CUET Mathematics Solved Paper-2023

Held on 22 May 2023, (Shift-III)

SECTION: COMMON

1. The probability that a student is not a swimmer is $\frac{1}{5}$. Then the probability that out of five students, four are swimmers is:

(a) ${}^{5}C_4\left(\frac{4}{5}\right)^4\frac{1}{5}$ (b) $\left(\frac{4}{5}\right)^4\frac{1}{5}$

(c) ${}^{5}C_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)$ (d) $\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)$

2. If $P=\begin{bmatrix}1 & x & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A|$ is 4, then x is equal to:

(a) 4 (b) 0
(c) 11 (d) 5

3. $\int_1^2 \frac{x\,dx}{(x+1)(x+2)}=$

(a) $\tan^{-1}\frac{32}{27}$ (b) $\tan^{-1}\frac{27}{32}$

(c) $\log\frac{32}{27}$ (d) $\log\frac{27}{32}$

4. The angle of intersection between the curves $y=4-x^2$ and $y=x^2$ is:

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\tan^{-1}\frac{4}{3}$ (d) $\tan^{-1}\frac{4\sqrt{2}}{7}$

5. Which of the following statements is incorrect regarding matrices?
For any matrices A and B of suitable orders,
(a) $(A')'=A$
(b) $(KA)'=KA'$ (Where K is any constant)
(c) $(A+B)'=B'+A'$
(d) $(AB)'=A'B'$

6. If $f(x)=\begin{cases}ax^2+b, & x<-1\\ bx^2+ax+4 & x\geq-1\end{cases}$ is everywhere differentiable, then:

(a) $a=2, b=3$ (b) $a=3, b=2$
(c) $a=-2, b=3$ (d) $a=2, b=-3$

7. All points lying inside the triangle formed by the points (5, 0), (–1, 2) and (1, 3) satisfy:
(A) $3x+2y-18>0$ (B) $3x+2y>0$
(C) $2x+y+13<0$ (D) $2x-3y-12<0$
(E) $2x-3y+12>0$

Choose the **correct** answer from the options given below:
(a) (A) and (B) only (b) (B) and (C) only
(c) (D) and (E) only (d) (B), (D) and (E) only

8. If $f(x)=2x$ and $g(x)=\frac{x^2}{2}+1$, then which of the following can be a discontinuous function?

(a) $f(x)+g(x)$ (b) $f(x)-g(x)$

(c) $f(x).g(x)$ (d) $\frac{g(x)}{f(x)}$

9. Interval in which the function $f(x)=2x^3-3x^2-12x+10$ is decreasing is:

(a) $(-\infty,-1]$ (b) $(-\infty,-1]\cup[2,\infty)$
(c) $[-1,2]$ (d) $[2,\infty)$

10. For the following probability distribution:

X	1	2	3	4
P(X)	1/10	1/5	3/10	2/5

$E(X^2)$ is equal to:
(a) 3 (b) 5
(c) 7 (d) 10

11. The differential equation of the family of curves $y=a\sin(bx+c)$, a and c are parameters, is:

(a) $\frac{d^2y}{dx^2}+b^2y=0$ (b) $\frac{dy}{dx}+b^2y=0$

(c) $\frac{d^2y}{dx^2}-b^2y=0$ (d) $\frac{d^2y}{dx^2}+y=0$

12. $\int e^x\left(\frac{1-x}{1+x^2}\right)^2dx=$

(a) $\frac{e^x}{1+x^2}+C$ (b) $-\frac{e^x}{1+x^2}+C$

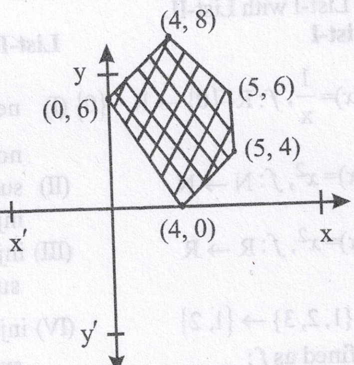
(c) $\frac{e^x}{(1+x^2)^2}+C$ (d) $-\frac{e^x}{(1+x^2)^2}+C$

13. The area enclosed by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ is given by:

(a) $3\int_0^4\sqrt{9-x^2}\,dx$ (b) $\frac{3}{4}\int_0^4\sqrt{9-x^2}\,dx$

(c) $3\int_0^4\sqrt{16-x^2}\,dx$ (d) $\frac{3}{4}\int_0^4\sqrt{16-x^2}\,dx$

14. The feasible region for an LPP is shown below. Let $Z = 3x - 4y$ be the objective function. Maximum of Z occurs at:



(a) (0,6)
(b) (5,4)
(c) (4,8)
(d) (4,0)

15. If $A=\begin{bmatrix}1 & 2\\ 4 & 2\end{bmatrix}$, then the value of K for which $|2A|=K|A|$ is:

(a) -24
(b) 4
(c) 4
(d) -6

SECTION: CORE MATHEMATICS

1. Which of the following statements are correct?

(A) $|A'| = |A|$, where A is the transpose of matrix A
(B) If $A = [a_{ij}]_{3\times 3}$, then $|4A| = 64|A|$
(C) $|A| = |\text{adj } A|^{n-1}$, where n is the order of the matrix
(D) If A is an invertible matrix of order 2, then det (A^{-1}) is equal to $\frac{1}{\det(A)}$

