be done is:

(*a*) 122 (*b*) 210
(*c*) 216 (*d*) 217

[Based on SNAP, 2009]

33. There are 10 stations on a railway line. The number of different journey tickets that are required by the authorities is:

(*a*) 92 (*b*) 90
(*c*) 91 (*d*) None of these

[Based on SNAP, 2010]

34. In how many ways can the letters of the word ABACUS be rearranged such that the vowels always appear together?

(*a*) $\frac{6!}{2!}$ (*b*) $3! \times 3!$
(*c*) $\frac{3! \times 3!}{2!}$ (*d*) $\frac{4! \times 3!}{2!}$

[Based on SNAP, 2010]

35. The Vice-Chancellor of University of Delhi decided to form a committee to look into the feasibility of introduction of semester systems at the undergraduate level in the University. 5 members from the Executive Council and 7 members of the Academic Council were found suitable for the job. In how many ways can the Vice-Chancellor form a committee of 6 members such that at least 4 members of the committee belong to the Academic Council?

(*a*) 462 (*b*) 422
(*c*) 412 (*d*) 442

[Based on FMS, 2009]

36. How many three-digit numbers are there, such that if one of the digits is 8, the following digit is 9?

(*a*) 666 (*b*) 665
(*c*) 17 (*d*) 19

37. A football team of 11 players is posing for a photograph along with its coach. The football players are standing in two rows in groups of six, one behind the other. The coach and the vice captain stand together in the centre in the first row while the captain stands behind the vice captain. The goalkeeper stands in the corner, while exactly two out of three defenders stand next to each other. In how many ways can it be done?

(*a*) 18,720 (*b*) 34,560
(*c*) 95,040 (*d*) 1,29,600

38. Eight people, A, B, C, D, E, F, G and H are sitting around a circular table numbered 1 to 8 in a clockwise order. A and F sit together. B and D never sit adjacent to each other. H and G sit opposite each other. What is the total number of ways in which these people can be seated?

(*a*) 1,280 ways (*b*) 2,560 ways
(*c*) 5,040 ways (*d*) None of these

39. Seven friends go to an electronics shop and buy items worth ₹500, ₹600, ₹800, ₹1500, ₹1200, ₹1600 and ₹1800. They are carrying only 100-rupee notes. If they are allowed to borrow and lend money from one another and the total money they have is exactly the amount required to buy these items, what is the total number of different ways they can contribute to the total money?

(*a*) 48,63,51,200 (*b*) 5,37,32,00,880
(*c*) 6,38,41,68,200 (*d*) None of these

40. The value of $\sum_{r=1}^{n} \frac{{}^{n}P_{r}}{r!}$ is:

(*a*) 2^n (*b*) $2^n - 1$
(*c*) 2^{n-1} (*d*) $2^n + 1$

[Based on IIFT, 2007]

41. If ${}^{n+2}C_8 : {}^{n-2}P_4 = 57:16$, then n is equal to:

(*a*) 20 (*b*) 22
(*c*) 15 (*d*) None of these

[Based on ITFT, 2008]

42. The number of ways in which a mixed double tennis game can be arranged amongst 9 married couples if no husband and wife play in the same game is:

(*a*) 1514 (*b*) 1512

(*c*) 3024 (*d*) None of these

[Based on ITFT, 2008]

43. While packing for a business trip Mr. Debashis has packed 3 pairs of shoes, 4 pants, 3 half-pants, 6 shirts, 3 sweater and 2 jackets.The outfit is defined as consisting of a pair of shoes, a choice of 'lower wear' (either a pant or a half-pant), a choice of 'upper wear' (it could be a shirt or a sweater or both) and finally he may or may not choose to wear a jacket. How many different outfits are possible?

(*a*) 567 (*b*) 1821

(*c*) 743 (*d*) None of these

[Based on ITFT, 2008]

44. If $F(x, n)$ be the number of ways of distributing 'x' toys to 'n' children so that each child receives at the most 2 toys, then $F(4, 3) =$

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 6

[Based on XAT, 2008]

45. Let X be a four-digit number with exactly three consecutive digits being same and it is a multiple of 9. How many such X's are possible?

(*a*) 12 (*b*) 16

(*c*) 19 (*d*) None of these

[Based on XAT, 2009]

46. In the country of twenty, there are exactly twenty cities and there is exactly one direct road between any two cities. No two direct roads have an overlapping road segment. After the election dates are announced, candidates from their respective cities start visiting the other cities. Following are the rules that the election commission has laid down for the candidates,

- Each candidate must visit each of the other cities exactly once.
- Each candidate must use only the direct roads between two cities for going from one city to another.
- The candidate must return to his own city at the end of the campaign.
- No direct road between two cities would be used by more than one candidate.

The maximum possible number of candidates is:

(*a*) 5 (*b*) 6

(*c*) 7 (*d*) 9

[Based on XAT, 2011]

47. The football league of a certain country is played according to the following rules

- Each team plays exactly one game against each of the other teams.
- The winning team of each game is awarded 1 point and the losing team gets 0 point.
- If a match ends in a draw, both the teams get 1/2 point.

After the league was over, the teams were ranked according to the points that they earned at the end of the tournament. Analysis of the points table revealed the following

- Exactly half of the points earned by each team were earned in games against the ten teams which finished at the bottom of the table.
- Each of the bottom ten teams earned half of their total points against the other nine teams in the bottom ten.

How many teams participated in the league?

(*a*) 16 (*b*) 18

(*c*) 19 (*d*) 25

[Based on XAT, 2011]

48. Suppose the English alphabet letters $A, B, C, \ldots, Z$ are denoted by the remainders obtained on dividing the numbers $2^0, 2^1, 2^2, \ldots, 2^{25}$, respectively, by 29, then the letter 'K' would be denoted by:

(*a*) 6 (*b*) 7

(*c*) 8 (*d*) 9

[Based on JMET, 2011]

49. A committee is to be formed from amongst 9 boys and 6 girls. In how many ways can the boys and girls divide themselves into groups of three so that no group has more than 2 girls and no group has all boys?

(*a*) $2025 \times 6!$ (*b*) $45 \times 9!$

(*c*) $\frac{15}{4} \times 9! \times 3!$ (*d*) None of these

50. Everyday 5 visitors enter a residential complex and park their cars in one of the 8 adjacent spaces available. How many different arrangements of these 5 cars are possible provided all the cars are always together?

(*a*) $4!$ (*b*) $5 \times 4!$

(*c*) $4 \times 5!$ (*d*) $5!$

51. How may zeroes are there at the end of $125! - 124! - 123!$?

(*a*) 27 (*b*) 28

(*c*) 29 (*d*) 30

52. A pack of 52 playing cards comprises four suites of 13 cards each. In each suite, there are cards of 13 distinct face values — from 1 to 13. In how many ways can a pack of 52 playing cards be distributed equally among four persons so that each person receives cards of all the possible face values?

(*a*) ${}^{52}C_{13}$ (*b*) $(24)^{13}$

(*c*) $\frac{52!}{(13!)^4}$ (*d*) $(13!)^4$

53. Let $n! = 1 \times 2 \times 3 \times \ldots \times n$ for integer $n \geq 1$. If $p = 1! + (2 + 2!) + (3 \times 3!) + \ldots + (10 + 10!)$, then $p + 2$ when divided by 11! leaves a remainder of

(*a*) 10 (*b*) 0

(*c*) 7 (*d*) 1

54. If there are 10 positive real numbers $n_1 < n_2 < n_3 \ldots < n_{10}$ ... How many triplets of these numbers (n_1, n_2, n_3), (n_2, n_3, n_4) ... can be generated such that in each triplet the first number is always less than the second number, and the second number is always less than the third number:

(*a*) 45 (*b*) 90

(*c*) 120 (*d*) 180

55. Boxes numbered 1, 2, 3, 4 and 5 are kept in a row, and they are to be filled with either a red or a blue ball, such that no two adjacent boxes can be filled with blue balls. Then, how many different arrangements are possible, given that all balls of a given colour are exactly identical in all respects?

(*a*) 8 (*b*) 10

(*c*) 15 (*d*) 22

56. *A*, *B*, *C*, and *D* are four towns, any three of which are non-collinear. Then, the number of ways to construct three roads each joining a pair of towns so that the roads do not form a triangle is:

(*a*) 7 (*b*) 8

(*c*) 9 (*d*) 24

57. An intelligence agency forms a code of two distinct digits selected from 0, 1, 2, ..., 9 such that the first digit of code is non zero. The code, handwritten on a slip, can however potentially create confusion when read upside down, for example, the code 91 may appear as 16. How many codes are there for which no such confusion can arise?

(*a*) 80 (*b*) 78

(*c*) 71 (*d*) 69

58. For a scholarship, at the most *n* candidates out of $2n + 1$ can be selected. If the number of different ways of selection of at least one candidate is 63, the maximum number of candidates that can be selected for the scholarship is:

(*a*) 3 (*b*) 4

(*c*) 6 (*d*) 5

59. In a certain laboratory, chemicals are identified by a colour-coding system. There are 20 different chemicals. Each one is coded with either a single colour or a unique two-colour pair. If the order of colours in the pairs does not matter, what is the minimum number of different colours needed to code all 20 chemicals with either a single colour or a unique pair of colours?

