

4.

MATHEMATICS

Congruence of Triangles

QUESTIONS

- **1.** Two students drew a line segment each. What is the condition for them to be congruent?
 - (a) They should be drawn with a scale.
 - (b) They should be drawn on the same sheet of paper.
 - (c) They should have different lengths.
 - (d) They should have the same length.
- **2.** In $\triangle ABC$, AB = AC and AD is perpendicular to BC. State the property by which $\triangle ADB \cong \triangle ADC$.
 - (a) S.A.S. property (b) S.S.S. property (c) R.H.S. property (d) A.S.A. property
- **3.** The given figure shows $\triangle ABC \& \triangle DBC$, AB = DB and CA = CD



What is the measure of $\angle DBC$?

(a) 115° (b) 100° (c) 150° (d) 155° In the given figure $PQ \parallel RS$



What is the value of (PR + QS)?

(a) 21 <i>cm</i> (b) 23	8 <i>cm</i> (c) 25	5 <i>cm</i> (d) 31 <i>cm</i>
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5. In the given figure, $\triangle ABC$ is congruent to $\triangle PQR$.



6. The given figure shows $\triangle ABC$ with equal altitudes BE and CF.



What is the perimeter of $\triangle ABC$?

(a) 19 cm (b) 21 cm (c) 24 cm (d) 25 cm

7. In the given figure, if AD = BC and $AD \parallel BC$ which of the following is true?





- **8.** Which of the following is a pair of congruent figures?
 - (a) A regular pentagon and a regular hexagon.
 - (b) A rhombus and a square.
 - (c) Two equilateral triangles of the same length of their sides.
 - (d) A quadrilateral and a rectangle.
- **9.** Two triangles, ΔPQR and ΔXYZ are of the same size and shape. What can we conclude about them?
 - (a) ΔPQR is smaller than ΔXYZ (b) ΔPQR is larger than ΔXYZ
 - (c) ΔPQR is congruent to ΔXYZ (d) ΔPQR is not congruent to ΔXYZ
- **10.** Which of the following examines the congruence of plane figures?
 - (a) Trial and error method (b) Superposition method
 - (c) Substitution method (d) Transposition method
- **11. Student A:** Two triangles are said to be congruent if two sides and an angle of one triangle are respectively equal to the two sides and an angle of the other.

Student R: Two triangles are congruent if two sides and the included angle of one are equal to the corresponding two sides and included angle of the other.

Given A & R, which of the following statements is correct?

- (a) A is false and R is the correct explanation of A.
- (b) A is true and R is the correct explanation of A.
- (c) A is true and R is false.
- (d) A is false and R is true.

12. In the given figure, AC is equal to BD.



What is the measure of $\angle CAD$?

(a)
$$20^{\circ}$$
 (b) 25° (c) 35° (d) 60°

13. The given figure shows $\triangle ABC$ and $\triangle PQR$ with AB = QR.



What is the value of (AB + 5BC + 3AC + 2RP - 3PQ - QR)?

(a) 60 (b) 55 (c) 70 (d) 25

14. In $\triangle ABC$, AB = AC and AD is perpendicular to BC. The property by which $\triangle ADB$ is congruent to $\triangle ADC$ is.(a) SAS property(b) SSS property(c) RHS property(d) AAA property

15. If the given two Ales arc congruent, then what is true?



16. In this given figure, PS = SR and PQ = QR. Which of the following is the correct conclusion?



- (a) ΔPQR and ΔRQS are isosceles triangle.
- (b) QS bisects $\angle PSR$.
- (c) QS bisects $\angle QPS$
- (d) ΔPQR and ΔRQS are equilateral triangles.

17. In the given figure, AB = AC and AD is the bisector of $\angle BAC$.



Which among the following statements is true?

- (a) $\triangle ADB \cong \triangle ABC$ (b) $\triangle ADC \cong \triangle ABC$ (c) $\angle B = \angle C$ (d) $\angle ABC = \angle CAB$
- **18.** Which of the following is an example of A.S.A. criterion of congruency for two triangles $\triangle ABC$ and $\triangle DEF$?
 - (a) AB = EF, $\angle B = \angle E$ and $\angle C = \angle F$
 - (b) BC = EF, $\angle B = \angle E$ and $\angle C = \angle F$
 - (c) AC = EF, $\angle B = \angle D$ and $\angle C = \angle F$
 - (d) AC = DE, $\angle B = \angle D$ and $\angle C = \angle F$
- **19.** State the property by which $\triangle ADB \cong \triangle ADC$, where AD is angle bisector of $\angle BAC$.