Choose the correct answer from the options given below:

(a) (A), (B), (D), only
(b) (A), (C) only
(c) (A), (B), (C) only
(d) (A), (D) only

2. The position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively in the ratio 2:1 externally is:

(a) $-\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$
(b) $-3\hat{i}+3\hat{k}$
(c) $3\hat{i}-3\hat{k}$
(d) $\frac{1}{3}\hat{i}-\frac{4}{3}\hat{j}-\frac{1}{3}\hat{k}$

3. Match List - I with List - II.

List-I	List-II
(A) $x = 2at^2, y = at^4$	(I) Inverse trignometric function
(B) $f(x) = (2x+3)^3$	(II) Implicit function
(C) $xy + y^2 = \tan(x+y)$	(III) Parametric function
(D) $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$	(IV) Composite function

Choose the correct answer from the options given below:

(a) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(b) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(c) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
(d) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

4. If $A=\begin{pmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{pmatrix}$, then $A^2 =$

(a) $\begin{pmatrix}\cos 4\theta & \sin 4\theta\\ -\sin 4\theta & \cos 4\theta\end{pmatrix}$
(b) $\begin{pmatrix}\cos^2 2\theta & \sin^2 2\theta\\ \sin^2 2\theta & \cos^2 2\theta\end{pmatrix}$
(c) $\begin{pmatrix}1 & 0\\ \sin 2\theta & \cos 2\theta\end{pmatrix}$
(d) $\begin{pmatrix}\cos 6\theta & \sin 6\theta\\ -\sin 6\theta & \cos 6\theta\end{pmatrix}$

5. Owner of a whole sale computers shop plans to sell 2 types of computers. A desktop and portable model. If x is the number of desktops and y is the number of portable model and the shop's capacity cannot exceed 250 units. Which of the following is correct?

(a) $x+y=250$
(b) $x+y\le 250$
(c) $x+y\ge 250$
(d) $x+y>250$

6. The value of C, in Rolle's theorem for the function $f(x) = e^x \sin x$, when $x \in [0, \pi]$ is:

(a) $\frac{\pi}{6}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$
(d) $\frac{3\pi}{4}$

7. Integrating factor of $(x \log_e x)\frac{dy}{dx} + y = 2\log_e x$ is:

(a) x
(b) e^x
(c) $\log_e x$
(d) $\log_e(\log_e x)$

8. Match List-I with List-II.

List-I	List-II
(A) $\lvert A^T\rvert$	(I) $B^{-1}A^{-1}$
(B) $A(\text{adj } A)$	(II) (adj A)
(C) $A^{-1}\lvert A\rvert$	(III) (adj A) A
(D) $(AB)^{-1}$	(IV) $\lvert A\rvert$

Choose the correct answer from the options given below:

(a) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)
(b) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)
(c) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(d) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

9. Match List-I with List - II.

List-I	List-II
(A) $y = \log(\sin x)$	(I) $\frac{d^2y}{dx^2} = -\frac{1}{x^2}$
(B) $y = e^{(1+\log x)}$	(II) $\frac{d^2y}{dx^2} = 2$
(C) $y = \log\lvert x\rvert$	(III) $\frac{d^2y}{dx^2} = 0$
(D) $y = x^2 + 4x - 1$	(IV) $\frac{d^2y}{dx^2} = -\text{cosec}^2 x$

Choose the correct answer from the options given below:

(a) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(b) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(c) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(d) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

10. Match List -I with List -II

List - I	List -II
(A) Area of triangle Δ with adjacent sides $\vec{a}$ and $\vec{b}$	(I) $\vec{a}\times\vec{b}$
(B) Area of parallelogram with adjacent side $\vec{a}\times\vec{b}$	(II) $\frac{1}{2}\lvert\vec{a}\times\vec{b}\rvert$
(C) $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$	(III) $\lvert\vec{a}\times\vec{b}\rvert$
(D) $\lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin\theta\hat{n}$, where symbols have their usual meaning	(IV) $2(\vec{a}\times\vec{b})$

Choose the correct answer from the options given below:
(a) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
(b) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)
(c) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)
(d) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

11. Let $y=\log_e\left(\frac{a+b\sin x}{a-b\sin x}\right)$, then value of $\frac{dy}{dx}$ is:
(a) $\frac{ab\cos x}{a^2-b^2\sin^2 x}$
(b) $\frac{ab\cos x}{a^2+b^2\sin^2 x}$
(c) $\frac{a\sin x}{a^2-b^2\sin^2 x}$
(d) $\frac{2ab\cos x}{a^2-b^2\sin^2 x}$

12. Cartesian equation of plane passing through the points (2,–4, 5) and perpendicular to the line with direction ratios (3, –1, 2) is:
(a) $3x-y+2z=0$
(b) $3x-y+2z=20$
(c) $2x-4y+5z=0$
(d) $2x-4y+5z=45$

13. The appropriate change in the volume V of a cube of side x metres caused by increasing the side by 2% is:
(a) $0.06x^3\,m^3$
(b) $0.02x^3\,m^3$
(c) $3x^2\,m^3$
(d) $0.02x\,m^3$