(*a*) 7 (*b*) 6

(*c*) 5 (*d*) 20

[Based on NMAT, 2005]

60. Eight chairs are numbered from 1 to 8. Two women and three men wish to occupy one chair each. First, the women chose the chairs from amongst the chairs marked 1 to 4. Then the men selected the chairs from amongst the remaining, marked 5 to 8. The number of possible arrangements is:

(*a*) ${}^6C_3 \times {}^4C_4$ (*b*) ${}^4P_2 \times {}^4P_3$

(*c*) ${}^4C_3 \times {}^4P_3$ (*d*) ${}^4C_2 \times {}^4C_3$

61. A box contains 10 balls out of which 3 are red and the rest are blue. In how many ways can a random sample of 6 balls be drawn from the bag so that at the most 2 red balls are included in the sample and no sample has all the 6 balls of the same colour?

(*a*) 105 (*b*) 168

(*c*) 189 (*d*) 120

62. How many integers, greater than 999 but not greater than 4000, can be formed with the digits 0, 1, 2, 3 and 4, repetition of digits is allowed?

(*a*) 499 (*b*) 500

(*c*) 375 (*d*) 376

[Based on CAT, 2010]

63. Rajat draws a 10×10 grid on the ground such that there are 100 identical squares numbered from 1 to 100. If he has to place two identical stones on any two separate squares in the grid, how many distinct ways are possible?

(*a*) 2475 (*b*) 4950

(*c*) 9900 (*d*) 1000

[Based on CAT, 2011]

64. Vaibhav wrote a certain number of positive prime numbers on a piece of paper. Vikram wrote down the product of all the possible triplets among those numbers. For every pair of numbers written by Vikram, Vishal wrote down the corresponding GCD. If 90 out of the total numbers written by Vishal were prime, how many numbers did Vaibhav write?

(*a*) 6 (*b*) 8

(*c*) 10 (*d*) Cannot be determined

[Based on CAT, 2011]

65. Letter of the word 'ATTRACT' are written on cards and are kept on a table. Manish is asked to lift three cards at a time, write all possible combinations of the three letters on a piece of paper and then replace the three cards. The exercise ends when all possible combinations of letters are exhausted. Then, he is asked to strike out all words in his list that look the same when seen in a mirror. How many words is he left with?

(*a*) 40

(*b*) 20

(*c*) 30

(*d*) None of these

[Based on CAT, 2012]

66. A student is asked to form numbers between 3000 and 9000 with digits 2, 3, 5, 7 and 9. If no digits is to be repeated, in how many ways can the student do so?

(*a*) 24 (*b*) 120

(*c*) 60 (*d*) 72

[Based on CAT, 2012]

67. In a chess tournament held this year in Kolkata, there were only two women participants among all the members that participated in the tournament. Every participant played two games with each other participant. The number of games that men played between themselves proved to exceed by 66, as compared to the number of games the men played with women. How many participants were there in the tournament?

(*a*) 156 (*b*) 13

(*c*) 610 (*d*) 108

[Based on CAT, 2013]

68. *ABC* is a three-digit number in which $A > 0$. The value of *ABC* is equal to the sum of the factorials of its three digits. What is the value of *B*?

(*a*) 9 (*b*) 7

(*c*) 4 (*d*) 2

[Based on CAT, 1997]

69. How many integers which are greater than 999 but not greater than 4000, can be formed with the digits 0, 1, 2, 3 and 4, if repetition of digits is allowed?

(*a*) 449

(*b*) 500

(*c*) 375

(*d*) 376

[Based on CAT, 2008]

70. A company CEO invited nine persons for a business meeting, where the host will be seated at a circular table. How many different arrangements are possible, if two invitees *X* and *Y* be seated on either side of the host CEO?

(*a*) 10080

(*b*) 10800

(*c*) 9200

(*d*) 4600

[Based on MAT, 2012]

71. Read the following instruction carefully and answer the question that follows:

Expression $\sum_{n=1}^{13} \frac{1}{n}$ can also be written as $\frac{x}{13!}$

What would be remainder if x is divided by 11?

(*a*) 2

(*b*) 4

(*c*) 7

(*d*) 9

(*e*) None of the above

[Based on XAT, 2014]

Answer Keys

Difficulty Level-1

1. (*b*)	**2.** (*a*)	**3.** (*b*)	**4.** (*b*)	**5.** (*b*)	**6.** (*a*)	**7.** (*c*)	**8.** (*a*)	**9.** (*b*)	**10.** (*a*)	**11.** (*c*)	**12.** (*b*)	**13.** (*d*)
14. (*c*)	**15.** (*a*)	**16.** (*d*)	**17.** (*a*)	**18.** (*d*)	**19.** (*b*)	**20.** (*a*)	**21.** (*b*)	**22.** (*b*)	**23.** (*d*)	**24.** (*a*)	**25.** (*a*)	**26.** (*d*)
27. (*d*)	**28.** (*a*)	**29.** (*b*)	**30.** (*c*)	**31.** (*d*)	**32.** (*b*)	**33.** (*b*)	**34.** (*b*)	**35.** (*c*)	**36.** (*d*)	**37.** (*d*)	**38.** (*d*)	**39.** (*a*)
40. (*c*)	**41.** (*c*)	**42.** (*b*)	**43.** (*a*)	**44.** (*d*)	**45.** (*b*)	**46.** (*c*)	**47.** (*c*)	**48.** (*b*)	**49.** (*d*)	**50.** (*a*)	**51.** (*d*)	**52.** (*b*)
53. (*d*)	**54.** (*a*)	**55.** (*d*)	**56.** (*d*)	**57.** (*b*)	**58.** (*b*)	**59.** (*b*)	**60.** (*d*)	**61.** (*a*)	**62.** (*a*)	**63.** (*b*)	**64.** (*a*)	**65.** (*b*)
66. (*a*)	**67.** (*c*)	**68.** (*a*)	**69.** (*d*)	**70.** (*d*)	**71.** (*b*)	**72.** (*a*)	**73.** (*a*)	**74.** (*c*)	**75.** (*b*)	**76.** (*a*)	**77.** (*b*)	**78.** (*d*)
79. (*a*)												

Difficulty Level-2

1. (*c*)	**2.** (*a*)	**3.** (*d*)	**4.** (*d*)	**5.** (*b*)	**6.** (*b*)	**7.** (*d*)	**8.** (*d*)	**9.** (*b*)	**10.** (*b*)	**11.** (*b*)	**12.** (*c*)	**13.** (*a*)
14. (*d*)	**15.** (*a*)	**16.** (*c*)	**17.** (*b*)	**18.** (*c*)	**19.** (*a*)	**20.** (*c*)	**21.** (*b*)	**22.** (*b*)	**23.** (*a*)	**24.** (*c*)	**25.** (*d*)	**26.** (*c*)
27. (*b*)	**28.** (*b*)	**29.** (*a*)	**30.** (*a*)	**31.** (*d*)	**32.** (*c*)	**33.** (*b*)	**34.** (*d*)	**35.** (*a*)	**36.** (*b*)	**37.** (*d*)	**38.** (*d*)	**39.** (*d*)
40. (*b*)	**41.** (*d*)	**42.** (*b*)	**43.** (*d*)	**44.** (*d*)	**45.** (*d*)	**46.** (*d*)	**47.** (*d*)	**48.** (*d*)	**49.** (*c*)	**50.** (*c*)	**51.** (*c*)	**52.** (*b*)
53. (*d*)	**54.** (*c*)	**55.** (*d*)	**56.** (*d*)	**57.** (*d*)	**58.** (*a*)	**59.** (*b*)	**60.** (*b*)	**61.** (*b*)	**62.** (*d*)	**63.** (*b*)	**64.** (*a*)	**65.** (*a*)
66. (*d*)	**67.** (*b*)	**68.** (*c*)	**69.** (*d*)	**70.** (*a*)	**71.** (*d*)							

Explanatory Answers

Difficulty Level-1

1. (*b*) The number of ways in which 1 green ball can be put = 6

The number of ways in which two green balls can be put such that the boxes are consecutive = 5 (i.e., (1, 2), (2, 3), (3, 4), (4, 5), (5, 6))

Similarly, the number of ways in which three green balls can be put = 4 (i.e., (1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6))

... ...

and so on.

∴ Total number of ways of doing this

$= 6 + 5 + 4 + 3 + 2 + 1 = 21.$

2. (*a*) Since every edge connects a pair of points, the given 12 points have to be joined using lines

We may have minimum number of edges if all the 12 points are collinear

No. of edges in this particular case = 12 – 1 = 11

Maximum number of edges are possible when all the 12 points are non-collinear. In this particular case, number of edges is equal to number of different straight lines that can be formed using 12 points, which is equal to $^{12}C_2 = \frac{12 \times 11}{2} = 66$

Therefore, following inequality holds for *e*:

$11 \le e \le 66.$

3. (*b*) The four constants can be written in 4! ways, i.e., 24 ways. The three vowels can be written in 3! ways, i.e., 6 ways. Since no two vowels can come together, therefore vowels can be inserted in any three places out of the five places available, such as, *P R M S*, i.e., in $^5C_3$ ways, i.e., 10 ways. Total number of arrangements required = 24 × 6 × 10 = 1440.