(c) RHS property

(d) ASA property

20. In the given figure if AD = BC and $AD \parallel BC$, then

21.



(c) RHS property (d) All i.e. (A), (B) and (C)

- **22.** Which type of triangle is formed by BC = 7.2 cm, AC = 6 cm and $\angle C = 120^{\circ}$?
 - (a) An acute angled triangle.

- (b) An obtuse angled triangle.
- (c) A right angled triangle. (d) An isosceles triangle.
- **23.** In the given figure, line PRT is parallel to SQN. If $RS = 8 \ cm$ what is the length of RO?



(c) 4 cm

(d) 6 cm

24. In the given figure, $\triangle ABC$ and $\triangle ADE$ are isosceles triangles with AB = AC and AD = AE.

If $CE = 2 \ cm$ and $DE = 6 \ cm$, then what is the length of side BC?



(a) 8 cm (b) 9 cm (c) 10 cm (d) 1 cm

- **25.** In $\triangle ABC$ and $\triangle DEF$, AC = DF, AB = EF and BC = DE. By which property are $\triangle ABC$ and $\triangle le$ (formed by three points D, E & F) congruent?
 - (a) R.H.S property (b) S.S.S. property (c) S.A.S. property (d) A.S.A. property
- **26.** In $\triangle ABC$ and $\triangle ADE$ If AC = EF, AB = DF and BC = DE, what is the correct way of writing congruency? (a) $\triangle ABC \cong \triangle DBF$ (b) $\triangle ACB \cong \triangle DEF$ (c) $\triangle ACB \cong \triangle DFE$ (d) $\triangle ABC \cong \triangle FDE$
- 27. If we apply Pythagoras theorem, R.H.S. property of congruency may be seen to be same as:(a) SSS property(b) AAA property(c) ASA property(d) None of above
- **28.** Which of the following criterion does not exist in the field of congruency of triangles.

(a) A.S.A. criterion (b) R.H.S. criterion (c) A.A.A. criterion (d) S.S.S. criterion

- **29.** In two triangles, the three angles of one triangle are correspondingly equal to three angles of another triangle. Which of the following is a correct statement?
 - (a) One triangle is an enlarged copy of other
 - (b) The two triangles are necessarily congruent.
 - (c) The two triangle are congruent by A.A.A. congruency criterion.
 - (d) All of the above.

30. \triangle ABX is congruent to \triangle XYZ. Find the measures of $\angle x$ and $\angle y$ respectively.







If $\triangle AOB$ is congruent to $\triangle AOC$, which of the following is correct?

(a) $\angle OBA = \angle OCA$ (b) $\angle AOC = \angle OCA$ (c) AO = AC (d) AB = OC



34. A regular hexagon is divided into four triangles as shown below:

37. In the given figure of a field, PQR and RST are right- angled at Q and S respectively. The measure of the sides are given (See figure)



ANSWER - KEY											
1.	D	2.	С	3.	А	4.	С	5.	D		
6.	В	7.	В	8.	С	9.	С	10.	В		
11.	А	12.	А	13.	С	14.	С	15.	D		
16.	В	17.	С	18.	В	19.	А	20.	В		
21.	D	22.	С	23.	С	24.	С	25.	В		
26.	D	27.	А	28.	С	29.	А	30.	А		
31.	А	32.	С	33.	А	34.	А	35.	С		
36.	В	37.	D	38.	В	39.	В	40.	С		

SOLUTIONS

- 1. (D) Not available
- **2.** (C) Not available
- **3.** (A): According to the given data, $\Delta ABC \cong \Delta \Delta BC$

$$\therefore \angle DBC = \angle ABC = \frac{1}{2} \angle ABD \quad (1 \text{ larger } \angle \text{le})$$
$$= \frac{1}{2} \times (360^\circ - 130^\circ)$$