14. The equation of tangent to the curve $x=a\cos^3 t$, $y=a\sin^3 t$ at t is:
(a) $x\sec t+y\,\text{cosec}\,t=a$
(b) $x\sec t-y\,\text{cosec}\,t=a$
(c) $x\,\text{cosec}\,t+y\sec t=a$
(d) $x\,\text{cosec}\,t-y\sec t=a$

15. The set of values of K for which the system of equations $\begin{bmatrix}2&3&1\\4&5&0\\1&K&3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}5\\6\\7\end{bmatrix}$ gives a unique solution is:
(a) $\left\{\frac{5}{4}\right\}$
(b) $\left\{-\frac{5}{4},\frac{5}{4}\right\}$
(c) $\left\{\frac{11}{4}\right\}$
(d) $R-\left\{\frac{11}{4}\right\}$

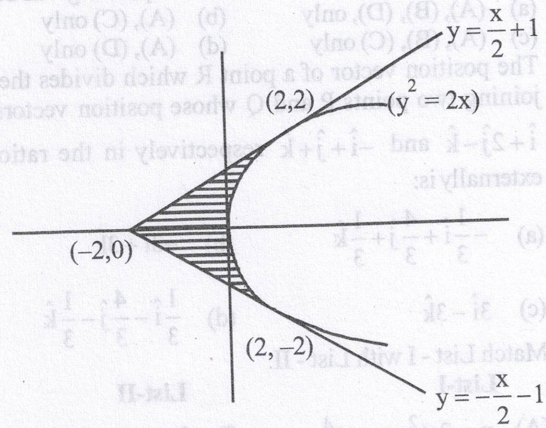
16. Match List-I with List-II.

List-I	List-II
(A) $f(x)=\frac{1}{x}$, $f:R-\{0\}\to R-\{0\}$	(I) neither injective nor surjective
(B) $f(x)=x^2$, $f:N\to N$	(II) surjective but not injective
(C) $f(x)=x^2$, $f:R\to R$	(III) injective but not surjective
(D) $f:\{1,2,3\}\to\{1,2\}$ defined as f: $\{(1,1),(2,2),(3,1)\}$	(IV) injective and surjective

Choose the correct answer from the options given below:
(a) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(b) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
(c) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)
(d) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

17. The black and red die are rolled. The conditional probability of obtaining a sum greater than 9 given that the black die resulted in a 5 is:
(a) $\frac{1}{3}$
(b) $\frac{1}{6}$
(c) $\frac{1}{18}$
(d) $\frac{1}{9}$

18. Calculate the shaded area as given below:



(a) $\frac{8}{3}$ sq. units
(b) $\frac{8}{5}$ sq. units
(c) 3 sq. units
(d) 8 sq. units

19. The region represented by the system of inequalities $x, y\geq 0$; $2x+3y\geq 4$; $x\geq 1$ is :
(a) unbounded in first quadrant
(b) unbounded in first and second quadrant
(c) bounded in first quadrant
(d) not feasible

20. The value of the determinant $\Delta = \begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}$ is:

(a) 2! (b) 3!
(c) 4! (d) 5!

21. $\int_0^{\pi/2} \sqrt{1-\sin 2x}\,dx$ is equal to:

(a) $2\sqrt{2}$ (b) $2(\sqrt{2}+1)$
(c) 2 (d) $2(\sqrt{2}-1)$

22. Let A = PQ. The elementary operation on A, that produces the same effect as it does on applying on P and keeping Q unchanged is:

(A) $R_i \leftrightarrow R_j$ (B) $R_i \to R_i + KR\hat{j}$
(C) $C_i \to KC_i$ (D) $C_i \to C_i + KC_j$

Choose the correct answer from the options given below:
(a) (A) and (B) only (b) (A), (B) and (D) only
(c) (A), (C) and (D) only (d) (B) and (D) only

23. The principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ is:

(a) $-\frac{2\pi}{3}$ (b) $\frac{2\pi}{3}$
(c) $\frac{\pi}{3}$ (d) $-\frac{\pi}{3}$

24. $\int e^x(\tan x + \log_e \sec x)\,dx =$

(a) $e^x \log_e \sec x + C$ (b) $\log_e \sec x + C$
(c) $e^x \tan x + C$ (d) $e^x \sec x + C$

25. Distance between the point (3, 4, 5) and the point where the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ meets the plane $x+y+z=17$ is:

(a) 1 (b) 2
(c) 3 (d) $\frac{3}{2}$

26. Urn I contains 6 red balls and 4 black balls and Urn II contains 4 red balls and 6 black balls. One ball is drawn at random from Urn I and placed in Urn II. If one ball is drawn at random from Urn II, then the probability that it is a red ball is:

(a) $\frac{3}{5}$ (b) $\frac{4}{11}$
(c) $\frac{23}{55}$ (d) $\frac{2}{5}$

27. The differential equation $y = xp + \sqrt{x^2p^3 + 4}$ where $p = \frac{dy}{dx}$ is:

(A) of order 1 (B) of degree 1
(C) of order 2 (D) of degree 3

Choose the correct answer from the options given below:
(a) (A) and (B) only (b) (A) and (D) only
(c) (B) and (C) only (d) (C) and (D) only