4. (*b*) $^4C_3 \times {}^4C_2 + {}^4C_2 \times {}^4C_3 = 4 \times 6 + 4 \times 6 = 48.$

5. (*b*) The possible ways are as follows:

(i) 1 red ball out of the three and 5 blue balls out of the seven

(ii) 2 red balls out of the three and 4 blue balls out of the seven

∴ Total number of ways in which a random sample of six balls can be drwan

$= {}^3C_1 \times {}^7C_5 + {}^3C_2 \times {}^7C_4 = 168.$

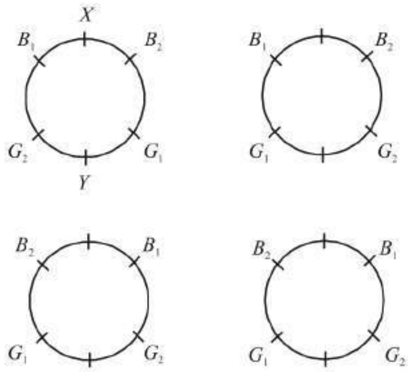
6. (*a*) There are 6 bowlers, 3 wicket keepers and 11 batsmen in all. The number of ways in which a team of 4 bowlers, 2 wicketkeepers and 5 batsmen can be chosen.

$= {}^6C_4 \times {}^3C_2 \times {}^{11}C_5$

$= {}^6C_2 \times {}^3C_1 \times {}^{11}C_5$

$= \frac{6 \times 5}{2 \times 1} \times \frac{3}{1} \times \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} = 20790.$

7. (*c*) Four

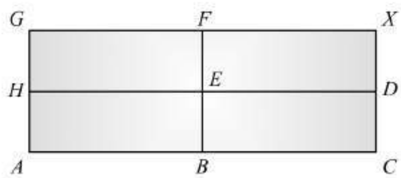


8. (*a*) $^{18}C_2 = \frac{18 \times 17}{2} = 153.$

9. (*b*) $^7C_2 = \frac{7!}{2!5!} = \frac{7 \times 6 \times 5!}{5! \times 2!} = 21.$

10. (*a*) $^{10}C_4 + {}^{10}C_5 = {}^{11}C_5 = 462.$

11. (*c*) There are six ways



AHGFX

AHEFX

AHEDX

ABEFX

ABEDX

ABCDX

12. (*b*) Word 'AGAIN' has letters 'A, A, G, I, N'.

When letter start with A, then number of ways

$= 4! = 24$

When letter start with G, then number of ways

$= \frac{4!}{2!} = 12$

When letter start with I, then number of ways

$= \frac{4!}{2!} = 12$

The next 49th word is NAAGI and the 50th word is NAAIG.

13. (*d*) $\therefore$ Required number of ways $= 3^4 \times 2^3 \times 5^1$

$= 81 \times 8 \times 5 = 3240.$

14. (*c*) Required number of ways $= 3^8$.

15. (*a*) The total six digit telephone numbers in which zero cannot initiate the number

$= 9 \times 10 \times 10 \times 10 \times 10 \times 10$

$= 900000$

Total-six digit telephone numbers in which zero cannot initiate the number and no digit is repeated

$= 9 \times 9 \times 8 \times 7 \times 6 \times 5$

$= 136080$

$\therefore$ Required six-digit telephone numbers if atleast one digit is repeated $= 900000 - 136080 = 763920$.

16. (*d*) Here balls are of different colours and boxes are of different sizes

Let, $n = 8$, $r = 3$

$\therefore$ Required number of ways

$n! \times {}^{n-1}C_{r-1} = 8! \times {}^{7}C_2$

$= 40320 \times 21$

$= 846720.$

17. (*a*) The total number of combinations which can be formed of 5 different green dyes, taking one or more of them is $2^5 - 1 = 31$

Similarly, by taking one or more of 4 different blue dyes $2^4 - 1 = 15$ and 3 different red dyes is $2^3 = 8$

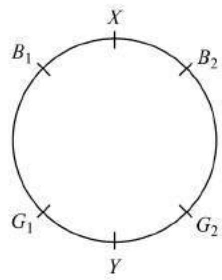
$\therefore$ The required number of combinations

$= 31 \times 15 \times 18 = 3720.$

18. (*d*) Out of 10 boxes we have to choose only 5 boxes because the balls are identical and the boxes are also identical

$\therefore$ Required number of ways $= {}^{10}C_5 = \frac{10!}{(5!)^2} = 252.$

19. (*b*) Required number of ways $= 2 \times 2 = 4$



[Since B_1 and B_2 interchange their position, also and G_2 interchange their position.]

20. (*a*) Required number of ways $= {}^{12}C_1 \times {}^{18}C_1 \times {}^{9}C_1$

$= 12 \times 18 \times 9$

$= 1944.$

21. (*b*) Required number of handshakes $= {}^{12}C_2$

$= \frac{12 \times 11}{2 \times 1} = 66.$

22. (*b*)

1. If one green ball in a box, then number of ways = 6
2. If two green balls in a box, then number of ways = 5
3. If three green balls in a box, then number of ways = 4
4. If four green balls in a box, then number of ways = 3
5. If five green balls in a box, then number of ways = 2
6. If six green balls in a box, then number of ways = 1

$\therefore$ Total number of ways $= 6 + 5 + 4 + 3 + 2 + 1 = 21$.

23. (*d*) A man invite the friends

$= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$

$= 2^5 - 1 = 31$ ways

His wife invite the friends

$= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

$= 2^4 - 1 = 15$ ways

$\therefore$ Total number of ways $= 31 + 15 = 46$.

24. (*a*) $\therefore$ Required number of ways $= 10^4$.

25. (*a*) In a number lock system, we can consider 0 as the beginning

$\therefore$ Required number of ways $= 10 \times 9 \times 8 = 720$.

26. (*d*) Total number of ways $= 4^3 \times 5^3$

$= 64 \times 125$

$= 8000.$

27. (*d*) Let the age's of three children be x_1, x_2 and x_3.

Then, $\frac{x_1 + x_2 + x_3}{3} = \frac{20}{100}\left(\frac{26 + x_3}{3}\right)$

$\Rightarrow \quad \frac{x_1 + x_2 + x_3}{3} = \frac{26 + x_3}{10} \qquad (1)$

Also, $\quad M + x_1 = 39 \qquad (2)$

From Eqs. (1) and (2), we connot determine the value of x_2.

28. (*a*) **Case I:** If Mr *Y* is a member, then Mrs *X* is refuse to serve a member.

Number of ways $= {}^7C_3 \times {}^6C_3 = 35 \times 20$

$= 700$

Case II: If Mr *Y* is not a member, then Mrs *X* may be a member, then

number of ways $= {}^8C_3 \times {}^6C_4$

$= 56 \times 15$

$= 840$

$\therefore \quad$ Total number of ways $= 700 + 840$

$= 1540$ ways.

29. (*b*) **Case I:** Choose four questions from first five questions

$= {}^5C_4 \times {}^8C_6$

$= 5 \times 28 = 140$

Case II: Choose five questions from first five questions $= {}^5C_5 \times {}^8C_5 = 1 \times 56 = 56$

$\therefore$ Total number of ways $= 140 + 56 = 196.$

30. (*c*) Required number of ways $= {}^{22-4-2}C_{11-2}$

$= {}^{16}C_9.$

31. (*d*) Total possible answers $= 3^4 \times 2^3 \times 5^1$

$= 81 \times 8 \times 5 = 3240.$

32. (*b*) **Case I:** If it includes the oldest, then we exclude the youngest. The number of ways

$= {}^{10-1-1}C_{5-1} = {}^8C_4$

$= \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = 70$

Case II: If we do not include the oldest, then number of ways $= {}^9C_5 = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 126$

$\therefore$ Total number of ways $= 70 + 126 = 196.$

33. (*b*) **Case I:** When 4 persons in tennis and rest in theatre

Number of ways $= {}^{11}C_4 \times {}^7C_7$

$= \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2} \times 1 = 330$

Case II: When 5 persons in tennis and rest in theatre, number of ways $= {}^{11}C_5 \times {}^6C_6$

$= \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} \times 1 = 462$

Case III: When 6 persons in tennis and rest in theatre

Number of ways $= {}^{11}C_6 \times {}^5C_5 = 462$

Case IV: When 7 persons in tennis and rest in theatre.

Number of ways $= {}^{11}C_7 \times {}^4C_4 = 330$

$\therefore \quad$ Total number of ways $= 2(330 + 462)$

$= 2(792) = 1584.$

34. (*b*) $\therefore$ Required number of ways

$= {}^8C_3 \times {}^4C_2 = 56 \times 6 = 336$

35. (*c*) Required number of ways

$= {}^{9-2}C_{5-2} = {}^7C_3$

$= \frac{7 \times 6 \times 5}{3 \times 2} = 35$ ways.

36. (*d*) Total number of seats

$= 1$ grandfather $+ 5$ sons and daughters

$+ 8$ grandchildren

$= 14$

The grandchildren can occupy the 4 seats on either side of the table in $8P_4 \times 4! = 8!$

The grandfather can occupy a seat in 4 ways (i.e., S_6, S_7, S_8, S_9)

And, the remaining seats can be occupied in $5! = 5 \times 4 \times 3 \times 2 = 120$ ways (5 seat for sons and daughters)

Hence, the total number of required ways

$= 8! \times 480 = 19353600.$

37. (*d*) The number of times the father would go to the zoological garden = Number of ways of selection of 3 children taken at a time

$= {}^8C_3 = \frac{8 \times 7 \times 6! \times 5!}{5! \times 3 \times 2} = 56$

Number of times a child will go to the zoological garden = Number of times he is accompanied by two other $1 \times {}^7C_2 = 21$

$\Rightarrow$ Number of times a child will not go to the zoological garden = 56 – 21 = 35.