- 4. (C): $PQ \parallel RS \Rightarrow \angle QPO = \angle SRO$ and $\angle OQP = \angle OSR$; thus, $\triangle OQP \cong \triangle OSR$ $OS = 7 \ cm$ and $PO = 5.5 \ cm$ $\therefore \ QS = 7 + 7 = 14 \ cm$; $PR = 5.5 + 5.5 = 11 \ cm$ $\therefore \ PR + QS = 11 + 14 = 25 \ cm$
- 5. (D): $\triangle ABC \cong \triangle PQR \Rightarrow \overline{AB} = \overline{PQ}, \ \overline{BC} = \overline{QR}; \ \overline{CA} = \overline{Rp} \Rightarrow 2m+1=5 \Rightarrow m=2; \ 2p+2=8 \Rightarrow p=3;$ $7 = 2n-1 \Rightarrow n=4$

$$\Rightarrow m + 2n + 3p = 2 + 8 + 9 = 19$$

6. (B): Perimeter = AB + BC + CA : BC is already known $= 6 \ cm$

From
$$\triangle BCF$$
, $CF = \sqrt{6^2 + 3^2} = 3\sqrt{3} = BE$

In
$$\triangle BEA$$
, $AB = \sqrt{45^2 \left(3\sqrt{3}\right)^2}$

Similarly, find CA and add AB + BC + CA

- 7. (B) Not available
- 8. (C): In case (C), the two Ales satisfy both SSS criterion and the SAS criterion (as included angle is 60° in equilateral Δle). In fact, even one of this criterion is sufficient so, we have extraneous data (data more than what is required to solve the problem)
- 9. (C): Same size \Rightarrow all corresponding sides are equal Same shape \Rightarrow all corresponding angles are equal This necessarily means that $\Delta PQR \cong \Delta XYZ$
- **10.** (B): Superposition means that if we correctly superimpose sides of one plane figure with sides of another plane figure, they completely overlap each after for e.g.;



Then, $\Delta BCD \cong QRSP$ if AB = QR : BC = SR CD = SP and DA = PQ.

- **11.** (A) Not available
- **12.** (A): By congruency (SAS), $\triangle ACD \cong \triangle BDC$; Now in $\triangle ACD$, $\angle C = 125^{\circ} \angle D = 35^{\circ}$ $\therefore \angle A = 180^{\circ} - 125^{\circ} - 35^{\circ} = 20^{\circ}$
- 13. (C): Tips: Sometimes the student needs to get into the thinking of the question setter. The question setter has framed this problem in order to trick the student. The two Ales do not look congruent. Not even do they look similar. But from the given data, they are seen to have their corresponding $\angle les$ as equal. Thus, they are similar. Now, also $AB = QR \Longrightarrow$ the Δ les are congruent because if two Ales are similar & if even one of corresponding sides are equal, they become congruent ($\Delta ABC \cong \Delta QRP$) in that order.
 - $AB = QR \Longrightarrow AB QR = O$ Similarly 3(AC - PQ) = 0Thus, only terms left are 5BC + 2RP and BC = 10 units $= 5 \times 10 + 2 \times 10$ (Sinec BC = RP) = 70

14. (C):



In Δles ADB and ADC,

AD = AD (common side)

AB = AC

 $\angle ADB = \angle ADC = 90^{\circ} \Longrightarrow$ congreency.

- **15.** (D): Actually, the correct way of showing congreency of these two Δles would be $\Delta ABC = \Delta DFE$
- **16.** (B): Not available
- **17.** (C): $\angle B = \angle C$ (by property of isosceles Ale)
- **18.** (B): Figuratively, we can show it as



19. (A) Not available

- **20.** (B) Not available
- **21.** (D): In two Δles , if two corresponding $\angle les$ are equal, then the remaining third (corresponding) $\angle les$ are also equal.
 - $\therefore \angle QPS = \angle RPS$



Thus, in Ales PSQ and PSR,

PS (common side) : $\angle PSQ = \angle PSR = 90^\circ$, $\angle QPS = \angle RPS$

By pytharos theorem, All three properties - ASA, SAS and RHS apply.