28. The area enclosed between the curve $x^2 + y^2 = 16$ and the coordinate axes in the first quadrant is:

(a) 12π sq. units (b) 8π sq. units
(c) 4π sq. units (d) 2π sq. units

29. Let R be a relation on the set of natural numbers N defined by nRm if n divides m. Then R is:

(A) Reflexive Relation (B) Symmetric Relation
(C) Transitive Relation (D) Identity Relation

Choose the correct answer from the option given below:
(a) (A) and (C) only (b) (A) and (B) only
(c) (A) and (D) only (d) (B) and (C) only

30. If the rate of change of area of a circle is equal to the rate of change of its diameter, then its radius is equal to:

(a) $\frac{1}{2\pi}$ unit (b) $\frac{1}{\pi}$ unit
(c) $\frac{\pi}{2}$ unit (d) π unit

31. If $f(x) = \sqrt{x}$, $g(x) = 2x-3$, then domain of $fog(x)$ is:

(a) $\left[\frac{3}{2}, \infty\right)$ (b) $\left[\frac{1}{2}, \infty\right)$
(c) $\left(\frac{3}{2}, \infty\right)$ (d) $\left[\frac{5}{2}, \infty\right)$

32. The given function $f(x) = [x]$ is discontinuous at:

(a) every integer (b) every even number
(c) every real number (d) at zero only

33. The maximum value of $2x^3 - 24x + 107$ in the interval [1, 3] is:

(a) 87 (b) 89
(c) 90 (d) 85

34. The equation of curve whose slope is given by $\frac{dy}{dx} = x$ and which passes through $\left(1, \frac{5}{2}\right)$ is:

(a) $y = x^2 + \frac{5}{2}$ (b) $y = \frac{x^2}{2} + 2$
(c) $y = \frac{x^2}{3} + \frac{2}{3}$ (d) $y = 3x^2 + 5$

35. If the shortest distance between the lines l_1 and l_2 given by $\vec{r} = a\hat{i} + 2\hat{j} - \hat{k} + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = \hat{i} - \hat{j} + \hat{k} + \mu(2\hat{i} - \hat{j} + \hat{k})$ is $\sqrt{\frac{35}{6}}$ units, the values of 'a' can be:

(a) 0, –8 (b) 0, 8
(c) 2, 6 (d) –2, 6

Hints & Explanations

1. (a) E_1 = Event that student is swimmer
E_2 = Event that student is not a swimmer

Given, $P(E_2) = \frac{1}{5}$. Then, $P(E_1) = 1 - \frac{1}{5} = \frac{4}{6}$

So, the required probability $= {}^5C_4\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$

2. (c) Given: $\text{Adj}(A) = P = \begin{bmatrix} 1 & x & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$

$|\text{Adj } A| = |A|^{3-1} = |A|^2$

$\Rightarrow \begin{vmatrix} 1 & x & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix} = 4^2 = 16.$

$\Rightarrow 1(12-12) - x(4-6) + 3(4-6) = 16$

$\Rightarrow 2x - 6 = 16 \Rightarrow x = 11$

3. (c) Let $I = \int_1^2 \frac{x\,dx}{(x+1)(x+2)} = \int_1^2 \left(\frac{2}{x+2} - \frac{1}{x+1}\right) dx.$

$= [2\log(x+2) - \log(x+1)]_1^2 = \left[\log\frac{(x+2)^2}{(x-1)}\right]_1^2$

$= \log\left(\frac{16}{3}\right) - \log\left(\frac{9}{2}\right) = \log\left(\frac{32}{27}\right)$

4. (d) $y = 4 - x^2 \Rightarrow \frac{dy}{dx} = -2x \Rightarrow m_1 = 2x.$

$y = x^2 \Rightarrow \frac{dy}{dx} = 2x \Rightarrow m_2 = 2x.$

Now, let calculate the point of interection of $y = 4 - x^2$ & $y = x^2$.

$y = 4 - y \Rightarrow y = 2$

$\therefore x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

So, the points are $(-\sqrt{2}, 2)$ and $(\sqrt{2}, 2)$

Now, the angle between the curves of $(-\sqrt{2}, 2)$ is

$\theta = \tan^1\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

Now, at $(-\sqrt{2}, 2) : m_1 = 2\sqrt{2}$ and $m_2 = -2\sqrt{2}$

$\therefore \theta = \tan^{-1}\left|\frac{2\sqrt{2}+2\sqrt{2}}{1-8}\right| = \tan^{-1}\left|\frac{4\sqrt{2}}{-7}\right|$

$\Rightarrow \theta = \tan^1\left(\frac{4\sqrt{2}}{7}\right)$

5. (d) Transpose of the product of matrices is: $(AB)' = B'A'$

6. (a) $\because f(x)$ is differentiable everywhere, then $f(x)$ is differentiable at $x = -1$ also.

L.H.D. $= 2a(-1) + 0 = -2a$

R.H.D. $= 2b(-1) + a = -2b + a$

$\because$ L.H.D. = R.H.D. $\Rightarrow -2a = -2b + a$

$\Rightarrow 3a = 2b$ ...(1)

$\because f(x)$ is continuous on $x = -1$.

$\therefore \lim_{x \to -1} f(x) = f(-1)$

$\Rightarrow a + b = b - a + 4 \Rightarrow a = 2$

from eqn.(1) $\Rightarrow 2b = 3 \times 2 \Rightarrow b = 3$

7. (c) (A) $\because 3x + 2y - 18 > 0$ does not satisfied by (5, 0). So, it does not satisfy..