38. (*d*) Each man gives the votes for any of the four candidates

$\therefore$ Total number of ways $= 4 \times 4 \times 4 \times 4 \times 4$

$= 1024.$

39. (*a*) Firstly we fix the alternate position of men in a round table is (6 – 1)! ways = 5!

In out of six positions 5 women can be seated in

$${}^6P_5 = 6!$$

Required no. of ways = 6! × 5!

40. (*c*) Let the no. of points be n

According to the question,

$${}^nC_2 = 10$$

or, $$\frac{\lfloor n}{\lfloor 2 \lfloor n-2} = 10$$

or, $$\frac{n(n-1)}{2} = 0$$

or, $$n^2 - n = 20$$

or, $$n = 5, -4$$

$\therefore$ $n = 5$ ($\because$ –4 is not possible)

41. (*c*) If there are 'm' horizontal and 'n' vertical lines, then the no. of different rectangles formed are given by ${}^mC_2 \times {}^nC_2$.

Here, $m = 7$ and, $n = 6$

$\therefore$ Required no. of parallelograms $= {}^7C_2 \times {}^6C_2$

$$= \frac{\lfloor 7}{\lfloor 2 \lfloor 7-2} \times \frac{\lfloor 6}{\lfloor 2 \lfloor 6-2}$$

$$= \frac{7 \times 6 \times 6 \times 5}{2 \times 2} = 315.$$

42. (*b*) With 1, 3, 5, 7; 4! = 24 numbers can be formed between 1,000 and 10,000, i.e., all four-digit numbers.

43. (*a*) One principal can be appointed in 36 ways

One vice-principal appointed in remaining 35 ways

$\therefore$ Total no. of ways = 36 × 35 = 1260.

44. (*d*) It is clear that out of these eight books two books are of chemistry

Now three books can be chosen in the following ways

He borrows chemistry part II or he does not borrow chemistry part II

If chemistry part II is borrowed then chemistry part I will also be borrowed. Hence the third book can be chosen out of remaining 6 books in 6 ways

If chemistry part II is not borrowed then three books are chosen in ${}^7C_3$ ways i.e., 35 ways

$\therefore$ Required no. of ways = 6 + 35 = 41.

45. (*b*) 100, 105, 110, … 995

The above numbers are in AP.

We have the following formula,

$$t_n = a + (n-1)d$$

Here we have to find n and

$t_n = 995$, $a = 100$ and $d = 5$ are given

Now,

$$995 = 100 + (n-1)5$$

$\therefore$ $$n = \frac{900}{5} = 180.$$

46. (*c*) Required no. of ways $= {}^6C_4 = \frac{6 \times 5}{2 \times 1} = 15$ ways.

47. (*c*) Number of arrangements

$$= \frac{6!}{2! \times 2!} \times \frac{5!}{2!} = 180 \times 60 = 10800$$

48. (*b*) 45 tickets from one side and 45 from the opposite side.

49. (*d*) The given word = SERIES, this contains 2S, 2E and rest are distinct

The number of ways of selecting the 4 L and the number of arrangements are as follows:

1. 4 L are distinct = S, E, R, I = 4! = 24
2. 2 L are same and rest are distinct
   = SSRI, SSRE, SSIE, EERI, EERS and EEIS
   $= \frac{4!}{2!} \times 6 = 72$
3. Two are same of one kind and two are same of the other kind
   $= \text{SSEE} = \frac{4!}{2! \times 2!} = 6$

Total ways = 24 + 72 + 6 = 102.

50. (*a*) **Case 1** = Number of ways of inviting friends, when no girls is invited $= {}^7C_4 \times {}^4C_6 = 35$ ways

Case 2 = Number of ways of inviting friends, when 1 girl is invited $= {}^7C_4 \times {}^4C_1 = 140$ ways

Case 3 = Number of ways of inviting the friends, when two girls are invited

$$= {}^7C_4 \times {}^4C_2 = 210 \text{ ways}$$

Case 4 = Number of ways of inviting the friends, when three girls are invited

$= {}^7C_4 \times {}^4C_3 = 140$ ways

Case 5 = Number of ways of inviting the friends, when four girls are invited

$= {}^7C_4 \times {}^4C_4 = 35$ ways

So, total ways = 35 + 140 + 210 + 140 + 35

= 560 ways.

51. (*d*) Number of ways of selecting exactly one ruling party member

$= {}^6C_1 = 6$ ways

Number of ways of selecting remaining 3 members from opposition party

$= {}^5C_3 = 10$ ways

Total ways = 6 × 10 = 60 ways.

52. (*b*) Selecting one opposition party member

$= {}^5C_1 \times {}^6C_3 = 5 \times 20 = 100$ ways

Selecting two opposition party members

$= {}^5C_2 \times {}^6C_2 = 10 \times 15 = 150$ ways

Selecting three opposition party members

$= {}^5C_3 \times {}^6C_1 = 10 \times 6 = 60$ ways

Selecting all four opposition party members

$= {}^5C_4 = 5$ ways

Total ways = 100 + 150 + 60 + 5 = 315 ways.

53. (*d*) Suppose total management book be 1 unit, then there are total 8 books, which are arrange in 8! ways

Also, 3 management books are arranged in 3! ways

∴ Number of ways = 3! × 8!

= 6 × 40320

= 241920.

54. (*a*) Total number of book = 10

∴ Number of ways arrangement = 10!

= 3628800.

55. (*d*) *X* and *Y* are two students and they

$\underline{X}$ _ _ $\underline{Y}$ = 2 ways can be arranged in the given ways

_ $\underline{X}$ _ $\underline{Y}$ = 1

$\underline{Y}$ _ $\underline{X}$ _ = 1

$\underline{Y}$ _ _ $\underline{X}$ = 2 ways

Total no. of ways = 2 + 1 + 1 + 2 = 6 ways.

56. (*d*) Number of ways

$${}^7C_4 \times {}^4C_2 + {}^7C_3 \times {}^4C_3 \times {}^7C_4 + {}^4C_1 = 371.$$

57. (*b*) 100! = 1 × 2 × 3 × 4 × 5 × ... × 100

also, 10 = 2 × 5 ∵ now

In any $n!$, number, of 10 = number of 5

∴ Higher power of 10 that will divide 100! is

$$\left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right] = 20 + 4 = 24.$$

58. (*b*) Number divisible by 5 will end with either '0' or '5'

Total numbers ending with '0' = 4 × 3 × 1 = 12

Total of numbers ending with '5' = 3 × 3 × 1 = 9

∴ Total such numbers = 21.

59. (*b*) Six pentagons *ABCDE*, *ABCDF*, *ABCEF*, *ABDEF*, *ACDEF* and *BCDEF* can be formed. All these six pentagons will have the same area.

60. (*d*) Since, 3 particular flowers are always together. So, considering these 3 flowers as 1, total number of flowers is 8. These 8 flowers can be arranged in 8! ways. Remaining 3 flowers (which we considered as 1) can be arranged in 3! ways. Hence, total number of ways = 8! × 3! = 241920.

61. (*a*) Required number of ways

$$= {}^3C_1{}^3C_1{}^1C_1 + {}^3C_1{}^3C_2 + {}^3C_2{}^3C_1 + {}^3C_2{}^1C_1 + {}^3C_2{}^1C_1$$

$$= 9 + 9 + 9 + 3 + 3 = 33$$

62. (*a*) From 4 bowlers, 2 wicket keepers and 10 players, total number of ways of required selection

$$= {}^4C_3{}^2C_1{}^{10}C_7 + {}^4C_4{}^2C_1{}^{10}C_6 + {}^4C_3{}^2C_2{}^{10}C_6 + {}^4C_4{}^2C_2{}^{10}C_5$$

$$= 4 \times 2 \times 120 + 1 \times 2 \times 210 + 4 \times 1 \times 201 + 1 \times 1 \times 252$$

$$= 960 + 420 + 840 + 252$$

$$= 2472.$$

63. (*b*) Student can choose 4 questions from first 5 questions, the number of ways $= {}^5C_4 = 5$

Similarly, student can choose 6 questions, form remaining 8 questions, then number of ways ${}^8C_6$

$$= \frac{8 \times 7}{2} 28$$

∴ Total number of choices available = 5 × 28 = 140.

64. (*a*) There are ten places, where the boys can be seated. Now, a_2 cannot come at 9th and 10th positions otherwise condition is not fulfilled

∴ Total number of ways the boys can be arranged in 8 places = 8!

Similarly for a_5, he cannot be seated in first place and at the 10th place, then possible ways of arrangement = 8!

Now, a_7 cannot be seated on 1st and 2nd places, then possible number of ways = 8!

∴ Total number of ways

$$= 8! + 8! + 8!$$

$$= 3 \times 8!$$

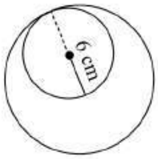
$$= \frac{3 \times 3 \times 8!}{3} = \frac{9 \times 8!}{3} = \frac{9!}{3}$$

65. (*b*) According to the question, 4 teams have to chosen form 8 players

Now, number of ways for choosing 4 teams form 8 players $= {}^8C_4 = \dfrac{8\times7\times6\times5}{4\times3\times2\times1} = 70$

Now, as there have to 2 players in each team, then possible number of ways are $\dfrac{70}{2!} = \dfrac{70}{2} = 35$ ways.