- **22.** (C) Not available
- **23.** (C): $\triangle POR \cong \triangle QOS$ (by ASA congmency) since $\angle RPO = \angle SQO : \angle POR = \angle QOS$ and PO = OQ

$$\therefore RO = SO = \frac{1}{2}RS = 4cm$$

24. (C): In Ales ABD and ACE, AB = AC; AD = AE and included $\angle les$ are 30° each Hence, they are congruent. $\Rightarrow BD = CE = 2 \ cm$

$$\therefore BC = BD + DE + CE; = 2 + 6 + 2 = 10cm$$

25. (B): When sides of $\triangle ABC$ and superimposed over sides of Ale FED, they are equal.

Here, we are not calling Ale formed by three points D, E & F as Ale DEF. It is actually ΔFED by theory of congruency.

- **26.** (D) Not available
- **27.** (A):



When we apply Pythagoras theorem in a Δle ABC, its third side (x or y or z) is obtained from two given sides. Thus, all sides are known.

28. (C): AAA criterion means corresponding $\angle les$ are equal. But this is easily seen to fail us.

For e.g.



$$\angle x = \angle x_1$$

 $\angle y = \angle y_1$; Hence, although shapes are same, sizes are different.

 $\angle z = \angle z_1$

Mind of a mathematician: In mathematics, if you can find even one example contrary to the given proposition, then the proposition is wrong. By the above example, we can see that AAA does not hold for congruency.

- **29.** (A) Not available
- **30.** (A) Not available
- **31.** (A): They are congruent $\Rightarrow \angle A = \angle F$; Now

 $\angle A = 180^{\circ} - 60^{\circ} - 70^{\circ} = 50^{\circ}$

$$\therefore \angle F = 50^{\circ}$$

- **32.** (C): As $\triangle PQR \cong \triangle PSR$, corresponding angles, $\angle QRP$ and $\angle SRP$ are equal.
- **33.** (A): $\angle OBA$ and $\angle OCA$ are corresponding of congruent Ales.
- **34.** (A): In $\triangle PQU \& \triangle TSU$

PQ = TS; UT = UP; $\angle UPQ = \angle UTS$

35. (C): Considering Ales PQT & RST,

$$QT = ST = 9; \ \angle PTQ = \angle RTS;$$

 $\angle PQT = 180^{\circ} - \angle PQV \Longrightarrow \angle PQT = \angle RST$

$$\therefore \ \Delta PQT \cong \Delta RST \Longrightarrow PQ = RS = 13$$

- \Rightarrow Perimeter $\Delta PQT = 13 + 11 + 9 = 33$
- **36.** (B): Perimeter ABCD $= 2 \times$ Perimeter

 $\Delta ABD - 2 \times BD$ $= 2 \times 30 - 2 \times 7 = 46$

37. (D): This is also an excellent problem to test the mind of students



 $\Delta PQR = \Delta RST$: Hence, if $\angle PRQ = x$ then $\angle SRT = 90 - x^{\circ}$

$$\Rightarrow \angle PRT = 90^{\circ}$$

Also $PR = RT \Longrightarrow (D)$

38. (B): This is again an excellent test of mind for the students:



From given data, $\triangle ABC \cong \triangle CDE$ $\Rightarrow AC = EC \Rightarrow \angle p = \angle q$ $\angle y = \angle m; \ \angle x = 180^\circ - 60^\circ - m = 120^\circ - m$ At point C: $\angle ACE = 180^\circ - x - y$ $= 180^\circ - (120 - m) - m = 60^\circ$ Also $\angle ACE = 180^\circ - \angle p - \angle q = 180^\circ - 2\angle p = 60^\circ$ $\Rightarrow \angle p = \angle q = 60^\circ \Rightarrow \triangle ACE$ is equilateral $\triangle le$.

39. (B):
$$AC = \sqrt{AB^2 + BC^2} = \sqrt{AB^2 + QR^2}$$
 (Since $BC = QR$ by congruency)
 $= \sqrt{70^2 + 240^2} = 250m$; $AP = AC - PC = 250 - 200 = 50$
Also, $CR = AP = 50$
 $\Rightarrow AR = AC + CR$
 $= 250 + 50$
 $= 300$

40. (C) Not available