(B) $(-1, 2)$ does not satisfy $3x + 2y > 0$.

(C) $(5, 0)$ does not satisfy $2x + y + 13 < 0$.

(D) $\because (5, 0), (-1, 2)$ and $(1, 3)$ satisfy $2x - 3y - 12 < 0$

Then, it will satisfy all the point of triangle formed by the points.

(E) $\because (5, 0), (-1, 2)$ and $(1, 3)$ satisfy $2x - 3y + 12 > 0$

Then, it will satisfy all the point of triangle formed by the points.

8. (d) $\because f(x) = 2x, g(x) = \frac{x^2}{2} + 1$

(a) $f(x) + g(x) = \frac{x^2}{2} + 2x + 1 \quad \Rightarrow$ continuous.

(b) $f(x) - g(x) = -\frac{x^2}{2} + 2x - 1 \quad \Rightarrow$ continuous

(c) $f(x) \cdot g(x) = 2x\left(\frac{x^2}{2} + 1\right) \quad \Rightarrow$ continuous

(d) $\frac{g(x)}{f(x)} = \frac{\frac{x^2}{2} + 1}{2x} = \frac{x^2 + 2}{4x}$

Which is discontinued at $x = 0$.

9. **(c)** $\because f(x) = 2x^3 - 3x^2 - 12x + 10$

$\Rightarrow f'(x) = 6x^2 - 6x - 12$

For decreasing function:

$f'(x) \le 0 \Rightarrow 6x^2 - 6x - 12 \le 0$

$\Rightarrow 6(x-2)(x+1) \le 0 \Rightarrow x \in [-1, 2]$

10. **(d)** $E(X)^2 = \Sigma X^2 P(X)$

$$= 1.\frac{1}{10} + 4.\frac{1}{5} + 3^2.\frac{3}{10} + 4^2.\frac{2}{5}$$

$$= \frac{1}{10} + \frac{4}{5} + \frac{27}{10} + \frac{32}{5} = 10.$$

11. **(a)** $y = a\sin(bx + c)$

$y' = ab.\cos(bx + c)$

$y'' = -ab^2\sin(bx + c)$

$\Rightarrow y'' = -b^2y \Rightarrow y'' + b^2y = 0$

12. **(b)** Let $I = \int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx.$

$$I = \int e^x\left[\frac{1+x^2-2x}{(1+x^2)^2}\right] dx.$$

$$I = \int e^x\left[\frac{1}{(1+x^2)} - \frac{2x}{(1+x^2)^2}\right] dx.$$

$$= e^x.\frac{1}{1+x^2} + C.$$

13. **(c)** $\because \frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow \frac{y^2}{9} = 1 - \frac{x^2}{16} \Rightarrow y = \frac{3}{4}\sqrt{16 - x^2}$

Required area = 4 × Area in first quadrant

$$= 4 \times \int_0^4 y\,dx.$$

$$= 4 \times \frac{3}{4}\int_0^4 \sqrt{16 - x^2}\,dx = 3\int_0^4 \sqrt{16 - x^2}\,dx$$

14. **(d)** At (0, 6), $Z = 3 \times 0 - 4 \times 6 = -24$

At (4, 8), $Z = 3 \times 4 - 4 \times 8 = -20$

At (5, 6), $Z = 3 \times 5 - 4 \times 6 = -9$

At (5, 4), $Z = 3 \times 5 - 4 \times 4 = -1$

At (4, 0), $Z = 3 \times 4 - 4 \times 0 = 12$

So, maximum of Z occurs at (4, 0)

15. **(c)** $\because A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} \Rightarrow$ A is a 2 × 2 matrix.

$\therefore |2A| = 2^2|A| = 4|A| \Rightarrow K = 4.$

Section : Core Mathematics

1. **(a)** (A) $|A'| = |A|$

(B) $|4A| = 4^3|A| = 64|A|$.

(C) $|A| \ne |\text{adj } A|^{n-1}$ But $|\text{adj } A| = |A|^{n-1}$

(D) $\det(A^{-1}) = \frac{1}{\det(A)}$

2. **(b)** Position vector of R

$$\vec{r} = \frac{1(\hat{i} + 2\hat{j} - \hat{k}) - 2(-\hat{i} + \hat{j} + \hat{k})}{1-2}$$

$$= -(3\hat{i} - 3\hat{k}) = -3\hat{i} + 3\hat{k}$$

3. **(a)** (A) $x = 2at^2$, $y = at^4 \rightarrow$ parametric function

(B) $f(x) = (2x + 3)^3 \rightarrow$ composite function

(C) $xy + y^2 = \tan(x + y) \rightarrow$ Implicit function

(D) $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) \rightarrow$ Inverse trigonometric function

4. **(a)** $A = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$

$$A^2 = A.A = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{pmatrix}$$

5. **(b)** $x \rightarrow$ no. of desktop model

$y \rightarrow$ no. of portable model

$\because$ shops capacity can't exceed 250 units.