66. (*a*) Let the radius of smaller circle be r cm and radius of larger circle = R cm.



Difference between the area = 116π

$$\pi R^2 + \pi r^2 = 116\pi$$

$$R^2 + r^2 = 116 \quad (1)$$

Also, the difference between their centres = 6 cm

i.e., $R - r = 6$

or, $R = (6 + r) \quad (2)$

From Eqs. (1) and (2),

$(6+r)^2 + r^2 = 116$

$\Rightarrow 36 + r^2 + 12r + r^2 = 116$

$\Rightarrow 2r^2 + 12r + 36 - 106 = 0$

$\Rightarrow 2r^2 + 12r - 80 = 0$

$\Rightarrow r^2 + 6r - 40 = 0$

$\Rightarrow r^2 + 10r - 4r - 40 = 0$

$\Rightarrow (r+10)(r-4) = 0$

$\therefore r = -10, 4$ (radius cannot be negative)

$\therefore$ Radius of smaller circle = 4 cm

and Radius of larger circle = 4 + 6 = 10 cm.

67. (*c*) The lock contains 4 rings, so any place can have number from 0 to 9

$\therefore$ Total ways to getting a code = 10^4

$\therefore$ So, maximum attempts before getting the right number is 1 less than total ways i.e., $10^4 - 1$.

68. (*a*) There are total of 10 persons of which P_1 is always selected and hence P_4 and P_5 are not be included in the arrangement

Therefore, persons left = 10 – 3 = 7 (as P_1 is already selected and P_4 and P_5 are not selected)

$\therefore$ To select 4 persons from 7 persons total ways $= {}^7C_4$

Also, these 5 persons can be arranged in 5! ways

$\therefore$ Required ways of arrangement $= {}^7C_4 \times 5!$

69. (*d*) We have no girls together, let us first arrange the 5 boys and after that we can arrange the girls in the space between the boys

Number of ways of arranging the boys around a circle $= [5-1]! = 24$

Number of ways of arranging the girls would be by placing them in the 5 spaces that are formed between the boys. This can be done in ${}^5P_3$ ways = 60 ways

Total arrangements $= 24 \times 60 = 1440$.

70. (*d*) 3 ladies can be appointed out of 7 ladies as 1 is not includid in ${}^7C_3$ ways

Also 3 gentlemen can be appointed out of 6 gentlemen as 1 is already member in ${}^6C_3$ ways

Hence, the required number of ways

$= {}^7C_3 \times {}^6C_3 = 700$

71. (*d*) Total no. of seats = 1 grandfather + 5 sons and daughters + 8 grand children

The grandchildren can occupy the 4 seats on either side of table in 4! = 24 ways

The grandfather can occupy a seat in $(5-1) = 4$ ways (4 gaps between 5 sons and daughters)

And the remaning seat can be occupied in $5! = 5\times4\times3\times2 = 120$ ways

(5 seats for sons and daughters)

Hence, the total number of required ways

$= 8! \times 480 = 193536$.

72. (*a*) **Case I:** When Y is a member.

Then no. of ways $= {}^6C_3 \times {}^7C_3 = 20 \times 35 = 700$

Case II: When Y is not a member.

Then, no. of ways $= {}^6C_4 \times {}^8C_3 = 15 \times 56 = 840$

$\therefore$ Total no. of ways = 700 + 840 = 1540.

73. (*a*) Let the number of teams participating be n. Then, we have

$$\frac{n(n-1)}{2} = 153$$

$$\Rightarrow n(n-1) = 153 \times 2$$

$$\Rightarrow n(n-1) = 18 \times 17$$

$$\Rightarrow n = 18.$$

74. (*c*) When the first two digits of the number is 41, then remaining four digits can be arranged in ${}^8P_4$ ways

Similarly, for 42, 46, 62, and 64, the remaining four digits can be arranged in ${}^8P_4$ ways

Hence, total number of numbers having all six digits distinct $= 5 \times {}^8P_4$

$$= 5 \times \frac{8!}{4!} = 5\times8\times7\times6\times5 = 8400.$$

75. (*b*) The three pairs of teachers can be arranged in 3! ways but every pair can be arranged in 2! ways

Thus, required number of ways

$= {}^6C_2{}^6C_5 + {}^6C_3{}^6C_4 + {}^6C_4 + {}^6C_3 + {}^6C_5{}^6C_2$

76. (*a*) Required number of ways

$= {}^6C_2{}^6C_5 + {}^6C_3{}^6C_4 + {}^6C_4 + {}^6C_3 + {}^6C_5{}^6C_2$

$= 2({}^6C_2{}^6C_5 + {}^6C_3{}^6C_4)$

$= 2({}^6C_2{}^6C_1 + {}^6C_3{}^6C_2)$

$= 2\left(\frac{6\times5}{2\times1}\times6 + \frac{6\times5\times4}{3\times2\times1}\times\frac{6\times5}{2\times1}\right)$

$= 2(90 + 300)$

$= 2\times390 = 780.$

77. (*b*) Required number of points $= {}^{20}C_2 = \frac{20\times19}{2\times1} = 190.$

78. (*d*) From 8 books, he can choose 2 book out of 6 and 1 book out of 2 Chemistry books

∴ Total number of ways

$= {}^6C_2{}^6C_2 = 15\times2 = 30.$

79. (*a*) The maximun sum would occur when we take the sum of all the positive terms of the series. The series 25, 24.5, 24, 23.5, 23,...,1, 0.5, 0 has 51 terms. The sum of the series would be given by $n\times$ average $= 51\times12.5 = 637.5.$

Difficulty Level-2

1. (*c*) The given digits are 4, 6, 0, 6, 7, 4, 6 which are 7 in all. So, numbers greater than a million can be formed by using all the digits

So, 4 occurs twice, 6 occurs thrice while 0 and 7 occur once

∴ Total number of arrangements

$$= \frac{7!}{3!\,2!} = \frac{7\times6\times5\times4}{2} = 420$$

But these also include the numbers starting with 0 which is not greater than a million. So, numbers starting with 0 have the remaining places filled with the remaining digits

Number of such numbers $= \frac{6!}{2!3!} = \frac{6\times5\times4}{2} = 60$

∴ Numbers greater than a million = 420 – 60 = 360.

2. (*a*) If Mr Y is selected, then 2 more students have to be selected out of remaining 7

Number of ways $= {}^7C_2 = \frac{7\times6}{2} = 21$ ways

If Mr Y is not selected, then Mr X cannot be selected. Then, 3 students have to be selected out of 6 students, which can be done in

${}^6C_3$ ways $= \frac{6\times5\times4}{3\times2} = 20$ ways

∴ Required number of ways = 21 + 20 = 41 ways.

3. (*d*) Number of black balls = 6

Number of non-black balls = 12

Number of ways of drawing at least 1 black ball = Number of ways of drawing 1 black ball + Number of ways of drawing 2 black balls + Number of ways of drawing 3 black balls

$= ({}^6C_1\times{}^{12}C_2) + ({}^6C_2\times{}^{12}C_1) + {}^6C_3$

$= \left(6\times\frac{12\times11}{2}\right) + \left(\frac{6\times5}{2}\times12\right) + \frac{6\times5\times4}{6}$

$= 396 + 180 + 20 = 596.$

4. (*d*) Total number of numbers from 1000 to 9999 = 9000

Number of numbers having 4 different digits

$= 9\times{}^9P_3 = 4536$

⇒ Required number = 90000 – 4536 = 4,464.

5. (*b*) 7! + 8! + 9! + ... + 100! is divisible by 7

6! + 5! + 4! + 3! + 2! + 1!

= 720 + 120 + 24 + 6 + 2 + 1 = 873

873, when divided by 7, leaves a remainder of 5.

6. (*b*) The number of ways of forming the groups

$= {}^5C_2\times{}^3C_1 = 30$

Numbers of each group can be arranged among members themselves in 3! ways = 6 ways

Therefore, the number of photographs

= 30 × 6 = 180.

7. (*d*) If all are of equal height, number of handshakes

$= {}^{40}C_2$

If all are of different heights, number of handshakes = 0

Difference $= {}^{40}C_2 - 0 = {}^{40}C_2.$

8. (*d*) Number of parallelograms $= {}^8C_2\times{}^6C_2 = 420$

Rhombuses of side 1 cm = (6 – 1) × (8 – 1) = 35

Rhombuses of side 2 cm = (6 – 2) × (8 – 2) = 24

Rhombuses of side 3 cm = (6 – 3) × (8 – 3) = 15

Rhombuses of side 4 cm = (6 – 4) × (8 – 4) = 8

Rhombuses of side 5 cm = (6 – 5) × (8 – 5) = 3

Total = 85

Thus, parallelograms which are not rhombuses

$= 420 - 85 = 335.$

9. (*b*) Total number of ways of sitting of 4 particular guests on a paticular side = ${}^9P_4 = \frac{9!}{5!}$

Similarly, total number of ways of sitting of 3 another guests on the other side = ${}^9P_3 = \frac{9!}{6!}$

After these total two operations, the remaining 11 guests can sit on remaining 11 seats in 11! ways. Hence, total number of seating arrangements

$= \frac{9!}{5!} \times \frac{9!}{6!} \times 11!$

10. (*b*) 4! × 2 ways, i.e., 24 × 2 = 48 ways.