$\therefore$ $x + y \le 250$

6. **(d)** By Rolle's theorem : $f'(c) = 0$ for some $c \in [0, \pi]$

$\because$ $f(x) = e^x \sin x \Rightarrow f'(x) = e^x(\sin x + \cos x)$

$\Rightarrow$ $f'(c) = e^c(\sin c + \cos c) = 0$

$\Rightarrow$ $\sin c + \cos c = 0 \Rightarrow \tan c = -1$

$\Rightarrow$ $c = \frac{3\pi}{4}$.

7. **(c)** Given : $(x\log_e x)\frac{dy}{dx} + y = 2\log_e x.$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x\log_e x}y = \frac{2}{x}$$

$$\therefore I.F. = e^{\int\frac{dx}{x\log_e x}} = e^{\log_e(\log_e x)} = \log_e x.$$

8. **(c)** (A) $|A^T| = |A|$

(B) $A(\text{adj } A) = (\text{adj } A)A$

(C) $A^{-1}|A| = \text{adj A}$

(D) $(AB)^{-1} = B^{-1}A^{-1}$

9. (d) (A) $y = \log(\sin x) \Rightarrow \dfrac{dy}{dx} = \cot x \Rightarrow \dfrac{d^2y}{dx^2} = -\operatorname{cosec}^2 x$

(B) $y = e^{(1+\log x)} \Rightarrow y = e.e^{\log x} = ex.$

$\Rightarrow \dfrac{dy}{dx} = e \Rightarrow \dfrac{d^2y}{dx^2} = 0$

(C) $y = \log|x| \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x} \Rightarrow \dfrac{d^2y}{dx^2} = -\dfrac{1}{x^2}$

(D) $y = x^2 + 4x - 1 \Rightarrow \dfrac{dy}{dx} = 2x + 4 \Rightarrow \dfrac{d^2y}{dx^2} = 2$

10. (c) (A) Area of triangle Δ with adjacent sides $\vec{a}$ and $\vec{b}$

$= \dfrac{1}{2}|\vec{a}\times\vec{b}|$

(B) Area of parallelogram with adjacent sides $\vec{a}$ and $\vec{b}$ $= |\vec{a}\times\vec{b}|$

(C) $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b}) = \vec{a}\times\vec{a} - \vec{b}\times\vec{a} + \vec{a}\times\vec{b} - \vec{b}\times\vec{b}$

$= 0 + \vec{a}\times\vec{b} + \vec{a}\times\vec{b} - 0 = 2(\vec{a}\times\vec{b})$

(D) $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$

11. (d) $y = \log\left(\dfrac{a+b\sin x}{a-b\sin x}\right) = \log(a+b\sin x) - \log(a-b\sin x)$

$\therefore \dfrac{dy}{dx} = \dfrac{b\cos x}{a+b\sin x} + \dfrac{b\cos x}{a-b\sin x}$

$= b\cos x\left[\dfrac{2a}{(a+b\sin x)(a-b\sin x)}\right]$

$= \dfrac{2ab\cos x}{a^2 - b^2\sin^2 x}$

12. (b) Equation of plane passing through the points (2, –4, 5) and perpendicular to line with direction ratios (3, –1, 2) is: $3(x-2) - 1(y+4) + 2(z-5) = 0$

$\Rightarrow 3x - y + 2z = 20$

13. (a) Volume of cube is $V = x^3$

Change in volume is $dV = 3x^2\, dx.$

$\Rightarrow dV = 3x^2 . \dfrac{2}{100} x = 0.06\, x^3.$

14. (a) $\because x = a\cos^3 t, y = a\sin^3 t.$

$\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t} = \dfrac{-\sin t}{\cos t} = -\tan t.$

Equation of tangent is

$(y - a\sin^3 t) = \left(\dfrac{dy}{dx}\right)(x - a\cos 3t)$

$\Rightarrow y - a\sin^3 t = -\tan t\,(x - a\cos^3 t)$

$\Rightarrow \cot y - a\sin^3 t\cos t = -\sin t\,.x + a\cos^3 t\sin t.$

$\Rightarrow x\sin t + y\cos t = a\sin t\cos t(\sin^2 t + \cos^2 t)$

$\Rightarrow x\sec t + y\operatorname{cosec} t = a.$

15. (d) System of equation $Ax = B$ has unique solution if $|A| \neq 0.$

$\Rightarrow |A| = \begin{vmatrix} 2 & 3 & 1 \\ 4 & 5 & 0 \\ 1 & k & 3 \end{vmatrix} = 2(15) - 3(12) + 1(4k - 5)$

$|A| = 30 - 36 + 4k - 5 = -11 + 4k$

$\because$ At $k = \dfrac{11}{4}, |A| = 0$

So, system of equations has unique solution for $R - \left\{\dfrac{11}{4}\right\}$

16. (d) (A) $f(x) = \dfrac{1}{x} \Rightarrow f'(x) = -\dfrac{1}{x^2} < 0$

$\therefore$ $f(x)$ is one-one or injective

and for all $x \in R - \{0\}$ there exist $\dfrac{1}{x}$ in $R - \{0\}$. So, $f(x)$ is surjective.

(B) $f(x) = x^2, f: N \to N.$

$f'(x) = 2x > 0, x \in N$

$\therefore$ $f(x)$ is injective.

But $3 \in N$ but there is no $x \in N$ s.t $x^2 = 3$

$\therefore$ $f(x)$ is not surjective.