11. (*b*) Octagon is 8 sided polygon and a triangle has 3 vertices

Of these, one vertex of a triangle is selected as the centre of the octagon in ${}^1C_1$ way

And, the other 2 vertices of a triangle can be selected from any of the 8 sides of the octagon in ${}^8C_2$ ways,

∴ The total number of ways = ${}^1C_1 \times {}^8C_2$

$= 1.\frac{8 \times 7}{2 \times 1} = 28.$

12. (*c*) If the couples want to sit together, there are 5 pairs

5 pairs can sit around a table in (5–1)! ways

Each couple can sit together in 2 ways

∴ Number of ways is $(4!)2^5$

13. (*a*) ${}^7C_3 \times {}^6C_2 + {}^7C_4 \times {}^6C_1 + {}^7C_5 \times {}^6C_0$

$= {}^7C_3 \times {}^6C_2 + {}^7C_3 \times 6 + {}^7C_2 \times 1$

$= \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{6 \times 5}{2 \times 1} + \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 6 + \frac{7 \times 6}{2 \times 1}$

$= 525 + 210 + 21 = 756.$

14. (*d*) Treating (AIE), i.e., all the vowels together as one letter. Therefore, the word capital, i.e., TRNR (AIE) can be arranged in $\frac{5!}{2}$ ways. (∵ *R* is repeated)

Since (AIE) can also be arranged in 3! ways, therefore, required number of ways

$= \frac{5!}{2} \times 3! = \frac{120 \times 6}{2} = 360.$

15. (*a*) Let the number of sides be *n*

∴ ${}^nC_2 - n = 44, n > 0$

⇒ $\frac{n(n-1)}{2} - n = 44$

⇒ $n^2 - 3n - 88 = 0$

⇒ $n^2 - 11n + 8n - 88 = 0$

⇒ $n(n - 11) + 8(n - 11) = 0$

⇒ $(n - 11)(n + 8) = 0$

⇒ $n = 11.$

16. (*c*) The total number of words that can be formed with five letters out of the ten given letters = 10^5 = 100000

The total number of words that can be formed with five distinct letters

$= 10 \times 9 \times 8 \times 7 \times 6 = 30240$

∴ The total number of words in which atleast one letter is repeated = 100000 – 30240 = 69760.

17. (*b*) All permutations formed with 1, 2, 3, 4, 5 (sum = 15) will be divisible by 3

There are 5! = 120 such permutations. Such numbers can also be formed using 0 and 1, 2, 4, 5. There are 4 × 4! such numbers, i.e., 96. (Factor of 4 for four positions of 0 and 4! for different permutations of these four numbers)

∴ Total of such numbers = 120 + 96 = 216

18. (*c*) ${}^7C_2 \times {}^6C_2 = 315.$

19. (*a*) $\frac{[1 \times 3 \times 5 \ldots (2n-1)][2 \times 4 \times 6 \ldots 2n]}{[2 \times 4 \times 6 \ldots 2n]^2}$

$= \frac{(2n)!}{[2^n (1, 2, 3, \ldots n)]^2}$

$= \frac{(2n)!}{[2^n \times n!]^2}.$

20. (*c*) The four-digit number formed from the digits 1, 2, 3, 4, 5 will be divisible by 4 if the last two digits are 12 or 24 or 32 or 44 or 52. The first two can be chosen in = 5 × 4 = 20 ways and also as 11, 22, 33, 44, 55, i.e., 25 ways in all. Hence these 125 four-digit numbers are divisible by 4.

21. (*b*) $\frac{6!}{2! \times 4!} = 15.$

22. (*b*) 5 bowlers can be selected out of 8 cm ${}^8C_5$ ways = $\frac{8 \times 7 \times 6}{3 \times 2 \times 1}$ = 56 ways

5 batsman can be selected out of 6 in ${}^6C_5$ ways = 6 ways

One wicket keeper can be selected in ${}^3C_1$ = 3 ways

Hence, total number of ways = 3 × 6 × 56

= 1,008 ways.

23. (*a*) 11 × 10 × 9 × 8 = 7920.

24. (*c*) Total number of passwords using all alphabets total number of passwords using no symmetric alphabets

= (26 × 25 × 24) – (15 × 14 × 13) = 12870.

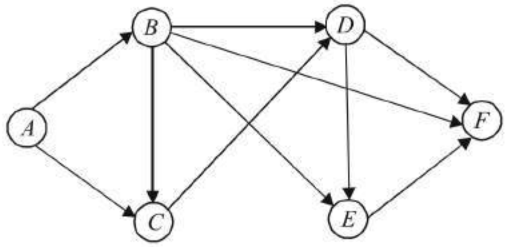
25. (*d*) A block square can be chosen in 32 ways. Once a black square is there, you cannot choose the 8 white squares in its row or column, so the number of white squares available = 24.

No. of ways = $32 \times 24 = 768$.

26. (*c*) One can draw red ball in first draw in 5 ways and black or white ball in second draw in $6 + 4 = 10$ ways, black ball in first draw in 6 and red or white ball in second draw in 9 ways and white ball in first draw in 4 and red or black ball in second draw in 11 ways. Hence, total number of ways

$= 5 \times 10 + 6 \times 9 + 4 \times 11 = 148$.

27. (*b*) The maximum routes from A to F are listed below.



(1) *ABDF* (2) *ACEF* (3) *ABF*
(4) *ABEF* (5) *ACDF* (6) *BCDEF*
(7) *ACDEF* (8) *ABDEF* (9) *ABCDF*
(10) *ABCEF*.

28. (*b*) Seven girls can sit around a circular table in $(7 - 1)!$ ways. Now six gaps can be chosen in ${}^7C_6$ ways in which boys can fit in then the boys can be arranged in 6! ways.

Hence, total number of ways $= 6!\ {}^7C_6 \times 6! = 6! \times 7!$

29. (*a*) Total number of ways

$= 2 \times 3 \times 4 \times 3 \times 2 \times 1$

$= 144$ ways.

30. (*a*) $6! = 720$, which when divided by 13 gives a remainder 5. The remainder when given expression is divided by 13 is the remainder obtained when $(5)^{7! \times 13333}$ is divided by 13.

$(25)^{7! \times 13333)/2} = (26 - 1)^{\text{even number}} = 26K + 1$

So, remainder when $(25)^{(7! \times 13333)/2}$ is divided by 13 is 1.

31. (*d*) Number of ways of arranging vowels $3!/2!$

Now, take vowels as a unit, then total no. of arrangement will be $= 4! \times 3!/ 2!$

32. (*c*) No. of ways of forming numbers without '*o*'

$= 5 \times 4 \times 3 \times 2 \times 1 = 120$

No. of ways of forming numbers with '*o*'

$= 4 \times 4 \times 3 \times 2 \times 1 = 96$

[This time only 0, 1, 2, 4 and 5 digits will be considered so as to make the number divisible by 3]

33. (*b*) From a certain station, there will be a ticket for each of the other 9 stations and there are 10 stations on the railway line.

$\therefore$ The number of different journey tickets

$= 9 \times 10 = 90$

Hence, option (*b*).

34. (*d*) The three vowels in ABACUS are A, A and U. These three can be arranged among themselves in $\frac{3!}{2!} = 3$ ways

As the three vowels are to appear together, we consider them as one entity. Thus we have four letters; (AAU), B, C and S to be arranged. This can be done in 4! ways

$\therefore$ Required number of ways $= \frac{4! \times 3!}{2}$

Hence, option (*d*).

35. (*a*) ${}^7C_4 \times {}^5C_2 + {}^7C_5 \times {}^5C_1 + {}^7C_6$

$= 350 + 105 + 7 = 462$.

36. (*b*) (*a*) From 890 to 899 (except 898) we have 8 in the hundreds place and 9 in tens place, i.e., 9 three-digit numbers.

(*b*) With 8 in the tens place and 9 in the units place we have 8 three-digit numbers (189, 289, etc except 889)

(*c*) If 8 is not any one of the three-digits, we have (1 to 9 except 8, 0 to 9 in the hundreds, tens and the units digits) i.e.,

$8 \times 9 \times 9 = 648$ three-digit numbers

$\therefore$ Total is $9 + 8 + 648 = 665$.