(C) $f(x) = x^2$, $f: R \to R$

$\because$ $f(-1) = 1$ & $f(1) = 1 \Rightarrow f$ is not injective

Since, $-1 \in R$ but there is no $x \in R$ s.t. $x^2 = -1$

$f(x)$ is not surjective.

(D) f: {1, 2, 3} $\to$ {1, 2} is defined as f: {(1, 1), (2, 2), (3, 1)}

Range (f) = {1, 2} $\Rightarrow f$ is surjective.

$\because f(1) = 1$ & $f(3) = 1 \Rightarrow f$ is not injective.

17. (a) A : Event of obtainng a sum greater than 9.

B : Event of obtaining 5 on black die.

A = {(4, 6) (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}

B = {(1, 5) (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)}

$P(A) = \dfrac{6}{36} = \dfrac{1}{6}, P(B) = \dfrac{6}{36} = \dfrac{1}{6}$

$$A \cap B = \{(5,5),(6,5)\} \Rightarrow P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{18}}{\frac{1}{6}} = \frac{1}{3}$$

18. **(a)** The required area $= 2\int_{y=0}^{2}[x_2 - x_1]dy$

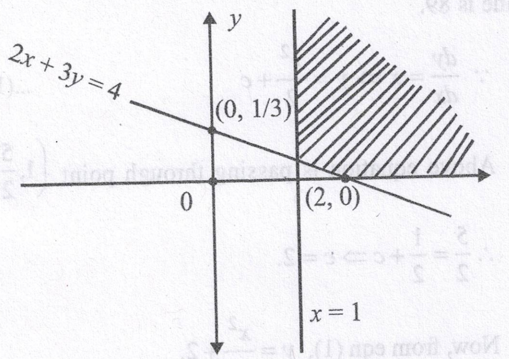
$$= 2\int_0^2\left(\frac{y^2}{2} - 2(y-1)\right)dy = 2\left[\frac{y^3}{6} - y^2 + 2y\right]_0^2$$

$$= 2\left[\frac{8}{6} - 4 + 4 - 0\right] = \frac{8}{3} \text{ sq. units.}$$

19. **(a)** Given inequalities are: $y \geq 0, 2x + 3y \geq 4; x \geq 1$

To draw the region, let us write the inequalities as equalities. $y = 0, 2x + 3y = 4, x = 1$

The shaded region is the region represented by the given inequalities which is unbounded in first quadrant.



20. **(c)** $\Delta = \begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} = \begin{vmatrix} 1 & 2 & 6 \\ 2 & 6 & 24 \\ 6 & 24 & 120 \end{vmatrix}$

$$= 1[720 - 576] - 2[240 - 144] + 6[48 - 36]$$
$$= 144 - 2 \times 96 + 6 \times 12 = 24 = 4!$$

21. **(c)** Let $I = \int_0^{\pi/2}\sqrt{1 - \sin 2x}\, dx$

$$= \int_0^{\pi/2}\sqrt{(\sin^2 x + \cos^2 x - 2\sin x\cos x)}\, dx$$

$$= \int_0^{\pi/2}(\sin x + \cos x)\, dx$$

$$= [-\cos x + \sin x]_0^{\pi/2} = [-0 + 1 + 1 - 0] = 2$$

22. **(a)** We know that in elementry row transformation using identity $A = IA$ and charge row by row operation. Change left side A and right side I only.

So, we can change only row-wise $\therefore R_i \to R_j$ and $R_i \to R_i + kR_j$ are valid.

23. **(b)** $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \cot^{-1}\left(\cot\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

24. **(a)** Let $I = \int e^x(\tan x + \log_e \sec x)\, dx$

$$= \int e^x(\log_e \sec x + \tan x)\, dx$$

Let $f(x) = \log_e \sec x \Rightarrow f'(x) = \frac{\sec x \tan x}{\sec x} = \tan x$

$$\therefore I = \int e^x(f(x) + f'(x))\, dx$$

$$= e^x f(x) + c = e^x \log_e \sec x + c.$$

25. **(c)** Equation of line is:

$$\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = \lambda \quad ...(1)$$

$\Rightarrow x = \lambda + 3, y = 2\lambda + 4, z = 2\lambda + 5$

Arbitrary point on line (i) is $(\lambda + 3, 2\lambda + 4, 2\lambda + 5)$

Let this point lies on the given plane also

$\therefore (\lambda + 3) + 2\lambda + 4 + 2\lambda + 5 = 17 \Rightarrow \lambda = 1$

So the point is $= (1 + 3, 2 + 4, 2 + 5) = (4, 6, 7)$

Distance between the point (4, 6, 7) and (3, 4, 5) is

$$d = \sqrt{(4-3)^2 + (6-4)^2 + (7-5)^2} = 3$$

26. **(c)** Let E_1 : Event of drawing red ball from Urn– I

E_2 : Event of drawing brack ball from Urn– I

E_3 : Event of drawing red ball from Urn– II

Then the required probability is:

$$P(F) = P(E_1).P\left(\frac{F}{E_1}\right) + P(E_2).P\left(\frac{F}{E_2}\right)$$

$$= \frac{6}{10} \times \frac{5}{10} + \frac{4}{10} \times \frac{4}{11} = \frac{46}{110} = \frac{23}{55}$$