37. (*d*) The vice captain and the coach can be arranged in the middle position of the first row in 2 ways. The position of the captain can be fixed thus

Two defenders out of 3 can be selected in ${}^3C_2$ ways

3 _ _ captain _ 2
a b *c*

4 *e* coach vice captain *d* 1

The goalkeeper can be placed at positions 1, 2, 3 or 4. Suppose he is at position 3

Two defenders out of three can be placed at positions (*a*, *b*) or (2, *c*) or (4, *e*) or (1, *d*) and rest of the players can be placed at 6 positions in 6! ways. The two defenders can be arranged among themselves in 2 ways

Thus, the total number of ways of arranging the team and coach with the goalkeeper at position 3 is $2 \times {}^3C_2 \times 2 \times 4 \times 6!$

Suppose the goalkeeper is at position 1

Two defenders can be placed at positions (3, *a*) or (*a*, *b*) or (*c*, 2) or (4, *e*)

If they are placed at (*c*, 2) or (4, *e*), rest of the players can be arranged in 6! ways

Thus total number of arrangements would be $2 \times {}^3C_2 \times 2 \times 2 \times 6!$

If the two defenders are placed at (3, a) or (a, b), then the third defender cannot be placed at b (or 3) as exactly two defenders can be placed together. The third defender would have to be placed at one of the positions from c, 2 4, e or d, i.e., in 5 ways. Thus, if the goal keeper were at position 1, the number of arrangements would be $(2 \times {}^3C_2 \times 2 \times 2 \times 6!) + (2 \times {}^3C_2 \times 2 \times 2 \times 5 \times 5!)$

The number of arrangements would be same if the goalkeeper were at positions 2 or 4

$\therefore$ Total number of possible arrangements would be $2 \times {}^3C_2 \times 2 \times 4 \times 6! + 3 \times [(2 \times {}^3C_2 \times 2 \times 2 \times 6!) + (2 \times {}^3C_2 \times 2 \times 2 \times 5 \times 5!)]$

$= 2 \times {}^3C_2 \times 2[4 \times 6! + 3 \times 2 \times 6! + 3 \times 2 \times 5 \times 5!)]$

$= 12[2880 + 4320 + 3600] = 1{,}29{,}600.$

38. (*d*) A can be seated in 8 ways and F in 2 ways

$\therefore$ A and F can together be seated in $8 \times 2 = 16$ ways

Now, B and D never sit together

For B and D:

Case 1: B sits adjacent to A or F

Note D cannot be adjacent to F or A as then G and H cannot sit opposite each other

$\therefore$ D can sit in 3 places. G can sit in 2 places and H has only 1 seat

$\therefore$ $2 \times 3 \times 2 = 12$ ways

Case 2: B sits opposite A or F

D can sit in 4 ways, G in 2 ways, H in 1 way

$\therefore$ Number of ways $= 2 \times 4 \times 2 = 16$

Case 3: B is neither opposite nor adjacent to A or F. B can sit in 2 ways. D can sit in 2 ways, G in 2 ways, H in one way

$\therefore$ Number of ways $= 2 \times 2 \times 2 = 8$

The remaining 2 people can be seated in 2 ways

$\therefore$ Total number of ways

$= 8 \times 2 \times (12 + 16 + 8) \times 2 \times 1 = 1{,}152.$

39. (*d*) Let the number of 100—rupee notes carried by them be and x^7

$\Rightarrow 100(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7)$

$= 500 + 600 + 800 + 1500 + 1200 + 1600 + 1800$

$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 + x_6$ and $x_7 =$ ₹80

The total number of solutions to the equation

$$= {}^{80+7-1}C_{7-1}$$

$$= \frac{86!}{80!6!}$$

$$= \frac{86 \times 85 \times 84 \times 83 \times 82 \times 81}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$= 47{,}01{,}55{,}077$

Note:

$86 \times 85 \times ... \times 81$ has only one zero at the end and $1 \times 2 \times ... \times 6$ will also have only one zero at the end. Hence, the answer will not end with 0.

40. (*b*) $\displaystyle\sum_{r=1}^{n} \frac{{}^nP_r}{r!} = \sum_{r=1}^{n} \frac{n!}{r!(n-r)!}$

$$= \sum_{r=1}^{n} {}^nC_r$$

$$= {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_{n-1} + {}^nC_n \quad (1)$$

We know that,

$${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_{n-1} + {}^nC_n = 2^n$$

$$\Rightarrow {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_{n-1} + {}^nC_1$$

$$= 2^n - {}^nC_0 = 2^n - 1.$$

41. (*d*) ${}^{n+2}C_8 : {}^{n-2}P_4 = 57:16$

$$\Rightarrow \frac{n+2!}{8!(n+2-8)!} : \frac{n-2!}{(n-2-4)!} = 57:16$$

$$\Rightarrow \frac{n+2!}{8!(n-6)!} \times \frac{n-6!}{(n-2)!} = \frac{57}{16}$$

$$\Rightarrow \frac{n+2!}{8!(n-2)!} = \frac{57}{16}$$

$$\Rightarrow (n-1)\,n(n+1)\,(n+2) = 8! \times \frac{57}{16}$$

$$\Rightarrow (n-1)\,n(n+1)\,(n+2) = 18 \times 19 \times 20 \times 21$$

On solving, we get

$\therefore$ $n = 19.$

42. (*b*) In a mixed double of tennis game there is a pair of man and woman on each ride

So, total 2 men and 2 women are in each game

There are 9 men and 9 women

Ways to select 2 men $= {}^9C_2$

Wives of these 2 men cannot play in same game

So ways to select 2 women $= {}^7C_2$

In a game these 4 persons can be paired by 2 types

$\therefore$ Total number of ways $= {}^9C_2 \times {}^7C_2 \times 2$

$$= \frac{9 \times 8}{2 \times 1} \times \frac{7 \times 6}{2 \times 1} \times 2$$

$= 1512.$

43. (*d*) Shoes can be selected = 3 ways

Lower wear can be selected = (4 + 3) ways = 7 ways

Upper wear can be selected = (6 + 3+ 9 × 2) = 27 ways

Jacket can be selected = 2 way

He may not wear jacket = 1 ways = 3 ways

∴ Total possible outputs = 3 × 7 × 27 × 3

= 1701 ways.

44. (*d*) $F(4, 3) = \frac{4 \times 3}{2} = 6.$

45. (*d*) Let x be represented as $pppq$, then $3p + q$ must be a multiple of 9

i.e., (1116), (6111), (2223), (3222), (3339), (9333) (4446), (6444), (5553), (3555), (6669), (9666), (7776), (6777), (8883), (3888) (9990), (9000), (3330) (6660)

Hence, there are 20 cases possible.

46. (*d*) If we consider the distance between any two cities as paths, there were a total of $^{20}C_2 = 190$ paths

Now, each candidate needed to visit all the cities and then come back to the city he started from i.e., each candidate needed to take 20 paths

Let the maximum number of candidates be n

Now, $20n < 190$

$n < 9.5$

Since, n is an integer, the maximum value of n is 9.

47. (*d*) There are 10 teams in the bottom group and say n teams in the top group. The bottom group gets 45 points (there are 45 matches and 1 point per match) playing amongst themselves. Therefore, they should get 45 points from their matches against the top group i.e., 45 out of the $10n$ points. The top group get $^nC_2$ points from the matches among themselves. They also get, $10n - 45$ points against the bottom group, which is half their total points.

∴ $^nC_2 = 10n - 45$

⇒ $n(n + 1) = 20n - 90$

⇒ $n^2 - 21n + 90 = 0$

⇒ $(n - 6)(n - 15) = 0$

If $n = 6$, the top group would get $^nC_2 + 10n - 45 = {}^nC_2 + 10(6) - 45 = 30$ points, or an average of 5 points per team, while the bottom group would get (45 + 45)/10 or an average of 9. This is not possible

∴ $n = 15$

The total number of teams is 10 + 15 or 25.

48. (*d*)

A	B	C	D	E	F	G	H	I	J	K
2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}

Letter '*K*' is represented by = $\frac{2^{10}}{29} = \frac{1024}{29}$

= 9 (Remainder)

49. (*c*) One group has 2 girls and others have 1 girl each. Total number of ways in which 5 groups can be formed

$= {}^6C_2 \times 9 \times 4 \times {}^8C_2 \times 3 \times {}^6C_2 \times 2 \times {}^4C_2$

$= {}^6C_2 \times 9 \times 4! \times \frac{8 \times 7}{2} \times \frac{6 \times 5}{2} \times \frac{4 \times 3}{2}$

$= \frac{15 \times 9! \times 4!}{16} = \frac{15}{4} \times 9! \times 3!$

50. (*c*) Since all the 5 cars must be adjacent, this can be done in 4 ways

Further the 5 cars can be arranged amongst themselves in 5! ways

Hence, the answer is 4 × 5!.

51. (*c*) $125! - 124! - 123! = (125 \times 124!) - 124! - 123!$

$= 124 \times 124! - 123!$

$= (124^2 - 1) \times 123!$

Now, last digit of $(124^2 - 1)$ is 5 and 123! has 28 zeroes. Hence, there are 29 zeroes in all.

52. (*b*) There are 4 cards of each denomination in a pack of 52 cards. Each set of four cards of the same denomination can be distributed among 4 persons in 4! ways. Hence, the required number of ways is $(4!)^{13} = (24)^{13}$.

53. (*d*) If $P = 1! = 1$

Then, $P + 2 = 3$, when divided by 2! remainder will be 1

If $P = 1! + 2 \times 2! = 5$

Then, $P + 2 = 7$ when divided by 3! remainder is still 1

Hence, $P = 1! + (2 \times 2!) + (3 + 3!) \ldots + (10 + 10!)$

When divided by 11! leaves remainder 1

Alternative method:

$P = 1 + 2.2! + 3.3! + \ldots + 10.10!$

$= (2 - 1)\,1! + (3 - 1)\,2! + (4 - 1)\,3! + \ldots (11 - 1)\,10!$

$= 2! - 1! + 3! - 2! + \ldots 11! - 10! = 11! + 1$

Hence, the remainder is 1.

54. (*c*) Three numbers can be selected and arranged out of 10 numbers in $^{10}P_3$ ways $= \frac{10!}{7!} = 10 \times 9 \times 8$. Now this arrangement is restricted to a given condition that first number is always less than the second number and second number is always less than the third number. Thus, three numbers can be arranged among themselves in 31 ways.