27. **(b)** Given, $y = xp + \sqrt{x^2p^3 + y}$

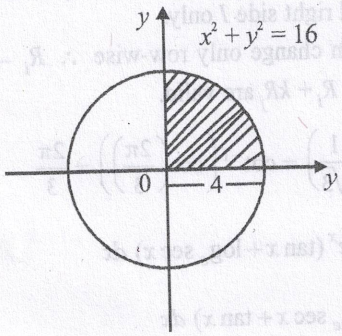
$\Rightarrow (y - xp)^2 = x^2p^3 + y$

$\Rightarrow y^2 + x^2p^2 - 2xyp = x^2p^3 + y$ where $P = \frac{dy}{dx}$

$$\Rightarrow x^2\left(\frac{dy}{dx}\right)^3 - x^2\left(\frac{dy}{dx}\right)^2 + 2xy\left(\frac{dy}{dx}\right) - y^2 + y = 0$$

So, order = 1, and degree = 3

28. **(c)** The area of the given circle is $= \pi(4)^2 = 16\pi$



Area in first quadrant $= \dfrac{16\pi}{4} = 4\pi$ sq. units

Another method : find area by

$$A = \int_0^4 \sqrt{4^2 - x^2}\, dx$$

29. **(a)** $R : N \to N$ defined as nRm if n divides m.

Reflexive $\Rightarrow \because$ for all $a \in n \Rightarrow a$ divides a
$\Rightarrow aRa. \therefore R$ is reflexive.

Symmetric $\Rightarrow 1, 2 \in N$
$\Rightarrow$ 1 divides 2 $\Rightarrow$ 2 divides 1
$\therefore$ R is not symmetric.

Transitive $\Rightarrow$ Let nRm and mRp.
Then n divides $m \Rightarrow n = mk$ $\quad k \in N$
& m divides $P \Rightarrow$ m = pk' $\quad k' \in N$
The $n = pk'k$.
$\Rightarrow n$ divides $p \Rightarrow nRp$.
$\therefore$ R is transitve.

30. **(b)** Let A, D and r be area, diameter & radius of circle respectively.

Given : $\dfrac{dA}{dt} = \dfrac{dD}{dt} \Rightarrow \dfrac{d(\pi r^2)}{dt} = \dfrac{d}{dt}(2r)$

$\Rightarrow \pi.2r\dfrac{dr}{dt} = 2\dfrac{dr}{dt}$

$\Rightarrow 2\pi r = 2 \Rightarrow r = \dfrac{1}{\pi}$

31. **(a)** Let $f(x) = \sqrt{x}, g(x) = 2x - 3$

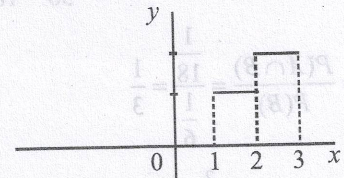
$fog\,(x) = f(g(x)) = f(2x-3) = \sqrt{2x-3}$

For domain of $fog\,(x)$:

$2x - 3 \geq 0 \Rightarrow 2x \geq 3 \Rightarrow x \in \left[\dfrac{3}{2}, \infty\right)$

32. **(a)** $f(x) = [x]$

$f(x)$ is greatest integer function whose graph is disconnected at each integral point



33. **(b)** Here, $f(x) = 2x^3 - 24x + 107$

$\Rightarrow f'(x) = 6x^2 - 24$

For critical points $\Rightarrow f'(x) = 0$

$\Rightarrow 6x^2 - 24 = 0$

$\Rightarrow x = \pm 2$

$\because$ $-2 \notin [1, 3] \Rightarrow$ the only critical point is $x = 2$.

Now, $f''(x) = 12x$.

At $x = 2 \Rightarrow f''(x) = 24 > 0$

$\therefore f(x)$ attains minimum at $x = 2$

Now, let us check the boundary points.

$f(1) = 2 - 24 + 107 = 85$

$f(3) = 54 - 72 + 107 = 89$

$\therefore$ $f(x)$ attains its maximum at $x = 3$ and its maximum value is 89.

34. **(b)** $\because \dfrac{dy}{dx} = x \Rightarrow y = \dfrac{x^2}{2} + c$ ...(1)

Above equation is passing through point $\left(1, \dfrac{5}{2}\right)$

$\therefore \dfrac{5}{2} = \dfrac{1}{2} + c \Rightarrow c = 2.$

Now, from eqn (1), $y = \dfrac{x^2}{2} + 2.$

35. **(a)** Equation of lines are-

$\vec{r} = a\hat{i} + 2\hat{j} - \hat{k} + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\vec{r} = \hat{i} - \hat{j} + \hat{k} + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\vec{a}_1 = a\hat{i} + 2\hat{j} - \hat{k}$, $\vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{a}_2 = (\hat{i} - \hat{j} + \hat{k})$, $\vec{b}_2 = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{a}_2 - \vec{a}_1 = (1-a)\hat{i} - 3\hat{j} + 2\hat{k}$

Shortest distance $= \left|\dfrac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|}\right|$

$\Rightarrow \sqrt{\dfrac{35}{6}} = \left|\dfrac{-\hat{i} + \hat{j}(a+3) + \hat{k}(a+5)}{\sqrt{4+1+1}}\right|$

$\Rightarrow 2a\,(a+8) = 0 \Rightarrow a = 0, -8.$