Hence, required number of arrangements

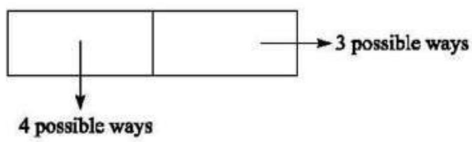
$$= \frac{10 \times 9 \times 8}{3 \times 2} = 120 \text{ ways.}$$

55. (*d*) Total number of ways of filling the 5 boxes numbered as (1, 2, 3, 4 and 5) with either blue or red balls = 2^5 = 32. Two adjacent boxes with blue can be obtained in 4 ways, i.e., (12), (23), (34) and (45). Three adjacent boxes with blue can be obtained in 3 ways, i.e., (123), (234) and (345). Four adjacent boxes with blue can be obtained in 2 ways, i.e., (1234) and (2345). And five boxes with blue can be got in 1 way. Hence, the total number of ways of filling the boxes such that adjacent boxes have blue = (4 + 3 + 2 + 1) = 10

Hence, the number of ways of filling up the boxes such that no two adjacent boxes have blue = 32 – 10 = 22.

56. (*d*) To construct 2 roads, three towns can be selected out of 4 in 4 × 3 × 2 = 24 ways. Now, if third road goes from the third town to the first town, a triangle is formed, and if it goes to the fourth town, a triangle is not formed. So, there are 24 ways to form a triangle and 24 ways of avoiding triangle.

57. (*d*) The available digits are 0, 1, 2, ...9. The first digit can be chosen in 9 ways (0 not acceptable), the second digit can be accepted in 9 ways (digit repretition not allowed). Thus the code can be made in 9 × 9 = 81 ways



Now, there are only 4 digits which can create confusion 1, 6, 8, 9. The same can be given in the following ways

Total number of ways confusion can arise

$= 4 \times 3 = 12$

Thus, required answer = 81 – 12 = 69.

58. (*a*) At least one candidate out of (2*n* + 1) candidates can be selected in $(2^{n+1} - 1)$ ways

$\therefore \quad 2^{2n+1} - 1 = 63$

$\Rightarrow \quad 2^{2n+1} = 64 = (2)^6$

$\Rightarrow \quad n = 2.5.$

Since *n* cannot be a fraction. Hence, *n* = 3.

59. (*b*) Each one is coded with either a single colour or unique two-colour pair. Therefore, total number of ways

$= n + {}^nC_2$

$\therefore$ Minimum number of different colours needed to code all 20 chemicals will be 6

$\therefore \quad 6 + {}^6C_2 = 6 + 15 = 21.$

60. (*b*)

61. (*b*) The possible ways are as follows:

(i) 1 red ball out of the three and 5 blue balls out of the seven

(ii) 2 red balls out of the three and 4 blue balls out of the seven

$\therefore$ Total number of ways in which a random sample of six balls can be drawn

$$= {}^3C_1 \times {}^7C_5 + {}^3C_2 \times {}^7C_4 = 168.$$

62. (*d*) The smallest number in the series is 1000, a four digit number

The largest number in the series is 4000, the only 4-digit number to start with 4

The left most digit (thousands place) of each of the four digit numbers other than 4000 can take one of the 3 values-1 or 2 or 3

The next three digits (hundreds, tens and units place) can take any of the 5 values 0 or 1or 2 or 3 or 4

Hence, there are $3 \times 5 \times 5 \times 5$ or 375 numbers from 1000 to 3999.

Including 4000, there will be 376 such numbers.

63. (*b*) Two identical stones are to be placed on any two separate squares in the grid

$\therefore \quad r = 2$

Total number of squares = 100

$\therefore$ Total number of stones (*n*) = 100

The number of ways of selecting 2 things out of 100 is ${}^nC_r = {}^{100}C_2 = 4950.$

64. (*a*) Suppose Vaibhav wrote *m* prime numbers

Vikram wrote down $n = {}^mC_3$ numbers of the form *pipjpk*, where *pi*, *pj* and *pk* are the numbers written by Vaibhav

Vishal wrote down *n*(*n* – 1)/2 instances of some numbers

Some of these were 1 (and hence not prime)

Some were of the form *pi* and others were of the form *pipj* (and hence not prime)

Each of the prime numbers (of vaibhav)were written down by vishal a certain number of times.

Consider one particular number, say, *pi*. Among the other *m* – 3 numbers, the number of ways of choosing 2 is ${}^{m-3}C_2$

Among the remaining *m* – 3 numbers, the number of ways of choosing 2 is ${}^{m-3}C_2$

But in the product $({}^{m-1}C_2)({}^{m-3}C_2)$, each such pair has been counted twice. Therefore, the number of distinct pair is,

$$\frac{({}^{m-1}C_2)({}^{m-3}C_2)}{2}$$

Vishal writes down so many numbers for each of the m primes of Vaibhav

∴ Number of instances of primes that vishal writes are

$$\frac{m({}^{m-1}C_2)({}^{m-3}C_2)}{2}$$

$$\Rightarrow \frac{m}{2}\times\frac{(m-1)(m-2)}{2}\times\frac{(m-3)(m-4)}{2}=90$$

$$\Rightarrow m(m-1)(m-2)(m-3)(m-4)=(90)(8)$$

$$\Rightarrow m(m-1)(m-2)(m-3)(m-4)=(6)(5)(4)(3)(2)$$

$$\Rightarrow m=6.$$

65. (*a*) There are 4 cases,

I: Number of words using 3 Ts = 1

II: Number of words using 2 Ts = 3 × 3 = 9

As the third letter may be anyone of A, C and R and can be placed in any one of the positions, e.g., CTT, TCT or TTC

III. Number of words using 2 As = 3 × 3 = 9

IV. Number of words with all 3 letters distinct

$= {}^4P_3 = 24$

∴ Total number of words $= 24 + 9 + 9 + 1 = 43$

Since the 3 words TTT, ATA and TAT look the same in a mirror, therefore they are striked out

Thus, the number of words left = 43 − 3 = 40 words.

66. (*d*) The first digit can be chosen from 3, 5 and 7 in 3 ways.

After choosing the first digit, the remaining three digits can be chosen from the remaining 4 numbers in

${}^4P_3 = 24$ ways

∴ Total number of ways = 3 × 24 = 72.

67. (*b*) Let the total number of women participants be x

As every participant played two games with each other, therefore, the total number of games played among men is

$$2\times{}^nC_2 = 2\times\frac{n!}{2!(n-2)} = n(n-1)$$

Number of games played with each women = $2n$

Since, each women must have played two games with each men

∴ Total matches played by woman = 2 × 2n = 4n

Now, according to the question,

$n(n-1) - 4n = 66$

$\Rightarrow\ n^2 - n - 4n = 66$

$\Rightarrow\ n^2 - 5n - 66 = 0$

$\Rightarrow\ (n-11)(n+6) = 0$

$\Rightarrow\ n = 11, -6$

But −6 is not possible as $n > 0$

∴ Total number of participants = 11 + 2 = 13.

68. (*c*) Given, $100A + 10B + C = A! + B! + C!$

From the options, value of B will be less than 7, because 7! = 5040 which is a 4 digit number. Hence, B can be either 4 or 2. Again value of A and C has to be any of 6! = 720 or = 144 because any other combination will produce a three digit number. Hence, required number will be 145 as 1! + 4! + 5!= 1 + 24 + 120 = 145.

69. (*d*) The number required is greater than 999, but less than and equal to 4000

Now out of given digits, 0, 1, 2, 3, and 4 form a number greater than 999 and less the 4000.

The digit at thousands place can be selected in 3 ways (As 0 and 4 cannot be taken)

The digit at hundreds place can be selected in 5 ways

The digit at tens place can be selected in 5 ways

The digit at units can be selected in 5 ways

∴ Total required number of ways

$= 3\times5\times5\times5 = 375$

Since, 4000 is also one of the required number.

Therefore, total number of ways = 375 + 1 = 376.

70. (*a*) Total number of persons seating at the circular table = 9 + 1 = 10

Considering CEO, X and Y, remaining 7 persons can be seated in 7! Ways. X and Y can sit on either side of the host CEO by 2 ways. Hence, total number of ways = 7! × 2 = 10080.

71. (*d*) $\sum_{n=1}^{13}\frac{1}{n} = \frac{x}{13!}$

$$\Rightarrow \frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{11}+\frac{1}{12}+\frac{1}{13}=\frac{x}{13!}$$

$$\therefore x = \frac{13!}{1}+\frac{13!}{2}+\frac{13!}{3}+\ldots+\frac{13!}{11}+\frac{13!}{2}+\frac{13!}{3}$$

All the terms in x are divisible by 11 expect 13!/11

$$\frac{13!}{11} = 1.2.3.4....10.12.13$$

According to Wilson theorem,

When, $(p-1)!$ Is divided by p, remainder will be −1

$$\text{i.e., rem}\left(\frac{(p-1)!}{p}\right) = -1$$

$$\text{Now, }\frac{13!}{11} = 1.2.3.4...10.12.13 = 10!(12)(13)$$

$$\therefore \text{rem}\left(\frac{\frac{13!}{11}}{11}\right) = \frac{10!(12)(13)}{11} = (-1).1.2 = -2 = 9.$$